Distributed Compressed Sensing for Accelerated MRI

R. Otazo[1], and D. K. Sodickson[1]

[1]Center for Biomedical Imaging, NYU School of Medicine, New York, NY, United States

INTRODUCTION: Compressed sensing has recently been introduced as a powerful method to reduce the number of required samples by exploiting signal compressibility [1,2]. Application to MRI was proposed for reconstruction of images that have a sparse representation in a known transform domain (e.g. wavelets) from randomly undersampled k-space data [3]. The maximum acceleration is limited by the image sparsity, e.g. for a K-sparse image and using a single coil, approximately 3 times to 5 times K samples are required [4]. Distributed compressed sensing [5] extends compressed sensing to multiple sensors by exploiting the idea of joint sparsity of the multi-signal ensemble to reduce the number of required samples to K, the signal sparsity level, when using a large number of sensors. In this work, we apply the principles of distributed compressed sensing to parallel MRI. We present a greedy reconstruction algorithm that takes advantage of both intra- and inter-coil correlations, and assess its performance with respect to the number of coils.

THEORY: The idea of distributed compressed sensing is to exploit joint sparsity of the multi-signal ensemble instead of individual signal sparsity to reconstruct one signal that represents the combination from all sensors. For parallel MRI, each coil acquires an image that can be represented as $\mathbf{y}_i = \mathbf{F}\mathbf{S}_i\mathbf{x}$, where $\mathbf{F}$ is the undersampled Fourier transform, $\mathbf{S}_i$ are the coil sensitivities and $\mathbf{x}$ is the image to reconstruct. If $\mathbf{x}$ is compressible under the transform $\mathbf{W}$, i.e. $\mathbf{d} = \mathbf{W}\mathbf{x}$ is K-sparse, the individual acquisition model can be also represented as $\mathbf{y}_i = \mathbf{F}\mathbf{S}_i\mathbf{W}^{-1}\mathbf{d}$. Instead of solving the system for each coil separately, we propose to solve the multi-coil image system $\mathbf{y} = \mathbf{E}\mathbf{W}^{-1}\mathbf{d}$ given by the concatenation of the individual models where the individual coil images are jointly sparsified by $\mathbf{W}$. To recover the sparse signal $\mathbf{d}$ we use a modified version of the Simultaneous Orthogonal Matching Pursuit (SOMP) algorithm [7] named Joint OMP (JOMP). The original SOMP algorithm aims to recover all the signals simultaneously; here we recover only one signal that represents the combination from all coils. Briefly, since only K components of $\mathbf{d}$ are different from zero, $\mathbf{y}$ can be represented as a linear combination of K columns of $\mathbf{E}\mathbf{W}^{-1}$. The algorithm searches iteratively for the set of columns that are most highly correlated with the set of measurements $\mathbf{y}$ to produce a constrained least-squares fit.

METHODS: To assess the performance of JOMP with respect to the number of coils, a one-dimensional MRI experiment was simulated, assuming a truly sparse signal in the image domain with random positioning of the non-zero components (number of points N = 512, number of non-zero points K = 32). A planar array with sensitivities computed according to the Biot-Savart law was simulated for different numbers of elements (N_c=2,4,6,8,12,16). For the single coil case, a constant sensitivity was assumed. The multi-coil signal was generated by multiplying the truly sparse signal by the coil sensitivities and optionally adding Gaussian noise. The average reconstruction error over 250 different random k-space undersampling realizations was computed for different numbers of measurements and coil elements.
To assess the performance of JOMP for compressible images which are nevertheless not rigorously sparse, a brain MRI experiment with acceleration along the k_y dimension was simulated using a fully-sampled brain image acquired with a single receiver. The multi-coil signal was generated by multiplying the brain image by the coil sensitivities and adding Gaussian noise (SNR = 100) for the 4-channel and 8-channel arrays. A Daubechies wavelet transform was employed to sparsify the image which provided a maximum compression ratio of 12.6-fold one-dimensional acceleration was simulated by decimating the data in k-space with a random pattern along k_y. JOMP was applied separately to each of the columns of the concatenated multi-coil accelerated image.

RESULTS: The number of required measurements to achieve error-free reconstruction decreases with the number of coils for the noise-free case (Fig. 1), e.g. around 120 measurements (~ 4 K) are required for N_c = 1 whereas only ~45 measurements are required for N_c = 12 and N_c = 16. A similar trend is observed for the noisy case where fewer measurements are required to approach the noise floor (Fig. 2). Note that the noise floor also decreases with increasing number of coils, suggesting a reduction in g-factor-like noise amplification. These trends confirm that for large numbers of coils, the required number of measurements per coil is very close to the signal sparsity. The simulated brain image reconstruction with the 8-channel array recovered more wavelet components and thus exhibited fewer artifacts than the one with the 4-channel array. The artifacts are mainly associated with small wavelet components that are deeply submerged in the interference created by the random undersampling pattern. Using more coils helps to separate better these small components from the interference, and thus higher accelerations are likely to be feasible.

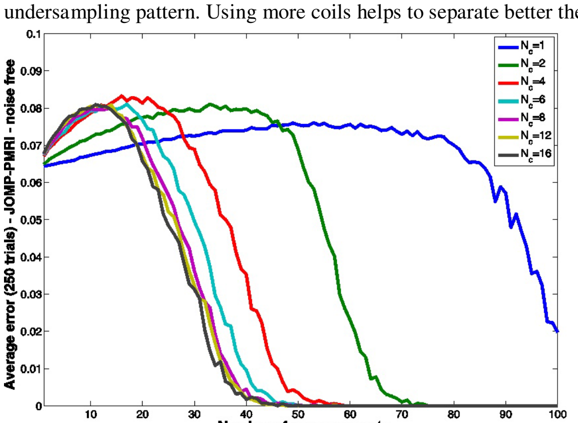

Fig. 1: Reconstruction error of the noise-free simulation experiment for different numbers of coils (N_c).

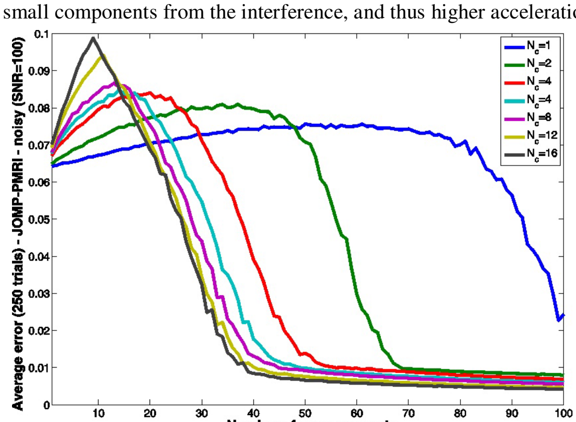

Fig. 2: Reconstruction error of the noisy simulation experiment (SNR = 100).

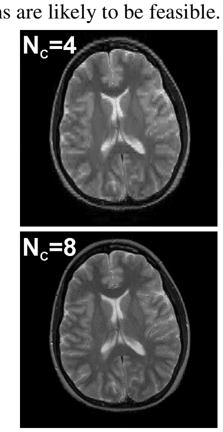

Fig. 3 : Simulated brain image reconstruction.

DISCUSSION: The proposed JOMP reconstruction algorithm is best suited for images that are highly sparse where a practical number of receiver coils will suffice to reduce the number of required measurements to the sparsity level. In the current implementation, the encoding matrix is computed explicitly, which might impose computational constraints for multi-dimensional acceleration. Future work will explore the implementation of the algorithm without computing the encoding matrix explicitly. The proposed method has some kinship with previously presented combinations of parallel imaging and compressed sensing using L_1-norm minimization [7,8]. However, in this work we make a specific connection to the theory of distributed compressed sensing to explore the idea of joint sparsity for parallel imaging. We also employ a reconstruction which approximates L_0-minimization, and which is expected to perform more robustly than L_1-minimization for highly sparse datasets.

GRANT SPONSOR: NIH R01-EB000447.

REFERENCES: [1] Candès E et al. IEEE Trans Inf Theory 2006; 52:489–509. [2] Donoho D. IEEE Trans Inf Theory 2006; 52:1289–1306. [3] Lustig M et al. Magn Reson Med. 2007; 58:1182-1195. [4] Tsaig Y et al. Signal Process. 2006; 86:533–548. [5] Duarte M et al. Proc Conf Signals, Systems and Computers 2005; 1537-1541. [6] Tropp et al. Proc IEEE ICASSP 2005; 721-724. [7] Zhao C et al. Proc ISMRM 2008; 1478. [8] Liu B et al. Proc ISMRM 2008; 3154.