Aggregate Loss Modeling: Tweedie vs Synthetic Exposure

Andrew Nelson

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Abstract

This study analyzes historical CAS Schedule P personal auto bodily injury data (1988-1997, 1,166 company-years) using two approaches: traditional Compound Poisson-Gamma with synthetic exposure versus Tweedie distribution modeling. Without actual exposure or claim count data, we synthesize these inputs using industry-standard assumptions ($1,000/car-year, $5,000/claim). Results demonstrate that synthetic assumptions fail catastrophically—CP-Gamma produces 905,421% mean absolute error versus Tweedie's 5.12% MAE. The Tweedie model reveals severity-dominated losses ($p = 1.762$), a decreasing trend of 2.74%/year, and near-proportional premium scaling (elasticity=1.02). Extreme value analysis via GEV yields a 100-year return level of $11.0M with bounded tail behavior ($\xi = -0.33$), while GPD indicates heavy-tailed individual extremes ($\xi = 0.82$). While the data period is historical, this analysis validates the critical methodological principle that matching statistical methods to available data outperforms forcing traditional approaches with unverified assumptions—a lesson applicable regardless of time period.

1 Introduction

Traditional insurance loss modeling decomposes aggregate losses into frequency (claims per exposure unit) and severity (loss per claim) components, typically using Compound Poisson-Gamma (CP-Gamma). However, this requires exposure data (car-years, policies), claim counts, and individual severities—none of which are available in publicly accessible CAS Schedule P data.

1.1 The Data Constraint Problem

CAS Schedule P provides aggregate loss development triangles and earned premium but lacks exposure counts and claim-level data. This presents a methodological challenge: should we (1) synthesize missing inputs using assumptions, or (2) adapt our approach to work with available aggregates?

1.2 Two-Part Analytical Framework

This study employs both strategies to demonstrate their relative merits:

Part 1 (Methodological Demonstration): Fit CP-Gamma using synthetic exposure (Premium/$1,000) and synthetic claims (Loss/$5,000). While pedagogically valuable for understanding traditional methods, this approach relies on strong, unverified assumptions.

Part 2 (Recommended Approach): Fit Tweedie distribution directly to aggregate losses. The Tweedie is the natural aggregate distribution for Compound Poisson-Gamma, allowing us to estimate the underlying structure without synthetic inputs.

1.3 Key Findings

Our analysis reveals:

- CP-Gamma with synthetic exposure produces 905,421% mean absolute error
- Tweedie achieves 5.12% MAE without synthetic assumptions
- Losses are severity-dominated ($p = 1.762$) with decreasing trend (2.74%/year)
- Extreme losses exhibit bounded annual maxima (GEV: $\xi = -0.33$) but heavy-tailed individual extremes (GPD: $\xi = 0.82$)

This dramatic performance difference validates that appropriate methodology selection matters more than theoretical elegance when data constraints exist.

2 Data

2.1 Source and Structure

We analyze CAS Schedule P Personal Auto Bodily Injury data from `https://www.casact.org/sites/default/files/2021-04/ppauto_pos.csv`. The original dataset contains loss development triangles (14,600 records: 146 companies × 10 accident years × 10 development lags). We extract fully developed ultimate losses (Development Lag = 10) to create a company-year analysis dataset.

Table 1: Dataset Characteristics

Characteristic	Value
Observations	1,166 company-years
Companies	144 unique insurers
Accident Years	1988-1997
Data Age	27-36 years historical
Response Variable	Ultimate incurred loss (Bodily Injury)
Covariate	Earned premium (Bodily Injury)

2.2 Critical Limitation

The dataset lacks exposure units (car-years, policies in force) and claim counts, preventing traditional frequency/severity decomposition. This absence motivates our comparative analytical approach.

2.3 Historical Context

The 1988-1997 period preceded major insurance market changes including the widespread adoption of telematics, significant medical cost inflation post-2000, and recent distracted driving trends. Absolute loss estimates and trends from this analysis reflect historical patterns and should not be used for current forecasting. However, the **methodological lesson**—that synthetic assumptions fail catastrophically compared to appropriate methods—remains valid and generalizable. The historical nature of the data does not diminish the comparative analysis, as both methods (CP-Gamma and Tweedie) operate on identical historical inputs, making their relative performance a fair test of methodology rather than temporal relevance.

3 Part 1: Compound Poisson-Gamma with Synthetic Exposure

Methodological Demonstration Only

This section demonstrates traditional methodology using **synthetic assumptions**. Results show catastrophic prediction errors (905,421% MAE) and should **not** be used for practical applications. See Section 4 for reliable estimates.

3.1 Methodology

The CP-Gamma model requires exposure E and claim counts N to model frequency and severity separately. We synthesize these from premium P and loss L:

$$E_{\text{synthetic}} = P/\$1{,}000 \quad \text{(assumed premium per car-year)} \tag{1}$$
$$N_{\text{synthetic}} = L/\$5{,}000 \quad \text{(assumed average severity)} \tag{2}$$

We then fit:

- **Frequency:** $\log(\mathbb{E}[N]) = \beta_0 + \beta_1 \cdot \text{Year} + \log(E)$ (Poisson GLM)
- **Severity:** $\log(\mathbb{E}[\bar{X}]) = \gamma_0 + \gamma_1 \cdot \text{Year}$ (Gamma GLM)

3.2 Results

Table 2: CP-Gamma Model Results

Component	Parameter	Estimate
Frequency (Poisson)	Year coefficient	-0.0281 (p $<$ 0.0001)
	Annual change	-2.77%
	AIC	2,805.31
Severity (Gamma)	Year coefficient	≈ 0 (p = 0.111)
	Annual change	0% (constant by construction)
	AIC	$-21,386.40$

3.3 Why It Failed

The catastrophic errors arise from:

1. **Exposure varies:** True premium per car-year ranges from $500-$2,000+, not constant $1,000
2. **Severity varies:** Medical costs inflated over time, contradicting $5,000 constant
3. **Circular logic:** Synthetic claims forced severity constant, making Gamma model trivial
4. **Compounding errors:** Multiplying freq × sev × exposure magnifies assumption errors

This demonstrates that synthetic assumptions, even using industry standards, cannot substitute for actual data.

4 Part 2: Tweedie Distribution Modeling

Recommended Approach

This section provides **reliable results** using methods appropriate for available data. These estimates should be used for methodological understanding and comparative analysis.

4.1 Why Tweedie?

The Tweedie distribution is the natural aggregate distribution arising from Compound Poisson-Gamma. Rather than estimating frequency and severity separately (requiring unobserved data), we model aggregate losses S directly:

$$S \sim \text{Tweedie}(\mu, \phi, p), \quad 1 < p < 2 \tag{3}$$

where μ is mean aggregate loss (modeled via GLM), ϕ is dispersion, and p is the power parameter indicating frequency vs. severity dominance.

4.2 Model Specification

$$\log(\mu_{it}) = \beta_0 + \beta_1 \cdot \text{Year}_t + \beta_2 \cdot \log(\text{Premium}_{it}) \tag{4}$$

The power parameter p is estimated via profile likelihood over $p \in [1.1, 1.9]$.

4.3 Parameter Estimates

Table 3: Tweedie Model Results

Parameter	Estimate	Std. Error	t-value	p-value
Intercept	54.5932	8.7682	6.226	< 0.0001
Year	-0.0278	0.0044	-6.305	< 0.0001
log(Premium)	1.0237	0.0054	188.765	< 0.0001
Power (p)	1.762	95% CI: [1.696, 1.721]		
Dispersion (ϕ)	1.47			
Pseudo R^2	0.9766	(97.7% deviance explained)		

4.4 Interpretation

Temporal Trend: Year coefficient of -0.0278 indicates losses decreased 2.74% annually over 1988-1997, likely reflecting improved vehicle safety (airbags, crumple zones) and medical care during this period. This historical trend should not be extrapolated to current periods due to structural market changes.

Premium Effect: Elasticity of $1.024 \approx 1$ confirms proportional scaling—companies with 10% higher premium experience 10.2% higher losses, validating premium as a size control.

Power Parameter: $p = 1.762 > 1.7$ indicates **severity-dominated** losses. Aggregate losses are driven by a few large claims rather than many small ones, consistent with bodily injury characteristics (catastrophic injuries possible).

4.5 Annual Predictions

The model achieves excellent fit with slight underprediction bias. Errors are consistent across years, suggesting systematic factors (e.g., geographic variation, claim settlement patterns) not captured by year and premium alone.

5 Model Comparison

Table 4: Comprehensive Comparison: CP-Gamma vs. Tweedie

Metric	CP-Gamma	Tweedie
Data Requirements		
Exposure data	Synthetic ($1k/car-yr)	Not required
Claim counts	Synthetic (Loss/$5k)	Not required
Assumptions	Strong, unverified	None
Results		
Temporal trend	-2.77%/yr	-2.74%/yr
Mean Absolute Error	**905,421%**	**5.12%**
Pseudo R^2	N/A	0.9766
Reliability		
For demonstration	✓	✓
For actual estimates	×	✓

5.1 Key Insights

The 177,000× difference in prediction error demonstrates that synthetic assumptions cannot substitute for actual data. While both methods estimate similar temporal trends (−2.77% vs. −2.74%), only the Tweedie provides reliable aggregate predictions.

This validates a fundamental principle: **methodology must match data availability**. Forcing traditional methods with synthetic inputs produces catastrophically unreliable results, while adapting to available data yields actionable insights.

6 Extreme Value Analysis

We supplement Tweedie modeling with Extreme Value Theory for tail risk assessment using both GEV (annual maxima) and GPD (threshold exceedances).

6.1 Generalized Extreme Value (Block Maxima)

We extract the maximum company loss for each of 10 accident years and fit the GEV distribution:

$$H(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \tag{5}$$

Table 5: GEV Parameter Estimates

Parameter	**Estimate**	**Interpretation**
Location (μ)	\$8,573,735	Central value
Scale (σ)	\$1,015,147	Dispersion
Shape (ξ)	−0.3291	**Weibull (bounded tail)**
Upper bound	\$11,657,682	$\mu - \sigma/\xi$

The negative shape parameter indicates bounded tail behavior—annual maximum losses have a finite upper limit, possibly due to policy limits or market constraints.

6.2 Generalized Pareto Distribution (Peaks Over Threshold)

We model exceedances above the 85th percentile (\$18,102, yielding 175 exceedances, 15% of data):

Table 6: GPD Parameter Estimates (Threshold = \$18,102)

Parameter	**Estimate**	**Interpretation**
Scale (σ)	\$642,144	Exceedance scale
Shape (ξ)	0.8238	**Heavy tail, infinite variance**

The positive shape ($\xi > 0.5$) indicates heavy-tailed individual extremes with infinite variance—consistent with bodily injury claims where catastrophic injuries create extreme severities.

6.3 Return Level Estimates

Table 7: Return Levels: GEV vs. GPD

Return Period	GEV ($M)	GPD ($M)
10-year	10.19	0.33
20-year	10.50	1.17
50-year	10.80	3.34
100-year	10.98	6.50

6.4 Reconciling GEV vs. GPD: Why Shape Parameters Differ

The dramatically different shape parameters ($\xi_{\text{GEV}} = -0.33$ vs. $\xi_{\text{GPD}} = 0.82$) are not contradictory—they reflect fundamentally different quantities and provide complementary risk insights.

GEV ($\xi = -0.33$): Models the maximum loss across all 144 companies in each year. The negative shape indicates a Weibull distribution with bounded tail behavior and finite upper endpoint (≈$11.7M). This suggests industry-wide maximum losses are naturally constrained, possibly due to regulatory policy limits, reinsurance structures, or market concentration. The 100-year return level of $11.0M represents the expected worst-case industry maximum in a century.

GPD ($\xi = 0.82$): Models individual company losses exceeding the 85th percentile ($18,102). The positive shape (> 0.5) indicates heavy Pareto-type tail with infinite variance. Individual companies can experience extreme losses without theoretical bound, consistent with bodily injury claims where catastrophic injuries, protracted litigation, and punitive damages can produce unlimited severity. The heavy tail reflects the reality that while most companies experience moderate losses, outlier companies can face devastating claims.

Why They Differ: The shape parameters would theoretically converge ($\xi_{\text{GEV}} \approx \xi_{\text{GPD}}$) if both methods modeled the *same* underlying distribution with infinite samples. Here, they model different aggregation levels:

- **GEV:** Industry maximum (best of 144 companies) → Bounded by market structure
- **GPD:** Individual extremes (any company having bad year) → Unbounded by catastrophic events

This divergence is **expected and valuable**: while the industry as a whole exhibits bounded maximum risk (diversification, reinsurance protection), individual companies face heavy-tailed concentration risk. Both perspectives inform comprehensive risk management—GEV for market-wide stress scenarios and regulatory capital, GPD for company-specific capital allocation and reinsurance treaty design.

The pattern observed (bounded industry maxima, heavy-tailed individual extremes) is common in insurance literature and reflects the protective effect of industry-level diversification versus company-level concentration risk.

7 Discussion

7.1 Summary of Findings

Methodological: Tweedie distribution modeling (5.12% MAE) vastly outperforms Compound Poisson-Gamma with synthetic exposure (905,421% MAE) when actual exposure and claim count data are unavailable. This 177,000× performance difference validates that appropriate method selection matters more than theoretical tradition.

Substantive (from Tweedie + EVT):

- Losses are severity-dominated ($p = 1.762$), driven by few large claims
- Significant decreasing trend of 2.74%/year over 1988-1997
- Near-proportional premium scaling (elasticity = 1.02)
- Annual maxima exhibit bounded behavior (GEV: $\xi = -0.33$, upper limit ≈\$11.7M)
- Individual extremes are heavy-tailed (GPD: $\xi = 0.82$, infinite variance)
- Divergent EVT shapes reflect different risk perspectives (industry vs. company)

7.2 Practical Implications

For Methodology: This analysis demonstrates general principles applicable to any time period: (1) synthetic assumptions cannot substitute for actual data, (2) adapting methods to available data outperforms forcing traditional approaches, and (3) comparative analysis reveals methodological strengths and weaknesses that remain valid regardless of data vintage.

For Current Application: While the 1988-1997 loss magnitudes and trends are historical, the severity-dominated structure ($p > 1.7$) and proportional premium scaling (elasticity ≈ 1) represent structural relationships likely to persist in modern data, though requiring empirical verification with recent data.

For Risk Management: The complementary EVT perspectives (bounded industry maxima, heavy-tailed company extremes) illustrate that effective capital allocation requires both market-level and entity-level tail analysis. Relying on a single EVT approach may miss critical risk dimensions.

7.3 Limitations

Temporal: Analysis covers 1988-1997 (27-36 years old). Absolute loss estimates, trends, and extreme value return levels reflect historical patterns and should not be used for current forecasting. Medical cost inflation, legal environment changes, vehicle safety improvements, and telematics adoption post-2000 fundamentally altered the loss landscape.

Data: Only 10 years limits EVT reliability, particularly for GEV (10 maxima is minimal for robust estimation). Company-level aggregates prevent examination of individual claim distributions. No geographic or economic covariates.

Model: Assumes stationary parameters over time. Treats companies as independent (no spatial correlation). Limited to year and premium as covariates.

7.4 Future Work

Priority improvements: (1) **update with 2010-2024 data** to assess current patterns and validate whether methodological conclusions hold in modern context, (2) obtain actual exposure data from Schedule P Part 2 or state filings to enable true frequency/severity analysis, (3) incorporate geographic and economic covariates, (4) implement time-varying parameter models, and (5) extend EVT analysis with longer time series for more robust extreme value estimation.

Most critically, replicating this comparative analysis with recent data would test whether the dramatic performance difference between Tweedie and synthetic CP-Gamma persists in modern insurance markets—a question with both methodological and practical significance.

8 Conclusions

This analysis demonstrates that when exposure and claim count data are unavailable, Tweedie distribution modeling provides reliable aggregate loss estimates (5.12% MAE) while traditional Compound

Poisson-Gamma with synthetic exposure fails catastrophically (905,421% MAE). The 177,000× performance difference validates the critical importance of matching statistical methods to available data.

While the analysis uses historical 1988-1997 data, the **methodological lesson is timeless**: synthetic assumptions cannot substitute for actual data, and forcing traditional methods onto inappropriate data structures produces unreliable results. This principle applies equally to historical and contemporary datasets.

Key substantive findings reveal severity-dominated losses ($p = 1.762$) with a decreasing trend of 2.74% annually, near-proportional premium scaling, and complementary tail behavior: bounded annual maxima (GEV: $\xi = -0.33$, 100-year return = \$11.0M) but heavy-tailed individual extremes (GPD: $\xi = 0.82$). The divergent EVT shapes are not contradictory but informationally rich, reflecting industry-level diversification protection versus company-level concentration risk. While these specific estimates reflect 1990s market conditions, the severity-dominated structure and proportional scaling represent relationships likely to persist in modern data.

Recommendations: (1) Use Tweedie methods when modeling aggregates without exposure data, (2) never rely on synthetic assumptions for production estimates, (3) apply both GEV and GPD perspectives for comprehensive tail risk assessment, (4) replicate this comparative framework with modern data to validate current applicability, and (5) when facing data constraints, adapt methodology rather than force traditional approaches with unverified assumptions.

This two-part framework demonstrates that negative results (Part 1 failure) combined with positive results (Part 2 success) provide more valuable methodological insights than either alone, illustrating not just what works, but why alternatives fail—a lesson independent of data vintage.

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