

## Week 5: (Ungraded) Challenge Problems 5

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## 5.1 About These Problems

- **This will not be graded. And it's not mandatory or necessary to do these problems.**
- You should focus on homework and lectures to do well in this class.
- These are merely meant to be supplementary challenge problems for those who want them.
- Consult Andrew Lizarraaga: andrewlizarraaga at g.ucla.edu for question or solutions.

## 5.2 Challenger! The trials have varied ...

**Problem 1 (A Blood Test):** Suppose that a laboratory test on a blood sample yields one of two results: positive or negative. It is known that 95% of people with a particular disease produce a positive result. But 2% of people without the disease will also produce a positive result (a false positive). Suppose that 1% of the population has the disease. What is the probability that a person chosen at random from the population will have the disease, given that the person's blood yields a positive result.

**Problem 2 (Biased Runs):** A biased coin is tossed  $n$  times, and heads shows with probability  $p$  on each toss. A run is a sequence of throws which result in the same outcome, so that, for example, the sequence  $HHTHTTH$  contains five runs. Show that the expected number of runs is  $1 + 2(n-1)p(1-p)$ . Find the variance of the number of runs.

**Problem 3 (Urn Removal):** An urn contains  $n$  balls numbered  $1, \dots, n$ . We remove  $k$  balls at random (without replacement) and add up their numbers. Find the mean and variance of the total.

**Problem 4 (Simpson's Game):** You have two bags, each with a mixture of green and red marbles. You can choose a bag, then randomly pick a marble from the bag. If it's green you win \$100, but if it's red, you get nothing.

**Game 1:** Bag  $A$  contains 5 green marbles and 6 red marbles. Bag  $B$  has 3 green marbles and 4 red marbles. What bag should you choose to increase your odds of winning?

**Game 2:** Bag  $C$  contains 6 green marbles and 3 red marbles. Bag  $D$  has 9 green marbles and 5 red marbles. What bag should you choose to increase your odds of winning?

**Game 3:** We combine bags  $A$  and  $C$  into bag  $E$  and we combine bags  $B$  and  $D$  into bag  $F$ . What bag should you choose to increase your odds of winning?