

π -Day Puzzles*Author: Andrew Lizarraaga***Problem 1: Buffon's Needle Experiment**

A table is lined with parallel strips of width t and we drop a needle of length $l < t$ onto the table. What is the probability the needle crosses a line?

Problem 2: A Bent Needle

Suppose now that the needle from problem 1 is bent into some planar curve shape. For example an "S" shape or a "C" shape. What is the probability that the bent needle crosses a line?

Problem 3: Two grids of lines

Now the table has a grid of parallel lines spaced a apart and another set of parallel lines (perpendicular to the first set) are imposed on the table and are spaced b apart. We drop a needle with length $l < \min(a, b)$ on the table. What is the probability the needle intersects a line?

Problem 4: Crossing a Cross

We have a table with parallel lines spaced 1 unit apart. We form a cross by welding two unit needles at their midpoints and drop it on the table. Let X denote the number of intersections the cross makes with a parallel line. What is $\mathbb{E}[\frac{X}{2}]$?

Problem 5: X-Ray Machine

An individual inadvertently swallowed a needle and is examined via an X-Ray machine. If the needle exhibits no preference for direction in the gut, what is the distribution of the length of its image on the X-Ray plate?

Problem 6: Digits of Pi

Some people believe that the successive digits in the decimal expansion of $\pi = 3.14159\dots$ are "uniformly" distributed. By this, they mean that the appearance of these digits are equally likely to be of any digit from 0 to 9. However, someone notices that starting with the 24,658,601st digit, there is a run of nine successive 7's. Is this information consistent with the hypothesis of a uniform distribution?

