Stats100A **Summer 2024**

Week 1: An (Ungraded) Quiz - Solutions

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1.1 A Reminder About the Quiz

- Don't stress if you didn't get a majority of the problems.
- The problems are merely to asses what you currently know, so the TA's have a better idea of what should be discussed in section.

1.2 Solutions

Problem 1 (Coin Flippers): I flip a fair coin 10 times and they all came up heads. Is it more likely that the 11^{th} coin flip will come up tails?

Solution: The coin is a fair coin. And it doesn't have any memory about the past flips, it doesn't matter if the previous 10 flips are all tails. The next coin flip is just as likely to be heads as it is to be tails.

Problem 2 (Coin Toss): I flip two fair coins simultaneously, when they hit the ground, what's the probability of getting one coin showing a heads and the other a tails?

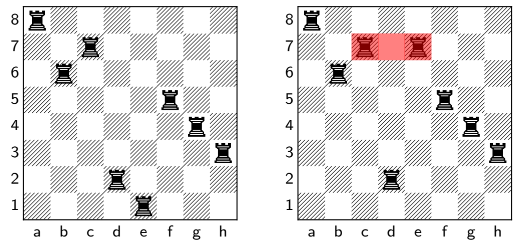
Solution: Here are the following outcomes for the two coins: $\{HH, HT, TH, TT\}$. Since we are interested in the HT and the TH outcomes, thus there is a $\frac{2}{4} = \frac{1}{2}$ chance of this occurring.

Remark: Notice that we distinguish HT from TH, even though we tossed the coins simultaneously. You might think that we shouldn't distinguish these two events because of this. However, it is essential to consider whether it makes sense to treat HT and TH as the same event. The answer is no.

Consider the probability of getting HH. There is a $\frac{1}{2}$ chance of getting H for one coin and the same probability for the other. Since the coins are independent, the probability of getting HH is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Likewise the probability for TT is $\frac{1}{4}$. It is an axiom of probability that the total probability of distinct outcomes must add up to 1. Since we consider $HT = TH$, we have $P(HH) + P(TT) + P(HT) = 1$, which implies that $P(HT) = \frac{1}{2}$. However, because the coins are independent, the probability of HT is $P(HT) = P(H)P(T) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Thus HT and TH must be treated as separate events.

Another way to understand why we care about the order is to note that when the coins are tossed, there is a first coin that hits the ground, followed by the second coin. Therefore we can order the outcomes: $\{HH, HT, TH, TT\}$ as stated earlier.

Problem 3 (Peaceful Rooks): You are given an 8×8 chess board and you need to place 8 rooks on the boards so that no two rooks can attack each other, i.e. a **peaceful** arrangement. How many peaceful arrangements are there? (Remember rooks can only go left, right, up and down. See below for an example of a peaceful and un-peaceful arrangement, respectively)



Since we have to place 8 rooks on the 8×8 chessboard, then it must be the case that each row of the chessboard requires exactly one rook. Start with row 1, we have 8 choices to place the rook. Now once the first rook is placed, there are 7 choices to place the second rook in the second row (because we can't place the rook in the column associated with the first rook we placed earlier). Once we place the second rook, we have 6 choices for the third rook to be placed in the third row, and we continue in this fashion until we get to the final row in which we only have one choice to place the rook. Thus there are $8! = 8 \times 7 \times 6 \ldots 3 \times 2 \times 1$ possible peaceful arrangements.

Problem 4 (The UCXB Students): Two students at UCXB, decided to party before the final. Unfortunately, due to their poor judgement they missed the final. They told the Professor that they were driving back from a families home but one of the tires went flat which is why they missed the final (this is a lie). The Professor agreed to let them take a different final. The two students were seated in separate rooms. There was only one question on the final worth 100 percent of the grade: "Which tire was it?"

What is the probability that the students give the same answer?

Solution: Let's think of the pairs of answers the students can give. They are in separate rooms, so we treat them as being independent and assume that they will pick any tire wih equal probability. We label the tires $1, 2, 3, 4$. There are $4 \times 4 = 16$ possible pairs of answers and only 4 of the pairs, namely $\{11, 22, 33, 44\}$, have both answers being the same, thus the probability of both students giving the same answer is $\frac{4}{16} = \frac{1}{4}$.

> *Remark:* This problem can also be solved with conditional probability. The first student will pick a tire, then the probability the second student will pick the same tire is $\frac{1}{4}$ since of the 4 options they have, only one of them matches the first student's answer.

Problem 5 (Aurora Borealis 1): Friday is going to have a clear night. There is a 60% chance that you can see the aurora borealis in any given hour. If you go outside and watch the sky for two hours, what's the probability that you'll see the aurora borealis?

Solution: Let S denote that we see the auroras in a given hour, and N denote that we didn't see the auroras in a given hour. Then, the possible states for the 2 hour period is $\{SS, SN, NS, NN\}$. Since we only care about the probability of seeing the Auroras, thus we want:

$P(\{SS, SN, NS\}) = P(SS) + P(SN) + P(NS) = P(S)P(S) + P(S)P(N) + P(N)P(S) = (0.6)^2 + (0.6)(0.4) + (0.4)(0.6) = 0.84$

There is an 84% chance of seeing the aurora borealis.

Problem 6 (Aurora Borealis 2): Saturday night is going to be clear as well! Conditions are even better this time, there is an 80% chance of seeing the aurora borealis at any given hour. Let's assume that the probability is uniform for the entire hour. What is the probability you'll see the aurora borealis in the first 15 minutes?

A Common Incorrect Answer:

You must avoid this trap: 15 minutes is $\frac{1}{4}$ of an hour, and there's an 80% chance of seeing the auroras in a given hour. Therefore the probability that you see the auroras in the first 15 minutes is $\frac{1}{4}80\% = 20\%$

Remark: The reasoning above can't be right. If we apply our knowledge from **problem 5**: We found that with a probability of 60% of seeing the auroras in a given hour led to a 84% probability of seeing the auroras in two hours. If we use the faulty logic from earlier, we'd conclude that there was only a 42% chance of seeing the auroras in 1 hour, which is clearly wrong.

Solution: An easier approach is to instead consider the probability you don't see the auroras for the entire hour. We know that that the probability of seeing the auroras in a given hour is 80%, so the probability that we don't see it in the entire hour is $100\% - 80\% = 20\%$. Equivalently, this occurs if you don't see the auroras for each of the 4-quarter hours that make up the entire hour.

Let p be the probability of seeing the auroras in the first 15 minutes. Also note that since the probability is uniform, the probability that you see the auroras for any particular 15 minute interval is also p.

Now we can denote the probability of not seeing the auroras in the first hour by:

$P(\text{No auroras in first hour}) = P(\text{No auroras in first 15 minutes}) \times P(\text{No auroras in second 15 minutes}) \times P(\text{No auroras in third 15 minutes}) \times P(\text{No auroras in fourth 15 minutes}) = (1-p)^4$

Thus $(1-p)^4 = 20\% = \frac{1}{5}$. Solving for p yields $p = 1 - \frac{1}{\sqrt[4]{5}} \approx 33.1\%$

Problem 7 (A Message?): Initially, lions often visit elephants, snakes take antelopes, tigers invade stables, toucans irk cheetahs steadily.

Solution: The keyword is *Initially*, i.e. think about the initials of each word:

Initially, **L**ions **O**ften **V**isit **E**lephants, **S**nakes **T**ake **A**ntelopes, **T**igers **I**nvade **S**tables, **T**oucans **I**rk **C**heetahs **S**teadily.

I LOVE STATISTICS

Well, at least I hope you do!

Problem 8 (Anything Else?): Is there anything that you'd like to see in this course? Anything I should be made aware of? Any concerns you may have? Also, what's your major and year by the way?

Solution: This is an exercise left for the reader.