Stats100A **Summer 2024**

Week 2: (Ungraded) Challenge Problems 2

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2.1 Solutions

Problem 1 (Bag Of Cards): A bag contains 3 cards. One card is black on both sides, another card is white on both sides, and the third card is black on one side while being white on the other side. The cards are mixed up in the bag. Then a single card is selected and placed on a table so that the visible side of the card is black. What is the probability of the other side of the card being white?

Solution: Label the faces of the cards, say b_1, b_2 for the double black card, w_1, w_2 for the double white card, and b_3, w_3 for the white and black card. Then the event of seeing a black side of the card on the table corresponds to one of the following evens: b_1, b_2, b_3. Only the b_3 card corresponds to the white face w_3, thus there is a $\frac{1}{3}$ chance that the other side of the card is white.

Problem 2 (In Between): I take three samples uniformly from the unit interval $[0, 1]$ and denote them A, B, C. What is the probability that $A < B < C$?

Solution: Let's say the three points A, B, C are chosen uniformly at random on the interval $[0, 1]$. Then there are $3! = 6$ possible orderings of the three unique values and therefore a $\frac{1}{6}$ chance that the numbers drawn or linearly ordered and coincide with $A < B < C$.

Problem 3 (An Obscuring Object?): A man in a gallery is asked about a portrait depicting a man with an object obscuring his face. He responds, 'Brothers and sisters I have none, but that man's father is my father's son.' What is the object obscuring the face of the man in the portrait?

Solution: Perhaps you are familiar with the original riddle which states *'Brothers and sisters I have none, but that man's father is my father's son.'* The answer to the original riddle is that the man in the portrait is the *son of the man*. If you know some art history or look up the portrait *The Son of Man*, then you'll know that this painting depicts a man with a green apple covering his face. So the the answer is a **green apple**.

Problem 4 (Three Slips):

Three different numbers are chosen at random, and one is written on each of three slips of paper. The slips are then placed face down on the table. If you choose the slip with the largest number you win!

Rules:

1. Initially, you can turn over any slip of paper and view the number written on it.
2. You may choose to keep the initial slip.
3. If unhappy with your first choice, you may discard your it and view another slip. You can choose to keep your second choice or discard it.
4. If you discard the second slip, then you must keep the third slip
5. You win if the slip you keep has the largest number out of all 3 slips.

What is the optimal strategy to increase your odds of winning?

Solution: For simplicity, let's just say the 3 randomly chosen numbers are $1, 2, 3$. Then the possibilities that you might face when turning the slips over are $\{123, 132, 213, 231, 312, 321\}$. Here is a proposed strategy, we will always turn two slips over. If the second slip is larger than the first, we keep it. There is $\frac{1}{2}$ chance of this occurring and of these cases $\{123, 132, 231\}$ 2 of the cases are winning.

If the second slip is less than the first slip, we flip the third slip and keep it. Again there is a $\frac{1}{2}$ chance of this occurring and among these cases $\{213, 312, 321\}$ only one is a winning case.

Following this strategy we have 3 winning cases and 3 losing cases, so we have a $\frac{1}{2}$ chance of winning.

Notice that if our strategy was to always pick the first slip, we'd win $\frac{2}{6} = \frac{1}{3} < \frac{1}{2}$. A similar argument holds for always choosing the second or third slip. This only leaves the strategy mentioned earlier in the solution, hence it's optimal.