Stats100A **Summer 2024**

Week 5: (Ungraded) Challenge Problems 5

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5.1 About These Problems

- **This will not be graded. And it's not mandatory or necessary to do these problems.**
- You should focus on homework and lectures to do well in this class.
- These are merely meant to be supplementary challenge problems for those who want them.
- Consult Andrew Lizarraga: andrewlizarraga at g.ucla.edu for question or solutions.

5.2 Challenger! The trials have varied ...

Problem 1 (A Blood Test): Suppose that a laboratory test on a blood sample yields one of two results: positive or negative. It is known that 95% of people with a particular disease produce a positive result. But 2% of people without the disease will also produce a positive result (a false positive). Suppose that 1% of the population has the disease. What is the probability that a person chosen at random from the population will have the disease, given that the person's blood yields a positive result.

Problem 2 (Biased Runs): A biased coin is tossed n times, and heads shows with probability p on each toss. A run is a sequence of throws which result in the same outcome, so that, for example, the sequence $HHTHTTH$ contains five runs. Show that the expected number of runs is $1 + 2(n-1)p(1-p)$. Find the variance of the number of runs.

Problem 3 (Urn Removal): An urn contains n balls numbered $1, \ldots, n$. We remove k balls at random (without replacement) and add up their numbers. Find the mean and variance of the total.

Problem 4 (Simpson's Game): You have two bags, each with a mixture of green and red marbles. You can choose a bag, then randomly pick a marble from the bag. If it's green you win \$100, but if it's red, you get nothing.

Game 1: Bag A contains 5 green marbles and 6 red marbles. Bag B has 3 green marbles and 4 red marbles. What bag should you choose to increase your odds of winning?

Game 2: Bag C contains 6 green marbles and 3 red marbles. Bag D has 9 green marbles and 5 red marbles. What bag should you choose to increase your odds of winning?

Game 3: We combine bags A and C into bag E and we combine bags B and D into bag F. What bag should you choose to increase your odds of winning?