Homework 2

EECE 5550

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1 Mobile Robot Configuration

1.1 Rotation Matrix

$$R_{robot}^{world} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$v_{world} = R_{robot}^{world} * \begin{bmatrix} vx_{robot} \\ vy_{robot} \\ v\theta_{robot} \end{bmatrix}$$

1.2 Velocity Problem 1

$$\theta = \pi/6$$

$$v_{world} = R(\theta) * \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}$$

$$v_{world} = \begin{bmatrix} 12.99 \\ 7.5 \\ 0 \end{bmatrix}$$

1.3 Velocity Problem 2

$$\theta = \pi/3$$

$$v_{world} = R(\theta) * \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$v_{world} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

1.4 Velocity Problem 3

$$\theta = \pi/4$$

$$v_{world} = R(\theta) * \begin{bmatrix} 25 \\ 0 \\ 4 \end{bmatrix}$$

$$v_{world} = \begin{bmatrix} 17.68 \\ 17.68 \\ 4 \end{bmatrix}$$

1.5 Velocity Problem 4

$$\theta = -3\pi/4$$

$$v_{world} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix} = R(\theta) * \begin{bmatrix} u \\ v \\ \omega \end{bmatrix}$$

$$v_{robot} = \begin{bmatrix} -14.14 \\ 0 \\ 10 \end{bmatrix}$$

2 Question 2

Rolling Constraints:

$$\begin{bmatrix} sin(\alpha+\beta) & -cos(\alpha+\beta) & -lcos(\beta) \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} - r\dot{\varphi} = 0$$

Sliding Constraints:

$$\begin{bmatrix} cos(\alpha+\beta) & sin(\alpha+\beta) & lsin(\beta) \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

2.1 ICC

$$ICC = \frac{l}{2} * \frac{(\phi_1 + \phi_2)}{\phi_1 - \phi_2}$$

2.2 Rolling Constraints

$$\alpha_1 = 90, \beta_1 = 0, l = b$$
$$\alpha_2 = 270, \beta_2 = 0, l = b$$

$$\begin{bmatrix} 1 & 0 & -b \\ -1 & 0 & -b \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = r \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

2.3 Sliding Constraints

$$\alpha_1 = 90, \beta_1 = 0, l = b$$

$$\alpha_2 = 270, \beta_2 = 0, l = b$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

2.4 Holonomic vs. Non-Holonomic

Non-holonomic since the robot cannot move in the in the y direction.

2.5 Mobility

Can move back and forth along x direction.

3 Triangular Robot 1

Rolling Constraints:

$$\begin{bmatrix} sin(\alpha+\beta) & -cos(\alpha+\beta) & -lcos(\beta) \\ & \vdots & \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} - r\dot{\varphi} = 0$$

Sliding Constraints:

$$\begin{bmatrix} cos(\alpha+\beta) & sin(\alpha+\beta) & lsin(\beta) \\ & \vdots & \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

3.1 Rolling Constraints

$$\alpha_1 = 90, \beta_1 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\alpha_2 = 225, \beta_2 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\alpha_2 = 315, \beta_2 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 & -17.32 \\ -.7 & .7 & -17.32 \\ -.7 & -.7 & -17.32 \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} - r \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 0$$

3.2 Sliding Constraints

$$\alpha_1 = 90, \beta_1 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\alpha_2 = 225, \beta_2 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\alpha_2 = 315, \beta_2 = 0, l = \frac{\sqrt{3}}{2} * 40cm * \frac{1}{2}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -.7 & -.7 & 0 \\ .7 & -.7 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

3.3 Mobility

Rotational movement, no lateral movement.

4 Triangular Robot 2

Rolling Constraints:

$$\begin{bmatrix} sin(\alpha + \beta) & -cos(\alpha + \beta) & -lcos(\beta) \\ & \vdots & \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} - r\dot{\varphi} = 0$$

Sliding Constraints:

$$\begin{bmatrix} cos(\alpha + \beta) & sin(\alpha + \beta) & lsin(\beta) \\ & \vdots & \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

4.1 Rolling Constraints

$$\alpha_1 = 90, \beta_1 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$

$$\alpha_2 = 225, \beta_2 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$

$$\alpha_3 = 315, \beta_3 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -.7 & -.7 & 0 \\ .7 & -.7 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} - r \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 0$$

4.2 Sliding Constraints

$$\alpha_1 = 90, \beta_1 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$
$$\alpha_2 = 225, \beta_2 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$
$$\alpha_3 = 315, \beta_3 = 90, l = \frac{\sqrt{3}}{2} * 30cm * \frac{1}{2}$$
$$\begin{bmatrix} -1 & 0 & 12.99 \\ .7 & -.7 & 12.99 \\ .7 & .7 & 12.99 \end{bmatrix} \begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = 0$$

5 Properties of R

- $tranpose(R) = inverse(R)$
- $det(R) = 1$
- **columns are orthogonal (dot product of columns = 0)**

6 Skew

No work necessary

7 Numerical Solutions for Diff Eq

7.1 a

$$x_1 = \theta$$
$$x_2 = \dot{\theta}$$
$$\dot{x_1} = \dot{\theta} = x_2 = \int \dot{x_2} = \frac{g}{L} cos(x_1) + C$$
$$\dot{x_2} = \ddot{\theta} = -\frac{g}{L} sin\theta = -\frac{g}{L} sin(x_1)$$

7.2 e

- **'pend' represents the name of the diff eq being solved**
- **'[0, 10]' the time span/range this simulation is run over**
- **'[0; 1]' the initial condition of the state**

7.3 f + g

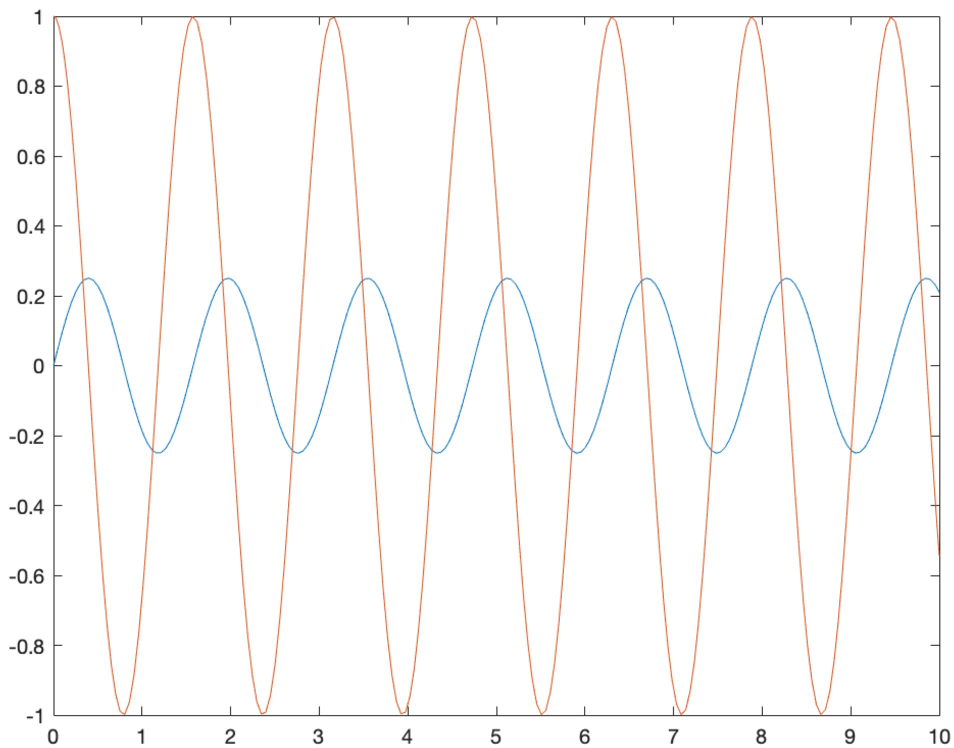


Figure 1: Orange: xdot(1), Blue: xdot(2)

See fig. 1.

7.4 h

7.5 f + g

See fig. 2.

8 8 + 9

See fig. 3.

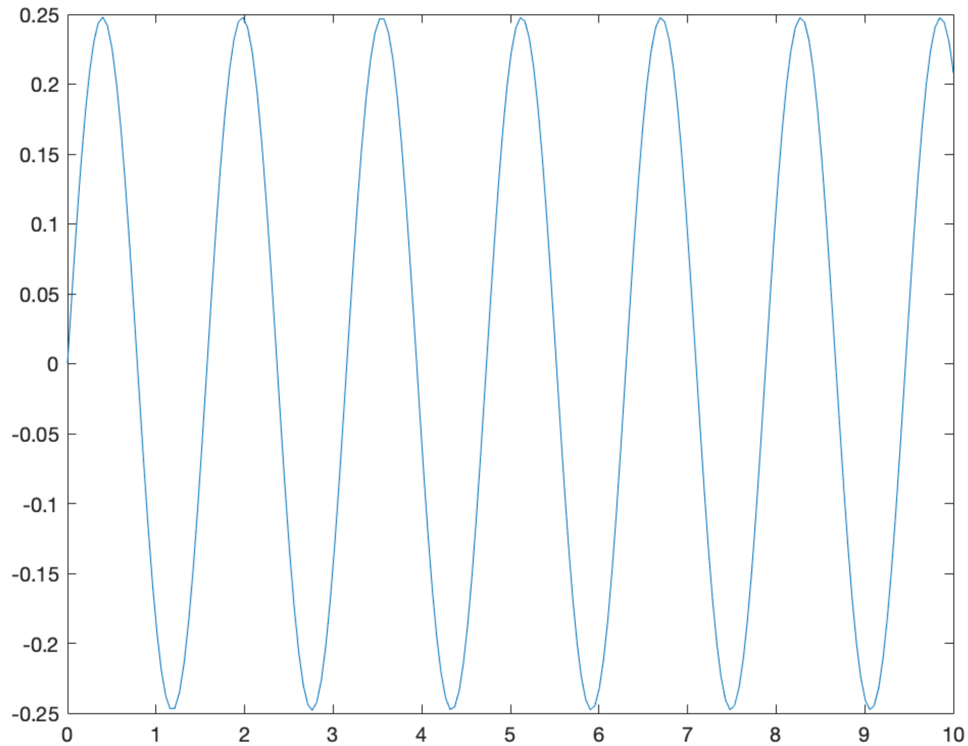


Figure 2: Y component given angle over time. Result should be scaled by L.

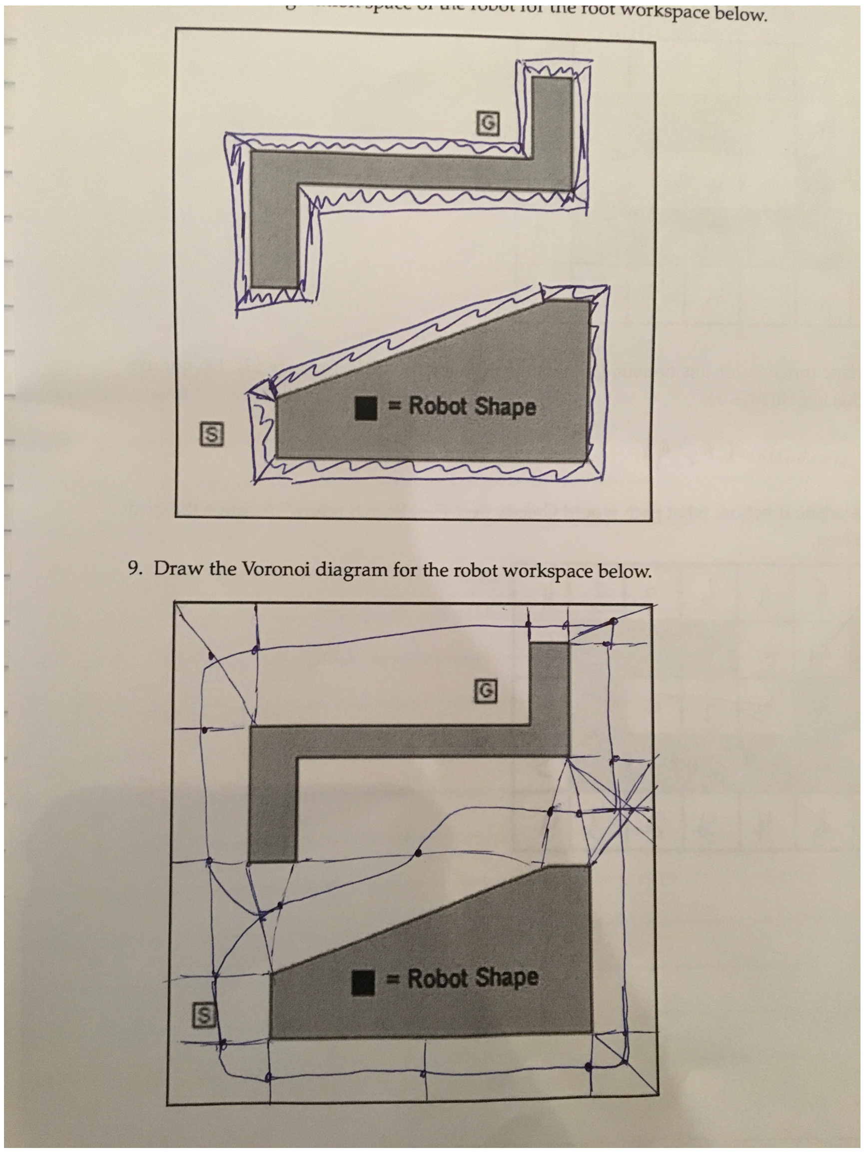


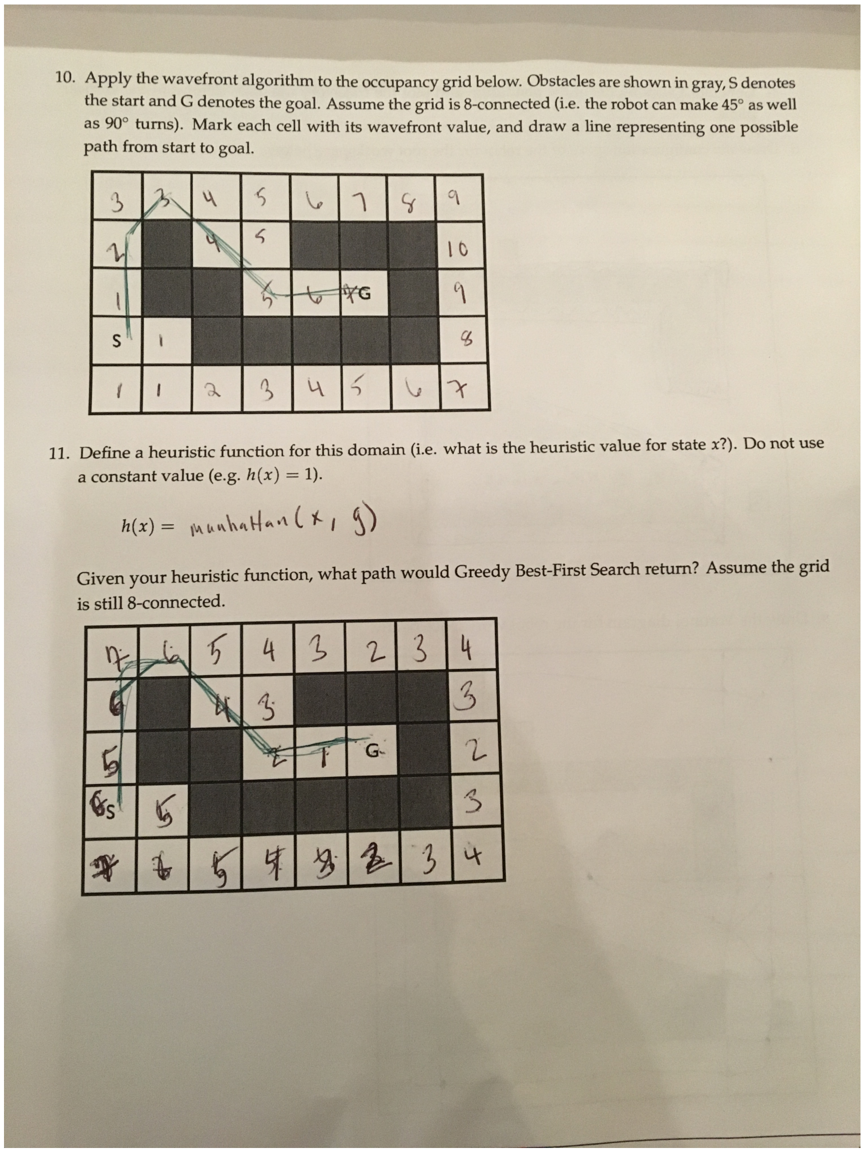
Figure 3: questions 8 and 9

9 10 + 11

See fig. 4

10 Graph Search

- **BFS: A, B, C, E, D, F, G**
- **DFS: A, B, D, F, E, C, G**



10. Apply the wavefront algorithm to the occupancy grid below. Obstacles are shown in gray, S denotes the start and G denotes the goal. Assume the grid is 8-connected (i.e. the robot can make 45° as well as 90° turns). Mark each cell with its wavefront value, and draw a line representing one possible path from start to goal.

3	3	4	5	6	7	8	9
2		4	5				10
1			5	6	~~7~~ G		9
S	1						8
1	1	2	3	4	5	6	7

11. Define a heuristic function for this domain (i.e. what is the heuristic value for state x?). Do not use a constant value (e.g. $h(x) = 1$).

$h(x) =$ manhattan(x, g)

Given your heuristic function, what path would Greedy Best-First Search return? Assume the grid is still 8-connected.

7	6	5	4	3	2	3	4
6		4	3				3
5			2	1	G		2
6 S	5						3
7	6	5	4	3	2	3	4

Figure 4: questions 10 and 11