Why Bundle When Consumers Can Resell?

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January 20, 2021

Abstract

Why do sellers offer discounted bundles when some consumers practice arbitrage through resale? I develop a model of monopoly bundling with costly resale to explain the coexistence of price discrimination and resale and illustrate the effects of resale on bundling. Larger bundle discounts can raise more revenue from consumers who do not resell, but also increase the number of less profitable resellers. In equilibrium, it can be optimal to allow resale among some consumers to increase profit earned from the rest. The ability to resell reduces the returns to bundling and has an ambiguous effect on consumer and total welfare.

Keywords: Bundling, price discrimination, pricing, resale

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I appreciate help and insights from Curtis Taylor, James Roberts, Allan Collard-Wexler, Bryan Bollinger, Jonathan Williams, Daniel Xu, and seminar participants in the theory and industrial organization groups at Duke University.

1 Introduction

Virtually all studies of price discrimination share one assumption: that resale is impossible.[1] There are good reasons why economists do not consider resale. Second-degree price discrimination relies on the ability to design different contracts for different consumer types, but resale breaks the self-selection process by letting consumers pool the contracts. Resale and price discrimination are thus seen as incompatible.

Yet sellers often use price discrimination when there are active resale markets. Consider tickets for sports, theater, and concerts. Sellers practice mixed bundling, offering a discounted season ticket bundle alongside tickets for each event, and consumers resell tickets frequently on services like StubHub. More generally, any durable good sold in a bundle or value pack can be resold. These include trading cards, cookware sets, and box sets of books, all of which are resold individually on internet platforms. The sellers of these goods are aware that some consumers will break up the bundle and resell, but they use price discrimination anyway.

Why do sellers use price discrimination knowing that consumers will resell? The puzzle has received little formal attention even though resale is common, leaving the implications for firm behavior unknown. How does resale affect the seller's problem? And how do predictions for profit and welfare change?

In this paper, I examine the puzzle using a model of monopoly bundling with costly resale and show that, in equilibrium, the monopolist bundles even though some consumers will resell parts of the bundle. Resale introduces a new tradeoff to the monopolist's problem, changes its incentives to bundle, reduces profit, and has an am-

[1] Tirole (1988) and McAfee (2008) mention resale as a determinant of whether price discrimination is feasible but do not consider it in their analysis. In a review of nonlinear pricing, Armstrong (2016) does not mention resale or arbitrage.

biguous effect on consumer and total welfare. The combination of bundling and resale in equilibrium is important because the seller's problem, and hence the implications of resale, are different than in a model without resale in equilibrium. Bundling is an important setting because of the ubiquity of bundles that can be resold, as in the examples discussed above, and because no prior studies have considered the effect of resale on bundling.

The central tension in the model is between the profitability of bundling and consumers' gains from sharing the bundle through resale. Both usually increase with the bundle discount, the difference between the price of the bundle and the price of buying its component goods separately. The essence of bundling is that discounting the bundle can raise profit, so increasing the discount is profitable on some interval. But larger discounts tempt consumers to share the bundle through resale, which lowers profit because they only pay the discounted bundle price.

The resulting tradeoff allows bundling and resale to occur simultaneously in equilibrium when consumers have heterogeneous costs of engaging in resale. With heterogeneous costs, a higher bundle discount increases the number of consumers practicing arbitrage through resale but lets the monopolist earn more from consumers who find resale too costly. As long as the profit earned from consumers with high costs exceeds losses to resale from those with low costs, it is optimal for the monopolist to allow some resale.

The tradeoff is a significant change to the monopolist's problem, forcing the monopolist to balance the profit made by bundling against the opportunity for consumer arbitrage. Importantly, the tradeoff is not present in simpler models of price discrimination and resale. With a homogeneous cost of resale, there is no tradeoff because all consumers move to the resale market when the bundle discount exceeds the costs of resale. In equilibrium, the monopolist chooses a discount just below the threshold and no consumers resell, simplifying the pricing problem. The difference also affects comparative statics for pricing. When resale becomes more costly, the monopolist never decreases its bundle discount if there is a homogeneous cost of resale. But if there are heterogeneous costs of resale, the monopolist might lower its discount—*reducing* the extent of bundling when resale becomes more costly. The finding suggests that resale is necessary in equilibrium to understand the monopolist's problem and pricing.

The model establishes that resale limits, but does not eliminate, the returns to bundling. Resale harms the monopolist's profit by interfering with its profit-maximizing allocation, but bundling remains profitable. The effects on welfare, however, are ambiguous. It is somewhat surprising that consumers do not benefit because, for a given allocation, resale improves welfare. The resolution is that resale causes the seller to change its prices, and price discrimination has ambiguous effects on consumer and total welfare. It follows that the ability to resell can harm consumers and society.

The analysis makes two main contributions. First, it advances our understanding of how resale affects price discrimination by providing an equilibrium in which the two coexist and by investigating the implications for firm behavior. Prior studies of price discrimination and resale often feature equilibria where the seller prevents all resale, such as Alger (1999). Other studies consider settings where the seller is able to profit from resale, like McManus (2001) and Gans and King (2007). But the case where the seller discriminates, consumers resell, and resale harms the seller has not been explored until now. I show that the omission matters because the seller's problem and pricing incentives differ when there is harmful resale in equilibrium.

The second contribution is to clarify the effects of resale on bundling. No prior study has considered how resale affects bundling despite the fact that the two coexist in numerous economically significant markets. Revenue from sports tickets, sold in mixed bundles, exceeded $10bn in 2019 (Statista (2020)). Revenue from just one trading card game is estimated to exceed $500m per year (Deaux (2019)). I clarify how resale affects pricing, profit, and welfare in these markets.

There are two parts to the analysis. In the first, I analyze a model in which a monopolist bundles and consumers have a homogeneous cost of participating in the resale market. The model provides a simple description of the effects of resale on bundling, but its equilibrium does not include resale. It therefore provides a benchmark for the equilibrium in which bundling and resale coexist. In the second part of the analysis, I allow consumers to have heterogeneous costs of resale. I show that bundling and resale can coexist in equilibrium and that their coexistence meaningfully changes the seller's problem.

The paper proceeds as follows. I start by reviewing the relevant literature. In Section 2, I introduce the model, and in Section 3 I study the benchmark case with homogeneous costs of resale. I consider heterogeneous costs of resale in Section 4. In Section 5, I conclude.

Related Literature. A primary focus of the bundling literature is to determine when monopoly bundling is profitable. Studies have considered how the value of bundling depends on the distribution of consumer values, as in McAfee et al. (1989) and Chen and Riordan (2013), the number of goods sold, as in Bakos and Brynjolfsson (1999), and uncertainty over future states of the world, as in Alexandrov and Bedre-Defolie

(2014). This paper contributes by demonstrating that resale is a determinant of the profitability of bundling.

Researchers have also considered bundling outside the traditional setting with a monopolist seller. Chen and Li (2018) consider the effect of bundling when a buyer must procure several products. Zhou (2017) and Zhou (2019) study the effects of bundling in a competitive environment. Pagnozzi (2009) considers bundling in auctions when consumers can resell after the auction. The implications of resale in his study are specific to auctions rather than monopoly bundling.

Bundling has also been studied empirically. Gandal et al. (2018) use data on computer software to determine the effect of correlations in consumer values on the profitability of bundling. Chu et al. (2011) use data on theater ticket sales to compare the performance of theoretically optimal bundle pricing to simpler rules. Crawford and Yurukoglu (2012) study bundling in a competitive setting, cable television, and consider its effect on upstream bargaining.

A separate literature considers the effects of resale markets. The effect of resale on sellers of durable goods has been widely studied, for instance in Chen et al. (2013). Sellers of durable goods can benefit from resale because it allocates past vintages to the consumers who value them most, but resale forces sellers to compete against past vintages. This paper features a similar tradeoff, with resale increasing consumers' willingness to pay for the bundle but introducing competition in the market for individual goods. I find that the harms of resale always outweigh the benefits. Sellers of perishable goods can also benefit from reallocation when there is limited capacity and consumers receive preference shocks, as in Cui et al. (2014).

Chen et al. (2019) consider how to price loot boxes, random prizes that can be thought of as bundles of award probabilities. Their analysis of salvage—letting consumers return an unwanted item for a partial refund—is similar to allowing resale, but it differs in that returned goods do not compete with the seller's offerings.

Finally, my paper is related to the literature on price discrimination with resale. The most notable paper in this literature is Alger (1999), who considers pricing when consumers can make joint purchases (equivalent to frictionless resale). In equilibrium, the seller sets prices that prevent all resale. Several other papers, such as Gans and King (2007) and McManus (2001), consider settings where the seller can profit from resale.

2 Model

A monopolist with zero fixed and marginal costs and no capacity constraints sells two goods, called 1 and 2. The monopolist sets primary market prices $P = (P_1, P_2, P_B)$, where P_1 and P_2 are the prices of goods 1 and 2 and P_B is the price of a bundle containing both goods. The bundle price satisfies $P_B \leq P_1 + P_2$ because consumers can buy each good separately.

The market includes a mass of consumers normalized to one. Each consumer has type (v_1, v_2), where v_1 and v_2 denote the consumer's value for each good and $v_1 + v_2$ is the consumer's value for the bundle. Values are drawn from the joint distribution $F(v_1, v_2)$. I assume that $F(v_1, v_2)$ has support on $[0, 1]^2$ with a strictly positive, atomless density.

The game proceeds as follows. First, the monopolist sets primary market prices P. Next, consumers purchase goods in the primary market. Finally, consumers participate in a resale market with a vector of endogenously determined clearing prices P^s. Consumers and the monopolist know the distribution of values $F(v_1, v_2)$ throughout the game.

Participating in the resale market, either as a buyer or reseller, is costly. Consumer k must pay a cost c_k to resell or buy from a reseller. Costs are independent of values, $c_k \perp (v_1, v_2)$, and follow the distribution $G(c)$, which has density $g(c)$ and satisfies $G(\underline{c}) = 0$ and $G(\bar{c}) = 1$ for some real numbers $0 \leq \underline{c} < \bar{c}$. The cost of resale can be interpreted as the time and effort needed to participate.[2]

I consider subgame perfect Nash equilibria of the model, which require consumers to have correct expectations for resale prices P^s when they make purchase decisions in the primary market. Equilibrium is a pair (P, P^s) of the monopolist's primary market prices P and resale market prices P^s such that (i) primary market prices P maximize profit given consumer demand, (ii) consumers make optimal primary market purchases under the expectation that the resale market will clear at prices P^s, and (iii) the prices P^s clear the resale market when consumers make optimal primary purchases anticipating P^s.

In the resale market, consumers can purchase either good or buy the bundle to resell either or both goods. For example, consumer k earns surplus $v_1 - (P_1^s + c_k)$ by buying good 1 in the resale market, but she can also earn $v_1 - (P_B - P_2^s + c_k)$ by purchasing the bundle, reselling good 2, and keeping good 1. Each consumer chooses the option maximizing surplus.

Surplus-maximizing choices define a map from types (v_1, v_2) to purchase decisions

[2] An alternative assumption would be that part of the cost is paid to the resale market operator, like the fees on sites like eBay and StubHub. Under this assumption, the cost would count towards total surplus. This would have no effect on the results in Section 3, but it would increase total welfare in Section 4.

for each cost c_k. Figure 1 depicts the purchase regions for the case without resale. The diagonal line, $v_1 + v_2 = P_B$, separates consumers with positive and negative surplus from the bundle. The horizontal and vertical lines $v_1 = P_1$ and $v_2 = P_2$ do the same for the individual goods. Some consumers have positive surplus for several choices, so the regions denote the surplus-maximizing option.

[Figure 1]

3 Homogeneous Costs of Resale

I begin the analysis by considering the case where all consumers share the same cost of participating in the resale market: $c_k = c$ for all consumers k. With a homogeneous cost, bundling and resale do not coexist in equilibrium. Nonetheless, the analysis is valuable because it provides easily interpreted predictions for the effect of resale on bundling. Many of the predictions—such as that resale harms profit and has an ambiguous effect on consumer and total welfare—apply in both models, but can be seen more clearly with a common cost of resale. Furthermore, the predictions in this section can be compared to those of the full model in Section 4 where both bundling and resale occur in equilibrium.

3.1 Resale Equilibrium

Suppose that the monopolist has announced its price vector P and consider the subgame in which consumers make primary and then resale market purchase decisions. An equilibrium of the subgame is a vector of secondary market prices $P^s(P)$ such that consumers make their optimal purchase decisions in the primary market believing that resale prices will be $P^s(P)$ and the vector of resale prices $P^s(P)$ clears the resale market after optimal purchases in the primary market. The goal of this subsection is to characterize equilibrium resale prices and the conditions necessary for resale in equilibrium.

The characterization of equilibrium relies on two supporting results. The first establishes a condition for resale prices.

Lemma 1. In any equilibrium with resale market transactions, $P_1^s + P_2^s = P_B$.

Lemma 1 follows from the need for a buyer and seller in each resale transaction. If the sum of resale prices is any higher, there will be no resale buyers, and if it is any lower, there will be no resellers. The only equilibria of interest satisfy $P_1^s + P_2^s = P_B$ because no other equilibrium has resale transactions.

The result is useful because it narrows the search for equilibrium resale prices. It also simplifies the consumer's choice: surplus is the same for consumers who buy the bundle to resell good 2 as for consumers who buy good 1 in the resale market.[3] For this reason, it only matters that a consumer acquires good 1 through resale, and I will hereafter discuss consumers who want to acquire good 1 or 2 through resale. Further, no consumer buys the bundle to resell in equilibrium because doing so results in a loss of $2c$.

The second supporting result establishes that, when the monopolist only sells a bundle, there is a unique vector of equilibrium resale prices.

Lemma 2. When the monopolist only sells the bundle ($P_1 = P_2 > 1$), there exists an equilibrium vector of resale market prices $\hat{P}^s$. It is unique if $\max\{c, P_B - 1 + c\} <$

[3]Observe that $v_1 - (P_1^s + c) = v_1 - (P_B - P_2^s + c)$.

$\min\{1-c, P_B - c\}$.

The prices $\hat{P}^s$ are called the pure-bundling resale prices and would prevail in the resale market if no consumer buys an individual good in the primary market. They are useful in finding resale prices in the mixed bundling equilibrium.

To see why an equilibrium exists, consider the case of pure bundling with resale depicted in Figure 2. Consumers know that the bundle price is P_B (defining the diagonal line as $v_1 + v_2 = P_B$) and believe that resale prices will be P^s. Types in the region B find it optimal to buy the bundle; those in regions 1 and 2 find it optimal to acquire one good through resale. The key difference from the case with no resale is that the individual good prices (P_1, P_2) have been replaced with fee-inclusive resale prices $(P_1^s + c, P_2^s + c)$. The dashed lines from the (fee-exclusive) resale prices meet at the diagonal by Lemma 1.

The primary market runs before the resale market. Consumers in the B region purchase the bundle and keep it. Half of consumers who want only one good purchase the bundle in order to resell.[4] Finally, consumers participate in the resale market. Consumers who bought the bundle but only want one good resell to consumers who only want one good and did not buy the bundle. The resale market only clears at the expected prices P^s if the mass of consumers in regions 1 and 2 are equal. If not, the market-clearing price differs from expectations and P^s is not an equilibrium.

Equilibrium therefore requires a vector of resale prices equating the mass of the two regions. Such a price vector exists under mild conditions. By Lemma 1, any increase in P_1^s must cause P_2^s to fall (and vice versa). The increase in P_1^s causes fewer consumers to want good 1, and more to want good 2. Graphically, the intersection of the dashed

[4]Each consumer could flip a coin, for example.

lines from P_1^s and P_2^s slides southeast along the diagonal, shrinking the 1 region and enlarging the 2 region. With a strictly positive, atomless density, there is a vector of resale prices making the mass of consumers in the two regions equal.

The condition required for uniqueness rules out the possibility of prices at which no consumer wants to participate in resale. Graphically, it requires that the regions labeled 1 and 2 cannot be empty simultaneously. For the remainder of the paper, I assume that c is small enough for the pure-bundling resale prices to be unique.

Lemmas 1 and 2 provide the tools needed to characterize the full resale equilibrium. I add one additional assumption to remove an open set problem: when all consumers are indifferent between the primary and resale markets, they choose to purchase in the primary market.

Theorem 1. Let $P = (P_1, P_2, P_B)$ be a vector of primary market prices. There is resale in equilibrium if and only if $P_1 + P_2 > P_B + 2c$. Resale prices are unique if there are resale transactions.

Theorem 1 shows that the existence of resale in equilibrium reduces to a simple condition: whether $P_B + 2c < P_1 + P_2$. The condition has a natural interpretation. Resale involves sharing the bundle at price P_B, but incurs the extra cost $2c$. Consumers only share the bundle through resale when it is strictly cheaper than buying each individual good from the seller.

The result also connects resale to the monopolist's ability to discriminate. Bundling is profitable because it lets the monopolist discount the bundle relative to buying each good individually, $P_B < P_1 + P_2$. Doing so lets it boost sales among consumers with high average valuations, like those in the triangle bounded by $v_1 + v_2 = P_B$, $v_1 < P_1$, and $v_2 < P_2$. But discounting the bundle lets consumers share the discount through the resale market, leading to resale whenever the discount exceeds $2c$.

3.2 The Monopolist's Problem

The monopolist's problem is to set its profit-maximizing price vector P given the resale equilibrium described in Theorem 1. The key insight is that it is never profitable for the monopolist to allow resale. To see why, suppose that some consumers resell at prices P^s. Consumers would make the same choices if primary market prices were $(P_1^s + c, P_2^s + c, P_B)$, but they would purchase in the primary market, letting the monopolist earn an additional $2c$ on each transaction that used to involve resale.

The conclusion that it is never optimal to allow resale, coupled with the conditions for resale in Theorem 1, allow a complete description of the monopolist's problem.

Theorem 2. Let $\pi_N(P)$ be the monopolist's profit when there is no resale. There are no resale transactions in equilibrium. The monopolist's problem is

$$\max_P \pi_N(P_1, P_2, P_B) \text{ subject to } P_1 + P_2 \leq P_B + 2c.$$

Theorem 2 establishes that the effect of resale is to limit the seller's bundle discount, and hence its ability to use price discrimination. Without resale, the monopolist is free to choose any bundle discount it likes, but with resale, it is limited to discounts smaller than $2c$. When resale is frictionless, the monopolist offers no discount at all.

The implication that resale is harmful for sellers rings true because sellers of bundles have attempted to prevent resale. The NFL set a price floor in the resale market before the New York Attorney General's office intervened (Belson (2016)). The Denver Broncos NFL team went so far as to revoke season tickets for consumers who resell too frequently (Thomas (2017)). Preventing the resale of durable goods is more difficult because consumers have a legal right to resell, but sellers can make resale more difficult, for example by selling sets of books in a single volume.

The condition for resale in equilibrium and the constraint on prices in the monopolist's problem are appealingly simple and demonstrate the effect of resale, but they rely on the assumption that all consumers have the same cost of resale c. With a homogeneous cost, the monopolist can prevent all resale by setting a discount less than $2c$. But the instant the discount exceeds $2c$, consumers flock to the resale market. Other studies of price discrimination and resale, such as Alger (1999), feature similar predictions but share the drawback that price discrimination and resale do not coexist in equilibrium.

3.3 Comparative Statics

To fully describe the effects of resale on bundling, I present comparative statics for profit, the bundle discount, and welfare as the cost of resale changes.

Corollary 1. The monopolist's profit and bundle discount are weakly increasing in the cost of resale c.

The conclusion of Corollary 1 follows from the monopolist's constraint in Theorem 2. It confirms that resale is harmful to monopoly bundling and that the ease of resale matters for the returns to bundling. It also establishes that, when resale becomes more costly, the seller weakly increases its use of price discrimination by raising the bundle discount.

The changes in consumer and total welfare are less clear-cut. When the initial allocation of goods is fixed, consumers benefit from resale by engaging in welfareenhancing trade. But consumers may not benefit because the monopolist revises its prices in response to resale, changing the initial allocation. As for price discrimination more generally, the effect of resale on consumer welfare is ambiguous.

To formalize the conclusion, assume that the monopolist's prices $P(c)$ are differentiable in c. Because the seller weakly increases its bundle discount as c increases, also assume that $P_B'(c) \geq 0$, $P_1'(c) \leq 0$, and $P_2'(c) \leq 0$.[5] When c increases, consumers buying the bundle benefit, but those buying the first two goods are harmed. Because marginal buyers have surplus zero, the change in consumer welfare depends only on inframarginal consumers.

Theorem 3. Let $\mu_1(c)$, $\mu_2(c)$, and $\mu_B(c)$ denote the masses of consumers buying good 1, good 2, and the bundle when the cost of resale is c. Consumer welfare weakly increases in c if and only if

$$\frac{\partial P_1(c)}{\partial c}\mu_1(c) + \frac{\partial P_2(c)}{\partial c}\mu_2(c) \leq -\frac{\partial P_B(c)}{\partial c}\mu_B(c). \tag{1}$$

The direction of the change in consumer welfare in Theorem 3 is necessarily ambiguous: bundling has an ambiguous effect, and the integral of the change in consumer welfare over c must equal the difference between component pricing and mixed bundling.

The change does not have to be monotone in the cost of resale. The sign of the change depends on how many consumers buy the bundle relative to the number buying the individual goods, and the share in each group varies with the cost of resale. The example in Section 3.4 illustrates the non-monotonicity.

Unlike the change in consumer welfare, the change in total welfare only depends

[5]These assumptions hold when $F(v_1, v_2)$ is uniform.

on marginal consumers. Inframarginal consumers make the same purchases when the cost changes and thus do not contribute to the welfare change.

Theorem 4. Total welfare weakly increases in c if and only if

$$\begin{aligned}
0 \leq\ & -\frac{\partial P_1(c)}{\partial c}\int_0^{P_B(c)-P_1(c)} P_1(c)f(P_1(c),v_2)\,dv_2 - \frac{\partial P_2(c)}{\partial c}\int_0^{P_B(c)-P_2(c)} P_2(c)f(v_1,P_2(c))\,dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c}-\frac{\partial P_1(c)}{\partial c}\right)\int_{P_1(c)}^{1} P_B(c)-P_1(c)f(v_1,P_B(c)-P_1(c))\,dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c}-\frac{\partial P_2(c)}{\partial c}\right)\int_{P_2(c)}^{1} P_B(c)-P_2(c)f(P_B(c)-P_2(c),v_2)\,dv_2 - \\
& \frac{\partial P_B(c)}{\partial c}\int_{P_B(c)-P_2(c)}^{P_1(c)} P_B(c)f(v_1,P_B(c)-v_1)\,dv_1.
\end{aligned}$$

Each term in Theorem 4 captures the effect of changing c for a line segment of marginal buyers separating the purchase regions in Figure 1. The first two terms describe the change due to consumers who switch to buying nothing when the prices of individual goods rise. The next two are the changes from consumers who switch from buying one good to the bundle. The last term comes from consumers who switch from buying nothing to buying the bundle.

As with consumer welfare, the change in total welfare is ambiguous because bundling has an ambiguous effect of total welfare.

3.4 An Example

I conclude the discussion of homogeneous costs with an example in which values are distributed uniformly on the unit square, $f(v_1,v_2)=1$. If there were no resale, the optimal prices would be $P=(\frac{2}{3},\frac{2}{3},\frac{4-\sqrt{2}}{3})$, constraining the monopolist whenever $c<\frac{\sqrt{2}}{6}$. I provide closed-form expressions for prices, profit, and welfare as a function of c

in the appendix.

Results are shown in Figure 3. Profit is increasing in the cost c, in line with Theorem 1. Further, the bundle price is decreasing in c and the prices of the individual goods are increasing in c, as assumed in the welfare discussion in the last subsection.

[Figure 3]

Total welfare is monotone in the cost of resale in this example, but the change in consumer welfare is not. It is decreasing at low values of c and reaches its minimum around $c = .08$. As c increases, consumer welfare starts to increase and attains its highest level at $c = \frac{\sqrt{2}}{6}$.

The non-monotonicity is consistent with Theorem 3. The change in consumer welfare is driven by inframarginal consumers, with an increase in c benefiting consumers who buy the bundle and harming consumers who buy individual goods. As c varies, the bundle price falls and the price of individual goods rises. At low values of c, the number of consumers buying individual goods is high relative to the number buying the bundle, so the left side of equation (10) is large and consumer welfare falls. But as c increases and more consumers purchase the bundle, the change in consumer welfare turns positive. The change in the set of consumers purchasing each good as c increases is illustrated for $c = .05$ and $c = .2$ in Figure 4.

[Figure 4]

4 Heterogeneous Costs of Resale

In this section, I show that the monopolist may allow resale in equilibrium when consumers have heterogeneous costs of resale. Heterogeneous costs introduce a new tradeoff to the monopolist's problem and break the relationship between costs of resale and the bundle discount. Still, the model shares some predictions with the homogeneous cost model, such as that resale harms the seller's profit and has an ambiguous effect on total and consumer welfare.

The monopolist allows resale in equilibrium because preventing more consumers from reselling reduces the profit earned from high-cost types. Some consumers may have very low costs, forcing the monopolist to set a minuscule bundle discount if it wants to prevent resale. Instead, it may prefer a higher discount that earns more from other consumers but allows low-cost consumers to resell. The tradeoff is presented in Example 1.

Example 1. Suppose that values are distributed uniformly, half of consumers have $c = .2$, the other half have $c = 0$, and that values and costs are independent. The monopolist has two choices: it can prevent all resale by using component pricing ($P_1 = P_2 = .5$, $P_B = 1$) and earn profit .5, or it can offer a larger bundle discount to those with $c = .2$ and allow consumers with $c = 0$ to resell.

The monopolist's optimal prices are $P_B = .9231$ and $P_1 = P_2 = .6615$. The monopolist earns $\pi = .5218 > .5$ and allows the types with $c = 0$ to resell.

To prevent resale, the monopolist in Example 1 would have to abandon bundling altogether. Instead, it prefers to allow half of consumers to resell so that it can discriminate among the other half. The example also demonstrates that the monopolist's optimal prices do not maximize profit earned from consumers who do not resell. The optimal prices if all consumers have $c = .2$ involve the lower bundle price $P_B = .875$; the monopolist adjusts its prices to earn more from the consumers who resell.

Heterogeneous costs are a plausible explanation for observed resale. Some consumers are keen to pocket a few dollars by purchasing resold goods while others do not bother. Sellers are almost certainly aware that consumers can resell parts of the bundle, demonstrated by efforts to prevent resale, but they may continue to bundle as long as there is relatively little resale.

4.1 Equilibrium

For simplicity, assume that $F(v_1, v_2)$ is symmetric so that the monopolist's optimal prices satisfy $P_1 = P_2$. Because they are equal, I use P_i to refer to P_1 and P_2. Let $c^* \equiv \frac{1}{2}P_B - P_i$ and

$$\mu_B(P_B, c) = \int_{\frac{1}{2}P_B - c}^{1} \int_{\frac{1}{2}P_B - c}^{1} f(v_1, v_2)\, dv_1\, dv_2 - \int_{\frac{1}{2}P_B - c}^{\frac{1}{2}P_B + c} \int_{\frac{1}{2}P_B - c}^{P_B - v_1} f(v_1, v_2)\, dv_2\, dv_1, \tag{2}$$

$$\mu_R(P_B, c) = \int_{\frac{1}{2}P_B + c}^{1} \int_{0}^{\frac{1}{2}P_B - c} f(v_1, v_2)\, dv_2\, dv_1, \tag{3}$$

$$\pi_R(P_B, G, c^*) = \int_{\underline{c}}^{c^*} P_B(\mu_B(P_B, c) + \mu_R(P_B, c))\, dG(c). \tag{4}$$

The critical cost c^* gives the lowest-cost type that does not engage in resale. The function μ_B gives the fraction of consumers with cost $c < c^*$ who purchase the bundle and do not resell when the bundle price is P_B. Similarly, μ_R gives the fraction with $c < c^*$ who purchase the bundle to resell. The function π_R gives profit earned from consumers with $c < c^*$ when c is distributed according to $G(c)$.

Theorem 5. The monopolist's problem is

$$\max_P (1 - G(P_i - \frac{1}{2}P_B))\pi_N(P) + \pi_R(P_B, G, P_i - \frac{1}{2}P_B). \tag{5}$$

Equilibrium resale prices are $P^s = (\frac{P_B}{2}, \frac{P_B}{2})$ and consumers with $c < c^*$ resell.

The monopolist now must consider the profit earned from consumers who do not resell (the first term), consumers who resell (the second), and the number of consumers in the two groups (which depends on c^*). The result is a tradeoff: the monopolist can generally earn more from consumers who do not resell by increasing its bundle discount, but by doing so it increases the number consumers who engage in resale and contribute less to profit.

The tradeoff substantially enriches the monopolist's problem. In Section 3, the effect of resale was to cap its bundle discount. Now, resale forces the monopolist to strike a balance between the profitability of its prices for consumers without resale and the number of consumers who engage in arbitrage at those prices. Only the model with heterogeneous costs describes the problem facing the seller. Moreover, the tradeoff is essential for understanding the market because it explains why the monopolist might allow resale in equilibrium.

4.2 Resale in Equilibrium

Example 1 suggests that resale is possible in equilibrium. I provide a sufficient condition for resale in equilibrium in Theorem 6.

Theorem 6. Assume that the prices maximizing $\pi_N(P)$ subject to $P_i \leq \frac{1}{2}P_B + \underline{c}$ have $P_i = \frac{1}{2}P_B + \underline{c}$. The monopolist allows the lowest-cost type $\underline{c}$ to resell in equilibrium if $2\underline{c}\mu_R(P_B, \underline{c})g(\underline{c}) < \partial\pi_N(P)/\partial P_i$.

Theorem 6 formalizes the intuition from Example 1. By preventing the lowest-cost type from reselling, the monopolist can gain $2\underline{c}$ from each transaction that used to involve resale. But it might not be worthwhile to do so because the monopolist must change its prices and earn less in profit from consumers who did not resell.

The result is illustrative but not necessary. It only applies to the lowest-cost type and there could be a global optimum where resale is tolerated even if there are no local improvements.[6] Nonetheless, the intuition explains why the monopolist might allow many types to resell. Table 1 presents examples in which more than a third of consumers have costs low enough to resell at the monopolist's optimal prices.

Theorem 6 requires a mild assumption, that the constraint $P_i \leq \frac{1}{2}P_B + \underline{c}$ binds. The assumption is needed to use the derivative at $\underline{c}$ and holds whenever the monopolist can increase profit by using a larger bundle discount.

4.3 Comparative Statics

Next I consider how pricing, profit, and welfare change when the distribution of costs of resale changes. Corollary 1 established an ironclad conclusion for pricing in the benchmark model: the monopolist's bundle discount is weakly increasing in the cost of resale, implying more intense price discrimination when resale becomes more costly. Surprisingly, the conclusion is no longer true with a distribution of costs.

Why might the monopolist reduce its discount and bundle less intensely when

[6] The complexity of the seller's problem prevents me from obtaining necessary conditions.

there are stronger barriers to resale? Unlike in Section 3, the monopolist's prices do not necessarily maximize profit from consumers who do not resell for a given bundle discount—they also affect sales to consumers who resell. It is therefore possible that, after resale becomes more costly, the seller might choose a smaller discount that earns more from consumers who do not resell.

Rows 3 and 4 of Table 1 provide an example. The distribution in row 4 first-order stochastically dominates the one in row 3, yet the monopolist sets a smaller discount when costs of resale are higher. The example demonstrates that simple models without resale in equilibrium do not capture the seller's incentives for pricing. However, it remains true that profit increases when resale becomes more difficult.

Lemma 3. At the monopolist's optimal prices P^*, for all $c \leq c^*$,

$$\pi_N(P^*) \geq P_B^* \left(\mu_B(P_B^*, c) + \mu_R(P_B^*, c) \right).$$

Theorem 7. Let P be the optimal prices for the distribution $G(c)$ and let $\tilde{G}$ be a distribution such that $\tilde{g}(c) \leq g(c)$ for all $c \leq c^*$. Then the firm's profit is higher at $\tilde{G}(c)$ than $G(c)$.

Theorem 7 is directly analogous to Corollary 1 in the original model. The assumption on the distributional shift is necessary because profit earned from resellers is not necessarily monotone in c.[7] The distributional shifts between each pair of rows in Table 1 satisfy the assumptions of Theorem 7. As expected, profit increases in each case.

Changes in consumer and total welfare remain ambiguous, but the expressions are far more complex than in Section 3. In the more stylized model, it is possible to

[7]With non-monotonicity, it is possible to have some $\tilde{G}(c) \geq_{FOSD} G(c)$ at which P is less profitable, for instance if profit from resellers is decreasing in c on some interval and $\tilde{G}(c)$ shifts mass upwards only on that interval.

consider the derivative of prices with respect to the cost, reducing welfare changes to the effect of price changes on marginal and inframarginal buyers. Such conclusions are not possible with a distribution of costs because a shift in the distribution does not lead to a smooth change in optimal prices.

Consider consumer welfare, which is now the integral over c of the welfare earned by consumers with each cost of resale c. Let $CW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Consumers with $c \geq c^*$ earn $CW_N(P_i, P_B)$ and all others earn $CW_N(\frac{P_B}{2} + c, P_B)$, making overall consumer welfare with distribution of costs $G(c)$

$$CW(G, P) = (1 - G(c^*))CW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} CW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \quad (6)$$

If the change in optimal prices were continuous, the change in consumer welfare for consumers with each cost c would depend only on inframarginal consumers, as in Theorem 3. Consumers with $c < c^*$ would only be affected by the change in P_B when the monopolist changes prices.

If prices jump when G shifts to G', however, some consumers will switch purchases and earn surplus. The overall change in consumer welfare includes both the inframarginal consumers and the switchers, adding the changes in surplus for each cost c weighted by the change in the number of consumers with that cost across the distributions. As before, the change is ambiguous: Table 1 contains examples in which consumer welfare moves in each direction as the distribution of costs shifts upwards.

The story is similar for total welfare. Let $TW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Total welfare $TW(G,P)$ for the distribution G is

$$TW(G,P) = (1 - G(c^*))TW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} TW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \quad (7)$$

Without a continuous change in prices when G shifts to G', the change in total welfare cannot be distilled to the values of marginal buyers. But the main insight of Theorem 4 applies because, for consumers with cost c, the change in total welfare is driven by consumers who change their purchase decisions at the new prices. The aggregate change is the sum of those changes, weighted by the change in the number of consumers with type c between the distributions. The change remains ambiguous, as demonstrated by the comparisons in Table 1.

4.4 Examples

To illustrate equilibrium and the comparative statics, I simulate the market for various distributions of costs of resale when values are distributed uniformly. The examples are paired, with resale becoming more costly from the first to the second distribution. The second distribution always dominates the first in the sense of first-order stochastic dominance; the changes also satisfy the stronger criterion in Theorem 7.

The examples use three types of distributions of costs: normal distributions, uniform distributions on $[0, .23]$ (ending at approximately the optimal bundle discount without resale), and a split uniform distribution. The split uniform distribution $SplitUnif(x, y)$ spreads probability y evenly on the interval $[0, x]$ and probability $1-y$ evenly on $[x, .23]$.

The split uniform is used in the third, fifth, and sixth rows. For example, in the third row, it spreads .3 of the probability between 0 and .02 and the rest between .02 and .23.

[Table 1]

In each shift, profit increases after resale becomes more difficult. In the shift from row 3 to row 4, the optimal bundle discount falls. Consumer and total welfare decline in the first three comparisons but increase in the last.

5 Conclusion

The goal of this paper has been to explain why sellers practice mixed bundling when consumers can resell. I showed that bundling and resale are possible in equilibrium when consumers have heterogeneous costs of participating in the resale market. The equilibrium with resale materially changes the monopolist's problem, introducing a tradeoff between the number of consumers who resell and the profit earned from consumers who do not. The depth of the monopolist's problem is not present the equilibrium of a benchmark model with a homogeneous cost of resale and no resale in equilibrium.

Both the full model and the benchmark shed light on the effect of resale on bundling, which had not yet been explored formally despite the number of bundles that are resold. The results establish that the ability to resell is an important determinant of the returns to bundling, and that resale has an ambiguous effect on consumer and total welfare in markets where the seller bundles.

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A Tables and Figures

	P_i	P_B	c^*	π	π_N	Consumer Welfare	Total Welfare
$N(.1,.05)$	0.5476	0.9431	0.0761	0.5210	0.5297	0.3354	0.8564
$N(.14,.05)$	0.5615	0.9271	0.0980	0.5282	0.5354	0.3072	0.8354
$SplitUnif(.02,.3)$	0.5696	0.9481	0.0956	0.5156	0.5345	0.4129	0.9285
$Unif(0,.23)$	0.5639	0.9374	0.0952	0.5206	0.5346	0.3675	0.8881
$SplitUnif(.1,.5)$	0.5569	0.9433	0.0853	0.5187	0.5321	0.3723	0.8910
$SplitUnif(.15,.5)$	0.5785	0.9258	0.1156	0.5245	0.5390	0.3569	0.8814
$N(.08,.001)$	0.5459	0.9372	0.0773	0.5300	0.5300	0.2504	0.7804
$N(.2,.001)$	0.6347	0.8764	0.1965	0.5483	0.5483	0.2525	0.8008

Table 1: Examples of equilibrium with a distribution of costs of resale on $[0,.23]$ when values are uniformly distributed. The distribution SplitUnif has mass uniformly distributed on either side of a tilt point. The tilt point is the first argument and the amount of mass to the left is the second.

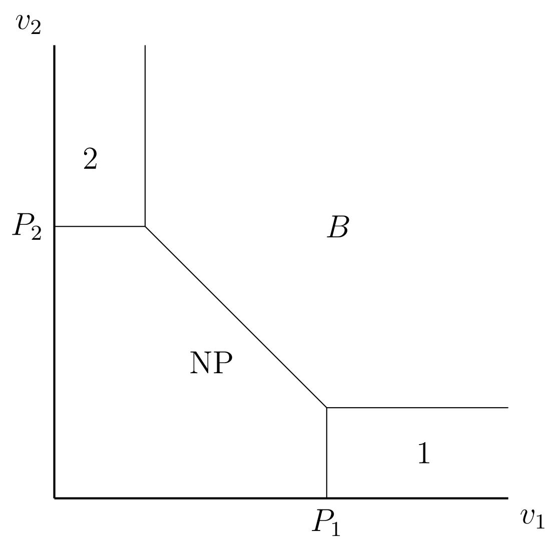


Figure 1: Allocations when there is no resale. Consumers in the B regions purchase the bundle, those in the 1 and 2 regions purchase only good 1 or good 2, and those in the NP region make no purchase.

B Uniform Example Expressions

Prices are

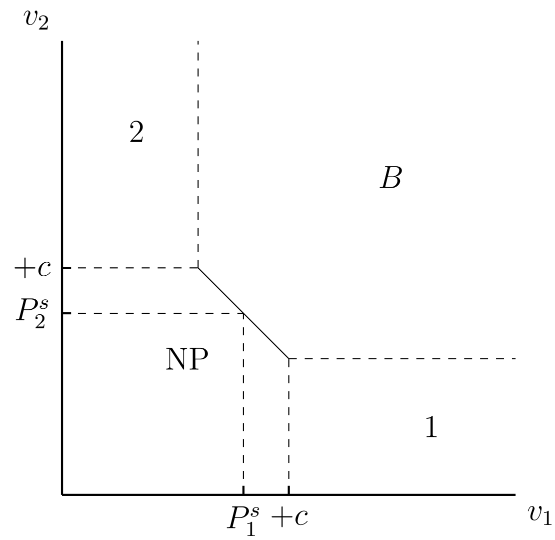


Figure 2: Pure bundling allocations when the resale price is P^s.

$$
\begin{aligned}
P_1(c) = P_2(c) &= \frac{1+4c+6c^2}{2(1+3c)} \\
P_B(c) = P_1(c) + P_2(c) - 2c &= \frac{1+2c}{1+3c}.
\end{aligned}
\tag{8}
$$

The expressions for prices lead to closed-forms for profit, total welfare, and consumer welfare,

$$
\begin{aligned}
TW(c) &= \frac{1}{4(1+4c)^2}\left(3 + 20c + 32c^2 - \frac{32}{3}c^3 - 4c^4 + 48c^5\right) \\
\Pi(c) &= \frac{1}{2(1+3c)}\left(1 + 4c - 8c^3 + 12c^4\right) \\
CW(c) &= TW(c) - \Pi(c).
\end{aligned}
\tag{9}
$$

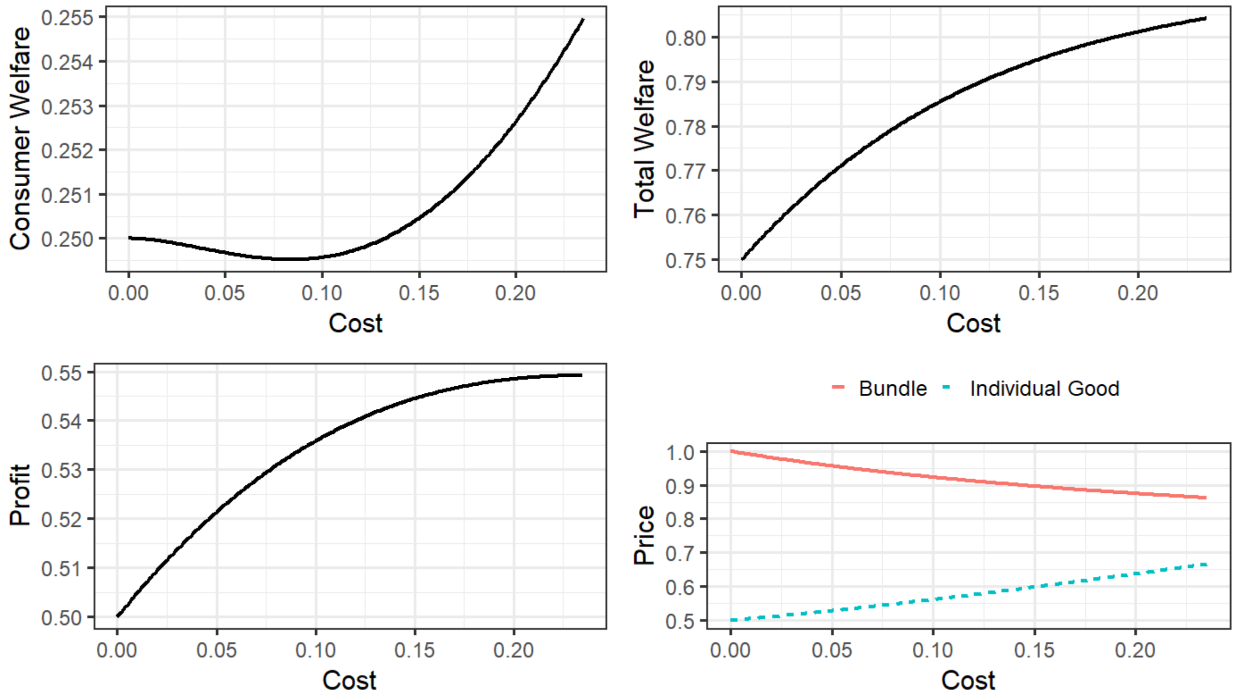
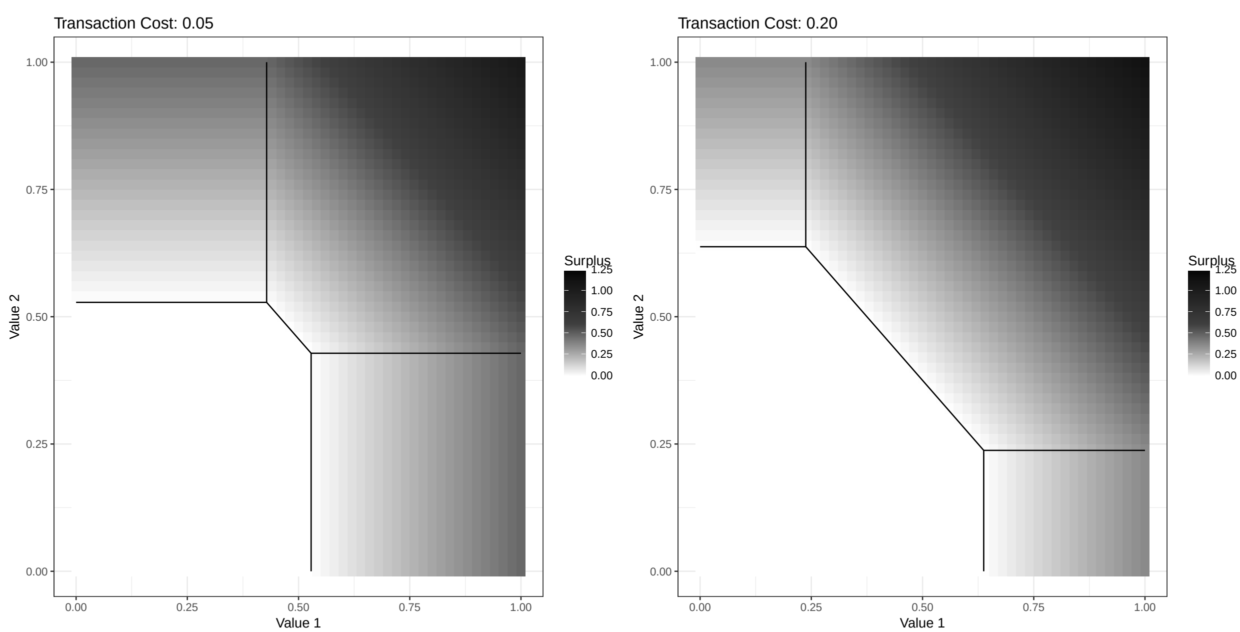


Figure 3: Consumer welfare, total welfare, profit, and prices when F is uniform.



(a) $c = .05$

(b) $c = .2$

Figure 4: Consumer welfare for $c = .05$ (left panel) and $c = .2$ (right panel) when F is uniform. The solid lines separate the regions purchasing each good.

C Proofs for Section 3 (Homogeneous Costs of Resale)

Lemma 1. In any equilibrium with resale market transactions, $P_1^s + P_2^s = P_B$.

Proof. Assume that P^s is an equilibrium price vector and that good 2 is resold in the secondary market. Resellers of 2 buy the bundle to resell good 2, effectively paying $P_B - (P_2^s - c)$ for good 1. To be willing to resell, this must be cheaper than buying good 1 in the resale market, $P_B - P_2^s + c \leq P_1^s + c$. The condition is reversed for secondary market buyers of good 2. Satisfying both conditions requires that $P_1^s + P_2^s = P_B$. $\square$

Lemma 2. When the monopolist only sells the bundle ($P_1 = P_2 > 1$), there exists an equilibrium vector of resale market prices $\hat{P}^s$. It is unique if $\max\{c, P_B - 1 + c\} < \min\{1 - c, P_B - c\}$.

Proof. Equilibrium requires the masses of consumers demanding each good in the resale market to be equal. When the monopolist only offers the bundle, the masses $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ of consumers who only want one good are

$$\mu_1(P_B, P_1^s) = \int_{P_1^s + c}^{1} \int_{0}^{P_B - P_1^s - c} f(v_1, v_2)\, dv_1\, dv_2$$

$$\mu_2(P_B, P_1^s) = \int_{0}^{P_1^s - c} \int_{P_B - P_1^s + c}^{1} f(v_1, v_2)\, dv_1\, dv_2.$$

Define $\delta(P_1^s) = \mu_1(P_B, P_1^s) - \mu_2(P_B, P_1^s)$ on $[0, P_B]$. There is some $\hat{P}_1^s$ satisfying $\delta(P_1^s) = 0$ because $\delta(\cdot)$ is a continuous (since F is atomless) decreasing function satisfying $\delta(0) \geq 0$ and $\delta(P_B) \leq 0$.

To be unique, $\delta(\cdot)$ can only cross zero at a single point and so $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ can never simultaneously be zero. Because F has a strictly positive atomless density, this only requires that the regions of integration not simultaneously be empty. For μ_1, this requires $P_1^s + c < 1$ and $0 < P_B - P_1^s - c$, or $P_1^s < \min\{1-c, P_B - c\}$. For μ_2, it requires $0 < P_1^s - c$ and $P_B - P_1^s + c < 1$, or $P_1^s > \max\{P_B - 1 + c, c\}$. Such a P_1^s exists when $\max\{c, P_B - 1 + c\} < \min\{1 - c, P_B - c\}$. □

Theorem 1. Let $P = (P_1, P_2, P_B)$ be a vector of primary market prices. There is resale in equilibrium if and only if $P_1 + P_2 > P_B + 2c$. Resale prices are unique if there are resale transactions.

Proof. ($\Rightarrow$) Without loss of generality, suppose that good 1 is resold. The reseller pays $P_B + c - P_1^s$ for good 2 and the buyer pays $P_1^s + c$ for good 1. Equilibrium requires $P_B + c - P_1^s \leq P_2$ and $P_1^s + c \leq P_1$. One inequality must be strict, implying that $P_B + 2c < P_1 + P_2$.

($\Leftarrow$) Suppose $P_1 + P_2 > P_B + 2c$ and let $(\hat{P}_1^s, \hat{P}_2^s)$ be the vector of pure-bundling resale prices corresponding to P_B. There are two cases. First, $\hat{P}_1^s + c < P_1$ and $\hat{P}_2^s + c < P_2$. All consumers who want one good strictly prefer the secondary market and $\hat{P}^s$ is the unique vector of resale prices. Second, suppose $\hat{P}_1^s + c \geq P_1$ and $\hat{P}_2^s + c < P_2$. Buyers of good 2 strictly prefer the resale market, so $\hat{P}_1^s$ must fall (and $\hat{P}_2^s$ must rise) until either $P_1 = P_1^s + c$ or $P_2 = P_2^s + c$. Since $P_1 + P_2 > P_B + 2c$, it must be that $\hat{P}_1^s + c - P_1 < P_2 - \hat{P}_2^s - c$, so prices adjust until $P_1^s + c = P_1$ and $P_2^s + c < P_2$. The resulting vector $P^s = (P_1 - c, P_B - P_1 + c)$ is an equilibrium: fewer consumers only want good 2 than good 1 (since $P_1^s < \hat{P}_1^s$), and consumers who want good 1 are indifferent between the two markets, making some willing to resell and clear the resale market. The equilibrium is unique. At any higher P_1^s, there would be no supply of good 1 in the resale market. At any lower P_1^s, there would be excess demand for good 1. $\square$

Theorem 2. Let $\pi_N(P)$ be the monopolist's profit when there is no resale. There are no resale transactions in equilibrium. The monopolist's problem is

$$\max_P \pi_N(P_1, P_2, P_B) \text{ subject to } P_1 + P_2 \leq P_B + 2c.$$

Proof. Suppose there is resale in equilibrium at prices P^s. The monopolist can deviate to set $P_1 = P_1^s + c$ and $P_2 = P_2^s + c$. All consumers will receive the same goods as before, but all purchases will be made in the primary market. The monopolist earns $2c$ from a positive measure of consumers, strictly increasing profit. The monopolist thus maximizes profit subject to the condition that there are no resale transactions.

$\square$

Corollary 1. The monopolist's profit and bundle discount are weakly increasing in the cost of resale c.

Proof. The monopolist's optimal prices for cost c are feasible at $c' > c$. $\square$

Theorem 3. Let $\mu_1(c)$, $\mu_2(c)$, and $\mu_B(c)$ denote the masses of consumers buying good 1, good 2, and the bundle when the cost of resale is c. Consumer welfare weakly increases in c if and only if

$$\frac{\partial P_1(c)}{\partial c}\mu_1(c) + \frac{\partial P_2(c)}{\partial c}\mu_2(c) \leq -\frac{\partial P_B(c)}{\partial c}\mu_B(c). \tag{10}$$

Proof. The regions are defined as

$$\mu_1(c) = \int_{P_1(c)}^{1} \int_{0}^{P_B(c)-P_1(c)} f(v_1, v_2)\, dv_2\, dv_1$$

$$\mu_2(c) = \int_{P_2(c)}^{1} \int_{0}^{P_B(c)-P_2(c)} f(v_1, v_2)\, dv_1\, dv_2$$

$$\mu_B(c) = \int_{P_B(c)-P_2(c)}^{1} \int_{P_B(c)-P_1(c)}^{1} f(v_1, v_2)\, dv_2\, dv_1 - \int_{P_B(c)-P_2(c)}^{P_1(c)} \int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} f(v_1, v_2)\, dv_2\, dv_1.$$

Consumer welfare is

$$\begin{aligned}
CW(c) = & \Big\{ \int_{P_1(c)}^{1} \int_{0}^{P_B(c)-P_1(c)} (v_1 - P_1(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\} + \\
& \Big\{ \int_{P_2(c)}^{1} \int_{0}^{P_B(c)-P_2(c)} (v_2 - P_2(c)) f(v_1, v_2)\, dv_1\, dv_2 \Big\} + \\
& \Big\{ \int_{P_B(c)-P_2(c)}^{1} \int_{P_B(c)-P_1(c)}^{1} (v_1 + v_2 - P_B(c)) f(v_1, v_2)\, dv_2\, dv_1 - \\
& \int_{P_B(c)-P_2(c)}^{P_1(c)} \int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} (v_1 + v_2 - P_B(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\}.
\end{aligned} \tag{11}$$

Taking the derivative with respect to c and comparing it to zero yields the result.

$\square$

Theorem 4. Total welfare weakly increases in c if and only if

$$\begin{aligned}
0 \leq & -\frac{\partial P_1(c)}{\partial c} \int_{0}^{P_B(c)-P_1(c)} P_1(c) f(P_1(c), v_2)\, dv_2 - \frac{\partial P_2(c)}{\partial c} \int_{0}^{P_B(c)-P_2(c)} P_2(c) f(v_1, P_2(c))\, dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c} \right) \int_{P_1(c)}^{1} P_B(c) - P_1(c) f(v_1, P_B(c) - P_1(c))\, dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c} \right) \int_{P_2(c)}^{1} P_B(c) - P_2(c) f(P_B(c) - P_2(c), v_2)\, dv_2 - \\
& \frac{\partial P_B(c)}{\partial c} \int_{P_B(c)-P_2(c)}^{P_1(c)} P_B(c) f(v_1, P_B(c) - v_1)\, dv_1.
\end{aligned}$$

Proof. Total welfare is

$$
\begin{aligned}
TW(c) =& \Big\{ \int_{P_1(c)}^{1} \int_{0}^{P_B(c)-P_1(c)} v_1 f(v_1, v_2)\, dv_2\, dv_1 \Big\} + \\
& \Big\{ \int_{P_2(c)}^{1} \int_{0}^{P_B(c)-P_2(c)} v_2 f(v_1, v_2)\, dv_1\, dv_2 \Big\} + \\
& \Big\{ \int_{P_B(c)-P_2(c)}^{1} \int_{P_B(c)-P_1(c)}^{1} v_1 + v_2 f(v_1, v_2)\, dv_2\, dv_1 - \\
& \int_{P_B(c)-P_2(c)}^{P_1(c)} \int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} v_1 + v_2 f(v_1, v_2)\, dv_2\, dv_1 \Big\}.
\end{aligned}
\tag{12}
$$

And so the change in total welfare with respect to c is

$$
\begin{aligned}
\frac{\partial TW(c)}{\partial c} = & -\frac{\partial P_1(c)}{\partial c} \int_{0}^{P_B(c)-P_1(c)} P_1(c) f(P_1(c), v_2)\, dv_2 + \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c} \right) \int_{P_1(c)}^{1} v_1 f(v_1, P_B(c) - P_1(c))\, dv_1 + \\
& -\frac{\partial P_2(c)}{\partial c} \int_{0}^{P_B(c)-P_2(c)} P_2(c) f(v_1, P_2(c))\, dv_1 + \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c} \right) \int_{P_2(c)}^{1} v_2 f(P_B(c) - P_2(c), v_2)\, dv_2 + \\
& -\left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c} \right) \int_{P_1(c)}^{1} v_1 + P_B(c) - P_1(c) f(v_1, P_B(c) - P_1(c))\, dv_1 + \\
& -\left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c} \right) \int_{P_2(c)}^{1} v_2 + P_B(c) - P_2(c) f(P_B(c) - P_2(c), v_2)\, dv_2 + \\
& -\frac{\partial P_B(c)}{\partial c} \int_{P_B(c)-P_2(c)}^{P_1(c)} P_B(c) f(v_1, P_B(c) - v_1)\, dv_1.
\end{aligned}
\tag{13}
$$

□

D Proofs for Section 4 (Heterogeneous Costs of Resale)

Theorem 5. The monopolist's problem is

$$\max_P (1 - G(P_i - \frac{1}{2}P_B))\pi_N(P) + \pi_R(P_B, G, P_i - \frac{1}{2}P_B). \tag{14}$$

Equilibrium resale prices are $P^s = (\frac{P_B}{2}, \frac{P_B}{2})$ and consumers with $c < c^*$ resell.

Proof. Lemma 1 applies with heterogeneity in c because its proof applies for each type c. By the symmetry of $F(\cdot)$, $P_1 = P_2$ and so equilibrium resale prices are $P^s = (\frac{P_i}{2}, \frac{P_i}{2})$ in any equilibrium with resale transactions.

For prices $P = (P_i, P_B)$, all types with $c \geq P_i - \frac{1}{2}P_B$ find resale too costly. The monopolist therefore earns $\pi_N(P)$ from a fraction $1 - G(P_i - \frac{1}{2}P_B)$ consumers.

All other consumers are willing to share the bundle through resale. Surplus maximization with prices $(\frac{1}{2}P_B + c_k, P_B)$ implies that for types c_k satisfying $c_k < P_i - \frac{1}{2}P_B$, a fraction $\mu_R(P_B, c_k)$ acquire good 1 through resale and a fraction $\mu_B(P_B, c)$ purchase the bundle. For type c_k, the monopolist sells $\mu_R(P_B, c_k)$ bundles to be shared and $\mu_B(P_B, c_k)$ to consumers who do not resell. Integrating over all such types, it earns $\pi_R(P_B, G, P_i - \frac{1}{2}P_B)$ from types who resell. □

Theorem 6. Assume that the prices maximizing $\pi_N(P)$ subject to $P_i \leq \frac{1}{2}P_B + \underline{c}$ have $P_i = \frac{1}{2}P_B + \underline{c}$. The monopolist allows the lowest-cost type $\underline{c}$ to resell in equilibrium if $2\underline{c}\mu_R(P_B, \underline{c})g(\underline{c}) < \partial\pi_N(P)/\partial P_i$.

Proof. Assume that the monopolist prevents consumers with cost $\underline{c}$ from reselling. By assumption, the monopolist's optimal prices have $P_i = \frac{1}{2}P_B + \underline{c}$. I show that the monopolist can increase profit by increasing P_i when $g(\underline{c})$ is small relative to the change in profit. The derivative of profit with respect to P_i at $\underline{c}$ is

$$
\begin{aligned}
\frac{\partial}{\partial P_i}\pi(P') &= g(\underline{c})\left[P_B\left(\mu_B(P_B,\underline{c})+\mu_R(P_B,\underline{c})\right)-\pi_N(P')\right]+(1-G(\underline{c}))\frac{\partial \pi_N(P')}{\partial P_i} \\
&= g(\underline{c})\left(-2\underline{c}\mu_R(P_B,\underline{c})\right)+(1-G(\underline{c}))\frac{\partial \pi_N(P)}{\partial P_i} \\
&= -2\underline{c}g(\underline{c})\mu_R(P_B,\underline{c})+\frac{\partial \pi_N(P)}{\partial P_i}.
\end{aligned}
$$

The second step relies on the fact that $P_B(\mu_B(P_B,\underline{c})+\mu_R(P_B,\underline{c}))-\pi_N(P') = -2\underline{c}\mu_R(P_B,\underline{c})$, which is true because the seller earns $2\underline{c}$ on each formerly resold transaction when type $\underline{c}$ moves to the primary market. The conclusion follows by setting the derivative greater than or equal to zero. $\square$

Lemma 3. At the monopolist's optimal prices P^*, for all $c \leq c^*$,

$$\pi_N(P^*) \geq P_B^*\left(\mu_B(P_B^*,c)+\mu_R(P_B^*,c)\right).$$

Proof. Let $\tilde{c} = \sup_{c\in[\underline{c},c^*]}\{P_B^*\left(\mu_B(P_B^*,c)+\mu_R(P_B^*,c)\right)\}$. Assume for contradiction that $\pi_N(P^*) < P_B^*\left(\mu_B(P_B^*,\tilde{c})+\mu_R(P_B^*,\tilde{c})\right)$. If the monopolist deviated to set $P = (\frac{1}{2}P_B^* + \tilde{c}, P_B^*)$, then it would strictly increase profit from consumers with $c \geq \tilde{c}$ and earn the same amount from consumers with $c < \tilde{c}$. $\square$

Theorem 7. Let P be the optimal prices for the distribution $G(c)$ and let $\tilde{G}$ be a distribution such that $\tilde{g}(c) \leq g(c)$ for all $c \leq c^*$. Then the firm's profit is higher at $\tilde{G}(c)$ than $G(c)$.

Proof. I show that the monopolist earns higher profit at P under $\tilde{G}$ than under G.

Let $\Delta G(c) = G(c) - \tilde{G}(c)$, which satisfies $\Delta G(c) \geq 0$ for $c \leq c^*$. Consider profit from resellers at G and use the fact that $\pi_N(P) \geq P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right)$ for $c \leq c^*$ by Lemma 3.

$$
\begin{aligned}
&\int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) dG(c) \\
&= \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\Delta G(c) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&\leq \int_{\underline{c}}^{c^*} \pi_N(P)\, d\Delta G(c) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= (G(c^*) - \tilde{G}(c^*))\pi_N(P) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c).
\end{aligned}
$$

Let $\pi^G(P)$ give profit for prices P under the distribution of costs G. Substituting into the profit function at G gives

$$
\begin{aligned}
\pi^G(P) &= (1 - G(c^*))\pi_N(P) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) dG(c) \\
&\leq (1 - G(c^*))\pi_N(P) + (G(c^*) - \tilde{G}(c^*))\pi_N(P) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= \pi^{\tilde{G}}(P).
\end{aligned}
$$

□