

5. Evaluate $\int \frac{e^x - 1}{x} dx$ as power series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$$

$$e^x - 1 = x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$\frac{e^x - 1}{x} = \frac{1}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$$

$$\int \frac{e^x - 1}{x} dx = \sum_{n=1}^{\infty} \int \frac{x^{n-1}}{n!} dx$$

we integrate term-by-term

$$= \sum_{n=1}^{\infty} \frac{x^{n-1+1}}{(n-1+1)n!} + C$$

power rule

$$= \left(C + \sum_{n=1}^{\infty} \frac{x^n}{n(n!)} \right)$$