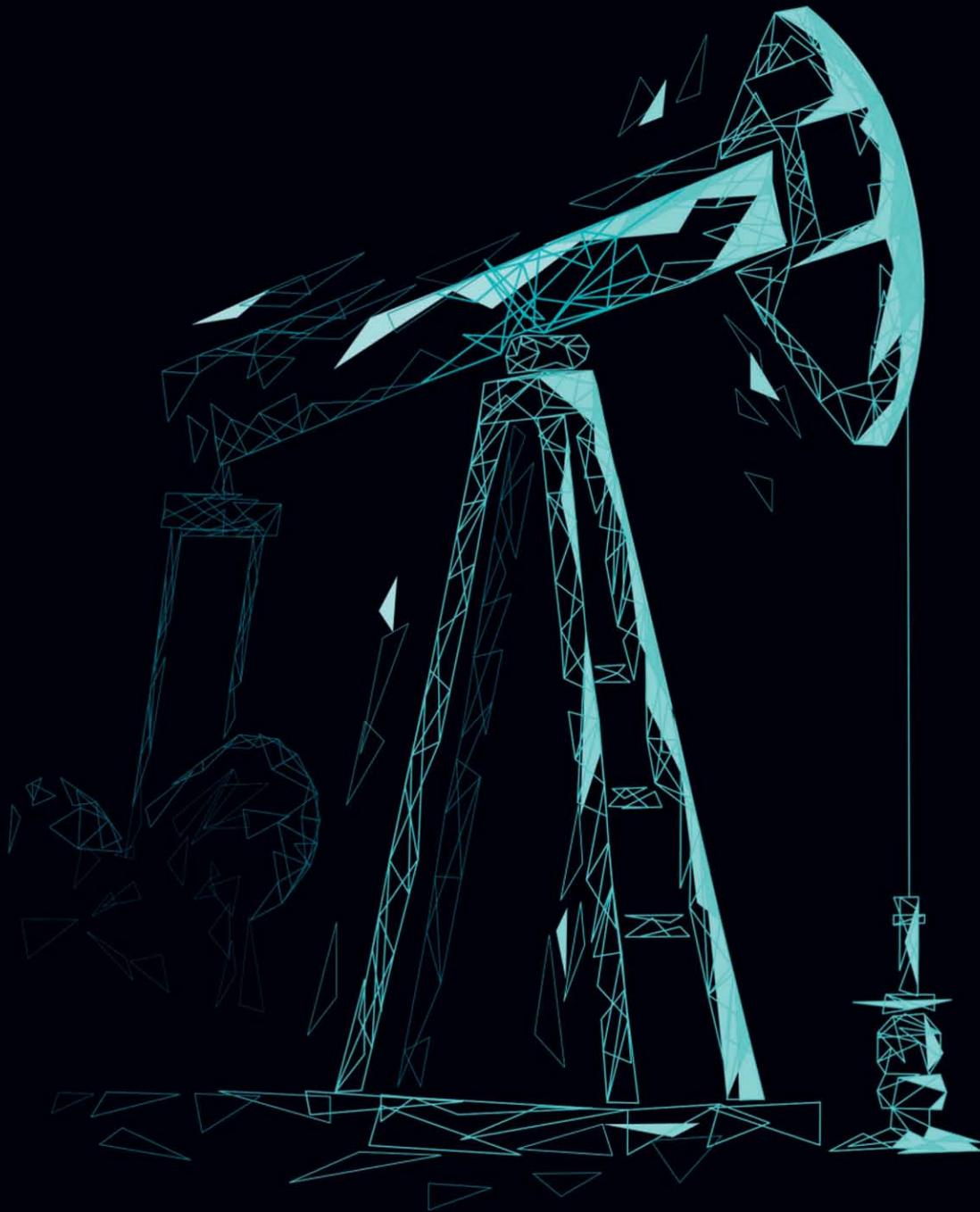


Cenk Temizel, Tayfun Tuna  
Mehmet Melih Oskay, Luigi A. Saputelli

# Formulas and Calculations for Petroleum Engineering



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**Mehmet Melih Oskay** earned his PhD from UT Austin, and he has been in academia and industry as advisors and managerial positions for more than 30 years at several major operators.

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**Luigi A. Saputelli** is a reservoir engineering senior advisor with over 28 years of experience. He worked in various operators and services companies around the world. He is a founding member of the Society of Petroleum Engineers' Real-time Optimization Technical Interest Group and the Petroleum Data-driven Analytics technical section. He is the recipient of the 2015 SPE International Production and Operations Award.

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# Foreword

***Formulas and Calculations for Petroleum Engineering*** unlocks the capability for any petroleum engineering individual, experienced or not, to solve problems and locate quick answers, eliminating nonproductive time spent searching for that right calculation. Enhanced with lab data experiments, practice examples, and a complimentary online software toolbox, the book presents the most convenient and practical reference for all oil and gas phases of a given project. Covering the full spectrum, this reference gives single-point reference to all critical modules, including drilling, production, reservoir engineering, well testing, well logging, enhanced oil recovery, well completion, fracturing, fluid flow, and even petroleum economics.

***ptlbx.com*** provides access to calculations of these formulas.

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## Authors

This book is dedicated to my wife, my love, Saule who has supported me unconditionally in my endeavors and has been an inspiration for me in life with her love, care, and understanding and to my daughter Ada Ayca who has brought joy and happiness to our life and to my parents Yuksel and Rasim Temizel and my brother Efe for their continuous support and love.

Cenk Temizel

I am indebted to my wife Suhendan and to my daughter Ceyda for their unflagging support to finish this book.

Mehmet Melih Oskay

I dedicate this book to my parents Julia and Emilio, who are eternal symbols of unconditional love and true parenthood, from whom I learned what exemplary human values.

Luigi A. Saputelli

## Reviewers

My effort that went into the completion of this book is dedicated to my wife Ezgi who assisted me with her love and patience, to Serkan who made me feel lucky to have an honest brother like him, and also to my beloved parents Fusun and Kaya Canbaz who gave their true love without any expectations and supported me with patience in any circumstances.

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Yildiray Palabiyik

# Chapter 1

# Reservoir engineering formulas and calculations

## Chapter Outline

1.1 API gravity	3	1.31 Effective wellbore radius of a horizontal well—van der Vlis et al. method	16
1.2 Average permeability for linear flow—Layered beds	3	1.32 Effective wellbore radius of a well in presence of uniform-flux fractures	16
1.3 Average permeability for linear flow—Series beds	4	1.33 Effective wellbore radius to calculate slant well productivity—van der Vlis et al.	16
1.4 Average permeability for parallel-layered systems	4	1.34 Estimation of average reservoir pressure—MDH method	17
1.5 Average permeability in radial systems	4	1.35 Formation temperature for a given gradient	17
1.6 Average temperature of a gas column	5	1.36 Fraction of the total solution gas retained in the reservoir as free gas	17
1.7 Calculation of fractional flow curve	5	1.37 Fractional gas recovery below the critical desorption pressure in coal bed methane reservoirs	18
1.8 Capillary number	6	1.38 Free gas in place	18
1.9 Capillary pressure	6	1.39 Gas adsorbed in coal bed methane reservoirs	19
1.10 Characteristic time for linear diffusion in reservoirs	6	1.40 Gas bubble radius	19
1.11 Cole plot	7	1.41 Gas cap ratio	19
1.12 Communication between compartments in tight gas reservoirs	7	1.42 Gas cap shrinkage	20
1.13 Communication factor in a compartment in tight gas reservoirs	7	1.43 Gas drive index in gas reservoirs	20
1.14 Compressibility drive in gas reservoirs	8	1.44 Gas expansion factor	20
1.15 Correction factor—Hammerlindl	8	1.45 Gas expansion term in gas reservoirs	21
1.16 Critical rate for horizontal wells in edge-water drive reservoirs	8	1.46 Gas flow rate into the wellbore	21
1.17 Crossflow index	9	1.47 Gas flow under laminar viscous conditions	22
1.18 Cumulative effective compressibility—Fetkovich	9	1.48 Gas formation volume factor	22
1.19 Cumulative gas production—Tanner's method	10	1.49 Gas hydrate dissociation pressure	22
1.20 Cumulative oil production—Undersaturated oil reservoirs	10	1.50 Gas material balance equation	23
1.21 Deliverability equation for shallow gas reservoirs	10	1.51 Gas produced by gas expansion	23
1.22 Dimensionless pressure—Kamal and Brigham	11	1.52 Gas saturation—Water-drive gas reservoirs	24
1.23 Dimensionless radius of radial flow—Constant-rate production	11	1.53 Gas solubility in coalbed methane reservoirs	24
1.24 Dimensionless time—Myhill and Stegemeier's method	11	1.54 Geertsma's model for porosity/transit-time relationship	25
1.25 Dimensionless time for interference testing in homogeneous reservoirs—Earlougher	12	1.55 Geothermal gradient	25
1.26 Dimensionless vertical well critical rate correlations—Hoyland, Papatzacos, and Skjaeveland	12	1.56 Hagen Poiseuille equation	26
1.27 Dimensionless wellbore storage coefficient of radial flow—Constant-rate production	13	1.57 Hagoort and Hoogstra gas flow in tight reservoirs	26
1.28 Effective compressibility in undersaturated oil reservoirs—Hawkins	13	1.58 Hammerlindl method for gas in place	26
1.29 Effective wellbore radius of a horizontal well—Method 1—Anisotropic reservoirs	13	1.59 High-pressure region gas flow rate	27
1.30 Effective wellbore radius of a horizontal well—Method 1—Isootropic reservoirs	14	1.60 Horizontal well breakthrough time—with gas cap or bottom water	27
	14	1.61 Horizontal well critical rate correlation—Chaperon	28
	14	1.62 Horizontal well critical rate correlations—Efros	28
	15	1.63 Horizontal well critical rate correlations—Giger and Karcher	29

1.64 Horizontal well critical rate correlations—Joshi method for gas coning	29	1.108 Porosity determination—IES and FDC logs	47
1.65 Hydrocarbon pore volume occupied by evolved solution gas	30	1.109 Produced gas-oil ratio	47
1.66 Hydrocarbon pore volume occupied by gas cap	30	1.110 Productivity index for a gas well	48
1.67 Hydrocarbon pore volume occupied by remaining oil	31	1.111 Pseudo-steady state productivity of horizontal wells—Method 1	48
1.68 Hydrostatic pressure	31	1.112 Pseudo-steady state productivity of horizontal wells—Method 2	49
1.69 Incremental cumulative oil production in undersaturated reservoirs	31	1.113 Pseudo-steady state productivity of horizontal wells—Method 3	50
1.70 Ineffective porosity	32	1.114 Pseudo-steady state radial flow equation	50
1.71 Initial gas cap	32	1.115 Relative permeability—Corey exponents	51
1.72 Initial gas in place for water-drive gas reservoirs	32	1.116 Remaining gas in place in coalbed methane reservoirs	51
1.73 Injectivity index	33	1.117 Roach plot for abnormally pressured gas reservoirs	52
1.74 Instantaneous gas-oil ratio	33	1.118 Rock expansion term in abnormally pressured gas reservoirs	52
1.75 Interporosity flow coefficient	34	1.119 Shape factor—Earlougher	52
1.76 Interstitial velocity	34	1.120 Solution gas oil ratio—Beggs-Standing correlation— $p < p_b$	53
1.77 Isothermal compressibility of oil—Vasquez-Beggs correlation— $p > p_b$	34	1.121 Solution gas oil ratio—Standing's correlation	53
1.78 Isothermal compressibility of oil—villena-Lanzi correlation— $p < p_b$	35	1.122 Solution gas water ratio	54
1.79 Isothermal compressibility of water—Osif correlation	35	1.123 Somerton method for formation permeability in coalbed methane reservoirs	54
1.80 Kerns method for gas flow in a fracture	35	1.124 Specific gravity of gas hydrate forming components	54
1.81 Klinkenberg gas effect	36	1.125 Time to reach the semi-steady state for a gas well in a circular or square drainage area	55
1.82 Kozeny equation	36	1.126 Time to the end of infinite-acting period for a well in a circular reservoir	55
1.83 Kozeny-Carman relationship	36	1.127 Torcaso and Wyllie's correlation for relative permeability ratio prediction	55
1.84 Leverett J-function	37	1.128 Total compressibility	56
1.85 Line-source solution for damaged or stimulated wells	37	1.129 Total pore volume compressibility	56
1.86 Low-pressure region gas flow rate for non-circular drainage area	38	1.130 Transmissibility between compartments	57
1.87 Material balance for cumulative water influx—Havlena and Odeh	38	1.131 Transmissibility of a compartment	57
1.88 Maximum height of oil column in cap rock	38	1.132 Transmissivity	57
1.89 Modified Cole plot	39	1.133 Trapped gas volume in water-invaded zones	58
1.90 Modified Kozeny-carman relationship	39	1.134 Two-phase formation volume factor	58
1.91 Normalized saturation	39	1.135 Underground fluid withdrawal—Havlena and Odeh	59
1.92 Oil bubble radius of the drainage area of each well represented by a circle	40	1.136 Vertical well critical rate correlations—Craft and Hawkins method	59
1.93 Oil density—Standing's correlation	40	1.137 Vertical well critical rate correlations—Hoyland, Papatzacos, and Skjaeveland—Isotropic reservoirs	60
1.94 Oil formation volume factor—Standing's correlation	41	1.138 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Simultaneous gas and water coning	60
1.95 Oil formation volume factor—Beggs-standing correlation— $p < p_b$	41	1.139 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Water coning	61
1.41 Oil formation volume factor—Beggs-standing correlation— $p > p_b$	42	1.140 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Gas coning	61
1.42 Oil in place for undersaturated oil reservoirs without fluid injection	42	1.141 Viscosity	62
1.98 Oil in place in saturated oil reservoirs	42	1.142 Viscosity of crude oil through API	62
1.99 Oil lost in migration	42	1.143 Viscosity of dead oil—Standing's correlation	62
1.100 Oil saturation at any depletion state below the bubble point pressure	42	1.144 Viscosity of dead-oil—Egbogah correlation— $p < p_b$	63
1.101 Original gas in place	42	1.145 Viscosity of live oil—Beggs/Robinson correlation	63
1.102 Payne method for intercompartmental flow in tight gas reservoirs	42	1.146 Viscosity of oil—Vasquez/Beggs correlation— $p > p_b$	63
1.103 Performance coefficient for shallow gas reservoirs	42	1.147 Viscosity of water at atmospheric pressure—McCain correlation	64
1.104 Poisson's ratio	42	1.148 Viscosity of water at reservoir pressure—McCain correlation	64
1.105 Pore throat sorting	42		
1.106 Pore volume occupied by injection of gas and water	42		
1.107 Pore volume through squared method in tight gas reservoirs	42		

<b>1.149 Volume of gas adsorbed in coalbed methane reservoirs</b>	<b>64</b>	<b>1.156 Water-drive recovery</b>	<b>68</b>
<b>1.150 Volumetric heat capacity of a reservoir</b>	<b>65</b>	<b>1.157 Water expansion term in gas reservoirs</b>	<b>68</b>
<b>1.151 Water breakthrough correlation in vertical wells—Bournazel and Jeanson</b>	<b>65</b>	<b>1.158 Water formation volume factor—McCain correlation</b>	<b>68</b>
<b>1.152 Water breakthrough correlations in vertical wells—Sobociński and Cornelius</b>	<b>66</b>	<b>1.159 Water influx—Pot aquifer model</b>	<b>69</b>
<b>1.153 Water content of sour gas</b>	<b>66</b>	<b>1.160 Water influx constant for the van Everdingen and Hurst unsteady-state model</b>	<b>69</b>
<b>1.154 Water cut—Stiles</b>	<b>67</b>	<b>1.161 Water two-phase formation volume factor</b>	<b>70</b>
<b>1.155 Water-drive index for gas reservoirs</b>	<b>67</b>	<b>1.162 Waxman and smits model—Clean sands</b>	<b>70</b>
		<b>1.163 Welge extension—Fractional flow</b>	<b>70</b>

## 1.1 API gravity

### Input(s)

$SG_o$ : Specific Gravity of Oil Phase (fraction)

### Output(s)

API: API Gravity (dimensionless)

### Formula(s)

$$API = \frac{141.5}{SG_o} - 131.5$$

Notes:  $SG_o = \frac{\rho_{oil}}{\rho_{water}}$  at 60 F.

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.2 Average permeability for linear flow—Layered beds

### Input(s)

- $k_1$ : Permeability for Layer 1 (mD)
- $k_2$ : Permeability for Layer 2 (mD)
- $k_3$ : Permeability for Layer 3 (mD)
- $A_1$ : Area of Layer 1 ( $\text{ft}^2$ )
- $A_2$ : Area of Layer 2 ( $\text{ft}^2$ )
- $A_3$ : Area of Layer 3 ( $\text{ft}^2$ )

### Output(s)

$k_{avg}$ : Average Permeability in Linear Systems when there is no crossflow between layers (mD)

### Formula(s)

$$k_{avg} = \frac{k_1 * A_1 + k_2 * A_2 + k_3 * A_3}{A_1 + A_2 + A_3}$$

Reference: Ahmed, T. (2006). *Reservoir Engineering Handbook*. Elsevier, Page: 238.

### 1.3 Average permeability for linear flow—Series beds

#### Input(s)

- $k_1$ : Permeability for layer 1 (mD)
- $k_2$ : Permeability for layer 2 (mD)
- $k_3$ : Permeability for layer 3 (mD)
- $L_1$ : Length of layer 1 (ft)
- $L_2$ : Length of layer 2 (ft)
- $L_3$ : Length of layer 3 (ft)

#### Output(s)

- $k_{avg}$ : Average Permeability in Linear Systems Series (mD)

#### Formula(s)

$$k_{avg} = \frac{L_1 + L_2 + L_3}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

Reference: Ahmed, T. (2006). *Reservoir Engineering Handbook*. Elsevier, Page: 240.

### 1.4 Average permeability for parallel-layered systems

#### Input(s)

- $k_1$ : Permeability for Layer 1 (mD)
- $k_2$ : Permeability for Layer 2 (mD)
- $k_3$ : Permeability for Layer 3 (mD)
- $h_1$ : Height of Layer 1 (ft)
- $h_2$ : Height of Layer 2 (ft)
- $h_3$ : Height of Layer 3 (ft)

#### Output(s)

- $k_{avg}$ : Average Permeability for Parallel-layered Systems (mD)

#### Formula(s)

$$k_{avg} = \frac{k_1 * h_1 + k_2 * h_2 + k_3 * h_3}{h_1 + h_2 + h_3}$$

Reference: Ahmed, T. (2006). *Reservoir Engineering Handbook*. Elsevier, Page: 237.

### 1.5 Average permeability in radial systems

#### Input(s)

- $k_a$ : Permeability between  $r_w$  and  $r_a$  (mD)
- $k_e$ : Permeability between  $r_e$  and  $r_a$  (mD)
- $r_e$ : Drainage radius (ft)
- $r_w$ : Well bore radius (ft)
- $r_a$ : Radius lesser than  $r_e$  (ft)

**Output(s)**

$k_{avg}$ : Average Permeability in Radial Systems Series (mD)

**Formula(s)**

$$k_{avg} = \frac{k_a * k_e * \ln\left(\frac{r_e}{r_w}\right)}{k_a * \ln\left(\frac{r_e}{r_a}\right) + k_e * \ln\left(\frac{r_a}{r_w}\right)}$$

Reference: *Applied Reservoir Engineering Vol. 1, Smith, Tracy & Farrar, Equation 7-7.*

**1.6 Average temperature of a gas column****Input(s)**

$T_t$ : Tubing Head Temperature (°R)

$T_b$ : Wellbore Temperature (°R)

**Output(s)**

$T$ : Arithmetic Average Temperature (°R)

**Formula(s)**

$$T = \frac{T_t + T_b}{2}$$

Reference: *Ahmed, T., McKinney, P.D.2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 199.*

**1.7 Calculation of fractional flow curve****Input(s)**

$\mu_w$ : Water Viscosity (cP)

$k_{rw}$ : Relative Permeability to Water (dimensionless)

$k_{ro}$ : Relative Permeability to Oil (dimensionless)

$\mu_o$ : Oil Viscosity (cP)

**Output(s)**

$f_w$ : Fraction of Total Flowing Stream Composed of Water (dimensionless)

**Formula(s)**

$$f_w = \frac{1}{1 + \frac{\mu_w * k_{ro}}{k_{rw} * \mu_o}}$$

Reference: *Craig Jr. F., 2004, the Reservoir Engineering Aspects of Waterflooding, Vol. 3. Richardson, Texas: Monograph Series, SPE, Page: 112.*

## 1.8 Capillary number

### Input(s)

$\mu_w$ : Viscosity of Displacing Fluid (cP)  
 $V$ : Characteristic Velocity (ft/D)  
 $\sigma_{ow}$ : Surface or Interfacial Tension of Oil and Water Phases (dyn/cm)

### Output(s)

$N_c$ : Capillary Number (dimensionless)

### Formula(s)

$$N_c = \frac{\mu_w * V}{\sigma_{ow}}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.9 Capillary pressure

### Input(s)

$\sigma$ : Fluid interfacial Tension (dyn/cm)  
 $\theta$ : Angle of Wettability (degree)  
 $r$ : Radius of Capillary (cm)

### Output(s)

$P_C$ : Capillary Pressure (dyn/cm)

### Formula(s)

$$P_C = \frac{2 * \sigma * \cos(\theta)}{r}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.10 Characteristic time for linear diffusion in reservoirs

### Input(s)

$\Phi$ : Porosity (fraction)  
 $\beta_f$ : Fluid Compressibility (1/psi)  
 $\beta_r$ : Rock Compressibility (1/psi)  
 $\mu$ : Viscosity (cP)  
 $l$ : Characteristic Length Scale of Diffusion (ft)  
 $k$ : Permeability (mD)

### Output(s)

$\tau$ : Time (s)

**Formula(s)**

$$\tau = \frac{(\Phi * \beta_f + \beta_r) * \mu * I^2}{k}$$

Reference: Zoback, M. D. *Reservoir Geomechanics*, Cambridge University Express, UK, Page: 41.

**1.11 Cole plot****Input(s)**

- G: GIP (MSCF)
- $E_g$ : Gas Expansion Term (bbl/MSCF)
- $W_e$ : Water influx (bbl)

**Output(s)**

- F: Underground Water Withdrawal (bbl)

**Formula(s)**

$$F = G * E_g + W_e$$

Reference: Ahmed, T., McKinney, P. D. *Advanced Reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

**1.12 Communication between compartments in tight gas reservoirs****Input(s)**

- G: Gas in Place (MSCF)
- $E_g$ : Gas Expansion Term (bbl/MSCF)
- $W_e$ : Cumulative Water Influx (bbl)

**Output(s)**

- F: Underground Fluid Withdrawal (bbl)

**Formula(s)**

$$F = G * E_g + W_e$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 209.

**1.13 Communication factor in a compartment in tight gas reservoirs****Input(s)**

- K: Permeability (mD)
- A: Area ( $\text{ft}^2$ )
- T: Temperature (R)
- L: Length of Compartment (ft)

**Output(s)**

- C: Communication Factor ( $\text{SCF/d/psi}^2/\text{cP}$ )

### Formula(s)

$$C = \frac{0.111924 * k * A}{T * L}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 235.

## 1.14 Compressibility drive in gas reservoirs

### Input(s)

- $G$ : Gas in place (MSCF)
- $G_P$ : Gas Produced (MSCF)
- $B_g$ : Gas Formation Volume Factor (MSCF/ft<sup>3</sup>)
- $E_f$ : Gas Compressibility Drive (ft<sup>3</sup>/MSCF)

### Output(s)

- CI: Compressibility Index (dimensionless)

### Formula(s)

$$CI = \frac{G * E_f}{B_g * G_P}$$

Reference: Ahmed, T. & McKinney, P. D. *Advanced Reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

## 1.15 Correction factor—Hammerlindl

### Input(s)

- $G$ : Gas in Place (MSCF)
- $G_p$ : Gas Produced (MSCF)
- $B_g$ : Gas Formation Volume Factor (bbl/MSCF)
- $E_{f,w}$ : Rock and Water Expansion Term (bbl/MSCF)

### Output(s)

- CDI: Compressibility Drive Index (dimensionless)

### Formula(s)

$$CDI = \frac{G * E_{f,w}}{G_p * B_g}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 211.

## 1.16 Critical rate for horizontal Wells in edge-water drive reservoirs

### Input(s)

- $e1$ : Constant for C1 Equals +0.023 or - 0.023 (dimensionless)
- $e2$ : Constant for C2 equals +0.0013 or - 0.0013 (dimensionless)
- $e3$ : Constant for C3 equals +0.022 or - 0.022 (dimensionless)
- $e4$ : Constant for C4 equals +0.0013 or - 0.0013 (dimensionless)
- $\Delta\rho$ : Density Difference between water and oil or, oil and gas (gm/cc)

- h: Pay Zone Thickness (ft)  
 L: Length of Well (ft)  
 $x_e$ : Distance between Horizontal Well and Constant Pressure Boundary (ft)  
 $\mu_o$ : Oil Viscosity (cP)  
 $k_h$ : Vertical Permeability (mD)  
 $k_v$ : Horizontal Permeability (mD)

### Output(s)

- $c_1$ : Dimensionless Constant for calculation (dimensionless)  
 $c_2$ : Dimensionless Constant for calculation (dimensionless)  
 $c_3$ : Dimensionless Constant for calculation (dimensionless)  
 $c_4$ : Dimensionless Constant for calculation (dimensionless)  
 $q_c$ : Dimensionless Critical Rate per Unit length (STB/day/ft)  
 $q_o$ : Critical Rate (STB/day)  
 $z_c$ : Critical Height Representing the Difference between the Apex of the Gas/Water Crest from the Well Elevation (ft)

### Formula(s)

$$c_1 = 1.4426 + e1$$

$$c_2 = -0.9439 + e2$$

$$c_3 = 0.4812 + e3$$

$$c_4 = -0.9534 + e4$$

$$q_c = c_1 * \left( \frac{x_e}{h * \left( \frac{k_h}{k_v} \right)^{0.5}} \right)^{c_2}$$

$$q_o = (4.888 * 10^{-4}) * \Delta_p * h * (k_h * k_v)^{0.5} * L * \frac{q_c}{\mu_o}$$

$$z_c = c_3 * h * \left( \frac{x_e}{h * \left( \frac{k_h}{k_v} \right)^{0.5}} \right)^{c_4}$$

Reference: Joshi, S.D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 309, 310.

## 1.17 Crossflow index

### Input(s)

- $N_{pcf}$ : Oil Recovery from Layered System with Crossflow (STB)  
 $N_{pncf}$ : Oil Recovery from Stratified System with No Crossflow (STB)  
 $N_{pu}$ : Oil Recovery from Uniform System with Average Permeability (STB)

### Output(s)

- $CI$ : Crossflow Index (dimensionless)

### Formula(s)

$$CI = \frac{N_{pcf} - N_{pncf}}{N_{pu} - N_{pncf}}$$

Reference: Willhite, G.P. 1986. *Waterflooding*, Vol. 3. Richardson, Texas: Textbook Series, SPE, Chapter: 2, Page: 166.

## 1.18 Cumulative effective compressibility—Fetkovich

### Input(s)

- $S_{wi}$ : Initial Water Saturation (fraction)
- $\bar{c}_w$ : Cumulative Total Water Compressibility (1/psi)
- $M$ : Dimensionless Volume Ratio (dimensionless)
- $\bar{c}_f$ : Total PV (Formation) Compressibility ( $\text{psi}^{-1}$ )

### Output(s)

- $\bar{c}_e$ : Effective Compressibility (1/psi)

### Formula(s)

$$\bar{c}_e = \frac{S_{wi} * \bar{c}_w + M * (\bar{c}_f + \bar{c}_w) + \bar{c}_f}{1 - S_{wi}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 215,216.

## 1.19 Cumulative gas production—Tarners's method

### Input(s)

- $N$ : Initial Oil-in Place (STB)
- $R_s$ : Gas Solubility (SCF/STB)
- $R_{si}$ : Initial Gas Solubility (SCF/STB)
- $B_o$ : Oil Formation Volume Factor at the Assumed Reservoir Pressure (bbl/STB)
- $B_{oi}$ : Oil Formation Volume Factor at Initial Reservoir Pressure (bbl/STB)
- $B_g$ : Gas Formation Volume Factor at the Assumed Reservoir Pressure (bbl/SCF)
- $N_p$ : Cumulative Oil Production (STB)

### Output(s)

- $G_p$ : Cumulative Gas Production (SCF)

### Formula(s)

$$G_p = N * \left[ (R_{si} - R_s) - \left( \frac{B_{oi} - B_o}{B_g} \right) \right] - N_p * \left[ \frac{B_o}{B_g} - R_s \right]$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 5, Page: 340.

## 1.20 Cumulative oil production—Undersaturated oil reservoirs

### Input(s)

- $N$ : Initial Oil-in Place (STB)
- $c_e$ : Effective Compressibility (1/psi)
- $B_o$ : Oil Formation Volume Factor at the Assumed Reservoir Pressure (bbl/STB)
- $B_{oi}$ : Oil Formation Volume Factor at Initial Reservoir Pressure (bbl/STB)
- $\Delta P$ : Pressure Differential (psi)

### Output(s)

- $N_p$ : Cumulative Oil Production (STB)

### Formula(s)

$$N_p = N * c_e * \left( \frac{B_o}{B_{oi}} \right) * \Delta P$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 5, Page: 333.

## 1.21 Deliverability equation for shallow gas reservoirs

### Input(s)

- $k$ : Permeability (mD)
- $h$ : Thickness (ft)
- $T$ : Temperature ( $^{\circ}$ R)
- $\mu$ : Viscosity (cP)
- $z$ : Compressibility Factor (dimensionless)
- $r_e$ : Radius of Drainage Area (ft)
- $r_w$ : Wellbore Radius (ft)

### Output(s)

- $C$ : Performance Coefficient (dimensionless)

### Formula(s)

$$C = \frac{k * h}{1422 * T * \mu_g * Z * \left( \ln\left(\frac{r_e}{r_w}\right) - 0.5 \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 287.

## 1.22 Dimensionless pressure—Kamal and Brigham

### Input(s)

- $Q$ : Flow Rate (STB/day)
- $\bar{k}$ : Average Permeability (mD)
- $h$ : Thickness (ft)

## 12 Formulas and calculations for petroleum engineering

B: Formation Volume Factor (bbl/STB)

$\mu$ : Viscosity (cP)

$\Delta P$ : Pressure Difference (psi)

### Output(s)

$\Delta P_d$ : Dimensionless Pressure (dimensionless)

### Formula(s)

$$\Delta P_d = \frac{\bar{k} * h * \Delta P}{141.2 * Q * \mu * B}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 125.

## 1.23 Dimensionless radius of radial flow—Constant-rate production

### Input(s)

r: Effective Radius/Reservoir Radius (ft)

$r_w$ : Wellbore Radius (ft)

### Output(s)

$r_d$ : Dimensionless Radius (dimensionless)

### Formula(s)

$$r_d = \frac{r}{r_w}$$

Reference: Lee, J., Rollins, J.B., & Spivey, J.P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 8.

## 1.24 Dimensionless time—Myhill and Stegemeier's method

### Input(s)

$M_s$ : Volumetric Heat Capacity of Steam (btu/ft<sup>3</sup> K)

$M_R$ : Volumetric Heat Capacity of the Reservoir (btu/ft<sup>3</sup> K)

$\alpha_s$ : Overburden Heat Transfer Coefficient (ft<sup>2</sup>/d)

$h_t$ : Thickness of Column (ft)

t: Time (day)

### Output(s)

$t_D$ : Dimensionless Time (dimensionless)

### Formula(s)

$$t_D = 4 * \left( \frac{M_s}{M_R} \right)^2 * \left( \frac{\alpha_s}{h_t^2} \right) * t$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 5, Page: 44.

## 1.25 Dimensionless time for interference testing in homogeneous reservoirs—Earlougher

### Input(s)

- k: Permeability (mD)
- $\phi$ : Porosity (fraction)
- t: Time (h)
- k: Overall Production (mD)
- $\mu$ : Viscosity (cP)
- $c_t$ : Total Compressibility (1/psi)
- $r_w$ : Wellbore Radius (ft)

### Output(s)

- $t_D$ : Dimensionless Time (dimensionless)

### Formula(s)

$$t_D = \frac{0.0002637 * k * t}{\phi * c_t * \mu * (r_w^2)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 117.

## 1.26 Dimensionless vertical well critical rate correlations—Hoyland, Papatzacos, and Skjaeveland

### Input(s)

- h: Oil Column Thickness (ft)
- $k_h$ : Effective Oil Permeability (mD)
- $\rho_w$ : Water Density (g/cc)
- $\mu_o$ : Oil Viscosity (cP)
- $\rho_o$ : Oil Density (g/cc)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $q_o$ : Critical Oil Rate (STB/day)

### Output(s)

- $Q_{oD}$ : Dimensionless Critical Rate (dimensionless)

### Formula(s)

$$Q_{oD} = 651.4 * \mu_o * B_o * \frac{q_o}{h^2 * (\rho_w - \rho_o) * k_h}$$

Reference: *Reservoir Engineering Handbook*, Fourth Edition, Ahmed, Page: 607.

## 1.27 Dimensionless wellbore storage coefficient of radial flow—Constant-rate production

### Input(s)

- h: Reservoir Thickness (ft)
- C: Wellbore Storage Coefficient (STB/psi)

## 14 Formulas and calculations for petroleum engineering

- $\emptyset$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- $r_w$ : Wellbore Radius (ft)

### Output(s)

- $C_d$ : Dimensionless Wellbore-Storage Coefficient (dimensionless)

### Formula(s)

$$C_d = \frac{0.8936 * C}{\emptyset * c_t * h * r_w^2}$$

Reference: *Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineer, Page: 8.*

## 1.28 Effective compressibility in undersaturated oil reservoirs—Hawkins

### Input(s)

- $S_{oi}$ : Initial Oil Saturation (fraction)
- $S_{wi}$ : Initial Water Saturation (fraction)
- $c_o$ : Oil Compressibility (1/psi)
- $c_w$ : Water Compressibility (1/psi)
- $c_f$ : Formation Compressibility (1/psi)

### Output(s)

- $c_e$ : Effective Compressibility (1/psi)

### Formula(s)

$$c_e = \frac{S_{oi} * c_o + S_{wi} * c_w + c_f}{1 - S_{wi}}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 5, Page: 334.*

## 1.29 Effective wellbore radius of a horizontal well—Method 1—Anisotropic reservoirs

### Input(s)

- $L$ : Horizontal Well Length (ft)
- $h$ : Pay Zone Thickness (ft)
- $r_w$ : Wellbore Radius (ft)
- $k_h$ : Horizontal Permeability (mD)
- $k_v$ : Vertical Permeability (mD)
- $A$ : Drainage Area (acre)

### Output(s)

- $r_{eh}$ : Effective Drainage Radius (ft)
- $a$ : Horizontal wellbore variable from Joshi (dimensionless)
- $\beta$ : Permeability Ratio constant (dimensionless)
- $r_{wd}$ : Effective Wellbore Radius (ft)

**Formula(s)**

$$\begin{aligned}
 reh &= \sqrt{A * \frac{43560}{3.14}} \\
 a &= \left( \frac{L}{2} \right) * \sqrt{0.5 + \sqrt{0.25 + 2 * \left( \frac{reh}{L} \right)^4}} \\
 \beta &= \sqrt{\frac{kh}{kv}} \\
 rwd &= reh * \frac{\frac{L}{2}}{a * \left( \left( 1 + \sqrt{1 - \left( \frac{L}{2 * a} \right)^2} \right) * \left( h * \frac{\beta}{2 * rw} \right)^{\frac{h}{L}} \right)}
 \end{aligned}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 90.

### 1.30 Effective wellbore radius of a horizontal well—Method 1—Isotropic reservoirs

**Input(s)**

- L: Horizontal Well Length (ft)
- h: Pay Zone Thickness (ft)
- $r_w$ : Wellbore Radius (ft)
- $k_h$ : Horizontal Permeability (mD)
- $k_v$ : Vertical Permeability (mD)
- A: Drainage Area (acre)

**Output(s)**

- $r_{eh}$ : Effective Drainage Radius (ft)
- a: Horizontal wellbore variable from Joshi (dimensionless)
- $r_{wd}$ : Effective Wellbore Radius (ft)

**Formula(s)**

$$\begin{aligned}
 reh &= \sqrt{A * \frac{43560}{3.14}} \\
 a &= \left( \frac{L}{2} \right) * \sqrt{0.5 + \sqrt{0.25 + 2 * \left( \frac{reh}{L} \right)^4}} \\
 rwd &= reh * \frac{\frac{L}{2}}{a * \left( \left( 1 + \sqrt{1 - \left( \frac{L}{2 * a} \right)^2} \right) * \left( \frac{h}{2 * rw} \right)^{\frac{h}{L}} \right)}
 \end{aligned}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 90.

### 1.31 Effective wellbore radius of a horizontal well—van der Vlis et al. method

#### Input(s)

- h: Pay Zone Thickness (ft)
- $r_w$ : Wellbore Radius (ft)
- $\alpha$ : Slant Angle (degrees)

#### Output(s)

- L: Length of Slant Wellbore (ft)
- $r_w$ : Effective Wellbore Radius (ft)

#### Formula(s)

$$L = \frac{h}{\cos(\alpha)}$$

$$r_w = \frac{L}{4} * \left[ 0.454 * \sin \left( 360 * \left( \frac{r_w}{h} \right) \right) \right]^{\frac{h}{L}}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 3, Page: 96.

### 1.32 Effective wellbore radius of a well in presence of uniform-flux fractures

#### Input(s)

- $x_f$ : Fracture Half Length (ft)
- e: Logarithmic Constant = 2.718 (dimensionless)

#### Output(s)

- $r_w$ : Effective Wellbore Radius (ft)

#### Formula(s)

$$r_w = \frac{x_f}{e}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 5, Page: 135.

### 1.33 Effective wellbore radius to calculate slant well productivity—van der Vlis et al.

#### Input(s)

- h: Pay Zone Thickness (ft)
- $r_w$ : Wellbore Radius (ft)
- $\alpha$ : Slant Angle (degrees)

#### Output(s)

- L: Length of Slant Wellbore (ft)
- $r_w$ : Effective Wellbore Radius (ft)

**Formula(s)**

$$L = \frac{h}{\cos(\alpha)}$$

$$r_w = \frac{L}{4} * \left[ 0.454 * \sin \left( 360 * \left( \frac{r_w}{h} \right) \right) \right]^{\frac{h}{L}}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 3, Page: 96.

**1.34 Estimation of average reservoir pressure—MDH method****Input(s)**

- $p_{ws}$ : Shut-In Pressure (psi)  
 $p_{DMDH}$ : MDH Pressure (dimensionless)  
 $m$ : Semi-log Straight Line of the MDH Plot (psi/cycle)

**Output(s)**

- $P_r$ : Average Reservoir Pressure (psi)

**Formula(s)**

$$P_r = p_{ws} + m * \frac{p_{DMDH}}{1.1513}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 59.

**1.35 Formation temperature for a given gradient****Input(s)**

- $T_s$ : Temperature Near Surface (degree °F)  
 $D$ : Total Depth (ft)  
 $g_G$ : Geothermal Gradient (degree °F/100 ft)

**Output(s)**

- $T_f$ : Formation Temperature (degree °F)

**Formula(s)**

$$T_f = T_s + g_G * \left( \frac{D}{100} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 4, Page: 75.

**1.36 Fraction of the total solution gas retained in the reservoir as free gas****Input(s)**

- $N$ : Oil in Place (STB)  
 $R_p$ : Produced Gas-Oil Ratio (SCF/STB)  
 $N_p$ : Cumulative Oil Production (STB)  
 $R_{si}$ : Initial Gas Solubility (SCF/STB)  
 $R_s$ : Gas Solubility (SCF/STB)

### Output(s)

$\alpha_g$ : Retained Gas Volume of the Total Gas (fraction)

### Formula(s)

$$\alpha_g = 1 - \left( \frac{N_p * R_p}{N * R_{si} - (N - N_p) * R_s} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 314.

## 1.37 Fractional gas recovery below the critical desorption pressure in coal bed methane reservoirs

### Input(s)

$V_m$ : Langmuir (constant)  
 $G_c$ : Gas Content at Critical Desorption Pressure (SCF/ton)  
 $b$ : Langmuir (constant)  
 $P$ : Pressure of Reservoir (psi)  
 $a$ : Recovery Exponent (dimensionless)

### Output(s)

RF: Recovery Factor (fraction)

### Formula(s)

$$RF = 1 - \left( \left( \frac{V_m}{G_c} \right) * \left( \frac{b * P}{1 + b * P} \right) \right)^a$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 223.

## 1.38 Free gas in place

### Input(s)

$A$ : Drainage Area (acres)  
 $h$ : Thickness (ft)  
 $\emptyset$ : Porosity (fraction)  
 $S_{wi}$ : Initial Water Saturation (fraction)  
 $E_{gi}$ : Gas Expansion Factor at Initial Reservoir Pressure (SCF/bbl)

### Output(s)

$G_f$ : Original Free Gas-in-Place (SCF)

### Formula(s)

$$G_f = 7758 * A * h * \emptyset * (1 - S_{wi}) * E_{gi}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 227.

## 1.39 Gas adsorbed in coal bed methane reservoirs

### Input(s)

- A: Area (acres)
- h: Height (ft)
- $\rho_b$ : Density (g/cc)
- V: Adsorption Gas (SCF/ton)

### Output(s)

- $G_a$ : Gas Adsorbed (SCF)

### Formula(s)

$$G_a = 1359.7 * A * h * \rho_b * V$$

Reference: Ahmed, T. & McKinney, P.D. *Advanced Reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

## 1.40 Gas bubble radius

### Input(s)

- A: Drainage Area (acres)
- h: Thickness (ft)
- $\sigma_B$ : Bulk (g/cc)
- $G_c$ : Gas Content (SCF/ton)

### Output(s)

- G: Gas-in-Place (SCF)

### Formula(s)

$$G = 1359.7 * A * h * \sigma_B * G_c$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 227.

## 1.41 Gas cap ratio

### Input(s)

- G: Initial Gas Cap Volume (SCF)
- $B_{gi}$ : Initial Gas Formation Volume Factor (bbl/SCF)
- N: Initial Oil in Place (STB)
- $B_{oi}$ : Initial Oil Formation Volume Factor (bbl/STB)

### Output(s)

- m: Gas Cap Ratio (dimensionless)

### Formula(s)

$$m = \frac{G * B_{gi}}{N * B_{oi}}$$

Reference: Ahmed,T., McKinney,P. *Advanced Reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015, Chapter: 4, Page: 317.

## 1.42 Gas cap shrinkage

### Input(s)

- $G_{pc}$ : Cumulative Gas Production from Gas Cap (SCF)
- $B_g$ : Gas Formation Volume Factor (bbl/SCF)
- $m$ : Gas Cap Ratio (fraction)
- $N$ : Oil in Place (STB)
- $B_{oi}$ : Initial Oil Formation Volume Factor (bbl/STB)
- $B_{gi}$ : Initial Gas Formation Volume Factor (bbl/SCF)

### Output(s)

- $G_s$ : Gas Cap Shrinkage (bbl)

### Formula(s)

$$G_s = G_{pc} * B_g - m * N * B_{oi} * \left( \left( \frac{B_g}{B_{gi}} \right) - 1 \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 5, Page: 333.

## 1.43 Gas drive index in gas reservoirs

### Input(s)

- $G$ : Gas Initially in Place (SCF)
- $G_p$ : Cumulative Gas Production (SCF)
- $B_{gi}$ : Initial Gas Formation Volume Factor ( $\text{ft}^3/\text{SCF}$ )
- $B_g$ : Gas Formation Volume Factor ( $\text{ft}^3/\text{SCF}$ )

### Output(s)

- $GDI$ : Gas (dimensionless)

### Formula(s)

$$GDI = \left( \frac{G}{G_p} \right) * \left( 1 - \frac{B_{gi}}{B_g} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 211.

## 1.44 Gas expansion factor

### Input(s)

- $E_{gi}$ : Initial Gas Expansion Factor (SCF/bbl)
- $A$ : Drainage Area (acres)
- $h$ : Thickness (ft)
- $\emptyset$ : Porosity (fraction)
- $S_{wi}$ : Initial Water Saturation (fraction)
- $G_p$ : Gas Produced (SCF)

**Output(s)**

$E_g$ : Gas Expansion Factor (SCF/bbl)

**Formula(s)**

$$E_g = E_{gi} - \left( \frac{1}{43560 * A * h * \emptyset * (1 - S_{wi})} \right) * G_p$$

Reference: *Ahmed, T., McKinney, P.D.2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 202.*

**1.45 Gas expansion term in gas reservoirs****Input(s)**

$B_g$ : Gas Formation Volume Factor ( $\text{ft}^3/\text{SCF}$ )

$B_{gi}$ : Initial Gas Formation Volume Factor ( $\text{ft}^3/\text{SCF}$ )

**Output(s)**

$E_g$ : Gas Expansion Term ( $\text{ft}^3/\text{SCF}$ )

**Formula(s)**

$$E_g = B_g - B_{gi}$$

Reference: *Ahmed, T., McKinney, P.D.2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 209.*

**1.46 Gas flow rate into the wellbore****Input(s)**

$k$ : Permeability (mD)

$\partial P$ : Pressure Differential (psi)

$L$ : Length of Section Open to Wellbore (ft)

$u$ : Viscosity of intruding Gas (cP)

$R_e$ : Radius of Drainage (ft)

$R_w$ : Radius of Wellbore (ft)

**Output(s)**

$Q$ : Flow Rate (bbl/min)

**Formula(s)**

$$Q = \frac{0.007 * k * (\partial P) * L}{u * \ln\left(\frac{R_e}{R_w}\right) * 1440}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 136.*

## 1.47 Gas flow under laminar viscous conditions

### Input(s)

- k: Permeability (mD)
- h: Thickness (ft)
- $\varphi_r$ : Average Reservoir Real-Gas Pseudo-Pressure (psi)
- $\varphi_{wf}$ : Real-Gas Pseudo-flowing Pressure (psi)
- T: Temperature (R)
- A: Drainage Area ( $\text{ft}^2$ )
- $C_A$ : Shape Factor (dimensionless)
- $r_w$ : Wellbore Radius (ft)
- S: Skin (dimensionless)

### Output(s)

- $Q_g$ : Gas Flow Rate (MSCF/d)

### Formula(s)

$$Q_g = \frac{k * h * (\varphi_r - \varphi_{wf})}{1422 * T * \left( 0.5 * \ln \left( \frac{4 * A}{1.781 * C_A * r_w^2} \right) + S \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 188.

## 1.48 Gas formation volume factor

### Input(s)

- z: Gas Deviation Factor (dimensionless)
- T: Temperature (R)
- P: Pressure (psi)

### Output(s)

- $B_g$ : Gas Formation Volume Factor (bbl/SCF)

### Formula(s)

$$B_g = \frac{0.005035 * z * T}{P}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 15.

## 1.49 Gas hydrate dissociation pressure

### Input(s)

- $\gamma_h$ : Specific Gravity of Hydrate-forming Components (dimensionless)
- $F_m$ : Molar Ratio between the Non-hydrate-forming and Hydrate-forming Components (dimensionless)
- T: Temperature ( $^{\circ}\text{R}$ )

**Output(s)**

$p_h$ : Disassociation Pressure (psi)

**Formula(s)**

$$p_h = 0.1450377 * \exp \left\{ \left[ \frac{2.50744 * 10^{-3}}{(\gamma_h + 0.46852)^3} + F_m * (1.214644 * 10^{-2}) + (-4.676111 * 10^{-4}) * F_m^2 + 0.0720122 \right] * T + \frac{3.6625 * 10^{-4}}{(\gamma_h + (-0.485054))^3} + F_m * (-5.44376) + F_m^2 * (3.89 * 10^{-3}) + (-29.9351) \right\}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 274.

**1.50 Gas material balance equation****Input(s)**

$\frac{P_i}{z_i}$ : Ratio of Pressure to Compressibility Factor at Initial Conditions (psi)

$P_{SC}$ : Pressure at Standard Conditions (psi)

$T_{SC}$ : Temperature at Standard Conditions ( $^{\circ}$ R)

$T$ : Current Temperature ( $^{\circ}$ R)

$V$ : Original Gas Volume ( $ft^3$ )

$G_p$ : Cumulative Gas Production (SCF)

**Output(s)**

$\frac{P}{z}$ : Ratio of Pressure to Compressibility Factor at Current Conditions for  $P/z$  vs  $G_p$  plot (psi)

**Formula(s)**

$$\frac{P}{z} = \left( \frac{P_i}{z_i} \right) - \left( \frac{P_{SC} * T}{T_{SC} * V} \right) * G_p$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 203.

**1.51 Gas produced by gas expansion****Input(s)**

$A$ : Drainage Area (acres)

$h$ : Thickness (ft)

$\phi$ : Porosity (fraction)

$S_{wi}$ : Initial Water Saturation (fraction)

$B_g$ : Gas Formation Volume Factor ( $ft^3/SCF$ )

$B_{gi}$ : Initial Gas Formation Volume Factor ( $ft^3/SCF$ )

**Output(s)**

$G_p$ : Gas Produced (SCF)

### Formula(s)

$$G_p = 43560 * A * h * \emptyset * (1 - S_{wi}) * \left( \frac{1}{B_{gi}} - \frac{1}{B_g} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 202.

## 1.52 Gas saturation—Water-drive gas reservoirs

### Input(s)

- $G$ : Gas Initially in Place (SCF)
- $G_p$ : Cumulative Gas Production (SCF)
- $B_g$ : Volume Factor (bbl/SCF)
- $B_{gi}$ : Initial Gas Formation Factor (bbl/SCF)
- $W_e$ : Cumulative Water Influx (bbl)
- $W_p$ : Cumulative Water Production (STB)
- $B_w$ : Water Formation Volume Factor (rb/STB)
- $S_{wi}$ : Initial Water Saturation (fraction)
- $S_{grw}$ : Residual Gas Saturation to Water Displacement (fraction)

### Output(s)

- $S_g$ : Gas Saturation (fraction)

### Formula(s)

$$S_g = \frac{\left( G - G_p \right) * B_g - \frac{W_e - W_p * B_w * S_{grw}}{1 - S_{wi} - S_{grw}}}{\left( \frac{G * B_{gi}}{1 - S_{wi}} \right) - \left( \frac{W_e - W_p * B_w}{1 - S_{wi} - S_{grw}} \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 208.

## 1.53 Gas solubility in coalbed methane reservoirs

### Input(s)

- $\rho_B$ : Bulk Coal Steam Density (g/cc)
- $\emptyset_m$ : Actual Coalbed Cleat Porosity (fraction)
- $S_{om}$ : Initial Oil Saturation (fraction)
- $V$ : Gas Content (SCF/STB)

### Output(s)

- $R_s$ : Equivalent Gas Solubility (dimensionless)

**Formula(s)**

$$R_s = \left( \frac{0.17525 * \rho_B}{\phi_m * S_{om}} \right) * V$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 222.

**1.54 Geertsma's model for porosity/transit-time relationship****Input(s)**

- $K_b$ : Bulk Modulus Constant for Formation (dimensionless)
- $\sigma_b$ : Bulk Density ( $\text{kg}/\text{m}^3$ )
- $\mu_b$ : Poisson Ratio for Formation (dimensionless)
- $K_{ma}$ : Bulk Modulus Constant for Matrix (dimensionless)
- $\sigma_{ma}$ : Matrix Density ( $\text{kg}/\text{m}^3$ )
- $\mu_{ma}$ : Poisson (dimensionless)

**Output(s)**

- $V_b$ : Acoustic Velocity in Bulk Formation (m/s)
- $V_{ma}$ : Acoustic Velocity in Matrix (m/s)

**Formula(s)**

$$V_b = \left( \left( 3 * \frac{K_b}{\sigma_b} \right) * \left( \frac{1 - \mu_b}{1 + \mu_b} \right) \right)^{0.5}$$

$$V_{ma} = \left( \left( 3 * \frac{K_{ma}}{\sigma_{ma}} \right) * \left( \frac{1 - \mu_{ma}}{1 + \mu_{ma}} \right) \right)^{0.5}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 55.

**1.55 Geothermal gradient****Input(s)**

- $T_{bh}$ : Maximum Recorded Temperature (degree °F)
- $T_s$ : Temperature Near Surface (degree °F)
- $D_{bh}$ : Total Depth of Logged Borehole (ft)

**Output(s)**

- $g_G$ : Geothermal Gradient (degree °F/100 ft)

**Formula(s)**

$$g_G = \left( \frac{T_{bh} - T_s}{D_{bh}} \right) * 100$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 4, Page: 75.

## 1.56 Hagen Poiseuille equation

### Input(s)

$P_o$ : Input Pressure (psi)  
 $P_L$ : Output Pressure (psi)  
 $R$ : Radius (ft)  
 $w$ : Mass rate of Flow (lb/s)  
 $\rho$ : Density (ppg)  
 $L$ : Length (ft)

### Output(s)

$\mu$ : Viscosity (cP)

### Formula(s)

$$\mu = \frac{(P_o - P_L) * \pi * (R^4) * \rho}{8 * w * L}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 51.*

## 1.57 Hagoort and Hoogstra gas flow in tight reservoirs

### Input(s)

$\Gamma$ : Transmissibility between Compartments (dimensionless)  
 $P_1$ : Pressure of Compartment 1 (psi)  
 $P_2$ : Pressure of Compartment 2 (psi)  
 $\mu_{gavg}$ : Average Viscosity (cP)  
 $B_{gavg}$ : Average Gas Formation Factor (MSCF/STB)

### Output(s)

$Q$ : Gas Flow (MSCF/d)

### Formula(s)

$$Q = \frac{\Gamma * (P_1^2 - P_2^2)}{2 * P_1 * \mu_{gavg} * B_{gavg}}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 236.*

## 1.58 Hammerlindl method for gas in place

### Input(s)

$G_{app}$ : Apparent Gas in Place (SCF)  
 $R$ : Ratio of the effective Total System Compressibility to gas Compressibility (dimensionless)

### Output(s)

$G$ : Gas in Place (SCF)

**Formula(s)**

$$G = \frac{G_{app}}{R}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 216.

## 1.59 High-pressure region gas flow rate

**Input(s)**

- k: Permeability (mD)
- h: Thickness (ft)
- $P_r$ : Average Reservoir Pressure (psi)
- $P_{wf}$ : Bottom-hole Flowing Pressure (psi)
- $\mu_{gavg}$ : Average Gas Viscosity (cP)
- $B_{gavg}$ : Average Gas Formation Volume Factor (bbl/SCF)
- $r_e$ : Drainage Radius (ft)
- $r_w$ : Wellbore Radius (ft)
- S: Skin (dimensionless)

**Output(s)**

- $Q_g$ : Gas Flow Rate (MSCF/d)

**Formula(s)**

$$Q_g = \frac{7.08 * (10^{-6}) * k * h * (P_r - P_{wf})}{\mu_{gavg} * B_{gavg} * \left( \ln \left( \frac{r_e}{r_w} \right) - 0.75 + S \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 189.

## 1.60 Horizontal well breakthrough time—With gas cap or bottom water

**Input(s)**

- $\rho_o$ : Oil Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)
- $t_{bt}$ : Breakthrough Time (days)
- $\phi$ : Porosity (fraction)
- h: Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $q_o$ : Flow Rate (cP)
- $B_o$ : Oil Formation Volume Factor (rb/STB)
- $k_h$ : Vertical Permeability (mD)
- $k_v$ : Horizontal Permeability (mD)

**Output(s)**

- $t_{dbt}$ : Breakthrough Time if breakthrough time is given (days)
- $q_d$ : Dimensionless Flow Rate (dimensionless)

**Formula(s)**

$$t_{dbt} = k_v * (\rho_o - \rho_g) * \frac{t_{bt}}{364.72 * h * \phi * \mu_o}$$

$$q_d = 325.86 * \mu_o * q_o * \frac{B_o}{(k_v * k_h)^{0.5} * h * (\rho_o - \rho_g)}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 301.

**1.61 Horizontal well critical rate correlation—Chaperon****Input(s)**

- $\rho_o$ : Oil Density (g/cc)
- $\rho_w$ : Water Density (g/cc)
- $x_A$ : Location of a Constant Pressure Boundary (ft)
- $h$ : Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $F$ :  $F = 5.48$  for  $k_v/k_h = 1$ ,  $F = 4.8$  for  $k_v/k_h = 0.01$  and  $F = 4.16$  for  $k_v/k_h = 0.01$  (mD)
- $k_h$ : Horizontal Permeability (mD)
- $L$ : Horizontal Well Length (m)

**Output(s)**

- $Q_c$ : Critical Oil Rate ( $m^3/h$ )

**Formula(s)**

$$Q_c = (3.486 * 10^{-5}) * \frac{L}{x_A} * h^2 * (\rho_w - \rho_o) * \frac{F}{k_h * \mu_o}$$

Reference: Chaperon, I. 1986. *Theoretical Study of Coning Toward Horizontal and Vertical in Anisotropic Formations: Subcritical and Critical Rates*. SPE ATCE, New Orleans, Louisiana.

**1.62 Horizontal well critical rate correlations—Efros****Input(s)**

- $\rho_o$ : Oil Density (g/cc)
- $\rho_w$ : Water Density (g/cc)
- $y_e$ : Half of Horizontal Well Spacing (ft)
- $h$ : Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_h$ : Horizontal Permeability (mD)
- $L$ : Length of Reservoir (ft)

**Output(s)**

- $q_o$ : Critical Oil Rate (STB/day)

**Formula(s)**

$$q_o = (4.888 * 10^{-4}) * k_h * h^2 * (\rho_w - \rho_o) * \frac{L}{B_o * \mu_o * \left( \left( 2 * y_e + (2 * y_e)^2 + \frac{h^2}{3} \right)^{0.5} \right)}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 286–295.

**1.63 Horizontal well critical rate correlations—Giger and Karcher****Input(s)**

- $\Delta\rho$ : Specific Gravity Difference (g/cc)
- $h$ : Thickness (m)
- $\mu$ : Oil Viscosity (mPa s)
- $B$ : Oil Formation Volume Factor (RB/STB)
- $k_h$ : Horizontal Permeability (mD)
- $L$ : Distance Between Lines of Horizontal Wells (m)
- $g$ : Acceleration of Gravity (m/s<sup>2</sup>)

**Output(s)**

- $q_c$ : Critical Oil Rate (m<sup>3</sup>/day)

**Formula(s)**

$$q_c = \left( \frac{k_h * h^2 * g * \Delta\rho}{B * \mu * L} \right) * \left( 1 - \left( \left( \frac{1}{6} \right) * \left( \frac{h}{L} \right)^2 \right) \right)$$

Reference: *B.J. Karcher, E.F. Aquitaine, F.M. Giger, Some Practical Formulas to Predict Horizontal Well Behavior*. 1986. SPE ATCE, New Orleans, Louisiana.

**1.64 Horizontal well critical rate correlations—Joshi method for gas coning****Input(s)**

- $\rho_o$ : Oil Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)
- $r_e$ : Effective Radius of drainage (ft)
- $r_w$ : Radius of Wellbore (ft)
- $l_v$ : Distance between Gas/Oil interphase and perforated top of Vertical Well (ft)
- $h$ : Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_h$ : Horizontal Permeability (mD)

**Output(s)**

- $q_o$ : Critical Oil Rate (STB/day)

### Formula(s)

$$q_o = 1.535 * 10^{-3} * (\rho_o - \rho_g) * k_h * \frac{h^2 - (h - l_v)^2}{B_o * \mu_o * \ln\left(\frac{r_e}{r_w}\right)}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 286–295.

## 1.65 Hydrocarbon pore volume occupied by evolved solution gas

### Input(s)

- N: Initial Oil in Place (STB)
- $N_p$ : Cumulative Oil Production (STB)
- $R_{si}$ : Gas Solubility at Initial Reservoir Pressure (SCF/STB)
- $R_s$ : Current Gas Solubility (SCF/STB)
- $B_g$ : Current Gas Formation Volume Factor (bbl/SCF)
- $R_p$ : Net Cumulative Produced Gas-Oil Ratio (SCF/STB)

### Output(s)

- V: Volume of the Evolved Gas that Remains in the PV (PV)

### Formula(s)

$$V = \left( N * R_{si} - N_p * R_p - (N - N_p) * R_s \right) * B_g$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 301.

## 1.66 Hydrocarbon pore volume occupied by gas cap

### Input(s)

- m: Ratio of Initial Gas Cap Gas Reservoir Volume to Initial Reservoir Volume (bbl/bbl)
- N: Initial Oil in Place (STB)
- $B_{oi}$ : Initial Oil Formation Volume Factor (bbl/STB)
- $B_{gi}$ : Initial Gas Formation Volume Factor (bbl/SCF)
- $B_g$ : Current Gas Formation Volume Factor (bbl/SCF)

### Output(s)

- V: Volume of the Gas Cap at Current Pressure (bbl)

### Formula(s)

$$V = \frac{m * N * B_{oi} * B_g}{B_{gi}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 301.

## 1.67 Hydrocarbon pore volume occupied by remaining oil

### Input(s)

- N: Oil in Place (STB)  
 N<sub>p</sub>: Cumulative Oil Production (STB)  
 B<sub>o</sub>: Oil Formation Volume Factor (bbl/STB)

### Output(s)

- V<sub>ro</sub>: Volume of the Remaining Oil (bbl)

### Formula(s)

$$V_{ro} = (N - N_p) * B_o$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 301.

## 1.68 Hydrostatic pressure

### Input(s)

- mw: Mud weight (ppg)  
 TVD: True Vertical Depth (ft)

### Output(s)

- HP: Hydrostatic Pressure (psi)

### Formula(s)

$$HP = mw * 0.052 * TVD$$

Reference: [Wikipedia.org](https://en.wikipedia.org/).

## 1.69 Incremental cumulative oil production in undersaturated reservoirs

### Input(s)

- N<sub>p</sub>: Cumulative Oil Production (STB)  
 φ<sub>o</sub>: Oil PVT Function (rb/STB)  
 G<sub>p</sub>: Cumulative Gas Production (MSCF)  
 φ<sub>g</sub>: Gas PVT Function (rb/MSCF)  
 GOR: Average Gas Oil Ratio (SCF/STB)

### Output(s)

- ΔN<sub>p</sub>: Incremental Oil Produced (STB)

### Formula(s)

$$\Delta N_p = \frac{1 - N_p * \phi_o - G_p * \phi_g}{(\phi_o) + (GOR * \phi_g)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 5, Page: 335.

## 1.70 Ineffective porosity

### Input(s)

$V_{dis}$ : Volume of Completely Disconnected Pores ( $\text{cm}^3$ )

$V_b$ : Bulk Volume ( $\text{cm}^3$ )

### Output(s)

$\phi_{in}$ : Ineffective Porosity (fraction)

### Formula(s)

$$\phi_{in} = \frac{V_{dis}}{V_b}$$

Reference: Dandekar, A. Y. 2006. *Petroleum Reservoir Rock and Fluid Properties*. Boca Raton, FL: CRC Press Taylor & Francis Group, Chapter: 3, Page: 15.

## 1.71 Initial gas cap

### Input(s)

$m$ : Ratio of Initial Gas Cap Gas Reservoir Volume to Initial Reservoir Oil Volume (bbl/bbl)

$N$ : Original Oil in Place (STB)

$B_{oi}$ : Oil Formation Volume Factor (bbl/STB)

$B_{gi}$ : Initial Gas Formation Volume Factor (bbl/SCF)

### Output(s)

$G$ : Initial Gas Cap Gas (SCF)

### Formula(s)

$$G = \frac{m * N * B_{oi}}{B_{gi}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 300.

## 1.72 Initial gas in place for water-drive gas reservoirs

### Input(s)

$G_p$ : Cumulative Gas Production at Depletion Pressure (SCF)

$B_g$ : Gas Formation Volume Factor at Depletion Pressure (bbl/SCF)

$W_e$ : Cumulative Water Influx (bbl)

$W_p$ : Cumulative Water Production at Depletion Pressure (STB)

$B_w$ : Water Formation Volume Factor (bbl/STB)

$B_{gi}$ : Initial Gas Formation Volume Factor (bbl/SCF)

**Output(s)**

G: Gas Initially in Place (MSCF)

**Formula(s)**

$$G = \frac{G_p * B_g - (W_e - W_p * B_w)}{B_g - B_{gi}}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 207.*

**1.73 Injectivity index****Input(s)**

q: Flow Rate of injection Well (STB/day)

p: Reservoir Pressure (psi)

$p_{wf}$ : Well Flow Pressure (psi)

**Output(s)**

I: Injectivity Index (STB/day/psi)

**Formula(s)**

$$I = \frac{q}{p_{wf} - p}$$

Reference: *Horizontal Well Technology, Joshi, Page: 10.*

**1.74 Instantaneous gas-oil ratio****Input(s)**

$R_s$ : Gas Solubility (SCF/STB)

$k_{rg}$ : Relative Gas Permeability (dimensionless)

$k_{ro}$ : Relative Oil Permeability (dimensionless)

$\mu_o$ : Viscosity of Oil (cP)

$\mu_g$ : Viscosity of Gas (cP)

$B_o$ : Oil Formation Volume Factor (bbl/STB)

$B_g$ : Gas Formation Volume Factor (bbl/SCF)

**Output(s)**

GOR: Gas Oil Ratio (SCF/bbl)

**Formula(s)**

$$GOR = R_s + \frac{k_{rg} * \mu_o * B_o}{k_{ro} * \mu_g * B_g}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 16.*

## 1.75 Interporosity flow coefficient

### Input(s)

- A: Surface Area of the Matrix Block ( $\text{ft}^2$ )
- V: Volume of the Matrix Block ( $\text{ft}^3$ )
- x: Characteristic Length of the Matrix Block (ft)
- $k_m$ : Permeability of Matrix (mD)
- $k_f$ : Permeability of Fracture (mD)
- $r_w$ : Wellbore Radius (ft)

### Output(s)

- $\alpha$ : Block-Shape Parameter ( $1/\text{ft}^2$ )
- $\lambda$ : Interporosity Flow Coefficient (dimensionless)

### Formula(s)

$$\alpha = \frac{A}{V * x}$$

$$\lambda = \frac{\alpha * k_m * (r_w^2)}{k_f}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 82.

## 1.76 Interstitial velocity

### Input(s)

- q: Flow Rate ( $\text{cm}^3/\text{s}$ )
- $\phi$ : Porosity (fraction)
- A: Area ( $\text{cm}^2$ )

### Output(s)

- V: Interstitial Velocity ( $\text{cm}/\text{s}$ )

### Formula(s)

$$V = \frac{q}{\phi * A}$$

Reference: Civan, F. *Reservoir Formation Damage: Fundamentals, Modeling, Assessment, and Mitigation*. Gulf Publishing Company, Houston, Texas, Page: 493.

## 1.77 Isothermal compressibility of oil—Vasquez-Beggs correlation—P > Pb

### Input(s)

- p: Pressure (psi)
- T: Temp (F)
- $\gamma_g$ : Specific Gravity of Gas (fraction)
- $R_{sob}$ : Solution oil gas ratio at bubble point (fraction)
- $\rho$ : Density of oil (API)

**Output(s)**

$c_o$ : Isothermal Compressibility of Oil (/psi)

**Formula(s)**

$$c_o = \frac{5 * R_{sob} + 17.2 * T - 1180 * \gamma_g + 12.61 * \rho - 1433}{p * 10^5}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 39.*

**1.78 Isothermal compressibility of oil—Villena-Lanzi correlation—P < Pb****Input(s)**

p: Pressure (psi)

$p_b$ : Bubble point pressure (psi)

T: Temp (F)

$R_{sob}$ : Solution oil gas ratio at bubble point (fraction)

$\rho$ : Density of oil (API)

**Output(s)**

$lc_o$ : Isothermal Compressibility of Oil (/psi)

**Formula(s)**

$$lc_o = -0.664 - 1.430 * \ln(p) - 0.395 * \ln(p_b) + 0.39 * \ln(T) + 0.455 * \ln(R_{sob}) + 0.262 * \ln(\rho)$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 39.*

**1.79 Isothermal compressibility of water—Osif correlation****Input(s)**

$C_{NaCl}$ : Salinity (g NaCl/L)

T: Temperature (F)

p: Pressure (psi)

**Output(s)**

$c_w$ : Isothermal Compressibility of Water (Osif Correlation) (SCF/STB)

**Formula(s)**

$$c_w = \frac{1}{7.033 * p + 541.5 * C_{NaCl} - 537.0 * T + 403300}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 46.*

**1.80 Kerns method for gas flow in a fracture****Input(s)**

k: Permeability (mD)

$\mu$ : Viscosity (cP)

P: Pressure (psi)

### Output(s)

$V_g$ : Gas Velocity (cc)

### Formula(s)

$$V_g = \frac{k * P^2}{2 * \mu}$$

Reference: Ahmed, T. & McKinney, P.D. *Advanced Reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

## 1.81 Klinkenberg gas effect

### Input(s)

$k_l$ : Permeability of liquid (mD)  
 $p$ : Mean flowing pressure of the gas (atm)  
 $b$ : Klinkenberg factor, fixed for a gas (constant)

### Output(s)

$k_g$ : Apparent permeability of gas (mD)

### Formula(s)

$$k_g = k_l * \left( 1 + \frac{b}{p} \right)$$

Reference: [Wikipedia.org](#).

## 1.82 Kozeny equation

### Input(s)

$\phi$ : Porosity (fraction)  
 $k_z$ : Kozeny Constant (dimensionless)  
 $S_p$ : Specific Surface Area ( $\text{cm}^{-1}$ )

### Output(s)

$k$ : Permeability ( $\text{cm}^2$ )

### Formula(s)

$$k = \frac{\phi}{k_z * S_p^2}$$

Reference: Dandekar, A. Y. 2006. *Petroleum Reservoir Rock and Fluid Properties*. Boca Raton, FL: CRC Press Taylor & Francis Group, Page: 52.

## 1.83 Kozeny-Carman relationship

### Input(s)

$B$ : Geometric factor (dimensionless)  
 $\phi$ : Porosity (fraction)  
 $d$ : Diameter of Particle (cm)  
 $\tau$ : Tortuosity (dimensionless)

**Output(s)**

k: Permeability (mD)

**Formula(s)**

$$k = \frac{B * \phi^3 * d^2}{\tau}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Express, UK, Page: 41.*

**1.84 Leverett J-function****Input(s)**

$\sigma$ : Fluid interfacial tension (dyn/cm<sup>2</sup>)  
 $\theta$ : Angle of wettability (fraction)  
 $P_c$ : Capillary Pressure (dyn/cm)  
k: Permeability (mD)  
 $\phi$ : Porosity (fraction)

**Output(s)**

J: Leverett J-function (dimensionless)

**Formula(s)**

$$J = \frac{P_c}{\sigma * \cos(\theta)} * \left( \frac{k}{\phi} \right)^{0.5}$$

Reference: *Reservoir Engineering Handbook, Fourth Edition, Ahmed, Page: 224.*

**1.85 Line-source solution for damaged or stimulated wells****Input(s)**

$P_i$ : Initial Pressure (psi)  
t: Time of production (h)  
k: Permeability (mD)  
B: Volume factor (RB/STB)  
 $\phi$ : Porosity (fraction)  
 $c_r$ : Compressibility (1/psi)  
h: Thickness of reservoir (ft)  
 $\mu$ : Viscosity of Oil (cP)  
r: Radius of wellbore (ft)  
q: Flow Rate (STB/day)  
s: Skin Factor (dimensionless)

**Output(s)**

$P_{wf}$ : Line-Source Solution for Damaged or Stimulated Wells (psi)

### Formula(s)

$$P_{wf} = P_i + 70.6 * q * B * \mu * \frac{\left( \ln \left( \frac{1688 * \emptyset * \mu * c_t * r^2}{k * t} \right) \right) - 2 * s}{k * h}$$

Reference: *Pressure Transient Testing, Lee, Rollins & Spivey, Page: 11.*

## 1.86 Low-pressure region gas flow rate for non-circular drainage area

### Input(s)

- k: Permeability (mD)
- h: Thickness (ft)
- $P_r$ : Average Reservoir Pressure (psi)
- $P_{wf}$ : Well Flowing Pressure (psi)
- $\mu_{gavg}$ : Average Gas Viscosity (cP)
- $Z_{avg}$ : Average Gas Compressibility Factor (Dimensionless)
- A: Drainage Area ( $\text{ft}^2$ )
- $C_A$ : Shape Factor (dimensionless)
- $r_w$ : Wellbore Radius (ft)
- S: Skin (dimensionless)
- T: Temperature (R)

### Output(s)

- $Q_g$ : Gas Flow Rate (MSCF/day)

### Formula(s)

$$Q_g = \frac{k * h * \left( (P_r^2) - \left( (P_{wf})^2 \right) \right)}{1422 * \mu_{gavg} * T * Z_{avg} * \left( 0.5 * \ln \left( \frac{4 * A}{1.781 * C_A * (r_w^2)} \right) + S \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 189.*

## 1.87 Material balance for cumulative water influx—Havlena and Odeh

### Input(s)

- G: Gas in Place (SCF)
- $E_G$ : Gas Expansion Term (bbl/SCF)
- $W_e$ : Cumulative Water Influx (bbl)

### Output(s)

- F: Fluid Withdrawal (bbl)

### Formula(s)

$$F = G * E_G + W_e$$

Reference: Ahmed, T., McKinney P.D. 2005. *Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 209.*

## 1.88 Maximum height of oil column in cap rock

### Input(s)

- $P_a$ : Total Potential energy of accumulation (psi)
- $P_w$ : Water Potential in Reservoir (psi)
- $G_o$ : Initial oil pressure gradient in Reservoir rock ( $\text{cm}^2$ )
- $\alpha$ : Height Constant (dimensionless)
- $h$ : Depth (ft)
- $P_c$ : Capillary Pressure (psi)
- $\rho$ : Density differential between fluids (ppg)

### Output(s)

- H: Maximum Height of Oil Column (ft)

### Formula(s)

$$H = \frac{P_a - P_w + G_o * h * \alpha + P_c}{\rho - G_o}$$

Reference: *Tarek Ahmed, Paul McKinney, Advanced Reservoir Engineering, Gulf Publishing House, Burlington, MA, 2015.*

## 1.89 Modified Cole plot

### Input(s)

- G: Gas in Place (SCF)
- $W_e$ : Cumulative Water Influx (bbl)
- $E_t$ : Total Expansion Term (bbl/SCF)

### Output(s)

- $\frac{F}{E_t}$ : Fluid Withdrawal (bbl)

### Formula(s)

$$\frac{F}{E_t} = G + \left( \frac{W_e}{E_t} \right)$$

Reference: *Ahmed, T., McKinney P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 212.*

## 1.90 Modified Kozeny-Carman relationship

### Input(s)

- $\phi$ : Porosity (fraction)
- $\phi_c$ : Percolation Porosity (fraction)
- B: Geometric Factor (fraction)
- d: Average Grain Diameter (m)

### Output(s)

$k$ : Permeability ( $\text{m}^2$ )

### Formula(s)

$$k = B * \frac{(\emptyset - \emptyset c)^3}{(1 + \emptyset c - \emptyset)} * d^2$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.91 Normalized saturation

### Input(s)

$S_w$ : Saturation of Water (fraction)  
 $S_{or}$ : Residual Water Saturation (fraction)  
 $S_{wi}$ : Initial Water Saturation (fraction)

### Output(s)

$Son$ : Normalized Saturation (fraction)

### Formula(s)

$$Son = \frac{1 - S_w - S_{or}}{1 - S_{wi} - S_{or}}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.92 Oil bubble radius of the drainage area of each well represented by a circle

### Input(s)

$N_p$ : Well Current Cumulative Oil Production (bbl)  
 $\emptyset$ : Porosity (fraction)  
 $h$ : Thickness (ft)  
 $S_{wi}$ : Initial Water Saturation at bubble point pressure (fraction)  
 $B_o$ : Oil Formation Volume factor (bbl/STB)  
 $B_{oi}$ : Initial Oil Formation Volume Factor (bbl/STB)  
 $S_o$ : Current Oil Saturation (fraction)

### Output(s)

$r_{ob}$ : Oil Bubble Radius (ft)

### Formula(s)

$$r_{ob} = \left( \frac{5.615 * N_p}{\pi * \emptyset * h * \left( \frac{1 - S_{wi}}{B_{oi}} - \frac{S_o}{B_o} \right)} \right)^{0.5}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 4, Page: 313.

## 1.93 Oil density—Standing's correlation

### Input(s)

- $\gamma_o$ : Oil Specific Gravity (fraction)
- $R_s$ : Solution Gas Oil Ratio (SCF/STB)
- $\gamma_g$ : Gas Specific Gravity (fraction)
- t: Temperature (F)

### Output(s)

- $\rho_o$ : Oil density (lbm/ft<sup>3</sup>)

### Formula(s)

$$\rho_o = \frac{62.4 * \gamma_o + 0.0136 * R_s * \gamma_g}{0.972 + 0.000147 * \left( R_s * \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 * t \right)^{1.175}}$$

Reference: Boyun, G., William, C., & Ali Ghalmabor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 2/20.

## 1.94 Oil formation volume factor—Standing's correlation

### Input(s)

- $R_s$ : Gas Oil Ratio (SCF/STB)
- $\gamma_o$ : Specific Gravity of Oil Phase (fraction)
- $\gamma_g$ : Specific Gravity of Gas Phase (fraction)
- t: Temperature (F)

### Output(s)

- $B_o$ : Oil FVF (RB/STB)

### Formula(s)

$$B_o = 0.9759 + 0.00012 * \left( R_s * \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 * t \right)^{1.2}$$

Reference: Boyun, G., William, C., & Ali Ghalmabor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 2/20.

## 1.95 Oil formation volume factor—Beggs-standing correlation—P < P<sub>b</sub>

### Input(s)

- $R_s$ : Solution Gas oil ratio (fraction)
- $\gamma_g$ : Specific Gravity of Gas (fraction)
- $\gamma_o$ : Specific Gravity of Oil (fraction)
- T: Temperature (F)

**Output(s)**

- F: Dimensionless Factor (fraction)  
 $B_o$ : Oil formation factor (fraction)

**Formula(s)**

$$F = R_s * \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 * T$$

$$B_o = 0.972 + 0.000147 * F^{1.175}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 37.*

**1.96 Oil formation volume factor—Beggs-standing correlation—P > P<sub>b</sub>****Input(s)**

- $B_b$ : Oil formation vol factor at bubble pt pressure (fraction)  
 $c_o$ : Oil compressibility (/psi)  
 $p_b$ : Bubble point pressure (psi)  
 $p$ : Pressure (psi)

**Output(s)**

- $B_o$ : Oil formation factor (fraction)

**Formula(s)**

$$B_o = B_b * \exp(c_o * (p_b - p))$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 37.*

**1.97 Oil in place for undersaturated oil reservoirs without fluid injection****Input(s)**

- $N_p$ : Produced Oil (STB)  
 $B_o$ : Oil Formation Volume Factor (rb/STB)  
 $B_{oi}$ : Initial Oil Formation Volume Factor (rb/STB)  
 $S_{wi}$ : Initial Water Saturation (fraction)  
 $c_w$ : Water Compressibility (1/psi)  
 $c_f$ : Formation Compressibility (1/psi)  
 $\Delta P$ : Pressure Differential (psi)

**Output(s)**

- $N$ : Oil in Place (STB)

**Formula(s)**

$$N = \frac{N_p * B_o}{B_o - B_{oi} + B_{oi} * \left( \frac{S_{wi} * c_w + c_f}{1 - S_{wi}} \right) * \Delta P}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 5, Page: 333.

## 1.98 Oil in place in saturated oil reservoirs

**Input(s)**

- $N_p$ : Cumulative Oil Production (STB)
- $B_o$ : Oil Formation Volume Factor (bbl/STB)
- $G_p$ : Cumulative Gas Production (MSCF)
- $R_s$ : Gas Solubility (SCF/STB)
- $B_g$ : Gas Formation Volume Factor (bbl/SCF)
- $B_{oi}$ : Initial Oil Formation Volume Factor (bbl/STB)
- $R_{si}$ : Initial Gas Solubility (SCF/STB)

**Output(s)**

- N: Oil in Place (bbl)

**Formula(s)**

$$N = \frac{N_p * B_o + (G_p - N_p * R_s) * B_g}{B_o - B_{oi} + (R_{si} - R_s) * B_g}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 5, Page: 334.

## 1.99 Oil lost in migration

**Input(s)**

- h: Average Change in Depth of the Gas-Oil Contact (ft)
- $\emptyset$ : Porosity (fraction)
- A: Average Cross-Sectional Area of the Gas-Oil Contact (acres)
- $S_{org}$ : Residual Oil Saturation in The Gas Cap Shrinking Zone (fraction)
- $B_{oa}$ : Oil Formation Volume Factor at Abandonment (bbl/STB)

**Output(s)**

- O: Volume of Oil Lost (bbl)

**Formula(s)**

$$O = 7758 * A * h * \emptyset * \left( \frac{S_{org}}{B_{oa}} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 5, Page: 333.

## 1.100 Oil saturation at any depletion state below the bubble point pressure

### Input(s)

- S<sub>wi</sub>: Initial Water Saturation (fraction)
- N<sub>p</sub>: Cumulative Oil Production (STB)
- N: Oil in Place (STB)
- B<sub>oi</sub>: Initial Oil Formation Volume Factor (bbl/STB)
- B<sub>o</sub>: Oil Formation Volume Factor (bbl/STB)

### Output(s)

- S<sub>o</sub>: Oil Saturation (fraction)

### Formula(s)

$$S_o = (1 - S_{wi}) * \left(1 - \frac{N_p}{N}\right) * \left(\frac{B_o}{B_{oi}}\right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 312.

## 1.101 Original gas in place

### Input(s)

- A: Area of Reservoir (acres)
- h: Average Reservoir Thickness (ft)
- Ø: Porosity (fraction)
- S<sub>wi</sub>: Initial Water Saturation (fraction)
- B<sub>gi</sub>: Gas Formation Volume Factor at Initial Pressure (ft<sup>3</sup>/SCF)

### Output(s)

- G: Gas-in-Place (SCF)

### Formula(s)

$$G = \frac{43560 * A * h * \emptyset * (1 - S_{wi})}{B_{gi}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 202.

## 1.102 Payne method for intercompartmental flow in tight gas reservoirs

### Input(s)

- k: Permeability (mD)
- T: Temperature (°R)
- A: Cross-Sectional Area (ft<sup>2</sup>)
- L: Distance between the Center of the Two Compartments (ft)
- m(P<sub>1</sub>): Gas Pseudo pressure in Compartment (Tank) 1 (psi<sup>2</sup>/cP)
- m(P<sub>2</sub>): Gas Pseudo pressure in Compartment (Tank) (psi<sup>2</sup>/cP)

**Output(s)**

$Q_{12}$ : Flow Rate between the Two Compartments (SCF/day)

**Formula(s)**

$$Q_{12} = \left( \frac{0.111924 * k * A}{T * L} \right) * (m(P_1) - m(P_2))$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 234, 235.

**1.103 Performance coefficient for shallow gas reservoirs****Input(s)**

- k: Permeability (mD)
- h: Thickness (ft)
- T: Temperature ( $^{\circ}$ R)
- $\mu$ : Viscosity (cP)
- z: Compressibility Factor (dimensionless)
- $r_e$ : Radius of Drainage Area (ft)
- $r_w$ : Wellbore Radius (ft)

**Output(s)**

C: Performance Coefficient (dimensionless)

**Formula(s)**

$$C = \frac{k * h}{1422 * T * \mu_g * Z * \left( \ln\left(\frac{r_e}{r_w}\right) - 0.5 \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 287.

**1.104 Poisson's ratio****Input(s)**

- $\xi_t$ : Transverse Strain (dimensionless)
- $\xi_l$ : Longitudinal Strain (dimensionless)

**Output(s)**

$\mu$ : Poisson (dimensionless)

**Formula(s)**

$$\mu = \frac{\xi_t}{\xi_l}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 55.

### 1.105 Pore throat sorting

#### Input(s)

- $Q_3$ : Capillary pressure at 75% saturation (psi)  
 $Q_1$ : Capillary pressure at 25% saturation (psi)

#### Output(s)

- PTS: Pore Throat Sorting (fraction)

#### Formula(s)

$$\text{PTS} = \left( \frac{Q_3}{Q_1} \right)^{0.5}$$

Reference: Dandekar, A. Y. 2006. *Petroleum Reservoir Rock and Fluid Properties*. Boca Raton, FL: CRC Press Taylor & Francis Group, Chapter: 8, Page: 176.

### 1.106 Pore volume occupied by injection of gas and water

#### Input(s)

- $W_{\text{INJ}}$ : Cumulative Water Injected (STB)  
 $B_w$ : Water Formation Volume Factor (bbl/STB)  
 $G_{\text{INJ}}$ : Cumulative Gas Injected (SCF)  
 $B_{G\text{INJ}}$ : Injected Gas Formation Volume Factor (bbl/SCF)

#### Output(s)

- $V_t$ : Total Volume (bbl)

#### Formula(s)

$$V_t = W_{\text{INJ}} * B_w + G_{\text{INJ}} * B_{G\text{INJ}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 302.

### 1.107 Pore volume through squared method in tight gas reservoirs

#### Input(s)

- $\mu_{g, \text{avg}}$ : Average Gas Viscosity (cP)  
 $z_{\text{avg}}$ : Compressibility Factor (dimensionless)  
 $q_i$ : Initial Gas Rate (MSCF/day)  
 $\mu_{gi}$ : Initial Gas Viscosity (cP)  
 $c_{ti}$ : Initial Total Compressibility (1/psi)  
 $P_i$ : Initial Pressure (psi)  
 $P_{wf}$ : Bottom-hole Flowing Pressure (psi)  
 $D_i$ : Decline Rate ( $\text{day}^{-1}$ )  
 $T$ : Temperature ( $^{\circ}\text{R}$ )

**Output(s)**

PV: Pore Volume (dimensionless)

**Formula(s)**

$$PV = \frac{28.27 * T * \mu_{g,avg} * z_{avg}}{\mu_{gi} * c_{ti} * (P_i^2 - P_{wf}^2)} * \left( \frac{q_i}{D_i} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 252.

**1.108 Porosity determination—IES and FDC logs****Input(s)**

$\rho_b$ : Bulk Density ( $\text{g/cm}^3$ )

$\rho_{ma}$ : Matrix Density ( $\text{g/cm}^3$ )

$\rho_f$ : Average Fluid Density in Pore Spaces ( $\text{g/cm}^3$ )

**Output(s)**

$\phi$ : Porosity (fraction)

**Formula(s)**

$$\phi = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 11, Page: 210.

**1.109 Produced gas-oil ratio****Input(s)**

$N_p$ : Cumulative Oil Production (STB)

$G_p$ : Cumulative Gas Production (SCF)

**Output(s)**

$R_p$ : Net Cumulative Produced Gas-Oil Ratio (SCF/STB)

**Formula(s)**

$$R_p = \frac{G_p}{N_p}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 302.

## 1.110 Productivity index for a gas well

### Input(s)

- k: Permeability (mD)
- h: Thickness (ft)
- T: Temperature ( $^{\circ}$ R)
- A: Drainage Area ( $ft^2$ )
- $C_A$ : Shape Factor (dimensionless)
- $r_w$ : Wellbore Radius (ft)
- S: Skin (dimensionless)

### Output(s)

- J: Productivity Index (MSCF/day/psi $^2$ /cP)

### Formula(s)

$$J = \frac{k * h}{1422 * T * \left( 0.5 * \ln \left( \frac{4 * A}{1.781 * C_A * r_w^2} \right) + S \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 188.

## 1.111 Pseudo-steady state productivity of horizontal Wells—Method 1

### Input(s)

- A: Drainage Area ( $ft^2$ )
- s: Skin Factor (dimensionless)
- D: Turbulence Coefficient (1/BOPD for oil and 1/MSCFD for gas)
- C: Shape Factor Conversion Constant = 1.386 (dimensionless)
- $s_{CAh}$ : Shape-Related Skin Factor (dimensionless)
- h: Thickness of Reservoir (ft)
- $k_v$ : Vertical Permeability (mD)
- $k_h$ : Horizontal Permeability (mD)
- k: Permeability (mD)
- q: Flow Rate (BOPD for oil and MSCFD for gas)
- L: Fracture Length (ft)
- $r_w$ : Radius of Wellbore (ft)
- $\mu_o$ : Viscosity of Oil (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)

### Output(s)

- $r_e$ : Effective Radius of Drainage Area (ft)
- $s_m$ : Mechanical Skin Factor (dimensionless)
- $s_f$ : Skin Factor of an Infinite-conductivity, Fully Penetrating Fracture of Length L (dimensionless)
- $J_h$ : Pseudo-steady State Productivity of a Horizontal Well (bbl/day/psi)

**Formula(s)**

$$r_e = \left( \frac{A}{\pi} \right)^{0.5}$$

$$s_m = s * \frac{h}{L} * \left( \frac{k_h}{k_v} \right)^{0.5}$$

$$s_f = - \ln \left( \frac{L}{4 * r_w} \right)$$

$$J_h = \frac{0.007078 * k * \frac{h}{\mu_o * B_o}}{\ln \left( \frac{r_e}{r_w} \right) - A + s_f + s_m + s_{CAh} - C + D * q}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 221.

**1.112 Pseudo-steady state productivity of horizontal Wells—Method 2****Input(s)**

- $\mu_o$ : Viscosity of Oil (cP)
- $B_o$ : Oil Formation Volume (RB/STB)
- $A_t$ : Horizontal Well Drainage Area in the Vertical Plane =  $2 y_e h$  ( $\text{ft}^2$ )
- $s_R$ : Skin Factor Due to Partial Penetration of Horizontal Well (dimensionless)
- $x_e$ : Half Length of the Short Side of the Rectangular Drainage Area (ft)
- $y_e$ : Half Length of the Long Side of the Rectangular Drainage Area (ft)
- $y_w$ : Distance from the Horizontal Well to the Closest Boundary in Y Direction (ft)
- $h$ : Thickness of Reservoir (ft)
- $k_v$ : Vertical Permeability (mD)
- $k_y$ : Horizontal Permeability (mD)
- $r_w$ : Radius of Wellbore (ft)
- $z_w$ : Vertical Distance between Horizontal Well and Bottom Boundary of Reservoir (ft)

**Output(s)**

- $InC_h$ : A Constant Related to the Natural Logarithm of Shape Factor (dimensionless)
- $J_h$ : Pseudo-steady State Productivity of Horizontal Wells (bbl/day/psi)

**Formula(s)**

$$InC_h = 6.28 * \left( 2 * \frac{y_e}{h} \right) * \left( \frac{k_v}{k_y} \right)^{0.5} * \left( \frac{1}{3} - \frac{y_w}{2 * y_e} + \left( \frac{y_w}{2 * y_e} \right)^2 \right) - \ln \left( \sin \left( 180 * \frac{z_w}{h} \right) \right) - 0.5 * \ln \left( \left( 2 * \frac{y_e}{h} \right) * \left( \frac{k_v}{k_y} \right)^{0.5} \right) - 1.088$$

$$J_h = 0.007078 * 2 * x_e * \frac{\left( \frac{(k_y * k_v)^{0.5}}{\mu_o * B_o} \right)}{\ln \left( \frac{(A_t)^{0.5}}{r_w} \right) + InC_h - 0.75 + s_R}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 22.

### 1.113 Pseudo-steady state productivity of horizontal wells—Method 3

#### Input(s)

- $\mu_o$ : Viscosity of Oil (cP)
- $s_x$ : Skin Factor (dimensionless)
- $h$ : Thickness of Reservoir (ft)
- $k_v$ : Vertical Permeability (mD)
- $k_h$ : Horizontal Permeability (mD)
- F: Dimensionless Function (ft)
- L: Length of Fracture (ft)

#### Output(s)

- $J_h$ : Pseudo-steady State Productivity of Horizontal Wells (bbl/day psi)

#### Formula(s)

$$J_h = k_h * \frac{\frac{h}{70.6 * \mu_o}}{F + \left( \frac{h}{0.5 * L} \right) * \left( \frac{k_h}{k_v} \right)^{0.5} * s_x}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 224.

### 1.114 Pseudo-steady state radial flow equation

#### Input(s)

- k: Permeability (mD)
- h: Thickness of Reservoir (ft)
- $P_i$ : Average Reservoir Pressure (psi)
- $P_{wf}$ : Well Flowing Pressure (psi)
- B: Formation Volume Factor (bbl/STB)
- $\mu$ : Viscosity (cP)
- $r_e$ : Drainage Radius (ft)
- $r_w$ : Wellbore Radius (ft)

#### Output(s)

- $Q$ : Flow Rate (STB/day)

#### Formula(s)

$$Q = \frac{0.00708 * k * h * (P_i - P_{wf})}{B * \mu * \left( \ln \left( \frac{r_e}{r_w} \right) - 0.75 \right)}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 33.

## 1.115 Relative permeability—Corey exponents

### Input(s)

- $k_{roe}$ : Permeability at end point (mD)
- $S_w$ : Saturation of Water (fraction)
- $S_{or}$ : Residual Water Saturation (fraction)
- $S_{wi}$ : Initial Water Saturation (fraction)
- $N_o$ : Corey Coefficient (fraction)

### Output(s)

- $k_{ro}$ : Relative Permeability at given Saturation (mD)

### Formula(s)

$$k_{ro} = k_{roe} * \left( \frac{1 - S_w - S_{or}}{1 - S_{wi} - S_{or}} \right)^{N_o}$$

Reference: Dandekar, A. Y. 2006. *Petroleum Reservoir Rock and Fluid Properties*. Boca Raton, FL: CRC Press Taylor & Francis Group, Chapter: 9, Page: 233.

## 1.116 Remaining gas in place in coalbed methane reservoirs

### Input(s)

- A: Drainage Area (acres)
- $\emptyset$ : Porosity (fraction)
- h: Thickness (ft)
- $B_w$ : Water Formation Volume Factor (bbl/STB)
- $W_p$ : Cumulative Water Produced (bbl)
- $S_{wi}$ : Water Saturation (fraction)
- $P_i$ : Initial Pressure (psi)
- P: Current Reservoir Pressure (psi)
- $c_f$ : Formation Compressibility (1/psi)
- $c_w$ : Water Compressibility (1/psi)
- $E_g$ : Gas Expansion Factor (SCF/bbl)

### Output(s)

- $G_R$ : Remaining Gas at Reservoir Pressure (SCF)

### Formula(s)

$$G_R = 7758 * A * h * \emptyset * E_g * \left( \frac{\left( \frac{B_w * W_p}{7758 * A * h * \emptyset} \right) + (1 - S_{wi}) - (P_i - P) * (c_f + c_w * S_{wi})}{1 - (P_i - P) * c_f} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 227.

### 1.117 Roach plot for abnormally pressured gas reservoirs

#### Input(s)

- $p_i/Z_i$ : Initial P/z Ratio (psi)
- $c_r$ : Rock Compressibility (1/psi)
- $G_p$ : Gas Produced (SCF)
- $G$ : Gas in Place (SCF)

#### Output(s)

- $\frac{p}{Z}$ : Current P/z Ratio (psi)

#### Formula(s)

$$\frac{p}{Z} = \left( \frac{p_i}{Z_i * c_r} \right) * \left( 1 - \left( \frac{G_p}{G} \right) \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 213.

### 1.118 Rock expansion term in abnormally pressured gas reservoirs

#### Input(s)

- $c_f$ : Formation Compressibility (1/psi)
- $c_w$ : Water Compressibility (1/psi)
- $S_{wi}$ : Initial Water Saturation (fraction)

#### Output(s)

- $E_R$ : Rock Expansion Term (1/psi)

#### Formula(s)

$$E_R = \frac{c_f + c_w * S_{wi}}{1 - S_{wi}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 213.

### 1.119 Shape factor—Earlougher

#### Input(s)

- $m$ : Slope of Transient Semi-log Straight Line (psi/log cycle)
- $m'$ : Slope of Semi-steady State Cartesian Straight Line (psi/h)
- $P_{1hr}$ : Pressure at  $t = 1$  h from Transient Semi-log Straight Line (psi)
- $P_f$ : Pressure at  $t = 0$  from Pseudo-steady State Cartesian Straight Line (psi)

#### Output(s)

- $C_A$ : Shape Factor (dimensionless)

**Formula(s)**

$$C_A = 5.456 * \left( \frac{m}{m'} \right) * \exp \left( 2.303 * \frac{P_{1hr} - P_f}{m'} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 47.

**1.120 Solution gas oil ratio—Beggs-standing correlation—P < Pb****Input(s)**

- T: Temperature (F)
- $\gamma_g$ : Specific gravity of produced gas (fraction)
- p: Pressure1 (psi)
- $p_o$ : Pressure at given API (psi)

**Output(s)**

- $Y_g$ : Constant (fraction)
- $R_{so}$ : Solution Gas Oil Ratio (fraction)

**Formula(s)**

$$Y_g = 0.00091 * T - 0.0125 * p_o$$

$$R_{so} = \gamma_g * \left( \frac{p}{18 * 10^{Y_g}} \right)^{1.204}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition*, Craft & Hawkins, Page: 33.

**1.121 Solution gas oil ratio—Standing's correlation****Input(s)**

- $\gamma_g$ : Gas Specific Gravity (fraction)
- p: Pressure (psi)
- API: API Gravity of Oil (API)
- t: Temperature (F)

**Output(s)**

- $R_s$ : Solution Gas Oil Ratio (SCF/STB)

**Formula(s)**

$$R_s = \left( \gamma_g \right) * \left( \left( \frac{p}{18} \right) * \left( \frac{10^{0.0125 * \text{API}}}{10^{0.00091 * t}} \right) \right)^{1.2048}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach* Page: 2/20.

## 1.122 Solution gas water ratio

### Input(s)

T: Temperature (F)

S: Salinity (% by weight solids)

$R_{swp}$ : Solution gas to pure water ratio (SCF/STB)

### Output(s)

$R_{sw}$ : Solution Gas Water Ratio (SCF/STB)

### Formula(s)

$$R_{sw} = R_{swp} * 10^{-0.0840655 * S * T^{-0.285854}}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 46.*

## 1.123 Somerton method for formation permeability in coalbed methane reservoirs

### Input(s)

$k_o$ : Original Permeability at Zero Net Stress (mD)

$\Delta\sigma$ : Net Stress (psi)

### Output(s)

k: Permeability (mD)

### Formula(s)

$$k = k_o * \left( \exp \left( \frac{(-0.003) * \Delta\sigma}{k_o^{0.1}} \right) + 0.0002 * \left( (\Delta\sigma)^{\frac{1}{3}} \right) * \left( (k_o)^{\frac{1}{3}} \right) \right)$$

Reference: *Ahmed, T., McKinney, P.D. 2005, Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 226.*

## 1.124 Specific gravity of gas hydrate forming components

### Input(s)

$m_h$ : Molar Mass (g)

### Output(s)

$\Gamma_h$ : Specific Gravity of Hydrate-forming Components (dimensionless)

### Formula(s)

$$\Gamma_h = \frac{m_h}{28.96}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 274.*

### 1.125 Time to reach the semi-steady state for a gas well in a circular or square drainage area

#### Input(s)

$\mu_{gi}$ : Initial Gas Viscosity (cP)  
 $\phi$ : Porosity (fraction)  
 $c_{ti}$ : Initial Total Compressibility (1/psi)  
 $A$ : Drainage Area ( $\text{ft}^2$ )  
 $k$ : Effective Gas Permeability (mD)

#### Output(s)

$t_{pss}$ : Time (h)

#### Formula(s)

$$t_{pss} = \frac{15.8 * \phi * \mu_{gi} * c_{ti} * A}{k}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 194.

### 1.126 Time to the end of infinite-acting period for a well in a circular reservoir

#### Input(s)

$\phi$ : Porosity (fraction)  
 $\mu$ : Viscosity (cP)  
 $c_t$ : Total Compressibility (1/psi)  
 $A$ : Well Drainage Area ( $\text{ft}^2$ )  
 $k$ : Permeability (mD)

#### Output(s)

$t_{eia}$ : Time to the End of Infinite-acting Period (h)

#### Formula(s)

$$t_{eia} = \frac{380 * \phi * \mu * c_t * A}{k}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 50.

### 1.127 Torcaso and Wyllie's correlation for relative permeability ratio prediction

#### Input(s)

$S_o$ : Oil Saturation (fraction)  
 $S_{wi}$ : Initial Water Saturation (fraction)  
 $k_{rg}$ : Gas Relative Permeability (dimensionless)

**Output(s)**

S: Saturation (fraction)  
 kog: Oil Relative Permeability Ratio (fraction)

**Formula(s)**

$$S = \frac{S_o}{1 - S_{wi}}$$

$$kog = krg * \frac{(1 - S)^2 * (1 - S^2)}{S^4}$$

Reference: Ahmed, T. & McKinney, P.D. *Advanced reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

**1.128 Total compressibility****Input(s)**

$c_g$ : Compressibility of gas (/psi)  
 $c_o$ : Compressibility of oil (/psi)  
 $c_w$ : Compressibility of water (/psi)  
 $c_f$ : Compressibility of formation (/psi)  
 $S_g$ : Saturation of gas (fraction)  
 $S_o$ : Saturation of oil (fraction)  
 $S_w$ : Saturation of water (fraction)

**Output(s)**

$c_t$ : Total Compressibility (/psi)

**Formula(s)**

$$c_t = c_g * S_g + c_o * S_o + c_w * S_w + c_f$$

Reference: [Petrowiki.org](http://Petrowiki.org).

**1.129 Total pore volume compressibility****Input(s)**

$P_i$ : Initial Pressure (psi)  
 $P$ : Pressure (psi)  
 $(PV)_i$ : Pore Volume at Initial Reservoir Pressure (bbl)  
 $(PV)_p$ : Pore Volume at Pressure P (bbl)

**Output(s)**

$c_f$ : Cumulative Pore Volume (Formation or Rock) Compressibility (1/psi)

**Formula(s)**

$$c_f = \left( \frac{1}{(PV)_i} \right) * \left( \frac{(PV)_i - (PV)_p}{P_i - P} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 213.

**1.130 Transmissibility between compartments****Input(s)**

- $\gamma_a$ : Transmissibility of Compartment 1 ( $\text{mD ft}^2/\text{cP}$ )
- $\gamma_b$ : Transmissibility of Compartment 2 ( $\text{mD ft}^2/\text{cP}$ )
- $L_1$ : Length of Compartment 1 (ft)
- $L_2$ : Length of Compartment 2 (ft)

**Output(s)**

- $\gamma$ : Transmissibility Between Compartments ( $\text{mD ft}^2/\text{cP}$ )

**Formula(s)**

$$\gamma = \frac{\gamma_a * \gamma_b * (L_1 + L_2)}{\gamma_a * L_2 + L_1 * \gamma_b}$$

Reference: Ahmed, T., McKinney, P.D. 2015. *Advanced Reservoir Engineering*, Gulf Publishing Elsevier, Chapter: 3, Page: 236.

**1.131 Transmissibility of a compartment****Input(s)**

- $k$ : Permeability ( $\text{mD}$ )
- $A$ : Cross-sectional Area ( $\text{ft}^2$ )
- $z$ : Compressibility Factor (dimensionless)
- $\mu_g$ : Gas Viscosity ( $\text{cP}$ )

**Output(s)**

- $\gamma$ : Transmissibility ( $\text{mD ft}^2/\text{cP}$ )

**Formula(s)**

$$\gamma = \frac{k * A}{z * \mu_g}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 236.

**1.132 Transmissivity****Input(s)**

- $C$ : Hazen (dimensionless)
- $D$ : Diameter of 10 percentile grain size (mm)

### Output(s)

$K$ : Transmissivity ( $\text{mm}^2$ )

### Formula(s)

$$K = C * (D)^2$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 1.133 Trapped gas volume in water-invaded zones

### Input(s)

$W_e$ : Cumulative Water Influx (bbl)  
 $W_p$ : Cumulative Water Produced (bbl)  
 $B_w$ : Water Formation Volume Factor (bbl/STB)  
 $s_{wi}$ : Initial Water Saturation (fraction)  
 $s_{grw}$ : Relative Gas Saturation (fraction)

### Output(s)

$TG$ : Trapped Gas Volume (MSCF)

### Formula(s)

$$TG = \left( \frac{W_e - W_p * B_w}{1 - s_{wi} - s_{grw}} \right) * s_{grw}$$

Reference: Ahmed, T., McKinney, P.D.2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 208.

## 1.134 Two-phase formation volume factor

### Input(s)

$B_o$ : Oil formation vol factor (fraction)  
 $B_g$ : Gas formation vol factor (fraction)  
 $R_{soi}$ : Initial Solution oil gas ratio (fraction)  
 $R_{so}$ : Solution oil gas ratio (fraction)

### Output(s)

$B_t$ : Oil formation factor (fraction)

### Formula(s)

$$B_t = B_o + (B_g * (R_{soi} - R_{so}))$$

Reference: [Petrowiki.org](https://petrowiki.org).

### 1.135 Underground fluid withdrawal—Havlena and Odeh

#### Input(s)

- $G_p$ : Cumulative Gas Production (SCF)
- $B_g$ : Gas Formation Volume Factor (bbl/SCF)
- $W_p$ : Cumulative Water Production (STB)
- $B_w$ : Water Formation Volume Factor (bbl/STB)

#### Output(s)

- F: Fluid Withdrawal (bbl)

#### Formula(s)

$$F = G_p * B_g + W_p * B_w$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 259.

### 1.136 Vertical well critical rate correlations—Craft and Hawkins method

#### Input(s)

- b: Penetration Ratio, ( $h_p/h$ ) (dimensionless)
- $r_e$ : Radius of Drainage (ft)
- $r_w$ : Radius of Wellbore (ft)
- $h_p$ : Thickness of Perforated interval (ft)
- h: Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_o$ : Effective Oil Permeability (mD)
- $p_{ws}$ : Static Well Pressure Corrected to the Middle of the Producing Interval (psi)
- $p_{wf}$ : Flowing Well Pressure at the Middle of the Producing Interval (psi)

#### Output(s)

- PR: Productivity Ratio (dimensionless)
- $q_o$ : Critical Rate (Maximum Oil Rate without Coning) (STB/day)

#### Formula(s)

$$PR = b * \left( 1 + 7 * \left( \frac{r_w}{2 * b * h} \right)^{0.5} * \cos(b * 90^\circ) \right)$$

$$q_o = \frac{0.007078 * k_o * h * (p_{ws} - p_{wf})}{\mu_o * B_o * \ln\left(\frac{r_e}{r_w}\right)} * PR$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 8, Page: 254.

### 1.137 Vertical well critical rate correlations—Hoyland, Papatzacos, and Skjaeveland—Isotropic reservoirs

#### Input(s)

- h: Oil Column Thickness (m)
- $h_p$ : Perforated Interval (m)
- $r_e$ : Drainage Radius (m)
- $k_o$ : Effective Oil Permeability (mD)
- $\rho_w$ : Water Density ( $\text{kg}/\text{m}^3$ )
- $\mu_o$ : Oil Viscosity (cP)
- $\rho_o$ : Oil Density ( $\text{kg}/\text{m}^3$ )
- $B_o$ : Oil Formation volume Factor (RB/STB)

#### Output(s)

- $Q_{oc}$ : Critical Rate ( $\text{m}^3/\text{day}$ )

#### Formula(s)

$$Q_{oc} = \frac{(\rho_w - \rho_o) * k_o}{B_o * \mu_o * 10822} * \left( 1 - \left( \frac{h_p}{h} \right)^2 \right)^{1.325} * h^{2.238} * (\ln(r_e))^{-1.99}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 257.

### 1.138 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Simultaneous gas and water coning

#### Input(s)

- $\rho_o$ : Oil Density (g/cc)
- $\rho_w$ : Water Density (g/cc)
- $r_e$ : Drainage Radius (ft)
- $r_w$ : Radius of Wellbore (ft)
- $h_p$ : Thickness of Perforated Interval (ft)
- h: Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_o$ : Effective Oil Permeability (mD)

#### Output(s)

- $q_o$ : Critical Rate (Maximum Oil Rate without Water Coning) (STB/day)

#### Formula(s)

$$q_o = 0.001535 * \frac{\rho_w - \rho_o}{\ln\left(\frac{r_e}{r_w}\right)} * \left( \frac{k_o}{\mu_o * B_o} \right) * \left( h^2 - (h_p)^2 \right)$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 8, Page: 255.

### 1.139 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Water coning

#### Input(s)

- $\rho_o$ : Oil Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)
- $r_e$ : Radius of Drainage (ft)
- $r_w$ : Radius of Wellbore (ft)
- $h_p$ : Thickness of Perforated interval (ft)
- $h$ : Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_o$ : Effective Oil Permeability (mD)

#### Output(s)

- $q_o$ : Critical Rate (Maximum Oil Rate without Gas Coning) (STB/day)

#### Formula(s)

$$q_o = 0.001535 * \left( \frac{\rho_o - \rho_g}{\ln \left( \frac{r_e}{r_w} \right)} \right) * \left( \frac{k_o}{\mu_o * B_o} \right) * \left( h^2 - (h - h_p)^2 \right)$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 8, Page: 255.

### 1.140 Vertical well critical rate correlations—Meyer, Gardner, and Pirson—Gas coning

#### Input(s)

- $\rho_o$ : Oil Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)
- $r_e$ : Radius of Drainage (ft)
- $r_w$ : Radius of Wellbore (ft)
- $h_p$ : Thickness of Perforated interval (ft)
- $h$ : Oil Column Thickness (ft)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $k_o$ : Effective Oil Permeability (mD)

#### Output(s)

- $q_o$ : Critical Rate (Maximum Oil Rate without Gas Coning) (STB/day)

#### Formula(s)

$$q_o = 0.001535 * \left( \frac{\rho_o - \rho_g}{\ln \left( \frac{r_e}{r_w} \right)} \right) * \left( \frac{k_o}{\mu_o * B_o} \right) * \left( h^2 - (h - h_p)^2 \right)$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 8, Page: 255.

### 1.141 Viscosity

#### Input(s)

- $\mu$ : Viscosity of displacing fluid (cP)
- $d\mu$ : Change in viscosity (cP)
- $dP$ : Change in pressure (psi)

#### Output(s)

- $cv$ : Viscosity (1/psi)

#### Formula(s)

$$cv = \left( \frac{1}{\mu} \right) * \left( \frac{d\mu}{dP} \right)$$

Reference: [Petrowiki.org](http://Petrowiki.org).

### 1.142 Viscosity of crude oil through API

#### Input(s)

- $a$ : Coefficient (dimensionless)
- $A$ : API Gravity (dimensionless)

#### Output(s)

- $\mu$ : Logarithmic Viscosity (cP)

#### Formula(s)

$$\mu = a - 0.035 * A$$

Reference: Campbell, J. M., (1992, Houston, TX (United States)), Gas Conditioning and Processing, Vol. 1, Campbell Petroleum Series, Page: 74.

### 1.143 Viscosity of dead oil—Standing's correlation

#### Input(s)

- API: API Gravity of Oil (API)
- t: Temperature (F)

#### Output(s)

- $\mu_{od}$ : Viscosity of Dead Oil (cP)

#### Formula(s)

$$\mu_{od} = \left( 0.32 + \left( 1.8 * \frac{10^7}{API^{4.53}} \right) \right) \left( \frac{360}{t+200} \right)^{10^{0.43 + \frac{8.33}{API}}}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 2/20.

**1.144 Viscosity of dead-oil—Egbogah correlation—P < Pb****Input(s)**

$\rho$ : Density of oil (API)  
 $T$ : Temp (F)

**Output(s)**

$\mu_{od}$ : Viscosity of Dead-Oil (cP)

**Formula(s)**

$$\mu_{od} = 10^{1.8653 - 0.025086 * \rho - 0.5644 * \log(T)}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 41.*

**1.145 Viscosity of live oil—Beggs/Robinson correlation****Input(s)**

$\mu_{od}$ : Viscosity of dead oil (cP)  
 $R_{so}$ : Solution oil gas ratio (fraction)

**Output(s)**

A: Constant A for Beggs Robinson Correlation (fraction)  
B: Constant B for Beggs Robinson Correlation (fraction)  
 $\mu_o$ : Viscosity of Live-Oil (cP)

**Formula(s)**

$$A = 10.715 * (R_{so} + 100)^{-0.515}$$

$$B = 5.44 * (R_{so} + 150)^{-0.338}$$

$$\mu_o = A * \mu_{od}^B$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 41.*

**1.146 Viscosity of oil—Vasquez/Beggs correlation—P > Pb****Input(s)**

$\mu_{od}$ : Oil viscosity at bubble point (cP)  
 $R_{so}$ : Solution oil gas ratio (fraction)  
 $p$ : Pressure (psi)  
 $p_b$ : Bubble Point Pressure (psi)

**Output(s)**

m: Exponential Constant for Beggs Correlation (dimensionless)  
 $\mu_o$ : Viscosity of Live-Oil (cP)

### Formula(s)

$$m = 2.6 * (p^{1.187}) * \exp(-11.513 - 8.98 * (10^{-5}) * p)$$

$$\mu_o = \mu_{od} * \left(\frac{p}{p_b}\right)^m$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 41.*

### 1.147 Viscosity of water at atmospheric pressure—McCain correlation

#### Input(s)

- S: Salinity (% weight by solids)
- T: Temperature (F)

#### Output(s)

- $\mu_{w1}$ : Viscosity of water (cP)

### Formula(s)

$$\begin{aligned} \mu_{w1} = & (109.574 - 8.40564 * S + 0.313314 * S^2 + 8.72213 * 10^8 - 3 * S^3) \\ & * T^8 (-1.12166 + 2.63951 * 10^{-2} * S - 6.79461 * 10^{-4} * S^2 - 5.47119 \\ & * 10^{-5} * S^3 + 1.55586 * 10^{-6} * S^4) \end{aligned}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft and Hawkins, Page: 48.*

### 1.148 Viscosity of water at reservoir pressure—McCain correlation

#### Input(s)

- p: Pressure (psi)
- $\mu_{w1}$ : Viscosity of Water at Atmospheric Pressure (cP)

#### Output(s)

- $\mu_w$ : Viscosity of water at Reservoir Pressure (cP)

### Formula(s)

$$\mu_w = \mu_{w1} * (0.9994 + 4.0295 * 10^{-5} * p + 3.1062 * 10^{-9} * p^2)$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 48.*

### 1.149 Volume of gas adsorbed in coalbed methane reservoirs

#### Input(s)

- $V_m$ : Langmuir's Isotherm Constant (SCF/ton)
- b: Langmuir's Pressure Constant (1/psi)
- p: Pressure (psi)

#### Output(s)

- V: Volume of Gas Currently Adsorbed (SCF/ton)

**Formula(s)**

$$V = \frac{V_m * b * p}{1 + b * p}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 4, Page: 320.

**1.150 Volumetric heat capacity of a reservoir****Input(s)**

- $M_s$ : Volumetric Heat Capacity of Solids (btu/ft<sup>3</sup> F)
- $\phi$ : Porosity (fraction)
- $S_o$ : Oil Saturation (fraction)
- $M_o$ : Volumetric Heat Capacity of Oil (btu/ft<sup>3</sup> F)
- $S_w$ : Water Saturation (fraction)
- $M_w$ : Volumetric Heat Capacity of Water (btu/ft<sup>3</sup> F)
- $S_g$ : Saturation of Gas (fraction)
- $f$ : Fraction of non-condensable Gases (fraction)
- $M_g$ : Volumetric Heat Capacity of Gases (btu/ft<sup>3</sup> F)
- $\rho_s$ : Density of Solids (g/cc)
- $C_w$ : Isobaric Specific Heat of Water (btu/lb F)
- $\Delta T$ : Temperature Differential (K)
- $L_v$ : Latent Heat of Vaporization (btu/lb)

**Output(s)**

- $M_r$ : Volumetric Heat Capacity of Reservoir (btu/ft<sup>3</sup> F)

**Formula(s)**

$$M_r = (1 - \phi) * M_s + \phi * M_o * S_o + \phi * S_w * M_w + \phi * S_g * \left( f * M_g + (1 - f) * \left( \frac{L_v * \rho_s}{\Delta T} + \rho_s * C_w \right) \right)$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 12, Page: 164.

**1.151 Water breakthrough correlation in vertical wells—Bournazel and Jeanson****Input(s)**

- $h$ : Oil Column Thickness (ft)
- $g$ : Gravitational Acceleration (ft/s<sup>2</sup>)
- $k_v$ : Vertical Permeability (mD)
- $\rho_w$ : Water Density (g/cc)
- $\mu_o$ : Oil Viscosity (cP)
- $\rho_o$ : Oil Density (g/cc)
- $t_{BT}$ : Water Breakthrough Time (s)
- $\phi_e$ : Porosity (fraction)
- $f_m$ : Mobility Function Ratio (dimensionless)

**Output(s)**

- $t_d$ : Dimensionless Breakthrough Time (days)

**Formula(s)**

$$t_d = \frac{(\rho_w - \rho_o) * g * k_v * t_{BT} * f_m}{\mu_o * \phi_e * h}$$

Reference: *Bournazel, C., Jeanson, B. 1971. Fast Water-coning Evaluation Method. SPE AIME, New Orleans, Louisiana.*

**1.152 Water breakthrough correlations in vertical wells—Sobociński and Cornelius****Input(s)**

- h: Oil Column Thickness (ft)
- $h_t$ : Height of the Apex of the Water Cone above the Average Water-Oil Contact (ft)
- $k_v$ : Vertical Permeability (mD)
- $k_h$ : Horizontal Permeability (mD)
- $\rho_w$ : Water Density (g/cc)
- $\mu_o$ : Oil Viscosity (cP)
- $\rho_o$ : Oil Density (g/cc)
- $q_o$ : Oil Production Rate (STB/D)
- $B_o$ : Oil Formation Volume Factor (RB/STB)
- $\alpha$ : Constant Value of 0.5 for M<1 and 0.6 for M between 1 and 10 (RB/STB)
- M: Water Oil Mobility Ratio (fraction)
- t: Breakthrough Time (days)
- $\phi$ : Porosity (fraction)

**Output(s)**

- Z: Dimensionless Cone Height (feet)
- $t_D$ : Dimensionless Breakthrough Time (days)

**Formula(s)**

$$Z = \frac{0.00307 * (\rho_w - \rho_o) * k_h * h * h_t}{\mu_o * q_o * B_o}$$

$$t_D = \frac{0.00137 * (\rho_w - \rho_o) * k_h * (1 + M^\alpha) * t}{\mu_o * \phi * h * \left(\frac{k_h}{k_v}\right)}$$

Reference: *Sobociński, D.P., Cornelius, A.J. 1965. A Correlation for Predicting Water Coning Time. SPE ATCE, Houston, Texas.*

**1.153 Water content of sour gas****Input(s)**

- y: Mole Content of Hydrocarbon (fraction)
- Whc: Water Content of hydrocarbon Part (fraction)
- ycō: Mole Content of Carbon Dioxide (fraction)
- Wco: Water Content of Carbon Dioxide (fraction)
- yhs: Mole Content of Hydrogen Sulfide (fraction)
- Whs: Water Content of Hydrogen Sulfide (fraction)

**Output(s)**

W: Water Content (fraction)

**Formula(s)**

$$W = y * Whc + yco * Wco + yhs * Whs$$

Reference: *Campbell, J. M., (1992, Houston, TX (United States)), Gas Conditioning and Processing, Vol. 1, Campbell Petroleum Series, Page: 149.*

**1.154 Water cut—Stiles****Input(s)**

k: Permeability (mD)

h: Cumulative Thickness for Breakthrough Layer & Above (ft)

$M_{wo}$ : Mobility Ratio (fraction)

$k_t$ : Cumulative Flow Capacity for Breakthrough Layer & Above (mD ft)

$h_t$ : Cumulative Flow Capacity for the Whole Model (mD ft)

**Output(s)**

$f_w$ : Water Cut (fraction)

**Formula(s)**

$$f_w = \frac{k * h * M_{wo}}{k * h * M_{wo} + k_t * h_t - k * h}$$

Reference: *Ehrlich, R. 2016. Enhanced Oil Recovery, Lecture Notes.*

**1.155 Water-drive index for gas reservoirs****Input(s)**

$W_e$ : Cumulative Water Influx (bbl)

$W_p$ : Cumulative Water Production (bbl)

$B_w$ : Water Formation Volume Factor (rb/STB)

$G_p$ : Cumulative Gas Production at Depletion Pressure (MSCF)

$B_g$ : Gas Formation Volume Factor at Depletion Pressure (bbl/SCF)

**Output(s)**

WDI: Water Drive Index (dimensionless)

**Formula(s)**

$$WDI = \frac{W_e - W_p * B_w}{G_p * B_g}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 211.*

### 1.156 Water-drive recovery

#### Input(s)

- $\phi$ : Porosity (fraction)
- $S_w$ : Water Saturation (fraction)
- $B_{oi}$ : FVF at Initial Conditions of Reservoir (RB/STB)
- $k$ : Permeability (D)
- $\mu_{wi}$ : Water Viscosity (cP)
- $\mu_{oi}$ : Oil Viscosity (cP)
- $P_i$ : Initial Pressure (psi)
- $P_a$ : Pressure at Depletion (psi)

#### Output(s)

- $E_R$ : Fractional Recovery Efficiency (fraction)

#### Formula(s)

$$E_R = 54.898 * \left( \frac{\phi * (1 - S_w)}{B_{oi}} \right)^{0.0422} * \left( \frac{k * \mu_{wi}}{\mu_{oi}} \right)^{0.0770} * (S_w)^{-0.1903} * \left( \frac{P_i}{P_a} \right)^{-0.2159}$$

Reference: *Craig Jr. F. F., 2004, The Reservoir Engineering Aspects of Waterflooding, Vol. 3. Richardson, Texas: Monograph Series, SPE, Page: 83.*

### 1.157 Water expansion term in gas reservoirs

#### Input(s)

- $B_g$ : Initial Gas Formation Factor (MSCF/ft<sup>3</sup>)
- $c_w$ : Water Compressibility (1/psi)
- $s_w$ : Initial Water Saturation (fraction)
- $c_f$ : Formation Compressibility (1/psi)
- $P$ : Pressure Differential (psi)

#### Output(s)

- $E_f$ : Expansion Term (fraction)

#### Formula(s)

$$E_f = \frac{B_g * (c_w * s_w - c_f) * P}{1 - s_w}$$

Reference: *Ahmed, T. & McKinney, P. Advanced Reservoir Engineering, Gulf Publishing House, Burlington, MA, 2015.*

### 1.158 Water formation volume factor—McCain correlation

#### Input(s)

- $T$ : Temperature (F)
- $p$ : Pressure (psi)

**Output(s)**

$B_w$ : Water Formation Volume Factor (McCain Correlation) (bbl/STB)

**Formula(s)**

$$B_w = \left( 1 - 1.00010 * 10^{-2} + 1.33391 * (10^{-4}) * T + 5.50654 * (10^{-7}) * T^2 \right) \\ * \left( 1 - 1.95301 * 10^{-9} * p * T - 1.72834 * 10^{-13} * p^2 * T - 3.58922 * 10^{-7} * p - 2.25341 * 10^{-10} * p^2 \right)$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 45.*

**1.159 Water influx—Pot aquifer model****Input(s)**

$c_t$ : Aquifer Total Compressibility (1/psi)

$W_i$ : Initial Volume of Water in Aquifer (bbl)

$P_i$ : Initial Reservoir Pressure (psi)

$P$ : Current Reservoir Pressure (Pressure at Oil-Water Contact) (psi)

**Output(s)**

$W_e$ : Cumulative Water Influx (bbl)

**Formula(s)**

$$W_e = c_t * W_i * (P_i - P)$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 2, Page: 152.*

**1.160 Water influx constant for the van Everdingen and Hurst unsteady-state model****Input(s)**

$c_t$ : Total Aquifer Compressibility (1/psi)

$\emptyset$ : Porosity of the Aquifer (fraction)

$r_e$ : Radius of Reservoir (ft)

$h$ : Thickness of the Aquifer (ft)

$f$ : Encroachment Angle (Divided by 360)

**Output(s)**

$B$ : Water Influx (bbl)

**Formula(s)**

$$B = 1.119 * \emptyset * c_t * (r_e^2) * h * f$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 4, Page: 310.*

### 1.161 Water two-phase formation volume factor

#### Input(s)

- $B_w$ : Water Formation Volume Factor (one phase) (rb/STB)
- $B_g$ : Gas Formation Volume Factor (MSCF/STB)
- $R_s$ : Gas Oil ratio (rb/MSCF)
- $R_i$ : Initial Gas Oil Ratio (rb/MSCF)

#### Output(s)

- $B_t$ : Water Formation Factor (rb/STB)

#### Formula(s)

$$B_t = B_w + B_g * (R_s - R_i)$$

Reference: Ahmed, T. & McKinney, P. *Advanced reservoir Engineering*, Gulf Publishing House, Burlington, MA, 2015.

### 1.162 Waxman and Smits model—Clean sands

#### Input(s)

- $F$ : Shaly-Sand Formation Resistivity Factor (dimensionless)
- $\phi_e$ : Effective Porosity (fraction)
- $C_{cl}$ : Clay Exchange Cation Conductivity (1/Ω m)
- $C_w$ : Formation Water Conductivity (1/Ω m)

#### Output(s)

- $C_o$ : Brine Saturated Rock Conductivity (1/Ω m)

#### Formula(s)

$$C_o = \left( \frac{1}{F} \right) * (C_{cl} + C_w)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 17.

### 1.163 Welge extension—Fractional flow

#### Input(s)

- $f_o$ : Fractional Flow of Oil Obtained (fraction)
- $\mu_w$ : Viscosity of Water (cP)
- $\mu_o$ : Viscosity of Oil (cP)

#### Output(s)

- relpr: Relative Permeability Ratio of Water to Oil (fraction)

#### Formula(s)

$$\text{relpr} = \frac{\mu_w}{\mu_o} * \left( \frac{1-f_o}{f_o} \right)$$

Reference: Dandekar, A. Y. 2006. *Petroleum Reservoir Rock and Fluid Properties*. Boca Raton, FL: CRC Press Taylor & Francis Group, Chapter: 9, Page: 215.

## Chapter 2

# Drilling engineering formulas and calculations

### Chapter Outline

2.1 Accumulator capacity	73	2.35 Control drilling—Maximum drilling rate	89
2.2 Accumulator precharge pressure	74	2.36 Conversion of pressure into the mud weight	90
2.3 Amount of additive required to achieve a required cement slurry density	74	2.37 Cost per foot during drilling	90
2.4 Amount of cement to be left in casing	75	2.38 Cost per foot of coring	91
2.5 Amount of mud required to displace cement in drillpipe	75	2.39 Critical annular velocity and critical flow rate	91
2.6 Angle of twist—Rod subjected to torque	75	2.40 Critical flow rate for flow regime change	92
2.7 Annular capacity between casing and multiple strings of tubing	76	2.41 Critical velocity for change in flow regime	92
2.8 Annular capacity between casing and multiple tubing strings	76	2.42 Crown block capacity	93
2.9 Annular velocity—Using circulation rate in GPM	77	2.43 Current drag force—Offshore	93
2.10 Annular velocity—Using pump output in bbl/min	77	2.44 Curvature radius for a borehole	94
2.11 Annular velocity for a given circulation rate	78	2.45 Cutting slip velocity	94
2.12 Annular velocity for a given pump output	78	2.46 Cuttings produced per foot of hole drilled—bbls	94
2.13 Annular volume capacity of pipe	78	2.47 Cuttings produced per foot of hole drilled—lbs	95
2.14 API water loss calculations	79	2.48 D—Exponent	95
2.15 Area below the casing shoe	79	2.49 Depth of a washout—Method 1	96
2.16 Axial loads in slips	79	2.50 Depth of a washout—Method 2	96
2.17 Beam force	80	2.51 Derrick efficiency factor	96
2.18 Bit nozzle pressure loss	80	2.52 Difference in pressure gradient between the cement and mud	97
2.19 Bit nozzle selection—Optimized hydraulics for two and three jets	80	2.53 Differential hydrostatic pressure between cement in the annulus and mud inside the casing	97
2.20 Borehole torsion—Cylindrical helical method	81	2.54 Dilution of a mud system	98
2.21 Bottomhole annulus pressure	82	2.55 Direction of dip	98
2.22 Bottomhole assembly length required for a desired weight on bit	83	2.56 Directional curvature for a deviated well	99
2.23 Bulk density of cuttings—Using the mud balance	83	2.57 Downward force or weight of casing	99
2.24 Bulk modulus using Poisson's ratio and Young's modulus	83	2.58 Drill pipe or drill collar capacity	99
2.25 Buoyancy weight	84	2.59 Drill pipe or drill collar displacement and weight	100
2.26 Buoyancy factor	84	2.60 Drill string design—Drill pipe length for bottomhole assembly	100
2.27 Buoyancy factor using mud weight	84	2.61 Drilled gas entry rate	101
2.28 Calculations for the number of feet to be cemented	85	2.62 Drilling cost per foot	101
2.29 Calculations required for spotting pills	85	2.63 Drilling ton miles—Coring operation ton miles	102
2.30 Capacity formulas—bbl/ft	85	2.64 Drilling ton miles—Drilling/connection ton miles	102
2.31 Capacity formulas—gal/ft	86	2.65 Drilling ton miles—Round trip ton miles	102
2.32 Capacity of tubulars and open-hole	86	2.66 Drilling ton miles—While making short trip ton miles	103
2.33 CO <sub>2</sub> solubility in oil and oil-mud emulsifiers	87	2.67 Drilling ton miles—Setting casing ton miles	103
2.34 Combined solubility—Hydrocarbon gas, CO <sub>2</sub> , and H <sub>2</sub> S—in each of the mud components	88	2.68 Duplex pump factor	104
	88	2.69 Duplex pump output—Using liner diameter	104
	88	2.70 Duplex pump output—Using rod diameter	104
	89	2.71 Duplex pump output by using liner and rod diameters	105
	89	2.72 Dynamically coupled linear flow—Formation invasion	105

<b>2.73 Effective weight during drilling</b>	<b>106</b>	<b>2.120 Kick analysis—Maximum pit gain from a gas kick in water-based mud</b>	<b>126</b>
<b>2.74 Effective wellbore radius for finite-conductivity fractures</b>	<b>106</b>	<b>2.121 Kick analysis—Maximum surface pressure from a gas kick in water-based mud</b>	<b>126</b>
<b>2.75 Effective wellbore radius in infinite-conductivity fractures</b>	<b>107</b>	<b>2.122 Kick analysis—Shut-in drill pipe pressure</b>	<b>127</b>
<b>2.76 Efficiency of block and tackle system</b>	<b>107</b>	<b>2.123 Kick analysis—Height of influx</b>	<b>127</b>
<b>2.77 Equivalent area of pipe subject to uniform axial force</b>	<b>108</b>	<b>2.124 Kill weight mud determination—Moore equation</b>	<b>127</b>
<b>2.78 Equivalent circulating density</b>	<b>108</b>	<b>2.125 Kinetic friction</b>	<b>128</b>
<b>2.79 Equivalent density of a wellbore fluid</b>	<b>109</b>	<b>2.126 Laser specific energy</b>	<b>128</b>
<b>2.80 Equivalent formation water resistivity from SP log</b>	<b>109</b>	<b>2.127 Lateral load imposed on a casing centralizer—Cementing</b>	<b>129</b>
<b>2.81 Equivalent mud weight—Deviated well</b>	<b>109</b>	<b>2.128 Lateral load imposed on a casing centralizer with a dogleg—Cementing</b>	<b>129</b>
<b>2.82 Equivalent mud weight—Vertical well</b>	<b>110</b>	<b>2.129 Linear annular capacity of pipe</b>	<b>129</b>
<b>2.83 Evaluation of centrifuge</b>	<b>110</b>	<b>2.130 Linear capacity of pipe</b>	<b>130</b>
<b>2.84 Evaluation of hydrocyclone</b>	<b>111</b>	<b>2.131 Load to break cement bond—Cementing</b>	<b>130</b>
<b>2.85 Fluid volume required to spot a plug</b>	<b>111</b>	<b>2.132 Mass rate of flow through annulus</b>	<b>131</b>
<b>2.86 Force applied to stretch material</b>	<b>112</b>	<b>2.133 Matching conditions at the cake-to-rock interface—Formation invasion</b>	<b>131</b>
<b>2.87 Force exerted by the fluid on the solid surface of flow through an annulus</b>	<b>112</b>	<b>2.134 Maximum allowable mud weight</b>	<b>131</b>
<b>2.88 Friction factor in drill pipe</b>	<b>113</b>	<b>2.135 Maximum drilling rate—Larger holes</b>	<b>132</b>
<b>2.89 Front displacement of a particle in the reservoir—Formation invasion</b>	<b>113</b>	<b>2.136 Maximum equivalent derrick load</b>	<b>132</b>
<b>2.90 Gas migration velocity</b>	<b>114</b>	<b>2.137 Maximum length of a slanted well in a given reservoir thickness</b>	<b>133</b>
<b>2.91 Gas solubility in a mud system</b>	<b>114</b>	<b>2.138 Maximum length of drillpipe for a specific bottomhole assembly</b>	<b>133</b>
<b>2.92 Gas/mud ratio</b>	<b>115</b>	<b>2.139 Maximum recommended low-gravity solids</b>	<b>133</b>
<b>2.93 Gel strength—Optimal solid removal efficiency</b>	<b>115</b>	<b>2.140 Maximum recommended solids fractions in drilling fluids</b>	<b>134</b>
<b>2.94 Gel strength—Solid control efficiency</b>	<b>116</b>	<b>2.141 Maximum weight on bit</b>	<b>134</b>
<b>2.95 Gel strength—Solids build-up in system</b>	<b>116</b>	<b>2.142 Mechanical energy balance for wellbore fluids</b>	<b>134</b>
<b>2.96 Height of cement in the annulus</b>	<b>117</b>	<b>2.143 Mechanical specific energy</b>	<b>135</b>
<b>2.97 Hydraulic horsepower</b>	<b>117</b>	<b>2.144 Mud rheology—Herschel and Buckley law</b>	<b>135</b>
<b>2.98 Hydraulics analysis</b>	<b>118</b>	<b>2.145 Mud rheology—Power-law model—Consistency index</b>	<b>136</b>
<b>2.99 Hydromechanical specific energy</b>	<b>118</b>	<b>2.146 Mud rheology—Power-law model—Power-law index</b>	<b>136</b>
<b>2.100 Hydrostatic pulling</b>	<b>119</b>	<b>2.147 Mud rheology—Power-law</b>	<b>136</b>
<b>2.101 Hydrostatic pulling wet pipe out of the hole</b>	<b>119</b>	<b>2.148 Mud rheology calculations—Bingham plastic model</b>	<b>137</b>
<b>2.102 Hydrostatic pressure in annulus due to slug</b>	<b>120</b>	<b>2.149 Mud weight increase required to balance pressure</b>	<b>137</b>
<b>2.103 Hydrostatic pressure decrease at total depth caused by gas-cut mud</b>	<b>120</b>	<b>2.150 Mud weight reduction by dilution—Water/diesel/any liquid</b>	<b>137</b>
<b>2.104 Impact force—Nozzle hydraulic analysis</b>	<b>121</b>	<b>2.151 Mudcake growth equation—Formation invasion</b>	<b>138</b>
<b>2.105 Impringing jet</b>	<b>121</b>	<b>2.152 Mudcake growth equation-2—Formation invasion</b>	<b>138</b>
<b>2.106 Increase mud density by barite</b>	<b>122</b>	<b>2.153 Mudcake permeability—Formation invasion</b>	<b>139</b>
<b>2.107 Increase mud density by calcium carbonate</b>	<b>122</b>	<b>2.154 New pump circulating pressure</b>	<b>139</b>
<b>2.108 Increase mud density by hematite</b>	<b>123</b>	<b>2.155 Nozzle area calculation</b>	<b>139</b>
<b>2.109 Increase volume by barite</b>	<b>123</b>	<b>2.156 Number of sacks of cement required</b>	<b>140</b>
<b>2.110 Increase volume by calcium carbonate</b>	<b>123</b>	<b>2.157 Number of sacks of cement required for a given length of plug</b>	<b>141</b>
<b>2.111 Increase volume by hematite</b>	<b>124</b>	<b>2.158 Number of sacks of lead cement required for annulus</b>	<b>141</b>
<b>2.112 Initial volume required to achieve a volume with barite</b>	<b>124</b>	<b>2.159 Number of sacks of tail cement required for casing</b>	<b>141</b>
<b>2.113 Initial volume required to achieve a volume with calcium carbonate</b>	<b>124</b>	<b>2.160 Open-ended displacement volume of pipe</b>	<b>142</b>
<b>2.114 Initial volume required to achieve a volume with hematite</b>	<b>124</b>	<b>2.161 Overall efficiency—Diesel engines to mud pump</b>	<b>142</b>
<b>2.115 Injection/casing pressure required to open valve</b>	<b>125</b>	<b>2.162 Overall power system efficiency</b>	<b>143</b>
<b>2.116 Input power of a pump—Using fuel consumption rate</b>	<b>125</b>	<b>2.163 Penetration rate—Drill-rate model—Alternative equation</b>	<b>143</b>
<b>2.117 Jet velocity—Nozzle hydraulic analysis</b>	<b>126</b>	<b>2.164 Penetration rate—Drill-rate model—Basic equation</b>	<b>143</b>
<b>2.118 Kick analysis—Influx</b>			
<b>2.119 Kick analysis—Formation pressure with well shut-in on a kick</b>			

2.165 Percentage of bit nozzle pressure loss	
2.166 Plastic viscosity—Bingham plastic model	
2.167 Plug length to set a balanced cement plug	
2.168 Polar moment of inertia	
2.169 Polished rod horsepower—Sucker-Rod pump	
2.170 Pore-pressure gradient—Rehm and Mcclendon	
2.171 Pore-pressure gradient—Zamora	
2.172 Pressure analysis—Pressure by each barrel of mud in casing	
2.173 Pressure analysis—Surface pressure during drill stem test	
2.174 Pressure gradient	
2.175 Pressure required to break circulation—Annulus	
2.176 Pressure required to break circulation—Drill string	
2.177 Pressure required to overcome gel strength of mud inside the drill string	
2.178 Pressure required to overcome mud's gel strength in annulus	
2.179 Pump calculation—Pump pressure	
2.180 Pump calculations—Power required	
2.181 Pump displacement	
2.182 Pump flow rate	
2.183 Pump head rating	
2.184 Pump output—gpm	
2.185 Pump output triplex pump	
2.186 Pump pressure/pump stroke relationship	
2.187 Radial force related to axial load—Cementing	
2.188 Range of load—Sucker-Rod pump	
2.189 Rate of fuel consumption by a pump	
2.190 Rate of gas portion that enters the mud	
2.191 Relationship between traveling block speed and fast line speed	
2.192 Rock removal rate	
2.193 Rotating horsepower	
2.194 Side force at bit in anisotropic formation	
2.195 Sinusoidal buckling	
2.196 Slurry density for cementing calculations	
2.197 Solids analysis—High-salt content muds	
2.198 Solids analysis low-salt content muds	
2.199 Spacer volume behind slurry required to balance the plug	
2.200 Specific gravity of cuttings by using mud balance	
2.201 Stripping/snubbing calculations—Breakover point between stripping and snubbing	
2.202 Stripping/snubbing calculations—Height gain and casing pressure from stripping into influx	
144      2.203 Stripping/snubbing calculations—Maximum allowable surface pressure governed by casing burst pressure	159
145      2.204 Stripping/snubbing calculations—Maximum allowable surface pressure governed by formation	160
145      2.205 Stripping/snubbing calculations—Minimum surface pressure before stripping	160
146      2.206 Stripping/snubbing calculations—Constant bhp with a gas bubble rising	161
147      2.207 Stroke per minute required for a given annular velocity	161
147      2.208 Stuck pipe calculations—Method-1	161
147      2.209 Stuck pipe calculations—Method-2	162
148      2.210 Subsea considerations—Adjusting choke line pressure loss for higher mud weight	162
148      2.211 Subsea considerations—Casing burst pressure-subsea stack	162
148      2.212 Subsea considerations—Choke line pressure loss	163
149      2.213 Subsea considerations—Maximum allowable mud weight—Subsea stack from leakoff test	163
149      2.214 Subsea considerations—Casing pressure decrease when bringing well on choke	164
150      2.215 Subsea considerations—Velocity through choke line	164
151      2.216 Surface test pressure required to frac the formation	164
151      2.217 Total amount of solids generated during drilling	165
151      2.218 Total heat energy consumed by the engine	165
152      2.219 Total number of sacks of tail cement required	166
152      2.220 Total water requirement per sack of cement	166
152      2.221 Triplex pump factor	166
153      2.222 Upward force acting at the bottom of the casing shoe	167
153      2.223 Vertical curvature for deviated wells	167
153      2.224 Viscous shear stress at the outer mudcake boundary	167
154      2.225 Volume of cuttings generated per foot of hole drilled	168
154      2.226 Volume of dilution water or mud required to maintain circulating volume	168
155      2.227 Volume of fluid displaced for duplex pumps	168
155      2.228 Volume of fluid displaced for single-acting pump	169
156      2.229 Volume of fluid displaced for triplex pump	169
157      2.230 Volume of liquid (oil plus water) required to prepare a desired volume of mud	170
157      2.231 Volume of slurry per sack of cement	170
158      2.232 Volumes and strokes—Annular volume	171
158      2.233 Volumes and strokes—Drill string volume	171
158      2.234 Volumes and strokes—Total strokes	171
158      2.235 Weight of additive per sack of cement	172
159      2.236 Weighted cementing calculations	172

## 2.1 Accumulator capacity

### Input(s)

- BC: Bottle Volume per capacity (gallons)  
 $P_p$ : Pre-charge Pressure (psi)  
 $P_s$ : System Pressure (psi)  
 $P_f$ : Final Pressure (psi)

### Output(s)

V: Accumulator Capacity (gallon)

### Formula(s)

$$V = BC * \left( \frac{P_p}{P_f} - \frac{P_p}{P_s} \right)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 39.*

## 2.2 Accumulator precharge pressure

### Input(s)

$V_a$ : Total Accumulator Volume (bbl)  
 $P_s$ : Starting Accumulator Pressure (psi)  
 $V_r$ : Volume of Fluid Removed (bbl)  
 $P_f$ : Final Accumulator Pressure (psi)

### Output(s)

P: Accumulator Pressure (psi)

### Formula(s)

$$P = \frac{V_r}{V_a} * \left( \frac{P_f * P_s}{P_s - P_f} \right)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 41.*

## 2.3 Amount of additive required to achieve a required cement slurry density

### Input(s)

$\rho$ : Required Slurry Density (lb/gal)  
 $S_{Gc}$ : Specific Gravity of Cement (unitless)  
 $C$ : Water Requirement of Cement (gal/stroke)  
 $A$ : Water Requirement of Additive (gal/stroke)  
 $S_{Ga}$ : Specific Gravity of Additive (unitless)

### Output(s)

x: Amount of Additive Required (lb/stroke)

**Formula(s)**

$$x = \frac{\left( \frac{\rho * 11.207983}{S_{Gc}} \right) + (\rho * C) - 94 - (8.33 * C)}{\left( 1 + \left( \frac{A}{100} \right) \right) - \left( \frac{\rho}{8.33 * S_{Ga}} \right) - \left( \rho + \frac{A}{100} \right)}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 53.*

## 2.4 Amount of cement to be left in casing

**Input(s)**

- L<sub>c</sub>: Casing Length (ft)
- D: Setting Depth of Cementing Tool (ft)
- C<sub>c</sub>: Casing Capacity (ft<sup>3</sup>/ft)

**Output(s)**

- AC: Amount of Cement (ft<sup>3</sup>)

**Formula(s)**

$$AC = (L_c - D) * C_c$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 58.*

## 2.5 Amount of mud required to displace cement in drillpipe

**Input(s)**

- L<sub>d</sub>: Length of Drillpipe (ft)
- C<sub>p</sub>: Drill Pipe Capacity (bbl/ft)

**Output(s)**

- A<sub>m</sub>: Amount of Mud Required (bbl)

**Formula(s)**

$$A_m = L_d * C_p$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 58.*

## 2.6 Angle of twist—Rod subjected to torque

**Input(s)**

- E: Modulus of Elasticity (psi)
- v: Poisson (dimensionless)

D<sub>o</sub>: Outer Diameter (ft)  
 D<sub>i</sub>: Inner Diameter (ft)  
 T: Torque (ft lbf)  
 L: Length of Section (ft)

### Output(s)

G: Modulus of Rigidity (psi)  
 J: Polar Moment of Inertia (ft<sup>4</sup>)  
 θ: Angle of Twist (rad)

### Formula(s)

$$G = \frac{E}{2 * (1 + \nu)}$$

$$J = \frac{3.142 * ((D_o)^4 - (D_i)^4)}{32}$$

$$\theta = \frac{T * L}{G * J}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 369.*

## 2.7 Annular capacity between casing and multiple strings of tubing

### Input(s)

D<sub>i</sub>: Inner Diameter of Casing (in.)  
 T<sub>1</sub>: Diameter of Tubing 1 (in.)  
 T<sub>2</sub>: Diameter of Tubing 2 (in.)

### Output(s)

C<sub>a</sub>: Annular Capacity (gal/ft)

### Formula(s)

$$C_a = \frac{D_i^2 - (T_1^2 + T_2^2)}{1029.4}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 15.*

## 2.8 Annular capacity between casing and multiple tubing strings

### Input(s)

D<sub>h</sub>: Hole Size (in.)  
 T<sub>a</sub>: Outer Diameter of Tubing No.1 (in.)  
 T<sub>b</sub>: Outer Diameter of Tubing No.2 (in.)

**Output(s)**

AC: Annular Capacity (gal/ft)

**Formula(s)**

$$AC = \frac{D_h^2 - (T_a^2 + T_b^2)}{1029.4}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 14.*

**2.9 Annular velocity—Using circulation rate in Gpm****Input(s)**

Q: Circulation Rate (Pump Output) (gpm)

D<sub>h</sub>: Inside Diameter of Casing (in.)

D<sub>p</sub>: Outside Diameter of Pipe, Tubing or Collars (in.)

**Output(s)**

AV: Annular Velocity (ft/min)

**Formula(s)**

$$AV = \frac{24.5 * Q}{(D_h^2) - (D_p^2)}$$

Notes: The Formula Calculates Annular Velocity (ft/min) using Circulation Rate (Pump Output) (gal/min)

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 10.*

**2.10 Annular velocity—Using pump output in bbl/min****Input(s)**

Q: Circulation Rate (gpm)

D<sub>h</sub>: Inside Diameter of Casing or Hole Size (in.)

D<sub>p</sub>: Outside Diameter of Pipe, Tubing or Collars (in.)

**Output(s)**

AV: Annular Velocity (ft/min)

**Formula(s)**

$$AV = \frac{[24.5 * Q]}{\left[(D_h)^2 - (D_p)^2\right]}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 10.*

## 2.11 Annular velocity for a given circulation rate

### Input(s)

- Q: Circulation Rate (gpm)
- $d_h$ : Inner Diameter of Casing (in.)
- $d_p$ : Outer Diameter of Casing (in.)

### Output(s)

- AV: Annular Velocity (ft/min)

### Formula(s)

$$AV = \frac{24.5 * Q}{(d_h^2 - d_p^2)}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 21.*

## 2.12 Annular velocity for a given pump output

### Input(s)

- $O_p$ : Pump Output (bbl/min)
- $d_h$ : Inner Diameter of Casing (in.)
- $d_p$ : Outer Diameter of Casing (in.)

### Output(s)

- AV: Annular Velocity (bbl/min)

### Formula(s)

$$AV = \frac{O_p * 1029.4}{d_h^2 - d_p^2}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 21.*

## 2.13 Annular volume capacity of pipe

### Input(s)

- $D_h$ : Inner Diameter of Casing against Pipe (in.)
- $D_o$ : Outside Diameter of Pipe (in.)
- L: Length of Pipe (ft)

### Output(s)

- $V_a$ : Annular Volume Capacity (bbl)

**Formula(s)**

$$V_a = \frac{0.7854 * (D_h^2 - D_o^2) * L}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 44.*

## 2.14 API water loss calculations

**Input(s)**

- $V_a$ : Water Loss in 7.5 min ( $\text{cm}^3$ )  
 $V_{sp}$ : Spurt Loss ( $\text{cm}^3$ )

**Output(s)**

- $V_{30}$ : Water Loss ( $\text{cm}^3$ )

**Formula(s)**

$$V_{30} = (2 * V_a) - (V_{sp})$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 199.*

## 2.15 Area below the casing shoe

**Input(s)**

- $D_c$ : Casing Diameter (in.)

**Output(s)**

- $A$ : Area Below Shoe ( $\text{in.}^2$ )

**Formula(s)**

$$A = D_c^2 * 0.7854$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 67.*

## 2.16 Axial loads in slips

**Input(s)**

- $A$ : Cross Sectional Area of the Pipe body ( $\text{in.}^2$ )  
 $\sigma$ : Yield Strength of the Casing (psi)  
 $r$ : Outside Casing Radius (in.)  
 $L$ : Slip Gripping Length (in.)  
 $K$ : Transverse Load Factor (dimensionless)

### Output(s)

- $F_c$ : Critical Axial Load (lbf)  
 $C$ : Crushing Factor (dimensionless)

### Formula(s)

$$F_c = C \cdot A \cdot \sigma$$

$$C = \frac{1}{\left[ 1 + \frac{rK}{L} + \left( \frac{rK}{L} \right)^2 \right]^{1/2}}$$

Reference: *Suman Jr, G. O., & Ellis, R. C. (1977). Cementing Handbook. World Oil, Page: 18.*

## 2.17 Beam force

### Input(s)

- H: Significant Wave Height (ft)  
B: Vessel Beam Length (ft)  
L: Vessel Length (ft)  
A: Wave Period (s)  
D: Vessel Draft (ft)

### Output(s)

- $F_{ba}$ : Beam Force When a Is Greater Than  $0.642 * (b + 2 * d) * 0.5$  ( $ft^5/s$ )  
 $F_{bb}$ : Beam Force When a Is Lower Than  $0.642 * (b + 2 * d) * 0.5$  ( $ft^5/s$ )

### Formula(s)

$$F_{ba} = \frac{2.1 * (H^2) * (B^2) * L}{A^4}$$

$$F_{bb} = \frac{2.1 * (H^2) * (B^2) * L}{\left( 1.28 * \left( (B + 2 * D)^{0.5} \right) - A \right)^4}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 457.*

## 2.18 Bit nozzle pressure loss

### Input(s)

- $N_a$ : Nozzle Area ( $in.^2$ )  
Q: Flow Rate (gpm)  
MW: Mud Weight (ppg)

**Output(s)**

$P_b$ : Bit Nozzle Pressure Loss (psi)

**Formula(s)**

$$P_b = (Q^2) * \frac{MW}{10858 * (N_a)^2}$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover, Second Edition*, Gulf Professional Publishing, Page: 165.

## 2.19 Bit nozzle selection—Optimized hydraulics for two and three jets

**Input(s)**

$N_a$ : Jet Size for nozzle 1 (in.)

$N_b$ : Jet Size for nozzle 2 (in.)

$N_c$ : Jet Size for nozzle 3 (in.)

$C_a$ : Circulation Rate (gpm)

$C_b$ : Circulation Rate (gpm)

$W_m$ : Mud Weight (ppg)

$C_{pa}$ : Circulating Pressure 1 (psi)

$C_{pb}$ : Circulating Pressure 2 (psi)

$P_{max}$ : Max surface pressure (psi)

$d$ : Dia of the nozzle chosen (in.)

$n$ : No. of nozzles (unit)

**Output(s)**

$A$ : Nozzle Area ( $\text{in.}^2$ )

$P_{ba}$ : Bit Nozzle Pressure Loss 1 (psi)

$P_{bb}$ : Bit Nozzle Pressure Loss 2 (psi)

$P_{ca}$ : Total Pressure Loss for pump pressure 1 except bit nozzle (psi)

$P_{cb}$ : Total Pressure Loss for pump pressure 2 except bit nozzle (psi)

$M$ : Slope of Line (unitless)

$P_{iopt}$ : Optimum Pressure for Impact Force (psi)

$P_{hopt}$ : Optimum Pressure for hydraulic Horsepower (psi)

$P_{ilb}$ : Pressure loss at the bit due to impact force (psi)

$P_{hlb}$ : Pressure loss at the bit due to hydraulic horsepower (psi)

$Q_{iopt}$ : Optimum flowrate for impact force (gpm)

$Q_{hopt}$ : Optimum flowrate for hydraulic horsepower (gpm)

$A_{in}$ : Area of nozzle for Impact force ( $\text{in.}^2$ )

$A_{hn}$ : Area of nozzle used for hydraulic horsepower ( $\text{in.}^2$ )

$A_n$ : Area of nozzle used ( $\text{in.}^2$ )

### Formula(s)

$$A = \frac{N_a^2 + N_b^2 + N_c^2}{1303.8}$$

$$Pba = (Ca)^2 * \frac{Wm}{10858 * A}$$

$$Pbb = (Cb)^2 * \frac{Wm}{10858 * A}$$

$$P_{ca} = Cpa - Pba$$

$$P_{cb} = Cpb - Pbb$$

$$M = \frac{\log\left(\frac{P_{ca}}{P_{cb}}\right)}{\log\left(\frac{Ca}{Cb}\right)}$$

$$P_{iopt} = 2 * \frac{P_{max}}{M + 1}$$

$$P_{hopt} = 2 * \frac{P_{max}}{P_{max} + 1}$$

$$Pilb = \left(\frac{P_{iopt}}{P_{max}}\right)^{1+M} * Ca$$

$$Phlb = \left(\frac{P_{hopt}}{P_{max}}\right)^{1+M} * Ca$$

$$Q_{iopt} = P_{max} - P_{iopt}$$

$$Q_{hopt} = P_{max} - P_{hopt}$$

$$A_{in} = \left(Q_{iopt}^2 * \frac{Wm}{10858 * P_{max}}\right)^{0.5}$$

$$A_{hn} = \left(Q_{hopt}^2 * \frac{Wm}{10858 * P_{max}}\right)^{0.5}$$

$$A_n = d * \left(\frac{A}{n * 0.7854}\right)^{0.5}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 166.*

## 2.20 Borehole torsion—Cylindrical helical method

### Input(s)

- k<sub>p</sub>: Horizontal Curvature (degree/100ft)
- k<sub>v</sub>: Vertical Curvature (degree/100ft)
- k: Curvature of Wellbore Trajectory (degree/100ft)
- a: Inclination Angle (degree)

### Output(s)

- t: Borehole Torsion—Cylindrical Helical Method (degree/100ft)

**Formula(s)**

$$t = k_h * \left( 1 + \left( 2 * \frac{(k_v)^2}{k^2} \right) \right) * \sin(a) * \cos(a)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 84.*

**2.21 Bottomhole annulus pressure****Input(s)**

$\rho_a$ : Annulus Fluid Density (ppg)

$D_{av}$ : Vertical Height in the Annulus (ft)

**Output(s)**

$P_{bh}$ : Annulus Pressure (psi)

**Formula(s)**

$$P_{bh} = 0.052 * (\rho_a) * (D_{av})$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 134.*

**2.22 Bottomhole assembly length required for a desired weight on bit****Input(s)**

$W_b$ : Desired Weight to be used while Drilling (lb)

f: Safety Factor (fraction)

$W_d$ : Drill Collar Weight (lb/ft)

BF: Buoyancy Factor (unitless)

**Output(s)**

L: Length of Bottomhole Assembly (ft)

**Formula(s)**

$$L = \frac{W_b * f}{W_d * BF}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 42.*

**2.23 Bulk density of cuttings—Using the mud balance****Input(s)**

$W_r$ : Resulting Mud Weight with Cuttings and Water (ppg)

### Output(s)

SG<sub>c</sub>: Specific Gravity (Average Bulk Density) of Cuttings (gm/cm<sup>3</sup>)

### Formula(s)

$$SG_c = \frac{1}{2 - (0.12 * W_r)}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 21.*

## 2.24 Bulk modulus using Poisson's ratio and Young's modulus

### Input(s)

E: Young's Modulus (N/m<sup>2</sup>)

$\mu$ : Poisson's Ratio (dimensionless)

### Output(s)

K: Bulk Modulus (N/m<sup>2</sup>)

### Formula(s)

$$K = \frac{E}{3 * (1 - 2 * \mu)}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 3, Page: 46.*

## 2.25 Buoyancy weight

### Input(s)

$\rho_m$ : Density of Mud (ppg)

$\rho_s$ : Density of Pipe Material (ppg)

d: Depth (ft)

m: Weight (ppf)

### Output(s)

B: Buoyancy Factor (fraction)

W: Buoyancy Weight (lbf)

### Formula(s)

$$B = \left( 1 - \left( \frac{\rho_m}{\rho_s} \right) \right)$$

$$W = B * m * d$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 20.*

## 2.26 Buoyancy factor

### Input(s)

MW: Mud Weight (ppg)

### Output(s)

BF: Buoyancy Factor (ppg)

### Formula(s)

$$BF = \frac{65.5 - MW}{65.5}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 20.*

## 2.27 Buoyancy factor using mud weight

### Input(s)

MW: Mud Weight (ppg)

65.5: Weight of Steel (Plain Carbon Steel 1020 (AISI and SAE) = 7.86 gm/cm<sup>3</sup> (ppg))

### Output(s)

BF: Buoyancy Factor (fraction)

### Formula(s)

$$BF = \frac{65.5 - MW}{65.5}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 24.*

## 2.28 Calculations for the number of feet to be cemented

### Input(s)

H: Hole Size (in.)

OD: Outer Diameter (in.)

ID: Inner Diameter (in.)

N<sub>s</sub>: Number of Sacks of Cement to be Used (sacks)

Y<sub>s</sub>: Slurry Yield (ft<sup>3</sup>/stroke)

h<sub>c</sub>: Feet of Casing (ft)

h<sub>sd</sub>: Setting Depth of Cementing Tool or Feet of Drill Pipe (ft)

DC: Drill Pipe Capacity (bbl/stroke)

E: Original+extra Volume Req in Percentage (fraction)

PO: Pump Output (bbl/stroke)

**Output(s)**

AC: Annular Capacity ( $\text{ft}^3/\text{ft}$ )  
 CC \* \*: Casing Capacity ( $\text{ft}^3/\text{ft}$ )  
 C<sub>c</sub>: Casing Capacity ( $\text{ft}^3$ )  
 V<sub>s</sub>: Volume of Slurry ( $\text{ft}^3$ )  
 h<sub>a</sub>: Height of Cement in Annulus (ft)  
 h<sub>tc</sub>: Depth of Top of Cement in Annulus ( $\text{ft}^3/\text{stroke}$ )  
 N<sub>m</sub>: Number of Barrels of Mud Required to Displace the Cement (bbl)  
 S: Number of Strokes Required to Displace the Cement (Strokes)

**Formula(s)**

$$\text{AC} = \frac{\text{H}^2 - \text{OD}^2}{183.35}$$

$$\text{CC} = \frac{(\text{ID})^2}{183.35}$$

$$C_c = N_s * Y_s$$

$$V_s = (h_c - h_{sd}) * CC$$

$$h_a = \frac{(V_s - C_c)}{\frac{AC}{E}}$$

$$h_{tc} = h_c - h_a$$

$$N_m = h_{sd} * DC$$

$$S = \frac{N_m}{PO}$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover*, Second Edition, Gulf Professional Publishing, Page: 57.

**2.29 Calculations required for spotting pills****Input(s)**

hd: Diameter of Hole (in.)  
 pd: Diameter of Drill Pipe (in.)  
 cd: Diameter of Drill Collar (in.)  
 L<sub>s</sub>: Length of Section (ft)  
 L<sub>e</sub>: Length Above the Collar (ft)  
 WF: Washout Factor (fraction)  
 V<sub>ds</sub>: Volume Left in Drill String (bbl)  
 C<sub>dc</sub>: Drill Collar Capacity (bbl/ft)  
 C<sub>dp</sub>: Drill Pipe Capacity (bbl/ft)  
 L: Length (ft)  
 PO: Pump Output (bbl/stroke)  
 S<sub>ds</sub>: Strokes of Drill Pipe (fraction)

**Output(s)**

AC<sub>dc</sub>: Annular Capacity of Drill Collar (bbl/ft)

- $AC_{dp}$ : Annular Capacity of Drill Pipe (bbl/ft)  
 $V_{dc}$ : Volume in Drill Collar (bbl)  
 $V_{dp}$ : Volume in Drill Pipe (bbl)  
 $V_a$ : Volume of Pill Required in Annulus (bbl)  
 $V_{ts}$ : Total Volume of Pill Required (bbl)  
 $C_t$ : Total Capacity of Drill String (bbl)  
 $S_{pp}$ : Strokes for Pump Pill (no.)  
 $V_{cp}$ : Volume Req for Chase Pill (bbl)  
 $S_{cp}$ : Strokes for Chase Pill (no.)  
 $S_t$ : Total Strokes to Spot the Pill (no.)

### Formula(s)

$$AC_{dc} = \frac{(hd)^2 - (cd)^2}{1029.4}$$

$$AC_{dp} = \frac{(hd)^2 - (pd)^2}{1029.4}$$

$$V_{dc} = AC_{dc} * L_s * WF$$

$$V_{dp} = AC_{dp} * L_e * WF$$

$$V_a = V_{dc} + V_{dp}$$

$$V_{ts} = V_a + V_{ds}$$

$$C_t = (C_{dc}) * L_s + (C_{dp}) * L$$

$$S_{pp} = \frac{V_{ts}}{PO}$$

$$V_{cp} = C_t - V_{ds}$$

$$S_{cp} = \left( \frac{V_{cp}}{PO} \right) + S_{ds}$$

$$S_t = (S_{pp}) + (S_{cp})$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 75.*

## 2.30 Capacity formulas—bbl/ft

### Input(s)

- $d_h$ : Inside Diameter of Casing (in.)  
 $d_p$ : Outside Diameter of Casing (in.)

### Output(s)

- AC: Annular Capacity (bbl/ft)

**Formula(s)**

$$AC = \frac{d_h^2 - d_p^2}{1029.4}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Work-over, Second Edition, Gulf Professional Publishing, Page: 12.*

**2.31 Capacity formulas—gal/ft****Input(s)**

- $d_h$ : Inside Diameter of Casing (in.)
- $d_p$ : Outside Diameter of Casing (in.)

**Output(s)**

- AC: Annular Capacity (gal/ft)

**Formula(s)**

$$AC = \frac{d_h^2 - d_p^2}{24.51}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 12.*

**2.32 Capacity of tubulars and open-hole****Input(s)**

- $D_i$ : Inner Diameter (in.)

**Output(s)**

- C: Capacity of any Cylindrical Object (drillpipe, drill collars, tubing, casing, openhole) (bbl/ft)

**Formula(s)**

$$C = \frac{D_i^2}{1029.4}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 16.*

**2.33 CO<sub>2</sub> solubility in oil and oil-mud emulsifiers****Input(s)**

- $p$ : Pressure (psi)
- $a$ : Solubility Equation Constant (dimensionless)
- $b$ : Solubility Equation Constant (dimensionless)
- $c$ : Solubility Equation Constant (dimensionless)

**Output(s)**

$dr_{s(CO_2)}$ : CO<sub>2</sub> Solubility in Oil and Oil-Mud Emulsifiers (SCF/bbl)

**Formula(s)**

$$dr_{s(CO_2)} = \left( \frac{p}{aT^b} \right)^c$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control* (Vol. 10). Society of Petroleum Engineers, Page: 16.

## 2.34 Combined solubility—Hydrocarbon gas, CO<sub>2</sub>, and H<sub>2</sub>S—in each of the mud components

**Input(s)**

- $f_h$ : Mole Fraction of Hydrocarbon Gas (dimensionless)
- $f_{CO_2}$ : Mole Fraction of Carbon Dioxide (dimensionless)
- $f_{H_2S}$ : Mole Fraction of Hydrogen Sulphure (dimensionless)
- $r_{sh}$ : Solubility of Hydrocarbon Gas (SCF/bbl)
- $r_{sCO_2}$ : Solubility of Carbon Dioxide (SCF/bbl)
- $r_{sH_2S}$ : Solubility of Hydrogen Sulphure (SCF/bbl)

**Output(s)**

$dr_{s(o,w,e)}$ : Combined Solubility of Hydrocarbon Gas, CO<sub>2</sub> and H<sub>2</sub>S (SCF/bbl)

**Formula(s)**

$$r_{s(o, e)} = f_h r_{sh} + f_{CO_2} r_{sCO_2} + f_{H_2S} r_{sH_2S}$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control* (Vol. 10). Society of Petroleum Engineers, Page: 16.

## 2.35 Control drilling—Maximum drilling rate

**Input(s)**

- q: Circulation Rate (gpm)
- D<sub>h</sub>: Hole Diameter (in.)
- MW<sub>o</sub>: Mud Wt. out (ppg)
- MW<sub>i</sub>: Mud Wt. in (ppg)

**Output(s)**

MDR: Maximum Drilling Rate (ft/h)

**Formula(s)**

$$MDR = 67 * (MW_o - MW_i) * \frac{q}{(D_h)^2}$$

*Notes:* The formula is valid when drilling in large diameter holes (14 3/4 in. and Larger).

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 24.*

**2.36 Conversion of pressure into the mud weight****Input(s)**

- TVD: True Vertical Depth (ft)  
 P: Hydrostatic Pressure (psi)

**Output(s)**

- M<sub>w</sub>: Mud Weight (ppg)

**Formula(s)**

$$M_w = \frac{P}{0.052 * TVD}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 4.*

**2.37 Cost per foot during drilling****Input(s)**

- C<sub>b</sub>: Bit Cost (\$)  
 C<sub>to</sub>: Tool Cost (\$)  
 C<sub>m</sub>: Mud Cost (\$)  
 T<sub>d</sub>: Drilling Time (h)  
 T<sub>t</sub>: Trip Time (h)  
 T<sub>c</sub>: Connection Time and Extras (h)  
 C<sub>r</sub>: Rig Rate (\$/h)  
 C<sub>s</sub>: Support Rate (\$/h)  
 C<sub>t</sub>: Tool Rental Rate (\$/h)  
 F: Footage (ft/h)  
 T<sub>d</sub>: Drilling Time (h)

**Output(s)**

- C<sub>d</sub>: Cost of Drilling Per Foot (\$/h)

**Formula(s)**

$$C_d = \frac{C_b + C_{to} + C_m + (T_d + T_t + T_c) * (C_r + C_s + C_t)}{F * T_d}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 468.*

**2.38 Cost per foot of coring****Input(s)**

- C<sub>b</sub>: Core Bit Cost (\$)
- C: Rig Cost (\$)
- T<sub>d</sub>: Total Coring Time (h)
- T<sub>t</sub>: Trip Time (h)
- T<sub>r</sub>: Core Recovery Time (h)
- F: Footage Cored (ft)
- R<sub>c</sub>: Core Recovery Percentage (fraction)

**Output(s)**

- C: Cost per Foot of Coring (\$/ft)

**Formula(s)**

$$C = \frac{C_b + C_r * (T_d + T_t + T_r)}{F * R_c}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 469.*

**2.39 Critical annular velocity and critical flow rate****Input(s)**

- θ600: 600 Viscometer Dial Reading (dimensionless)
- θ300: 300 Viscometer Dial Reading (dimensionless)
- D<sub>h</sub>: Diameter of Hole (in.)
- D<sub>p</sub>: Diameter of Drill Pipe (in.)
- MW: Mud Weight in Use (ppg)

**Output(s)**

- n: Power Constant (dimensionless)
- K: Constant (dimensionless)
- x: Constant (dimensionless)
- A<sub>Vc</sub>: Critical Annular Velocity (ppg)
- GPM<sub>c</sub>: Critical Flow Rate (ppg)

### Formula(s)

$$n = 3.32 * \log\left(\frac{\theta 600}{\theta 300}\right)$$

$$K = \frac{\theta 600}{1022^n}$$

$$x = 81600 * K * \frac{(n)^{0.387}}{((Dh - Dp)^n) * MW}$$

$$AVc = x^{\frac{1}{2-(n)}}$$

$$GPMc = AVc * \frac{Dh^2 - Dp^2}{24.5}$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 212.*

## 2.40 Critical flow rate for flow regime change

### Input(s)

$V_c$ : Critical Velocity (ft/s)  
 $D_i$ : Integral Diameter of Pipe (in.)

### Output(s)

$Q_c$ : Critical Flow Rate (gpm)

### Formula(s)

$$Q_c = 2.448 * V_c * (D_i^2)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 220.*

## 2.41 Critical velocity for change in flow regime

### Input(s)

$PV$ : Plastic Viscosity (cP)  
 $YP$ : Yield Stress or Yield Point (lb/100 ft<sup>2</sup>)  
 $\rho_m$ : Mud Density (ppg)  
 $D_i$ : inside Diameter of Pipe (in.)

### Output(s)

$V_c$ : Critical Velocity (ft/s)

**Formula(s)**

$$V_c = \frac{(1.08 * PV) + \left(1.08 * (PV^2 + 12.34 * \rho_m * (D_i^2) * YP)^{0.5}\right)}{\rho_m * D_i}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 220.*

**2.42 Crown block capacity****Input(s)**

- H<sub>I</sub>: Net Static Hook Load Capacity (lb)
- S: Effective Weight of Suspended Equipment (lb)
- n: No. of Lines Strung to the Traveling Block (unitless)

**Output(s)**

- R<sub>c</sub>: Crown Block Capacity (lb)

**Formula(s)**

$$R_c = \frac{(H_I + S) * (n + 2)}{n}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 26.*

**2.43 Current drag force—Offshore****Input(s)**

- g: Acceleration due to Gravity (ft/s<sup>2</sup>)
- V<sub>c</sub>: Current Velocity (ft/s)
- C<sub>s</sub>: Drag Coefficient (dimensionless)
- A: Area (ft<sup>2</sup>)

**Output(s)**

- F: Current Force (lbf)

**Formula(s)**

$$F = g * (V_c^2) * C_s * A$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 37.*

## 2.44 Curvature radius for a borehole

### Input(s)

- C: Constant Related to the Unit of Borehole Curvature (unitless)
- k: Curbature of Wellbore Trajectory (degree/100ft)

### Output(s)

- R: Curvature Radius of Borehole (ft)

### Formula(s)

$$R = \frac{180 * (C)}{3.1415 * k}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 77.*

## 2.45 Cutting slip velocity

### Input(s)

- PV: Plastic Viscosity (cP)
- MW: Mud Weight (ppg)
- Dp: Diameter of Particle (in.)
- DenP: Density of Particle (ppg)

### Output(s)

- V<sub>s</sub>: Slip Velocity (ft/min)

### Formula(s)

$$V_s = 0.45 * \left( \frac{PV}{MW * Dp} \right) * \left( \left( \left( \frac{36800}{\left( \frac{PV}{MW * Dp} \right)^2} \right) * Dp * \left( \frac{DenP}{MW} - 1 \right) \right) + 1^{-1} \right)^{0.5}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition Gulf Professional Publishing, Page: 176.*

## 2.46 Cuttings produced per foot of hole drilled—bbls

### Input(s)

- D<sub>h</sub>: Diameter of Bit or Hole Size (Plus Washout) (in.)
- Ø: Porosity (fraction)

### Output(s)

- V<sub>c</sub>: Amount of Cuttings Produced per each Foot of Hole Drilled (bbls)

**Formula(s)**

$$V_c = \frac{D_h^2}{1029.4} * (1 - \emptyset)$$

Reference: Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, *Formulas and Calculations for Drilling, Production and Workover, Third Edition*, Gulf Professional Publishing, Page: 18.

**2.47 Cuttings produced per foot of hole drilled—lbs****Input(s)**

- C<sub>a</sub>: Capacity of Hole (bbl/ft)
- L<sub>d</sub>: Footage Drilled (ft)
- SG: Specific Gravity of Cuttings (fraction)
- $\emptyset$ : Porosity (fraction)

**Output(s)**

- W<sub>cg</sub>: Amount of Cuttings Produced per each Foot of Hole Drilled (lb)

**Formula(s)**

$$W_{cg} = 350 * (C_a) * (L_d) * (1 - \emptyset) * SG$$

Reference: Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, *Formulas and Calculations for Drilling, Production and Workover, Third Edition*, Gulf Professional Publishing, Page: 19.

**2.48 D—Exponent****Input(s)**

- d: Penetration Rate (ft/h)
- N: Rotary Speed (rpm)
- W: Weight on Bit in 1000lb (lb)
- D: Bit Size (in.)

**Output(s)**

- d: D Exponent (dimensionless)

**Formula(s)**

$$d = \frac{\log \left( \frac{R}{60 * N} \right)}{\log \left( \frac{12 * W}{1000 * D} \right)}$$

Reference: Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, *Formulas and Calculations for Drilling, Production and Workover Third Edition*, Gulf Professional Publishing, Page: 214.

## 2.49 Depth of a washout—Method 1

### Input(s)

- $N_s$ : Number of Strokes Required (unitless)
- $P_o$ : Pump Output per Stroke (bbl/stroke)
- $C_p$ : Drillpipe Capacity (bbl/ft)

### Output(s)

- $D_w$ : Depth of Washout (ft)

### Formula(s)

$$D_w = N_s * \frac{P_o}{C_p}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition Gulf Professional Publishing, Page: 70.*

## 2.50 Depth of a washout—Method 2

### Input(s)

- $N_s$ : Number of Strokes Required (unitless)
- $P_o$ : Pump Output per Stroke (bbl/stroke)
- $C_p$ : Drillpipe Capacity (bbl/ft)
- $C_a$ : Annular Capacity (bbl/ft)

### Output(s)

- $D_w$ : Depth of Washout (ft)

### Formula(s)

$$D_w = N_s * \frac{P_o}{C_p + C_a}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition Gulf Professional Publishing, Page: 70.*

## 2.51 Derrick efficiency factor

### Input(s)

- $E$ : Power Efficiency (per cent)
- $n$ : Number of Lines strung between crown block and travelling block (dimensionless)

### Output(s)

- $E_d$ : Derrick Efficiency Factor (per cent)

**Formula(s)**

$$E_d = \frac{\left(\frac{1+E+En}{En}\right)W}{\left(\frac{n+4}{n}\right)W}$$

Reference: Bourgoyne, A. T., Millheim, K. K., Chenevert, M. E., & Young, F. S. (1986). *Applied Drilling Engineering*, Page: 10.

**2.52 Difference in pressure gradient between the cement and mud****Input(s)**

- $W_c$ : Cement Weight (ppg)
- $W_m$ : Mud Weight (ppg)

**Output(s)**

- PG: Pressure Gradient between Cement and Mud (psi/ft)

**Formula(s)**

$$PG = (W_c - W_m) * 0.052$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover*, Second Edition Gulf Professional Publishing, Page: 66.

**2.53 Differential hydrostatic pressure between cement in the annulus and mud inside the casing****Input(s)**

- $h_w$ : Well Depth (ft)
- $h_f$ : Float Collar (Feet Number Above the Shoe) (ft)
- $h_l$ : Lead Slurry Depth (ft)
- $h_t$ : Tail Slurry Depth (ft)
- $w_m$ : Mud Weight (lb/gal)
- $w_l$ : Lead Slurry Weight (lb/gal)
- $w_t$ : Tail Slurry Weight (lb/gal)

**Output(s)**

- $P_a$ : Total Pressure in Annulus (psi)
- $P_c$ : Total Pressure in Casing (psi)
- $P_d$ : Total Pressure Differential (psi)

### Formula(s)

$$P_a = 0.052 * (w_l * h_l + w_t * h_t + w_m * (h_w - h_l - h_t))$$

$$P_c = 0.052 * (w_m * (h_w - h_f) + w_t * h_f)$$

$$P_d = P_a - P_c$$

Reference: *Lyons, W. C., Carter, T., and Lapeyrouse, N. J., 2012, Formulas and Calculations for Drilling, Production and Workover, Third Edition, Gulf Professional Publishing, Page: 84.*

## 2.54 Dilution of a mud system

### Input(s)

- $V_m$ : Mud in Circulation System (bbl)
- $F_{ct}$ : Per cent of Low Gravity Solids in System (per cent)
- $F_{cop}$ : Per cent of Total Optimumlow Gravity Solids (per cent)
- $F_{ca}$ : Per cent of Low Gravity Solids (Bentonite and or Chemicals Added) (per cent)

### Output(s)

- $V_{wm}$ : Dilution Water or Mud Required (bbl)

### Formula(s)

$$V_{wm} = \frac{V_m * (F_{ct} - F_{cop})}{F_{cop} - F_{ca}}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 96.*

## 2.55 Direction of dip

### Input(s)

- $h$ : Height of Fracture (ft)
- $d$ : Diameter of Well (ft)

### Output(s)

- $D$ : Dip Direction (radian)

### Formula(s)

$$D = \tan\left(\frac{h}{d}\right)$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 2.56 Directional curvature for a deviated well

### Input(s)

$\partial \emptyset$ : Directional Change (degree)  
 $\partial L$ : Course Length (ft)

### Output(s)

DC: Directional Curvature (degree/ft)

### Formula(s)

$$DC = (\partial \emptyset) * \frac{100}{\partial L}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 84.*

## 2.57 Downward force or weight of casing

### Input(s)

W: Casing Weight (lb/ft)  
L: Length of Casing (ft)  
BF: Buoyancy Factor (unitless)

### Output(s)

$W_d$ : Downward Force of Casing (lb)

### Formula(s)

$$W_d = W * L * BF$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition Gulf Professional Publishing, Page: 67.*

## 2.58 Drill pipe or drill collar capacity

### Input(s)

ID: Inner Diameter (in.)

### Output(s)

C: Drill Collar Capacity (bbl/ft)

### Formula(s)

$$C = \frac{ID^2}{1029.4}$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover, second edition*, Gulf Professional Publishing, Page: 25.

## 2.59 Drill pipe or drill collar displacement and weight

### Input(s)

ID: Inner Diameter (in.)  
OD: Outer Diameter (in.)

### Output(s)

Disp: Drill Pipe/drill Collar Displacement (bbl/ft)  
W: Drill Pipe/drill Collar Weight (lb/bbl)

### Formula(s)

$$\text{Disp} = \frac{OD^2 - ID^2}{1029.4}$$
$$W = Disp * 2747$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover, second edition*, Gulf Professional Publishing, Page: 25.

## 2.60 Drill string design—Drill pipe length for bottomhole assembly

### Input(s)

BF: Buoyancy Factor (dimensionless)  
T: Tensile Strength for New Pipe (lb)  
f: Safety Factor to Correct New Pipe to No. 2 Pipe Equal to 1-safety Factor of Pipe (fraction)  
MOP: Margin of Overpull (lb)  
 $W_{bha}$ : Bha Weight in Air (lb)  
 $W_{dp}$ : Drill Pipe Wt in Air including Tool Joint (lb)  
 $BHA_I$ : Length of Bore Hole Assembly (dimensionless)

### Output(s)

$L_{max}$ : Drill Pipe Length for a Specific Bore Hole Assembly (fraction)  
 $Depth_t$ : Total Depth That Can be Reached with a Specific Bottomhole Assembly (ft)

**Formula(s)**

$$L_{\max} = \frac{65.5 - MW}{65.5}$$

$$\text{Depth}_t = [(T * f) - MOP - W_{bha}] * \left( \frac{BF}{W_{dp}} \right)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, second edition, Gulf Professional Publishing, Page: 43.*

**2.61 Drilled gas entry rate****Input(s)**

- $d_b$ : Bit Diameter (in.)
- $R$ : Penetration Rate (ft/h)
- $S_g$ : Gas Saturation (fraction)
- $\emptyset$ : Formation Porosity (per cent)
- $p$ : Pressure (psi)
- $z$ : Gas Compressibility Factor (dimensionless)
- $t$ : time (h)

**Output(s)**

- $q_{gsc}$ : Drilled Gas Entry Rate (SCF/min)

**Formula(s)**

$$q_{gsc} = \frac{d_b^2 R \emptyset S_g p}{310 z T}$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers., Page: 19.*

**2.62 Drilling cost per foot****Input(s)**

- $B$ : Bit Cost (\$)
- $C_R$ : Rig Cost (\$)
- $T$ : Rotating Time (spm)
- $t$ : Round Trip Time (spm)
- F:Footage per Bit (ft)

**Output(s)**

- $C_T$ :Drilling Cost (\$)

**Formula(s)**

$$C_T = \frac{B + C_R * (t + T)}{F}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 28.*

**2.63 Drilling ton miles—Coring operation ton miles****Input(s)**

- $T_4$ : Ton Miles for One Round Trip-depth Where Coring Stopped Before Coming out of Hole (ton miles)  
 $T_3$ : Ton-miles for One Round Trip-depth Where Coring Started After Going in Hole (ton miles)

**Output(s)**

- $T_c$ : Coring Operation Ton Miles (ton miles)

**Formula(s)**

$$T_c = 2 * (T_4 - T_3)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 46.*

**2.64 Drilling ton miles—Drilling/connection ton miles****Input(s)**

- $T_2$ : Ton Miles for One Round Trip-Depth Where Drilling Stopped Before Coming out of Hole (ton miles)  
 $T_1$ : Ton-miles for One Round Trip-Depth Where Drilling Started (ton miles)

**Output(s)**

- $T_D$ : Drilling/connection Ton Miles (ton miles)

**Formula(s)**

$$T_D = 3 * (T_2 - T_1)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, second edition, Gulf Professional Publishing, Page: 46.*

**2.65 Drilling ton miles—Round trip ton miles****Input(s)**

- $W_p$ : Buoyed Weight of Drill Pipe (lb/ft)  
 $D$ : Depth of Hole (ft)  
 $L_p$ : Length of One Stand of Drill Pipe (ft)  
 $W_b$ : Weight of Travelling Block (lb)  
 $Wdc$ : Weight of Drill Collar Per Feet (lb/ft)

Wdp: Drill Pipe Weight Per Feet (lb/ft)

L: Drill Collar Length (ft)

BF: Buoyancy Factor (dimensionless)

### **Output(s)**

W<sub>c</sub>: Buoyed Weight of Drill Collars-buoyed Weight of Drill Pipe (lb)

RT<sub>TM</sub>: Round Trip Ton Miles (ton miles)

### **Formula(s)**

$$W_c = L * BF * (Wdc - Wdp)$$

$$RT_{TM} = \frac{W_p * D * (L_p + D) + (2 * D) * (2 * W_b + W_c)}{5280 * 2000}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 44.*

## **2.66 Drilling ton miles—While making short trip ton miles**

### **Input(s)**

T<sub>6</sub>: Ton Miles for One Round Trip-at the Deeper Depth, the Depth of the Bit Before Starting Short Trip (ton miles)

T<sub>5</sub>: Ton-miles for One Round Trip-at Shallower Depth, the Depth that the Bit is Pulled up to (ton miles)

### **Output(s)**

T<sub>ST</sub>: Short Trip Ton Miles (ton miles)

### **Formula(s)**

$$T_{ST} = (T_6 - T_5)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 47.*

## **2.67 Drilling ton miles—Setting casing ton miles**

### **Input(s)**

W<sub>p</sub>: Buoyed Weight of Casing (lb/ft)

L<sub>cs</sub>: Length of One Joint of Casing (ft)

D: Depth of the Setting (ft)

W<sub>b</sub>: Weight of Travelling Block Assembly (lb)

### **Output(s)**

T<sub>c</sub>: Setting Casing Ton Miles (ton miles)

**Formula(s)**

$$T_c = \left( W_p * D * (L_{cs} + D) + D * W_b \right) * \frac{0.5}{5280 * 2000}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 47.*

**2.68 Duplex pump factor****Input(s)**

$D_L$ : Piston Diameter (in.)

$D_r$ : Rod Diameter (in.)

$L_S$ : Stroke Length (in.)

**Output(s)**

$PF_d$ : Duplex Pump Factor (bbl/stroke)

**Formula(s)**

$$PF_d = \frac{3.1415 * L_S * ((2 * D_L^2) - D_r^2)}{2 * 9702}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 84.*

**2.69 Duplex pump output—Using liner diameter****Input(s)**

$d_l$ : Liner Diameter (in.)

$l_s$ : Stroke Length (in.)

**Output(s)**

$q_o$ : Pump Output (bbl/stroke)

**Formula(s)**

$$q_o = 0.000324 * (d_l^2) * (l_s)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 8.*

**2.70 Duplex pump output—Using rod diameter****Input(s)**

$d_r$ : Rod Diameter (in.)

$l_s$ : Stroke Length (in.)

**Output(s)**

$q_o$ : Pump Output (bbl/stroke)

**Formula(s)**

$$q_o = 0.000162 * (d_r^2) * l_s$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 8.*

**2.71 Duplex pump output by using liner and rod diameters****Input(s)**

$d_l$ : Liner Diameter (in.)

$d_r$ : Rod Diameter (in.)

$l_s$ : Stroke Length (in.)

**Output(s)**

$q_o$ : Pump Output (bbl/stroke)

**Formula(s)**

$$q_o = 0.000162 * l_s * ((2 * (d_l^2)) - (d_r^2))$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 9.*

**2.72 Dynamically coupled linear flow—Formation invasion****Input(s)**

$x_f$ : Transient Invasion Front (in.)

$x_{f, o}$ : Initial Displacement, i.e., Spurt (in.)

$L$ : Lineal Core Length (in.)

$p_m$ : Constant Mud Pressure (psi)

$p_r$ : Constant Reservoir Pressure (psi)

$\phi_{eff}$ : Effective Rock Porosity (fraction)

$\phi_c$ : Mudcake Porosity (fraction)

$k_1$ : Mudcake Permeability to Filtrate (mD)

$k_2$ : Rock Permeability to Filtrate (mD)

$k_3$ : Rock Permeability to "Oil" (mD)

$\mu_f$ : Mud Filtrate Viscosity (cP)

$\mu_o$ : Viscosity of "Oil" or Formation Fluid (cP)

$f$ : Mud Solid Fraction (fraction)

**Output(s)**

$x_f(t)$ : Minimum Number of Jobs to Survive in a Minimum Chance Scenario (dimensionless)

### Formula(s)

$$x_f(t) = -H + \sqrt{\left\{ H^2 + 2 \left( H x_{f,o} + \frac{1}{2} x_{f,o}^2 + Gt \right) \right\}}$$

$$G = - \left\{ k_1 (p_m - p_r) / \mu_f \varnothing_{eff} \right\} / \left\{ \frac{\mu_o k_1}{\mu_f k_3} - \frac{k_1}{k_2} - \frac{\varnothing_{eff} f_s}{\{(1 - \varnothing_c)(1 - f_s)\}} \right\}$$

$$H = \left[ \frac{x_{f,o} \varnothing_{eff} f_s}{\{(1 - \varnothing_c)(1 - f_s)\}} - \frac{\mu_o k_1 L}{\mu_f k_3} \right] / \left\{ \frac{\mu_o k_1}{\mu_f k_3} - \frac{k_1}{k_2} - \frac{\varnothing_{eff} f_s}{\{(1 - \varnothing_c)(1 - f_s)\}} \right\}$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 16.*

## 2.73 Effective weight during drilling

### Input(s)

- $w_s$ : Weight of Drillstring Material (lbs)
- $\rho_o$ : Density of Mud (ppg)
- $\rho_s$ : Density of Drillstring Material (ppg)

### Output(s)

- $w_o$ : Effective Weight (lbs)

### Formula(s)

$$w_o = w_s * \left( 1 - \left( \frac{\rho_o}{\rho_s} \right) \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 26.*

## 2.74 Effective wellbore radius for finite-conductivity fractures

### Input(s)

- $k_f$ : Fracture Conductivity (mD)
- $b_f$ : Fracture Width (ft)
- $k$ : Formation Conductivity (mD)
- $x_f$ : Fracture Half Length (ft)

### Output(s)

- $F_{CD}$ : Dimensionless Fracture Conductivity (dimensionless)
- $r_w$ : Effective Wellbore Radius (ft)

**Formula(s)**

$$F_{CD} = \frac{k_f * b_f}{k * x_f}$$

$$r_w = 0.2807 * \frac{k_f * b_f}{k}$$

Notes:  $r_w$  is valid if  $F_{CD} < 0.1$ .

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 5, Page: 135.

**2.75 Effective wellbore radius in infinite-conductivity fractures****Input(s)**

$x_f$ : Fracture Half Length (ft)

$L$ : Total Fracture Length (ft)

**Output(s)**

$r_w$ : Effective Wellbore Radius (ft)

$r_w$ : Effective Wellbore Radius (ft)

**Formula(s)**

$$r_w = \frac{L}{4}$$

$$r_w = \frac{x_f}{2}$$

Notes: Check Validity, for  $\frac{x_f}{x_e} \leq 0.30$  where  $x_e$  is Half Length of a Side of a Drainage Area Square.

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 5, Page: 134.

**2.76 Efficiency of block and tackle system****Input(s)**

$F_h$ : Load Hoisted (lb)

$v_t$ : Traveling Block Velocity (fpm)

$F_f$ : Load in Fast Line (lb)

$v_f$ : Fast Line Speed (fpm)

**Output(s)**

$E$ : Efficiency (fraction)

### Formula(s)

$$E = \frac{F_h * v_t}{F_f * v_f}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 10.*

## 2.77 Equivalent area of pipe subject to uniform axial force

### Input(s)

$A_b$ : Area of Body (in.<sup>2</sup>)

$A_j$ : Area of Joint (in.<sup>2</sup>)

$\alpha$ : Length Factor for Pipe Body (fraction)

### Output(s)

$A_p$ : Area of Pipe (in.<sup>2</sup>)

### Formula(s)

$$A_p = \frac{A_b * A_j}{\alpha * A_b * (1 - \alpha) * A_j}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 44.*

## 2.78 Equivalent circulating density

### Input(s)

$\rho_m$ : Mud Weight (ppg)

$\delta P_a$ : Annulus Pressure Loss (psi)

$L_t$ : True Vertical Depth (ft)

### Output(s)

ECD: Equivalent Circulating Density (ppg) (ppg)

### Formula(s)

$$ECD = \rho_m + \left( \frac{\delta P_a}{0.052 * L_t} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 258.*

## 2.79 Equivalent density of a wellbore fluid

### Input(s)

M<sub>w</sub>: Mud Weight (ppg)  
 TVD: True Vertical Depth (ft)  
 APL: Annular Pressure Loss (psi)

### Output(s)

ECD: Equivalent Circulating Density (ppg)

### Formula(s)

$$\text{ECD} = \frac{\text{APL}}{0.052 * \text{TVD}} + M_w$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover, Second Edition*, Gulf Professional Publishing, Page: 6.

## 2.80 Equivalent formation water resistivity from SP log

### Input(s)

R<sub>mfeq</sub>: Equivalent Resistivity of Mud Filtrate (ohm m)  
 SSP: Static Spontaneous Potential (mV)  
 T: Temperature (F)

### Output(s)

R<sub>weq</sub>: Equivalent Formation Water Resistivity (ohm m)

### Formula(s)

$$R_{weq} = (-61 + 0.133 * T) * \frac{R_{mfeq}}{10^{-\frac{SSP}{61 + 0.133 * T}}}$$

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics*, Page: 74.

## 2.81 Equivalent mud weight—Deviated well

### Input(s)

P<sub>h</sub>: Pressure (psi)  
 D<sub>h</sub>: Measured Depth (ft)  
 α: Deviation Angle (degree)

### Output(s)

EMW: Equivalent Mud Weight (ppg)

### Formula(s)

$$EMW = \frac{P_h}{0.052 * D_h * \cos(\alpha)}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 139.*

## 2.82 Equivalent mud weight—Vertical well

### Input(s)

- P<sub>h</sub>: Pressure (psi)
- L<sub>tvd</sub>: True Vertical Depth (ft)

### Output(s)

- EMW: Equivalent Mud Weight (ppg)

### Formula(s)

$$EMW = \frac{P_h}{0.052 * (L_{tvd})}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 139.*

## 2.83 Evaluation of centrifuge

### Input(s)

- MW: Mud Density into centrifuge (ppg)
- QM: Mud Volume into centrifuge (gal/min)
- PW: Dilution water density (ppg)
- QW: Dilution Water volume (gal/min)
- PU: Underflow Mud density (ppg)
- PO: Overflow Mud Density (ppg)
- CC: Clay content in mud (lb/bbl)
- CD: Additive content in mud (lb/bbl)

### Output(s)

- QU: Underflow Mud Vol (gal/min)
- FU: Fraction of old mud in underflow (fraction)
- QC: Mass rate of Clay (lb/min)
- QD: Mass rate of Additive (lb/min)
- QP: Water flow rate into mixing pit (gal/min)
- QB: Mass rate of API Barite (lb/min)

**Formula(s)**

$$QU = \frac{QM * (MW - PO) - QW * (PO - PW)}{PU - PO}$$

$$FU = \frac{35 - PU}{35 - MW + \frac{QW}{QM} * (35 - PW)}$$

$$QC = CC * \frac{QM - (QU * FU)}{42}$$

$$QD = CD * \frac{QM - (QU * FU)}{42}$$

$$QP = \frac{(QM * (35 - MW)) - (QU * (35 - PU)) - 0.6129 * QC - 0.6129 * QD}{35 - PW}$$

$$QB = \left( QM - QU - QP - \frac{QC}{21.7} - \frac{QD}{21.7} \right) * 35$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 99.*

**2.84 Evaluation of hydrocyclone****Input(s)**

MW: Average Density of discarded mud (ppg)

V: Volume of Slurry sample collected (quarts)

T: Time to collect slurry sample (s)

**Output(s)**

SF: Fraction percentage of solids (fraction)

MS: Mass rate of solids removed by one cone of a hydroclone (lb/h)

WR: Volume of water ejected by one cone of a hydroclone (gal/h)

**Formula(s)**

$$SF = \frac{MW - 8.33}{13.37}$$

$$MS = 19530 * SF * \frac{V}{T}$$

$$WR = 900 * (1 - SF) * \frac{V}{T}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 97.*

**2.85 Fluid volume required to spot a plug****Input(s)**

L<sub>t</sub>: Length of Tubing or Pipe (ft)

L<sub>p</sub>: Length of Plug (ft)

C<sub>t</sub>: Tubing or Pipe Capacity (bbl/ft)

V<sub>s</sub>: Spacer Volume behind Slurry (bbl)

**Output(s)**

V: Fluid Volume Required (bbl)

**Formula(s)**

$$V = \left( (L_t - L_p) * C_t \right) - V_s$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 62.*

**2.86 Force applied to stretch material****Input(s)**

A: Area ( $m^2$ )  
 E: Young ( $N/m^2$ )  
 L<sub>a</sub>: New Length (m)  
 L<sub>b</sub>: Old Length (m)

**Output(s)**

F: Force (N)

**Formula(s)**

$$F = A * E * \left( \frac{L_a - L_b}{L_b} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 350.*

**2.87 Force exerted by the fluid on the solid surface of flow through an annulus****Input(s)**

R: Radius (m)  
 K: Boltzmann Constant ( $m^2 \text{ kg s}^{-2} \text{ K}^{-1}$ )  
 p<sub>o</sub>: Pressure at initial point (Pa)  
 p<sub>L</sub>: Pressure at point L (Pa)

**Output(s)**

F<sub>z</sub>: Force (Newton)

**Formula(s)**

$$F_z = \pi * R^2 * (1 - K^2) * (p_o - p_L)$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 2.88 Friction factor in drill pipe

### Input(s)

- $\mu_s$ : Static Viscosity (cP)
- $\sigma$ : Normal Stress at Contact (psi)
- $k$ : Exponential Constant (dimensionless)
- $t$ : Average Contact Time (s)
- $V_t$ : Trip Speed (ft/s)
- $\omega$ : Angular Speed (ft/s)

### Output(s)

- $\mu$ : Constant (dimensionless)

### Formula(s)

$$\mu = \frac{\mu_s}{1 + \left( \frac{\mu_s * \sigma * ((V_t^2 + \omega^2)^{0.5})}{k * t} \right)}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 375.*

## 2.89 Front displacement of a particle in the reservoir—Formation invasion

### Input(s)

- $x_o$ : Initial Marked Position (ft)
- $k$ : Permeability (mD)
- $\mu$ : Viscosity (cP)
- $\emptyset$ : Porosity (fraction)
- $P_r$ : Constant "Reservoir" Pressure at  $x=L$  (psi)
- $P_l$ : Pressure at  $x=0$  location (psi)
- $t$ : time (days)
- $L$ : Length (ft)

### Output(s)

- $x(t)$ : Front Displacement of an Initially Marked Particle in the Reservoir (ft)

### Formula(s)

$$x(t) = x_o - \left( \frac{k}{\mu \emptyset} \right) \left( \frac{(P_r - P_l)t}{L} \right)$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 30.*

## 2.90 Gas migration velocity

### Input(s)

- $\Delta p_{cs}$ : Pressure (psi)
- $g_m$ : Depth (ft)
- $\Delta t$ : Time over which the Rise in Casing Pressure Occurs (h)

### Output(s)

- $v_{sl}$ : Gas Migration Velocity (ft/h)

### Formula(s)

$$v_{sl} = \Delta p_{cs} / g_m \Delta t$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control (Vol. 10)*. Society of Petroleum Engineers, Page: 12.

## 2.91 Gas solubility in a mud system

### Input(s)

- $f_o$ : Volume Fraction of Base Oil (fraction)
- $f_w$ : Volume Fraction of Water (fraction)
- $f_e$ : Volume Fraction of Emulsifier (fraction)
- $r_{so}$ : Solution Gas/Component Ratio of Base Oil (fraction)
- $r_{sw}$ : Solution Gas/Component Ratio of Water (fraction)
- $r_{se}$ : Solution Gas/Component Ratio of Emulsifier (fraction)

### Output(s)

- $r_{sm}$ : Solution Gas/Component of the Mud (fraction)

### Formula(s)

$$r_{sm} = f_o r_{so} + f_w r_{sw} + f_e r_{se}$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control (Vol. 10)*. Society of Petroleum Engineers, Page: 16.

## 2.92 Gas/mud ratio

### Input(s)

- $d_b$ : Bit Diameter (in.)
- $R$ : Penetration Rate (ft/h)
- $S_g$ : Gas Saturation (fraction)
- $\emptyset$ : Formation Porosity (per cent)
- $p$ : Pressure (psi)
- $z$ : Gas Compressibility Factor (dimensionless)
- $t$ : time (h)
- $q_m$ : Mud Circulation Rate (bbl/min)

**Output(s)**

$r_m$ : Gas/Mud Ratio (SCF/bbl)

**Formula(s)**

$$r_m = \frac{d_b^2 R \emptyset S_g p}{310 z T q_m}$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control (Vol. 10)*. Society of Petroleum Engineers, Page: 19.

**2.93 Gel strength—Optimal solid removal efficiency****Input(s)**

$V_s$ : Expected Drilled Solids in Drilling Fluid (fraction)

$V_c$ : Drilled Solids in Discard (fraction)

**Output(s)**

$\eta_{sr}$ : Optimal Solid Removal Efficiency (fraction)

**Formula(s)**

$$\eta_{rs} = 1 - \left( \frac{1 - V_s}{1 - V_s + \left( \frac{V_c}{V_s} \right)} \right)$$

Reference: Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 192.

**2.94 Gel strength—Solid control efficiency****Input(s)**

$V_r$ : Fraction of the Solids Removed (unitless)

$V_h$ : Hole Volume (BBL)

$V_d$ : Volume of Solids Discarded (BBL)

**Output(s)**

$(\eta_s)ce$ : Solid Control Efficiency (fraction)

**Formula(s)**

$$(\eta_s)ce = (V_r) * \frac{V_d}{V_h}$$

Reference: Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 191.

## 2.95 Gel strength—Solids build-up in system

### Input(s)

$V_h$ : Hole Volume (BBL)  
 $\eta_s$ : Solid Control Efficiency (fraction)

### Output(s)

$V_{sb}$ : Solids Build Up (BBL)

### Formula(s)

$$V_{sb} = V_h * (1 - \eta_s)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 188.*

## 2.96 Height of cement in the annulus

### Input(s)

$V_s$ : Slurry Volume ( $\text{ft}^3$ )  
 $C_c$ : Casing Capacity ( $\text{ft}^3/\text{ft}$ )  
AC: Cement Remaining in Casing ( $\text{ft}^3$ )  
E: Excess Volume (fraction)

### Output(s)

H: Height of Cement in Annulus (ft)

### Formula(s)

$$H = \frac{V_s - AC + C_c}{E}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 58.*

## 2.97 Hydraulic horsepower

### Input(s)

q: Flow Rate (gpm)  
P: Pressure (psi)

### Output(s)

HHP: Hydraulic Horsepower (hp)

**Formula(s)**

$$HHP = \frac{q * P}{1714}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 46.*

## 2.98 Hydraulics analysis

**Input(s)**

- $N_1$ : Jet Size for nozzle 1 (in.)
- $N_2$ : Jet Size for nozzle 2 (in.)
- $N_3$ : Jet Size for nozzle 3 (in.)
- $Q$ : Circulation Rate (gpm)
- $D_h$ : Diameter of Hole (in.)
- $D_p$ : Dia of Drill Pipe (in.)
- $MW$ : Mud Weight (psi)
- $B$ : Bit Size (in.)
- $P_s$ : Surface pressure (psi)

**Output(s)**

- $AV$ : Annular Velocity (ft/min)
- $P_b$ : Bit Nozzle Pressure Loss (psi)
- $HHP$ : Hydraulic Horsepower at bit (hp)
- $HHPba$ : Power per unit area in sq inc ( $\text{hp/in.}^2$ )
- $P_{psib}$ : Percentage pressure loss at bit (percent)
- $V_n$ : Jet Velocity (ft/s)
- $IFa$ : Impact force per unit area in sq inc ( $\text{lb/in.}^2$ )

**Formula(s)**

$$AV = 24.5 * \frac{Q}{D_h^2 - D_p^2}$$

$$P_b = 156.5 * Q^2 * \frac{MW}{(N_1^2 + N_2^2 + N_3^2)^2}$$

$$HHP = P_s * \frac{Q}{1714}$$

$$HHPba = Q * P_b * \frac{1.27}{1714 * B^2}$$

$$P_{psib} = P_b * \frac{100}{P_s}$$

$$V_n = 417.2 * \frac{Q}{N_1^2 + N_2^2 + N_3^2}$$

$$IFa = MW * V_n * Q * \frac{1.27}{1930 * B^2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 170.*

## 2.99 Hydromechanical specific energy

### Input(s)

WOB: Weight on Bit (lbf)  
 $A_b$ : Area of Bit ( $\text{ft}^2$ )  
 N: Rotational Speed for a Bit (ft/s)  
 $\eta$ : Factor for Energy Reduction (dimensionless)  
 $P_b$ : Pressure Drop Across Bit (psi)  
 Q: Flow Rate ( $\text{ft}^3/\text{s}$ )  
 ROP: Rate of Penetration (s/ft)  
 T: Torque (lbf ft)

### Output(s)

HMSE: Hydro Mechanical Specific Energy (lbf/ $\text{ft}^2$ )

### Formula(s)

$$\text{HMSE} = \left( \frac{\text{WOB}}{A_b} \right) + \left( \frac{120 * 3.142 * N * T + 1154 * \eta * P_b * Q}{A_b * \text{ROP}} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 415.*

## 2.100 Hydrostatic pulling

### Input(s)

NS: Number of Stands Pulled (number)  
 Lavg: Average length per stand (ft)  
 PD: Pipe Displacement (bbl/ft)  
 mw: Mud Weight (ppg)  
 cc: Casing Capacity (bbl/ft)  
 pd: Pipe Disp. (bbl/ft)

### Output(s)

BD: Barrel Displaced (bbl)  
 HP: Hydrostatic Pressure Decrease (Pulling Dry Pipe out of the Hole) (psi)

### Formula(s)

$$BD = NS * Lavg * PD$$

$$HP = BD * 0.052 * \frac{mw}{cc - pd}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 20.*

## 2.101 Hydrostatic pulling wet pipe out of the hole

### Input(s)

NS: Number of Stands Pulled (number)  
 Lavg: Average length per stand (ft)  
 mw: Mud Weight (ppg)  
 cc: Casing Capacity (bbl/ft)  
 pd: Pipe disp. (bbl/ft)  
 pc: Pipe capacity. (bbl/ft)

### Output(s)

BD: Barrel Displaced (bbl)  
 HP: Hydrostatic Pressure Decrease (Pulling Wet Pipe out of the Hole) (psi)

### Formula(s)

$$BD = NS * Lavg * (pd + pc)$$

$$HP = BD * 0.052 * \frac{mw}{cc - (pd + pc)}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 21.*

## 2.102 Hydrostatic pressure in annulus due to slug

### Input(s)

V<sub>a</sub>: Volume of Annulus (ft/bbl)  
 V<sub>s</sub>: Volume of Slug (bbl)  
 W<sub>s</sub>: Slug Weight (ppg)  
 W<sub>m</sub>: Mud Weight (ppg)

### Output(s)

P: Hydrostatic Pressure (psi)

### Formula(s)

$$P = V_a * V_s * (W_s - W_m) * 0.052$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 36.*

## 2.103 Hydrostatic pressure decrease at total depth caused by gas-cut mud

### Input(s)

MG: Mud Gradient (psi/ft)  
 C: Annular Volume (bbl/ft)  
 V: Pit Gain (bbl)

### Output(s)

P: Reduction in Bottomhole Pressure (psi)

### Formula(s)

$$P = \left( \frac{MG}{C} \right) * V$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 129.*

## 2.104 Impact force—Nozzle hydraulic analysis

### Input(s)

MW: Mud Weight (ppg)

v<sub>n</sub>: Jet Velocity (ft/s)

Q: Flowrate (gpm)

### Output(s)

IF: Impact Force (lb)

### Formula(s)

$$IF = (MW) * (v_n) * \frac{Q}{1930}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 170.*

## 2.105 Impinging jet

### Input(s)

$\Psi_1$ : Mass Rate Of Flow for stream (kg/s)

v<sub>1</sub>: Velocity of Stream (m/s)

$\theta$ : Angle of inclination of plate (rad)

### Output(s)

$\Psi_{2a}$ : Mass Rate Of Flow for stream 2a (kg/s)

$\Psi_{2b}$ : Mass Rate Of Flow for stream 2b (kg/s)

v<sub>2a</sub>: Velocity of Stream 2a (m/s)

v<sub>2b</sub>: Velocity of Stream 2b (m/s)

**Formula(s)**

$$\Psi 2a = v1$$

$$\Psi 2b = v1$$

$$v2a = 0.5 * \Psi 1 * (1 + \cos(\theta))$$

$$v2b = \Psi 1 * (1 - \cos(\theta))$$

Reference: *Transport Phenomena, Second Edition, Bird, Page: 201.*

**2.106 Increase mud density by barite****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt (ppg)

**Output(s)**

B: Sacks of barite required per 100 bbl (stroke/100bbl)

**Formula(s)**

$$B = 1470 * \frac{W_2 - W_1}{35 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 81.*

**2.107 Increase mud density by calcium carbonate****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt (ppg)

**Output(s)**

B: Sacks of Calcium Carbonate required per 100 bbl (stroke/100bbl)

**Formula(s)**

$$B = 945 * \frac{W_2 - W_1}{22.5 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 82.*

**2.108 Increase mud density by hematite****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt. (ppg)

**Output(s)**

B: Sacks of Hematite required per 100 bbl (stroke/100bbl)

**Formula(s)**

$$B = 1680 * \frac{W_2 - W_1}{40 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 84.*

**2.109 Increase volume by barite****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt. (ppg)

**Output(s)**

V: Volume increase per 100 bbl (bbl/100bbl)

**Formula(s)**

$$V = 100 * \frac{W_2 - W_1}{35 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 82.*

**2.110 Increase volume by calcium carbonate****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt. (ppg)

**Output(s)**

V: Volume increase per 100 bbl (bbl/100bbl)

**Formula(s)**

$$V = 100 * \frac{W_2 - W_1}{22.5 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 82.*

**2.111 Increase volume by hematite****Input(s)**

$W_1$ : Initial Mud wt. (ppg)

$W_2$ : Required mud wt. (ppg)

**Output(s)**

$V$ : Volume increase per 100 bbl (bbl/100bbl)

**Formula(s)**

$$V = 100 * \frac{W_2 - W_1}{40 - W_2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 82.*

**2.112 Initial volume required to achieve a volume with barite****Input(s)**

- $W_1$ : Initial Mud wt. (ppg)
- $W_2$ : Required mud wt. (ppg)
- $V_f$ : Final Volume (ppg)

**Output(s)**

$V_i$ : Starting Volume (bbl)

**Formula(s)**

$$V_i = V_f * \frac{35 - W_2}{35 - W_1}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 82.*

**2.113 Initial volume required to achieve a volume with calcium carbonate****Input(s)**

- $W_1$ : Initial Mud wt. (ppg)
- $W_2$ : Required mud wt. (ppg)
- $V_f$ : Final Volume (ppg)

**Output(s)**

$V_i$ : Starting Volume (bbl)

**Formula(s)**

$$V_i = V_f * \frac{22.5 - W_2}{22.5 - W_1}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 83.*

## 2.114 Initial volume required to achieve a volume with hematite

### Input(s)

- $W_1$ : Initial Mud wt. (ppg)
- $W_2$ : Required mud wt. (ppg)
- $V_f$ : Final Volume (ppg)

### Output(s)

- $V_i$ : Starting Volume (bbl)

### Formula(s)

$$V_i = V_f * \frac{40 - W_2}{40 - W_1}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 84.*

## 2.115 Injection/casing pressure required to open valve

### Input(s)

- $P_{bt}$ : Bellows or Dome Pressure at Valve Depth (psi)
- $P_2$ : Production or Tubing Pressure at Valve Depth (psi)
- $A_p$ : Area of Valve Port or Seat (in.<sup>2</sup>)
- $A_b$ : Area of Bellows (in.<sup>2</sup>)

### Output(s)

- $P_1$ : Injection/Casing Pressure Existing at the Valve Depth (psi)

### Formula(s)

$$P_1 = \frac{P_{bt} - \left( P_2 * \left( \frac{A_p}{A_b} \right) \right)}{1 - \left( \frac{A_p}{A_b} \right)}$$

Reference: Beggs, H. D. 2003. *Production Optimization using Nodal Analysis*, OGCI and Petroskills Publications, Second Edition, Chapter 5, Page: 166.

## 2.116 Input power of a pump—Using fuel consumption rate

### Input(s)

- $Q_f$ : Rate of Fuel Consumption (lbm/h)
- $H$ : Fuel Heating Value (BTU/lb)

### Output(s)

- $P_i$ : Input Power (hp)

**Formula(s)**

$$P_i = \frac{Q_f * H}{2545}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 6.*

**2.117 Jet velocity—Nozzle hydraulic analysis****Input(s)**

- Q: Flow Rate (GPM)
- F: Jet Size (1) (in.)
- C: Jet Size (2) (in.)
- B: Jet Size (3) (in.)

**Output(s)**

- $v_c$ : Jet Velocity (ft/s)

**Formula(s)**

$$v_c = \frac{417.2 * Q}{F^2 + C^2 + B^2}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 170.*

**2.118 Kick analysis—Influx****Input(s)**

- SICP: Shut in Casing Pressure (psi)
- SIDPP: Shut in Drill Pipe Pressure (psi)
- $h_i$ : Height of Influx (ft)
- mw: Mud weight (ppg)

**Output(s)**

- I: Influx (ppg)

**Formula(s)**

$$I = mw - \frac{SICP - SIDPP}{h_i * 0.052}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 127.*

## 2.119 Kick analysis—Formation pressure with well shut-in on a kick

### Input(s)

SIDPP: Maximum allowable shut in casing pressure (psi)  
mw: Mud weight (ppg)  
h: Height (ft)

### Output(s)

$P_{fp}$ : Kick Analysis (Formation Pressure with well shut in on a kick) (psi)

### Formula(s)

$$P_{fp} = SIDPP + (mw * 0.052 * h)$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 125.*

## 2.120 Kick analysis—Maximum pit gain from a gas kick in water-based mud

### Input(s)

P: Formation Pressure (psi)  
V: Pit Gain (bbl)  
KWM: Kill weight mud (ppg)  
C: Annular Capacity (bbl/ft)

### Output(s)

MPG: Maximum pit gain resulting from a gas kick in a water-based mud (bbl)

### Formula(s)

$$MPG = 4 * \left( P * V * \frac{C}{KWM} \right)^{0.5}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 127.*

## 2.121 Kick analysis—Maximum surface pressure from a gas kick in water-based mud

### Input(s)

P: Formation Pressure (psi)  
V: Pit Gain (bbl)  
KWM: Kill weight mud (ppg)  
C: Annular Capacity (bbl/ft)

### Output(s)

MSP: Max surface pressure resulting from a gas kick in a water-based mud (psi)

**Formula(s)**

$$MSP = 0.2 * \left( P * V * \frac{KWM}{C} \right)^{0.5}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 129.*

**2.122 Kick analysis—Shut-in drill pipe pressure****Input(s)**

$P_{fp}$ : Formation Pressure (psi)  
 mw: Mud weight (ppg)  
 h: height (ft)

**Output(s)**

SIDPP: Shut in Drill Pipe Pressure (psi)

**Formula(s)**

$$SIDPP = P_{fp} - (mw * 0.052 * h)$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 131.*

**2.123 Kick analysis—Height of influx****Input(s)**

PG: Pit Gain (bbl)  
 AC: Annular Capacity (bbl/ft)

**Output(s)**

h: Height of Influx (ft)

**Formula(s)**

$$h = \frac{PG}{AC}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 126.*

**2.124 Kill weight mud determination—Moore equation****Input(s)**

SIDPP: Shut-in Drill Pipe Pressure (psi)  
 OMW: Original Mud Weight (ppg)  
 TVD: True Vertical Depth (ft)

**Output(s)**

KWM: Kill Weight Mud (ppg)

**Formula(s)**

$$KWM = \left( \frac{SIDPP}{0.052 * (TVD)} \right) + (OMW)$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 131.*

**2.125 Kinetic friction****Input(s)**

- $\mu_s$ : Static Viscosity (cP)
- a: Constant of Friction in X Direction (dimensionless)
- g: Acceleration Due to Gravity (ft/s<sup>2</sup>)

**Output(s)**

- $\varphi$ : Angle of Friction (rad)
- $\mu_k$ : Kinetic Viscosity (cP)

**Formula(s)**

$$\varphi = \tan(\mu_s)$$

$$\mu_k = \mu_s - \left( \frac{a}{g * \sin(\varphi)} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 375.*

**2.126 Laser specific energy****Input(s)**

- P: Power intensity (w/ft<sup>2</sup>)
- t: Time (s)
- T: Thermal Penetration Depth (ft)

**Output(s)**

LSE: Laser Specific Energy (W s/ft)

**Formula(s)**

$$LSE = \frac{P * t}{T}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 415.*

## 2.127 Lateral load imposed on a casing centralizer—Cementing

### Input(s)

- $m$ : Steel in Mud Buoyancy Factor (dimensionless)
- $W$ : Weight per foot of Casing (lbf)
- $L$ : Distance from Centralizer to next Lower Centralizer (ft)
- $\theta$ : Borehole Angle (degree)
- $T$ : Tension (Pulling Force) due to casing below Centralizer (lbf)
- $\delta$ : One-half the change in Angle between Centralizer and next Lower Centralizer (degrees)

### Output(s)

- $F_L$ : Lateral Load, Casing Weight Component  $\pm$  Tension Component (lbf)

### Formula(s)

$$F_L = m \cdot W \cdot L \cdot \sin \theta \pm 2(T) \sin \delta$$

Reference: Suman Jr, G. O., & Ellis, R. C. (1977). *Cementing handbook*. World Oil, Page: 44.

## 2.128 Lateral load imposed on a casing centralizer with a dogleg—Cementing

### Input(s)

- $m$ : Steel in Mud Buoyancy Factor (dimensionless)
- $W$ : Weight per foot of Casing (lbf)
- $L$ : Distance from Centralizer to next Lower Centralizer (ft)
- $\theta$ : Borehole Angle (degree)
- $T$ : Tension (Pulling Force) due to casing below Centralizer (lbf)
- $\delta$ : One-half the change in Angle between Centralizer and next Lower Centralizer (degrees/100 ft)

### Output(s)

- $F_L$ : Lateral Load, Casing Weight Component  $\pm$  Tension Component (lbf)

### Formula(s)

$$F_L = m \cdot W \cdot L \cdot \sin \theta \pm 2(T) \sin \delta$$

$$\delta = \frac{\text{Dogleg} \left( \frac{\text{degrees}}{100\text{ft}} \right) \cdot \text{Spacing(ft)}}{200}$$

$$T = \sum m \cdot W \cdot L \cdot \cos \theta$$

Reference: Suman Jr, G. O., & Ellis, R. C. (1977). *Cementing handbook*. World Oil, Page: 44.

## 2.129 Linear annular capacity of pipe

### Input(s)

- $D_h$ : Inside Diameter of Casing Against the Pipe (in.)
- $D_o$ : Outside Diameter of Pipe (in.)

### Output(s)

$C_o$ : Annular Linear Capacity of Pipe (bbl/ft)

### Formula(s)

$$C_o = \frac{0.7854 * ((D_h)^2 - (D_o)^2)}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 44.*

## 2.130 Linear capacity of pipe

### Input(s)

$D_i$ : Inside Diameter of Pipe (in.)

### Output(s)

$C_i$ : Linear Capacity of Pipe (bbl/ft)

### Formula(s)

$$C_i = \frac{0.7854 * (D_i)^2}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 44.*

## 2.131 Load to break cement bond—Cementing

### Input(s)

$S_c$ : Compressive Strength (psi)

$d$ : Outside Diameter of Casing (in.)

$H$ : Height of Cement Column (ft)

### Output(s)

$F$ : Force or Load to Break Cement Bond (lbfm)

### Formula(s)

$$F = 0.969 \cdot S_c \cdot d \cdot H$$

Reference: *Suman Jr, G. O., & Ellis, R. C. (1977). Cementing handbook. World Oil, Page: 6.*

## 2.132 Mass rate of flow through annulus

### Input(s)

- $p_o$ : Pressure at initial point (Pa)  
 $p_L$ : Pressure at point L (Pa)  
 $R$ : Radius (m)  
 $\rho$ : Density ( $\text{kg}/\text{m}^3$ )  
 $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )  
 $L$ : Length (m)  
 $K$ : Boltzmann Constant ( $\text{m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$ )

### Output(s)

- $\Psi$ : Mass Rate of Flow (kg/s)

### Formula(s)

$$\Psi = \frac{\pi * (p_o - p_L) * R^4 * \rho}{8 * \mu * L} * \left( (1 - K^4) - \frac{(1 - K^2)^2}{\ln\left(\frac{1}{K}\right)} \right)$$

Reference: *Transport Phenomena, Second Edition, Bird, Chapter 2.*

## 2.133 Matching conditions at the cake-to-rock interface—Formation invasion

### Input(s)

- $P_{i\_wall}$ : Pressure value at Cake to Rock Interface (psi)  
 $P_{i\_wall-1}$ : Pressure value at previous grid of Cake to Rock Interface (psi)  
 $P_{i\_wall+1}$ : Pressure value at the following grid of Cake to Rock Interface (psi)  
 $k_c$ : Permeability of the Mudcake (mD)  
 $k_r$ : Permeability of the Rock (mD)

### Output(s)

- $(k_c + k_r)P_{i\_wall}$ : Relationship for matching conditions at the Cake-to-Rock interface (mD psi)

### Formula(s)

$$(k_c + k_r)P_{i\_wall} = k_c P_{i\_wall-1} + k_r P_{i\_wall+1}$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 147.*

## 2.134 Maximum allowable mud weight

### Input(s)

- $mw$ : Mud weight (ppg)  
 $tvd$ : True Vertical Depth of Casing Shoe (ft)  
 $P_l$ : Leak-off Pressure (psi)

**Output(s)**

$mW_{max}$ : Max Allowable Mud wt (ppg)

**Formula(s)**

$$mW_{max} = \frac{P_l}{0.052 * tvd} + mw$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 120.*

**2.135 Maximum drilling rate—Larger holes****Input(s)**

$MW_o$ : Mud Weight out (ppg)

$MW_i$ : Mud Weight in (ppg)

$q_c$ : Circulation Rate (gpm)

$D_h$ : Hole Size (in.)

**Output(s)**

$MDR$ : Maximum Drilling Rate (ft/h)

**Formula(s)**

$$MDR = \frac{67 * (MW_o - MW_i) * q_c}{D_h^2}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 19.*

**2.136 Maximum equivalent derrick load****Input(s)**

$F_h$ : Load Hoisted (lb)

$n$ : No. of Lines Strung Between Crown Block and Traveling Block (unitless)

**Output(s)**

$F_{de}$ : Maximum Equivalent Derrick Load (lb)

**Formula(s)**

$$F_{de} = \left( \frac{n+4}{n} \right) * F_h$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 11.*

## 2.137 Maximum length of a slanted well in a given reservoir thickness

### Input(s)

- h: Pay Zone Thickness (ft)  
 $\alpha$ : Angle of inclination of drilling (degrees)

### Output(s)

- L: Length of Slant Wellbore (ft)

### Formula(s)

$$L = \frac{h}{\cos(\alpha * \frac{\pi}{180})}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 96.

## 2.138 Maximum length of drillpipe for a specific bottomhole assembly

### Input(s)

- T: Tensile Strength of Pipe (lb)  
f: Safety Factor (unitless)  
MOP: Margin of Overpull (unitless)  
 $W_b$ : Weight of Bha in Air (lb/ft)  
 $W_d$ : Weight of Drill Pipe in Air (lb/ft)  
BF: Buoyancy Factor (unitless)

### Output(s)

- $L_m$ : Maximum Length (ft)

### Formula(s)

$$L_m = \frac{((T * f) - MOP - W_b) * BF}{W_d}$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover*, Second Edition, Gulf Professional Publishing, Page: 43.

## 2.139 Maximum recommended low-gravity solids

### Input(s)

- SF: Maximum Recommended Solids Fractions (% by vol)  
MW: Mud Weight (ppg)

### Output(s)

- LGS: Maximum Recommended Lgs (% by vol)

**Formula(s)**

$$LGS = \left( \left( \frac{SF}{100} \right) - \left( 0.3125 * \left( \left( \frac{MW}{8.33} \right) - 1 \right) \right) \right) * 200$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 95.*

**2.140 Maximum recommended solids fractions in drilling fluids****Input(s)**

MW: Mud Weight (ppg)

**Output(s)**

SF: Maximum Recommended Solid Fraction (percent by volume)

**Formula(s)**

$$SF = (2.917 * MW) - 14.17$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 96.*

**2.141 Maximum weight on bit****Input(s)**

$L_d$ : Length of Drill Collar (ft)

$W_d$ : Unit Wight of Collar (lbf/ft)

SF: Safety Factor (dimensionless)

BF: Buoyancy Factor (dimensionless)

$\alpha$ : Wellbore Inclination (degree)

**Output(s)**

WOB: Maximum Weight on Bit (lbf)

**Formula(s)**

$$WOB = \frac{L_d * W_d * BF * \cos(\alpha)}{SF}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 380.*

**2.142 Mechanical energy balance for wellbore fluids****Input(s)**

$p_1, p_2$ : Pressure at positions 1 and 2 (psi)

$\rho_f$ : Fluid Density (lbm/ft<sup>3</sup>)

$g_c$ : Gravitational System Conversion Constant (32.17 (lbf/ft)/(lbf/s<sup>2</sup>))

$g$ : Acceleration of Gravity (ft/s<sup>2</sup>)

$Z_1, Z_2$ : Fluid elevation at Positions 1 and 2 (ft)

$v_1, v_2$ : Fluid velocity at Positions 1 and 2 (ft/s)

$E_l$ : Irreversible Energy Loss between Positions 1 and 2 (ft lbf/lbm)

### Output(s)

$W$ : Work done by the Fluid while in Flow (ft lbf/lbm)

### Formula(s)

$$W = - \left[ \int_{P_1}^{P_2} \frac{dp}{\rho_f} + \frac{g}{g_c} (Z_2 - Z_1) + \frac{\rho_f (v_2^2 - v_1^2)}{2g_c} + E_l \right]$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control* (Vol. 10). Society of Petroleum Engineers, Page: 8.

## 2.143 Mechanical specific energy

### Input(s)

WOB: Weight on Bit (lbf)

$A_b$ : Area of Bit (in.<sup>2</sup>)

N: Rotational Speed of Bit (in./m)

T: Torque of Bit (lbf in.)

ROP: Rate of Penetration (m/in.)

### Output(s)

MSE: Mechanical Specific Energy (lbf/in.<sup>2</sup>)

### Formula(s)

$$MSE = \left( \frac{WOB}{A_b} \right) + \left( \frac{120 * \pi * N * T}{A_b * ROP} \right)$$

Reference: Samuel. E Robello. *501 Solved Problems and Calculations for Drilling Operations*. Sigma Quadrant. 2015. Houston, Texas, Page: 414.

## 2.144 Mud rheology—Herschel and Buckley law

### Input(s)

$\tau_y$ : Yield Stress (lb/100 ft<sup>2</sup>)

K: Consistency Index (lb/100 ft<sup>2</sup>)

$\gamma$ : Shear Rate (1/s)

n: Power Law Index, Slope of Plot Between Log Shear Stress and Log Shear Rate (dimensionless)

### Output(s)

$\tau$ : Shear Stress (lb/100 ft<sup>2</sup>)

### Formula(s)

$$\tau = \tau_y + K * (\gamma^n)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 185.*

## 2.145 Mud rheology—Power-law model—Consistency index

### Input(s)

- $\theta_a$ : Fann Dial Reading at 300 Rpm (cP)
- n: Power-Law index (dimensionless)

### Output(s)

- K: Consistency index (cP)

### Formula(s)

$$K = \frac{510 * \theta_a}{511^n}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 194.*

## 2.146 Mud rheology—Power-law model—Power-law index

### Input(s)

- $\theta_a$ : Fann Dial Reading at 600 Rpm (cP)
- $\theta_b$ : Fann Dial Reading at 300 Rpm (cP)

### Output(s)

- n: Power Law index (dimensionless)

### Formula(s)

$$n = 3.322 * \log \left( \frac{\theta_b}{\theta_a} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 194.*

## 2.147 Mud rheology—Power-law

### Input(s)

- K: Consistency Index (cP)
- n: Power-Law Index (fraction)
- $\gamma$ : Shear Rate (1/s)

### Output(s)

- $\tau$ : Shear Stress (lbf/100 ft<sup>2</sup>)

**Formula(s)**

$$\tau = K * (\gamma^n)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 184.*

**2.148 Mud rheology calculations—Bingham plastic model****Input(s)**

- $\tau_y$ : Yield Stress (lb/100 ft<sup>2</sup>)
- $\mu_p$ : Bingham Plastic Viscosity (cP)
- $\gamma$ : Shear Rate (1/s)

**Output(s)**

- $\tau$ : Shear Stress (lb/100 ft<sup>2</sup>)

**Formula(s)**

$$\tau = \tau_y + (\mu_p * \gamma)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 185.*

**2.149 Mud weight increase required to balance pressure****Input(s)**

- $F$ : Differential Force (lb)
- $A$ : Area Below Casing Shoe (in.<sup>2</sup>)
- $L_c$ : Casing Length (ft)

**Output(s)**

- $W_m$ : Mud Weight increase Required (ppg)

**Formula(s)**

$$W_m = \frac{F}{A * 0.052 * L_c}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 67.*

**2.150 Mud weight reduction by dilution—Water/diesel/any liquid****Input(s)**

- $W_2$ : Required mud density (ppg)
- $W_1$ : Initial Mud density (ppg)
- $V_m$ : Initial Mud Volume (bbl)
- $D_w$ : Water/Diesel density (ppg)

### Output(s)

$V_r$ : Required Volume of water/diesel/any liq (bbl)

### Formula(s)

$$V_r = V_m * \frac{W_1 - W_2}{W_2 - D_w}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 85.*

## 2.151 Mudcake growth equation—Formation invasion

### Input(s)

$f_s$ : Mud Solid Fraction (fraction)

$\emptyset_c$ : Mudcake Porosity (fraction)

$v_n$ : Darcy Velocity of the Filtrate through the cake and past the paper (in./s)

### Output(s)

$\frac{dx_c(t)}{dt}$ : Mudcake Growth by time (in./s)

### Formula(s)

$$\frac{dx_c(t)}{dt} = \left( \frac{f_s}{(1-f_s)(1-\emptyset_c)} \right) |v_n|$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 39.*

## 2.152 Mudcake growth equation-2—Formation invasion

### Input(s)

$\emptyset_{eff}$ : Effective Porosity (fraction)

$f_s$ : Mud Solid Fraction (fraction)

$\emptyset_c$ : Mudcake Porosity (fraction)

$x_f$ : Transient Invasion Front (in.)

$x_{f,o}$ : Initial Displacement, i.e., Spurt (in.)

### Output(s)

$x_c(t)$ : Mudcake Growth by time (in./s)

### Formula(s)

$$x_c(t) = \left[ \emptyset_{eff} f_s / \{(1-\emptyset_c)(1-f_s)\} \right] (x_f - x_{f,o})$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 45.*

## 2.153 Mudcake permeability—Formation invasion

### Input(s)

$\mu$ : Viscosity (cP)  
 $h(t)$ : Filtrate Height (in.)  
 $x_c(t)$ : Mudcake Growth by time (in./s)  
 $\Delta p$ : Pressure Change (psi)  
 $t$ : Time (s)

### Output(s)

$k$ : Viscous Shear Stress at the outer Mudcake boundary (lb/in.<sup>2</sup>)

### Formula(s)

$$k = \frac{\mu \cdot h(t) x_c(t)}{(2t\Delta p)}$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 95.*

## 2.154 New pump circulating pressure

### Input(s)

$P_p$ : Present Circulating Pressure (psi)  
 $q_n$ : New Pump Rate (spm)  
 $q_o$ : Old Pump Rate (spm)

### Output(s)

$P_c$ : New Circulating Pressure (psi)

### Formula(s)

$$P_c = P_p * \left( \frac{q_n}{q_o} \right)^2$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 27.*

## 2.155 Nozzle area calculation

### Input(s)

F: 1st Nozzle Size (in.)  
C: 2nd Nozzle Size (in.)  
B: 3rd Nozzle Size (in.)

### Output(s)

$N_a$ : Nozzle Area (in.<sup>2</sup>)

**Formula(s)**

$$N_a = \frac{F^2 + C^2 + B^2}{1303.8}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 165.*

**2.156 Number of sacks of cement required****Input(s)**

- H: Hole Size (in.)
- OD: Outer Dia (in.)
- ID: Inner Dia (in.)
- $h_c$ : Feet to be Cemented (ft)
- E: Original+extra Volume Req in Percentage (fraction)
- $h_{fc}$ : Feet Between Float Collar and Shoe (ft)
- Y1: Yield of Lead/filler Cement ( $\text{ft}^3/\text{stroke}$ )
- Y2: Yield of Tail Cement ( $\text{ft}^3/\text{stroke}$ )
- FC: Float Collar (Number of Feet Above Shoe) (ft)
- Csd: Casing Setting Depth (ft)
- PO: Pump Output (bbl/stroke)

**Output(s)**

- AC: Annular Capacity ( $\text{ft}^3/\text{ft}$ ) C:Casing Capacity ( $\text{ft}^3/\text{ft}$ )
- $N_f$ : Number of Sacks of Cement Required (stroke)
- $N_a$ : Sacks Required for Annulus (stroke)
- $N_c$ : Sacks Required for Casing (stroke)
- $N_t$ : Total Number of Sacks Required (stroke)
- Cd: Casing Capacity in Barrels (bbl/ft)
- SS: Number of Strokes Required to Bump the Plug (strokes)

**Formula(s)**

$$AC = \frac{H^2 - OD^2}{183.35}$$

$$C = \frac{(ID)^2}{183.35}$$

$$N_f = \frac{h_c * AC * E}{Y1}$$

$$N_a = h_c * AC * \frac{E}{Y2}$$

$$N_c = h_{fc} * \frac{C}{Y2}$$

$$N_t = N_a + N_c$$

$$Cd = \frac{(ID)^2}{1029.4}$$

$$SS = \frac{Cd}{PO}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 54.*

## 2.157 Number of sacks of cement required for a given length of plug

### Input(s)

- L<sub>p</sub>: Plug Length (ft)
- C<sub>c</sub>: Casing Capacity (ft<sup>3</sup>/ft)
- E: Excess Volume (fraction)
- Y: Slurry Yield (ft<sup>3</sup>/stroke)

### Output(s)

- N<sub>s</sub>: Sacks of Cement Required (unitless)

### Formula(s)

$$N_s = \frac{L_p * C_c * E}{Y}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 61.*

## 2.158 Number of sacks of lead cement required for annulus

### Input(s)

- L<sub>c</sub>: Feet to be Cemented (ft)
- C<sub>a</sub>: Annular Capacity (ft<sup>3</sup>/ft)
- E: Excess Volume (fraction)
- Y: Slurry Yield of Lead Cement (ft<sup>3</sup>/stroke)

### Output(s)

- N<sub>s</sub>: Sacks Required (unitless)

### Formula(s)

$$N_s = \frac{L_c * C_a * E}{Y}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 54.*

## 2.159 Number of sacks of tail cement required for casing

### Input(s)

- L<sub>c</sub>: Distance between Float Collar and Shoe (ft)
- C<sub>c</sub>: Casing Capacity (ft<sup>3</sup>/ft)
- Y: Slurry Yield of Tail Cement (ft<sup>3</sup>/stroke)

### Output(s)

- N<sub>s</sub>: Sacks Required by Casing (unitless)

**Formula(s)**

$$N_s = \frac{L_c * C_c}{Y}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 56.*

**2.160 Open-ended displacement volume of pipe****Input(s)**

- D<sub>o</sub>: Outside Diameter of Pipe (in.)
- D<sub>i</sub>: Inside Diameter of Pipe (in.)
- L: Length of Pipe (ft)

**Output(s)**

- V<sub>o</sub>: Open-ended Displacement Volume of Pipe (bbl)

**Formula(s)**

$$V_o = \frac{0.7854 * (D_o^2 - D_i^2) * L}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 45.*

**2.161 Overall efficiency—Diesel engines to mud pump****Input(s)**

- n<sub>e</sub>: Engine Efficiency (unitless)
- n: Electric Motor Efficiency (unitless)
- n<sub>m</sub>: Mud Pump Mechanical Efficiency (unitless)
- n<sub>v</sub>: Mud Pump Volumetric Efficiency (unitless)

**Output(s)**

- n<sub>o</sub>: Overall Efficiency (unitless)

**Formula(s)**

$$n_o = n_e * n_l * n_m * n_v$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 49.*

## 2.162 Overall power system efficiency

### Input(s)

- $P$ : Power Output (hp)  
 $Q_i$ : Total Heat Energy consumed by the Engine (hp)

### Output(s)

- $E_i$ : Total Heat Energy consumed by the Engine (hp)

### Formula(s)

$$E_i = \frac{P}{Q_i}$$

Reference: Bourgoyne, A. T., Millheim, K. K., Chenevert, M. E., & Young, F. S. (1986). *Applied Drilling Engineering*, Page: 7.

## 2.163 Penetration rate—Drill-rate model—Alternative equation

### Input(s)

- $K'$ : Drill-Rate Model Proportionality Constant (dimensionless)  
 $N$ : Bit rotating Speed (rev/min)  
 $a_N$ : Rotating Speed Exponent (dimensionless)

### Output(s)

- $R$ : Penetration Rate (ft/h)

### Formula(s)

$$\log R = \log K' + a_N \log N$$

Reference: Watson, D., Brittenham, T., & Moore, P. L. (2003). *Advanced Well Control* (Vol. 10). Society of Petroleum Engineers, Page: 49.

## 2.164 Penetration rate—Drill-rate model—Basic equation

### Input(s)

- $d_b$ : Bit Diameter (in.)  
 $K$ : Drill-Rate Model Proportionality Constant (dimensionless)  
 $N$ : Bit rotating Speed (rev/min)  
 $W$ : Applied Bit Weight (lbf)  
 $a_W$ : Bit Weight Exponent (dimensionless)  
 $a_N$ : Rotating Speed Exponent (dimensionless)

### Output(s)

$R$ : Penetration Rate (ft/h)

### Formula(s)

$$R = K \left( \frac{W}{d_b} \right)^{a_w} N^{a_N}$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers, Page: 48.*

## 2.165 Percentage of bit nozzle pressure loss

### Input(s)

$P_b$ : Jet Pressure Nozzle (psi)

$P$ : Surface Pressure (psi)

### Output(s)

$\% \psi_b$ : Percent Pressure Loss (fraction)

### Formula(s)

$$\% \psi_b = \left( \frac{P_b}{P} \right) * 100$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 165.*

## 2.166 Plastic viscosity—Bingham plastic model

### Input(s)

$\theta_{600}$ : Fann Dial Reading at 600 Rpm (cP)

$\theta_{300}$ : Fann Dial Reading at 300 Rpm (cP)

### Output(s)

$PV$ : Plastic Viscosity (cP)

### Formula(s)

$$PV = (\theta_{600}) - (\theta_{300})$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 192.*

## 2.167 Plug length to set a balanced cement plug

### Input(s)

- N<sub>s</sub>: Number of Sacks of Cement to be Used (stroke)  
 AC: Annular Capacity (ft<sup>3</sup>/ft)  
 P<sub>c</sub>: Pipe or Tubing Capacity (ft<sup>3</sup>/ft)  
 E: Original+extra Volume Req in Percentage (fraction) Y<sub>s</sub>:Yield of Slurry (ft<sup>3</sup>/stroke)

### Output(s)

- L<sub>p</sub>: Length of Plug (stroke)

### Formula(s)

$$L_p = \frac{N_s * Y_s}{AC * E + P_c}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 61.*

## 2.168 Polar moment of inertia

### Input(s)

- J<sub>b</sub>: Polar Moment of Inertia of Pipe Body (ft<sup>4</sup>)  
 J<sub>j</sub>: Polar Moment of Inertia of Tool Joint (ft<sup>4</sup>)  
 L: Length of Pipe (ft)  
 l: Joint Tool Length (ft)

### Output(s)

- J<sub>p</sub>: Polar Moment of Inertia of Pipe (ft<sup>4</sup>)

### Formula(s)

$$J_p = \frac{J_b * J_j}{\left(\frac{L-l}{L}\right) * J_j * \left(\frac{l}{L}\right) * J_b}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 356.*

## 2.169 Polished rod horsepower—Sucker-rod pump

### Input(s)

- C: Calibration Constant of Dynamometer (lb/in.)  
 A: Upper Area of Card (in.<sup>2</sup>)  
 L: Length of Card (in.)  
 S: Maximum Theoretical Polished Rod Position (in.)  
 N: Strokes Per Minute (spm)

### Output(s)

PRHP: Polished Rod Horsepower (hp)

### Formula(s)

$$\text{PRHP} = C * S * N * \frac{A}{33000 * 12 * L}$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18.*

## 2.170 Pore-pressure gradient—Rehm and McClendon

### Input(s)

- $d_{cn}$ : Normal dc (Modified Bit-Weight exponent in Bingham equation) exponent (dimensionless)
- $d_{co}$ : Observed dc (Modified Bit-Weight exponent in Bingham equation) exponent (dimensionless)

### Output(s)

$g_p$ : Pore Pressure Gradient (psi/ft)

### Formula(s)

$$g_p = (0.398 \log(d_{cn} - d_{co})) + 0.86$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers, Page: 52.*

## 2.171 Pore-pressure gradient—Zamora

### Input(s)

- $d_{cn}$ : Normal dc (Modified Bit-Weight exponent in Bingham equation) exponent (dimensionless)
- $d_{co}$ : Observed dc (Modified Bit-Weight exponent in Bingham equation) exponent (dimensionless)
- $g_n$ : Normal Pore Pressure Gradient (psi/ft)

### Output(s)

$g_p$ : Pore Pressure Gradient (psi/ft)

### Formula(s)

$$g_p = g_n \left( \frac{d_{cn}}{d_{co}} \right)$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers, Page: 52.*

## 2.172 Pressure analysis—Pressure by each barrel of mud in casing

### Input(s)

Dh: Hole Dia (in.)  
 Dp: Pipe Dia (in.)  
 mw: Mud wt (ppg)

### Output(s)

P: Hydrostatic Pressure (psi/bbl)

### Formula(s)

$$P = 1029.4 * 0.052 * \frac{mw}{(Dh)^2 - (Dp)^2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 138.*

## 2.173 Pressure analysis—Surface pressure during drill stem test

### Input(s)

h: Total Vertical Depth (ft)  
 EMW: Equivalent Mud Weight for Formation Pressure (ppg)  
 SG: Oil Specific Gravity (fraction)

### Output(s)

P: Hydrostatic Pressure (psi/bbl)

### Formula(s)

$$P = 0.052 * h * (EMW - SG * 8.33)$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 138.*

## 2.174 Pressure gradient

### Input(s)

mw: Mud weight (ppg)

### Output(s)

PG: Pressure Gradient (psi/ft)

### Formula(s)

$$PG = mw * 0.052$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 1.*

## 2.175 Pressure required to break circulation—Annulus

### Input(s)

- y: Gel strength of drilling fluid ( $\text{lb}/100\text{ft}^2$ )
- L: Length of Drill String (ft)
- Dh: Diameter of hole (in.)
- Dp: Diameter of drill pipe (in.)

### Output(s)

- $P_{gs}$ : Pressure to overcome mud's gel strength inside annulus (psi)

### Formula(s)

$$P_{gs} = \frac{y}{300 * (Dh - Dp)} * L$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 79.*

## 2.176 Pressure required to break circulation—Drill string

### Input(s)

- y: Gel Strength of drilling fluid ( $\text{lb}/100\text{ft}^2$ )
- L: Length of Drill String (ft)
- d: Inside diameter of drill pipe (in.)

### Output(s)

- $P_{gs}$ : Pressure to overcome mud's gel strength inside drill string (psi)

### Formula(s)

$$P_{gs} = \left( y * \frac{L}{300 * d} \right)$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 79.*

## 2.177 Pressure required to overcome gel strength of mud inside the drill string

### Input(s)

- y: Gel Strength of Drilling Fluid ( $\text{lb}/100 \text{ ft}^2$ )
- d: Inside Diameter of Drill Pipe (in.)
- L: Length of Drill String (ft)

### Output(s)

- $P_m$ : Pressure Required (psi)

**Formula(s)**

$$P_m = \frac{y * L}{300 * d}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 79.*

**2.178 Pressure required to overcome mud's gel strength in annulus****Input(s)**

- y: Gel Strength of Drilling Fluid (lb/100 ft<sup>2</sup>)
- D<sub>h</sub>: Hole Diameter (in.)
- D<sub>p</sub>: Pipe Diameter (in.)
- L: Length of Drillstring (ft)

**Output(s)**

- P<sub>m</sub>: Pressure Required (psi)

**Formula(s)**

$$P_m = \frac{y * L}{300 * (D_h - D_p)}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 79.*

**2.179 Pump calculation—Pump pressure****Input(s)**

- P<sub>d</sub>: Frictional Pressure Losses (psi)
- ΔP: Bit Pressure Drop (psi)

**Output(s)**

- P<sub>p</sub>: Pump Pressure (psi)

**Formula(s)**

$$P_p = \Delta P + P_d$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 69.*

**2.180 Pump calculations—Power required****Input(s)**

- Q: Flow Rate (gpm)
- ΔP: Bit Pressure Drop (psi)

### Output(s)

$HP_p$ : Power Required (hp)

### Formula(s)

$$HP_p = \frac{Q * \Delta P}{1714}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 70.*

## 2.181 Pump displacement

### Input(s)

D: Rotor Diameter (in.)  
E: Rotor/Stator Eccentricity (in.)  
 $P_s$ : Pitch Length of Stator (ft)

### Output(s)

$V_o$ : Pump Displacement ( $\text{ft}^3$ )

### Formula(s)

$$V_o = 0.028 * D * E * P_s$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 14.*

## 2.182 Pump flow rate

### Input(s)

D: Rotor Diameter (in.)  
E: Rotor/Stator Eccentricity (in.)  
N: Rotary Speed (rpm)  
 $Q_s$ : Leak Rate (bbl/d)  
 $P_s$ : Pitch Length of Stator (ft)

### Output(s)

$Q_e$ : Pump Flow Rate (bbl/d)

### Formula(s)

$$Q_e = 7.12 * D * E * P_s * N - Q_s$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 14.*

## 2.183 Pump head rating

### Input(s)

- $n_p$ : Number of Pitches of Stator (unitless)  
 $\Delta p$ : Head Rating Developed into an Elementary Cavity (psi)

### Output(s)

- $\Delta P$ : Pump Head Rating (psi)

### Formula(s)

$$\Delta P = (2 * n_p - 1) * \Delta p$$

Reference: *Boyoun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 14.*

## 2.184 Pump output—gpm

### Input(s)

- d: Liner Diameter (in.)  
S: Stroke Length (in.)  
spm: Strokes per minute (dimensionless)

### Output(s)

- PO: Pump Output gpm (bbl/stroke)

### Formula(s)

$$PO = 3 * (d^2 * 0.7854) * S * 0.00411 * spm$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 8.*

## 2.185 Pump output triplex pump

### Input(s)

- ld: Liner Diameter (in.)  
sl: Stroke Length (in.)

### Output(s)

- PO: Pump Output (bbl/stroke)

### Formula(s)

$$PO = 0.000243 * ld^2 * sl$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 7.*

## 2.186 Pump pressure/pump stroke relationship

### Input(s)

- PCP: Present Circulation Pressure (psi)  
 NPR: New Pump Rate (spm)  
 OPR: Old Pump Rate (spm)

### Output(s)

- PP: Pump Pressure—Pump Stroke Relationship (psi)

### Formula(s)

$$PP = PCP * \left( \frac{NPR}{OPR} \right)^2$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, second edition, Gulf Professional Publishing, Page: 27.*

## 2.187 Radial force related to axial load—Cementing

### Input(s)

- F: Force or Load to Break Cement Bond (lbm)  
 $\alpha$ : Slip Bowl Taper Angle (degree)  
 $\mu$ : Constant (Usually 0.2) (dimensionless)

### Output(s)

- W: Radial Force related to Axial Load (lbm)

### Formula(s)

$$W = \frac{1 - (\mu \cdot \tan \alpha)}{\mu + \tan \alpha} \cdot F$$

Reference: *Suman Jr, G. O., & Ellis, R. C. (1977). Cementing Handbook. World Oil, Page: 18.*

## 2.188 Range of load—Sucker-Rod pump

### Input(s)

- PPRL: Peak Polished Rod Load (lb)  
 MPRL: Minimum Polished Rod Load (lb)

### Output(s)

- ROL: Range of Load (lb)

**Formula(s)**

$$ROL = PPRL - MPRL$$

Reference: *Boyuni, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18.*

**2.189 Rate of fuel consumption by a pump****Input(s)**

- N: Pump Rotary Speed (rpm)
- T: Output Torque (ft lbs)
- n: Pump Efficiency (%)
- H: Fuel Heating Value (BTU/lb)

**Output(s)**

- Q<sub>f</sub>: Fuel Consumption Rate (lbm/h)

**Formula(s)**

$$Q_f = 48.46 * \frac{N * T}{n * H}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 6.*

**2.190 Rate of gas portion that enters the mud****Input(s)**

- q<sub>r</sub>: Rock Removal Rate (SCF/min)
- S<sub>g</sub>: Gas Saturation (fraction)
- Ø: Formation Porosity (per cent)

**Output(s)**

- q<sub>g</sub>: The Rate of Gas Portion that enters the Mud with Bulk Rock (SCF/min)

**Formula(s)**

$$q_g = q_r \varnothing S_g$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers, Page: 19.*

**2.191 Relationship between traveling block speed and fast line speed****Input(s)**

- v<sub>f</sub>: Fast Line Speed (fpm)
- n: Number of Lines Strung B/w Crown and Traveling Block (unitless)

### Output(s)

$v_{tb}$ : Traveling Block Velocity (fpm)

### Formula(s)

$$v_{tb} = \frac{v_f}{n}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 10.*

## 2.192 Rock removal rate

### Input(s)

$d_b$ : Bit Diameter (in.)

$R$ : Penetration Rate (ft/h)

### Output(s)

$q_r$ : Rock Removal Rate (SCF/min)

### Formula(s)

$$q_r = \frac{\pi d_b^2 R(12)}{4(1,728)(60)} = \frac{d_b^2 R}{11000}$$

Reference: *Watson, D., Brittenham, T., & Moore, P. L. (2003). Advanced Well Control (Vol. 10). Society of Petroleum Engineers., Page: 19.*

## 2.193 Rotating horsepower

### Input(s)

$T$ : Torque (ft lbf)

$N$ : Speed (rpm)

### Output(s)

RHP: Rotating Horsepower (hp)

### Formula(s)

$$RHP = \frac{T * N}{5252}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 46.*

## 2.194 Side force at bit in anisotropic formation

### Input(s)

- p: Buoyed Weight of Drill Collar (ppf)
- E: Modulus of Elasticity (psi)
- I: Moment of inertia of Drill Collar ( $\text{in.}^4$ )
- r: Radial Clearance Between Hole and Collar (ft)
- $\theta$ : inclination Angle (degree)
- W: Weight on Bit (lbf)

### Output(s)

- F: Side Force on Bit (lbf)

### Formula(s)

$$F = p * \left( (E*I)^{0.5} \right) * \left( \left( \frac{W}{24} \right) * \left( \left( \frac{24*r}{E*I*p * \sin(\theta)} \right)^{0.75} \right) - \left( \left( \frac{1.5*r}{E*I*p * \sin(\theta)} \right)^{0.25} \right) \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 378.*

## 2.195 Sinusoidal buckling

### Input(s)

- E: Modulus of Elasticity (psi)
- I: Moment of Inertia ( $\text{ft}^4$ )
- W: Weight on Bit (lbf)
- $\theta$ : Angle of Wellbore Inclination (degree)
- r: Radial Clearance Between Wellbore and Component (ft)

### Output(s)

- $F_s$ : Buckling Force (lbf)

### Formula(s)

$$F_s = 2 * \left( \left( \frac{E * I * W * \sin(\theta)}{r} \right)^{0.5} \right)$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 426.*

## 2.196 Slurry density for cementing calculations

### Input(s)

- $V_s$ : Volume of Slurry (gal/stroke)
- $W_a$ : Weight of Additive Per Sack (lb/stroke)
- $Q_w$ : Total Water Requirement (gal/stroke)

### Output(s)

- $\sigma_s$ : Slurry Density (lb/gal)

**Formula(s)**

$$\sigma_s = \frac{94 + W_a + (8.33 * Q_w)}{V_s}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 48.*

**2.197 Solids analysis—High-salt content muds****Input(s)**

- $C_{Cl}$ : Concentration of Chlorine (ppm)
- $P_{vw}$ : Percentage by vol of water (percent)
- $P_{vo}$ : Percentage by vol of oil (percent)
- S: CEC of shale (lb/bbl)
- M: CEC of Mud (lb/bbl)
- mw: Mud Weight (ppg)

**Output(s)**

- SW: Percentage by vol of salt water (percent)
- SS: Percentage by vol of suspended solids (percent)
- $ASG_{sw}$ : Average specific gravity of salt water (fraction)
- ASG: Average specific gravity of solids (fraction)
- LGS: Percentage by volume of low gravity solids (% by vol)
- $P_b$ : Pounds per barrel of barite (% by vol)
- $P_{be}$ : Percentage of Bentonite (lb/bbl)
- $P_{ds}$ : Percentage of Drilled Solids (lb/bbl)

**Formula(s)**

$$SW = \left( (5.88 * 10^{-8}) * \left( (C_{Cl})^{1.2} \right) + 1 \right) * P_{vw}$$

$$SS = 100 - P_{vo} - SW$$

$$ASG_{sw} = \left( (C_{Cl})^{0.95} \right) * 1.94 * 10^{-6} + 1$$

$$ASG = \frac{(12 * mw) - (SW * ASG_{sw}) - 0.84 * P_{vo}}{SS}$$

$$LGS = SS * \frac{4.2 - ASG}{1.6}$$

$$P_b = (SS - LGS) * 14.71$$

$$P_{be} = \frac{\left( (M - 9) * \frac{S}{65} \right) * \frac{LGS}{1 - \frac{S}{65}}}{9.1}$$

$$P_{ds} = (LGS - P_{be}) * 9.1$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 91.*

## 2.198 Solids analysis low-salt content muds

### Input(s)

- $C_{Cl}$ : Concentration of Chlorine (ppm)  
 $P_{vw}$ : Percentage by vol of water (percent)  
 $P_{vo}$ : Percentage by vol of oil (percent)  
S: CEC of shale (lb/bbl)  
M: CEC of Mud (lb/bbl)  
mw: Mud Weight (ppg)

### Output(s)

- SW: Percentage by vol of salt water (percent)  
SS: Percentage by vol of suspended solids (percent)  
 $ASG_{sw}$ : Average specific gravity of salt water (fraction)  
ASG: Average specific gravity of solids (fraction)  
LGS: Percentage by volume of low gravity solids (% by vol)  
 $P_b$ : Pounds per barrel of barite (% by vol)  
 $P_{be}$ : Percentage of Bentonite (lb/bbl)  
 $P_{ds}$ : Percentage of Drilled Solids (lb/bbl)

### Formula(s)

$$SW = \left( (5.88 * 10^{-8}) * \left( (C_{Cl})^{1.2} \right) + 1 \right) * P_{vw}$$

$$SS = 100 - P_{vo} - SW$$

$$ASG_{sw} = \left( (C_{Cl})^{0.95} \right) * 1.94 * 10^{-6} + 1$$

$$ASG = \frac{(12 * mw) - (SW * ASG_{sw}) - 0.84 * P_{vo}}{SS}$$

$$LGS = SS * \frac{4.2 - ASG}{1.6}$$

$$P_b = (SS - LGS) * 14.71$$

$$P_{be} = \frac{\left( (M - 9) * \frac{S}{65} \right) * \frac{LGS}{1 - \frac{S}{65}}}{9.1}$$

$$P_{ds} = (LGS - P_{be}) * 9.1$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 92.*

## 2.199 Spacer volume behind slurry required to balance the plug

### Input(s)

- $C_a$ : Annular Capacity (ft/bbl)  
E: Excess Volume (fraction)  
 $V_a$ : Spacer Volume Ahead (bbl)  
 $C_p$ : Capacity of Pipe or Tubing (bbl/ft)

### Output(s)

$V_s$ : Spacer Volume (bbl)

### Formula(s)

$$V_s = \left( \frac{C_a}{E} \right) * V_a * C_p$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 62.*

## 2.200 Specific gravity of cuttings by using mud balance

### Input(s)

$R_w$ : Resulting Weight with Cuttings Plus Water (ppg)

### Output(s)

$SG$ : Specific Gravity of Cuttings (unitless)

### Formula(s)

$$SG = \frac{1}{2 - (0.12 * R_w)}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 41.*

## 2.201 Stripping/snubbing calculations—Breakover point between stripping and snubbing

### Input(s)

$D_p$ : Pipe or collar OD (in.)

$P$ : Wellbore Pressure (psi)

$W_{dc}$ : Weight of Drill Collar (lb/ft)

$L$ : Drill Collar Length (ft)

$BF$ : Buoyancy Factor (dimensionless)

$W_{dp}$ : Drill Pipe Weight (lb/ft)

### Output(s)

$F$ : Force created by wellbore pressure on Drill collar or pipe (lb)

$W$ : Weight of Drill Collar (lb)

$W_{adp}$ : Additional weight required from Drill Pipe (lb)

$L_{bp}$ : Length of Drill Pipe required to reach over breakover point (ft)

$L_{ds}$ : Length of Drill string req to reach breakover point (ft)

**Formula(s)**

$$F = Dp^2 * 0.7854 * P$$

$$W = W_{dc} * L * BF$$

$$W_{adp} = F - W$$

$$L_{bp} = \frac{W_{adp}}{BF * W_{dp}}$$

$$L_{ds} = L + L_{bp}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 139.*

## **2.202 Stripping/snubbing calculations—Height gain and casing pressure from stripping into influx**

**Input(s)**

- $C_{dp}$ : Capacity of Drill Pipe or tubing (bbl/ft)
- $D_{dp}$ : Displacement of Drill Pipe, Drill Collar or tubing (bbl/ft)
- $C_a$ : Annular Capacity (bbl/ft)
- L: Length of Drill Pipe Stripped (ft)
- G: Gradient of Mud (psi/ft)
- $G_i$ : Gradient of Influx (psi/ft)

**Output(s)**

- H: Height gain from stripping into influx (ft)
- P: Casing Pressure increase from stripping into influx (psi)

**Formula(s)**

$$H = L * \frac{C_{dp} + D_{dp}}{C_a}$$

$$P = H * (G - G_i)$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 140.*

## **2.203 Stripping/snubbing calculations—Maximum Allowable surface pressure governed by casing burst pressure**

**Input(s)**

- $W_o$ : Mud Weight behind casing (ppg)
- $W_u$ : Mud Weight in use (ppg)
- H: Casing Shoe TVD (ft)
- $P_{bc}$ : Casing Burst Pressure (psi)
- S: Safety Factor (e.g., 80% represent 0.8) (dimensionless)

### Output(s)

MASP: Maximum Allowable Surface Pressure governed by Casing Burst Pressure (psi)

### Formula(s)

$$MASP = (P_{bc} * S) - (W_u - W_o) * 0.052 * H$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 143.*

## 2.204 Stripping/snubbing calculations—Maximum allowable surface pressure governed by formation

### Input(s)

$W_{max}$ : Max allowable mud Weight (ppg)

$W_u$ : Mud Weight in use (ppg)

H: Casing Shoe TVD (ft)

### Output(s)

MASP: Maximum Allowable Surface Pressure governed by Formation (psi)

### Formula(s)

$$MASP = (W_{max} - W_u) * 0.052$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 143.*

## 2.205 Stripping/snubbing calculations—Minimum surface pressure before stripping

### Input(s)

$W_c$ : Weight per feet of one stand of Drill Collar (lb/ft)

L: Length of one stand (feet)

D: Drill Collar Dia (in.)

### Output(s)

$P_{min}$ : Minimum surface Pressure before stripping is possible (psi)

### Formula(s)

$$P_{min} = W_c * \frac{L}{D^2 * 0.7854}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 141.*

## 2.206 Stripping/snubbing calculations—Constant BHP with a gas bubble rising

### Input(s)

- D<sub>p</sub>: Incremental pressure steps that the casing pressure will be allowed to increase (psi)  
 Ca: Annular Capacity (in.<sup>2</sup>)  
 G: Gradient of Mud (psi/ft)

### Output(s)

- V: V for constant BHP with a Gas Bubble Rising (bbl)

### Formula(s)

$$V = (D_p) * \frac{Ca}{G}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 142.*

## 2.207 Stroke per minute required for a given annular velocity

### Input(s)

- AV: Annular Velocity (ft/min)  
 AC: Annular Capacity (bbl/ft)  
 q<sub>o</sub>: Pump Output (bbl/stroke)

### Output(s)

- SPM: Strokes Per Minute Required (unitless)

### Formula(s)

$$SPM = \frac{AV * AC}{q_o}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 12.*

## 2.208 Stuck pipe calculations—Method-1

### Input(s)

- OD: Outer dia of tubing (in.)  
 ID: Inner dia of tubing (in.)  
 S: Stretch (in.)  
 PF: Pull force in thousands of pounds (1000lb)

### Output(s)

- fpc: Free Point Constant (dimensionless)  
 h<sub>f</sub>: Feet of Free pipe (ft)

**Formula(s)**

$$fpc = (OD^2 - ID^2) * 0.7854 * 2500$$

$$h_f = S * \frac{fpc}{PF}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 74.*

**2.209 Stuck pipe calculations—Method-2****Input(s)**

- e: Pipe Stretch (in.)
- Wdp: Drill Pipe Wt. (lb/ft)
- Pd: Differential Pull (lb)

**Output(s)**

- $h_f$ : Feet of Free pipe (ft)

**Formula(s)**

$$h_f = 735294 * e * \frac{Wdp}{Pd}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 74.*

**2.210 Subsea considerations—Adjusting choke line pressure loss for higher mud weight****Input(s)**

- $W_o$ : Old mud weight (ppg)
- $W_n$ : Higher mud weight (ppg)
- CLPLO: Old Choke line Pressure Loss for higher mud weight (psi)

**Output(s)**

- CLPL: Adjusted CLPL for higher mud weight (psi)

**Formula(s)**

$$CLPL = W_n * \frac{CLPLO}{W_o}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 144.*

**2.211 Subsea considerations—Casing burst pressure-subsea stack****Input(s)**

- YP: Internal Yield Pressure (psi)

SF: Safety Factor (fraction)  
 $W_u$ : Mud Weight in use (ppg)  
H: Depth from Rotary Kelly Bushing to Mudline (ft)  
 $H_s$ : Depth of Seawater (ft)  
 $W_s$ : Seawater Weight (ppg)

**Output(s)**

$YP_c$ : Corrected Internal Yield Pressure (psi)  
HP: Hydrostatic Pressure of Mud in Use (psi)  
 $HP_{sw}$ : Hydrostatic Pressure exerted by sea water (psi)  
CBP: Casing Burst Pressure (psi)

**Formula(s)**

$$\begin{aligned} YP_c &= YP * SF \\ HP &= W_u * 0.052 * H \\ HP_{sw} &= H_s * 0.052 * W_s \\ CBP &= YP_c - HP + (HP_{sw}) \end{aligned}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 146.*

**2.212 Subsea considerations—Choke line pressure loss****Input(s)**

$W_m$ : Mud Weight (ppg)  
L: Choke Line Length (ft)  
 $R_c$ : Circulation Rate (gpm)  
 $D_i$ : Choke Line ID (in.)

**Output(s)**

CLPL: Choke Line Pressure Loss (psi)

**Formula(s)**

$$CLPL = \frac{0.000061 * W_m * L * (R_c^{1.86})}{D_i^{4.86}}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 148.*

**2.213 Subsea considerations—Maximum allowable mud weight—Subsea stack from leakoff test****Input(s)**

$P_{lo}$ : Leak off test pressure (psi)  
H: TVD Rotary Bushing to Casing Shoe (ft)  
 $W_u$ : Mud weight in use (ppg)

### Output(s)

$W_{max}$ : Maximum Allowable Mud Weight (ppg)

### Formula(s)

$$W_{max} = \frac{P_{lo}}{0.052 * H} + W_u$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 150.*

## 2.214 Subsea considerations—Casing pressure decrease when bringing well on choke

### Input(s)

SICP: Shut in casing Pressure (psi)

CLPL: Choke Line Pressure Loss (psi)

### Output(s)

$P_r$ : Casing Pressure Decrease when bringing well on choke (psi)

### Formula(s)

$$P_r = SICP - CLPL$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 144.*

## 2.215 Subsea considerations—Velocity through choke line

### Input(s)

Gpm: Mud Circulation rate (gpm)

ID: Choke line ID (in.)

### Output(s)

V: Velocity through Choke Line (ft/min)

### Formula(s)

$$V = 24.5 * \frac{Gpm}{ID^2}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 148.*

## 2.216 Surface test pressure required to frac the formation

### Input(s)

$FG$ : Fracture Gradient (psi/ft)

D: Depth (ft)

$\rho$ : Mud Density (ppg)

**Output(s)**

$P_{ST}$ : Surface Test Pressure (psi)

**Formula(s)**

$$P_{ST} = (FG \cdot D) - (0.052 \cdot \rho \cdot D)$$

Reference: Suman Jr, G. O., & Ellis, R. C. (1977). *Cementing Handbook*. World Oil, Page: 69.

**2.217 Total amount of solids generated during drilling****Input(s)**

$C_h$ : Capacity of Hole (bbl/ft)

$L$ : Footage Drilled (ft)

$SG$ : Specific Gravity of Cuttings (unitless)

$\emptyset$ : Porosity (fraction)

**Output(s)**

$W_s$ : Amount of Solids Generated (pounds)

**Formula(s)**

$$W_s = 350 * C_h * L * (1 - \emptyset) * SG$$

Reference: Lapeyrouse, N. J., 2002, *Formulas and Calculations for Drilling, Production and Workover*, Second Edition, Gulf Professional Publishing, Page: 19.

**2.218 Total heat energy consumed by the engine****Input(s)**

$w_f$ : Mass Rate of Fuel Consumption (lbm/min)

$H$ : Heating value of Fuel (ft lbf/Btu)

**Output(s)**

$Q_i$ : Total Heat Energy consumed by the Engine (hp)

**Formula(s)**

$$Q_i = \frac{w_f H}{33,000}$$

Reference: Bourgoyne, A. T., Millheim, K. K., Chenevert, M. E., & Young, F. S. (1986). *Applied Drilling Engineering*, Page: 6.

## 2.219 Total number of sacks of tail cement required

### Input(s)

- $N_a$ : Sacks Required by Annulus (unitless)  
 $N_c$ : Sacks Required by Casing (unitless)

### Output(s)

- $N$ : Total Number of Sacks of Tail Cement Required (unitless)

### Formula(s)

$$N = N_a + N_c$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 55.*

## 2.220 Total water requirement per sack of cement

### Input(s)

- $Q_c$ : Cement Water Requirement Per Sack of Cement (gal/stroke)  
 $Q_a$ : Additive Water Requirement Per Sack of Cement (gal/stroke)

### Output(s)

- $Q_w$ : Total Water Requirement Per Sack of Cement (gal/stroke)

### Formula(s)

$$Q_w = Q_c + Q_a$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 48.*

## 2.221 Triplex pump factor

### Input(s)

- $D_L$ : Piston diameter (in.)  
 $L_s$ : Stroke length (in.)

### Output(s)

- $PF_t$ : Triplex pump factor (bbl/stroke)

### Formula(s)

$$PF_t = \frac{3 * 3.1415 * D_L^2 * L_s}{4 * 9702}$$

Reference: *501 Solved Problems and Calculations for Drilling Operations; Page: 48.*

## 2.222 Upward force acting at the bottom of the casing shoe

### Input(s)

- A: Area Below Casing Shoe (in.<sup>2</sup>)  
 $\partial P$ : Differential Pressure B/w Cement and Mud (psi)

### Output(s)

- $F_u$ : Upward Force (lb)

### Formula(s)

$$F_u = A * \partial P$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 67.*

## 2.223 Vertical curvature for deviated wells

### Input(s)

- $I_a$ : Original Hole Inclination (degree)  
 $I_b$ : Desired Hole Inclination (degree)  
 $\partial L$ : Course Length (ft)

### Output(s)

- VC: Vertical Curvature (degree/ft)

### Formula(s)

$$VC = (I_b - I_a) * \frac{100}{\partial L}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 84.*

## 2.224 Viscous shear stress at the outer mudcake boundary

### Input(s)

- $R_c$ : Radius from the Center of Drillpipe to the beginning of Mudcake (in.)  
 $R_p$ : Radius from the Center to the Inner Boundary of the Drillpipe (in.)  
 $v_z$ : Axial Velocity Parallel to the Wellbore Axis (in/s)  
 $x_f$ : Transient Invasion Front (in.)  
 $x_{f,o}$ : Initial Displacement, i.e., Spurt (in.)  
 $\frac{dp}{dz}$ : Pressure change with Z direction that is parallel to the Wellbore Axis (psi/in.)  
 $\mu$ : Viscosity (cP)

### Output(s)

- $\tau_{(R_c)}$ : Viscous Shear Stress at the outer Mudcake boundary (lb/in.<sup>2</sup>)

### Formula(s)

$$\tau_{(Rc)} = \mu (dv_z/dr)_{(Rc)}$$

$$\tau_{(Rc)} = \frac{1}{4} \left[ 2R_c + \left\{ \left( R_c^2 - R_p^2 \right) / \left( R_c \log \left( R_p/R_c \right) \right) \right\} \right] \frac{dp}{dz}$$

Reference: *Chin, W. C. (1995). Formation Invasion, Page: 59.*

### 2.225 Volume of cuttings generated per foot of hole drilled

#### Input(s)

- D<sub>h</sub>: Hole Size (in.)  
 Ø: Porosity (fraction)

#### Output(s)

- V<sub>c</sub>: Volume of Cuttings (bbl/ft)

### Formula(s)

$$V_c = \frac{D_h^2 * (1 - \varnothing)}{1029.4}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover (2nd Edition), Page: 18.*

### 2.226 Volume of dilution water or mud required to maintain circulating volume

#### Input(s)

- V<sub>m</sub>: Volume of Mud in Circulating System (bbl)  
 F<sub>l</sub>: % Low Gravity Solids in System (%)  
 F<sub>o</sub>: % Optimum Low Gravity Solids Desired (%)  
 F<sub>a</sub>: % Low Gravity Solids Added (%)

#### Output(s)

- V<sub>wm</sub>: Volume of Dilution Water or Mud Required (bbl)

### Formula(s)

$$V_{wm} = \frac{V_m * (F_l - F_o)}{F_l - F_a}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 96.*

### 2.227 Volume of fluid displaced for duplex pumps

#### Input(s)

- D<sub>L</sub>: Piston Diameter (in.)

- D<sub>r</sub>: Rod Diameter (in.)  
 L<sub>s</sub>: Stroke Length (in.)  
 N<sub>C</sub>: Number of Cylinders (unitless)  
 n<sub>v</sub>: Volumetric Efficiency (unitless)

**Output(s)**

V<sub>t</sub>: Volume of Fluid Displaced (bbl/stroke)

**Formula(s)**

$$V_t = \frac{N_C * L_s * ((2 * D_L^2) - D_r^2) * n_v}{42 * 294}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 49.*

**2.228 Volume of fluid displaced for single-acting pump****Input(s)**

- D<sub>L</sub>: Piston Diameter (in.)  
 L<sub>s</sub>: Stroke Length (in.)  
 N<sub>C</sub>: Number of Cylinders (unitless)

**Output(s)**

V<sub>t</sub>: Volume of Fluid Displaced (bbl)

**Formula(s)**

$$V_t = \frac{\pi * D_L^2 * L_s * N_C}{4}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 47.*

**2.229 Volume of fluid displaced for triplex pump****Input(s)**

- D<sub>L</sub>: Piston Diameter (in.)  
 L<sub>s</sub>: Stroke Length (in.)  
 n<sub>v</sub>: Volumetric Efficiency (unitless)

**Output(s)**

V<sub>t</sub>: Volume of Fluid Displaced (bbl/stroke)

### Formula(s)

$$V_t = \frac{L_s * D_L^2 * n_v}{42 * 98.03}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 49.*

## 2.230 Volume of liquid (oil plus water) required to prepare a desired volume of mud

### Input(s)

- $W_1$ : Initial Density of oil water mix (ppg)
- $W_2$ : Desired Density of oil water mix (ppg)
- DV: Desired Volume (bbl)

### Output(s)

- SV: Starting Volume Of Liquid (Oil Plus Water) Required To Prepare A Desired Volume Of Mud (ppg)

### Formula(s)

$$SV = \frac{35 - W_2}{35 - W_1} * DV$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 88.*

## 2.231 Volume of slurry per sack of cement

### Input(s)

- $S_c$ : Specific Gravity of Cement (unitless)
- $W_a$ : Weight of Additive per Sack of Cement (lb/stroke)
- $S_a$ : Specific Gravity of Additive (unitless)
- $Q_w$ : Total Water Requirement per Sack of Cement (gal/stroke)

### Output(s)

- $V_s$ : Volume of Slurry (gal/stroke)

### Formula(s)

$$V_s = \left( \frac{94}{S_c * 8.33} \right) + \left( \frac{W_a}{S_a * 8.33} \right) + Q_w$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 48.*

## 2.232 Volumes and strokes—Annular volume

### Input(s)

- D<sub>h</sub>: Drill Hole Diameter (in.)  
D<sub>p</sub>: Drillpipe Outer Diameter (in.)

### Output(s)

- B: Annular Volume (bbl)

### Formula(s)

$$B = \frac{D_h^2 - D_p^2}{1029.4}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 39.*

## 2.233 Volumes and strokes—Drill string volume

### Input(s)

- ID: Internal Diameter (in.)  
PL: Pipe Length (ft)

### Output(s)

- B: Drill String Volume (bbl)

### Formula(s)

$$B = \left( \frac{(ID)^2}{1029.4} \right) * PL$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, second edition, Gulf Professional Publishing, Page: 31.*

## 2.234 Volumes and strokes—Total strokes

### Input(s)

- V<sub>DS</sub>: drill String Volume (bbl)  
V<sub>AV</sub>: Annular Volume (bbl)  
O: Pump Output (bbl/stroke)

### Output(s)

- S: Total Strokes (Surface to Bit+Bit to Surface) (No)

**Formula(s)**

$$S = \frac{V_{DS} + V_{AV}}{O}$$

Reference: *Formulas and Calculations for Drilling, Production and Workover, Second Edition, Lapeyrouse, Page: 104.*

**2.235 Weight of additive per sack of cement****Input(s)**

- P<sub>a</sub>: Percentage of Additive (fraction)  
 W<sub>c</sub>: Weight of Cement Per Sack (lb/stroke)

**Output(s)**

- W<sub>a</sub>: Weight of Additive (lb)

**Formula(s)**

$$W_a = P_a * W_c$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Workover, Second Edition, Gulf Professional Publishing, Page: 47.*

**2.236 Weighted cementing calculations****Input(s)**

- wt: Req. Slurry Density (lb/gal)  
 SG<sub>c</sub>: Specific Gravity of Cement (fraction)  
 CW: Water Req of Cement (gal/stroke)  
 AW: Water Req of Additive (gal/stroke)  
 SG<sub>a</sub>: Specific Gravity of Additive (fraction)

**Output(s)**

- x: Additive Req. Pounds Per Sack of Cement (lb/stroke)

**Formula(s)**

$$x = \frac{\left( \frac{wt * 11.207983}{SG_c} \right) + ((wt) * (CW)) - 94 - (8.33 * (CW))}{\left( 1 + \frac{AW}{100} \right) - \frac{wt}{(SG_a) * 8.33} - \left( wt + \frac{AW}{100} \right)}$$

Reference: *Lapeyrouse, N. J., 2002, Formulas and Calculations for Drilling, Production and Work-over, Second Edition, Gulf Professional Publishing, Page: 47.*

## Chapter 3

# Well test analysis formulas and calculations

### Chapter Outline

3.1 Analysis of a flow test with smoothly varying rates	174	3.27 Dimensionless rate (radial flow/constant pressure production)	187
3.2 Analysis of a post-fracture—Constant-rate flow test with boundary effects	174	3.28 Dimensionless shut-in time for MDH method	188
3.3 Analysis of a post-fracture pressure buildup test with wellbore-storage distortion	175	3.29 Dimensionless storage constant for gases	188
3.4 Analysis of a well from a PI test	176	3.30 Dimensionless storage constant for liquids	188
3.5 Analysis of DST flow data with Ramey type curves	177	3.31 Dimensionless time (linear flow/constant rate production/general case)	189
3.6 Average fracture permeability (pseudo-steady state case for pressure build-up test)	178	3.32 Dimensionless time (linear flow/constant rate production/hydraulically fractured wells)	189
3.7 Bottomhole flowing pressure during infinite-acting pseudoradial flow	178	3.33 Dimensionless time (radial flow/constant rate production)	190
3.8 Calculation of pressure beyond the wellbore (line-source solution)	179	3.34 Dimensionless time function (transient heat transfer to the formation)	190
3.9 Conventional DST design without a water cushion (collapse pressure calculation)	179	3.35 Dimensionless wellbore storage coefficient (compressible fluids for pressure build-up test)	191
3.10 Diffusion depth in a geothermal well	180	3.36 Flow period duration (hydraulically fractured wells)	191
3.11 Dimensionless buildup pressure for field calculations	180	3.37 Fracture conductivity (bilinear-flow regime in gas wells)	191
3.12 Dimensionless buildup pressure for liquid flow	180	3.38 Fracture conductivity during bilinear flow	192
3.13 Dimensionless buildup pressure for steam or gas flow	181	3.39 Inflow performance relationship (IPR) for horizontal wells in solution gas-drive reservoirs (Fetkovich)	192
3.14 Dimensionless buildup time	181	3.40 Inflow performance relationship (IPR) for horizontal wells in solution gas-drive reservoirs (Vogel)	193
3.15 Dimensionless cumulative production (radial flow constant-pressure production)	182	3.41 Interporosity flow coefficient in pressure build-up test	193
3.16 Dimensionless drawdown correlating parameter by carter	182	3.42 Minimum shut-in time to reach pseudo-steady state for tight gas reservoirs being hydraulically fractured	194
3.17 Dimensionless length (linear flow constant rate production/hydraulically fractured wells)	183	3.43 Permeability and reservoir pressure from buildup tests	194
3.18 Dimensionless length (linear flow/constant-rate production/general case)	183	3.44 Permeability and skin factor from a constant-rate flow test	195
3.19 Dimensionless pressure (linear flow/constant rate production/general case)	183	3.45 Pressure buildup equation (Horner equation)	196
3.20 Dimensionless pressure (linear flow/constant rate production/hydraulically fractured wells)	184	3.46 Radius of investigation	196
3.21 Dimensionless pressure (radial-flow/constant pressure production)	184	3.47 Radius of investigation (flow time)	196
3.22 Dimensionless pressure (radial-flow/constant rate production)	184	3.48 Radius of investigation (shut-in time)	197
3.23 Dimensionless pressure drop across a skin at the well face	184	3.49 Raymer hunt transform (porosity/transit time relationship)	197
3.24 Dimensionless pressure drop during pseudo-steady state flow for a fractured vertical well in a square drainage area	185	3.50 Reservoir permeability	198
3.25 Dimensionless pressure drop during pseudo-steady state flow for a horizontal well in a bounded reservoir	185	3.51 Shut-in time for pressure build-up test (Dietz method)	198
3.26 Dimensionless production time	186	3.52 Skin during infinite-acting pseudoradial flow for vertical wells	199
	187	3.53 Skin estimation type-1 (pressure buildup test)	199
		3.54 Slope of Horner plot in pressure buildup test	200
		3.55 Slope of pseudo-steady state flow in pressure buildup test	200

<b>3.56 Time to pseudo-steady state (single well-circular reservoir)</b>	<b>3.58 True wellbore storage coefficient (pressure build-up test)</b>	<b>201</b>
<b>3.57 Time to reach the semi-steady state for a gas well in a circular or square drainage area</b>	<b>3.59 Well flow efficiency (geothermal well)</b>	<b>202</b>
	<b>3.60 Well shut-in pressure during buildup (Horner plot)</b>	<b>202</b>

### 3.1 Analysis of a flow test with smoothly varying rates

#### Input(s)

- $t_1$ : Time at  $P_{wf1}$  from Given Values or Trendline (h)  
 $t_2$ : Time at  $P_{wf2}$  from Given Values or Trendline (h)  
 $q_1$ : Flow Rate at  $P_{wf1}$  (STB/day)  
 $q_2$ : Flow Rate at  $P_{wf2}$  (STB/day)  
 $p_i$ : Initial Pressure (psi)  
 $p_{wf2}$ : Wellflow Pressure at Point 2 from Given Values or Trendline (psi)  
 $p_{wf1}$ : Wellflow Pressure at Point 1 from Given Values or Trendline (psi)  
 $p_{wf}$ : Pressure Value at  $t = 1$  h (psi)  
 $q$ : Flow Rate (STB/day)  
 $B$ : Volume Factor (RB/STB)  
 $h$ : Thickness of Reservoir (ft)  
 $\mu$ : Viscosity of Oil (cP)  
 $\emptyset$ : Porosity (fraction)  
 $c_t$ : Compressibility (1/psi)  
 $r_w$ : Wellbore Radius (ft)

#### Output(s)

- $m$ : Slope of Line (dimensionless)  
 $k$ : Permeability (mD)  
 $s$ : Skin Factor (dimensionless)

#### Formula(s)

$$m = \frac{\left( \frac{p_i - p_{wf2}}{q_2} \right) - \left( \frac{p_i - p_{wf1}}{q_1} \right)}{\log(t_2) - \log(t_1)}$$

$$k = 162.6 * B * \frac{\mu}{m * h}$$

$$s = 1.151 * \left( \frac{1}{m \left( \frac{p_i - p_{wf}}{q} \right)} - \log \left( \frac{k}{\emptyset * \mu * c_t * r_w^2} \right) + 3.23 \right)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 31.

### 3.2 Analysis of a post-fracture—Constant-rate flow test with boundary effects

#### Input(s)

- $q_g$ : Gas Flow Rate (MSCF/day)  
 $B_g$ : Gas formation Volume Factor (RB/MSCF)

m: Slope from Curve (psi/cycle)  
 $m_L$ : Slope from linear Region of Curve (psi/cycle)  
 $p_{ai}$ : Initial Adjusted Well Pressure (psi)  
 $p_{ahr}$ : Adjusted Well pressure at  $t = 1$  h (psi)  
 $p_D$ : Dimensionless Pressure (dimensionless)  
 $\phi$ : Porosity (dimensionless)  
 $c_t$ : Compressibility (1/psi)  
 $r_w$ : Radius of Wellbore (ft)  
 $\mu$ : Viscosity (cP)  
 $h$ : Formation Thickness (ft)  
 $t_{LFD}$ : Time of end of Linear or Pseudo Radial flow from Plot (dimensionless)  
 $\Delta t_a$ : Adjusted Delta Time from Derivative Curve (h)

## Output(s)

k: Permeability (mD)  
 $L_{fPR}$ : Length of Fracture for Pseudo Radial Flow (ft)  
 $L_{fL}$ : Length of Fracture for Linear Flow (ft)  
 $L_{fMP}$ : Length of Fracture from Match Point Analysis (ft)  
 s: Skin Factor (dimensionless)  
 $\Delta P_{a_{MP}}$ : Adjusted Pressure Difference at Match Point from Plot (psi)

## Formula(s)

$$\begin{aligned}
 k &= 162.6 * q_g * B_g * \frac{\mu}{m * h} \\
 L_{fPR} &= 1.151 * \left( \left( \frac{p_{ai} - p_{ahr}}{m} \right) - \log \left( \frac{k}{\phi * \mu * c_t * r_w^2} \right) + 3.23 \right) \\
 L_{fL} &= 2 * r_w * 2.71^{-s} \\
 L_{fMP} &= 4.064 * q_g * \frac{B_g}{m_L * h * k^{0.5}} * \left( \left( \frac{\mu * c_t}{\phi} \right)^{\frac{1}{2}} \right) \\
 s &= \left( 141.2 * q_g * B_g * \frac{\mu}{k} * h \right) * (p_D) \\
 \Delta P_{a_{MP}} &= \left( \left( \frac{0.0002637 * k}{\phi * \mu * c_t} \right) * \left( \frac{\Delta t_a}{t_{LFD}} \right) \right)^{\frac{1}{2}}
 \end{aligned}$$

Reference: Lee, J., Rollins J.B., and Spivey J.P. 2003, *Pressure Transient Testing*, Vol. 9, SPE Textbook Series, Vol. 9, Henry L. Doherty Memorial Fund of AIME, Richardson, Texas, SPE, Chapter: 6, Page: 121.

## 3.3 Analysis of a post-fracture pressure buildup test with wellbore-storage distortion

### Input(s)

$q_g$ : Gas flow rate (MSCF/day)  
 $B_g$ : Gas formation Volume Factor (RB/MSCF)  
 $p_D$ : Dimensionless Pressure (dimensionless)  
 $\phi$ : Porosity (dimensionless)  
 $c_t$ : Compressibility (1/psi)  
 $\mu$ : Viscosity (cP)  
 $h$ : Formation Thickness (ft)

$t_{(L_f)_D}$ : Time of end of linear or pseudo radial flow from plot (dimensionless)  
 $C_{rD}$ : Dimensionless fracture conductivity (dimensionless)  
 $C_{fD}$ : Dimensionless Wellbore storage coefficient (dimensionless)  
 $L_f$ : Length of fracture (ft)  
 $k$ : Permeability (mD)  
 $\Delta t_{AE}$ : Equivalent Adjusted delta time from derivative curve (h)

## Output(s)

$C$ : Well bore storage coefficient (bbl/psi)  
 $w_{fk_f}$ : Min Fracture conductivity for infinite conductive fracture (mD ft)  
 $L_{fMP}$ : Length of fracture from match point analysis (ft)  
 $(\Delta P_a)_{MP}$ : Adjusted Pressure difference at match point from plot (psi)

## Formula(s)

$$\begin{aligned}
 C &= \left( 141.2 * q_g * B_g * \frac{\mu}{k * h} \right) * (p_D)_{MP} \\
 w_{fk_f} &= \left( \left( \frac{0.0002637 * k}{\emptyset * \mu * c_t} \right) * \left( \frac{\Delta t_{AE}}{t_{(L_f)_D}} \right)_{MP} \right)^{\frac{1}{2}} \\
 L_{fMP} &= \left( \emptyset * h * c_t * \frac{L_f^2}{0.8936} \right) * C_{fD} \\
 (\Delta P_a)_{MP} &= 3.14 * k * C_{rD} * L_f
 \end{aligned}$$

Reference: Lee, J., Rollins J.B., and Spivey J.P. 2003, *Pressure Transient Testing*, Vol. 9, SPE Textbook Series, Vol. 9, Henry L. Doherty Memorial Fund of AIME, Richardson, Texas, SPE, Chapter: 6, Page: 127.

## 3.4 Analysis of a well from a PI test

### Input(s)

$P$ : Average Reservoir Pressure (psi)  
 $q$ : Flow Rate (STB/day)  
 $P_{wf}$ : Well Flowing Pressure (psi)  
 $k$ : Permeability (mD)  
 $B$ : Volume Factor (RB/STB)  
 $h$ : Thickness of Reservoir (ft)  
 $\mu$ : Oil Viscosity (cP)  
 $r_e$ : Drainage Radius (ft)  
 $r_w$ : Wellbore Radius (ft)

### Output(s)

$J$ : Productivity index (STB/day/psi)  
 $k_j$ : Average Permeability (mD)  
 $s$ : Skin Factor (dimensionless)

**Formula(s)**

$$J = \frac{q}{P - P_{wf}}$$

$$k_j = 141.2 * J * B * \mu * \frac{\ln\left(\frac{r_e}{r_w}\right) - 0.75}{h}$$

$$s = \left( \frac{k}{k_j} - 1 \right) * \left( \ln\left(\frac{r_e}{r_w}\right) - 0.75 \right)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing (Vol. 9)*. Richardson, Texas: Society of Petroleum Engineers, Page: 16.

### 3.5 Analysis of DST flow data with Ramey type curves

**Input(s)**

- $p_i$ : Initial Pressure (psi)
- $P_{wf}$ : Well Flowing Pressure (psi)
- $p_o$ : Pressure at Time  $T = 0$  (psi)
- $\phi$ : Porosity (dimensionless)
- $c_t$ : Total Compressibility (1/psi)
- $h$ : Formation Thickness (ft)
- $C$ : Wellbore Storage Coefficient (bbl/psi)
- $r_w$ : Radius of Wellbore (ft)
- $\mu$ : Viscosity (cP)
- $t_c$ : Dimensionless Parameter from Curve Fitting as  $\frac{T_d}{C_d}$  (dimensionless)
- $t$ : Time (h)
- CES: Match Point for Dimensionless Well Bore Coefficient from Curve as  $(C_D E \gamma(2 s))_{(mp)}$  (dimensionless)

**Output(s)**

- $p_{DR}$ : Dimensionless Pressure (dimensionless)
- $q_{DR}$ : Dimensionless Flow Rate (dimensionless)
- $C_D$ : Dimensionless Well Bore Storage Coefficient (dimensionless)
- $k$ : Permeability (mD)
- $s$ : Skin Factor (dimensionless)

**Formula(s)**

$$p_{DR} = \frac{p_i - (p_{wf})(t)}{p_i - p_o}$$

$$q_{DR} = 1 - p_{DR}$$

$$C_D = 0.8936 * \frac{C}{\phi * h * c_t * (r_w^2)}$$

$$k = \left( 3390 * \mu * \frac{C}{h} \right) * \left( \frac{t_c}{t} \right)$$

$$s = 0.5 * \ln\left(\frac{CES}{C_D}\right)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing (Vol. 9)*. Richardson, Texas: Society of Petroleum Engineers, Page: 155.

### 3.6 Average fracture permeability (pseudo-steady state case for pressure build-up test)

#### Input(s)

- q: Flow Rate (bbl/d)
- $\mu$ : Viscosity (cP)
- B: Formation Volume Factor (BBL/STB)
- m: Slope of Horner Plot (psi/cycles)
- h: Formation Thickness (ft)

#### Output(s)

- $k_f$ : Average Fracture Permeability (mD)

#### Formula(s)

$$k_f = 162.6 * q * B * \frac{\mu}{m * h}$$

Notes: Horner Plot Analysis (Semi-Log Plot).

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 107.

### 3.7 Bottomhole flowing pressure during infinite-acting pseudoradial flow

#### Input(s)

- $p_i$ : Initial Reservoir Pressure (psi)
- $Q_o$ : Well Flow Rate (STB/day)
- $B_o$ : Oil Formation Volume Factor (bbl/STB)
- $\mu_o$ : Oil Viscosity (cP)
- k: Permeability (mD)
- h: Thickness (ft)
- t: Flowing Time (h)
- $\phi$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- $r_w$ : Wellbore Radius (ft)
- S: Skin (dimensionless)

#### Output(s)

- $P_{wf}$ : Wellbore Flowing Pressure (psi)

#### Formula(s)

$$P_{wf} = p_i - \left( \frac{162.6 * Q_o * \mu_o * B_o}{k * h} \right) * \left( \log(t) + \log\left(\frac{k}{\phi * \mu_o * c_t * r_w^2}\right) - 3.23 + 0.87 * S \right)$$

$$p_i - p_{wf} = \Delta p = a + m \log(t)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 98.

### 3.8 Calculation of pressure beyond the wellbore (line-source solution)

#### Input(s)

$P_i$ : Initial Pressure (psi)  
 $t$ : Time of Production (h)  
 $k$ : Permeability (mD)  
 $B$ : Volume Factor (RB/STB)  
 $\phi$ : Porosity (fraction)  
 $c_t$ : Compressibility (1/psi)  
 $h$ : Thickness of Reservoir (ft)  
 $\mu$ : Viscosity of Oil (cP)  
 $r$ : Radius of Wellbore (ft)  
 $q$ : Flow Rate (STB/day)

#### Output(s)

P: Pressure (psi)

#### Formula(s)

$$P = P_i + 70.6 * q * B * \mu * \frac{\ln\left(1.688 * \phi * \mu * c_t * \frac{r^2}{k * t}\right)}{k * h}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: SPE, Page: 11.

### 3.9 Conventional DST design without a water cushion (collapse pressure calculation)

#### Input(s)

$g_w$ : Water Gradient (psi/ft)  
 $W_m$ : Mud Weight (lbm/gal)  
 $D_c$ : Depth (ft)  
 $F_d$ : Design Factor (dimensionless)  
 $W_w$ : Water Weight (lbm/gal)

#### Output(s)

$p_{collapse}$ : Collapse Pressure (psi)

#### Formula(s)

$$p_{collapse} = \frac{g_w * W_m * D_c * F_d}{W_w}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 153.

### 3.10 Diffusion depth in a geothermal well

#### Input(s)

- $w$ : Mass Flow Rate (lb/h)
- $c$ : Thermal Heat Capacity of the Fluid (BTU/lbm °F)
- $k$ : Thermal Conductivity of Earth (= 33.6 BTU/(ft·day °F))
- $f(t)$ : Dimensionless Time Function that Represents the Transient Heat Transfer to the formation (dimensionless)

#### Output(s)

- $A(t)$ : Diffusion Depth as a Function of Time (ft)

#### Formula(s)

$$A(t) = \frac{wcf(t)}{2\pi k}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Equation (6-2).

### 3.11 Dimensionless buildup pressure for field calculations

#### Input(s)

- $P_i$ : Initial Reservoir Pressure (psi)
- $P_{ws}$ : Shut in Pressure (psi)
- $m$ : Slope of Semi-Log Graph ( $\text{kg}/\text{cm}^2$ )/ Log cycle for Liquid

#### Output(s)

- $P_{Ds}$ : Dimensionless Pressure (dimensionless)

#### Formula(s)

$$P_{Ds} = \frac{(P_i - P_{ws})}{0.87m}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.8).

### 3.12 Dimensionless buildup pressure for liquid flow

#### Input(s)

- $k$ : Effective Permeability of Flowing Phase (D)
- $h$ : Net Formation Thickness (m)
- $P_i$ : Initial Reservoir Pressure (psi)
- $P_{ws}$ : Shut in Pressure (psi)
- $v_{sc}$ : Specific Volume at Standard Conditions (cc/g)
- $q$ : Production Rate (tons/h (1000 kg/h))
- $B$ : Formation Volume Factor (Reservoir Volume/Standard Volume)
- $\mu$ : Viscosity of Flowing Fluid (cP)

**Output(s)**

$P_{Ds}$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$P_{Ds} = \frac{kh(P_i - P_{ws})}{0.4568v_{sc}qB\mu}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.7).

**3.13 Dimensionless buildup pressure for steam or gas flow****Input(s)**

$k$ : Effective Permeability of Flowing Phase (D)

$h$ : Net Formation Thickness (m)

$P_i$ : Initial Reservoir Pressure (psi)

$P_{ws}$ : Shut in Pressure (psi)

$M$ : Molecular Weight (g/g mol)

$q$ : Production Rate (tons/h (1000 kg/h))

$Z$ : Real Gas Law Deviation Factor (Ratio)

$\mu$ : Viscosity of Flowing Fluid (cP)

$T$ : Absolute Formation Temperature (°K)

**Output(s)**

$P_{Ds}$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$P_{Ds} = \frac{Mkh(P_i^2 - P_{ws}^2)}{0.01291q\mu ZT}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.7).

**3.14 Dimensionless buildup time****Input(s)**

$k$ : Effective Permeability of Flowing Phase (D)

$t$ : Time (h)

$\emptyset$ : Porosity (Per cent)

$\mu$ : Viscosity of Flowing Fluid (cP)

$c_t$ : Total System Effective Isothermal Compressibility ( $\text{kg}/\text{cm}^2$ ) $^{-1}$

$r_w$ : Well Radius (m)

**Output(s)**

$t_D$ : Dimensionless time (dimensionless)

**Formula(s)**

$$t_D = \frac{0.3604 k t}{\phi \mu c_i r_w^2}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.7).

**3.15 Dimensionless cumulative production (radial flow constant-pressure production)****Input(s)**

- $p_i$ : Initial Reservoir Pressure (psi)
- $Q_p$ : Flow Rate of Production (STB/day)
- B: Volume Factor (RB/STB)
- $\mu$ : Oil Viscosity (cP)
- k: Permeability (mD)
- $r_w$ : Radius of Wellbore (ft)
- $\phi$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- h: Thickness of Reservoir (ft)
- $p_{wf}$ : Well Flowing Pressure (psi)

**Output(s)**

- $Q_{pd}$ : Dimensionless Cumulative Production (dimensionless)

**Formula(s)**

$$Q_{pd} = \frac{B}{1.119 * \phi * c_t * h * r_w^2 * (p_i - p_{wf})} * Q_p$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

**3.16 Dimensionless drawdown correlating parameter by Carter****Input(s)**

- $\mu_{gi}$ : Initial Gas Viscosity (cP)
- $\mu_{gavg}$ : Average Gas of Viscosity (cP)
- $c_{gi}$ : Initial Gas Compressibility (1/psi)
- $c_{gavg}$ : Average Gas Compressibility (1/psi)

**Output(s)**

- $\lambda$ : Dimensionless Drawdown Correlating Parameter (dimensionless).

**Formula(s)**

$$\lambda = \frac{\mu_{gi} * c_{gi}}{\mu_{gavg} * c_{gavg}}$$

Reference: Ahmed, T., *McKinney*, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 256.

### **3.17 Dimensionless length (linear flow constant rate production/hydraulically fractured wells)**

**Input(s)**

- x: Length of Flow (ft)  
 $L_f$ : Half Length of Fracture (ft)

**Output(s)**

- $L_D$ : Dimensionless Length (dimensionless)

**Formula(s)**

$$L_D = \frac{x}{L_f}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 10.

### **3.18 Dimensionless length (linear flow/constant-rate production/general case)**

**Input(s)**

- x: Fracture Length (ft)  
A: Cross Sectional Area (ft)

**Output(s)**

- $x_D$ : Dimensionless Length (dimensionless)

**Formula(s)**

$$x_D = \frac{x}{A^{0.5}}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

### **3.19 Dimensionless pressure (linear flow/constant rate production/general case)**

**Input(s)**

- $p_i$ : Initial Reservoir Pressure (psi)

- B: Volume Factor (RB/STB)
- $\mu$ : Oil Viscosity (cP)
- k: Permeability (mD)
- A: Cross Section Area (ft)
- q: Flow Rate (STB/day)
- p: Pressure (psi)

**Output(s)**

$p_D$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$p_D = \frac{k * A^{0.5}}{141.2 * q * B * \mu} * (p_i - p)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

**3.20 Dimensionless pressure (linear flow/constant rate production/hydraulically-fractured wells)****Input(s)**

- $p_i$ : Initial Reservoir Pressure (psi)
- B: Formation Volume Factor (RB/STB)
- $\mu$ : Oil Viscosity (cP)
- k: Permeability (mD)
- h: Reservoir Thickness (ft)
- q: Flow Rate (STB/day)
- p: Pressure (psi)

**Output(s)**

$p_d$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$p_d = \frac{k * h}{141.2 * q * B * \mu} * (p_i - p)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 10.

**3.21 Dimensionless pressure (radial-flow/constant pressure production)****Input(s)**

- $P_i$ : Initial Reservoir Pressure (psi)
- P: Final Reservoir Pressure (psi)
- $P_{wf}$ : Well Flowing Pressure (psi)

**Output(s)**

$P_d$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$P_d = \frac{P_i - P}{P_i - P_{wf}}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

**3.22 Dimensionless pressure (radial-flow/constant rate production)****Input(s)**

- $p_i$ : Initial Reservoir Pressure (psi)
- $p$ : Reservoir Thickness (psi)
- $k$ : Permeability (mD)
- $h$ : Reservoir Thickness (ft)
- $q$ : Formation Volume Factor (STB/day)
- $B$ : Volume Factor (RB/STB)
- $\mu$ : Oil Viscosity (cP)

**Output(s)**

$P_d$ : Dimensionless Pressure (dimensionless)

**Formula(s)**

$$P_d = \frac{k * h}{141.2 * q * B * \mu} * (p_i - p)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 8.

**3.23 Dimensionless pressure drop across a skin at the well face****Input(s)**

- $k$ : Effective Permeability of Flowing Phase (D)
- $h$ : Net Formation Thickness (m)
- $P_i$ : Initial Reservoir Pressure (psi)
- $P_{1hr}$ : Pressure at 1 hour on semi-log straight line or its extension (psi)
- $P_{ws}$ : Shut in Pressure (psi)
- $P_{wf}$ : Bottomhole Pressure at a flowing well (psi)
- $v_{sc}$ : Specific Volume at Standard Conditions (cc/g)
- $q$ : Production Rate ( $m^3/h$ )
- $B$ : Formation Volume Factor (Reservoir Volume/Standard Volume)
- $\mu$ : Viscosity of Flowing Fluid (cP)
- $c_t$ : Total System Effective Isothermal Compressibility ( $kg/cm^2$ ) $^{-1}$
- $r_w$ : Well Radius (m)
- $m$ : Slope of Semi-Log Graph ( $kg/cm^2$ )/ Log cycle for Liquid
- $\emptyset$ : Porosity (Per cent)

**Output(s)**

$P_{Dw}$ : Dimensionless Pressure at the well face (dimensionless)  
 $s$ : Skin Effect (dimensionless)

**Formula(s)**

$$P_{Dw} + s = \frac{kh(P_i - P_{ws})}{0.4568v_{sc}qB\mu}$$

$$s = 1.151 \left[ \frac{\left( P_{1hr}^2 - P_{wf}^2 \right)}{m} - \log_{10} \frac{k}{\emptyset \mu c_t r_w^2} + 0.0919 \right]$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.13).

**3.24 Dimensionless pressure drop during pseudo-steady state flow for a fractured vertical well in a square drainage area****Input(s)**

$t_{DA}$ : Dimensionless Time (dimensionless)  
 $C_f$ : Shape Factor for a Fractured Vertical Well (dimensionless)  
 $x_e$ : Half the Side of Square Drainage Area (ft)  
 $x_f$ : Fracture Half Length (ft)

**Output(s)**

$P_D$ : Wellbore Dimensionless Pressure Drop (dimensionless)

**Formula(s)**

$$P_D = 2 * \pi * t_{DA} + 0.5 * \ln \left( \frac{x_e}{x_f} \right)^2 + 0.5 * \ln \left( \frac{2.2458}{C_f} \right) + 0.5 * 2.77$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 211.

**3.25 Dimensionless pressure drop during pseudo-steady state flow for a horizontal well in a bounded reservoir****Input(s)**

$t_{DA}$ : Dimensionless Time (dimensionless)  
 $C_{Ah}$ : Shape Factors for Horizontal Wells (dimensionless)  
 $L$ : Length of Horizontal Well (ft)  
 $A$ : Drainage Area (ft)

**Output(s)**

$P_D$ : Wellbore Dimensionless Pressure Drop (dimensionless)

**Formula(s)**

$$P_D = 2 * \pi * t_{DA} + 0.5 * \ln \left( \frac{A}{4 * \left( \left( \frac{L}{2} \right)^2 \right)} \right) + 0.5 * \ln \left( \frac{2.2458}{C_{Ah}} \right) + 0.5 * 2.77$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 216.

**3.26 Dimensionless production time****Input(s)**

- $t_D$ : Dimensionless time (dimensionless)
- $A$ : Drainage Area ( $\text{m}^2$ )
- $r_w$ : Well Radius (m)

**Output(s)**

- $t_{DA}$ : Dimensionless Production time (dimensionless)

**Formula(s)**

$$t_{DA} = t_D \frac{r_w^2}{A}$$

Reference: Ramey Jr., H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.7).

**3.27 Dimensionless rate (radial flow/constant pressure production)****Input(s)**

- $p_i$ : Initial Reservoir Pressure (psi)
- $q$ : Flow Rate (STB/day)
- $B$ : Formation Volume Factor (RB/STB)
- $\mu$ : Oil Viscosity (cP)
- $k$ : Permeability (mD)
- $h$ : Reservoir Thickness (ft)
- $p_{wf}$ : Well Flowing Pressure (psi)

**Output(s)**

- $q_d$ : Dimensionless Rate (dimensionless)

**Formula(s)**

$$q_d = \frac{q * B * \mu}{0.00708 * k * h * (p_i - p_{wf})}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

### 3.28 Dimensionless shut-in time for MDH method

#### Input(s)

- $k$ : Permeability (mD)
- $\Delta t$ : Total Shut-in Time (day)
- $\phi$ : Porosity (fraction)
- $\mu$ : Viscosity (cP)
- $c_t$ : Total Compressibility (1/psi)
- $A$ : Drainage Area ( $\text{ft}^2$ )

#### Output(s)

- $\Delta t_{DA}$ : Dimensionless Shut-in Time (dimensionless)

#### Formula(s)

$$\Delta t_{DA} = \frac{0.0002637 * k * \Delta t}{\phi * \mu * c_t * A}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 59.

### 3.29 Dimensionless storage constant for gases

#### Input(s)

- $C'$ : Wellbore Storage ( $\text{tons}/(\text{kg/cm}^2)^2$ )
- $Z$ : Real Gas Law Deviation Factor (ratio)
- $T$ : Absolute Formation Temperature ( $^\circ\text{K}$ )
- $M$ : Molecular Weight (g/g mol)
- $\phi$ : Porosity (Per cent)
- $h$ : Net Formation Thickness (m)
- $c_t$ : Total System Effective Isothermal Compressibility ( $\text{kg/cm}^2$ ) $^{-1}$
- $r_w$ : Well Radius (m)

#### Output(s)

- $C_D$ : Dimensionless Storage Constant for Gases (dimensionless)

#### Formula(s)

$$C_D = \frac{27 C' Z T}{M \phi h c_t r_w^2}$$

Reference: Ramey Jr., H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.18).

### 3.30 Dimensionless storage constant for liquids

#### Input(s)

- $C$ : Storage
- $B$ : Formation Volume Factor (Reservoir Volume/Standard Volume)

- $V_{sc}$ : Specific Volume at Standard Conditions (cc/g)  
 $\emptyset$ : Porosity (fraction)  
 $h$ : Net Formation Thickness (m)  
 $c_t$ : Total System Effective Isothermal Compressibility ( $\text{kg}/\text{cm}^2$ )<sup>-1</sup>  
 $r_w$ : Well Radius (m)

**Output(s)**

- $C_D$ : Dimensionless Storage Constant for Liquids (dimensionless)

**Formula(s)**

$$C_D = \frac{CBV_{sc}}{2\pi\emptyset h c_t r_w^2}$$

Reference: Ramey Jr., H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.18).

**3.31 Dimensionless time (linear flow/constant rate production/general case)****Input(s)**

- $t$ : Production Time (h)  
 $k$ : Permeability (mD)  
 $\emptyset$ : Porosity (fraction)  
 $c_t$ : Total Compressibility (1/psi)  
 $\mu$ : Oil Viscosity (cP)  
 $A$ : Cross Sectional Area (ft)

**Output(s)**

- $t_{AD}$ : Dimensionless Time (dimensionless)

**Formula(s)**

$$t_{AD} = \frac{0.0002637 * k * t}{\emptyset * \mu * c_t * A}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 9.

**3.32 Dimensionless time (linear flow/constant rate production/hydraulically fractured wells)****Input(s)**

- $t$ : Production Time (h)  
 $k$ : Permeability (mD)  
 $\emptyset$ : Porosity (fraction)  
 $c_t$ : Total Compressibility (1/psi)  
 $\mu$ : Oil Viscosity (cP)  
 $L_f$ : Half Length of Fracture (ft)

### Output(s)

$t_{L_{jp}}$ : Dimensionless Time (dimensionless)

### Formula(s)

$$t_{L_{jp}} = \frac{0.0002637 * k * t}{\phi * \mu * c_t * L_f^2}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 10./

## 3.33 Dimensionless time (radial flow/constant rate production)

### Input(s)

t: Production Time (h)  
 k: Permeability (mD)  
 φ: Porosity (fraction)  
 $c_t$ : Total Compressibility (1/psi)  
 μ: Oil Viscosity (cP)  
 $r_w$ : Wellbore Radius (ft)

### Output(s)

$t_d$ : Dimensionless Time (dimensionless)

### Formula(s)

$$t_d = \frac{0.0002637 * k * t}{\phi * \mu * c_t * r_w^2}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 8.

## 3.34 Dimensionless time function (transient heat transfer to the formation)

### Input(s)

$\alpha$ : Thermal Diffusivity of the Formation (ft<sup>2</sup>/day)  
 r: Diffusion Depth (ft)  
 t: Production Time (day)

### Output(s)

$f(t)$ : Dimensionless Time Function that Represents the Transient Heat Transfer to the formation (dimensionless)

### Formula(s)

$$f(t) = -\ln \frac{r}{2\sqrt{\alpha t}} - 0.290$$

Reference: Ramey Jr, H. J. (1981). Reservoir Engineering Assessment of Geothermal Systems. *Department of Petroleum Engineering, Stanford University*, Equation no (6.3).

### 3.35 Dimensionless wellbore storage coefficient (compressible fluids for pressure build-up test)

#### Input(s)

- C: Well Bore Storage Coefficient (bbl/psi)
- $\phi$ : Porosity (fraction)
- $c_f$ : Formation Compressibility (1/psi)
- h: Pay zone Thickness (ft)
- $r_w$ : Well Bore Radius (ft)

#### Output(s)

- $C_d$ : Dimensionless Wellbore Storage Coefficient (dimensionless)

#### Formula(s)

$$C_d = \frac{0.8936 * C}{\phi * c_f * h * r_w^2}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 105.

### 3.36 Flow period duration (hydraulically fractured wells)

#### Input(s)

- $\phi$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- $\mu$ : Viscosity (cP)
- $L_f$ : Fracture Length (ft)
- $t_{LJD}$ : Time of End of Linear or Start Pseudo Radial Flow from Plot (dimensionless)
- k: Permeability (mD)

#### Output(s)

- t: Duration of Flow Periods (h)

#### Formula(s)

$$t = \frac{\phi * \mu * c_t * (L_f^2) * t_{LJD}}{0.0002637 * k}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 115.

### 3.37 Fracture conductivity (bilinear-flow regime in gas wells)

#### Input(s)

- q: Flow Rate (bbl/d)
- $\phi$ : Porosity (fraction)
- $\mu$ : Viscosity (cP)
- B: Formation Volume Factor (BBL/STB)

$m_b$ : Slope of Pressure vs Fourth Root of Time (Bi-linear Flow Analysis) (psi/h<sup>0.25</sup>)  
 $c_t$ : Total Compressibility (1/psi)  
 $k$ : Permeability (mD)  
 $h$ : Reservoir Thickness (ft)

### Output(s)

$w_f \cdot k_f$ : Fracture Conductivity (ft mD)

### Formula(s)

$$w_f \cdot k_f = \left( \frac{44.1 * q * \mu * B}{h * m_b} \right)^2 * \left( \frac{1}{\phi * \mu * c_t * k} \right)^{0.5}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). Pressure Transient Testing (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 117.

## 3.38 Fracture conductivity during bilinear flow

### Input(s)

$Q$ : Flow Rate (STB/day)  
 $B$ : Formation Volume Factor (bbl/STB)  
 $\mu$ : Viscosity (cP)  
 $m_{bf}$ : Slope of Bilinear Flow during Build-up Test (psi/cycle)  
 $h$ : Thickness (ft)  
 $\phi$ : Porosity (fraction)  
 $c_t$ : Total Compressibility (1/psi)  
 $k$ : Permeability (mD)

### Output(s)

$F_C$ : Fracture Conductivity (mD ft)

### Formula(s)

$$F_C = \left( \frac{44.1 * Q * \mu * B}{m_{bf} * h * (\phi * \mu * c_t * k)^{0.25}} \right)^2$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 95.

## 3.39 Inflow performance relationship (IPR) for horizontal wells in solution gas-drive reservoirs (Fetkovich)

### Input(s)

$q_{omax}$ : Maximum Flow Rate for Maximum Drawdown (STB/day)  
 $P_{wf}$ : Flowing Bottomhole Pressure (psi)  
 $P$ : Shut in Pressure or Average Reservoir Pressure (psi)  
 $n$ : Exponent (dimensionless)

**Output(s)**

$q_o$ : Oil Flow Rate (STB/day)

**Formula(s)**

$$q_o = q_{omax} * \left( 1 - \left( \frac{P_{wf}}{P} \right)^2 \right)^n$$

Reference: Joshi, S. D. 1991, Horizontal Well Technology. Tulsa, Oklahoma: *PennWell Publishing Company*. Chapter: 7, Page: 240.

### 3.40 Inflow performance relationship (IPR) for horizontal wells in solution gas-drive reservoirs (Vogel)

**Input(s)**

$q_{omax}$ : Maximum Flow Rate for 100% Drawdown (STB/day)

$P_{wf}$ : Flowing Bottomhole Pressure (psi)

$P$ : Shut in Pressure or Average Reservoir Pressure (psi)

**Output(s)**

$q_o$ : Oil Flow Rate (STB/day)

**Formula(s)**

$$q_o = q_{omax} * \left( 1 - 0.2 * \left( \frac{P_{wf}}{P} \right) - 0.8 * \left( \frac{P_{wf}}{P} \right)^2 \right)$$

Reference: Joshi, S. D. 1991, Horizontal Well Technology. Tulsa, Oklahoma: *PennWell Publishing Company*. Chapter: 7, Page: 240.

### 3.41 Interporosity flow coefficient in pressure build-up test

**Input(s)**

$\omega$ : Storativity of the Fractures (dimensionless)

$\phi_m$ : Matrix Porosity (fraction)

$h_m$ : Matrix Thickness (ft)

$c_m$ : Matrix Compressibility (1/psi)

$\mu$ : Viscosity (cP)

$r_w$ : Wellbore Radius (ft)

$k_f$ : Fracture Permeability (mD)

$t_p$ : Production Time (day)

$\Delta t$ : Shut-in Time (h)

**Output(s)**

$\lambda$ : Interporosity Flow Coefficient (dimensionless)

**Formula(s)**

$$\lambda = \left( \frac{w}{1-\omega} \right) * \left( \frac{\phi_m * h_m * c_m * \mu * r_w^2}{1.781 * k_f * t_p} \right) * \left( \frac{tp + \Delta t}{\Delta t} \right)_1$$

or

$$\lambda = \left( \frac{1}{1-\omega} \right) * \left( \frac{\phi_m * h_m * c_m * \mu * r_w^2}{1.781 * k_f * t_p} \right) * \left( \frac{tp + \Delta t}{\Delta t} \right)_2$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 84.

### 3.42 Minimum shut-in time to reach pseudo-steady state for tight gas reservoirs being hydraulically fractured

**Input(s)**

- $\mu_g$ : Gas Viscosity (cP)
- $\phi$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- $x_f$ : Fracture Half Length (ft)
- $k$ : Permeability (mD)

**Output(s)**

- $t_{pss}$ : Time (h)

**Formula(s)**

$$t_{pss} = \frac{474 * \phi * \mu_g * c_t * (x_f^2)}{k}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 23.

### 3.43 Permeability and reservoir pressure from buildup tests

**Input(s)**

- $t_p$ : Time till which drawdown was done (h)
- $dt_a$ : Time of shut in 1 from given values or trendline (h)
- $dt_b$ : Time of shut in 2 from given values or trendline (h)
- $pws_a$ : Well flowing Pressure at shut in point 2 from given values or trendline (psi)
- $pws_b$ : Well flowing Pressure at shut in point 1 from given values or trendline (psi)
- $h$ : Thickness of Reservoir (ft)
- $\mu$ : Viscosity of Oil (cP)
- $p_{hr}$ : Pressure at  $t = 1$  h (psi)
- $p_{wf}$ : Pressure flowing well (psi)
- $\phi$ : Porosity (fraction)

$c_t$ : Compressibility (1/psi)  
 $r_w$ : Wellbore Radius (ft)

### Output(s)

s: Skin Factor (dimensionless)

### Formula(s)

$$s = 1.151 * \left( (phr - pwf) - \log \left( \frac{\frac{pws_a - pws_b}{\log \left( \frac{tp + dt_b}{dt_b} \right) - \log \left( \frac{tp + dt_a}{dt_a} \right)}}{\varnothing * \mu * c_t * r_w^2} \right) + 3.23 \right)$$

Reference: *Pressure Buildup and Flow Tests in Wells*, Matthews & Russell, Page: 21.

## 3.44 Permeability and skin factor from a constant-rate flow test

### Input(s)

$t_a$ : Time at Pwf1 from given values or trendline (h)  
 $t_b$ : Time at Pwf2 from given values or trendline (h)  
 $p_w$ : Well flowing Pressure at point 2 from given values or trend line (psi)  
 $p_{wf}$ : Well flowing Pressure at point 1 from given values or trendline (psi)  
 $p_i$ : Initial Pressure (psi)  
 $phr$ : Pressure Value at  $t = 1$  h (psi)  
 $q$ : Flow Rate (STB/day)  
 $B$ : Volume factor (RB/STB)  
 $h$ : Thickness of reservoir (ft)  
 $\mu$ : Viscosity of Oil (cP)  
 $\varnothing$ : Porosity (fraction)  
 $c_t$ : Compressibility (1/psi)  
 $r_w$ : Wellbore Radius (ft)

### Output(s)

s: Skin Factor (dimensionless)

### Formula(s)

$$s = 1.151 * \left( \frac{\frac{p_i - phr}{p_{wf} - p_w}}{\log(t_b) - \log(t_a)} - \log \left( \frac{162.6 * q * B * \frac{\mu}{\left( \frac{p_{wf} - p_w}{\log(t_b) - \log(t_a)} \right) * h}}{\varnothing * \mu * c_t * r_w^2} \right) + 3.23 \right)$$

Reference: *Pressure Buildup and Flow Tests in Wells*, Matthews & Russell, Page: 57.

### 3.45 Pressure buildup equation (Horner equation)

#### Input(s)

- $P_i$ : Initial Reservoir Pressure (psi)
- $Q_o$ : Stabilized Well Flow Rate before Shut-in (bbl/d)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (rb/stb)
- $k$ : Permeability (mD)
- $h$ : Height (ft)
- $t_p$ : Flowing Time before Shut-in (day)
- $\Delta t$ : Shut-in Time (day)

#### Output(s)

- $P_{ws}$ : Pressure (psi)

#### Formula(s)

$$P_{ws} = P_i - \left( \frac{162.6 * Q_o * B_o * \mu_o}{k * h} \right) * \log \left( \frac{t_p + \Delta t}{\Delta t} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 54.

### 3.46 Radius of investigation

#### Input(s)

- $k$ : Permeability (mD)
- $t$ : Time (h)
- $\phi$ : Porosity (fraction)
- $\mu$ : Viscosity (cP)
- $c_t$ : Total Compressibility (1/psi)

#### Output(s)

- $r_i$ : Radius of Investigation at the end of Injection Time (ft)

#### Formula(s)

$$r_i = 0.0359 * \left( \left( \frac{k * t}{\phi * \mu * c_t} \right)^{0.5} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 134.

### 3.47 Radius of investigation (flow time)

#### Input(s)

- $t$ : Flow time (h)
- $k$ : Permeability (mD)

- $\mu$ : Viscosity of Oil (cP)
- $\phi$ : Porosity (fraction)
- $c_t$ : Compressibility (1/psi)

**Output(s)**

- $r_i$ : Radius of investigation (ft)

**Formula(s)**

$$r_i = \left( k * \frac{t}{948 * \phi * \mu * c_t} \right)^{0.5}$$

Reference: *Pressure Buildup and Flow Tests in Wells*, Matthews & Russell, Page: 116.

**3.48 Radius of investigation (shut-in time)****Input(s)**

- $dt$ : Time of shut in (h)
- $k$ : Permeability (mD)
- $\mu$ : Viscosity of Oil (cP)
- $\phi$ : Porosity (fraction)
- $c_t$ : Compressibility (1/psi)

**Output(s)**

- $r_i$ : Radius of investigation (ft)

**Formula(s)**

$$r_i = \left( k * \frac{dt}{948 * \phi * \mu * c_t} \right)^{0.5}$$

Reference: *Pressure Buildup and Flow Tests in Wells*, Matthews & Russell, Page: 117.

**3.49 Raymer hunt transform (porosity/transit time relationship)****Input(s)**

- $\phi$ : Porosity (fraction)
- $\Delta t_{ma}$ : Travel Time in Matrix ( $\mu\text{s}/\text{ft}$ )
- $\Delta t_f$ : Travel Time in Liquid/fluid ( $\mu\text{s}/\text{ft}$ )

**Output(s)**

- $\Delta t$ : Total Travel Time ( $\mu\text{s}/\text{ft}$ )

**Formula(s)**

$$\Delta t = \left( \frac{(1-\phi)^2}{\Delta t_{ma}} + \left( \frac{\phi}{\Delta t_f} \right) \right)^{-1}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 56.

**3.50 Reservoir permeability****Input(s)**

- $q_o$ : Oil Flow Rate (bbl/d)
- $B_o$ : Oil Formation Volume Factor (bbl/STB)
- $\mu_o$ : Oil Viscosity (cP)
- $m$ : Slope of Horner Plot (psi/cycle)
- $h$ : Pay zone of Thickness (ft)

**Output(s)**

- $k_o$ : Reservoir Permeability (mD)

**Formula(s)**

$$k_o = 162.6 * (q_o) * (B_o) * \frac{\mu_o}{m * h}$$

Notes: Permeability Evaluated from Horner Plot (Semi-Log Plot).

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 104.

**3.51 Shut-in time for pressure build-up test (Dietz method)****Input(s)**

- $\phi$ : Porosity (fraction)
- $A$ : Drainage Area ( $\text{ft}^2$ )
- $C_A$ : Shape Factor (dimensionless)
- $\mu$ : Viscosity (cP)
- $c_t$ : Total Compressibility (1/psi)
- $k$ : Permeability (mD)

**Output(s)**

- $t_s$ : Shut-in Time (h)

**Formula(s)**

$$t_s = \frac{\phi * \mu * c_t * A}{0.0002637 * C_A * k}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 63.

### 3.52 Skin during infinite-acting pseudoradial flow for vertical wells

#### Input(s)

$p_i$ : Initial Pressure (psi)  
 $p_{1hr}$ : Pressure After 1 Hour (psi)  
 $m$ : Slope (psi/h)  
 $k$ : Permeability (mD)  
 $\phi$ : Porosity (fraction)  
 $\mu_{Total}$ : Viscosity (cP)  
 $c_t$ : Total Compressibility (1/psi)  
 $r_w$ : Wellbore Radius (ft)

#### Output(s)

S: Skin (dimensionless)

#### Formula(s)

$$S = 1.151 * \left( \left( \frac{p_i - p_{1hr}}{m} \right) - \log \left( \frac{k}{\phi * \mu_{Total} * c_t * r_w^2} \right) + 3.23 \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 99.

### 3.53 Skin estimation type-1 (pressure buildup test)

#### Input(s)

$P_{hr}$ : Pressure After One Hour of Well Shut-in (psi)  
 $P_{wf}$ : Well Flowing Pressure (psi)  
 $m$ : Slope of Horner Plot (psi/cycle)  
 $k$ : Permeability (mD)  
 $\phi$ : Porosity (fraction)  
 $c_t$ : Total Compressibility (1/psi)  
 $r_w$ : Wellbore Radius (ft)  
 $\mu_o$ : Oil Viscosity (cP)

#### Output(s)

s: Skin Factor (dimensionless)

#### Formula(s)

$$s = 1.151 * \left( \left( \frac{(P_{hr}) - P_{wf}}{m} \right) - \log \left( \frac{k}{\phi * \mu_o * c_t * r_w^2} \right) + 3.23 \right)$$

Notes: Skin Estimation from Horner Plot (Semi Log Plot).

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 104.

### 3.54 Slope of Horner plot in pressure buildup test

#### Input(s)

- $Q_o$ : Stabilized Well Flow Rate before Shut-in (STB/day)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (bbl/STB)
- $k$ : Permeability (mD)
- $h$ : Thickness (ft)

#### Output(s)

- $m$ : Slope (psi/cycle)

#### Formula(s)

$$m = \frac{162.6 * Q_o * \mu_o * B_o}{k * h}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 54.

### 3.55 Slope of pseudo-steady state flow in pressure buildup test

#### Input(s)

- $Q$ : Flow Rate (STB/day)
- $B$ : Formation Volume Factor (bbl/STB)
- $c_t$ : Total Compressibility (1/psi)
- $A$ : Drainage Area ( $\text{ft}^2$ )
- $\phi$ : Porosity (fraction)
- $h$ : Thickness (ft)

#### Output(s)

- $m_{pss}$ : Slope (psi/h)

#### Formula(s)

$$m_{pss} = \frac{0.23396 * Q * B}{c_t * A * h * \phi}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 33.

### 3.56 Time to pseudo-steady state (single well-circular reservoir)

#### Input(s)

- $\phi$ : Porosity (fraction)
- $\mu$ : Viscosity (cP)
- $c_t$ : Compressibility (/psi)
- $r_e$ : Radius of reservoir (ft)
- $k$ : Permeability (mD)

**Output(s)**

$t_{ps}$ : Pseudo steady state (h)

**Formula(s)**

$$tpss = 1200 * \phi * \mu * c_t * \frac{r_e^2}{k}$$

Reference: *Pressure Buildup and Flow Tests in Wells*, Matthews & Russell, Page: 13.

### 3.57 Time to reach the semi-steady state for a gas well in a circular or square drainage area

**Input(s)**

$\mu_{gi}$ : Initial Gas Viscosity (cP)

$\phi$ : Porosity (fraction)

$A$ : Drainage Area ( $\text{ft}^2$ )

$k$ : Effective Gas Permeability (mD)

$c_{ti}$ : Initial Total Compressibility (1/psi)

**Output(s)**

$t_{ps}$ : Time (h)

**Formula(s)**

$$t_{ps} = \frac{15.8\phi * \mu_{gi} * c_{ti} * A}{k}$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 194.

### 3.58 True wellbore storage coefficient (pressure build-up test)

**Input(s)**

$A_{wb}$ : Cross Sectional Area of Well Bore ( $\text{ft}^2$ )

$\rho_{wb}$ : Density of Fluid in Well Bore (lbm/ $\text{ft}^3$ )

**Output(s)**

$C$ : Wellbore Storage Coefficient (BBL/PSI)

**Formula(s)**

$$C = 25.65 * \frac{A_{wb}}{\rho_{wb}}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 105.

### 3.59 Well flow efficiency (geothermal well)

#### Input(s)

- $p^*$ : Horner's False Pressure at  $(t + \Delta t = 1)$  (psi)  
 $P_{wf}$ : Bottomhole Pressure at a flowing well (psi)  
 $\Delta p_{skin}$ : Pressure Drop caused by the Skin Effect (psi)

#### Output(s)

- FE: Flow Efficiency (per cent)

#### Formula(s)

$$FE = \frac{p^* - p_{wf} - \Delta p_{skin}}{p^* - p_{wf}}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University, Page: (5.14).

### 3.60 Well shut-in pressure during buildup (Horner plot)

#### Input(s)

- $p_i$ : Initial False Pressure (psi)  
 $Q_o$ : Stabilized Well Flow Rate before Shut-in (STB/day)  
 $B_o$ : Oil Formation Volume Factor (bbl/STB)  
 $k$ : Permeability (mD)  
 $\mu_o$ : Oil Viscosity (cP)  
 $t_p$ : Flowing Time before Shut-in (h)  
 $\Delta t$ : Shut-in Time (h)  
 $h$ : Thickness (ft)

#### Output(s)

- $p_{ws}$ : Shut-in Pressure (psi)

#### Formula(s)

$$p_{ws} = p_i - \left( \frac{162.6 * Q_o * \mu_o * B_o}{k * h} \right) * \log \left( \frac{t_p + \Delta t}{\Delta t} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 56.

## Chapter 4

# Production engineering formulas and calculations

### Chapter Outline

4.1 Acid penetration distance (acidizing)	204	4.28 Filter cake on the fracture (acidizing)	216
4.2 Additional pressure drop in the skin zone		4.29 Flow coefficient during drawdown	217
4.3 Additive crystalline salt amount to increase the density—Method I (single-salt systems)		4.30 Flow rate through orifice	217
4.4 Additive crystalline salt amount to increase the density—Method II (single-salt systems)	205	4.31 Flow through fracture in response to pressure gradient	217
4.5 Additive crystalline salt and water amount to increase the density—Method I (two-salt systems)	206	4.32 Formation fluid compressibility (acidizing)	218
4.6 Annulus pressure loss due to friction during hydraulic fracturing (laminar flow)	206	4.33 Fracture area of a hydraulically fractured formation	218
4.7 Annulus pressure loss due to friction during hydraulic fracturing (turbulence flow)	206	4.34 Fracture coefficient of a hydraulically fractured reservoir	219
4.8 Approximate ideal counterbalanced load	207	4.35 Fracture fluid coefficient for reservoir-controlled liquids	219
4.9 Average downstroke load (sucker-rod pump)	207	4.36 Fracture fluid coefficient for viscosity-controlled liquids	220
4.10 Average fracture width (acidizing)	208	4.37 Fracture geometry (acidizing)	220
4.11 Average permeability of a hydraulically fractured formation	208	4.38 Fracture gradient (hydraulic fracturing)	220
4.12 Average specific weight of the formation (hydraulic fracturing)	209	4.39 Fracture-fluid invasion of the formation (acidizing)	221
4.13 Average upstroke load (sucker-rod pump)	209	4.40 Frictional pressure drop (Economides and Nolte)	221
4.14 Average wellbore fluid density (completion and workover fluids)	209	4.41 Gas velocity under sonic flow conditions (through choke)	222
4.15 Capacity ratio of a hydraulically fractured surface	210	4.42 Hydraulic fracture efficiency	222
4.16 Choke discharge coefficient	210	4.43 Hydraulic horse power for a hydraulic fracturing operation	223
4.17 Close-ended displacement volume of pipe	210	4.44 Ideal fracture conductivity created by acid reaction (acidizing)	223
4.18 Convective mass transfer for laminar flow (acidizing)	211	4.45 Incremental density in any wellbore interval (completion and workover fluids)	223
4.19 Convective mass transfer for turbulent flow (acidizing)	211	4.46 Initial rate following a hydraulic fracturing operation	224
4.20 Correct counterbalance (sucker-rod pump)	212	4.47 Injection pressure for hydraulic fracturing	224
4.21 Corresponding reciprocal rate (post-fracture production—Constant bottomhole flowing conditions)	213	4.48 Lifetime of a hydraulically fractured well	225
4.22 Damaged/undamaged zone productivity comparison (acidizing)	213	4.49 Mass of rock dissolved per unit mass of acid (acidizing)	225
4.23 Density of brine (completion and workover fluids)	213	4.50 Mass transfer in acid solutions by Fick's law (acidizing)	225
4.24 Dimensionless fracture width for linear vertical fracture	214	4.51 Maximum treatment pressure (hydraulic fracturing)	226
4.25 Downhole operating pressure (hydraulic fracturing)	214	4.52 Mechanical resistant torque (PCP)	226
4.26 Entrance hole size (perforation)	214	4.53 Minimum polished rod load (sucker rod pump)	226
4.27 Equivalent skin factor in fractured wells	214	4.54 Peclet number for fluid loss (acidizing)	227
	215	4.55 Perforation friction factor	227
	215	4.56 Perforation friction pressure	228
	216	4.57 Perforation hole size (perforation)	228

<b>4.58 Perforation length in formation</b>	<b>228</b>	<b>4.89 Sucker rod—Peak polished rod load</b>	<b>244</b>
<b>4.59 Perforation penetration ratio (formation of interest/ reference formation)</b>	<b>229</b>	<b>4.90 Suspension property of static fluids (completion and workover fluids)</b>	<b>245</b>
<b>4.60 Perforation skin factor</b>	<b>229</b>	<b>4.91 Tangential annular flow of a power law fluid</b>	<b>245</b>
<b>4.61 Pore growth function (acidizing)</b>	<b>230</b>	<b>4.92 Temperature at choke outlet</b>	<b>246</b>
<b>4.62 Pressure drop across perforations in gas wells</b>	<b>230</b>	<b>4.93 The z component of the force of the fluid on the wetted surface of the pipe</b>	<b>246</b>
<b>4.63 Pressure drop across perforations in oil wells</b>	<b>231</b>	<b>4.94 Total skin in partially depleted wells for a buildup test</b>	<b>246</b>
<b>4.64 Pressure loss due to perforations during hydraulic fracturing</b>	<b>232</b>	<b>4.95 Velocity distribution in the annular slit of a falling-cylinder viscometer</b>	<b>247</b>
<b>4.65 Pressure loss due to perforations during hydraulic fracturing—2</b>	<b>232</b>	<b>4.96 Velocity distribution in the narrow annular region in annular flow with inner cylinder moving axially</b>	<b>247</b>
<b>4.66 Principal stress due to petro-static pressure (hydraulic fracturing)</b>	<b>232</b>	<b>4.97 Velocity distribution of flow through an annulus</b>	<b>248</b>
<b>4.67 Productivity index (for generating composite IPR curve)</b>	<b>233</b>	<b>4.98 Velocity of fluid in annulus</b>	<b>248</b>
<b>4.68 Productivity ratio</b>	<b>233</b>	<b>4.99 Velocity of fluid in pipe</b>	<b>249</b>
<b>4.69 Productivity ratio calculation of a hydraulically- fractured formation</b>	<b>234</b>	<b>4.100 Viscous force acting on the rod over the narrow annular region</b>	<b>249</b>
<b>4.70 Pseudo skin factor due to partial penetration (Brons and Marting method)</b>	<b>234</b>	<b>4.101 Volume capacity of pipe</b>	<b>250</b>
<b>4.71 Pseudo-skin factor due to partial penetration (Yeh and Reynolds correlation)</b>	<b>235</b>	<b>4.102 Volume of fluid loss per unit area measured in a dynamic test (acidizing)</b>	<b>250</b>
<b>4.72 Pseudo-skin factor due to partial penetration (Odeh correlation)</b>	<b>236</b>	<b>4.103 Volume of fluid loss per unit area measured in a static test (acidizing)</b>	<b>250</b>
<b>4.73 Pseudo-skin factor due to partial penetration (Papatzacos correlation)</b>	<b>236</b>	<b>4.104 Volume of rock dissolved per unit volume of acid (acidizing)</b>	<b>251</b>
<b>4.74 Pseudo-skin factor due to perforations</b>	<b>237</b>	<b>4.105 Water quantity that dilutes the original brine with assumed density (two-salt systems)</b>	<b>251</b>
<b>4.75 Quantifying formation damage and improvement</b>	<b>238</b>	<b>4.106 Weight of crystalline <math>\text{CaCl}_2</math> and <math>\text{CaBr}_2</math> salt addition to brine (two-salt systems)</b>	<b>252</b>
<b>4.76 Recommended underbalanced environment for perforation</b>	<b>239</b>	<b>4.107 Well flowing pressure (line-source solution by including skin factor)</b>	<b>252</b>
<b>4.77 Reynolds number for acid flow into the fracture (acidizing)</b>	<b>239</b>	<b>4.108 Well flowing pressure under pseudo-steady state flow for non-circular reservoirs</b>	<b>253</b>
<b>4.78 Reynolds number for fluid loss (acidizing)</b>	<b>240</b>	<b>4.109 Wellbore pressure loss due to friction during hydraulic fracturing (laminar flow)</b>	<b>253</b>
<b>4.79 Sand weight needed to refill a hydraulically- fractured reservoir volume</b>	<b>240</b>	<b>4.110 Wellbore pressure loss due to friction during hydraulic fracturing (turbulence flow)</b>	<b>254</b>
<b>4.80 Shape factor expressed as skin factor for vertical wells</b>	<b>240</b>	<b>4.111 Wellbore storage</b>	<b>254</b>
<b>4.81 Single-phase gas flow (subsonic)</b>	<b>241</b>	<b>4.112 Wellbore storage due to fluid level</b>	<b>254</b>
<b>4.82 Single-phase liquid flow through choke</b>	<b>241</b>	<b>4.113 Wellhead pressure (multiphase flow across the choke)</b>	<b>255</b>
<b>4.83 Skin factor</b>	<b>242</b>	<b>4.114 Workover operations (maximum allowed tubing pressure)</b>	<b>255</b>
<b>4.84 Skin factor by Hawkins method</b>	<b>242</b>	<b>4.115 Young modulus by using sonic travel time (acidizing)</b>	<b>256</b>
<b>4.85 Skin factor due to partial penetration</b>	<b>242</b>		
<b>4.86 Skin factor due to reduced crushed-zone permeability</b>	<b>243</b>		
<b>4.87 Skin factor for a deviated well</b>	<b>243</b>		
<b>4.88 Slope of semilog plot for bottom-hole flowing pressure vs time for drawdown test</b>	<b>244</b>		

## 4.1 Acid penetration distance (acidizing)

### Input(s)

$\bar{w}$ : Average Fracture Width (ft)

$L_{aD}$ : Dimensionless Acid Penetration Distance (dimensionless)

$N_{Re}$ : Reynolds Number (dimensionless)

$N_{Re^*}$ : Reynolds Number for Fluid Loss (dimensionless)

**Output(s)**

$xL$ : Acid Penetration Distance (ft)

**Formula(s)**

$$xL = \frac{\bar{w} L_{aD}}{2} \left( \frac{N_{re}}{N_{Re^*}} \right)$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 62.*

**4.2 Additional pressure drop in the skin zone****Input(s)**

- $Q_o$ : Well Flow Rate (STB/day)
- $B_o$ : Oil Formation Volume Factor (bbl/STB)
- $\mu_o$ : Oil Viscosity (cP)
- $k$ : Reservoir Permeability (mD)
- $h$ : Thickness (ft)
- $S$ : Skin (dimensionless)

**Output(s)**

$\Delta P_{skin}$ : Pressure Drop Due to Skin (psi)

**Formula(s)**

$$\Delta P_{skin} = \left( \frac{141.2 * Q_o * B_o * \mu_o}{k * h} \right) * S$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 37.*

**4.3 Additive crystalline salt amount to increase the density—Method I (single-salt systems)****Input(s)**

- $V_i$ : Volume of Initial Brine Formulation (bbl)
- $\rho_i$ : Density of Fluid (g/cm<sup>3</sup>)
- $C_{sf}$ : Concentration of Salt in Final Formulation (%)
- $C_{si}$ : Concentration of Salt in Initial Formulation (%)
- $P_s$ : Purity of Salt to be added (%)

**Output(s)**

$m_s$ : Additional Salt need to raise the Density (lbm)

**Formula(s)**

$$m_s = \frac{V_i(42 \text{ gal/bbl})(\rho_i)(C_{sf} - C_{si})}{P_s - mC_{sf}}$$

Reference: Bridges, K. L. (2000). *Completion and Workover Fluids* (Vol. 19). Society of Petroleum Engineers, Page: 54.

#### **4.4 Additive crystalline salt amount to increase the density—Method II (single-salt systems)**

**Input(s)**

- $V_i$ : Volume of Initial Brine Formulation (bbl)
- $V_f$ : Volume of Brine Following the Adjustment (bbl)
- $C_{sf}$ : Concentration of Salt in Final Brine Formulation (lbm/bbl)
- $C_{si}$ : Concentration of Salt in Initial Brine Formulation (lbm/bbl)

**Output(s)**

- $m_s$ : Additional Salt need to raise the Density (lbm)

**Formula(s)**

$$m_s = (C_{sf}V_f) - (C_{si}V_i)$$

Reference: Bridges, K. L. (2000). *Completion and Workover Fluids* (Vol. 19). Society of Petroleum Engineers, Page: 55.

#### **4.5 Additive crystalline salt and water amount to increase the density—Method I (two-salt systems)**

**Input(s)**

- $V_i$ : Volume of Initial Brine (bbl)
- $V_w$ : Total Water Volume to add to achieve the Final Brine Density (bbl)
- $C_{msf}$ : Concentration of More Soluble Salt in Final Brine Formulation (%) (lbm/bbl)
- $C_{lsf}$ : Concentration of Less Soluble Salt in Final Brine Formulation (lbm/bbl)
- $C_{msi}$ : Concentration of More Soluble Salt in Initial Brine Formulation (lbm/bbl)
- $C_{lsi}$ : Concentration of Less Soluble Salt in Initial Brine Formulation (lbm/bbl)
- $W_i$ : Water Fraction in Initial Brine Formulation (fraction)
- $W_f$ : Water Fraction in Final Brine Formulation (fraction)
- $V_f$ : Final Volume of Brine after Adjustment (bbl)

**Output(s)**

- $m_{ms}$ : Additional More Soluble Salt need to raise the Brine Density (lbm)

**Formula(s)**

$$m_{ms} = (V_i) \frac{C_{lsi} C_{msf}}{C_{lsf}} - C_{msi}$$

$$V_w = (V_i) \frac{C_{lsi} W_f}{C_{lsf}} - W_i$$

$$V_f = \frac{C_{lsi} V_i}{C_{lsf}}$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 56.*

**4.6 Annulus pressure loss due to friction during hydraulic fracturing (laminar flow)****Input(s)**

$v$ : Average Velocity (ft/s)

$L$ : Pipe Length (ft)

$d_0$ : Inner Diameter of the Outer Pipe (in.)

$d_1$ : Outer Diameter of the Inner Pipe (in.)

$\mu_p$ : Plastic Viscosity (cP)

$\tau_y$ : Yield Point of the Liquid (lb/100ft<sup>2</sup>)

**Output(s)**

$\Delta P_f$ : Pressure Loss in the wellbore due to Frictions during Hydraulic Fracturing (psi)

**Formula(s)**

$$\Delta P_f = \frac{\mu_p Lv}{1000 (d_0 - d_i)^2} + \frac{\tau_y \cdot L}{200 \cdot (d_0 - d_i)}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 31.*

**4.7 Annulus pressure loss due to friction during hydraulic fracturing (turbulence flow)****Input(s)**

$\rho$ : Liquid Density (lb/gal)

$v$ : Average Velocity (ft/s)

$f$ : Fanning Friction Factor (dimensionless)

$L$ : Pipe Length (ft)

$d_0$ : Inner Diameter of the Outer Pipe (in.)

$d_1$ : Outer Diameter of the Inner Pipe (in.)

**Output(s)**

$\Delta P_f$ : Pressure Loss in the wellbore Sourced by Frictions during Hydraulic Fracturing (psi)

**Formula(s)**

$$\Delta P_f = \frac{f \cdot L \cdot \rho \cdot v^2}{25.80 \cdot (d_0 - d_i)}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 31.*

**4.8 Approximate ideal counterbalanced load****Input(s)**

- PPRL:* Peak Polished Rod Load (lb)  
*MPRL:* Minimum Polished Rod Load (lb)

**Output(s)**

- AICB:* Ideal Counterbalance (lb)

**Formula(s)**

$$AICB = \frac{PPRL + MPRL}{2}$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 12/169.*

**4.9 Average downstroke load (sucker-rod pump)****Input(s)**

- C: Calibration Constant of Dynamometer Card (lb/in.)  
A: Lower Area of Card (in.<sup>2</sup>)  
L: Length of Card (in.)

**Output(s)**

- ADL:* Average Downstroke Load (lb)

**Formula(s)**

$$ADL = C * \frac{A}{L}$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18/269.*

**4.10 Average fracture width (acidizing)****Input(s)**

- $\pi$ :* Pi Number (dimensionless)  
 *$w_w$ :* Fracture width at the Wellbore (m)

**Output(s)**

$\bar{w}$ : Average Fracture width at the Wellbore (m)

**Formula(s)**

$$\bar{w} = \frac{\pi w_w}{4}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 41.*

**4.11 Average permeability of a hydraulically fractured formation****Input(s)**

$k_{avgz}$ : Average Permeability of a Hydraulically Fractured Zone (mD)

$k$ : Permeability of the Formation (mD)

$h$ : Formation Thickness (ft)

$W$ : Fracture Thickness (ft)

$r_e$ : Drainage Radius of the Well (in.)

$r_w$ : Radius of the Well (in.)

$r_f$ : Radius of the Fracture (in.)

**Output(s)**

$k_{avg}$ : Average Permeability of a Hydraulically Fractured Formation (mD)

**Formula(s)**

$$k_{avg} = \frac{k \cdot k_{avgz} \cdot \ln\left(\frac{r_e}{r_w}\right)}{k_{avgz} \cdot \ln\left(\frac{r_e}{r_f}\right) + k \cdot \ln\left(\frac{r_f}{r_w}\right)}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 7.*

**4.12 Average specific weight of the formation (hydraulic fracturing)****Input(s)**

$\gamma_{min}$ : Specific Weight of Minerals (p/cm<sup>3</sup>)

$\gamma_{Liq}$ : Specific Weight of Liquid Phase (p/cm<sup>3</sup>)

$\phi$ : Porosity (fraction)

**Output(s)**

$\gamma_{formation}$ : Average Specific Weight of the Formation (p/cm<sup>3</sup>)

### Formula(s)

$$\gamma_{formation} = (1 - \emptyset) \gamma_{min} + \emptyset \cdot \gamma_{Liq}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 5.*

## 4.13 Average upstroke load (sucker-rod pump)

### Input(s)

- C: Calibration Constant of Dynamometer (lb/in.)
- a: Lower Area of Card (in.<sup>2</sup>)
- A: Area of Upper Card (in.<sup>2</sup>)
- L: Length of Dynamometer Card (in.)

### Output(s)

- AUL: Average Upstroke Load (lb)

### Formula(s)

$$AUL = C * \frac{A + a}{L}$$

Notes: Calculations Based on Dynamometer Card.

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18/269.*

## 4.14 Average wellbore fluid density (completion and workover fluids)

### Input(s)

- $\rho_m$ : Measured Ambient (or Surface) Fluid Density (lbm/bbl)
- $\Delta\rho_i$ : Incremental Wellbore Fluid Density Change (lbm/bbl)
- n: Number of Linear Depth intervals (dimensionless)

### Output(s)

- $\bar{\rho}$ : Average Wellbore Fluid Density (lbm/bbl)

### Formula(s)

$$\bar{\rho} = \rho_m - \left[ \sum_{i=1}^n \frac{(\Delta\rho_i)}{n} - 1 \right]$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 46.*

## 4.15 Capacity ratio of a hydraulically fractured surface

### Input(s)

- $k_f$ : Fracture Permeability (mD)
- $k$ : Average Formation Permeability (mD)
- $h$ : Formation Thickness (ft)
- $W$ : Fracture Thickness (ft)

**Output(s)**

$c_f$ : Cumulative Present Value of Production (\$)

**Formula(s)**

$$c_f = \frac{k_f \cdot W}{k \cdot h}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 92.*

**4.16 Choke discharge coefficient****Input(s)**

$d$ : Upstream Pipe Diameter (in.)

$d_c$ : Choke Diameter (in.)

$N_R$ : Reynold's Number based on  $d_c$  (dimensionless)

**Output(s)**

$C_d$ : Choke Discharge Coefficient (dimensionless)

**Formula(s)**

$$C_d = \left( \frac{d_c}{d} \right) + \left( \frac{0.3167}{\left( \frac{d_c}{d} \right)^{0.6}} \right) + 0.025 * (\log(N_R) - 4)$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 5/60.*

**4.17 Close-ended displacement volume of pipe****Input(s)**

$D_o$ : Outside Diameter of Pipe (in.)

L: Length of Pipe (ft)

**Output(s)**

$V_c$ : Close-ended Diameter of Pipe (bbl)

**Formula(s)**

$$V_c = (0.7854 * D_o^2 * L) / 808.5$$

$$V_c = \frac{0.7854 * D_o^2 * L}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 45.*

## 4.18 Convective mass transfer for laminar flow (acidizing)

### Input(s)

$D_A$ : Diffusion Coefficient of Component A ( $\text{cm}^2/\text{s}$ )

$\partial c_a / \partial Y$ : Concentration Gradient ( $\text{mol}/\text{cm}^4$ )

$c_a$ : Concentration of Flowing Acid ( $\text{mol}/\text{cm}^3$ )

$V_N$ : Fluid Velocity Normal to the Surface ( $\text{cm}/\text{s}$ )

### Output(s)

$U_{a,y}$ : Diffusion Flux of a Component A in Y direction ( $\text{mol}/\text{cm}^2 \text{ s}$ )

### Formula(s)

$$U_{a,y} = -D_A \frac{\partial c_a}{\partial Y} + c_a V_N$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page 22.*

## 4.19 Convective mass transfer for turbulent flow (acidizing)

### Input(s)

$D_A$ : Diffusion Coefficient of Component A ( $\text{cm}^2/\text{s}$ )

$D_E$ : Effective Diffusion Coefficient of Component A ( $\text{cm}^2/\text{s}$ )

$dc_a/dY$ : Concentration Gradient ( $\text{mol}/\text{cm}^4$ )

$c_a$ : Concentration of Flowing Acid ( $\text{mol}/\text{cm}^3$ )

$\langle c_a \rangle$ : Average Concentration of Flowing Acid ( $\text{mol}/\text{cm}^3$ )

$\langle c_a(w) \rangle$ : Average Concentration of Flowing Acid ( $\text{mol}/\text{cm}^3$ )

$V_N$ : Fluid Velocity Normal to the Surface ( $\text{cm}/\text{s}$ )

$\langle V_N \rangle$ : Average Fluid Velocity Normal to the Surface ( $\text{cm}/\text{s}$ )

$K_g$ : Effective Mass Transfer Coefficient ( $\text{cm}/\text{s}$ )

### Output(s)

$U_{a,y}$ : Diffusion Flux of a Component A in Y direction ( $\text{mol}/\text{cm}^2 \text{ s}$ )

$\langle U_{a,y} \rangle$ : Average Diffusion Flux of a Component A in Y direction ( $\text{mol}/\text{cm}^2 \text{ s}$ )

### Formula(s)

$$\begin{aligned} \langle U_{a,y} \rangle &= -D_A \frac{d \langle c_a \rangle}{dY} + \langle c_a \rangle V_N \\ \langle U_{a,y} \rangle &= -D_E \frac{d \langle c_a \rangle}{dY} + \langle c_a \rangle \langle V_N \rangle \\ \langle U_{a,y} \rangle &= K_g [\langle c_a \rangle - \langle c_a(w) \rangle] + \langle c_a \rangle \langle V_N \rangle \end{aligned}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page 23.*

## 4.20 Correct counterbalance (sucker-rod pump)

### Input(s)

- AUL: Average Upstroke Load (lb)  
ADL: Average Downstroke Load (lb)

### Output(s)

- CCB: Correct Counterbalance (lb)

### Formula(s)

$$CCB = \frac{AUL + ADL}{2}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 18/269.

## 4.21 Corresponding reciprocal rate (post-fracture production—Constant Bottomhole flowing conditions)

### Input(s)

- $q_D$ : Dimensionless Reciprocal Rate from Match Point (1/q\_d) (dimensionless)  
 $B_g$ : Gas Formation Volume Factor (RB/MSCF)  
 $p_{ai}$ : initial Adjusted Pressure (psi)  
 $p_{awf}$ : Adjusted Well Flowing Pressure (psi)  
 $\phi$ : Porosity (dimensionless)  
 $c_t$ : Total Compressibility (1/psi)  
 $\mu$ : Viscosity (cP)  
 $h$ : Formation Thickness (ft)  
 $C_{rD}$ : Dimensionless Fracture Conductivity (dimensionless)  
 $L_f$ : Fracture Length (ft)  
 $t_{LJD}$ : Ending of Linear Part or Starting Time of Pseudo Radial Flow from Plot (dimensionless)  
 $k$ : Permeability (mD)  
 $\Delta t$ : Time Difference from Data (h)

### Output(s)

- $\left(\frac{1}{q}\right)_{MP}$ : Reciprocal Rate from Match Point (1/q)\_mp (Days/MSCF)  
 $L_{fMP}$ : Length of Fracture from Match Point Analysis (ft)  
 $w_{fk_f}$ : Fracture Conductivity from Dimensionless Fracture Conductivity (mD ft)

### Formula(s)

$$\begin{aligned} \left(\frac{1}{q}\right)_{MP} &= \left( 141.2 * B_g * \frac{\mu}{k * h * (p_{ai} - p_{awf})} \right) * \left(\frac{1}{q_D}\right) \\ L_{fMP} &= \left( \left( \frac{0.0002637 * k}{\phi * \mu * c_t} \right) * \left( \frac{\Delta t}{t_{LJD}} \right) \right)^{\frac{1}{2}} \\ w_{fk_f} &= \pi * k * C_{rD} * L_f \end{aligned}$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 126.

## 4.22 Damaged/undamaged zone productivity comparison (acidizing)

### Input(s)

- $F_k$ : Permeability Ratio (dimensionless)
- $r_e$ : Drainage Radius (in.)
- $r_w$ : Wellbore Radius (in.)
- $r_s$ : Damaged Zone Radius (in.)

### Output(s)

- $J_s$ : Damaged or Stimulated Formation Productivity Index (STB/day/psi)
- $J_o$ : Undamaged Formation Productivity Index (STB/day/psi)

### Formula(s)

$$\frac{J_s}{J_o} = \frac{F_k \log r_e / r_w}{\log r_s / r_w + F_k \log r_e / r_s}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 6.*

## 4.23 Density of brine (completion and workover fluids)

### Input(s)

- $\rho_m$ : Density of Brine at ambient Surface Pressure and Temperature (lbm/gal)
- $C_{te}$ : Coefficient of Thermal Expansion or Volume Expansion Factor (dimensionless)
- $T_m$ : Ambient Temperature of Brine-Density Measurement ( $^{\circ}$ F)
- $T_s$ : Standard Reference Temperature ( $70^{\circ}$ F)

### Output(s)

- $\rho_s$ : Density of Brine at Standard Reference Temperature (lbm/gal)

### Formula(s)

$$\rho_s = \rho_m [1 + C_{te} (T_m - T_s)]$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 45.*

## 4.24 Dimensionless fracture width for linear vertical fracture (Geertsma & Klerk)

### Input(s)

- $C$ : Over-all Fluid-Loss Coefficient ( $m/(s^{0.5})$ )
- $h$ : Fracture Height (m)
- $E$ : Young's Modulus of the Formation Rock ( $kg/m \cdot s^2$ )
- $i$ : Fluid Injection Rate ( $m^3/s$ )
- $t$ : Total time for Fluid Injection (s)
- $w_w$ : Fracture width at the Wellbore (m)
- $V_{spf}$ : The Spurt Volume ( $m^3/m^2$ )

**Output(s)**

- $K_L$ : Dimensionless Fracture Length (dimensionless)  
 $K_u$ : Reciprocal Dimensionless Fracture Width (dimensionless)  
 $K_s$ : Dimensionless Fluid-Loss Coefficient (dimensionless)  
 $K_{\eta L}$ : Dimensionless Variable (dimensionless)

**Formula(s)**

$$\begin{aligned} K_L &= CLh/i\sqrt{t} \\ K_u &= C\sqrt{t}/w_w \\ K_s &= C\sqrt{t}/V_{spt} \\ K_{\eta L} &= 21.8 \left( \frac{i}{hC^2} \right)^3 \left( \frac{\mu}{Et} \right) \end{aligned}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 40.*

**4.25 Downhole operating pressure (hydraulic fracturing)****Input(s)**

- $\Delta P_f$ : Pressure Loss Sourced by Friction (psi)  
 $\Delta P_p$ : Pressure Loss sourced by Perforations(psi)  
 $\Delta P_h$ : Hydrostatic Pressure Change (psi)  
 $P_{inj}$ : Injection Pressure (psi)  
 $D$ : Fracture Elevation Depth (ft)  
 $G_f$ : Fracture Gradient (psi/ft)

**Output(s)**

- $P_F$ : Downhole Operating Pressure (psi)

**Formula(s)**

$$\begin{aligned} P_F &= G_f \cdot D \\ G_f &= \frac{P_{inj} + \Delta P_h - \Delta P_f - \Delta P_p}{D} \end{aligned}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 25.*

**4.26 Entrance hole size (perforation)****Input(s)**

- $\sigma_y$ : Yield Strength of Casing of Interest (ksi)  
 $\sigma_{yr}$ : Yield Strength of Reference Casing (ksi)  
 $d_r$ : Entrance Hole Diameter in Reference Casing (in.)

### Output(s)

$d$ : Entrance Hole Diameter in Casing of Interest (in.)

### Formula(s)

$$d = \left( \frac{\sigma_{yr}}{\sigma_y} \right)^{0.5} d_r$$

Reference: Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). *Perforating*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 50.

## 4.27 Equivalent skin factor in fractured wells

### Input(s)

$x_f$ : Fracture Half Length (ft)

$r_w$ : Wellbore Radius (ft)

### Output(s)

$S_f$ : Equivalent Skin (unitless)

### Formula(s)

$$S_f = 0.7 - \ln \left( \frac{x_f}{r_w} \right)$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 17/257.

## 4.28 Filter cake on the fracture (acidizing)

### Input(s)

$C_w$ : Fluid Loss Coefficient, ( $\text{m}/\text{s}^{1/2}$ )

$\sqrt{t}$ : Time root ( $\text{s}^{1/2}$ )

### Output(s)

$v_N$ : Fluid Loss Velocity (m/s)

### Formula(s)

$$v_N = \frac{C_w}{\sqrt{t}}$$

Reference: Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). *Acidizing fundamentals*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 41.

## 4.29 Flow coefficient during drawdown

### Input(s)

- $p_{wf}$ : Wellbore Flowing Pressure (psi)  
 $P_i$ : Average Pressure (psi)  
 $\Delta p_{skin}$ : Pressure Drop Due to Skin (psi)

### Output(s)

- E: Flow Coefficient (fraction)

### Formula(s)

$$E = \frac{P_i - p_{wf} - \Delta p_{skin}}{P_i - p_{wf}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 64.

## 4.30 Flow rate through orifice

### Input(s)

- $Cd$ : Dimensionless Flow Factor (dimensionless)  
 $\rho_1$ : Initial Density (ppg)  
 $\rho_2$ : Final Density (ppg)  
 $P_1$ : Initial Pressure (psi)  
 $P_2$ : Final Pressure (psi)  
 $\gamma$ : Adiabatic Constant (dimensionless)  
 $S_o$ : Output Cross Section ( $\text{ft}^2$ )  
 $S_1$ : Input Cross Section ( $\text{ft}^2$ )

### Output(s)

- w: Flow Rate ( $\text{ft}^3/\text{s}$ )

### Formula(s)

$$w = Cd * \rho_2 * S_o * \left( \frac{2 * \left( \frac{P_1}{\rho_1} \right) * \left( \frac{\gamma}{\gamma-1} \right) * \left( 1 - \left( \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right) \right)}{1 - \left( \left( \frac{S_o}{S_1} \right)^2 \right) * \left( \left( \frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} \right)} \right)^{0.5}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 15, Page: 472.

## 4.31 Flow through fracture in response to pressure gradient

### Input(s)

- $\mu$ : Fluid Viscosity (cP)  
 $L$ : Length (ft)

$v$ : Poisson (dimensionless)  
 $P_f$ : Fracture Pressure (psi)  
 $S_c$ : Least Principle Stress (psi)  
 $E$ : Young Modulus (psi)  
 $\Delta P$ : Pressure Gradient (psi)

### Output(s)

$Q$ : Flow Rate ( $\text{ft}^3/\text{s}$ )

### Formula(s)

$$Q = \left( \frac{\pi * \Delta P}{8 * \mu} \right) * \left( \left( \frac{L * (1 - v^2) * (P_f - S_c)}{E} \right)^3 \right)$$

Reference: *Mark D. Zoback., Reservoir Geomechanics, Cambridge University Express, UK, Page: 142.*

## 4.32 Formation fluid compressibility (acidizing)

### Input(s)

$K_o, K_w, K_g$ : Isothermal coefficient of Compressibility of Reservoir Oil. Water and Gas ( $\text{psi}^{-1}$ )  
 $S_o, S_w, S_g$ : Oil, Water and Gas Saturation Fractions (dimensionless)

### Output(s)

$K_{fl}$ : Fracture Conductivity (mD in.)

### Formula(s)

$$K_{fl} = S_o(K_o) + S_w(K_w) + S_g(K_g)$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing Fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 56.*

## 4.33 Fracture area of a hydraulically fractured formation

### Input(s)

$q_i$ : Injection Rate ( $\text{m}^3/\text{min}$ )  
 $W$ : Fracture Thickness (ft)  
 $t$ : Injection Time (min)  
 $C$ : Fracture Liquid Coefficient ( $\text{m}/\sqrt{\text{min}}$ )

### Output(s)

$A(t)$ : Fracture Area in time "t" ( $\text{m}^2$ )

**Formula(s)**

$$A(t) = \left[ \frac{q_i \cdot W}{4\pi \cdot C^2} \right] \cdot \left[ e^{x^2} erfc(x) + \frac{2x}{\sqrt{\pi}} - 1 \right]$$

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$$

$$x = 2C \frac{\sqrt{\pi t}}{W}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 12.*

### **4.34 Fracture coefficient of a hydraulically fractured reservoir**

**Input(s)**

$m$ : Slope of the Production Decline Curve ( $\text{ft}^3/\text{min}$ )

$A_f$ : Fractured Surface Area ( $\text{ft}^2$ )

**Output(s)**

$C_f$ : Fracture Coefficient of a Hydraulically Fractured Well ( $\text{ft}/\text{min}$ )

**Formula(s)**

$$C_f = \frac{0.0164 \cdot m}{A_f}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 99.*

### **4.35 Fracture fluid coefficient for reservoir-controlled liquids**

**Input(s)**

$k$ : Effective Permeability (Darcy)

$\emptyset$ : Effective Porosity (%)

$\mu$ : Viscosity of Reservoir Fluid (cP)

$\Delta P$ : Pressure difference in Fracture Face (psi)

$c_f$ : Isothermal Compressibility of Reservoir Fluid ( $\text{psi}^{-1}$ )

$c$ : Conversion Coefficient (dimensionless)

**Output(s)**

$C_r$ : Fracture Fluid Coefficient for Viscosity-Controlled Liquids (dimensionless)

**Formula(s)**

$$C_r = c \cdot \Delta P \left( \frac{k \cdot c_f \cdot \emptyset}{\mu} \right)^{1/2}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 20.*

## 4.36 Fracture fluid coefficient for viscosity-controlled liquids

### Input(s)

- $k$ : Effective Permeability (Darcy)
- $\emptyset$ : Effective Porosity (%)
- $\mu_f$ : Viscosity of Fracturing Fluid in Reservoir Conditions (cP)
- $\Delta P$ : Pressure difference in Fracture Face (psi)
- $c$ : Conversion Coefficient (dimensionless)

### Output(s)

- $C_v$ : Fracture Fluid Coefficient for Viscosity-Controlled Liquids (dimensionless)

### Formula(s)

$$C_v = c \left( \frac{k \cdot \Delta P \cdot \emptyset}{\mu_f} \right)^{1/2}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 17.*

## 4.37 Fracture geometry (acidizing)

### Input(s)

- $i/h$ : Injection Rate per unit of Fracture Height ( $\text{m}^2/\text{s}$ )
- $\mu$ : Fluid Viscosity ( $\text{kg}/\text{m s}$ )
- $E$ : Young's Modulus ( $\text{kg}/\text{m s}^2$ )
- $L$ : Fracture Length (m)

### Output(s)

- $W_w$ : Fracture width at the Wellbore (m)

### Formula(s)

$$\frac{W_w}{L} \approx \left( \frac{\mu i}{E h L^2} \right)^{0.25}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 29.*

## 4.38 Fracture gradient (hydraulic fracturing)

### Input(s)

- $\Delta P_f$ : Pressure Loss Sourced by Friction (psi)
- $\Delta P_p$ : Pressure Loss sourced by Perforations (psi)
- $\Delta P_h$ : Hydrostatic Pressure Change (psi)
- $P_{inf}$ : Injection Pressure (psi)

**Output(s)**

$G_f$ : Fracture Gradient (psi/ft)

**Formula(s)**

$$G_f = \frac{P_{inj} + \Delta P_h - \Delta P_f - \Delta P_p}{D}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 25.*

**4.39 Fracture-fluid invasion of the formation (acidizing)****Input(s)**

$\Delta P_v$ : Pressure difference between the fracture wall and the interface between the penetrating fracture fluid-formation fluids (psi)

$\mu$ : Viscosity (cP)

$\emptyset$ : Porosity (per cent)

$k$ : Permeability (Darcy)

$t$ : Time (min)

**Output(s)**

$v_N$ : Fluid Loss Velocity (ft/min)

**Formula(s)**

$$v_N = 0.0374 \sqrt{\frac{k\emptyset(\Delta P_v)}{\mu t}}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 41.*

**4.40 Frictional pressure drop (Economides and Nolte)****Input(s)**

$q$ : Injection Rate (bbl/min)

$\mu$ : Fluid Viscosity (cP)

$\rho$ : Density of Fluid (g/cc)

$D$ : Tubing Diameter (in.)

$L$ : Tubing Length (ft)

**Output(s)**

$\Delta P_f$ : Frictional Pressure Drop (psi)

**Formula(s)**

$$\Delta P_f = 518 * \rho^{0.79} * q^{1.79} * \mu^{0.207} * \frac{L}{1000 * D^{4.79}}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 16/246.

**4.41 Gas velocity under sonic flow conditions (through choke)****Input(s)**

- $K_s$ : Empirical Gas Constant (ft/s)
- $\rho_l$ : Liquid Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)

**Output(s)**

- v: Allowable Gas Velocity (ft/s)

**Formula(s)**

$$v = K_s * \left( \frac{\rho_l - \rho_g}{\rho_g} \right)^{0.5}$$

Reference: John M. Campbell, 1992. *Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma*, Vol. 2, Page: 73.

**4.42 Hydraulic fracture efficiency****Input(s)**

- x: Fracture Length (ft)
- t: Injection Time (min)
- C: Fracture Liquid Coefficient ( $m/\sqrt{\text{min}}$ )

**Output(s)**

- $\eta$ : Hydraulic Fracture Efficiency (%)

**Formula(s)**

$$\eta = \left[ \frac{1}{x^2} \right] \cdot \left[ e^{x^2} \operatorname{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right]$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$$

$$x = 2C \frac{\sqrt{\pi t}}{W}$$

Reference: Saydam, T., (1967). *Principles of Hydraulic Fracturing*, ARI Publishing Co., Page: 23.

## 4.43 Hydraulic horse power for a hydraulic fracturing operation

### Input(s)

$P_{inj}$ : Injection Pressure (psi)  
 $q_t$ : Injection Rate (bbl/day)

### Output(s)

$H_h$ : Hydraulic Horse Power (hp)

### Formula(s)

$$H_h = 0.0245 \cdot P_{inj} \cdot q_t$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 24.*

## 4.44 Ideal fracture conductivity created by acid reaction (acidizing)

### Input(s)

$h$ : Fracture Height (ft)  
 $i$ : Injection rate ( $\text{ft}^3/\text{min}$ )  
 $t$ : Time (min)  
 $xL$ : Acid Penetration Distance (ft)  
 $X$ : Acid Dissolving Power ( $\text{ft}^3/\text{ft}^3$ )  
 $\emptyset$ : Porosity (per cent)

### Output(s)

$w_a$ : Opened Channel of (Dissolved) Width (ft)

### Formula(s)

$$w_a = \frac{X \cdot i \cdot t}{2xL \cdot h(1 - \emptyset)}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 49.*

## 4.45 Incremental density in any wellbore interval (completion and workover fluids)

### Input(s)

$g_p$ : Pressure Gradient (psi/ft)  
 $\Delta D$ : Depth Interval (ft)  
 $B$ : Pressure Compressibility Coefficient (lbm/gal/1000 psi)  
 $A$ : Thermal Expansion Coefficient ( $^{\circ}\text{F}^{-1}$ )  
 $g_T$ : Temperature Gradient ( $^{\circ}\text{F}/\text{ft}$ )

### Output(s)

$\Delta\rho_i$ : Incremental Wellbore Fluid Density Change (lbm/bbl)

**Formula(s)**

$$\Delta\rho_i = (B) \cdot (g_p) \cdot (\Delta D) - (A) \cdot (g_T) \cdot (\Delta D)$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 46.*

**4.46 Initial rate following a hydraulic fracturing operation****Input(s)**

- $Q_{pre}$ : Rate before the Hydraulic Fracturing Operation (bbl/d)
- $r_e$ : Drainage Radius of the Well (in.)
- $r_w$ : Hole Radius of the Well (in.)
- $r_f$ : Radius of the Fracture (in.)
- $k_f$ : Fracture Permeability (mD)
- $k_e$ : Average Formation Permeability (mD)

**Output(s)**

- $Q$ : Initial Rate Following a Hydraulic Fracturing Operation (bbl/d)

**Formula(s)**

$$Q = Q_{pre} \left[ \frac{\left( k_f / k_e \right) \log \left( r_e / r_w \right)}{\log \frac{r_f}{r_w} + \left( \frac{k_f}{k_e} \cdot \log \frac{r_e}{r_f} \right)} \right]$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 84.*

**4.47 Injection pressure for hydraulic fracturing****Input(s)**

- $P_F$ : Downhole Fracturing Operation Pressure (psi)
- $\Delta P_f$ : Pressure Loss Sourced by Friction (psi)
- $\Delta P_p$ : Pressure Loss sourced by Perforations (psi)
- $\Delta P_h$ : Hydrostatic Pressure Change (psi)

**Output(s)**

- $P_{inj}$ : Injection Pressure (psi)

**Formula(s)**

$$P_{inj} = P_F + \Delta P_f + \Delta P_p - \Delta P_h$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 24.*

## 4.48 Lifetime of a hydraulically fractured well

### Input(s)

- $m$ : Slope of the Production Decline Curve (bbl/day)  
 $Q_a$ : Production Rate just before the Abandonment of the Well (bbl/d)  
 $Q_0$ : Initial Production Rate following the Hydraulic Fracturing Operation (bbl/d)

### Output(s)

- $t$ : Lifetime of a Hydraulically Fractured Well (days)

### Formula(s)

$$t = \ln\left(\frac{Q_a}{Q_0}\right) \cdot \frac{1}{|m|}$$

Reference: Saydam, T., (1967). *Principles of Hydraulic Fracturing*, ARI Publishing Co., Page: 86.

## 4.49 Mass of rock dissolved per unit mass of acid (acidizing)

### Input(s)

- $M_{wm}$ : Molecular Weight of Mineral (Rock) (g/mol)  
 $M_{wa}$ : Molecular Weight of Acid (g/mol)  
 $A$ : Stoichiometric Coefficient of Mineral (Rock) (Number of Molecules)  
 $B$ : Stoichiometric Coefficient of Acid (Number of Molecules)

### Output(s)

- $\beta$ : Acid Dissolving Power (g/g)

### Formula(s)

$$\beta = \frac{M_{wm} \times A}{M_{wa} \times B}$$

Reference: Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). *Acidizing Fundamentals*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 13.

## 4.50 Mass transfer in acid solutions by Fick's law (acidizing)

### Input(s)

- $D_A$ : Diffusion Coefficient of Component A ( $\text{cm}^2/\text{s}$ )  
 $\partial c_a / \partial Y$ : Concentration Gradient ( $\text{mol}/\text{cm}^4$ )

### Output(s)

- $U_{a,y}$ : Diffusion Flux of a Component A in Y direction ( $\text{mol}/\text{cm}^2 \text{ s}$ )

**Formula(s)**

$$U_{a,y} = -D_A \frac{\partial c_a}{\partial Y}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing Fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 21.*

**4.51 Maximum treatment pressure (hydraulic fracturing)****Input(s)**

- $P_b$ : Formation Breakdown Pressure (psi)
- $\Delta P_h$ : Hydrostatic Pressure Drop (psi)
- $\Delta P_f$ : Frictional Pressure Drop (psi)

**Output(s)**

- $P_{si}$ : Surface Injection Pressure (psi)

**Formula(s)**

$$P_{si} = P_b - \Delta P_h + \Delta P_f$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 17/259.*

**4.52 Mechanical resistant torque (PCP)****Input(s)**

- $V_o$ : Pump Displacement ( $\text{ft}^3$ )
- $\Delta P$ : Pump Head Rating (psi)
- $e_p$ : Efficiency (fraction)

**Output(s)**

- $T_m$ : Surface Injection Pressure (psi)

**Formula(s)**

$$T_m = 144 * V_o * \frac{\Delta P}{e_p}$$

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 14/214.*

**4.53 Minimum polished rod load (sucker rod pump)****Input(s)**

- C: Dynamometer Calibration Constant (lb/in.)
- d: Minimum Deflection (in.)

**Output(s)**

MPRL: Minimum Polished Rod Load (lb)

**Formula(s)**

$$MPRL = C * d$$

*Notes:* Based on Dynamometer Card.

Reference: *Boyun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18/269.*

**4.54 Peclet number for fluid loss (acidizing)****Input(s)**

$\bar{w}$ : Average Fracture Width (ft)

$\bar{v}_N$ : Average Fluid Loss Velocity (ft/min)

$D_e^\infty$ : Effective Diffusivity (ft<sup>2</sup>/min)

**Output(s)**

$N_{Pe^*}$ : Peclet Number (dimensionless)

**Formula(s)**

$$N_{Pe^*} = \frac{\bar{w} \bar{v}_N}{2 D_e^\infty}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 62.*

**4.55 Perforation friction factor****Input(s)**

$q_t$ : Total Flow Rate (bbl/min)

$n$ : Number of Perforations (dimensionless)

$d$ : Casing Entrance-Hole Diameter (in.)

$C_d$ : Discharge Coefficient (dimensionless)

$\rho_o$ : Oil Density (lbf/in.<sup>2</sup>)

**Output(s)**

$p_f$ : Perforation Friction (Differential Pressure across Perforations) (psi)

**Formula(s)**

$$p_f = \frac{0.2369 q_t^2 \rho_o}{n^2 d^4 C_d^2}$$

Reference: *Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). Perforating. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 79.*

## 4.56 Perforation friction pressure

### Input(s)

- Q: Flow Rate ( $\text{m}^3/\text{min}$ )
- $\rho$ : Fluid Density ( $\text{kg}/\text{L}$ )
- n: Number of Perforations (unitless)
- $D_p$ : Diameter of Perforation (cm)
- C: Perforation Coefficient (0.6–1.0) (unitless)

### Output(s)

- $\Delta P_{pf}$ : Perforation Friction Pressure (MPa)

### Formula(s)

$$\Delta P_{pf} = 22.335 * \frac{Q^2 * \rho}{n^2 * C^2 * D_p^4}$$

Reference: Daneshy, A. 2013. *Fundamentals of Hydraulic Fracturing*, Daneshy Consultants International, Page: 93.

## 4.57 Perforation hole size (perforation)

### Input(s)

- x: Brinell Hardness of Casing of Interest (dimensionless)
- $d_r$ : Entrance Hole Diameter in Reference Casing (in.)
- $x_r$ : Brinell Hardness of Reference Casing (dimensionless)

### Output(s)

- $d$ : Entrance Hole Diameter in Casing of Interest (in.)

### Formula(s)

$$d = \left( \frac{[2,250 + 4.2x_r]}{[2,250 + 4.2x]} \right)^{0.5} d_r$$

Reference: Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). *Perforating*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers., Page: 50.

## 4.58 Perforation length in formation

### Input(s)

- $L_{pr}$ : Total Target Penetration in Reference Target (in.)
- $d_{wb}$ : Wellbore Diameter (in.)
- $d_{ci}$ : Casing Inner Diameter (in.)

### Output(s)

- $L_p$ : Total Target Penetration in Formation of Interest (in.)

**Formula(s)**

$$L_p = (L_{pc}) - 0.5(d_{wb} - d_{ci})$$

Reference: *Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). Perforating. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 52.*

**4.59 Perforation penetration ratio (formation of interest/reference formation)****Input(s)**

- $c_r$ : Compressive Strength of reference Formation (ksi)
- $c$ : Compressive Strength of Formation of Interest (ksi)

**Output(s)**

- $L_p$ : Total Target Penetration in Formation of Interest (in.)
- $L_{pr}$ : Total Target Penetration in Reference Formation (in.)

**Formula(s)**

$$\frac{L_p}{L_{pr}} = \exp [0.086(c_r - c)]$$

Reference: *Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). Perforating. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 51.*

**4.60 Perforation skin factor****Input(s)**

- $s_H$ : Horizontal or Plane Flow Skin (dimensionless)
- $s_v$ : Vertical or Converging Flow Skin (dimensionless)
- $s_{wb}$ : Skin resulted by the Wellbore Effect (dimensionless)
- $s_{pd}$ : Skin caused by the Damaged Zone around Perforation (dimensionless)

**Output(s)**

- $s_p$ : Perforation Skin (dimensionless)

**Formula(s)**

$$s_p = s_H + s_v + s_{wb} + s_{pd}$$

Reference: *Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). Perforating. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 65.*

## 4.61 Pore growth function (acidizing)

### Input(s)

- $r_{ave}$ : Average Wall Reaction Rate taken over the entire Reactive Surface Area (mol/cm<sup>2</sup> s)  
 $\beta$ : Acid gravimetric dissolving power (mass rock/mass acid)  
 $\rho_{ma}$ : Formation Matrix Density (g/cm<sup>3</sup>)  
 $\Gamma$ : Perimeter of the Pore ( $\mu\text{m}$ )  
 $A$ : Cross-sectional Area of Pore (cm<sup>2</sup>)  
 $l$ : Length of Pore ( $\mu\text{m}$ )  
 $t$ : time (s)

### Output(s)

- $\psi$ : Pore Growth Function (cm<sup>2</sup>/s)

### Formula(s)

$$\psi = \frac{r_{ave} \beta \Gamma}{\rho_{ma}}$$

or

$$\rho_{ma} l \frac{dA}{dt} = r_{ave} \beta \Gamma l$$

Reference: Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 69.

## 4.62 Pressure drop across perforations in gas wells

### Input(s)

- $q_g$ : Gas Flow Rate through Perforation (bbl/d)  
 $n$ : Number of Perforations (dimensionless)  
 $\mu_g$ : Gas Viscosity (cP)  
 $Z$ : Gas Supercompressibility Factor (dimensionless)  
 $T$ : Formation Temperature (R°)  
 $L_p$ : Perforation Length (ft)  
 $k_{pd}$ : Perforation Damaged Zone Permeability (mD)  
 $r_p$ : Perforation Radius (ft)  
 $r_{pd}$ : Perforation Damaged Zone Radius (ft)  
 $\gamma_g$ : Gas Specific Gravity (air = 1.0)

### Output(s)

- $p_{sf}$ : Pressure at the Sandface (psi)  
 $p_{wb}$ : Pressure in the Wellbore (psi)  
 $\beta_{pd}$ : Velocity Coefficient of Turbulence Factor (1/ft)

**Formula(s)**

$$\begin{aligned}
 p_{sf}^2 - p_{wb}^2 &= \left( A \cdot \left( \frac{q_g}{n} \right) \right) + \left( B \cdot \left( \frac{q_g}{n} \right)^2 \right) \\
 A &= \frac{1.424 \cdot 10^3 \cdot \mu_g(Z)(T)}{L_p k_{pd}} \ln \left( \frac{r_{pd}}{r_p} \right) \\
 B &= \frac{(3.16)(10^{-12})\beta_{pd}\gamma_g(Z)(T)}{L_p^2} \left( \frac{1}{r_p} - \frac{1}{r_{pd}} \right) \\
 \beta_{pd} &= (2.33)(10^{10}) k^{-1.201}
 \end{aligned}$$

Reference: Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). *Perforating*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 62.

**4.63 Pressure drop across perforations in oil wells****Input(s)**

- $q_o$ : Oil Flow Rate through Perforation (bbl/d)
- $n$ : Number of Perforations (dimensionless)
- $\mu_o$ : Oil Viscosity (cP)
- $B_o$ : Oil Formation Volume Factor (dimensionless)
- $L_p$ : Perforation Length (ft)
- $k_{pd}$ : Perforation Damaged Zone Permeability (mD)
- $r_p$ : Perforation Radius (ft)
- $r_{pd}$ : Perforation Damaged Zone Radius (ft)
- $\rho_o$ : Oil Density (lbm/ft<sup>3</sup>)

**Output(s)**

- $\Delta p_p$ : Pressure Drop across Perforations (psi)
- $\beta_{pd}$ : Velocity Coefficient of Turbulence Factor (1/ft)

**Formula(s)**

$$\begin{aligned}
 \Delta p_p &= \left( A \cdot \left( \frac{q_o}{n} \right) \right) + \left( B \cdot \left( \frac{q_o}{n} \right)^2 \right) \\
 A &= \frac{141.2 \cdot \mu_o B_o}{L_p k_{pd}} \ln \left( \frac{r_{pd}}{r_p} \right) \\
 B &= \frac{2.3(10^{-14})\beta_{pd}B_o^2\rho_o}{L_p^2} \left( \frac{1}{r_p} - \frac{1}{r_{pd}} \right) \\
 \beta_{pd} &= \frac{(2.33)(10^{10})}{k_{pd}^{1.201}}
 \end{aligned}$$

Reference: Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). *Perforating*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 61.

## 4.64 Pressure loss due to perforations during hydraulic fracturing

### Input(s)

- $\rho$ : Specific Gravity ( $\text{lb}/\text{ft}^3$ )
- $q$ : Real Flowing Rate ( $\text{ft}^3/\text{s}$ )
- $g_c$ : Unit Conversion Factor ( $32.2 \ (\text{lb}\cdot\text{ft})/(\text{lbf s}^2)$ )
- $A_2$ : Fracture Area in time “t” ( $\text{ft}^2$ )

### Output(s)

- $\Delta P_p$ : Pressure Loss in Sourced by Perforations during Hydraulic Fracturing (psi)

### Formula(s)

$$\Delta P_p = \frac{\rho \cdot q^2}{1.345 \cdot g_c \cdot A_2^2}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 28.*

## 4.65 Pressure loss due to perforations during hydraulic fracturing—2

### Input(s)

- $\rho$ : Specific Gravity ( $\text{lb}/\text{gal}$ )
- $q$ : Real Flowing Rate ( $\text{gal}/\text{min}$ )
- $A_2$ : Fracture Area in time “t” ( $\text{in}^2$ )

### Output(s)

- $\Delta P_p$ : Pressure Loss in Sourced by Perforations during Hydraulic Fracturing (psi)

### Formula(s)

$$\Delta P_p = \frac{\rho \cdot q^2}{8090 \cdot A_2^2}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 30.*

## 4.66 Principal stress due to petro-static pressure (hydraulic fracturing)

### Input(s)

- $\gamma$ : Average Specific Weight of Formation ( $\text{p/cm}^3$ )
- $h$ : Depth (m)

**Output(s)**

$\sigma_z$ : Principal Stress in z-axis (kp/cm<sup>2</sup>)

**Formula(s)**

$$\sigma_z = \frac{\gamma \cdot h}{10}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 5.*

**4.67 Productivity index (for generating composite IPR curve)****Input(s)**

q: Tested Flow Rate (STB/D)

$P_b$ : Bubble Point Pressure (psi)

$P_w$ : Wellbore Pressure (psi)

P: Reservoir Pressure (psi)

**Output(s)**

$J_i$ : Productivity Index (STB/D/psi)

**Formula(s)**

$$J_i = \frac{q}{(P - P_b) + \left(\frac{P_b}{1.8}\right) \left(1 - 0.2 * \left(\frac{P_w}{P}\right) - 0.8 * \left(\frac{P_w}{P}\right)^2\right)}$$

Reference: *Boyun, G., William, C., & Ali Ghalmor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 3/38.*

**4.68 Productivity ratio****Input(s)**

J: Productivity Index of a well in any condition (fraction)

$J_{sw}$ : Productivity index of a standard well (fraction)

**Output(s)**

PR: Productivity Ratio (fraction)

**Formula(s)**

$$PR = \frac{J}{J_{sw}}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 46.

**4.69 Productivity ratio calculation of a hydraulically-fractured formation****Input(s)**

- $k_{avgz}$ : Average Permeability of a Hydraulically Fractured Zone (mD)
- $k_f$ : Permeability of the Fracture (mD)
- $k$ : Permeability of the Formation (mD)
- $h$ : Formation Thickness (ft)
- $W$ : Fracture Thickness (ft)
- $r_e$ : Drainage Radius of the Well (in.)
- $r_w$ : Radius of the Well (in.)
- $r_f$ : Radius of the Fracture (in.)
- $k_{avg}$ : Average Permeability of a Hydraulically Fractured Formation (mD)

**Output(s)**

- $PR$ : Productivity Ratio of a Hydraulically Fractured Formation (mD)

**Formula(s)**

$$PR = \frac{k_{avg}}{k}$$

or,

$$PR = \left[ \frac{k_f \cdot W}{k \cdot h} \right] \cdot \frac{\left[ \frac{k \cdot h}{k_f \cdot W} + 1 \right] \cdot \ln\left(\frac{r_e}{r_w}\right)}{\left[ \frac{k_f \cdot W}{k \cdot h} + 1 \right] \cdot \ln\left(\frac{r_e}{r_f}\right) + \ln\left(\frac{r_f}{r_w}\right)}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 7.*

**4.70 Pseudo skin factor due to partial penetration (Brons and Marting method)****Input(s)**

- $h_p$ : Perforation interval (ft)
- $h$ : Total pay thickness (ft)
- $kh$ : Vertical permeability (mD)
- $kv$ : Horizontal permeability (mD)
- $rw$ : Wellbore diameter (ft)

**Output(s)**

- $h_d$ : Dimensionless Payzone Thickness (dimensionless)  
**b**: Penetration Ratio (dimensionless)  
**Gb**: Function for penetration ratio (dimensionless)  
 $s_p$ : Skin due to partial penetration (dimensionless)

**Formula(s)**

$$h_d = \frac{h_p}{h}$$

$$b = \frac{h}{rw} * \left( \frac{kh}{kv} \right)^{0.5}$$

$$Gb = 2.948 - 7.363 * b + 11.45 * b^2 - 4.675 * b^3$$

$$s_p = \left( \frac{1}{b} - 1 \right) * (\ln(h_d) - Gb)$$

Reference: *Horizontal Well Technology*, Joshi, Page: 493.

### 4.71 Pseudo-skin factor due to partial penetration (Yeh and Reynolds correlation)

**Input(s)**

- $h_p$ : Perforation Interval (ft)  
**h**: Total pay thickness (ft)  
 $C_d$ : Location of open interval from graphical co-relation by Yeh and Ronald (dimensionless)  
 $k_v$ : Horizontal Permeability (mD)  
 $k_h$ : Vertical Permeability (mD)  
 $r_w$ : Wellbore Dia (ft)

**Output(s)**

- b**: Penetration ratio (dimensionless)  
 $C_1$ : Location of open interval from graphical co-relation by Yeh and Ronald (dimensionless)  
 $h_d$ : Dimensionless payzone thickness (dimensionless)  
 $h_{\{wd\}}$ : Effective Wellbore depth (ft)  
 $s_p$ : Skin due to partial penetration (dimensionless)

**Formula(s)**

$$b = \frac{h_p}{h}$$

$$C_1 = \frac{h}{r_w} * \sqrt{\left( \frac{k_h}{k_v} \right)}$$

$$h_d = 0.481 + 1.01 * b - 0.838 * b^2$$

$$h_{wd} = C_d * b * (1 - b) * \frac{h_d}{2.71^C_1}$$

$$s_p = \frac{1 - b}{b} * \ln(h_{\{wd\}})$$

Reference: *Horizontal Well Technology*, Joshi, Page: 496.

## 4.72 Pseudo-skin factor due to partial penetration (Odeh correlation)

### Input(s)

- $h_1$ : Distance from payzone top to top of open interval (ft)
- $h_p$ : Perforation interval (ft)
- $h$ : Total pay thickness (ft)
- $r_w$ : Wellbore Dia (ft)
- $k_v$ : Horizontal Permeability (mD)
- $k_h$ : Vertical Permeability (mD)

### Output(s)

- $z_m$ : Well Perforation Distance (ft)
- $h_d$ : Dimensionless Payzone Thickness (dimensionless)
- $b$ : Penetration Ratio (dimensionless)
- $r_{wc}$ : Effective Wellbore dia (ft)
- $s_p$ : Skin due to partial penetration (dimensionless)

### Formula(s)

$$z_m = \frac{h_p}{h}$$

$$h_d = \frac{h}{r_w} * \sqrt{\left(\frac{k_h}{k_v}\right)}$$

$$b = h_1 + \frac{h_p}{2}$$

$$r_{wc} = r_w * 2.71^{0.2126 * \left(2.753 + \frac{z_m}{h}\right)}$$

$$s_p = 1.35 * \left(\frac{1}{b} - 1\right)^{0.825} * \left( \ln(r_w * h_d + 7) - 1.95 - \ln(r_{\{wc\}}) * (0.49 + 0.1 * \ln(r_w * h_d)) \right)$$

Reference: *Horizontal Well Technology*, Joshi, Page: 495.

## 4.73 Pseudo-skin factor due to partial penetration (Papatzacos correlation)

### Input(s)

- $h_p$ : Perforation interval (ft)
- $h$ : Total pay thickness (ft)
- $h_1$ : Distance from top of reservoir to top of open interval (ft)

- $k_v$ : Horizontal permeability (mD)
- $k_h$ : Vertical permeability (mD)
- $r_w$ : Wellbore dia (ft)

### Output(s)

- b: Penetration ratio (dimensionless)
- A: Constant for calc (dimensionless)
- B: Constant for calc (dimensionless)
- $h_d$ : Dimensionless payzone thickness (dimensionless)
- $s_p$ : Skin due to partial penetration (dimensionless)

### Formula(s)

$$\begin{aligned} b &= \frac{h_p}{h} \\ A &= \frac{h}{r_w} * \sqrt{\left(\frac{k_h}{k_v}\right)} \\ B &= \frac{h}{h_1 + 0.25 * h_p} \\ h_d &= \frac{h}{h_1 + 0.75 * h_p} \\ s_p &= \frac{1-b}{b} * \ln\left(\pi * \frac{h_d}{2}\right) + \frac{1}{b} * \ln\left(\frac{b}{b+2} * \sqrt{\left(\frac{A-1}{B-1}\right)}\right) \end{aligned}$$

Reference: *Horizontal Well Technology*, Joshi, Page: 498.

## 4.74 Pseudo-skin factor due to perforations

### Input(s)

- $h_p$ : Perforation interval (ft)
- $h$ : Total pay thickness (ft)
- $h_1$ : Distance from top of reservoir to top of open interval (ft)
- $k_v$ : Horizontal permeability (mD)
- $k_h$ : Vertical permeability (mD)
- $r_w$ : Wellbore dia (ft)

### Output(s)

- b: Penetration ratio (dimensionless)
- A: Constant for calc (dimensionless)
- B: Constant for calc (dimensionless)
- $h_d$ : Dimensionless payzone thickness (dimensionless)
- $s_p$ : Skin due to partial penetration (dimensionless)

### Formula(s)

$$b = \frac{h_p}{h}$$

$$A = \frac{h}{r_w} * \sqrt{\left(\frac{k_h}{k_v}\right)}$$

$$B = \frac{h}{h_1 + 0.25 * h_p}$$

$$h_d = \frac{h}{h_1 + 0.75 * h_p}$$

$$s_p = \frac{1-b}{b} * \ln\left(\pi * \frac{h_d}{2}\right) + \frac{1}{b} * \ln\left(\frac{b}{b+2} * \sqrt{\left(\frac{A-1}{B-1}\right)}\right)$$

Reference: *Horizontal Well Technology*, Joshi, Page: 499.

## 4.75 Quantifying formation damage and improvement

### Input(s)

- $p_{wf}$ : Pressure of Wellflow (psi)
- $p$ : Pressure of Reservoir (psi)
- $s$ : Skin Factor (dimensionless)
- $r_s$ : Radius of skin (ft)
- $r_w$ : Radius of Wellbore (ft)
- $q$ : Flow Rate (STB/day)
- $k$ : Permeability of Formation (mD)
- $\mu$ : Viscosity of Oil (cP)
- $B$ : Volume factor (RB/STB)
- $h$ : Thickness of reservoir (ft)

### Output(s)

- $k_s$ : Permeability of altered zone (mD)
- $r_{wa}$ : Effective Radius of Wellbore (ft)
- $E$ : Flow Efficiency (fraction)

### Formula(s)

$$k_s = \frac{k}{1 + \frac{s}{\ln\left(\frac{r_s}{r_w}\right)}}$$

$$r_{wa} = r_w * 2.71828^{-s}$$

$$E = \frac{p - p_{wf} - \left(141.2 * q * B * \mu * \frac{s}{k * h}\right)}{p - p_{wf}}$$

Reference: *Pressure Transient Testing*, Lee, Rollins & Spivey, Page: 43.

## 4.76 Recommended underbalanced environment for perforation

### Input(s)

$k$ : Formation Permeability (mD)

### Output(s)

$p_u$ : Underbalance Pressure (psi)

### Formula(s)

$$\log_{10} p_u = 3.46055 - 0.3812 \log_{10} k$$

Reference: Bell, W. T., Sukup, R. A., & Tariq, S. M. (1995). *Perforating*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, Page: 74.

## 4.77 Reynolds number for acid flow into the fracture (acidizing)

### Input(s)

$\rho$ : Acid Density (lb/ft<sup>3</sup>)

$i$ : Injection Rate (ft<sup>3</sup>/min)

$\mu$ : Viscosity of the Reacted Acid (lb/ft min)

$h_g$ : Formation Thickness (ft)

### Output(s)

$N_{Re}$ : Reynolds Number (dimensionless)

### Formula(s)

$$N_{Re} = \frac{\rho i}{\mu h_g}$$

Reference: Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). *Acidizing Fundamentals*. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 62.

## 4.78 Reynolds number for fluid loss (acidizing)

### Input(s)

$\bar{w}$ : Average Fracture Width (ft)

$\bar{v}_N$ : Average Fluid Loss Velocity (ft/min)

$\rho$ : Acid Density (lb/ft<sup>3</sup>)

$\mu$ : Viscosity of the Reacted Acid (lb/ft min)

$r_c$ : Radius of Wormhole (ft)

### Output(s)

$N_{Re}$ : Reynolds Number (dimensionless)

### Formula(s)

For Fracture:

$$N_{Re} = \frac{2\bar{w}\bar{v}_N\rho}{\mu} \text{ For Wormhole :}$$

$$N_{Re} = \frac{2r_c\bar{v}_N\rho}{\mu}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 111.*

## 4.79 Sand weight needed to refill a hydraulically fractured reservoir volume

### Input(s)

$V$ : Volume of Unit Area of the Fracture ( $\text{ft}^3/\text{ft}^2$ )

$\emptyset$ : Porosity (fraction)

$\rho_{sand}$ : Density of the Sand ( $\text{lb}/\text{ft}^3$ )

### Output(s)

$S$ : Sand Weight needed to Refill the Hydraulically Fractured Reservoir Volume ( $\text{lb}/\text{ft}^2$ )

### Formula(s)

$$S = V \cdot (1 - \emptyset) \cdot \rho_{sand}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 98.*

## 4.80 Shape factor expressed as skin factor for vertical wells

### Input(s)

$C_A$ : Shape Factor (dimensionless)

### Output(s)

$s_{CA}$ : Shape Related Skin Factor (dimensionless)

### Formula(s)

$$s_{CA} = \ln \left( \left( \frac{31.62}{C_A} \right)^{0.5} \right)$$

Reference: *Joshi, S. D. 1991, Horizontal Well Technology. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 7, Page: 208.*

## 4.81 Single-phase gas flow (subsonic)

### Input(s)

$P_d$ : Downstream Pressure (psi)

$P_u$ : Upstream Pressure at Choke (psi)

- k: Specific Heat Ratio (dimensionless) A: Cross Sectional Area of Choke (in.<sup>2</sup>)  
 $\gamma_g$ : Gas Specific Gravity related to Air (dimensionless)  
 $T_u$ : Upstream Temperature (R)  
 $C_d$ : Choke Discharge Coefficient (dimensionless)

**Output(s)**

$q_{sc}$ : Gas Flow Rate (Mscf/D)

**Formula(s)**

$$q_{sc} = 1248 * C_d * A * P_u * \left( \left( \frac{k}{(k-1)*\gamma_g*T_u} \right) * \left( \left( \frac{P_d}{P_u} \right)^{\frac{2}{k}} - \left( \frac{P_d}{P_u} \right)^{\frac{k+1}{k}} \right) \right)^{0.5}$$

Reference: Boyun, G., William, C., & Ali Ghalmab, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 5/61.

**4.82 Single-phase liquid flow through choke****Input(s)**

- $C_d$ : Upstream Pressure at Choke (psi)  
A: Choke Area (ft<sup>2</sup>)  
 $g_c$ : Unit Conversion Factor (Constant = 32.17) (lb ft/ lbf s<sup>2</sup>)  
 $\Delta P$ : Pressure Drop (lbf/ft<sup>2</sup>)  
 $\rho$ : Fluid Density (lbm/ft<sup>3</sup>)

**Output(s)**

$q$ : Flow Rate (ft<sup>3</sup>/s)

**Formula(s)**

$$q = C_d * A * \left( 2 * g_c * \frac{\Delta P}{\rho} \right)^{0.5}$$

Reference: Boyun, G., William, C., & Ali Ghalmab, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 5.

**4.83 Skin factor****Input(s)**

- $k_s$ : Permeability of damaged zone (mD)  
 $r_s$ : Radius of damaged zone (ft)  
 $r_w$ : Radius of wellbore (ft)  
k: Permeability (mD)

**Output(s)**

s: Skin (dimensionless)

**Formula(s)**

$$s = \left( \frac{k}{k_s} - 1 \right) * \ln \left( \frac{r_s}{r_w} \right)$$

Reference: *Fundamental Principles of Reservoir Engineering*, Towler, Page: 62.

**4.84 Skin factor by Hawkins method****Input(s)**

- $k$ : Permeability of Reservoir (mD)
- $k_s$ : Permeability of Skin Zone Near Wellbore (mD)
- $r_w$ : Radius of Wellbore (ft)
- $r_s$ : Radius of Skin Zone (ft)

**Output(s)**

- $S$ : Skin Factor (dimensionless)

**Formula(s)**

$$S = \left( \left( \frac{k}{k_s} \right) - 1 \right) * \ln \left( \frac{r_s}{r_w} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 37.

**4.85 Skin factor due to partial penetration****Input(s)**

- $h_t$ : Total Formation Thickness (ft)
- $r_w$ : Radius of Wellbore (ft)
- $k_v$ : Vertical Permeability (mD)
- $k_h$ : Horizontal Permeability (mD)
- $h_p$ : Height of Perforated interval (ft)
- $h_a$ : Height (ft)
- $h_{pd}$ : Dimensionless Perforated Thickness (fraction)
- $h$ : Thickness of reservoir (ft)

**Output(s)**

- $r_d$ : Permeability of altered zone (mD)
- $A$ : Dimensionless Constant A (dimensionless)
- $B$ : Dimensionless Constant B (dimensionless)
- $s_p$ : Skin Factor from partial penetration (dimensionless)

**Formula(s)**

$$r_d = \left( \frac{r_w}{h_t} \right) * \left( \frac{k_v}{k_h} \right)^{0.5}$$

$$A = \frac{h_p}{h_t}$$

$$B = \frac{h_a}{h_t}$$

$$s_p = \left( \frac{1}{h_{pd}} - 1 \right) * 2.303 * \log \left( 0.5 * \frac{3.142}{\left( \frac{rw}{h_t} \right) * \left( \frac{k_v}{k_h} \right)^{0.5}} \right) + \frac{1}{h_{pd}} * \ln \left( \frac{h_{pd}}{2 + h_{pd}} * \left( \left( \frac{\left( \frac{h_p}{h_t} \right) - 1}{\left( \frac{h_a}{h_t} \right) - 1} \right)^{0.5} \right) \right)$$

Reference: *Pressure Transient Testing, Lee, Rollins & Spivey, Page: 44.*

## 4.86 Skin factor due to reduced crushed-zone permeability

### Input(s)

- $h_p$ : Perforation interval (ft)
- $k_d$ : Damage Zone permeability (mD)
- $k_{dp}$ : Crushed Zone Permeability (mD)
- $k$ : Permeability of Formation (mD)
- $r_{dp}$ : Crushed Zone radius (ft)
- $r_p$ : Perforation Radius (ft)
- $L_p$ : Depth of Penetration (ft)
- N: Total Number of Perforations (number)

### Output(s)

- sc: Skin Factor due to Reduced Crushed-Zone Permeability (dimensionless)

### Formula(s)

$$sc = \left( \frac{k}{k_{dp}} - \frac{k}{k_d} \right) * 12 * \frac{h_p}{N * L_p} * \ln \left( \frac{r_{dp}}{r_p} \right)$$

Reference: *Horizontal Well Technology, Joshi, Page: 504.*

## 4.87 Skin factor for a deviated well

### Input(s)

- $\theta$ : Angle of Inclination (degree)
- $k_v$ : Vertical Permeability (mD)
- $k_h$ : Horizontal Permeability (mD)
- $h$ : Formation Thickness (ft)
- $rw$ : Radius of Wellbore (ft)
- s: Total Skin Factor (dimensionless)

### Output(s)

- $tw$ : Angle of Inclination due to drilling (degree)

- $h_d$ : Dimensionless Effective Thickness (dimensionless)  
 $s_{th}$ : Directional Drilling Skin Factor (dimensionless)  
 $s_t$ : Skin Factor other than Directional Drilling (dimensionless)

**Formula(s)**

$$tw = \left( \tan \left( \left( \frac{kv}{kh} \right)^{0.5} * \tan \left( \theta * \frac{\pi}{180} \right) \right) \right) * \frac{180}{\pi}$$

$$h_d = \left( \frac{h}{rw} \right) * \left( \frac{kh}{kv} \right)^{0.5}$$

$$s_{th} = - \left( \frac{tw}{41} \right)^{2.06} - \left( \frac{tw}{56} \right)^{1.865} * \log \left( \frac{h_d}{100} \right)$$

$$s_t = s - s_{th}$$

Reference: *Pressure Transient Testing, Lee, Rollins & Spivey, Page: 44.*

**4.88 Slope of Semilog plot for bottom-hole flowing pressure vs time for drawdown test****Input(s)**

- $Q_o$ : Oil Flow Rate (STB/day)  
 $\mu_o$ : Oil Viscosity (cP)  
 $B_o$ : Oil Formation Volume Factor (bbl/STB)  
 $k$ : Permeability (mD)  
 $h$ : Thickness (ft)

**Output(s)**

- m: Slope (psi/cycle)

**Formula(s)**

$$m = \frac{162.6 * Q_o * \mu_o * B_o}{k * h}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 45.*

**4.89 Sucker rod—Peak polished rod load****Input(s)**

- C: Calibration Constant of Dynamometer (lb/in.)  
D: Maximum Deflection (in.)

**Output(s)**

- PPRL: Peak Polished Rod Load (lb)

**Formula(s)**

$$PPRL = C * D$$

Reference: *Boyoun, G., William, C., & Ali Ghalambor, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach, Page: 18/269.*

**4.90 Suspension property of static fluids (completion and workover fluids)****Input(s)**

- $d$ : Particle Diameter ( $\mu\text{m}$ )
- $\rho_p$ : Density of Solid Particle ( $\text{g}/\text{cm}^3$ )
- $\rho_f$ : Density of Fluid ( $\text{g}/\text{cm}^3$ )
- $g$ : Acceleration Owing to Gravity ( $\text{ft}/\text{s}^2$ )
- $\mu$ : Fluid Viscosity ( $\text{cP}$ )

**Output(s)**

- $v$ : Particle Settling Velocity (in/s)

**Formula(s)**

$$v = \frac{d^2(\rho_p - \rho_f)g}{(\mu)(4.5 \cdot 10^6)}$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 49.*

**4.91 Tangential annular flow of a power law fluid****Input(s)**

- $m$ : Power Law Constant (dimensionless)
- $n$ : Power Law Constant 2 (dimensionless)
- $\Omega$ : Collision Integral (dimensionless)
- $\kappa$ : Dilatational Viscosity ( $\text{cP}$ )
- $R$ : Radius (ft)
- $L$ : Length (ft)

**Output(s)**

- $T_z$ : Torque Exerted ( $\text{lb ft}^2/\text{s}^2$ )

**Formula(s)**

$$T_z = 2 * \pi * m * \Omega * \left( (\kappa * R)^2 \right) * L * \left( \left( \frac{\frac{2}{n}}{1 - \left( \frac{2}{\kappa^n} \right)} \right)^n \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 244.*

## 4.92 Temperature at choke outlet

### Input(s)

- $T_u$ : Upstream Temperature (R)
- $z_u$ : Z Factor of Upstream Gas Flow (dimensionless)
- $z_o$ : Z Factor of Upstream Gas Flow in Outlet (dimensionless)
- $k$ : Specific Heat Ratio (dimensionless)
- $P_o$ : Outlet Pressure (psi)
- $P_u$ : Upstream Pressure at Choke (psi)

### Output(s)

- $T_{dn}$ : Downstream Temperature (R)

### Formula(s)

$$T_{dn} = T_u * \left( \frac{z_u}{z_o} \right) * \left( \frac{P_o}{P_u} \right)^{\frac{k-1}{k}}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 5/62.

## 4.93 The z component of the force of the fluid on the wetted surface of the pipe

### Input(s)

- $p_0$ : Pressure at initial point (Pa)
- $p_L$ : Pressure at point L (Pa)
- $R$ : Radius (m)
- $L$ : Length (m)
- $\rho$ : Density ( $\text{kg}/\text{m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m}/\text{s}^2$ )

### Output(s)

- $F_z$ : z-Component of Force (Newton)

### Formula(s)

$$F_z = \pi * R^2 * (p_0 - p_L) + \pi * R^2 * L * \rho * g$$

Reference: *Transport Phenomena*, Second Edition, Bird, Page: 51.

## 4.94 Total skin in partially depleted wells for a buildup test

### Input(s)

- $r_{ew}$ : Dimensionless Parameter in terms of Perforated Length and Wellbore Radius (dimensionless)
- $\phi$ : Porosity (fraction)
- $\mu$ : Viscosity (cP)
- $c_t$ : Total Compressibility (1/psi)
- $k$ : Permeability (mD)

- $p_s$ : Shut-in Pressure at any Shut-in Time (psi)  
 $p_w$ : Well Flowing Pressure at the Instant of Shut-in (psi)  
 $\Delta t$ : Shut-in Time (day)  
 $m$ : Slope of Cartesian Pressure Build-up Plot (psi/cycle)

**Output(s)**

S: Skin (dimensionless)

**Formula(s)**

$$S = 34.7 * r_{ew} * \left( \left( \phi * \mu * \frac{c}{k} \right)^{0.5} \right) * \left( \left( \frac{p_s - p_w}{m} \right) + \left( \frac{1}{(\Delta t)^{0.5}} \right) \right) - 1$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 81.

**4.95 Velocity distribution in the annular slit of a falling-cylinder viscometer****Input(s)**

- $\kappa$ : Ratio of inner Radius to Outer Radius (fraction)  
 $v_o$ : Velocity (cm/s)  
 $\xi$ : Radial Coordinate (dimensionless)

**Output(s)**

$v_z$ : Velocity Profile (cm/s)

**Formula(s)**

$$\frac{v_z}{v_o} = - \frac{(1 - \xi^2) - (1 + \kappa^2) * \ln\left(\frac{1}{\xi}\right)}{(1 - \kappa^2) - (1 + \kappa^2) * \ln\left(\frac{1}{\kappa}\right)}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 70.

**4.96 Velocity distribution in the narrow annular region in annular flow with inner cylinder moving axially****Input(s)**

- $\rho$ : Density (ppg)  
 $g$ : Acceleration due to Gravity (ft/s<sup>2</sup>)  
 $R$ : Radius (ft)  
 $r$ : Position from Point (ft)  
 $a$ : Ratio of Distance of Rod from Centre (fraction)  
 $\mu$ : Viscosity (cP)

**Output(s)**

$v_z$ : Velocity (ft/s)

**Formula(s)**

$$v_z = \left( \frac{\rho * g * (R^2)}{4 * \mu} \right) * \left( 1 - \left( \frac{r}{R} \right)^2 + 2 * (a^2) * \ln \left( \frac{r}{R} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 65.*

**4.97 Velocity distribution of flow through an annulus****Input(s)**

- po: Pressure at initial point (Pa)
- pL: Pressure at point L (Pa)
- R: Radius (m)
- $\mu$ : Viscosity (kg/(ms))
- L: Length (m)
- r: Cylindrical Shell of Thickness (m)
- K: Boltzmann Constant ( $\text{m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$ )

**Output(s)**

$v_z$ : Velocity Distribution (m/s)

**Formula(s)**

$$v_z = \frac{(p_o - p_L) * R^2}{4 * \mu * L} * \left( 1 - \left( \frac{r}{R} \right)^2 - \frac{1 - K^2}{\ln \left( \frac{1}{K} \right)} * \ln \left( \frac{R}{r} \right) \right)$$

Reference: *Transport Phenomena, Second Edition, Bird, Page: 55.*

**4.98 Velocity of fluid in annulus****Input(s)**

- Q: Flow rate (gpm)
- $D_o$ : Open Hole Diameter (in.)
- $D_p$ : Outside Diameter of Pipe (in.)

**Output(s)**

$v_a$ : Velocity of Fluid in Annulus (ft/s)

**Formula(s)**

$$v_a = \frac{Q}{2.448 * (D_o^2 - D_p^2)}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 321.*

**4.99 Velocity of fluid in pipe****Input(s)**

- $Q$ : Flowrate (gpm)  
 $D_i$ : Inside Diameter of Pipe (in.)

**Output(s)**

- $v_p$ : Velocity of Fluid (ft/s)

**Formula(s)**

$$v_p = \frac{Q}{2.448 * D_i^2}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 219.*

**4.100 Viscous force acting on the rod over the narrow annular region****Input(s)**

- $L$ : Length (ft)  
 $\mu$ : Viscosity (cP)  
 $v_o$ : Velocity (ft/s)  
 $\kappa$ : Ratio of Inner to Outer Ratio (fraction)

**Output(s)**

- $F$ : Force (N)

**Formula(s)**

$$F = \frac{-2 * \pi * L * \mu * v_o}{\ln\left(\frac{1}{\kappa}\right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 65.*

### 4.101 Volume capacity of pipe

#### Input(s)

$D_i$ : Inner Diameter of Pipe (in.)  
 $L$ : Length of Pipe (ft)

#### Output(s)

$V$ : Volume Capacity of Pipe (bbl)

#### Formula(s)

$$V = \frac{0.7854 * (D_i^2) * L}{808.5}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 44.*

### 4.102 Volume of fluid loss per unit area measured in a dynamic test (acidizing)

#### Input(s)

$t$ : Time (s)  
 $V_{spt}$ : The Spurt Volume ( $\text{m}^3/\text{m}^2$ )  
 $v_N$ : Fluid Loss Velocity (m/s)

#### Output(s)

$V$ : Volume of Fluid Loss per Unit Ares ( $\text{m}^3/\text{m}^2$ )

#### Formula(s)

$$V = V_{spt} + v_N t$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 42.*

### 4.103 Volume of fluid loss per unit area measured in a static test (acidizing)

#### Input(s)

$C_w$ : Fluid Loss Coefficient, ( $\text{m}/\text{s}^{1/2}$ )  
 $\sqrt{t}$ : Time root ( $\text{s}^{1/2}$ )  
 $V_{spt}$ : The Spurt Volume ( $\text{m}^3/\text{m}^2$ )

#### Output(s)

$V$ : Volume of Fluid Loss per Unit Ares ( $\text{m}^3/\text{m}^2$ )

**Formula(s)**

$$V = V_{spt} + 2C_w\sqrt{t}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 42.*

**4.104 Volume of rock dissolved per unit volume of acid (acidizing)****Input(s)**

$\rho_{15 \text{ percent HCl}}$ : Density of 15 Weight percent HCl Solution, (g/cc)

$\rho_{CaCO_3}$ : Density of Calcium Carbonate (g/cc)

$\beta_{15 \text{ percent HCl}}$ : Mass of Dissolved Rock with 15 percent of HCl (g/g)

**Output(s)**

$X_{15}$ : Acid Dissolving Power (cc/cc)

**Formula(s)**

$$X_{15} = \frac{\rho_{15 \text{ percent HCl}} \times \beta_{15 \text{ percent HCl}}}{\rho_{CaCO_3}}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 13.*

**4.105 Water quantity that dilutes the original brine with assumed density (two-salt systems)****Input(s)**

$V_d$ : Volume of Diluted Brine (bbl)

$\rho_i$ : Density of initial, undiluted Brine (lbm/bbl)

$\rho_d$ : Density of Diluted Brine (lbm/bbl)

$\rho_{8.33}$ : Density of Water diluting the Original Brine (lbm/bbl)

**Output(s)**

$V_{8.33}$ : Volume of Water Diluting the Original Brine (bbl)

**Formula(s)**

$$V_{8.33} = (V_d) \frac{(\rho_i - \rho_d)}{(\rho_i - \rho_{8.33})}$$

Reference: *Bridges, K. L. (2000). Completion and Workover Fluids (Vol. 19). Society of Petroleum Engineers, Page: 56.*

#### 4.106 Weight of crystalline CaCl<sub>2</sub> and CaBr<sub>2</sub> salt addition to brine (two-salt systems)

##### Input(s)

- $C_{95}$ : Concentration of 95% CaBr<sub>2</sub> in the Initial undiluted Brine (lbm/bbl)
- $m_{94}$ : Mass of 94% CaCl<sub>2</sub> needed to weight up diluents water (lbm)
- $C_{94}$ : Concentration of 94% CaCl<sub>2</sub> in the Initial undiluted Brine (lbm/bbl)
- $W_i$ : Water Fraction in the initial undiluted Brine (bbl/bbl)

##### Output(s)

- $m_{95}$ : Mass of 95% CaBr<sub>2</sub> needed to weight up diluents water (lbm)

##### Formula(s)

$$m_{95} = (V_{8.33}) \frac{(C_{95})}{(W_i)}$$

$$m_{94} = (V_{8.33}) \frac{(C_{94})}{(W_i)}$$

Reference: Bridges, K. L. (2000). *Completion and Workover Fluids* (Vol. 19). Society of Petroleum Engineers, Page: 56.

#### 4.107 Well flowing pressure (line-source solution by including skin factor)

##### Input(s)

- $P_i$ : Initial Reservoir Pressure (psi)
- $q$ : Flow Rate (bbl/d)
- $\mu$ : Viscosity (cP)
- $B$ : Formation Volume Factor (bbl/STB)
- $k$ : Permeability (mD)
- $h$ : Payzone Thickness (ft)
- $\phi$ : Porosity (fraction)
- $c_t$ : Total Compressibility (1/psi)
- $r$ : Radius of Reservoir (ft)
- $t$ : Time (days)
- $S$ : S Variable (psi)

##### Output(s)

- $P_{wf}$ : Well Flowing Pressure (psi)

##### Formula(s)

$$P_{wf} = P_i + \left( 70.6 * q * \mu * \frac{B}{k * h} \right) * \left( \ln \left( 1688 * \phi * \mu * c_t * \frac{r^2}{k * t} \right) - 2 * S \right)$$

Reference: Lee, J., Rollins, J. B., & Spivey, J. P. (2003). *Pressure Transient Testing* (Vol. 9). Richardson, Texas: Society of Petroleum Engineers, Page: 13.

### 4.108 Well flowing pressure under Pseudo-steady state flow for non-circular reservoirs

#### Input(s)

- $p_i$ : Initial Reservoir Pressure (psi)
- $k$ : Permeability (mD)
- $h$ : Thickness (ft)
- $B$ : Formation Volume Factor (bbl/STB)
- $\mu$ : Viscosity (cP)
- $C_A$ : Shape Factor (dimensionless)
- $r_w$ : Wellbore Radius (ft)
- $Q$ : Flow Rate (STB/day)
- $c_t$ : Total Compressibility (1/psi)
- $\phi$ : Porosity (fraction)
- $t$ : Time (hour) A: Area ( $\text{ft}^2$ )

#### Output(s)

- $P_{wf}$ : Well Flowing Pressure (psi)

#### Formula(s)

$$P_{wf} = \left( p_i - \left( \frac{162.6 * Q * B * \mu}{k * h} \right) * \log \left( \frac{2.2458 * A}{C_A * r_w^2} \right) \right) - \left( \frac{0.23396 * Q * B * t}{A * h * \phi * c_t} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 33.

### 4.109 Wellbore pressure loss due to friction during hydraulic fracturing (laminar flow)

#### Input(s)

- $v$ : Average Velocity (ft/s)
- $L$ : Pipe Length (ft)
- $d$ : Inner Diameter of Pipe (in.)
- $\mu_p$ : Plastic Viscosity (cP)
- $\tau_y$ : Yield Point of the Liquid (lb/100ft<sup>2</sup>)

#### Output(s)

- $\Delta P_f$ : Pressure Loss in the wellbore Sourced by Frictions during Hydraulic Fracturing (psi)

#### Formula(s)

$$\Delta P_f = \frac{\mu_p L v}{1500 d^2} \cdot \frac{\tau_y \cdot L}{225 \cdot d}$$

Reference: Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 30.

## 4.110 Wellbore pressure loss due to friction during hydraulic fracturing (turbulence flow)

### Input(s)

- $\rho$ : Liquid Density (lb/gal)
- $v$ : Average Velocity (ft/s)
- $f$ : Fanning Friction Factor (dimensionless)
- $L$ : Pipe Length (ft)
- $d$ : Inner Diameter of Pipe (in.)

### Output(s)

- $\Delta P_f$ : Pressure Loss in the wellbore Sourced by Frictions during Hydraulic Fracturing (psi)

### Formula(s)

$$\Delta P_f = \frac{f \cdot L \cdot \rho \cdot v^2}{25.80 \cdot d}$$

Reference: *Saydam, T., (1967). Principles of Hydraulic Fracturing, ARI Publishing Co., Page: 31.*

## 4.111 Wellbore storage

### Input(s)

- $\Delta V_m$ : Change in the Volume of Fluid in the Wellbore (bbl)
- $\Delta P$ : Change in Pressure (psi)

### Output(s)

- $C$ : Wellbore Storage Coefficient (bbl/psi)

### Formula(s)

$$C = \frac{\Delta V_m}{\Delta P}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 1, Page: 49.*

## 4.112 Wellbore storage due to fluid level

### Input(s)

- $A_a$ : Annulus Cross Sectional Area ( $\text{ft}^2$ )
- $\rho$ : Wellbore Fluid Density ( $\text{lb}/\text{ft}^3$ )

### Output(s)

- $C_{FL}$ : Wellbore Storage Coefficient (bbl/psi)

**Formula(s)**

$$C_{FL} = \frac{144 * A_a}{5.615 * \rho}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 1, Page: 49.

**4.113 Wellhead pressure (multiphase flow across the choke)****Input(s)**

- C: Empirical Constant (dimensionless)
- n: Empirical Constant (dimensionless)
- m: Empirical Constant (dimensionless)
- R: Producing Gas Liquid Ratio (SCF/BBL)
- S: Choke Size (1/64 in.)
- q: Gross Liquid Rate (bbl/d)

**Output(s)**

$P_{wh}$ : Upstream Wellhead Pressure (psi)

**Formula(s)**

$$P_{wh} = C * R^m * \frac{q}{S^n}$$

Reference: Boyun, G., William, C., & Ali Ghalmab, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 5/64.

**4.114 Workover operations (maximum allowed tubing pressure)****Input(s)**

- FG: Fracture Gradient (psi/ft)
- $P_t$ : Tubing Pressure (psi)
- H: Depth of Perforations (ft)

**Output(s)**

MATP: Maximum Allowable Tubing Pressure (psi)

**Formula(s)**

$$MATP = FG * H - P_t$$

Reference: [Petrowiki.org](http://Petrowiki.org).

### 4.115 Young Modulus by using sonic travel time (acidizing)

#### Input(s)

$t_s$ : Sonic Travel Time ( $\mu\text{s}/\text{ft}$ )  
 $\emptyset$ : Porosity (per cent)  
 $v$ : Poisson's Ratio (dimensionless)  
 $\rho_{ma}$ : Density of Formation Matrix ( $\text{lb}/\text{ft}^3$ )  
 $\rho_f$ : Density of Formation fluids ( $\text{lb}/\text{ft}^3$ )

#### Output(s)

$E$ : Young's Modulus (psi)

#### Formula(s)

$$E = 2.16 \times 10^8 \frac{[\rho_{ma}(1 - \emptyset) + \rho_f \emptyset](1 - 2v)(1 + v)}{(1 - v)t_s^2}$$

Reference: *Williams, B. B., Gidley, J. L., & Schechter, R. S. (1979). Acidizing Fundamentals. Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers of AIME, Page: 56.*

## Chapter 5

# Fluid flow and transport phenomena formulas and calculations

### Chapter Outline

5.1 Archimedes number	258	5.40 Drag coefficient	275
5.2 Average number of collisions to reduce neutron energy	259	5.41 Drag force	275
5.3 Average velocity of a falling film with variable viscosity	259	5.42 Draining of a cylindrical tank	276
5.4 Average velocity of flow through a circular tube	260	5.43 Draining of a spherical tank	276
5.5 Average velocity of flow through an annulus	260	5.44 Eckert number	277
5.6 Average velocity of fluids in flow of two adjacent immiscible fluids		5.45 Effective emissivity of a hole	277
5.7 Average velocity over the cross section of a falling film		261 5.46 Effective thermal conductivity for a solid with spherical inclusions	277
5.8 Blowdown time in unsteady gas flow		261 5.47 Efflux time for draining a conical tank	278
5.9 Boltzmann equation		262 5.48 Ekman number	278
5.10 Boussinesq approximation—Buoyancy		262 5.49 Elimination of circulation in a rising gas bubble	279
5.11 Brinkman number		262 5.50 Energy emitted from the surface of a black body	279
5.12 Buckingham Reiner equation		263 5.51 Estimation of diffusivity of liquids	279
5.13 Calculation of mass flow rate		263 5.52 Estimation of self diffusivity at high density	280
5.14 Calculation of momentum flux		264 5.53 Estimation of the viscosity of a pure liquid	280
5.15 Combined momentum flux tensor		264 5.54 Euler number	281
5.16 Combined radiation and convection		265 5.55 Fanning friction factor (laminar flow)	281
5.17 Compressible flow in a horizontal circular tube		265 5.56 Fanning's friction factor (turbulent flow)	282
5.18 Compton scattering		265 5.57 Fick's law of binary diffusion	282
5.19 Correction factor for stagnant film according to the penetration model		266 5.58 Film condensation on vertical pipes	282
5.20 Darcy Weisbach equation (head loss form)		266 5.59 Film condensation on vertical tubes	283
5.21 Darcy Weisbach equation (pressure loss form)		266 5.60 Film thickness of a falling film on a conical surface	284
5.22 Dean number		267 5.61 Flow in a liquid-liquid ejector pump	284
5.23 Deborah number		267 5.62 Flow in a slit with uniform cross flow	285
5.24 Decay of thermal neutrons		267 5.63 Flow near a corner	285
5.25 Determination of the controlling resistance		268 5.64 Flow of power law fluid through a narrow slit	286
5.26 Determination of the diameter of a falling sphere		268 5.65 Fluid kinetic force in conduits	286
5.27 Diffusion from an instantaneous point source		269 5.66 Fluid kinetic force in flow around submerged objects	286
5.28 Diffusion in a moving film		269 5.67 Form drag	287
5.29 Diffusion in polymers		270 5.68 Free air correction—Gravity survey	287
5.30 Diffusion into a falling liquid film (gas absorption)		270 5.69 Free batch expansion of a compressible fluid	288
5.31 Diffusion of low-density gases with equal mass		270 5.70 Free convection heat transfer from a vertical plate	288
5.32 Diffusion potential		271 5.71 Friction drag	288
5.33 Diffusion through a non-isothermal spherical film		271 5.72 Friction factor for creeping flow around a sphere	289
5.34 Diffusion through a stagnant film		272 5.73 Friction factor in flow around submerged objects	289
5.35 Diffusion through a stagnant gas film		272 5.74 Friction factor in flow through conduits	290
5.36 Diffusion through cleat spacing in cased methane reservoirs		273 5.75 Friction factor in packed column (laminar)	290
5.37 Diffusion with a heterogeneous chemical reaction		273 5.76 Friction factor in packed column (turbulent)	290
5.38 Diffusion with a homogeneous chemical reaction		5.77 Galilei number	291
5.39 Diffusion, convection, and chemical reaction	275	273 5.78 Gas absorption from rising bubbles for creeping flow	291
		274 5.79 Gas absorption through bubbles	292
		274 5.80 Gas absorption with chemical reaction in an agitated tank	292

5.81 Gas absorption with rapid reaction	293	5.115 Momentum flux distribution of flow through an annulus	307
5.82 Gas mass rate flow in compressible tube flow	293	5.116 Momentum flux profile of fluids in flow of two adjacent immiscible fluids	307
5.83 Graetz number	293	5.117 Momentum fluxes for creeping flow into a slot	308
5.84 Graham equation viscosity ratio	294	5.118 Mooney equation viscosity	308
5.85 Grashof number	294	5.119 Non-Newtonian flow in annulus	309
5.86 Hagen number	295	5.120 Nusselt number	309
5.87 Hagen-Poiseuille equation	295	5.121 Ohnesorge number	310
5.88 Influence of changing interfacial area on mass transfer	296	5.122 Potential flow around a cylinder	310
5.89 Knudsen flow	296	5.123 Prandtl number	310
5.90 Krieger Dougherty equation viscosity ratio	296	5.124 Pressure distribution in a creeping flow around a sphere	311
5.91 Laminar flow along a flat plate (approximate solution)	297	5.125 Pressure drop per length of the adsorption unit	311
5.92 Laminar flow of an incompressible power-law fluid in a circular tube	297	5.126 Pressure loss due to sudden enlargement	312
5.93 Laplace number	298	5.127 Reynolds number	312
5.94 Lewis number	298	5.128 Schmidt number	313
5.95 Mach number	298	5.129 Sherwood number	313
5.96 Manning formula	299	5.130 Slit flow in Bingham fluid	313
5.97 Marangoni number	299	5.131 Smoluchowski equation	314
5.98 Mass absorption (attenuation) coefficient	300	5.132 Stanton number	314
5.99 Mass flow rate as a function of the modified pressure drop in a network of tubes	300	5.133 Stefan number	315
5.100 Mass flow rate in a rotating cone pump	300	5.134 Stokes number	315
5.101 Mass rate of flow	300	5.135 Strouhal number	315
5.102 Mass rate of flow in a squared duct	301	5.136 Taylor dispersion (axial dispersion coefficient)	316
5.103 Mass rate of flow of a falling film	301	5.137 Taylor equation viscosity	316
5.104 Mass rate of flow through a circular tube	302	5.138 Taylor number	317
5.105 Mass transfer for creeping flow around a gas bubble	302	5.139 Theory of diffusion in colloidal suspensions	317
5.106 Mass transfer to drops and bubbles	303	5.140 Toricelli equation	317
5.107 Maximum flow rate (Vogel's equation)	303	5.141 Total force of the fluid on the sphere in a creeping flow around a sphere	318
5.108 Maximum velocity of a falling film	303	5.142 Velocity distribution in a creeping flow around a sphere	318
5.109 Maximum velocity of flow through a circular tube	304	5.143 Velocity distribution of a falling film with variable viscosity	319
5.110 Maximum-velocity $v_z$ -maximum of a falling film	305	5.144 Velocity distribution of flow through a circular tube	319
5.111 Method for separating helium from natural gas	305	5.145 Velocity profile of fluids in flow of two adjacent immiscible fluids	319
5.112 Modified capillary number	306	5.146 Viscosity by a falling-cylinder viscometer	320
5.113 Modified Van Driest equation	306	5.147 Winsauer equation	320
5.114 Momentum flux distribution of flow through a circular tube	306		

## 5.1 Archimedes number

### Input(s)

- g: Local External Field (For Example Gravitational Acceleration) ( $\text{m/s}^2$ )
- $\rho_l$ : Density of the Fluid ( $\text{kg/m}^3$ )
- $\rho$ : Density of the Body ( $\text{kg/m}^3$ )
- $\mu$ : Dynamic Viscosity ( $\text{kg/(s*m)}$ )
- L: Characteristic Length of Body (m)

### Output(s)

- Ar: Archimedes Number (dimensionless)

**Formula(s)**

$$Ar = \frac{g * L^3 * \rho_l * (\rho - \rho_l)}{\mu^2}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.2 Average number of collisions to reduce neutron energy****Input(s)**

$E_o$ : Initial Energy Level (eV)

$E$ : Final Energy Level (eV)

$\xi$ : Average Energy Decrement per Collision (dimensionless)

**Output(s)**

$n$ : Average Number of Collisions to Reduce Neutron Energy (dimensionless)

**Formula(s)**

$$n = \left( \frac{1}{\xi} \right) * \ln \left( \frac{E_o}{E} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 39.

**5.3 Average velocity of a falling film with variable viscosity****Input(s)**

$\rho$ : Density (g/cc)

$g$ : Acceleration due to Gravity (cm/s<sup>2</sup>)

$\delta$ : Film Thickness (cm)

$\beta$ : Inclination (rad)

$\mu$ : Viscosity (cP)

$x$ : Position in Film (cm)

**Output(s)**

$v$ : Velocity (cm/s)

**Formula(s)**

$$v = \left( \frac{\rho * g * (\delta^2) * \cos(\beta)}{2 * \mu} \right) * \left( 1 - \left( \frac{x}{\delta} \right)^2 \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons., Chapter: 2, Page: 48.

## 5.4 Average velocity of flow through a circular tube

### Input(s)

$P_o$ : Pressure at Initial Point (Pa)

$P_L$ : Pressure at Point L (Pa)

$R$ : Radius (m)

$\mu$ : Viscosity (kg/(ms))

$L$ : Length (m)

### Output(s)

$v_z$ : Average Velocity (m/s)

$v_{z,max}$ : Maximum Velocity (Occurs at  $R = 0$ ) (m/s)

### Formula(s)

$$v_z = \frac{(P_o - P_L) * R^2}{8 * \mu * L}$$

$$v_{z,max} = \frac{(P_o - P_L) * R^2}{4 * \mu * L}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons., Chapter: 2, Page: 51.

## 5.5 Average velocity of flow through an annulus

### Input(s)

$P_o$ : Pressure at Initial Point (Pa)

$P_L$ : Pressure at Point L (Pa)

$R$ : Radius (m)

$\mu$ : Viscosity (kg/(ms))

$L$ : Length (m)

$\kappa$ : Ratio of Inner Pipe's Radius to Outer Pipe's Radius (fraction)

### Output(s)

$v_z$ : Average Velocity (m/s)

### Formula(s)

$$v_z = \frac{(P_o - P_L) * R^2}{8 * \mu * L} * \left[ \frac{1 - \kappa^4}{1 - \kappa^2} - \frac{1 - \kappa^2}{\ln\left(\frac{1}{\kappa}\right)} \right]$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second ed.). John Wiley & Sons, Chapter: 2, Page: 55.

## 5.6 Average velocity of fluids in flow of two adjacent immiscible fluids

### Input(s)

$P_o$ : Pressure at Initial Point (Pa)

$P_L$ : Pressure at Point L (Pa)

$b$ : Distance (m)

$\mu_I$ : Viscosity of Phase I, Denser, More Viscous Fluid (kg/(ms))

$\mu_{II}$ : Viscosity of phase II, Less Dense, Less Viscous Fluid (kg/(ms))

$L$ : Length (m)

### Output(s)

$v_{zI}$ : Average Velocity for Phase I (m/s)

$v_{zII}$ : Average Velocity for Phase II (m/s)

### Formula(s)

$$v_{zI} = \frac{(P_o - P_L) * b^2}{12 * \mu_I * L} * \left( \frac{7 * \mu_I + \mu_{II}}{\mu_I + \mu_{II}} \right)$$

$$v_{zII} = \frac{(P_o - P_L) * b^2}{12 * \mu_{II} * L} * \left( \frac{\mu_I + 7 * \mu_{II}}{\mu_I + \mu_{II}} \right)$$

Reference: Bird, R.B., Stewart, W.E., and Lightfoot, E.N. (2002). *Transport Phenomena* (Second ed.). John Wiley & Sons, Chapter: 2, Page: 58.

## 5.7 Average velocity over the cross section of a falling film

### Input(s)

$\rho$ : Density (kg/m<sup>3</sup>)

$g$ : Gravitational Acceleration (m/s<sup>2</sup>)

$\delta$ : Film Thickness (m)

$\mu$ : Viscosity (kg/(ms))

$\beta$ : Angle of Inclination w.r.t Direction of Gravity (rad)

$v_{z,max}$ : The Maximum Velocity at x = 0 (m/s)

### Output(s)

$v_z$ : Average Velocity (m/s)

### Formula(s)

$$v_z = \frac{\rho * g * \delta^2 * \cos(\beta)}{3 * \mu}$$

Reference: Bird, R.B., Stewart, W.E., and Lightfoot, E.N. (2002). *Transport Phenomena* (Second ed.). John Wiley & Sons, Chapter: 2, Page: 45.

## 5.8 Blowdown time in unsteady gas flow

### Input(s)

- V: Volume (ft<sup>3</sup>)
- $C_d$ : Valve Discharge Coefficient (dimensionless)
- $A_v$ : Valve Area (in.<sup>2</sup>)
- $\gamma$ : Gas Specific Density (dimensionless)
- $z$ : Average Gas Compressibility (dimensionless)
- T: Temperature (K)
- $P_a$ : Initial Pressure (psi)
- $P_b$ : Output Pressure (psi)
- B: Constant

### Output(s)

- t: Interstitial Velocity2 (cm/s)

### Formula(s)

$$t = \left( \frac{B * V}{C_d * A_v} \right) * \left( \frac{\gamma}{z * T} \right)^{0.5} * \ln \left( \frac{P_a}{P_b} \right)$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page 29.*

## 5.9 Boltzmann equation

### Input(s)

- $C_K$ : Boltzmann Constant, 1.38042\*10<sup>-23</sup> (J/K)
- $T_a$ : Absolute Temperature (Kelvin)

### Output(s)

- $E_{mp}$ : Most Probable Energy (eV)

### Formula(s)

$$E_{mp} = C_K * T_a$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 37.*

## 5.10 Boussinesq approximation—Buoyancy

### Input(s)

- $g$ : Acceleration due to Earth (m/s<sup>2</sup>)
- $\rho_2$ : Final Density (kg/m<sup>3</sup>)
- $\rho_1$ : Initial Density (kg/m<sup>3</sup>)
- $\rho$ : Single Density (kg/m<sup>3</sup>)

**Output(s)**

$g_r$ : Reduced Acceleration due to Gravity ( $\text{m/s}^2$ )

**Formula(s)**

$$g_r = g * \frac{\rho_2 - \rho_1}{\rho}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.11 Brinkman number****Input(s)**

- $\mu$ : Viscosity ( $\text{Pa s}$ )
- $v_b$ : Velocity at Outer Perimeter ( $\text{cm/s}$ )
- $k$ : Thermal Conductivity ( $\text{W/cm K}$ )
- $T_b$ : Temperature at Outer Perimeter ( $\text{K}$ )
- $T_o$ : Temperature at Inner Perimeter ( $\text{K}$ )

**Output(s)**

$Br$ : Brinkman Number (dimensionless)

**Formula(s)**

$$Br = \mu * \frac{v_b^2}{k * (T_b - T_o)}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 300.

**5.12 Buckingham Reiner equation****Input(s)**

- $P_o$ : Input Pressure ( $\text{psi}$ )
- $P_L$ : Output Pressure ( $\text{psi}$ )
- R: Radius ( $\text{ft}$ )
- $\rho$ : Density of Fluid ( $\text{ppg}$ )
- $\mu$ : Viscosity of Fluid ( $\text{cP}$ )
- L: Length ( $\text{ft}$ )
- $\tau_o$ : Torque ( $\text{psi}$ )

**Output(s)**

- $\tau_R$ : Shear Stress at the Tube Wall ( $\text{psi}$ )
- $Q$ : Mass Flow Rate ( $\text{lb/s}$ )

**Formula(s)**

$$\tau_R = \frac{(P_o - P_L) * R}{2 * L}$$

$$Q = \left[ \frac{3.142 * (P_o - P_L) * R^4 * \rho}{8 * \mu * L} \right] * \left( 1 - \frac{4 * \tau_o}{3 * \tau_R} + 0.333 * \left( \frac{\tau_o}{\tau_R} \right)^4 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 260.*

**5.13 Calculation of mass flow rate****Input(s)**

- $\rho$ : Density ( $\text{kg/m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m/s}^2$ )
- $\delta$ : Film Thickness (m)
- W: Width (m)
- $\nu$ : Kinematic Viscosity ( $\text{m}^2/\text{s}$ )

**Output(s)**

- w: Mass Flow Rate ( $\text{kg/s}$ )

**Formula(s)**

$$w = \frac{\rho * g * \delta^3 * W}{3 * \nu}$$

Reference: *Bird, R.B., Stewart, W.E., and Lightfoot, E.N. (2002). Transport Phenomena (Second ed.). John Wiley & Sons, Chapter: 2, Page: 47.*

**5.14 Calculation of momentum flux****Input(s)**

- $\mu$ : Fluid Viscosity (cP)
- $\frac{dv_x}{dy}$ : Change in Velocity in the x-direction (ft/s)
- $y$ : Plate Separation in the y-direction (ft)

**Output(s)**

- $\tau_{yx}$ : Momentum Flux ( $\text{lb f/ft}^2$ )

**Formula(s)**

$$\tau_{yx} = -(2.0886 * 10^{-5}) * \mu * \frac{dv_x}{dy}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second ed.). John Wiley & Sons, Chapter: 1, Page: 15.*

## 5.15 Combined momentum flux tensor

### Input(s)

- $\rho V$ : Convective Momentum Flux ( $\text{kg}/\text{m s}^2$ )  
 $\pi$ : Molecular Momentum Flux ( $\text{kg}/\text{m s}^2$ )

### Output(s)

- $\phi$ : Sum of the Momentum Flux Tensors ( $\text{kg}/\text{m s}^2$ )

### Formula(s)

$$\phi = \pi + \rho V$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 36.*

## 5.16 Combined radiation and convection

### Input(s)

- $A_i$ : Area ( $\text{ft}^2$ )  
 $\sigma$ : Stephen Boltzmann Constant ( $\text{lb}/\text{h}^3\text{R}^4$ )  
 $e_i$ : Emissivity (fraction)  
 $T_i$ : Maximum Air Temperature at the Water Surface (R)

### Output(s)

- $Q$ : Heat Transfer Rate (Btu/s)

### Formula(s)

$$Q = \sigma * A_i * e_i * T_i^4$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 505.*

## 5.17 Compressible flow in a horizontal circular tube

### Input(s)

- $p_0$ : Pressure at Initial Point (Pa)  
 $p_L$ : Pressure at Point L (Pa)  
 $R$ : Radius (m)  
 $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )  
 $L$ : Length (m)  
 $\rho_{avg}$ : Average Density ( $\text{kg}/\text{m}^3$ )

### Output(s)

- $\omega$ : Mass Rate of Flow (kg/s)

**Formula(s)**

$$\omega = \frac{\pi * (p_0 - p_L) * R^4 * \rho_{avg}}{8 * \mu * L}$$

Reference: *Bird, R.B., Stewart, W.E., and Lightfoot, E.N., (2002). Transport Phenomena (Second ed.). John Wiley & Sons, Chapter: 2, Page: 53.*

**5.18 Compton scattering****Input(s)**

- $E_0$ : Energy of the Photon Before Scattering (MeV)
- $m_e$ : Mass of the Electron (kg)
- C: Velocity of Light Considered as  $3*10^8$  (m/s)
- $\theta$ : Scattering Angle (degree)

**Output(s)**

- E: Energy of the Photon After Scattering (MeV)

**Formula(s)**

$$E = \frac{E_0}{1 + \left( \frac{E_0}{m_e * C^2} \right) * (1 - \cos(\theta))}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 33.*

**5.19 Correction factor for stagnant film according to the penetration model****Input(s)**

- $\phi$ : Rate Factor (rad)

**Output(s)**

- $\theta$ : Correction Factor (dimensionless)

**Formula(s)**

$$\theta = \frac{\exp\left(\frac{-(\phi)^2}{3.142}\right)}{1 + \operatorname{erf}\left(\frac{\phi}{(3.142)^{0.5}}\right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 707.*

## 5.20 Darcy Weisbach equation (head loss form)

### Input(s)

- L: Length of the Pipe (m)
- g: Acceleration due to Gravity (m/s<sup>2</sup>)
- V: Average Velocity of the Fluid Flow, Equal to the Volumetric Flow Rate per Unit Cross-sectional Wetted Area (m/s)
- D: Hydraulic Diameter of the Pipe (For a Pipe of Circular Section, this Equals the Internal Diameter of the Pipe) (m)
- $f_D$ : Dimensionless Coefficient Called the Darcy Friction Factor (dimensionless)

### Output(s)

- $\Delta h$ : Head Loss Due to Friction (m)

### Formula(s)

$$\Delta h = f_D * \frac{1}{2 * g} * \left( \frac{V^2}{D} \right)$$

Reference: [Wikipedia.org](#).

## 5.21 Darcy Weisbach equation (pressure loss form)

### Input(s)

- L: Length of the Pipe (m)
- $\rho$ : Density of Fluid (kg/m<sup>3</sup>)
- V: Average Velocity of the Fluid Flow, Equal to the Volumetric Flow Rate per Unit Cross-sectional Wetted Area (m/s)
- L: Lift Force (Newton)
- D: Hydraulic Diameter of the Pipe (for a Pipe of Circular Section, equals the Internal Diameter of the Pipe) (m)
- $f_D$ : Dimensionless Coefficient Called the Darcy Friction Factor (dimensionless)

### Output(s)

- $\Delta P$ : Pressure Loss due to Friction (m)

### Formula(s)

$$\Delta P = L * f_D * \frac{\rho}{2} * \left( \frac{V^2}{D} \right)$$

Reference: [Wikipedia.org](#).

## 5.22 Dean number

### Input(s)

- $\rho$ : Density of the Fluid (kg/m<sup>3</sup>)
- $\mu$ : Dynamic Viscosity (kg/ms)
- V: Axial Velocity Scale (m/s)
- d: Diameter (m)
- $R_c$ : Radius of Curvature of the Path of Channel (m)

### Output(s)

De: Dean Number (dimensionless)

### Formula(s)

$$De = \frac{\rho * V * d}{\mu} * \left( \sqrt[2]{\left( \frac{d}{2 * R_c} \right)} \right)$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.23 Deborah number

### Input(s)

$t_c$ : Relaxation Time Scale (h)

$t_p$ : Time Scale of Observation (h)

### Output(s)

De: Deborah Number (dimensionless)

### Formula(s)

$$De = \frac{t_c}{t_p}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.24 Decay of thermal neutrons

### Input(s)

$n_0$ : Number of Thermal Neutrons at Time t0 (dimensionless)

t: Time (s)

$\tau$ : Thermal Decay Time (s)

### Output(s)

n: Number of Thermal Neutrons at Time t (fraction)

### Formula(s)

$$n = n_0 * \exp \left( -\frac{t}{\tau} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 9, Page: 180.

## 5.25 Determination of the controlling resistance

### Input(s)

- S: Solubility (mol/cc)
- M: Mass (g)
- $x_a$ : Partial Pressure (kPa)
- $S_o$ : Solubility at NTP (mol/cc)
- $c_l$ : Concentration of Liquid (g mol/cc)
- $c_g$ : Concentration of Gas (g mol/cc)
- $D_{AB}$ : Diffusivity in the Liquid Phase (cm<sup>2</sup>/s)
- $D_{AC}$ : Diffusivity in the Gas Phase (cm<sup>2</sup>/s)

### Output(s)

- m: Slope (dimensionless)
- R: Resistance (ohm)

### Formula(s)

$$m = \frac{S}{M} / \frac{x_a}{S_o}$$

$$R = m * \left( \frac{c_l}{c_g} \right) * \left( \left( \frac{D_{AB}}{D_{AC}} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 691.*

## 5.26 Determination of the diameter of a falling sphere

### Input(s)

- Re: Reynolds Number (dimensionless)
- v: Velocity of Fluid (ft/s)
- $\rho$ : Density of Fluid (g/cc)
- $\mu$ : Viscosity of Fluid (cP)

### Output(s)

- D: Diameter (ft)

### Formula(s)

$$D = \frac{Re * \mu}{\rho * v}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 650.*

## 5.27 Diffusion from an instantaneous point source

### Input(s)

$m_A$ : Mass of Species A (g)  
 $D_{AB}$ : Binary Diffusivity for System A-B ( $\text{cm}^2/\text{s}$ )  
 t: Time (s)  
 r: Radial Coordinate, L (m)

### Output(s)

$\rho_A$ : Density of Species A ( $\text{g}/\text{cm}^3$ )

### Formula(s)

$$\rho_A = \left( \frac{m_A}{(4 * \pi * D_{AB} * t)^{\frac{3}{2}}} \right) * \exp \left( -\frac{r^2}{4 * D_{AB} * t} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second ed.). John Wiley & Sons, Chapter: 20, Page: 650.*

## 5.28 Diffusion in a moving film

### Input(s)

H: Initial Height of Gas Column (cm)  
 C: Concentration (g/cc)  
 t: Time (s)

### Output(s)

$h(t)$ : Height of Gas Interface (cm)

### Formula(s)

$$h(t) = H * \left( \left( 1 + \left( \frac{Ct}{H^2} \right)^{0.5} \right) - 1 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 549.*

## 5.29 Diffusion in polymers

### Input(s)

M: Molar Weight (g/mol)

### Output(s)

$D_{AB}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )

**Formula(s)**

$$D_{AB} = \frac{1}{M^{0.5}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 532.*

### 5.30 Diffusion Into a falling liquid film (gas absorption)

**Input(s)**

c: Concentration (mol/cm<sup>3</sup>)

$D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)

$v_{max}$ : Velocity of Dropping Particle (cm/s)

z: Height of Column (cm)

**Output(s)**

N: Molar Flux (mol/cm<sup>2</sup>s)

**Formula(s)**

$$N = c * \left( \frac{D_{AB} * v_{max}}{\pi * z} \right)^{0.5}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 560.*

### 5.31 Diffusion of low-density gases with equal mass

**Input(s)**

T: Temperature (K)

$M_A$ : Mass of Component A (g)

$M_B$ : Mass of Component B (g)

$\sigma_{AB}$ : Collision Diameter (cm)

P: Pressure (atm)

$\Omega$ : Collision integral (dimensionless)

**Output(s)**

$D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)

**Formula(s)**

$$D_{AB} = 0.0018583 * \left( \left( (T^3) * \left( \left( \frac{1}{M_A} \right) + \left( \frac{1}{M_B} \right) \right) \right)^{0.5} \right) * \left( \frac{1}{P * (\sigma_{AB}^2) * \Omega} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 526.*

### 5.32 Diffusion potential

#### Input(s)

- $t_{Cl}$ : Chlorine Anion Transference Number (dimensionless)
- R: Gas Constant (8.31) (J/mol K)
- $T_a$ : Absolute Temperature (degree K)
- F: Faraday Constant (96516) (C)
- $a_1$ : Activities of 1st Electrolyte (dimensionless)
- $a_2$ : Activities of 2nd Electrolyte (dimensionless)

#### Output(s)

- $E_d$ : Diffusion Potential (V)

#### Formula(s)

$$E_d = (2 * t_{Cl} - 1) * \left( R * \frac{T_a}{F} \right) * \ln \left( \frac{a_1}{a_2} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 130.

### 5.33 Diffusion through a non-isothermal spherical film

#### Input(s)

- P: Pressure (atm)
- $D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)
- R: Gas Constant (J/mol K)
- $T_1$ : Temperature (K)
- n: Exponent (dimensionless)
- $r_1$ : Radius of Gas Film (cm)
- $r_2$ : Radius of Film (cm)
- $x_{A1}$ : Position of Gas (fraction)
- $x_{A2}$ : Position of Film (fraction)

#### Output(s)

- $W_A$ : Mass Rate (mol/s)

#### Formula(s)

$$W_A = \frac{4 * \pi * \left( \frac{P * D_{AB}}{R * T_1} \right) * \left( 1 + \left( \frac{n}{2} \right) \right)}{\left( \frac{n}{r_1^2} \right) * \left( \left( \frac{1}{r_1} \right)^{1+\frac{n}{2}} - \left( \frac{1}{r_2} \right)^{1+\frac{n}{2}} \right)} * \ln \left( \frac{1 - x_{A2}}{1 - x_{A1}} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 551.

### 5.34 Diffusion through a stagnant film

#### Input(s)

- $D_{AB}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )  
 $z_1$ : Initial Position (ft)  
 $z_2$ : Output Position (ft)  
 $x_{A1}$ : Mole Fraction of A1 (g/ft)  
 $x_{A2}$ : Mole Fraction of A2 (g/ft)  
 $x_b$ : Mole Fraction of B in the Logarithmic Mean of Thermal Concentrations ( $\text{mol}/\text{ft}^3$ )  
 $c$ : Total Molar Concentration ( $\text{mol}/\text{ft}^3$ )

#### Output(s)

- N: Combined Molar Flux ( $\text{mol}/\text{ft}^2\text{s}$ )

#### Formula(s)

$$N = \frac{c * D_{AB} * (x_{A1} - x_{A2})}{(z_2 - z_1) * (x_b)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 548.*

### 5.35 Diffusion through a stagnant gas film

#### Input(s)

- $x_{B1}$ : Mole Fraction of B1 (fraction)  
 $x_{B2}$ : Mole Fraction of B2 (fraction)

#### Output(s)

- $x_B$ : Average Value of Mole Fraction of B in the Logarithmic Mean of Thermal Concentrations (fraction)

#### Formula(s)

$$x_B = (x_{B2} - x_{B1}) / \ln(x_{B2}/x_{B1})$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 547.*

### 5.36 Diffusion through cleat spacing in coalbed methane reservoirs

#### Input(s)

- s: Coal Cleat Spacing (ft)  
t: Desorption Time from the Canister Test (day)

#### Output(s)

- D: Diffusion Coefficient ( $\text{ft}^2/\text{day}$ )

**Formula(s)**

$$D = \frac{s^2}{8 * \pi * t}$$

Reference: Ahmed, T., Paul McKinney, P. D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter:3 Page: 233.

**5.37 Diffusion with a heterogeneous chemical reaction****Input(s)**

- $D_{AB}$ : Diffusivity ( $\text{ft}^2/\text{s}$ )
- $\delta$ : Position of Catalytic Film (cm)
- $x_a$ : Main Stream Concentration ( $\text{mol}/\text{ft}^3$ )
- c: Total Molar Concentration ( $\text{mol}/\text{ft}^3$ )

**Output(s)**

- N: Molar Flux ( $\text{mol}/\text{ft}^2\text{s}$ )

**Formula(s)**

$$N = \left( \frac{2 * c * D_{AB}}{\delta} \right) * \ln \left( \frac{1}{1 - 0.5 * x_a} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 553.

**5.38 Diffusion with a homogeneous chemical reaction****Input(s)**

- k: Rate Constant ( $\text{mol cm/s}$ )
- L: Length (cm)
- $D_{AB}$ : Diffusivity ( $\text{cm cm/s}$ )
- c: Total Molar Concentration ( $\text{mol}/\text{ft}^3$ )

**Output(s)**

- $\phi$ : Thiele Modulus (dimensionless)
- N: Molar Flux ( $\text{mol L/cm}^2$ )

**Formula(s)**

$$\phi = \left( \frac{k * (L^2)}{D_{AB}} \right)^{0.5}$$

$$N = \left( \frac{c * D_{AB}}{L} \right) * \phi * \tanh(\pi)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 555.

## 5.39 Diffusion, convection, and chemical reaction

### Input(s)

- $k_1$ : Homogeneous Chemical Reaction Rate Coefficient (1/s)  
 $D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)  
 $v_o$ : Velocity (cm/s)  
 $c_{AO}$ : Fraction of Initial Concentration (dimensionless)  
 $z$ : Vertical Distance (cm)

### Output(s)

- $c_A$ : Fraction of Initial Concentration remaining (dimensionless)

### Formula(s)

$$c_A = (c_{AO}) * \exp \left( - \left( \left( 1 + \frac{4 * k_1 * D_{AB}}{v_o^2} \right)^{0.5} \right) - 1 \right) * \left( \frac{v_o * z}{2 * D_{AB}} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 19, Page: 586.*

## 5.40 Drag coefficient

### Input(s)

- $\rho$ : Mass Density of the Fluid (kg/m<sup>3</sup>)  
 $u$ : Flow Speed of the Object Relative to the Fluid (m/s)  
 $F_d$ : Drag Force (N)  
 $A$ : Cross Sectional Area (m<sup>2</sup>)

### Output(s)

- $c_d$ : Drag Coefficient (dimensionless)

### Formula(s)

$$c_d = \frac{2 * F_d}{\rho * u^2 * A}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.41 Drag force

### Input(s)

- $\rho$ : Density of the Fluid (kg/m<sup>3</sup>)  
 $v$ : Speed of the Object Relative to the Fluid (m/s)  
 $C_d$ : Drag Coefficient (dimensionless)  
 $A$ : Cross Sectional Area (m<sup>2</sup>)

### Output(s)

$F_D$ : Drag Force (N)

### Formula(s)

$$F_D = \left(\frac{1}{2}\right) * \rho * v^2 * C_d * A$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.42 Draining of a cylindrical tank

### Input(s)

$\mu$ : Fluid Viscosity (kg/(ms))  
 $L$ : Height of the Pipe (m)  
 $H$ : Height of the Cylindrical Tank (m)  
 $D$ : Diameter of the Pipe (m)  
 $R$ : Radius of the Cylindrical Tank (m)  
 $\rho$ : Fluid Density (kg/m<sup>3</sup>)  
 $g$ : Gravitational Acceleration (m/s<sup>2</sup>)

### Output(s)

$t_{efflux}$  : Efflux Time (s)

### Formula(s)

$$t_{efflux} = \frac{128 * \mu * L * R^2}{\rho * g * D^4} * \ln \left( 1 + \frac{H}{L} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 228.

## 5.43 Draining of a spherical tank

### Input(s)

$R$ : Radius of the Sphere (m)  
 $L$ : Length of the Pipe (m)  
 $A$ : A Constant Related to Length, Radius, and Height (m<sup>2</sup>/s)

### Output(s)

$t_{efflux}$ : Efflux Time (s)

**Formula(s)**

$$t_{\text{efflux}} = \left( \frac{L^2}{A} \right) * \left( \left( 2 * \frac{R}{L} * \left( 1 + \frac{R}{L} \right) - \left( 1 + 2 * \frac{R}{L} \right) * \ln \left( 1 + 2 * \frac{R}{L} \right) \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 200.*

**5.44 Eckert number****Input(s)**

- V: Characteristic Velocity of Flow (m/s)
- $c_p$ : Constant-Pressure Specific Heat of Flow (m/s<sup>2</sup> K)
- $\Delta T$ : Temperature Difference (K)

**Output(s)**

Ec: Eckert Number = Advection Transport/(Heat Dissipation Potential) (dimensionless)

**Formula(s)**

$$Ec = \frac{V^2}{c_p * \Delta T}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.45 Effective emissivity of a hole****Input(s)**

- e: Emissivity of the Cavity Walls (dimensionless)
- f: Friction Factor (dimensionless)

**Output(s)**

$e_h$ : Emissivity of Hole (dimensionless)

**Formula(s)**

$$e_h = \frac{e}{e + f * (1 - e)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 493.*

**5.46 Effective thermal conductivity for a solid with spherical inclusions****Input(s)**

- T: Initial Temperature (K)
- $k_{\text{eff}}$ : Effective Thermal Conductivity (J/ft s K)

- $k_o$ : Thermal Conductivity of Solid (J/ft s K)
- R: Effective Radius (ft)
- r: Distance from Centre (ft)
- A: Temperature Gradient (K/ft)
- $\Theta$ : Position of Substance from Centre (rad)

### Output(s)

- $T_r$ : Temperature at R (K)

### Formula(s)

$$T_r = T + \left( 1 - \left( \frac{k_{eff} - k_o}{k_{eff} + 2 * k_o} \right) * \left( \left( \frac{R}{r} \right)^3 \right) \right) * A * r * \cos(\Theta)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 11, Page: 371.*

## 5.47 Efflux time for draining a conical tank

### Input(s)

- $z_0$ : Vertical Coordinate at the Top of the Cone (m)
- $z_2$ : Vertical Coordinate just Above the Datum Plane for Potential Energy (m)
- g: Gravitational Acceleration ( $m/s^2$ )

### Output(s)

- $t_{efflux}$ : Efflux Time (s)

### Formula(s)

$$t_{efflux} = \left( \frac{1}{5} \right) * \left( \frac{z_0}{z_2} \right)^2 * \left( 2 * \frac{z_0}{g} \right)^{0.5}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 229.*

## 5.48 Ekman number

### Input(s)

- D: Characteristic Length Scale (m)
- v: Kinematic Eddy Viscosity ( $m^2/s$ )
- $\omega$ : Angular Velocity of Planetary Motion (1/s)
- $\vartheta$ : Latitude (degree)

### Output(s)

- Ek: Ekman Number (dimensionless)

**Formula(s)**

$$Ek = \frac{v}{2 * D^2 * \omega * \sin\left(\theta * \frac{\pi}{180}\right)}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.49 Elimination of circulation in a rising gas bubble****Input(s)**

- R: Radius of bubble (cm)
- $\mu$ : Viscosity (cP)
- $v_\infty$ : Velocity (cm/s)
- $\theta$ : Angle of bubble (rad)

**Output(s)**

- $\tau$ : Stress (kPa)

**Formula(s)**

$$\tau = \frac{3 * \mu * v_\infty * \sin(\theta)}{2 * R}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 701.*

**5.50 Energy emitted from the surface of a black body****Input(s)**

- S: Stephan-Boltzmann Constant ( $\text{lb}/(\text{hr}^3 \text{R}^4)$ )
- T: Temperature (R) A: Area ( $\text{ft}^2$ )

**Output(s)**

- $(q_b)^e$ : Emitted Heat Flux ( $\text{Btu}/\text{h ft}^2$ )

**Formula(s)**

$$(q_b)^e = \rho * (T^4)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 494.*

**5.51 Estimation of diffusivity of liquids****Input(s)**

- $\phi_B$ : Association Number (dimensionless)
- T: Temperature (K)

**M<sub>B</sub>**: Molar Weight (g/g mol)

**μ**: Viscosity (cP)

**V<sub>A</sub>**: Molar Volume (cm<sup>3</sup>/g mol)

### Output(s)

**D<sub>AB</sub>**: Diffusivity (cm<sup>2</sup>/s)

### Formula(s)

$$D_{AB} = 0.000000074 * \frac{((\phi_B * M_B)^{0.5}) * T}{\mu * (V_A^{0.6})}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 530.*

## 5.52 Estimation of self diffusivity at high density

### Input(s)

**a**: Concentration (g mol/cm<sup>3</sup>)

**Daa**: Prediction of Self-Diffusivity (g mol/cm s)

**cDaa<sub>r</sub>**: Reduced Diffusivity (g mol/cm s)

### Output(s)

**cDaa<sub>c</sub>**: Critical Diffusivity (g mol/cm s)

**CDaa**: Prediction of Self-Diffusivity (g mol/cm s)

### Formula(s)

$$cDaa_c = \frac{c * Daa}{cDaa_r}$$

$$CDaa = (cDaa_c) * (cDaa_r)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 523.*

## 5.53 Estimation of the viscosity of a pure liquid

### Input(s)

**N**: Avogadro Number (g/mol)

**h**: Planck Constant (g cm<sup>2</sup>/s)

**V**: Volume of a Mole of Liquid (cm<sup>3</sup>/g mol)

**T<sub>b</sub>**: Boiling Point (Centigrade)

**T**: Ambient Temperature (Celsius)

### Output(s)

**μ**: Viscosity of a Pure Liquid (g/(cm s))

**Formula(s)**

$$\mu = \frac{N * h}{V} * \exp\left(3.8 * \frac{T_b}{T}\right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 31.*

**5.54 Euler number****Input(s)**

- $\rho$ : Density of Fluid ( $\text{kg/m}^3$ )
- $P_u$ : Upstream Pressure ( $\text{kg/ms}^2$ )
- $P_d$ : Downstream Pressure ( $\text{kg/ms}^2$ )
- $V$ : Velocity of Flow ( $\text{m/s}$ )

**Output(s)**

- $\text{Eu}$ : Euler Number (dimensionless)

**Formula(s)**

$$Eu = (P_u - P_d) * \frac{1.0}{\frac{1}{2} * \rho * V^2}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.55 Fanning friction factor (laminar flow)****Input(s)**

- $D$ : Diameter (ft)
- $v$ : Velocity (ft/s)
- $\rho$ : Density (g/cc)
- $\mu$ : Viscosity of Fluid (cP)

**Output(s)**

- $f$ : Friction Factor (dimensionless)

**Formula(s)**

$$f = \frac{16 * \mu}{D * v * \rho}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 181.*

## 5.56 Fanning's friction factor (turbulent flow)

### Input(s)

- D: Diameter (ft)
- v: Velocity of Fluid (ft/s)
- $\rho$ : Density of Fluid (g/cc)
- $\mu$ : Viscosity of Fluid (cP)

### Output(s)

- f: Friction Factor (dimensionless)

### Formula(s)

$$f = \frac{0.0791 * \mu^{0.25}}{(D * v * \rho)^{0.25}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 181.*

## 5.57 Fick's law of binary diffusion

### Input(s)

- $\rho$ : Density (g/cc)
- $D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)
- $dw_a$ : Mass Fraction of A (fraction)
- dy: Difference in Distance (cm)

### Output(s)

- $j_{Ay}$ : Mass Flux (g/cm<sup>2</sup> s)

### Formula(s)

$$j_{Ay} = D_{AB} * (-\rho) * \frac{dw_a}{dy}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 515.*

## 5.58 Film condensation on vertical pipes

### Input(s)

- k: Thermal Conductivity (W/mK)
- $\rho$ : Density (g/cc)
- g: Acceleration due to Gravity (m/s<sup>2</sup>)
- $H_{vap}$ : Heat of Vaporization (N)
- $\mu$ : Viscosity (cP)
- L: Length (m)
- $T_d$ : Dew Point Temperature (K)
- $T_o$ : Outlet Temperature (K)

**Output(s)**

- $h_{ml}$ : Laminar Coefficient of Heat Transfer (W/m K)  
 $h_{mt}$ : Turbulent Coefficient of Heat transfer (W/m K)

**Formula(s)**

$$h_{ml} = \left( \frac{2 * (\sqrt{2})}{3} \right) * \left( \left( \frac{(k^3) * (\rho * 2) * g * H_{vap}}{(\mu^3) * (Td - To) * L} \right)^{0.25} \right)$$

$$h_{mt} = 0.003 * \left( \left( \frac{(k^3) * (\rho * 2) * g * (Td - To) * L}{(\mu^3) * H_{vap}} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 14, Page: 447.*

**5.59 Film condensation on vertical tubes****Input(s)**

- k: Thermal Conductivity (g m/s s s K)  
 $\rho$ : Density (g/cc)  
g: Gravitational Acceleration (m/s s)  
 $\mu$ : Viscosity (cP)  
 $\tau$ : Rate of Condensate Flow per Unit Width to Bottom (m<sup>2</sup>/s)  
*Td*: Dew Point Temperature (K)  
*To*: Temperature Outlet (K)  
L: Length (m)  
 $H_{vap}$ : Heat of Vaporization (N)

**Output(s)**

- $h_{ml}$ : Laminar Coefficient of Heat Transfer (W/m K)  
 $h_{mt}$ : Turbulent Coefficient of Heat Transfer (W/m K)

*Formula(s)*

$$h_{ml} = \frac{4}{3} * \left( \left( \frac{(k^3) * (\rho^2) * g}{3 * \mu * \tau} \right)^{0.334} \right)$$

$$h_{mt} = 0.003 * \left( \left( \frac{(k^3) * (\rho^2) * g * (Td - To) * L}{(\mu^3) * H_{vap}} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 14, Page: 447.*

## 5.60 Film thickness of a falling film on a conical surface

### Input(s)

- $\mu$ : Viscosity (cP)
- $w$ : Mass Rate (lbs/s)
- $L$ : Length (ft)
- $\rho$ : Density (lb)
- $g$ : Acceleration of Gravity (ft/s<sup>2</sup>)
- $\beta$ : Inclination (rad)
- $s$ : Distance from Cone Apex (ft)

### Output(s)

- $\delta$ : Film Thickness (ft)

### Formula(s)

$$\delta = \left( \left( \frac{3 * \mu * L * w}{\pi * (\rho^2) * g * L * \sin(2 * \beta) * s} \right)^{\frac{1}{3}} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 71.*

## 5.61 Flow in a liquid-liquid ejector pump

### Input(s)

- $v_2$ : Outlet Velocity (ft/s)
- $\rho$ : Density of Fluid (g/cc)

### Output(s)

- $v_o$ : Inlet Velocity (ft/s)
- $E_v$ : Energy Dissipation (ft<sup>2</sup>/s<sup>2</sup>)
- $p_2 - p_1$ : Pressure Drop (psi)

### Formula(s)

$$\begin{aligned} v_o &= 1.5 * v_2 \\ p_2 - p_1 &= \left( \frac{1}{18} \right) * \rho * (v_o^2) \\ E_v &= \left( \frac{5}{144} \right) * (v_o^2) \end{aligned}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 211.*

## 5.62 Flow in a slit with uniform cross flow

### Input(s)

- B: Thickness (cm)
- L: Length (cm)
- $W_p$ : Width (cm)
- $v_o$ : Cross-flow Velocity (cm/s)
- $\rho$ : Fluid Density (g/cc)
- $\mu$ : Viscosity (Pa s)
- $P_o$ : Input Pressure (Pa)
- $P_L$ : Output Pressure (Pa)

### Output(s)

- A: Constant (dimensionless)
- w: Mass Flow Rate (g/s)

### Formula(s)

$$A = \frac{B * v_o * \rho}{\mu}$$

$$w = \left( \frac{(P_o - P_L) * (B^3) * W_p}{\mu * L * A} \right) * \left( \frac{1}{2} - \frac{1}{A} + \frac{1}{(\exp(A) - 1)} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 3, Page: 110.*

## 5.63 Flow near a corner

### Input(s)

- c: Constant (dimensionless)
- B: Velocity Gradient at a Surface (1/s)
- x: Length (ft)

### Output(s)

- $v_e$ : External Flow Velocity (ft/s)

### Formula(s)

$$v_e = c * x \left( \frac{B}{2 - B} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 4, Page: 139.*

## 5.64 Flow of power law fluid through a narrow slit

### Input(s)

W: Width (cm)  
 B: Breadth (cm)  
 L: Length (cm)  
 $\rho$ : Density of Fluid (g/cc)  
 $P_o$ : Input Pressure (Pa)  
 $P_L$ : Output Pressure (Pa)  
 m: Power Law Constant (dimensionless)  
 n: Power Law Constant 2 (dimensionless)

### Output(s)

w: Mass Rate of Flow (g/s)

### Formula(s)

$$w = \left( \frac{2 * W * (B^2) * \rho}{\left(\frac{1}{n}\right) + 2} \right) * \left( \left( \frac{(P_o - P_L) * B}{m * L} \right)^{\frac{1}{n}} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 243.

## 5.65 Fluid kinetic force in conduits

### Input(s)

R: Radius (ft)  
 L: Length (ft)  
 $\rho$ : Density of Fluid (lb/ft<sup>3</sup>)  
 v: Velocity of Fluid (ft/s)  
 f: Friction Factor (dimensionless)

### Output(s)

$F_k$ : Force of Fluid Flow (N)

### Formula(s)

$$F_k = \pi * R * L * \rho * (v^2) * f$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 178.

## 5.66 Fluid kinetic force in flow around submerged objects

### Input(s)

R: Radius of Sphere (ft)  
 $\rho$ : Density of Fluid (lb/ft<sup>3</sup>)

- v: Velocity of Fluid (ft/s)  
f: Friction Factor (dimensionless)

**Output(s)**

$F_k$ : Fluid Kinetic Force (N)

**Formula(s)**

$$F_k = \pi * 0.5 * (R^2) * \rho * (v^2) * f$$

Reference: Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons. Chapter: 3, Page: 110.*

**5.67 Form drag****Input(s)**

- R: Radius (ft)  
ρ: Density (ppg)  
g: Acceleration due to Gravity (ft/s<sup>2</sup>)  
μ: Viscosity (cP)  
v: Velocity (ft/s)

**Output(s)**

F: Normal Force (N)

**Formula(s)**

$$F = \left(\frac{4}{3}\right) * \pi * (R^3) * \rho * g + 2 * \pi * \mu * R * v$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 60.*

**5.68 Free air correction—Gravity survey****Input(s)**

$\Delta h$ : Elevation change (m)

**Output(s)**

$\Delta g_z$ : Free Air Correction (mGal)

**Formula(s)**

$$\Delta g_z = 0.3086 * \Delta h$$

Reference: <https://sites.ualberta.ca/~unsworth/UA-classes/210/exams210/210-final-2008-formula-sheet.pdf>.

## 5.69 Free batch expansion of a compressible fluid

### Input(s)

$S_2$ : Output Surface Area ( $\text{ft}^2$ )

$P_1$ : Input Pressure (psi)

$\rho_1$ : Density (ppg)

$\gamma$ : Adiabatic Constant (dimensionless)

### Output(s)

$w_{\max}$ : Discharge Rate (lb/s)

### Formula(s)

$$w_{\max} = S_2 * \left( \left( P_1 * \rho_1 * \gamma * \left( \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \right) \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 15, Page: 474.*

## 5.70 Free convection heat transfer from a vertical plate

### Input(s)

C: Constant of Convection (dimensionless)

Gr: Grashof Number (dimensionless)

Pr: Prandtl (dimensionless)

k: Thermal Conductivity ( $\text{J/m s K}$ )

H: Height (ft)

$T_o$ : Outer Temperature (K)

$T_i$ : Inner Temperature (K)

### Output(s)

Q: Average Heat Transfer Flux ( $\text{J/m}^2 \text{ s}$ )

### Formula(s)

$$Q = \frac{C * k * (T_o - T_i) * ((Gr * Pr)^{0.25})}{H}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 11, Page: 348.*

## 5.71 Friction drag

### Input(s)

$\mu$ : Viscosity (cP)

R: Radius (ft)

$v_\infty$ : Velocity as r Goes to Infinity (ft/s)

**Output(s)**

F: Force (N)

**Formula(s)**

$$F = 4 * \pi * \mu * R * v_\infty$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 60.*

**5.72 Friction factor for creeping flow around a sphere****Input(s)**

D: Diameter (ft)

v: Velocity of Fluid (ft/s)

$\rho$ : Density of Fluid (g/cc)

$\mu$ : Viscosity of Fluid (cP)

**Output(s)**

f: Friction Factor (dimensionless)

**Formula(s)**

$$f = \frac{24 * \mu}{D * v * \rho}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 186.*

**5.73 Friction factor in flow around submerged objects****Input(s)**

g: Gravitational Acceleration (ft/s<sup>2</sup>)

v: Fluid Velocity (ft/s)

D: Diameter (ft)

$\rho$ : Fluid Density (g/cc)

$\rho_{sph}$ : Density of Sphere (g/cc)

**Output(s)**

f: Friction Factor (dimensionless)

**Formula(s)**

$$f = \left( \frac{4 * g * D}{3 * v^2} \right) * \left( \frac{\rho_{sph} - \rho}{\rho} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 179.*

## 5.74 Friction factor in flow through conduits

### Input(s)

D: Diameter of Conduit (ft)  
 L: Length of Conduit (ft)  
 v: Fluid Velocity (ft/s)  
 $P_o$ : Inlet Pressure (psi)  
 $P_L$ : Outlet Pressure (psi)  
 ρ: Density (g/cc)

### Output(s)

f: Friction Factor (dimensionless)

### Formula(s)

$$f = \frac{0.25 * D * (P_o - P_L)}{0.5 * L * \rho * (v^2)}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 178.

## 5.75 Friction factor in packed column (laminar)

### Input(s)

ε: Fractional Void Space (dimensionless)  
 $D_P$ : Diameter of Particles (ft)  
 $v_o$ : Velocity of Fluid (ft/s)  
 ρ: Density of Fluid (g/cc)  
 μ: Viscosity of Fluid (cP)

### Output(s)

f: Friction Factor (dimensionless)

### Formula(s)

$$f = \left( \frac{(1-\varepsilon)^2}{\varepsilon^3} \right) * \left( \frac{75 * \mu}{\rho * D_P * v_o} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 190.

## 5.76 Friction factor in packed column (turbulent)

### Input(s)

ε: Fractional Void Space (dimensionless)

**Output(s)**

f: Friction Factor (dimensionless)

**Formula(s)**

$$f = 0.875 * \left( \frac{1 - \epsilon}{\epsilon^3} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 6, Page: 191.*

**5.77 Galilei number****Input(s)**

v: Characteristic Kinematic Viscosity ( $\text{m}^2/\text{s}$ )  
L: Characteristic Length (m)  
g: Gravitational Acceleration ( $\text{m}/\text{s}^2$ )

**Output(s)**

Ga: Galilei Number (dimensionless)

**Formula(s)**

$$Ga = g * L^3 * \frac{1.0}{v^2}$$

Reference: [wikipedia.org](https://en.wikipedia.org).

**5.78 Gas absorption from rising bubbles for creeping flow****Input(s)**

$D_{AB}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )  
 $v_t$ : Terminal Velocity ( $\text{cm}/\text{s}$ )  
D: Bubble Diameter (cm) ( $N_A$ )  
 $c_{A0}$ : Solubility ( $\text{g}/\text{cm}^3$ )

**Output(s)**

$N_A$ : Average Molar Absorption Rate ( $\text{g mol}/\text{cm}^2 \text{ s}$ )

**Formula(s)**

$$N_A = \sqrt{\left( \frac{4 * D_{AB} * v_t}{3 * \pi * D} \right) * c_{A0}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 561.*

## 5.79 Gas absorption through bubbles

### Input(s)

$D_{AB}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )  
 $v_t$ : Terminal Velocity ( $\text{cm/s}$ )  
 $D$ : Diameter of Bubble (cm)  
 $C_{A0}$ : Concentration ( $\text{g mol/cm}^3$ )

### Output(s)

$N_a$ : Molar Flux ( $\text{g mol/s}$ )

### Formula(s)

$$N_a = \sqrt{\left(\frac{4 * D_{AB} * v_t}{3 * \pi * D}\right) * C_{A0}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 637.*

## 5.80 Gas absorption with chemical reaction in an agitated tank

### Input(s)

$k_1$ : Diffusion Rate ( $\text{g mol/s}$ )  
 $L$ : Length (cm)  
 $D_{AB}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )  
 $V$ : Volume (cc)  
 $S$ : Surface Area ( $\text{cm}^2$ )  
 $\delta$ : Location of Catalytic Bed (fraction)

### Output(s)

$\phi$ : Thiele Modulus (dimensionless)  
 $N$ : Flux ( $\text{g mol/s}$ )

### Formula(s)

$$\phi = \left( \frac{k_1 * (L^2)}{D_{AB}} \right)^{0.5}$$

$$N = \left( \frac{\phi}{\sinh(\phi)} \right) * \left( \cosh(\phi) - \left( \frac{1}{\cosh(\phi) + \left( \frac{V}{\delta * S} \right) * \phi * \sinh(\phi)} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 557.*

## 5.81 Gas absorption with rapid reaction

### Input(s)

c: Concentration (mol/cm<sup>3</sup>)  
 γ: Adiabatic Constant (dimensionless)  
 $D_{AS}$ : Diffusivity (cm<sup>2</sup>/s)  
 t: Time (s)

### Output(s)

$N_{AZO}$ : Rate of Absorption (cc/s)

### Formula(s)

$$N_{AZO} = \left( \frac{c}{\operatorname{erf}\left(\frac{\gamma}{D_{AS}}\right)^{0.5}} \right) * \left( \left( \frac{D_{AS}}{\pi * t} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 618.*

## 5.82 Gas mass rate flow in compressible tube flow

### Input(s)

m: Molecular Mass (g)  
 κ: Boltzmann (W/cm K<sup>4</sup>)  
 T: Temperature (K)  
 $P_o$ : Input Pressure (psi)  
 $P_L$ : Output Pressure (psi)  
 R: Radius (cm)  
 L: Length (cm)

### Output(s)

w: Mass Rate (g/cm)

### Formula(s)

$$w = \left( \left( \frac{2 * m}{\pi * \kappa * T} \right)^{0.5} \right) * \left( \frac{4}{3} * \pi * (R^3) \right) * \left( \frac{P_o - P_L}{L} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 66.*

## 5.83 Graetz number

### Input(s)

DH: Diameter in Round Tubes or Hydraulic Diameter in Arbitrary Cross-section Ducts (m)

- L: Length (m)
- Re: Reynold Number (dimensionless)
- Pr: Prandtl Number (dimensionless)

### Output(s)

- Gz: Graetz Number (dimensionless)

### Formula(s)

$$Gz = DH * Re * \frac{Pr}{L}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.84 Graham equation viscosity ratio

### Input(s)

- $\phi$ : Volume Fraction (dimensionless)
- $\Psi$ : Stream Function (dimensionless)

### Output(s)

- $\mu_r$ : Graham Equation Viscosity Ratio (dimensionless)

### Formula(s)

$$\mu_r = 1 + \frac{5}{2} * \phi + \frac{9}{4} * \left( \frac{1}{\Psi * (1 + 0.5 * \Psi) * (1 + \Psi)^2} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second ed.). John Wiley & Sons. Chapter: 1, Page: 33.

## 5.85 Grashof number

### Input(s)

- $g$ : Acceleration Due to Earth ( $m/s^2$ )
- $\beta$ : Volumetric Thermal Expansion Coefficient ( $1/K$ )
- $T_1$ : Surface Temperature (K)
- $T_o$ : Bulk Temperature (K)
- $I_o$ : Diameter (m)
- $v$ : Kinematic Viscosity ( $m^2/s$ )

### Output(s)

- Gr: Grashof Number (dimensionless)

**Formula(s)**

$$Gr = \frac{g * \beta * (T_1 - T_o) * l_o^3}{v^2}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 11, Page: 355.*

**5.86 Hagen number****Input(s)**

- PG: Pressure Gradient (Pa/m)
- $\rho$ : Fluid Density ( $\text{kg}/\text{m}^3$ )
- L: Characteristic Length (m)
- v: Kinematic Viscosity ( $\text{m}^2/\text{s}$ )

**Output(s)**

- Hg: Hagen Number (dimensionless)

**Formula(s)**

$$Hg = -\frac{1}{\rho} * PG * \frac{L^3}{v^2}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.87 Hagen-Poiseuille equation****Input(s)**

- $P_o$ : Input Pressure (psi)
- $P_L$ : Output Pressure (psi)
- R: Radius (ft)
- w: Mass rate of Flow (lb/s)
- $\rho$ : Density (ppg)
- L: Length (ft)

**Output(s)**

- $\mu$ : Viscosity (cP)

**Formula(s)**

$$\mu = \frac{(P_o - P_L) * \pi * (R^4) * \rho}{8 * w * L}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 51.*

## 5.88 Influence of changing interfacial area on mass transfer

### Input(s)

$c_{AO}$ : Concentration (g mol/cm)

$D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)

t: Time (s)

a: Constant (dimensionless)

### Output(s)

$M_A$ : Molar Rate (mol/s)

### Formula(s)

$$M_A = c_{AO} * \left( \frac{4 * D_{AB} * (t^{2*0.667+1})}{\pi * (2 * 0.667 + 1)} \right)^{0.5}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons. Chapter: 20, Page: 623.

## 5.89 Knudsen flow

### Input(s)

m: Molecular Mass (g)

K: Boltzmann Constant (m<sup>2</sup> kg s<sup>-2</sup> K<sup>-1</sup>)

T: Temperature (Kelvin)

R: Radius (m)

p<sub>o</sub>: Pressure at Initial point (Pa)

p<sub>L</sub>: Pressure at point L (Pa)

L: Length (m)

### Output(s)

w: Mass Rate Flow (kg/s)

### Formula(s)

$$w = \sqrt{2 * \frac{m}{K * T}} * \left( \frac{4}{3} * R^3 \right) * \frac{p_o - p_L}{L},$$

Reference: Wikipedia.org.

## 5.90 Krieger Dougherty equation viscosity ratio

### Input(s)

$\phi$ : Porosity (fraction)

A: Dimensionless Constant (dimensionless)

$\phi_{max}$ : Volume Fraction for the Spheres (fraction)

**Output(s)**

$\mu_r$ : Krieger Dougherty Equation Viscosity Ratio (fraction)

**Formula(s)**

$$\mu_r = \left(1 - \frac{\phi}{\phi_{max}}\right)^{(-A * \phi_{max})}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 33.*

**5.91 Laminar flow along a flat plate (approximate solution)****Input(s)**

$v_\infty$ : Fluid Velocity (ft/s)  
 L: Length of Plate (ft)  
 W: Width of Plate (ft)  
 $\rho$ : Density of Fluid (g/cc)  
 $\mu$ : Viscosity (cP)

**Output(s)**

$F_x$ : Momentum Flux (psi)

**Formula(s)**

$$F_x = 1.293 * \left( (\rho * \mu * L * (W^2) * (v_\infty^3))^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 4, Page: 137.*

**5.92 Laminar flow of an incompressible power-law fluid in a circular tube****Input(s)**

$\rho$ : Density of Fluid (g/cc)  
 $P_o$ : Input Pressure (Pa)  
 $P_L$ : Output Pressure (Pa)  
 R: Radius of Tube (cm)  
 L: Length of Tube (cm)  
 m: Power Law Constant (dimensionless)  
 n: Power Law Constant 2 (dimensionless)

**Output(s)**

w: Mass Rate (g/s)

**Formula(s)**

$$w = \left( \frac{\pi * (R^3) * \rho}{\left(\frac{1}{n}\right) + 3} \right) * \left( \left( \frac{(P_o - P_L) * R}{2 * m * L} \right)^{\frac{1}{n}} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 243.*

**5.93 Laplace number****Input(s)**

- $\rho$ : Density ( $\text{kg/m}^3$ )
- $L$ : Characteristic Length (m)
- $\sigma$ : Particle Hard Shell Dia (N/m)
- $\mu$ : Liquid Viscosity (Pa s)

**Output(s)**

- La: Laplace number (dimensionless)

**Formula(s)**

$$La = \sigma * \rho * \frac{L}{\mu^2}$$

Reference: *Wikipedia.org.*

**5.94 Lewis number****Input(s)**

- $\alpha$ : Thermal Diffusivity ( $\text{m}^2/\text{s}$ )
- $D$ : Mass diffusivity ( $\text{m}^2/\text{s}$ )

**Output(s)**

- Le: Lewis number (dimensionless)

**Formula(s)**

$$Le = \frac{\alpha}{D}$$

Reference: *Wikipedia.org.*

**5.95 Mach number****Input(s)**

- $v$ : Velocity of source relative to medium (m/s)
- $vs$ : Speed of sound in the medium (m/s)

**Output(s)**

M: Mach number (dimensionless)

**Formula(s)**

$$M = \frac{v}{vs}$$

Reference: *Wikipedia.org.*

**5.96 Manning formula****Input(s)**

k: Conversion factor of  $(L^{1/3})/T$ , 1 for SI  $((m^{1/3})/s)$

Rh: Hydraulic radius (m)

S: Slope of the hydraulic grade line or the linear ( $\text{kg}/\text{m}^3$ )

n: Manning coefficient (dimensionless)

**Output(s)**

V: Cross-sectional average velocity (m/s)

**Formula(s)**

$$V = k * Rh^{\frac{2}{3}} * \frac{S^{\frac{1}{2}}}{n}$$

Reference: *Wikipedia.org.*

**5.97 Marangoni number****Input(s)**

SG: Change in Surface Tension per unit Temp (N/m/K)

L: Characteristic length (m)

dT: Speed of sound in the medium (K)

$\alpha$ : Thermal diffusivity ( $\text{m}^2/\text{s}$ )

$\sigma$ : Dynamic viscosity ( $\text{kg}/(\text{s m})$ )

**Output(s)**

Mg: Marangoni number (dimensionless)

**Formula(s)**

$$Mg = SG * L * \frac{dT}{\alpha * \sigma}$$

Reference: *Wikipedia.org.*

## 5.98 Mass absorption (attenuation) coefficient

### Input(s)

- N: Number of Atoms per Unit Volume ( $m^{-3}$ )
- $\sigma$ : Thin Cross Section Expressed in Barns per Atom (1 barn is  $10^{-28} m^2$ ) ( $m^2/atom$ )
- $\rho$ : Density ( $kg/m^3$ )

### Output(s)

- $\alpha_l$ : Linear Absorption Coefficient (1/m)
- $\alpha_m$ : Mass Absorption Coefficient ( $m^2/kg$ )

### Formula(s)

$$\alpha_l = \sigma * N$$

$$\alpha_m = \frac{\alpha_l}{\rho}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 33–34.

## 5.99 Mass flow rate as a function of the modified pressure drop in a network of tubes

### Input(s)

- $P_A$ : Pressure at A (psi)
- $P_B$ : Pressure at B (psi)
- $\mu$ : Viscosity (cP)
- $\rho$ : Density (ppg)
- L: Length (ft)
- R: Radius (ft)

### Output(s)

- w: Mass Flow Rate (lb/s)

### Formula(s)

$$w = \frac{3 * \pi * (P_A - P_B) * (R^4) * \rho}{20 * \mu * L}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 68.

## 5.100 Mass flow rate in a rotating cone pump

### Input(s)

- $P_o$ : Input Pressure (psi)
- $P_L$ : Output Pressure (psi)
- L: Length (ft)
- B: Distance from Centre (ft)

- $\rho$ : Density (ppg)
- $z$ : Distance from Input Side (ft)
- $\beta$ : Inclination (rad)
- $\mu$ : Viscosity (cP)

**Output(s)**

- w: Mass Flow rate (lb/s)

**Formula(s)**

$$w = \frac{-2 * (P_o - P_L) * (B^3) * \rho * 2 * \pi * z * \sin(\beta)}{3} * L * \mu$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 72.*

**5.101 Mass rate of flow****Input(s)**

- $g$ : Acceleration due to Gravity (cm/s<sup>2</sup>)
- $\rho$ : Density (g/cc)
- $\delta$ : Film Thickness (cm)
- $v$ : Kinematic (cm<sup>2</sup>/s)
- W: Width (cm)

**Output(s)**

- w: Mass Rate of Flow (g/s)

**Formula(s)**

$$w = \frac{\rho * g * (\delta^3) * W}{3 * v}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 47.*

**5.102 Mass rate of flow in a squared duct****Input(s)**

- $P_o$ : Input Pressure (psi)
- B: Duct Boundary (ft)
- $\rho$ : Fluid Density (g/cc)
- $\mu$ : Viscosity (cP)
- L: Length (ft)
- $P_L$ : Output Pressure (psi)

**Output(s)**

- w: Mass Rate in Square Duct (g/s)

**Formula(s)**

$$w = \frac{0.563 * (P_o - P_L) * B^4 * \rho}{\mu * L}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 3, Page: 106.*

**5.103 Mass rate of flow of a falling film****Input(s)**

- $\rho$ : Density ( $\text{kg}/\text{m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m}/\text{s}^2$ )
- $\delta$ : Film Thickness (m)
- $W$ : Width (m)
- $\mu$ : Kinematic Viscosity ( $\text{kg}/(\text{ms})$ )
- $\beta$ : Angle of Inclination w.r.t Direction of Gravity (rad)

**Output(s)**

- $w$ : Mass Rate of Flow ( $\text{kg}/\text{s}$ )

**Formula(s)**

$$w = \left( \rho^2 * g * \delta^3 * W * \frac{\cos(\beta)}{3 * \mu} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 46.*

**5.104 Mass rate of flow through a circular tube****Input(s)**

- $P_o$ : Input Pressure (psi)
- $P_L$ : Output Pressure (psi)
- $R$ : Radius (ft)
- $\rho$ : Density (ppg)
- $\mu$ : Viscosity (cP)
- $L$ : Length (ft)

**Output(s)**

- $w$ : Mass Rate ( $\text{lbf}/\text{s}$ )

**Formula(s)**

$$w = \pi * ((P_o - P_L) * (R^4) * \rho) / (8 * \mu * L)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 55.*

### 5.105 Mass transfer for creeping flow around a gas bubble

#### Input(s)

$D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)  
 $v_\infty$ : Velocity (cm/s)  
 $D$ : Diameter of Bubble (cm)  
 $C_{AO}$ : Concentration (g mol/cm)

#### Output(s)

$N_{AOavg}$ : Molar Flux Average (mol/cm<sup>2</sup> s)

#### Formula(s)

$$N_{AOavg} = C_{AO} * \left( \left( \frac{4 * D_{AB} * v_\infty}{3 * \pi * D} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 637.*

### 5.106 Mass transfer to drops and bubbles

#### Input(s)

$\sigma$ : Interfacial Tension (N)  
 $D$ : Diameter (cm)  
 $\rho_d$ : Density of Drops (g/cc)  
 $\rho_c$ : Density of Continuous Medium (g/cc)

#### Output(s)

$\omega$ : Angular Frequency of Oscillation (rad/s)

#### Formula(s)

$$\omega = \left( \left( \frac{192 * \sigma}{(D^3) * (3 * \rho_d + 2 * \rho_c)} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 687.*

### 5.107 Maximum flow rate (Vogel's equation)

#### Input(s)

$q$ : Tested Production Rate (stb/d)  
 $P$ : Reservoir Pressure (psi)  
 $P_w$ : Tested Flowing Bottom Hole Pressure (psi)

### Output(s)

$q_{max}$ : Maximum Flow Rate (stb/d)

### Formula(s)

$$q_{max} = \frac{q}{1 - 0.2 * \left(\frac{P_w}{P}\right) - 0.8 * \left(\frac{P_w}{P}\right)^2}$$

Reference: Boyun, G., William, C., & Ali Ghalambor, G. (2007). *Petroleum Production Engineering: A Computer-Assisted Approach*, Page: 3/34.

## 5.108 Maximum velocity of a falling film

### Input(s)

- $\rho$ : Density ( $\text{kg}/\text{m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m}/\text{s}^2$ )
- $\delta$ : Film Thickness (m)
- $\beta$ : Angle of Inclination w.r.t. Direction of Gravity (rad)
- $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )

### Output(s)

$V_{zmax}$ : Maximum Velocity (m/s)

### Formula(s)

$$V_{zmax} = \frac{\rho * g * (\delta^2) * \cos(\beta)}{2 * \mu}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 45.

## 5.109 Maximum velocity of flow through a circular tube

### Input(s)

- $p_o$ : Pressure at Initial Point (Pa)
- $p_L$ : Pressure at Point L (Pa)
- R: Radius (m)
- $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )
- L: Length (m)

### Output(s)

$v_{zmax}$ : Maximum Velocity (m/s)

**Formula(s)**

$$v_{z\max} = \frac{(p_o - p_L) * R^2}{4 * \mu * L}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 51.*

**5.110 Maximum-velocity V<sub>z</sub>-maximum of a falling film****Input(s)**

- $\rho$ : Density ( $\text{kg}/\text{m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m}/\text{s}^2$ )
- $\delta$ : Film Thickness (m)
- $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )
- $\beta$ : Angle of inclination w.r.t direction of gravity (rad)

**Output(s)**

- $v_z$ : Maximum Velocity (m/s)

**Formula(s)**

$$v_z = \frac{\rho * g * \delta^2 * \cos(\beta)}{2 * \mu}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Page: 45.*

**5.111 Method for separating helium from natural gas****Input(s)**

- $D_{AB}$  : Diffusivity of Helium and Natural gas ( $\text{cm}^2/\text{s}$ )
- $c_{He1}$  : Concentration of Helium (g/s)
- $c_{He2}$  : Concentration of Natural gas (g/s)
- $R_2$ : Outer Radius (cm)
- $R_1$ : Inner Radius (cm)
- $L$ : Length (cm)

**Output(s)**

- $W_{He}$ : Mass Flux (g/cm)

**Formula(s)**

$$W_{He} = 2 * \pi * L * \frac{D_{AB} * (c_{He1} - c_{He2})}{\ln\left(\frac{R_2}{R_1}\right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 573.*

### 5.112 Modified capillary number

#### Input(s)

- $\mu_w$ : Viscosity of water (cP)
- $\mu_o$ : Viscosity of oil (cP)
- $\sigma$ : Interfacial tension (dyn/cm)
- $v$ : Characteristic velocity (ft/D)

#### Output(s)

- Mc: Modified Capillary Number (dimensionless)

#### Formula(s)

$$Mc = \left( \mu_w * \frac{v}{\sigma} \right) * \left( \frac{\mu_w}{\mu_o} \right)^{0.4}$$

Reference: [Petrowiki.org](https://petrowiki.org/).

### 5.113 Modified Van Driest equation

#### Input(s)

- $v$ : Velocity (ft/s)
- $V_m$ : Friction Velocity (ft/s)
- $y$ : Length (ft)

#### Output(s)

- $L$ : Mixing Length (ft)

#### Formula(s)

$$L = 0.4 * y * \left( \frac{1 - \left( -y * \frac{v}{26} * V_m \right)}{\left( 1 - \left( -0.26 * y * \frac{v}{V_m} \right) \right)^{0.5}} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 5, Page: 164.

### 5.114 Momentum flux distribution of flow through a circular tube

#### Input(s)

- $p_0$ : Pressure at Initial Point (Pa)
- $p_L$ : Pressure at Point L (Pa)
- $r$ : Cylindrical Shell of Thickness (m)
- $L$ : Length (m)

**Output(s)**

$\tau_{rz}$ : Momentum Flux Distribution (Pa)

**Formula(s)**

$$\tau_{rz} = \frac{p_o - p_L}{2 * L} * r$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 50.*

**5.115 Momentum flux distribution of flow through an annulus****Input(s)**

$p_o$  : Pressure at Initial Point (Pa)

$p_L$  : Pressure at Point L (Pa)

$R$  : Radius (m)

$L$ : Length (m)

$r$ : Cylindrical Shell of Thickness (m)

$K$ : Ratio of Inner Pipe's Radius to Outer Pipe's Radius (fraction) ( $m^2 \text{ kg s}^{-2} \text{ K}^{-1}$ )

**Output(s)**

$\tau_{rz}$ : Momentum Flux Distribution (Pa)

**Formula(s)**

$$\tau_{rz} = (p_o - p_L) * \frac{R}{2 * L} * \left( r - \frac{1 - K^2}{2 * \ln\left(\frac{1}{K}\right)} * \left( \frac{R}{r} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 55.*

**5.116 Momentum flux profile of fluids in flow of two adjacent immiscible fluids****Input(s)**

$p_o$ : Pressure at Initial Point (Pa)

$p_L$ : Pressure at point L (Pa)

$b$ : Distance (m)

$x$  : Distance in Cartesian Coordinate x (m)

$L$  : Length (m)

$\mu_I$ : Viscosity of More Dense and Viscous Fluid (kg/(ms))

$\mu_{II}$ : Viscosity of Less Dense and Viscous Fluid (kg/(ms))

**Output(s)**

$\tau_{rz}$ : Momentum Flux (Pa)

**Formula(s)**

$$\tau_{rz} = \frac{(p_o - p_L) * b}{L} * \left( \left( \frac{x}{b} \right) - \frac{1}{2} * \frac{\mu_I - \mu_{II}}{\mu_I + \mu_{II}} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 57.*

**5.117 Momentum fluxes for creeping flow into a slot****Input(s)**

- $P_i$ : Pressure input (psi)
- $P_o$ : Pressure output (psi)
- R: Radius of sphere (cm)
- $\kappa$ : Turbulance coefficient (dimensionless)
- $\rho$ : Fluid density (g/cc)
- $\mu$ : viscosity of fluid (cP)
- $\xi$ : angle of contact between two entities (rad)

**Output(s)**

- W: Mass Flow Rate (g/s)

**Formula(s)**

$$W = \frac{\pi * (P_i - P_o) * (R^3) * ((1 - \kappa)^3) * \rho}{12 * \mu * \ln \left( \cot \left( \frac{\xi}{2} \right) \right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.), John Wiley & Sons, Page: 107.*

**5.118 Mooney equation viscosity****Input(s)**

- $\mu_0$ : Viscosity of Suspending Medium (cP)
- $\phi$ : Volume Fraction (fraction)
- $\phi_0$ : Empirical Constant Between 0.74 and 0.52 (fraction)

**Output(s)**

- $\mu_{eff}$ : Effective Viscosity (cP)

**Formula(s)**

$$\mu_{\text{eff}} = \mu_0 * \left( \exp \left( \frac{\frac{5}{2} * \phi}{1 - \left( \frac{\phi}{\phi_0} \right)} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 32.*

**5.119 Non-Newtonian flow in annulus****Input(s)**

- r: Inner Radius (ft)
- R: Outer Radius (ft)
- n: Power Law Constant 2 (dimensionless)
- $\kappa$ : Dilatational Viscosity (cP)
- $v_o$ : Input Velocity (ft/s)
- $\rho$ : Fluid Density (lb/ft<sup>3</sup>)

**Output(s)**

- $v_z$ : Output Velocity (ft/s)
- w: Mass Flow Rate (lb/s)

**Formula(s)**

$$v_z = v_o * \left( \frac{\left( \left( \frac{r}{R} \right)^{1-\left(\frac{1}{n}\right)} \right) - 1}{\left( \kappa^{1-\left(\frac{1}{n}\right)} \right) - 1} \right)$$

$$w = \left( \frac{2 * \pi * R^2 * \rho * v_o}{\left( \kappa^{1-\left(\frac{1}{n}\right)} \right) - 1} \right) * \left( \frac{1 - \left( \kappa^{3-\left(\frac{1}{n}\right)} \right)}{3 - \left( \frac{1}{n} \right)} - \left( \frac{1 - \kappa^2}{2} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 258.*

**5.120 Nusselt number****Input(s)**

- h: Convective Heat Transfer Coefficient of the Fluid (W/(m<sup>2</sup> K))
- L: Characteristic Length (m)
- k: Thermal Conductivity of Fluid (W/m K)

**Output(s)**

- $NuL$ : Nusselt Number (dimensionless)

### Formula(s)

$$NuL = h * \frac{L}{k}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.121 Ohnesorge number

### Input(s)

- $\mu$ : Liquid Viscosity (Pa s or kg/(m s))
- $L$ : Characteristic length (m)
- $\rho$ : Liquid Density (kg/m<sup>3</sup>)
- $\sigma$ : Surface Tension (N/m)

### Output(s)

- Oh: Ohnesorge Number (dimensionless)

### Formula(s)

$$Oh = \frac{\mu}{(\rho * \sigma * L)^{0.5}}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 5.122 Potential flow around a cylinder

### Input(s)

- $v_\infty$ : Fluid Velocity (ft/s)
- $z$ : Length (ft)
- $R$ : Radius of Cylinder (ft)

### Output(s)

- $W(z)$ : Flow Potential (ft<sup>2</sup>/s)

### Formula(s)

$$W(z) = (-v_\infty * R) * \left( \left( \frac{z}{R} \right) + \left( \frac{R}{z} \right) \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 4, Page: 128.

## 5.123 Prandtl number

### Input(s)

- $\nu$ : Kinematic Viscosity (m<sup>2</sup>/s)
- $\alpha$ : Thermal Diffusivity (m<sup>2</sup>/s)

**Output(s)**

Pr: Prandtl Number (dimensionless)

**Formula(s)**

$$\text{Pr} = \frac{\nu}{\alpha}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 268.*

**5.124 Pressure distribution in a creeping flow around a sphere****Input(s)**

$p_o$ : Pressure in the Plane  $z = 0$  far away from the Sphere (Pa)

$\rho$ : Density ( $\text{kg}/\text{m}^3$ )

$g$ : Gravitational Acceleration ( $\text{m}/\text{s}^2$ )

$z$ : Direction (m)

$\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )

$v_\infty$ : Velocity as  $r$  Goes to Infinity (m/s)

$R$ : Radius (m)

$r$ : Cylindrical Shell of Thickness (m)

**Output(s)**

$p$ : Pressure Distribution (Pa)

**Formula(s)**

$$p = p_o - \left( \left( \frac{3}{2} \right) * \left( \mu * \frac{v_\infty}{R} \right) * \left( \frac{R}{r} \right)^2 \right) * \cos(\theta)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 59.*

**5.125 Pressure drop per length of the adsorption unit****Input(s)**

$B$ : Constant (dimensionless)

$C$ : Constant (dimensionless)

$\mu$ : Viscosity (cP)

$v_g$ : Gas Velocity (ft/m)

$\rho_g$ : Gas Density ( $\text{lb}/\text{ft}^3$ )

**Output(s)**

$(\Delta P/L)$ : Pressure Drop per Length (psi/ft)

### Formula(s)

$$\left( \frac{\Delta P}{L} \right) = B * \mu * v_g + C * \rho_g * (v_g^2)$$

Reference: *John M. Campbell., Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page 393.*

### 5.126 Pressure loss due to sudden enlargement

#### Input(s)

- $\rho$ : Density of Fluid (g/cc)
- $v_1$ : Input Velocity (m/s)
- $v_2$ : Output Velocity (m/s)

#### Output(s)

- $\beta$ : Velocity Ratio (dimensionless)
- $p_2 - p_1$ : Pressure Drop (Pa)

### Formula(s)

$$\beta = \frac{v_o}{v_i}$$

$$p_2 - p_1 = \rho * (v_2^2) * \left( \left( \frac{1}{\beta} \right) - 1 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 7, Page: 210.*

### 5.127 Reynolds number

#### Input(s)

- $\rho$ : Fluid Density (ppg)
- $v$ : Bulk Flow Velocity (ft/min)
- $d$ : Diameter (ft)
- $\mu$ : Viscosity (cP)

#### Output(s)

- $N_r$ : Reynolds Number (dimensionless)

### Formula(s)

$$N_r = 1488 * \frac{\rho * v * d}{\mu}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

### 5.128 Schmidt number

#### Input(s)

- v: Kinematic Viscosity ( $\text{m}^2/\text{s}$ )  
D: Mass Diffusivity ( $\text{m}^2/\text{s}$ )

#### Output(s)

- Sc: Schmidt Number (dimensionless)

#### Formula(s)

$$\text{Sc} = \frac{v}{D}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

### 5.129 Sherwood number

#### Input(s)

- $L_o$ : Characteristic Length (m)  
 $k_x$ : Single-phase Mass Transfer Coefficient ( $\text{mol}/(\text{m}^2 \text{ s})$ )  
 $D_{AB}$ : Diffusivity of the Binary System ( $\text{m}^2/\text{s}$ )  
c: Total Molar Concentration ( $\text{mol}/\text{m}^3$ )

#### Output(s)

- Sh: Sherwood number (dimensionless)

#### Formula(s)

$$\text{Sh} = \frac{(k_x * L_o)}{(c * D_{AB})}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 675.*

### 5.130 Slit flow in Bingham fluid

#### Input(s)

- $P_o$ : Input Pressure (psi)  
 $P_L$ : Output Pressure (psi)  
B: Breadth (ft)  
L: Length (ft)  
 $\mu_o$ : Viscosity (cP)  
 $\tau_o$ : Torque ( $\text{lb}/\text{ft s}^2$ )  
W: Width (ft)  
ρ: Density (ppg)

### Output(s)

w: Mass Flow Rate (lb/s)

### Formula(s)

$$w = \left( \frac{2 * (P_o - P_L) * W * B^3 * \rho}{3 * \mu_o * L} \right) * \left( 1 - \left( \frac{3 * \tau_o * L}{2 * (P_o - P_L) * B} \right) + \left( 0.5 * \left( \frac{\tau_o * L}{(P_o - P_L) * B} \right)^3 \right) \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 8, Page: 260.

## 5.131 Smoluchowski equation

### Input(s)

- $\mu_o$ : Viscosity of the Suspension (cP)
- $\phi$ : Volume Fraction of Spheres (dimensionless)
- D: Dielectric Constant ( $A^2 s^2/cm^3$ )
- R: Particle Radius (cm)
- $\zeta$ : Electro kinetic Potential of the Particles (J)
- $k_e$ : Specific Electrical Conductivity of the Suspension (ohm/cm)

### Output(s)

$\mu_{eff}$ : Effective Viscosity (cP)

### Formula(s)

$$\mu_{eff} = \mu_o * \left( 1 + 2.5 * \phi \left( 1 + \frac{\left( \frac{D * \zeta}{2 * \pi * R} \right)^2}{\mu_o * k_e} \right) \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 34.

## 5.132 Stanton number

### Input(s)

- h: Convective heat transfer coefficient of the fluid ( $W/(m^2 K)$ )
- $\rho$ : Density of the fluid ( $kg/m^3$ )
- u: Speed of the fluid (m/s)
- cp: Specific heat of the fluid ( $J/kg K$ )

### Output(s)

St: Stanton number (dimensionless)

**Formula(s)**

$$St = \frac{h}{\rho * u * cp}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.133 Stefan number****Input(s)**

- dt: Temperature difference between phases (Kelvin)
- L: Latent Heat of Melting (Joules)
- cp: Specific heat of the fluid (J/kg K)

**Output(s)**

- Ste.: Stefan number (dimensionless)

**Formula(s)**

$$Ste = cp * \frac{dt}{L}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.134 Stokes number****Input(s)**

- $\tau$ : Relaxation Time of the Particle (s)
- $U_o$ : Fluid Velocity of the Flow Well away from the Obstacle (m/s)
- dc: Characteristic Dimension of the Obstacle (m)

**Output(s)**

- Stk: Stokes Number (dimensionless)

**Formula(s)**

$$Stk = \tau * \frac{U_o}{dc}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**5.135 Strouhal number****Input(s)**

- f: Frequency of vortex shedding (/s)
- L: Characteristic length (m)
- v: Velocity of the fluid (m/s)

### Output(s)

St: Strouhal number (dimensionless)

### Formula(s)

$$St = f * \frac{L}{v}$$

Reference: *Wikipedia.org*.

## 5.136 Taylor dispersion (axial dispersion coefficient)

### Input(s)

R: Radius (cm)

$v_z$ : Average Velocity (cm/s)

$D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)

### Output(s)

K: Axial Dispersion Coefficient (cm<sup>2</sup>/s)

### Formula(s)

$$K = \frac{(R^2) * (v_z^2)}{48 * D_{AB}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 645.*

## 5.137 Taylor equation viscosity

### Input(s)

$\mu_o$ : Viscosity of Suspending Medium (cP)

$\phi$ : Volume Fraction (fraction)

$\mu_1$ : Viscosity of Disperse Phase (cP)

### Output(s)

$\mu_{eff}$ : Effective Viscosity (cP)

### Formula(s)

$$\mu_{eff} = \mu_o * \left( 1 + \left( \frac{\mu_o + \frac{5}{2} * \mu_1}{\mu_o + \mu_1} \right) * \phi \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 34.*

### 5.138 Taylor number

#### Input(s)

- $\omega$ : Characteristic angular velocity (rad/s)
- $R$ : Characteristic linear dimension perpendicular to the rotation axis (m)
- $v$ : Kinematic viscosity (Pas s)

#### Output(s)

- $Ta$ : Taylor number (dimensionless)

#### Formula(s)

$$Ta = 4 * \omega^2 * \frac{R^4}{v^2}$$

Reference: *Wikipedia.org*.

### 5.139 Theory of diffusion in colloidal suspensions

#### Input(s)

- $K$ : Molar Transfer Coefficient (mol/s cm<sup>2</sup>)
- $T$ : Temperature (K)
- $\mu_B$ : Viscosity (cP)
- $R_A$ : Radius (cm)

#### Output(s)

- $D_{AB}$ : Diffusivity (cm<sup>2</sup>/s)

#### Formula(s)

$$D_{AB} = \frac{K * T}{6 * \pi * \mu_B * R_A}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 532.*

### 5.140 Toricelli equation

#### Input(s)

- $g$ : Gravitational Acceleration (ft/(s<sup>2</sup>))
- $h$ : Height (ft)

#### Output(s)

- $V$ : Efflux Velocity (ft/s)

### Formula(s)

$$V = (2 * g * h)^{0.5}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 3, Page: 110.*

## 5.141 Total force of the fluid on the sphere in a creeping flow around a sphere

### Input(s)

- R: Radius (m)
- $\rho$ : Density ( $\text{kg}/\text{m}^3$ )
- $\mu$ : Viscosity ( $\text{kg}/(\text{ms})$ )
- g: Gravitational Acceleration ( $\text{m}/\text{s}^2$ )
- vs: Apparent Velocity (m/s)

### Output(s)

- F: Total Force of Fluid (N)

### Formula(s)

$$F = \frac{4}{3} * \pi * R^3 * \rho * g + 6 * \pi * \mu * R * vs$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Page: 60.*

## 5.142 Velocity distribution in a creeping flow around a sphere

### Input(s)

- $P_i$ : Input Pressure (psi)
- $P_o$ : Output Pressure (psi)
- L: Length (ft)
- $\mu_a$ : Viscosity of A (cP)
- $\mu_b$ : Viscosity of B (cP)
- b: Radius (ft)
- x: Distance from center (ft)

### Output(s)

- v: Velocity (ft/s)

### Formula(s)

$$v = \left( \frac{(P_i - P_o) * (b^2)}{2 * \mu_a * L} \right) * \left( \frac{2 * \mu_a}{\mu_a + \mu_b} + \frac{(\mu_a - \mu_b) * x}{(\mu_a + \mu_b) * b} - \left( \frac{x}{b} \right)^2 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Page: 59.*

### 5.143 Velocity distribution of a falling film with variable viscosity

#### Input(s)

- $\rho$ : Density ( $\text{kg/m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{m/s}^2$ )
- $\delta$ : Film Thickness (m)
- $x$ : Distance in Cartesian Coordinate (x)
- $\mu$ : Viscosity ( $\text{kg/(ms)}$ )
- $\beta$ : Angle of Inclination w.r.t Direction of Gravity (rad)

#### Output(s)

- $v_z$ : Velocity Distribution (m/s)

#### Formula(s)

$$v_z = \frac{(\rho * g * \delta^2) * \cos(\beta) * \left(1 - \left(\frac{x}{\delta}\right)^2\right)}{2 * \mu}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 45.*

### 5.144 Velocity distribution of flow through a circular tube

#### Input(s)

- $p_o$ : Pressure at Initial Point (Pa)
- $p_L$ : Pressure at Point L (Pa)
- $R$ : Radius of the Tube (m)
- $\mu$ : Viscosity ( $\text{kg/(ms)}$ )
- $L$ : Length of the Tube (m)
- $r$ : Cylindrical Shell of Thickness (m)

#### Output(s)

- $v_z$ : Velocity Distribution (m/s)

#### Formula(s)

$$v_z = \frac{(p_o - p_L) * R^2}{4 * \mu * L} * \left(1 - \left(\frac{r}{R}\right)^2\right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 51.*

### 5.145 Velocity profile of fluids in flow of two adjacent immiscible fluids

#### Input(s)

- $P_o$ : Pressure at Initial Point (Pa)
- $P_L$ : Pressure at Point L (Pa)

- b: Half Plane Thickness (m)
- x: Vertical Distance (m)
- L: Length (m)
- $\mu_I$ : Viscosity of More Dense and Viscous Fluid (kg/(ms))
- $\mu_{II}$ : Viscosity of Less Dense and Viscous Fluid (kg/(ms))

### Output(s)

- $v_{zI}$ : Average Velocity (m/s)
- $v_{zII}$ : Average Velocity (m/s)

### Formula(s)

$$v_{zI} = \frac{(P_o - P_L) * b^2}{2 * \mu_I * L} * \left( \frac{2 * \mu_I}{\mu_I + \mu_{II}} + \frac{\mu_I - \mu_{II}}{\mu_I + \mu_{II}} * \frac{x}{b} - \left( \frac{x}{b} \right)^2 \right)$$

$$v_{zII} = \frac{(P_o - P_L) * b^2}{2 * \mu_{II} * L} * \left( \frac{2 * \mu_{II}}{\mu_I + \mu_{II}} + \frac{\mu_I - \mu_{II}}{\mu_I + \mu_{II}} * \frac{x}{b} - \left( \frac{x}{b} \right)^2 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 57.*

## 5.146 Viscosity by a falling-cylinder viscometer

### Input(s)

- $\rho$ : Density of Fluid (ppg)
- $\rho_o$ : Density of Slug (ppg)
- g: Acceleration due to Gravity ( $ft/s^2$ )
- $\kappa$ : Ratio of Inner to Outer Radii (fraction)
- R: Outer Radius (ft)
- $v_o$ : Slug Velocity (ft/s)

### Output(s)

- $\mu$ : Viscosity (cP)

### Formula(s)

$$\mu = \left( \frac{(\rho_o - \rho) * g * ((\kappa * R)^2)}{2 * v_o} \right) * \left( \left( \ln \left( \frac{1}{\kappa} \right) \right) - \frac{1 - \kappa^2}{1 + \kappa^2} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 2, Page: 70.*

## 5.147 Winsauer equation

### Input(s)

- a: Pore Geometry Coefficient (range of 0.35 to 4.78) (dimensionless)
- m: Pore Geometry Coefficient2 (range of 1.14 to 2.52) (dimensionless)
- $\phi$ : Porosity (fraction)

**Output(s)**

F: Formation resistivity factor (dimensionless)

**Formula(s)**

$$F = a * (\phi)^{-m}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 7.

## Chapter 6

# Well log analysis, geophysics, petrophysics formulas, and calculations

### Chapter Outline

6.1 Acoustic transit time	324	6.31 Fresh water-filled porosity (fresh-water-bearing limestones)	336
6.2 Amplitude transmission coefficient in seismic reflection and refraction	324	6.32 $F_{x_0}/F_s$ approach	336
6.3 Apparent intensity reflected by recorder (gamma ray)	325	6.33 Gamma ray log shale index	336
6.4 Apparent resistivity	325	6.34 General form of the Archie equation—Water saturation from resistivity logs	337
6.5 Apparent sorption compressibility	325	6.35 Generalized relationship between formation resistivity factor and porosity (Chevron formula)	337
6.6 Atlas wireline neutron lifetime log	326	6.36 Geometric coefficient for the electrode	338
6.7 Barenblatt-Chorin universal velocity distribution	326	6.37 Geometric coefficient for the lateral device	338
6.8 Coefficient of reflection	327	6.38 Geometric coefficient for the normal sonde	338
6.9 Compaction correction factor for sonic logs in shale lithology	327	6.39 Half thickness value	339
6.10 Composite capture cross section of the formation (Schlumberger thermal decay time tool)	327	6.40 Hingle nonlinear-resistivity/linear-porosity crossplot	339
6.11 Correlation of mud cake resistivity to mud resistivity	328	6.41 Humble equation (formation resistivity factor vs porosity)	340
6.12 Correlation of mud filtrate resistivity to mud resistivity	328	6.42 Integrated radial geometric factor	340
6.13 Diffuse-layer thickness	328	6.43 Lennard Jones potential	340
6.14 Effect of clay on conductivity	329	6.44 Linear absorption (attenuation) coefficient	341
6.15 Effective photoelectric absorption cross section index	329	6.45 Maximum potential for self-potential (SP) log	341
6.16 Electric resistance to a radial current from a wellbore	329	6.46 Mean free path (photon absorption)	342
6.17 Electrochemical potential (SP log)	330	6.47 Membrane potential	342
6.18 Electrokinetic potential (developed across a mud cake)	330	6.48 Neutron lethargy (logarithmic energy decrement)	342
6.19 Electron density index (GR absorption logging)	331	6.49 Neutron porosity of shale zone	343
6.20 Epithermal neutron diffusion coefficient	331	6.50 Oil saturation determination (IE and CDN logs)	343
6.21 Epithermal neutron distribution (epithermal neutron flux)	331	6.51 Pair production (gamma ray interactions)	343
6.22 Fertl and Hammack equation	332	6.52 Phillips equation (sandstones)	344
6.23 Formation conductivity in dual water model	332	6.53 Photoelectric absorption cross sectional area	344
6.24 Formation factor—Archie's equation	332	6.54 Pickett crossplot	345
6.25 Formation factor (Archie's equation with resistivity logs)	333	6.55 Poisson's ratio (seismic arrival time method)	345
6.26 Formation resistivity and permeability (Carothers) relation for limestones	332	6.56 Porosity by using density log data	345
6.27 Formation resistivity and permeability (Carothers) relation for sandstones	332	6.57 Porosity corrected for gas effect	346
6.28 Formation resistivity and porosity relations for carbonate rocks	333	6.58 Porosity-neutron flux relationship	346
6.29 Formation resistivity and porosity relations from well log data based on porter and Carothers data	333	6.59 Rate of radioactive decay	346
6.30 Fraction of total porosity occupied by clays	333	6.60 Relation between concentration of K, Th, or U and recorded total gamma ray signal	347
	334	6.61 Relationship between rock resistivity and water saturation	347
	334	6.62 Relationship between SSP and $R_w$ (NaCl predominant)	348
	334	6.63 Relationship between SSP and $R_w$ (non-ideal shale membrane)	348
	335	6.64 Relationship between SSP and $R_w$ for water containing salts (non-NaCl predominant)	348

<b>6.65 Resistivity of a partially saturated shaly sand with hydrocarbons (<math>V_{sh}</math> models)</b>	<b>349</b>	<b>6.75 Time-average relation in uncompacted formations (porosity/transit time relationships)</b>	<b>353</b>	
<b>6.66 Resistivity of a water-saturated shaly sand (<math>V_{sh}</math> models)</b>	<b>349</b>	<b>6.76 Tortuosity (resistivity logs)</b>	<b>354</b>	
<b>6.67 Rock conductivity (relatively clean water bearing rocks)</b>		<b>6.77 Total rock conductivity</b>	<b>354</b>	
<b>6.68 Shale index from gamma ray spectrometry</b>		<b>6.78 True porosity from sonic log (corrected for compaction)</b>	<b>355</b>	
<b>6.69 Simandoux (total shale) equation</b>		<b>350</b>	<b>6.79 True resistivity—Archie</b>	<b>355</b>
<b>6.70 Sonic porosity (Raymer Hunt Gardner method)</b>		<b>351</b>	<b>6.80 Volumetric photoelectric absorption cross section</b>	<b>355</b>
<b>6.71 Spacing between transmitter and receiver</b>		<b>351</b>	<b>6.81 Water salinity index ratio</b>	<b>356</b>
<b>6.72 Static self potential</b>		<b>352</b>	<b>6.82 Water saturation determination (IE and CDN logs)</b>	<b>356</b>
<b>6.73 Time between the initiation of the pulse and the first arrival acoustic energy at the receiver</b>		<b>352</b>	<b>6.83 Water saturation from neutron tools</b>	<b>356</b>
<b>6.74 Time-average relation in compacted formations (porosity/transit time relationships)</b>	<b>353</b>	<b>353</b>	<b>6.84 Water saturation—Resistivity logs</b>	<b>357</b>
			<b>6.85 Wavelength equation</b>	<b>357</b>
			<b>6.86 Wellbore electric voltage generation</b>	<b>358</b>

## 6.1 Acoustic transit time

### Input(s)

$v$ : Velocity (ft/s)

### Output(s)

$\Delta_t$ : Acoustic Transit Time (micros/ft)

### Formula(s)

$$\Delta_t = \frac{10^6}{v}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 45.

## 6.2 Amplitude transmission coefficient in seismic reflection and refraction

### Input(s)

$Z_1$ : Acoustic Impedance of Layer 1 ( $\text{kg}/\text{m}^2 \text{s}$ )  
 $Z_2$ : Acoustic Impedance of Layer 2 ( $\text{kg}/\text{m}^2 \text{s}$ )

### Output(s)

$T$ : Amplitude Transmission Coefficient (dimensionless)

### Formula(s)

$$T = \frac{2 * Z_1}{Z_2 + Z_1}$$

Reference: <https://sites.ualberta.ca/~unsworth/UA-classes/210/exams210/210-final-2008-formula-sheet.pdf>.

### 6.3 Apparent intensity reflected by recorder (gamma ray)

#### Input(s)

- $J_1$ : First Pulse Rate Response (counts/s)
- $J_2$ : Second Pulse Rate Response (counts/s)
- $t$ : Time (s)
- $r_c$ : Time Constant of Electric Circuit (s)

#### Output(s)

- $J_{a(t)}$ : Apparent Intensity Reflected by Recorder (counts/s)

#### Formula(s)

$$J_{a(t)} = J_1 + (J_2 - J_1) * \left(1 - \exp\left(-\frac{t}{r_c}\right)\right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 7, Page: 148.*

### 6.4 Apparent resistivity

#### Input(s)

- $G_t$ : Geometric Coefficient (m)
- $I$ : Current (Ampere)
- $\Delta V_{12}$ : Potential Difference between two Points (V)

#### Output(s)

- $R$ : Apparent Resistivity (ohm m)

#### Formula(s)

$$R = G_t * \left(\frac{\Delta V_{12}}{I}\right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 5, Page: 93.*

### 6.5 Apparent sorption compressibility

#### Input(s)

- $B_g$ : Gas Formation Factor (bbl/SCF)
- $V_m$ : Langmuir's Constant (dimensionless)
- $\rho_B$ : Bulk Density of the Coal Deposit (gm/cm<sup>3</sup>)
- $b$ : Langmuir's Constant (dimensionless)
- $\phi$ : Porosity (fraction)
- $p$ : Pressure (psi)

### Output(s)

$c_s$ : Apparent Sorption Compressibility (1/psi)

### Formula(s)

$$c_s = \frac{0.17525 * B_g * V_m * \rho_B * b}{\phi * (1 + b * p)^2}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter: 3, Page: 222.

## 6.6 Atlas wireline neutron lifetime log

### Input(s)

$\tau$ : Half-Decay time for Neutron (s)

### Output(s)

$\Sigma$ : Composite Capture Cross Section of the Formation (dimensionless)

### Formula(s)

$$\Sigma = \frac{3.15}{\tau}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4, Chapter 9, Page: 181.

## 6.7 Barenblatt-Chorin universal velocity distribution

### Input(s)

$v_*$ : Velocity of Fluid (ft/s)

Re: Reynolds Number (ft)

$v$ : Molar Velocity (ft/s)

L: Length (ft)

### Output(s)

$v_x$ : Velocity in X-direction (dimensionless)

### Formula(s)

$$\frac{v_x}{v_*} = \left( \left( \frac{1}{3^{0.5}} \right) * \ln(Re) + \frac{5}{2} \right) * \left( \left( L * \frac{v_*}{v} \right)^{\frac{3}{2 * \ln(Re)}} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 5, Page: 161.

## 6.8 Coefficient of reflection

### Input(s)

- $R_1$ : Resistivity of 1st Formation Bed (ohm m)  
 $R_2$ : Resistivity of 2nd Formation Bed (ohm m)

### Output(s)

- $C_R$ : Coefficient of Reflection (dimensionless)

### Formula(s)

$$C_R = \frac{R_1 - R_2}{R_1 + R_2}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 5, Page: 96.

## 6.9 Compaction correction factor for sonic logs in shale lithology

### Input(s)

- $\Delta t_{sh}$ : Adjacent Shale Bed's Transit Time (μs/ft)  
 $c$ : Shale Compaction Coefficient (dimensionless)

### Output(s)

- $C_p$ : Compaction Correction Factor (dimensionless)

### Formula(s)

$$C_p = (\Delta t_{sh}) * \frac{c}{100}$$

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics*, Page: 87.

## 6.10 Composite capture cross section of the formation (Schlumberger thermal decay time tool)

### Input(s)

- $\tau$ : Time Required for Neutron to Diminish to 37% (s)

### Output(s):

- $\Sigma$ : Composite Capture Cross Section of Formation (dimensionless)

**Formula(s)**

$$\Sigma = \frac{4.55}{\tau}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 9, Page: 181.

**6.11 Correlation of mud cake resistivity to mud resistivity****Input(s)**

- $R_m$ : Mud Resistivity (ohm m)  
 $R_{mf}$ : Mud Filtrate Resistivity (ohm m)

**Output(s)**

- $R_{mc}$ : Mud Cake Resistivity (ohm m)

**Formula(s)**

$$R_{mc} = 0.69 * R_{mf} * \left( \frac{R_m}{R_{mf}} \right)^{2.65}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 4, Page: 67.

**6.12 Correlation of mud filtrate resistivity to mud resistivity****Input(s)**

- $K_m$ : Mud Coefficient Varies with Mud Weight (dimensionless)  
 $R_m$ : Mud Resistivity (ohm m)

**Output(s)**

- $R_{mf}$ : Mud Filtrate Resistivity (ohm m)

**Formula(s)**

$$R_{mf} = K_m * (R_m)^{1.07}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 4, Page: 67.

**6.13 Diffuse-layer thickness****Input(s)**

- $n$ : Salt Concentration (moles per liter)

**Output(s)**

$x_d$ : Diffuse-Layer Thickness (angstroms)

**Formula(s)**

$$x_d = 3 * n^{\left(-\frac{1}{2}\right)}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 14.

**6.14 Effect of clay on conductivity****Input(s)**

$C_w$ : Conductivity of Water (1/ohm m)  
 $F$ : Formation Factor (unitless)  
 $C_s$ : Conductivity due to Salinity (1/ohm m)

**Output(s)**

$C_o$ : Conductivity of Oil (1/ohm m)

**Formula(s)**

$$C_o = \left( \frac{C_w}{F} \right) + C_s$$

Reference: Ellis, D.V., Singer, J.M. 2008. *Well Logging for Earth Scientists*. Second Edition. Springer. Chapter: 4, Page: 74.

**6.15 Effective photoelectric absorption cross section index****Input(s)**

$Z$ : Atomic Number (dimensionless)

**Output(s)**

$Pe$ : Effective Photoelectric Absorption Cross-Section Index for the Formation (Barns per Electron)

**Formula(s)**

$$Pe = \left( \frac{Z}{10} \right)^{3.6}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 8, Page: 167.

## 6.16 Electric resistance to a radial current from a wellbore

### Input(s)

- h: Height of Reservoir (ft)
- $r_e$ : Drainage Radius (ft)
- $r_w$ : Wellbore Radius (ft)
- $\sigma$ : Electric Conductivity (ohm/ft)

### Output(s)

- $R_e$ : Electric Resistivity (ohm)

### Formula(s)

$$R_e = \frac{\ln\left(\frac{r_e}{r_w}\right)}{2 * \pi * h * \sigma}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 14, Page: 188.*

## 6.17 Electrochemical potential (SP log)

### Input(s)

- $t_{Cl}$ : Chloride Ion Transference Number (dimensionless)
- R: Gas Constant (8.31) (J/mol K)
- $T_a$ : Absolute Temperature (degree K)
- F: Faraday Constant (96485.336) (s A/mol)
- $a_1$ : Activities of 1st Electrolyte (dimensionless)
- $a_2$ : Activities of 2nd Electrolyte (dimensionless)

### Output(s)

- $E_c$ : Electrochemical Potential (V)

### Formula(s)

$$E_c = 2 * t_{Cl} * \left( \frac{R * T_a}{F} \right) * \ln\left(\frac{a_1}{a_2}\right)$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 6, Page: 131.*

## 6.18 Electrokinetic potential (developed across a mud cake)

### Input(s)

- p: Differential Pressure (psi)
- x: Constants Related to Mud Composition (dimensionless)
- y: Constants Related to Resistivity (dimensionless)

**Output(s)**

$E_k$ : Electrokinetic Potential (V)

**Formula(s)**

$$E_k = x * p^y$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 131.

**6.19 Electron density index (GR absorbtion logging)****Input(s)**

$N_e$ : Number of Electrons Per Unit Volume (per m<sup>3</sup>)

$N_A$ : Avogadro's Number (per mole)

**Output(s)**

$\rho_e$ : Electron Density Index (mole per m<sup>3</sup>)

**Formula(s)**

$$\rho_e = \left( \frac{2 * N_e}{N_A} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 36.

**6.20 Epithermal neutron diffusion coefficient****Input(s)**

$L_e$ : Slowing-Down Length (cm)

$\xi$ : Logarithmic Energy Decrement Per Collision (dimensionless)

$\Sigma_e$ : Macroscopic Scattering of Cross Section (cm<sup>2</sup>/cm<sup>3</sup>)

**Output(s)**

$D_e$ : Epithermal Diffusion Coefficient (dimensionless)

**Formula(s)**

$$D_e = \xi * L_e^2 * \Sigma_e$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 40.

## 6.21 Epithermal neutron distribution (epithermal neutron flux)

### Input(s)

- $N_N$ : Source Strength (neutrons/s)  
 $r$ : Distance from Source (cm)  
 $L_e$ : Slowing Down Length (cm)  
 $D_e$ : Epithermal Diffusion Coefficient (cm)

### Output(s)

$\psi_e(r)$ : Epithermal Neutron Flux at a Distance ( $r$ ) from the Source (neutrons/cm<sup>2</sup>/s)

### Formula(s)

$$\psi_e(r) = \left( \frac{N_N}{4 * \pi * D_e} \right) * \frac{\exp\left(-\frac{r}{L_e}\right)}{r}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 39.

## 6.22 Fertl and Hammack equation

### Input(s)

- $F$ : Formation Resistivity Factor (dimensionless)  
 $\Phi_e$ : Effective Porosity (fraction)  
 $R_t$ : Rock Resistivity (ohm m)  
 $R_w$ : Formation Water Resistivity (ohm m)  
 $V_{sh}$ : Bulk Volume Fraction of Shale (fraction)  
 $R_{sh}$ : Shale Resistivity (ohm m)

### Output(s)

$S_w$ : Water Saturation (fraction)

### Formula(s)

$$S_w = \left( \frac{F * R_w}{R_t} \right)^{0.5} - \frac{V_{sh} * R_w}{0.4 * \Phi_e * (R_{sh})}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 16.

## 6.23 Formation conductivity in dual water model

### Input(s)

- $F_s$ : Shaly-Sand Formation Resistivity Factor (dimensionless)  
 $C_{wf}$ : Free Water Conductivity (m ohm meter)  
 $C_{wb}$ : Bound Water Apparent Conductivity (m ohm meter)  
 $f_{dt}$ : Expansion Factor (dimensionless)  
 $Q_v$ : Volume Concentration of Clay Exchange Cations (meq/mL)  
 $v_Q$ : Volume Equivalent (cm<sup>3</sup>/meq)

**Output(s)**

- $F_o$ : Salinity Dependent Formation Resistivity Factor (dimensionless)  
 $C_o$ : Brine Saturated Rock Conductivity (m ohm meter)

**Formula(s)**

$$F_o = F_s * \left(1 - v_Q * Q_v\right)$$

$$C_o = \left(\frac{1}{F_o}\right) * \left(C_{wf} + f_{dl} * v_Q * Q_v * \left(C_{wb} - C_{wf}\right)\right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 19.*

**6.24 Formation factor—Archie's equation****Input(s)**

- a: Tortuosity Factor (dimensionless)  
 $\Phi$ : Porosity (fraction)  
m: Cementation Factor (dimensionless)

**Output(s)**

- F: Formation Factor (dimensionless)

**Formula(s)**

$$F = \frac{a}{\Phi^m}$$

Notes: a and m are measured from log-log plot of F vs Porosity.

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics, Page: 38.*

**6.25 Formation factor (Archie's equation with resistivity logs)****Input(s)**

- T: Tortuosity (unitless)  
 $\Phi$ : Porosity (fraction)

**Output(s)**

- F: Archie (unitless)

**Formula(s)**

$$F = \frac{T}{\Phi}$$

Reference: Ellis, D.V., Singer, J.M. 2008. *Well Logging for Earth Scientists. Second Edition. Springer. Chapter: 4, Page: 76.*

## 6.26 Formation resistivity and permeability (Carothers) relation for limestones

### Input(s)

F: Formation Resistivity Factor (dimensionless)

### Output(s)

k: Permeability (mD)

### Formula(s)

$$k = \frac{4 * 10^8}{F^{3.65}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 11.

## 6.27 Formation resistivity and permeability (Carothers) relation for sandstones

### Input(s)

F: Formation Resistivity Factor (dimensionless)

### Output(s)

k: Permeability (mD)

### Formula(s)

$$k = \frac{7 * 10^8}{F^{4.5}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 11.

## 6.28 Formation resistivity and porosity relations for carbonate rocks

### Input(s)

a: Constant (Range of 2.2 to 2.5) (dimensionless)

$\Phi$ : Porosity (dimensionless)

### Output(s)

$F_c$ : Formation Resistivity Factor for Chalky Rocks (dimensionless)

$F_{co}$ : Formation Resistivity Factor for Compact Rocks (dimensionless)

$F_f$ : Formation Resistivity Factor for Low Porosity and non-fractured Carbonates, Shell Equation (dimensionless)

**Formula(s)**

$$F_c = \frac{1}{(\Phi)^2}$$

$$F_{cO} = \frac{1}{(\Phi)^a}$$

$$F_l = \frac{1}{(\Phi)^{1.87 + \left(\frac{0.019}{\Phi}\right)}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 10.*

### **6.29 Formation resistivity and porosity relations from well log data based on Porter and Carothers data**

**Input(s)**

$\Phi$ : Porosity (dimensionless)

**Output(s)**

$F_g$ : Formation Resistivity Factor for California Pliocene (dimensionless)

$F_c$ : Formation Resistivity Factor for U.S Gulf Coast Miocene (dimensionless)

**Formula(s)**

$$F_g = \frac{2.45}{(\Phi)^{1.08}}$$

$$F_c = \frac{1.97}{(\Phi)^{1.29}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 9.*

### **6.30 Fraction of total porosity occupied by clays**

**Input(s)**

$\Phi_S$ : Sonic Log Porosity (fraction)

$\Phi_D$ : Density Log Porosity (fraction)

**Output(s)**

q: Fraction of the Total Porosity Occupied by Clays (fraction)

**Formula(s)**

$$q = \frac{(\Phi_S) - (\Phi_D)}{\Phi_S}$$

Reference: Core Laboratories. 2005: *Formation Evaluation and Petrophysics, Page: 98.*

### 6.31 Fresh water-filled porosity (fresh-water-bearing limestones)

#### Input(s)

- $\rho_b$ : Bulk Density ( $\text{g/cm}^3$ )
- $\rho_{ls}$ : Limestone Density ( $\text{g/cm}^3$ )
- $\rho_w$ : Water Density ( $\text{g/cm}^3$ )

#### Output(s)

- $\Phi$ : Water Filled Porosity (fraction)

#### Formula(s)

$$\Phi = \frac{\rho_{ls} - \rho_b}{\rho_{ls} - \rho_w}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 8, Page: 164.

### 6.32 $F_{xo}/F_s$ approach

#### Input(s)

- $F_s$ : Formation Resistivity Factor (dimensionless)
- $S_{xo}$ : Apparent Saturation (fraction)

#### Output(s)

- $F_{xo}$ : Apparent Formation Factor (dimensionless)

#### Formula(s)

$$F_{xo} = \frac{F_s}{S_{xo}^2}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 12, Page: 250.

### 6.33 Gamma ray log shale index

#### Input(s)

- $GR_{cl}$ : Log Response in Clean Beds (API)
- $GR_{sh}$ : Log Response in Shale Beds (API)
- GR: Log Response in Zone of Interest (API)

#### Output(s)

- $I_{GR}$ : Gamma Ray Shale Index (fraction)

**Formula(s)**

$$I_{GR} = \frac{GR - GR_{cl}}{GR_{sh} - GR_{cl}}$$

Reference: *Core Laboratories. 2015. Formation Evaluation and Petrophysics, Page: 63.*

**6.34 General form of the Archie equation—Water saturation from resistivity logs****Input(s)**

- m: Cementation Factor (dimensionless)
- a: Tortuosity Factor (dimensionless)
- $\Phi$ : Porosity (fraction)
- $R_w$ : Resistivity of Water (ohm m)
- $R_t$ : True Resistivity (ohm m)
- n: Saturation Exponent (dimensionless)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \left( \left( \frac{a}{\Phi^m} \right) * \left( \frac{R_w}{R_t} \right) \right)^{\frac{1}{n}}$$

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics, Page: 45.*

**6.35 Generalized relationship between formation resistivity factor and porosity (Chevron formula)****Input(s)**

- $\Phi$ : Porosity (dimensionless)

**Output(s)**

- F: Formation Resistivity Factor (dimensionless)

**Formula(s)**

$$F = \frac{1.13}{(\Phi)^{1.73}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 9.*

### 6.36 Geometric coefficient for the electrode

#### Input(s)

$r_1$ : Distance to 1st Electrodes (m)  
 $r_2$ : Distance to 2nd Electrodes (m)

#### Output(s)

$G_t$ : Geometric Coefficient for the Electrode Array (m)

#### Formula(s)

$$G_t = 4 * \pi * \left( r_1 * \frac{r_2}{r_2 - r_1} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 5, Page: 93.

### 6.37 Geometric coefficient for the lateral device

#### Input(s)

$AM$ : Midpoint Between Electrodes a and M (m)  
 $AN$ : Midpoint Between Electrodes a and N (m)  
 $MN$ : Midpoint Between Electrodes M and N (m)

#### Output(s)

$G_L$ : Geometric Coefficient for the Lateral Device (m)

#### Formula(s)

$$G_L = 4 * \pi * \left( (AM) * \frac{AN}{MN} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 5, Page: 95.

### 6.38 Geometric coefficient for the normal sonde

#### Input(s)

$AM$ : Midpoint Between Electrodes a and M (m)

#### Output(s)

$G_N$ : Geometric Coefficient for the Normal Sonde (m)

**Formula(s)**

$$G_N = 4 * \pi * AM$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 5, Page: 95.*

**6.39 Half thickness value****Input(s)**

$\alpha_l$ : Linear Absorption Coefficient (1/m)

**Output(s)**

$h_{\frac{1}{2}}$ : Half Thickness Value (m)

**Formula(s)**

$$h_{\frac{1}{2}} = \frac{0.693}{\alpha_l} = 0.693 * \bar{h}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 35.*

**6.40 Hingle nonlinear-resistivity/linear-porosity crossplot****Input(s)**

$S_w$ : Water Saturation (fraction)

$R_w$ : Formation Water Resistivity (ohm m)

$\Phi$ : Mud Filtrate Resistivity (fraction)

a: Pore Geometry Coefficient (Range of 0.35 to 4.78) (dimensionless)

n: Saturation Exponent (dimensionless)

m: Pore Geometry Coefficient2 (Range of 1.14 to 2.52) (dimensionless)

**Output(s)**

$R_t$ : Rock Resistivity (ohm m)

**Formula(s)**

$$(R_t)^{-\frac{1}{m}} = \Phi * \left( \frac{S_w^n}{a * R_w} \right)^{\frac{1}{m}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 13, Page: 268.*

## 6.41 Humble equation (formation resistivity factor vs porosity)

### Input(s)

$\Phi$ : Porosity (dimensionless)

### Output(s)

$F$ : Formation Resistivity Factor (dimensionless)

### Formula(s)

$$F = \frac{0.62}{(\Phi)^{2.15}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 8.

## 6.42 Integrated radial geometric factor

### Input(s)

$r$ : Radial Distance from Borehole Wall (in.)

$h$ : Mean Free Path (in.)

### Output(s)

$G(r)$ : Integrated Radial Geometric Factor (dimensionless)

### Formula(s)

$$G(r) = 1 - e^{-\frac{r}{h}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 7, Page: 152.

## 6.43 Lennard Jones potential

### Input(s)

$\epsilon$ : Potential of Well (N m)

$\sigma$ : Collision Diameter (m)

$r$ : Distance (m)

### Output(s)

$\phi_{ir}$ : Intermolecular potential energy (N m)

**Formula(s)**

$$phir = 4 * \varepsilon * \left( \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Page: 26.*

**6.44 Linear absorption (attenuation) coefficient****Input(s)**

$N$ : Number of Atoms Per Unit Volume ( $\text{m}^{-3}$ )

$\sigma$ : Thin Cross Section Expressed in Barns Per Atom (1 Barn Is  $10^{-28} \text{ m}^2$ ) ( $\text{m}^2/\text{atom}$ )

**Output(s)**

$\alpha_l$ : Linear Absorption Coefficient (1/m)

**Formula(s)**

$$\alpha_l = \sigma * N$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 33.*

**6.45 Maximum potential for self-potential (SP) log****Input(s)**

$R_{mfe}$ : Apparent Resistivity of Mud Filtrate (ohm m)

$R_{we}$ : Apparent Formation Water Resistivity (ohm m)

$K$ : Temperature-dependent Coefficient (Temp. Degree)

**Output(s)**

$SSP$ : Maximum Potential (millivolts)

**Formula(s)**

$$SSP = -K \log_{10} \left( \frac{R_{mfe}}{AR_{we}} \right)$$

$$K (\text{°C}) = 61 + 0.133 T$$

$$K (\text{°F}) = 65 + 0.24 T$$

Reference: *Ramey Jr, H. J. (1981). Reservoir Engineering Assessment of Geothermal Systems. Department of Petroleum Engineering, Stanford university, Page (4.5).*

## 6.46 Mean free path (photon absorption)

### Input(s)

$\alpha_l$ : Linear Absorption Coefficient (1/m)

### Output(s)

h: Mean Free Path (m)

### Formula(s)

$$h = \frac{1}{\alpha_l}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 35.

## 6.47 Membrane potential

### Input(s)

R: Gas Constant (8.31) (J/mol K)  
 $T_a$ : Absolute Temperature (degree K)  
F: Faraday Constant (96485.336) (s A/mol)  
 $a_1$ : Activities of 1st Electrolyte (dimensionless)  
 $a_2$ : Activities of 2nd Electrolyte (dimensionless)

### Output(s)

$E_m$ : Membrane Potential (V)

### Formula(s)

$$E_m = \left( R * \frac{T_a}{F} \right) * \ln \left( \frac{a_1}{a_2} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 131.

## 6.48 Neutron lethargy (logarithmic energy decrement)

### Input(s)

$E_o$ : Initial Energy Level (eV)  
E: Final Energy Level (eV)

### Output(s)

u: Neutron Lethargy (dimensionless)

**Formula(s)**

$$u = \ln\left(\frac{E_o}{E}\right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 38.

**6.49 Neutron porosity of shale zone****Input(s)**

- $\Phi_T$ : True Formation Porosity (fraction)
- $V_{Sh}$ : Shale Volume Factor (dimensionless)
- $\Phi_{NSh}$ : Neutron Porosity of a Nearby Shale Region (fraction)

**Output(s)**

- $\Phi_N$ : Observed Neutron Porosity in a Shaly Formation (fraction)

**Formula(s)**

$$\Phi_N = (\Phi_T) + (V_{Sh} * \Phi_{NSh})$$

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics*, Page: 108.

**6.50 Oil saturation determination (IE and CDN logs)****Input(s)**

- $R_o$ : Hydrocarbon Formation Resistivity (ohm m)
- $R_t$ : Rock Resistivity (ohm m)

**Output(s)**

- $S_w$ : Water Saturation (fraction)
- $S_o$ : Oil Saturation (fraction)

**Formula(s)**

$$S_w = \left(\frac{R_o}{R_t}\right)^{0.5}$$

$$S_o = 1 - S_w$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 11, Page: 213.

**6.51 Pair production (gamma ray interactions)****Input(s)**

- $E_c$ : Energy of Compton Electrons (MeV)

E: Energy Required to Remove Electron from Shell (MeV)

### Output(s)

$E_e$ : Energy of Each Photoelectron in Pair Production (MeV)

### Formula(s)

$$E_e = E_c - E$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 33.*

## 6.52 Phillips equation (sandstones)

### Input(s)

$\Phi$ : Porosity (dimensionless)

### Output(s)

F: Formation Resistivity Factor (dimensionless)

### Formula(s)

$$F = \frac{1.45}{(\Phi)^{1.54}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 8.*

## 6.53 Photoelectric absorption cross sectional area

### Input(s)

Z: Atomic Number (dimensionless)

$E_\gamma$ : Energy of Gamma Rays (keV)

### Output(s)

$\alpha_{pe}$ : Photoelectric Absorption Cross Sectional Area (barns/electron)

### Formula(s)

$$\alpha_{pe} = 12.1 * \frac{Z^{3.6}}{E_\gamma^{3.15}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 36.*

## 6.54 Pickett crossplot

### Input(s)

- $S_w$ : Water Saturation (fraction)
- $R_w$ : Formation Water Resistivity (ohm m)
- $\Phi$ : Mud Filtrate Resistivity (fraction)
- n: Saturation Exponent (dimensionless)
- m: Pore Geometry Coefficient2 (Range of 1.14 to 2.52) (dimensionless)

### Output(s)

- logR1: Rock Resistivity (ohm m)

### Formula(s)

$$\log R1 = -m * \log(\Phi) + \log(R_w) - n * \log(S_w)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 13, Page: 276.*

## 6.55 Poisson's ratio (seismic arrival time method)

### Input(s)

- $V_p$ : Velocity of Compressional Waves (ft/s)
- $V_s$ : Shear Waves (ft/s)

### Output(s)

- v: Poisson's Ratio (dimensionless)

### Formula(s)

$$v = \frac{\left(V_p^2\right) - 2 * \left(V_s^2\right)}{2 * \left(\left(V_p^2\right) - \left(V_s^2\right)\right)}$$

Reference: Mark D. Zoback., *Reservoir Geomechanics*, Cambridge University Express, UK, Page: 64.

## 6.56 Porosity by using density log data

### Input(s)

- $\rho_b$ : Bulk Density (g/cm<sup>3</sup>)
- $\rho_{ma}$ : Matrix Density(g/cm<sup>3</sup>)
- $\rho_f$ : Average Fluid Density in Pore Spaces (g/cm<sup>3</sup>)

### Output(s)

- $\Phi$ : Porosity (fraction)

**Formula(s)**

$$\Phi = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 8, Page: 165.

**6.57 Porosity corrected for gas effect****Input(s)**

$\Phi_D$ : Density Porosity (fraction)

$\Phi_N$ : Neutron Porosity (fraction)

**Output(s)**

$\Phi_c$  or  $r$ : Corrected Porosity for the Gas Effect (fraction)

**Formula(s)**

$$\Phi_{c \text{ or } r} = \left( \frac{(\Phi_D)^2 + (\Phi_N)^2}{2} \right)^{0.5}$$

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics*, Page: 111.

**6.58 Porosity-neutron flux relationship****Input(s)**

N: Neutron Tool Response (dimensionless)

$\alpha$ : Constant Related to Formation Properties and Tool Design (dimensionless)

$\beta$ : Constant Related to Formation Properties and Tool Design (dimensionless)

**Output(s)**

$\Phi$ : Porosity (fraction)

**Formula(s)**

$$\Phi = \alpha - \beta * \log(N)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 9, Page: 174.

**6.59 Rate of radioactive decay****Input(s)**

$N_o$ : Initial Parent Nuclei (dimensionless)

$C_d$ : Decay Constant (1/s)

t: Half-Life Time (s)

**Output(s)**

N: Parent Nuclei at a Particular Time (dimensionless)

**Formula(s)**

$$N = N_o * \exp(-C_d * t)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 28.*

## **6.60 Relation between concentration of K, Th, or U and recorded total gamma ray signal**

**Input(s)**

$C_{Th}$ : Thorium Concentration (ppm)

$C_U$ : Uranium Concentration (ppm)

$C_K$ : Potassium Concentration (wt %)

**Output(s)**

$\gamma$ : Total Gamma Ray (API units)

**Formula(s)**

$$\gamma = 4 * C_{Th} + 8 * C_U + C_K$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 7, Page: 152.*

## **6.61 Relationship between rock resistivity and water saturation**

**Input(s)**

F: Formation Resistivity Factor (dimensionless)

$R_w$ : Formation Water Resistivity (ohm m)

$R_t$ : Rock Resistivity (ohm m)

n: Saturation Exponent (dimensionless)

**Output(s)**

$S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \left( \frac{F * R_w}{R_t} \right)^{\frac{1}{n}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 11.*

## 6.62 Relationship between SSP and $R_w$ (NaCl predominant)

### Input(s)

$R_{(mf)_{eq}}$ : Equivalent Resistivity of Mud Filtrate (ohm m)

K: Equilibrium Constant (mV)

A: Proportionality Factor (dimensionless)

$a_w$ : Activity of NaCl (dimensionless)

### Output(s)

$(R_w)_{eq}$ : Equivalent Resistivity of Formation Water (ohm m)

SSP: Static Self Potential (V)

### Formula(s)

$$(R_w)_{eq} = \left( \frac{A}{a_w} \right)$$

$$SSP = -K * \log \left( \left( \frac{R_{(mf)_{eq}}}{(R_w)_{eq}} \right) \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 133.

## 6.63 Relationship between SSP and $R_w$ (non-ideal shale membrane)

### Input(s)

$R_{sh}$ : Shale Resistivity (ohm m)

### Output(s)

$m_{eff}$ : Membrane Efficiency (fraction)

### Formula(s)

$$m_{eff} = 0.47 + 0.3 * R_{sh}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 137.

## 6.64 Relationship between SSP and $R_w$ for water containing salts (non-NaCl predominant)

### Input(s)

$a_{Na}$ : Activity of Na Ions in Water (dimensionless)

K: Equilibrium Constant (mV)

$a_{Ca}$ : Activity of Ca Ions in Water (dimensionless)

$a_{Mg}$ : Activity of Mg Ions in Water (dimensionless)

$a_{Nam}$ : Activity of Na Ions in Mud Filtrate (dimensionless)

$a_{Cam}$ : Activity of Ca Ions in Mud Filtrate (dimensionless)

$a_{Mgm}$ : Activity of Mg Ions in Mud Filtrate (dimensionless)

**Output(s)**

$E_{ssp}$ : Static Self Potential (V)

**Formula(s)**

$$E_{ssp} = -K * \log \left( \frac{\left( a_{Na} + (a_{Ca} + a_{Mg})^{0.5} \right)}{\left( a_{Nam} + (a_{Cam} + a_{Mgm})^{0.5} \right)} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 135.

**6.65 Resistivity of a partially saturated shaly sand with hydrocarbons ( $V_{sh}$  models)****Input(s)**

- $S_w$ : Water Saturation (fraction)
- $R_w$ : Formation Water Resistivity (ohm m)
- $V_{sh}$ : Bulk Volume Fraction of Shale (fraction)
- $R_{sh}$ : Shale Resistivity (ohm meter)
- F: Formation Resistivity Factor (dimensionless)

**Output(s)**

- $\alpha$ : Clay Distribution Factor (dimensionless)
- $\beta$ : Clay Distribution Factor (dimensionless)
- $R_t$ : Rock Resistivity (ohm m)

**Formula(s)**

$$\begin{aligned}\alpha &= \frac{V_{sh}}{R_{sh}} \\ \beta &= 1/F \\ \frac{1}{R_t} &= (\alpha * S_w) + \frac{\beta * (S_w)^2}{R_w}\end{aligned}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 16.

**6.66 Resistivity of a water-saturated shaly sand ( $V_{sh}$  models)****Input(s)**

- $R_w$ : Formation Water Resistivity (ohm m)
- $V_{sh}$ : Bulk Volume Fraction of Shale (fraction)
- $R_{sh}$ : Shale Resistivity (ohm m)
- F: Formation Resistivity Factor (dimensionless)

**Output(s)**

- $\alpha$ : Clay Distribution Factor (dimensionless)
- $\beta$ : Clay Distribution Factor (dimensionless)
- $\frac{1}{R_o}$ : Formation Hydrocarbon Resistivity (per(ohm m))

**Formula(s)**

$$\alpha = \left( \frac{V_{sh}}{R_{sh}} \right)$$

$$\beta = \frac{1}{F}$$

$$\frac{1}{R_o} = (\alpha) + \left( \frac{\beta}{R_w} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 16.

**6.67 Rock conductivity (relatively clean water bearing rocks)****Input(s)**

- $F$ : Formation Resistivity Factor (dimensionless)
- $C_w$ : Formation Water Conductivity (micromhos per centimeter)

**Output(s)**

- $C_o$ : Rock Conductivity (micromhos per centimeter)

**Formula(s)**

$$C_o = \frac{C_w}{F}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 13.

**6.68 Shale index from gamma ray spectrometry****Input(s)**

- $C_{Th_{log}}$ : Log Response for Thorium Curve (ppm)
- $C_{Th_{min}}$ : Log Response in Zone with Min Radioactivity of Thorium (ppm)
- $C_{Th_{sh}}$ : Log Response of Thorium for Shale (ppm)
- $C_{K_{log}}$ : Log Response of Potassium Curve (ppm)
- $C_{K_{min}}$ : Log Response in Zone with Min Radioactivity of Potassium (ppm)
- $C_{K_{sh}}$ : Log Response of Potassium for Shale (ppm)
- $\gamma_{(uf)_{log}}$ : Log Response for Uranium Free Curve (API units)
- $\gamma_{(uf)_{min}}$ : Log Response in Zone with Min Radioactivity of Uranium (API units)
- $\gamma_{(uf)_{sh}}$ : Log Response of Uranium for Shale (API units)

**Output(s)**

- $I_{(sh)_{Th}}$ : Shale Index for Thorium (dimensionless)
- $I_{(sh)_K}$ : Shale Index for Potassium (dimensionless)
- $I_{(sh)_{Uf}}$ : Shale Index for Uranium (dimensionless)

**Formula(s)**

$$I_{(sh)_{Th}} = \frac{\left[ C_{Th_{log}} - C_{Th_{min}} \right]}{\left[ C_{Th_{sh}} - C_{Th_{min}} \right]}$$

$$I_{(sh)_K} = \frac{C_{K_{log}} - C_{K_{min}}}{C_{K_{sh}} - C_{K_{min}}}$$

$$I_{(sh)_{Uf}} = \frac{\left[ \gamma_{(uf)_{log}} - \gamma_{(uf)_{min}} \right]}{\left[ \gamma_{(uf)_{sh}} - \gamma_{(uf)_{min}} \right]}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 7, Page: 156.

**6.69 Simandoux (total shale) equation****Input(s)**

- $\Phi_e$ : Effective Porosity (fraction)
- $R_t$ : Rock Resistivity (dimensionless)
- $R_{vw}$ : Formation Water Resistivity (ohm m)
- $V_{sh}$ : Bulk Volume Fraction of Shale (fraction)
- $R_{sh}$ : Shale Resistivity (ohm m)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \left( 0.4 * \frac{R_w}{\Phi_e^2} \right) * \left( -\frac{V_{sh}}{R_{sh}} + \left( \left( \frac{V_{sh}}{R_{sh}} \right)^2 + \left( \frac{5 * (\Phi_e)^2}{R_w * R_t} \right) \right)^{0.5} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 1, Page: 16.

**6.70 Sonic porosity (Raymer Hunt Gardner method)****Input(s)**

- $\Delta t$ : Sonic Log Reading for Transit Time ( $\mu\text{mu s}/\text{ft}$ )
- $\Delta t_{ma}$ : Sonic Log Reading for Transit Time ( $\mu\text{-s}/\text{ft}$ )
- C: Empirical Constant (0.624-0.7) (dimensionless)

**Output(s)**

$\Phi_{sonic}$ : Sonic Porosity (fraction)

**Formula(s)**

$$\Phi_{sonic} = C * \frac{\Delta t - \Delta t_{ma}}{\Delta t}$$

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics*, Page: 87.

**6.71 Spacing between transmitter and receiver****Input(s)**

$s_{off}$ : Tool Standoff (in.)

$C_{mf}$ : Mud/Formation Velocity Contrast (dimensionless)

**Output(s)**

$(L_s)_c$ : Transmitter-to-Receiver Spacing (ft)

**Formula(s)**

$$(L_s)_c = 2 * s_{off} * \left( \frac{1 + C_{mf}}{1 - C_{mf}} \right)^{0.5}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 10, Page: 192.

**6.72 Static self potential****Input(s)**

$t_{Cl}$ : Chlorine Anion Transference Number (dimensionless)

R: Gas Constant (J/degree C)

$T_a$ : Absolute Temperature (degree K)

F: Faraday Constant (96485.336) (s A/mol)

$a_w$ : Activity of NaCl (dimensionless)

$a_{mf}$ : Activity of Dilute Solution (dimensionless)

**Output(s)**

K: Equilibrium Constant (mV)

$E_{ssp}$ : Static Self Potential (V)

**Formula(s)**

$$K = 4.606 * t_{Cl} * \left( R * \frac{T_a}{F} \right)$$

$$E_{ssp} = -K * \log \left( \frac{a_w}{a_{mf}} \right)$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 6, Page: 131–132.

## 6.73 Time between the initiation of the pulse and the first arrival acoustic energy at the receiver

### Input(s)

- $d_h$ : Borehole Diameter (in.)
- $d_t$ : Tool Diameter (in.)
- $L_s$ : Spacing (ft)
- $l_c$ : Displaced Distance (ft)
- $v$ : Formation Compressional Velocity ( $\mu\text{s}/\text{ft}$ )
- $v_m$ : Mud Compressional Velocity ( $\mu\text{s}/\text{ft}$ )

### Output(s)

- $t_{log}$ : Time Between Initiation of the Pulse and First Arrival Acoustic Energy at the Receiver ( $\mu\text{s}/\text{ft}$ )

### Formula(s)

$$t_{log} = \left( \frac{L_s}{v} \right) + \left( \frac{d_h - (d_t + 2 * l_c)}{v_m} \right) * \sqrt{1 - \left( \frac{v_m}{v} \right)^2}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 10, Page: 189.

## 6.74 Time-average relation in compacted formations (porosity/transit time relationships)

### Input(s)

- $\delta t$ : Total Travel Time ( $\mu\text{s}/\text{ft}$ )
- $\delta t_f$ : Travel Time in Liquid/Fluid ( $\mu\text{s}/\text{ft}$ )
- $\delta t_{ma}$ : Travel Time in Matrix ( $\mu\text{s}/\text{ft}$ )

### Output(s)

- $\phi$ : Porosity (fraction)

### Formula(s)

$$\phi = \frac{\delta t - \delta t_{ma}}{\delta t_f - \delta t_{ma}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 54.

## 6.75 Time-average relation in uncompacted formations (porosity/transit time relationships)

### Input(s)

- $\delta t$ : Total Travel Time ( $\mu\text{s}/\text{ft}$ )
- $\delta t_f$ : Travel Time in Liquid/Fluid ( $\mu\text{s}/\text{ft}$ )
- $\delta t_{ma}$ : Travel Time in Matrix ( $\mu\text{s}/\text{ft}$ )
- $\delta t_{sh}$ : Transit Time in Adjacent Shales ( $\mu\text{s}/\text{ft}$ )

**Output(s)**

$B_{cp}$ : Compaction Correction Factor ( $\mu\text{s}/\text{ft}$ )  
 $\emptyset$ : Porosity (fraction)

**Formula(s)**

$$B_{cp} = \frac{\delta t_{sh}}{100}$$

$$\emptyset = \frac{\frac{\delta t - \delta t_{ma}}{\delta t_f - \delta t_{ma}}}{B_{cp}}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 3, Page: 54.

**6.76 Tortuosity (resistivity logs)****Input(s)**

$L_a$ : Apparent Length (cm)  
 $L$ : Actual Length (cm)

**Output(s)**

$T$ : Tortuosity (unitless)

**Formula(s)**

$$T = \frac{L_a^2}{L^2}$$

Reference: Ellis, D.V., Singer, J.M. 2008. *Well Logging for Earth Scientists*. Second Edition. Springer. Chapter: 4, Page: 76.

**6.77 Total rock conductivity****Input(s)**

$C_w$ : Conductivity of Water ( $1/\text{ohm m}$ )  
 $S_w$ : Saturation of Water (fraction)  
 $\emptyset$ : Porosity (fraction)  
 $E$ : Electrical Efficiency (fraction)

**Output(s)**

$C_t$ : Total Conductivity of Rock ( $1/\text{ohm m}$ )

**Formula(s)**

$$C_t = C_w * S_w * \emptyset * E$$

Reference: *Ellis, D.V., Siger, J.M. 2008. Well Logging for Earth Scientists, Elsevier, 2nd Edition, Chapter: 4, Page: 77.*

**6.78 True porosity from sonic log (corrected for compaction)****Input(s)**

$\Phi_a$ : Calculated Sonic Porosity Without Compaction Correction (fraction)

$C_p$ : Compaction Factor (dimensionless)

**Output(s)**

$\Phi_t$ : True Porosity (fraction)

**Formula(s)**

$$\Phi_t = \frac{\Phi_a}{C_p}$$

Notes: Works best for unconsolidated sands where Wyllie Time Average Equation Overestimates.

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics, Page: 90.*

**6.79 True resistivity—Archie****Input(s)**

$F$ : Factor of Formation (dimensionless)

$R_w$ : Resistivity of the Saturating Brine (ohm length)

$RI$ : Resistivity Index of Saturation (dimensionless)

**Output(s)**

$R_t$ : True Resistivity (ohm length)

**Formula(s)**

$$R_t = F * R_w * RI$$

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics, Page: 38.*

**6.80 Volumetric photoelectric absorption cross section****Input(s)**

$\rho_e$ : Electron Density Index ( $\text{gm/cm}^3$ )

$P_e$ : Effective Photoelectric Absorption Cross-Section Index (Barns per Electron)

**Output(s)**

$U$ : Volumetric Photoelectric Absorption Cross-Section (Barns per Volume)

**Formula(s)**

$$U = P_e * \rho_e$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 8, Page: 167.*

**6.81 Water salinity index ratio****Input(s)**

- C: Water Salinity (ppm)
- $C_a$ : Apparent Water Salinity (ppm)
- $\phi$ : Porosity (fraction)
- $\phi_a$ : Apparent Porosity (fraction)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \frac{C_a}{C} = \frac{\phi_a}{\phi}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 9, Page: 183.*

**6.82 Water saturation determination (IE and CDN logs)****Input(s)**

- $R_o$ : Hydrocarbon Formation Resistivity (ohm m)
- $R_t$ : Rock Resistivity (ohm m)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \left( \frac{R_o}{R_t} \right)^{0.5}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 11, Page: 213.*

**6.83 Water saturation from neutron tools****Input(s)**

- $\Sigma_{log}$ : Measured Capture Cross Section of Formation (dimensionless)
- $\Sigma_{ma}$ : Capture Cross Section of Matrix (dimensionless)

- $\Sigma_w$ : Capture Cross Section of Water (dimensionless)  
 $\Sigma_h$ : Capture Cross Section of Hydrocarbon (dimensionless)  
 $\Phi$ : Porosity (fraction)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \frac{(\Sigma_{\log} - \Sigma_{ma}) - \Phi * (\Sigma_h - \Sigma_{ma})}{\Phi * (\Sigma_w - \Sigma_h)}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 9, Page: 181.

**6.84 Water saturation—Resistivity logs****Input(s)**

- n: Saturation Exponent (dimensionless)  
 $RI$ : Resistivity Index (dimensionless)

**Output(s)**

- $S_w$ : Water Saturation (fraction)

**Formula(s)**

$$S_w = \left( \frac{1}{RI} \right)^{\frac{1}{n}}$$

Reference: Core Laboratories. 2005. *Formation Evaluation and Petrophysics*, Page: 42.

**6.85 Wavelength equation****Input(s)**

- C: Speed of Light (m/s)  
f: Frequency (Hz)

**Output(s)**

- $\lambda$ : Wavelength (m)

**Formula(s)**

$$\lambda = \frac{C}{f}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 2, Page: 28.

## 6.86 Wellbore electric voltage generation

### Input(s)

- $V_{e,w}$ : Electric Voltage at Wellbore Radius (V)  
 $\Delta V_e$ : Voltage Drop Between Wellbore Radius and Drainage Radius (V)  
 $r$ : Specific Location (ft)  
 $r_e$ : Drainage Radius (ft)  
 $r_w$ : Wellbore Radius (ft)

### Output(s)

- $V$ : Voltage (V)

### Formula(s)

$$V = V_{e,w} - \Delta V_e * \left[ \frac{\ln\left(\frac{r}{r_w}\right)}{\ln\left(\frac{r_e}{r_w}\right)} \right]$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 14, Page: 188.

## Chapter 7

# Petroleum economics formulas and calculations

### Chapter Outline

7.1 Acceptable reliability level	359	7.21 Meterage model	368
7.2 Additional production estimation with new wells	360	7.22 Minimum number of jobs to survive in a minimum chance scenario	368
7.3 Annual gross revenue after royalties and wellhead taxes	360	7.23 Minimum profit ratio per a risky job	369
7.4 Annuity from future value	360	7.24 Net cash flow	369
7.5 Annuity from present value	361	7.25 Net present value	369
7.6 Average annual rate of return method	361	7.26 Operating cash income	370
7.7 Average book rate of return method	362	7.27 Payback period	370
7.8 Calculation of unknown interest rate	362	7.28 Present value of an annuity	370
7.9 Compound interest	363	7.29 Present value of a deferred annuity	371
7.10 Cost depletion	363	7.30 Present value of future sum	371
7.11 Cumulative interest on operational expenses during the lifetime of a well	364	7.31 Present value of profit/investment ratio for an oil well	372
7.12 Effective interest rate for periodic compounding	364	7.32 Present value of uniform gradient series	372
7.13 Exploration efficiency	364	7.33 Present worth expectation for a risky job	373
7.14 Future value of an annuity	365	7.34 Probability of an oilfield discovery	373
7.15 Future value of present sum	365	7.35 Profitability index	373
7.16 Generalized expected value calculation	366	7.36 Rate of growth per unit of exploration length	374
7.17 Growth rate of return for continuous compounding	366	7.37 Simple interest	374
7.18 Hoskold method for annual rate of return prediction-1	366	7.38 Total expected additional production discovery	375
7.19 Hoskold method for annual rate of return prediction-2	367	7.39 Total expected additional production discovery in constant production per unit area	375
7.20 Initial capital needed to survive in a minimum chance scenario	367	7.40 Total new production area estimation expected to be discovered	376

### 7.1 Acceptable reliability level

#### Input(s)

$N$ : Number of Exploration Wells (dimensionless)

$P_{(x)}$ : Binomial Probability of Discovering  $X$  number of Fields with Total  $N$  number of Exploration Wells (fraction)

$P_F$ : Probability of having  $X$  number of fields that averagely includes minimum  $F$  barrels of Oil (fraction)

#### Output(s)

$L_{Rel}$ : Acceptable Reliability Level (dimensionless)

### Formula(s)

$$L_{Rel} = \sum_{X=1}^{X=N} P_{(x)} \cdot P_F$$

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 89.

## 7.2 Additional production estimation with new wells

### Input(s)

$K_i$ : Number of Fields Discovered up to date in Selected Field Class (dimensionless)

$K_{i\_add}$ : Number of Fields estimated to be discovered with additional number of Exploration Wells (dimensionless)

$N_{p\_avg}$ : Average Annual Production of a Field in the Region (dimensionless)

### Output(s)

$N_{p\_add}$ : Additional Annual Production of Fields estimated to be explored (dimensionless)

### Formula(s)

$$N_{p\_add} = (K_i - K_{i\_add}) (N_{p\_avg})$$

Note: All Formulas need to be applied in each Field Class separately.

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 80.

## 7.3 Annual gross revenue after royalties and wellhead taxes

### Input(s)

$V_{u,j}$ : Average Unit Crude Price after Royalties and Wellhead Taxes (\$/bbl)

$\Delta N_{p,j}$ : Annual Oil Production (bbl)

### Output(s)

$V_j$ : Annual Gross Revenue After Royalties and Wellhead Taxes (\$)

### Formula(s)

$$V_j = V_{u,j} * \Delta N_{p,j}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 12, Page: 162.

## 7.4 Annuity from future value

### Input(s)

$i_e$ : Effective Interest or Discount Rate (fraction)

$t$ : Time (year)

$F_v$ : Future Value (currency unit)

**Output(s)**

$A_v$ : Annuity from Future Value (currency unit)

**Formula(s)**

$$A_v = F_v * \left( \frac{i_e}{((1 + i_e)^t) - 1} \right)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 67.

**7.5 Annuity from present value****Input(s)**

$i_e$ : Effective Interest or Discount Rate (fraction)

$t$ : Time (year)

$P_v$ : Annuity from Present Value (currency unit)

**Output(s)**

$A_v$ : Annuity from Present Value (currency unit)

**Formula(s)**

$$A_v = P_v * \left( \frac{i_e * ((1 + i_e)^t)}{((1 + i_e)^t) - 1} \right)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 56.

**7.6 Average annual rate of return method****Input(s)**

$C$ : Initial Investment/Capital (\$)

$I$ : Interest paid by "i" rate for the Capital (\$)

$i$ : Rate of Interest (per cent)

$B$ : Total Balance not Amortized (\$)

$D$ : Present worth Factor (dimensionless)

**Output(s)**

$P$ : Profit (\$)

$E$ : Total Net Undiscounted Cash Flow during the whole project (\$)

$r$ : Annual Average Rate of Return (\$)

**Formula(s)**

$$P = i \sum B$$

$$E = C \cdot I \cdot P$$

$$r = i \frac{D}{1 - D} \left( \frac{E}{C} - 1 \right)$$

$$C \leq \frac{DE}{\frac{r}{i} - D \left( \frac{r}{i} - 1 \right)}$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 40.

## 7.7 Average book rate of return method

### Input(s)

- $C$ : Initial Investment/Capital (\$)
- $E$ : Total Net Undiscounted Cash Flow during the whole project (\$)
- $n$ : Life of the Project (years)
- $N$ : Total Recoverable Oil Amount (bbl)
- $N_k$ : Total Recoverable Oil Amount in  $k^{\text{th}}$  year (bbl)
- $N_p$ : Total Recoverable Oil Amount in  $n^{\text{th}}$  year (bbl)
- $Q_k$ : Oil Production in  $k^{\text{th}}$  year (bbl)
- $D$ : Present worth Factor (dimensionless)
- $W_p$ : Working Profit per barrel of Oil (\$/bbl)

### Output(s)

- $P$ : Profit (\$)
- $\sum B$ : Total End of Year Balance not Amortized (\$)
- $\sum B_{\text{md}}$ : Total Mid-Year Balance not Amortized (\$)
- $r$ : Annual Average Rate of Return (\$)
- $r_{\text{md}}$ : Average Mid Year Rate of Return (\$)

### Formula(s)

$$\sum B = \frac{C}{N} \sum_{k=1}^n k \cdot Q_k$$

$$\sum B_{\text{md}} = C \left( n + \frac{1}{2} \right) - \frac{C}{N_p} \sum_{k=1}^n N_k$$

$$r = \frac{(E \cdot W_p) - C}{\sum B}$$

$$r_{\text{md}} = \frac{(E \cdot W_p) - C}{\sum B_{\text{md}}}$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 45.

## 7.8 Calculation of unknown interest rate

### Input(s)

- $F$ : Amount to be Payed After End of  $t$  Years (currency unit)
- $P$ : Amount Borrowed (money)
- $t$ : Time at Which  $F$  Needs to be Payed (years)

### Output(s)

- $i$ : Interest Rate (fraction)

**Formula(s)**

$$i = \exp\left(\frac{\ln\left(\frac{F}{P}\right)}{t}\right) - 1$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition.* Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 65.

## 7.9 Compound interest

**Input(s)**

- P: Principal Amount (currency unit)
- $i_n$ : Nominal Interest Rate (fraction per year)
- m: Compounding or Interest Periods per Year (where 1 for Annually, 2 for Semi-annually, 4 for quarterly, and 12 for monthly) (dimensionless)
- t : The Loan Period or Investment Period (years)

**Output(s)**

- I: Compound Interest (currency unit)

**Formula(s)**

$$I = \left( \left( 1 + \frac{i_n}{m} \right)^{t*m} - 1 \right) * P$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition.* Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 27.

## 7.10 Cost depletion

**Input(s)**

- AB: Adjusted Basis for The Taxable Year (currency unit)
- Q: Number of Units Sold in That Year (number)
- $R_r$ : Number of Remaining Reserves at the End of the Taxable Year (number)

**Output(s)**

- CD: Cost Depletion (currency unit)

**Formula(s)**

$$CD = AB * \left( \frac{Q}{R_r + Q} \right)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition.* Tulsa, Oklahoma: PennWell Corporation. Chapter 4, Page: 204.

## 7.11 Cumulative interest on operational expenses during the lifetime of a well

### Input(s)

- a: Interest Rate (%)
- L: Operational Expenses (\$/day)
- t: Operating Time (days)

### Output(s)

- $R_c$ : Cumulative Interest on Operation Expenses during the Lifetime of a Well (\$)

### Formula(s)

$$R_c = \frac{a \cdot L \cdot t^2}{2}$$

Reference: Saydam, T., (1967). *Principles of Hydraulic Fracturing*, ARI Publishing Co., Page: 86.

## 7.12 Effective interest rate for periodic compounding

### Input(s)

- m: Number of Compounding Periods Per Year (for example, 12 for Monthly Compounding) (time)
- $i_n$ : Nominal Interest Rate (fraction)

### Output(s)

- $i_e$ : Effective Interest Rate (fraction)

### Formula(s)

$$i_e = \left(1 + \frac{i_n}{m}\right)^m - 1$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 43.

## 7.13 Exploration efficiency

### Input(s)

- $F_D$ : Total Meterage Drilled up to the date of Discovery (ft)
- a: Ratio Constant (dimensionless)
- $E_o$ : Exploration Efficiency at Initial Conditions (bbl/ft)
- $R_u$ : Total Expected Additional Discovery of Resources in Selected Field Class (bbl)
- $r_u$ : Latest Rate of Growth for Discovered Resources (fraction)

### Output(s)

- E: Exploration Efficiency per unit Meterage (bbl/ft)

**Formula(s)**

$$E = E_0 \cdot e^{-a \cdot F_D}$$

$$E_0 = \frac{dR_u}{r_u}$$

*Note: All Formulas need to be applied in each Field Class separately.*

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 79.*

## 7.14 Future value of an annuity

**Input(s)**

$i_e$ : Effective Interest or Discount Rate (fraction)

t: Time (year)

$A_v$ : Annuity (currency unit)

**Output(s)**

$F_v$ : Future Value of an Annuity (currency unit)

**Formula(s)**

$$F_v = A_v * \frac{(1 + i_e)^t - 1}{i_e}$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 51.*

## 7.15 Future value of present sum

**Input(s)**

$i_e$ : Effective Interest or Discount Rate (fraction)

t: Time (year)

$P_v$ : Present Value of Future Sum (currency unit)

**Output(s)**

$F_v$ : Future Sum Received at Time t (currency unit)

**Formula(s)**

$$F_v = P_v * ((1 + i_e)^t)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 44.*

## 7.16 Generalized expected value calculation

### Input(s)

- $P_i$ : Possible Result of Probability from case “1” to “n” (fraction)  
 $V_i$ : Contingency Value of Investment from case “1” to “n” (\$)

### Output(s)

- $EV$ : Expected Value (dimensionless)

### Formula(s)

$$EV = \sum_{i=1}^n (P_i)(V_i)$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 72.*

## 7.17 Growth rate of return for continuous compounding

### Input(s)

- $t$ : Time (years)  
 $PI$ : Profitability Index (dimensionless)  
 $i_d$ : Reinvestment Rate (fraction)

### Output(s)

- $GRR$ : Growth Rate of Return (fraction)

### Formula(s)

$$GRR = \frac{1}{t} * \ln(PI) + i_d$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 6, Page: 351.*

## 7.18 Hoskold method for annual rate of return prediction-1

### Input(s)

- $C$ : Initial Investment/Capital (\$)  
 $r_H$ : Speculative Ratio (fraction)  
 $i$ : Rate of Interest (per cent)  
 $n$ : Life of the Project (years)  
 $D$ : Present worth Factor (dimensionless)

### Output(s)

- $DE$ : Present worth Factor with Total Net Undiscounted Cash Flow during the whole project (\$)  
 $r_H$ : Speculative Ratio (fraction)

**Formula(s)**

$$DE = r_H \cdot C \frac{1 - \left(\frac{1}{1+i}\right)^n}{i} + \left(\frac{1}{1+i}\right)^n \cdot C$$

$$r_H = i \cdot \frac{\frac{DE}{C} - (1+i)^{-n}}{1 - (1+i)^{-n}}$$

*Control Form:*

$$C \leq \frac{DE}{\frac{r_H}{i} - \left[ \left( \frac{r_H}{i} - 1 \right) \cdot (1+i)^{-n} \right]}$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 47.

**7.19 Hoskold method for annual rate of return prediction-2****Input(s)**

- C: Initial Investment/Capital (\$)
- i: Rate of Interest (per cent)
- n: Life of the Project (years)
- S: Present Value of Total Net Income (\$)

**Output(s)**

- $PV_i$ : Present Value of Income (\$)
- $r_H$ : Speculative Ratio (fraction)

**Formula(s)**

$$PV_i = S \cdot \frac{1 - (1+i)^{-n}}{i}$$

$$r_H = \frac{S}{C} - \frac{i}{(1+i)^n - 1}$$

*Control Form:*

$$C \leq \frac{S}{\frac{r_H}{i} + \frac{1}{(1+i)^n - 1}}$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 48.

**7.20 Initial capital needed to survive in a minimum chance scenario****Input(s)**

- Z: Number of Standard Deviation corresponds to a Certain Change (dimensionless)
- $\sigma$ : Standard Deviation of a Risky Job reduced to Present Value (fraction)
- $X_E$ : Present Worth Expectation per a Risky Job (\$)

### Output(s)

$M_G$ : Initial Capital Needed to Survive in a Minimum Chance Scenario (dimensionless)

### Formula(s)

$$M_G = \frac{(Z \cdot \sigma)^2}{4X_E}$$

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 99.

## 7.21 Meterage model

### Input(s)

$F_D$ : Total Meterage Drilled up to the date of Discovery (ft)

$a$ : Ratio Constant (dimensionless)

$R_u$ : Total Expected Additional Discovery of Resources in Selected Field Class (bbl)

$r_u$ : Latest Rate of Growth for Discovered Resources (fraction)

### Output(s)

$dR_D/F_D$ : Total Cumulative Discovered Resources per total drilled Meterage (bbl/ft)

$R_D$ : Total Cumulative Resources of Discovery (bbl)

### Formula(s)

$$\frac{dR_D}{dF_D} = a \left( \frac{R_u}{r_u} - R_D \right)$$

For Initial Conditions ( $F_D = 0; R_D = 0$ ):

$$R_D = \frac{R_u}{r_u} \left( 1 - e^{-a \cdot F_D} \right)$$

Note: All Formulas need to be applied in each Field Class separately.

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 83.

## 7.22 Minimum number of jobs to survive in a minimum chance scenario

### Input(s)

$Z$ : Number of Standard Deviation corresponds to a Certain Change (dimensionless)

$\sigma$ : Standard Deviation of a Risky Job reduced to Present Value (fraction)

$X_E$ : Present Worth Expectation per a Risky Job (\$)

### Output(s)

$n$ : Minimum Number of Jobs to Survive in a Minimum Chance Scenario (dimensionless)

**Formula(s)**

$$n = \left( \frac{-Z \cdot \sigma}{2X_E} \right)^2$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 99.*

### **7.23 Minimum profit ratio per a risky job**

**Input(s)**

- y: Wealth Position that could be opened in a certain Probability (\$)
- $y_o$ : Initial Wealth of the Company (\$)
- n: Total number of Independent Risky Jobs (dimensionless)
- C: Dry-well Drilling Cost in Initial Investment (\$)

**Output(s)**

- W: Minimum Profit Ratio per a Risky Job - Minimum Profit/Investment Ratio (fraction)

**Formula(s)**

$$W = \frac{y - y_o}{n \cdot C}$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 92.*

### **7.24 Net cash flow**

**Input(s)**

- R: Receipts (currency unit)
- D: Disbursements (currency unit)

**Output(s)**

- NCF: Net Cash Flow (currency unit)

**Formula(s)**

$$NCF = R - D$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition.* Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 45.

### **7.25 Net present value**

**Input(s)**

- t: the time of cash flow
- i: discount rate
- $R_t$ : the net cash flow

### Output(s)

NPV : Net Present Value

### Formula(s)

$$NPV = \frac{R_t}{(1+i)^t}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 7.26 Operating cash income

### Input(s)

$V_j$ : Annual Gross Revenue after Royalties and Wellhead Taxes (\$)  
 $O_j$ : Operating Charges (\$)

### Output(s)

$I_j$ : Operating Cash Income (\$)

### Formula(s)

$$I_j = V_j - O_j$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 7.27 Payback Period

### Input(s)

SN: Cumulative Negative Net Cash Flow (NCF) Years (year)  
 PN: Positive NCF (currency unit)  
 NN: Negative NCF (Positive Numeric Value) (currency unit)

### Output(s)

PP: Payback Period (year)

### Formula(s)

$$PP = SN + \frac{1}{PN + NN} * NN$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 6, Page: 306.

## 7.28 Present value of an annuity

### Input(s)

$i_e$ : Effective Interest or Discount Rate (fraction)  
 $t$  : Time (year)  
 $A_v$ : Annuity (currency unit)

**Output(s)**

$P_v$ : Present Value of an Annuity (currency unit)

**Formula(s)**

$$P_v = A_v * \frac{(1+i_e)^t - 1}{i_e * (1+i_e)^t}$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 57.

**7.29 Present value of a deferred annuity****Input(s)**

$i_e$ : Effective Interest or Discount Rate (fraction)

$t$ : Time (year)

$A_v$ : Annuity (currency unit)

**Output(s)**

$P_v$ : Present Value of Deferred (currency unit)

**Formula(s)**

$$P_v = A_v * \frac{(1+i_e)^t - 1}{i_e * (1+i_e)^t} * \left( \frac{1}{(1+i_e)^{t-2}} \right)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 62.

**7.30 Present value of future sum****Input(s)**

$i_e$ : Effective Interest or Discount Rate Fraction (fraction)

$t$ : Time (year)

$F_v$ : Future Sum Received at Time  $t$  (currency unit)

**Output(s)**

$P_v$ : Present Value of Future Sum (currency unit)

**Formula(s)**

$$P_v = F_v * \left( \frac{1}{(1+i_e)^t} \right)$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 49.

### 7.31 Present value of profit/investment ratio for an oil well

#### Input(s)

- $RF$ : Recovery Factor (bbl/acre ft)  
 $h$ : Formation Thickness (ft)  
 $A$ : Reservoir Area (acre)  
 $D$ : Present Worth Factor (dimensionless)  
 $W_p$ : Working Profit per barrel of Oil (\$/bbl)  
 $C_T$ : Total Cost of an Oil Well (\$)

#### Output(s)

- $R$ : Recovery (bbl)  
 $PV_i$ : Present Value of Income (\$)  
 $BE$ : Break-even Price (\$)  
 $PV_{PIR}$ : Present Value of Profit/Investment Ratio (dimensionless)

#### Formula(s)

$$R = A \cdot h \cdot RF$$

$$PV_i = (D) \cdot (R) \cdot (WP)$$

$$BE = (PV_i) - (C_T)$$

$$PV_{PIR} = \frac{(PV_i) - (C_T)}{C_T}$$

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 34.

### 7.32 Present value of uniform gradient series

#### Input(s)

- $i_e$ : Effective Interest or Discount Rate (fraction)  
 $t$ : Time (year)  
 $G$ : Annual Change (Positive or Negative) (currency unit)  
 $A_1$ : Cash Flow at the End of the First Year (currency unit)

#### Output(s)

- $A_v$ : Present Value of Uniform Gradient Series (currency unit)

#### Formula(s)

$$A_v = A_1 \pm G * \left( \frac{1}{i_e} - \frac{t}{((1 + i_e)^t) - 1} \right)$$

Reference: Mian, M. A. 2011. Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 68.

### 7.33 Present worth expectation for a risky job

#### Input(s)

- $\bar{X}_A$ : Present Values of Discounted Probable Outcomes that includes all Costs except Initial Speculative Cost (\$)  
 $C$ : Dry Well Drilling Cost in Initial Investment (\$)

#### Output(s)

- $X_E$ : Present Worth Expectation per a Risky Job (\$)

#### Formula(s)

$$X_E = \bar{X}_A - C$$

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 92.

### 7.34 Probability of an oilfield discovery

#### Input(s)

- $K_i$ : Number of Fields Discovered up to date in Selected Field Class (dimensionless)  
 $K_{oi}$ : Estimated Total Number of Fields in Selected Field Class in the Region (dimensionless)  
 $A_i$ : Total Area of Discovered Fields in Selected Field Class (acres)  
 $W$ : Number of Exploration Wells Drilled up to date (dimensionless)  
 $B$ : Total Area of the Region (acres)  
 $J$ : Relative Efficiency of Scientific Exploration compared with Random Drilling (dimensionless)  
 $K_{i\_add}$ : Number of Fields will be discovered with additional number of Exploration Wells (dimensionless)  
 $W_{add}$ : Additional Number of Exploration Wells will be Drilled (dimensionless)

#### Output(s)

- $P_D$ : Probability of an Oilfield Discovery (fraction)

#### Formula(s)

$$\begin{aligned} K_{oi} &= \frac{K_i}{(1 - e^{-JA_i W/B})} \\ K_{i\_add} &= K_{oi} \left( 1 - e^{-JA_i (W + W_{add})/B} \right) \\ P_D &= \frac{K_{i\_add} - K_i}{W_{add}} \end{aligned}$$

Note: All Formulas need to be applied in each Field Class separately.

Reference: Serpen, U., Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 79.

### 7.35 Profitability index

#### Input(s)

- NPV: Net Present Value of investment (currency unit)  
PVC: Present Value of Capital investments (currency unit)

### Output(s)

PI: Profitability Index (dimensionless)

### Formula(s)

$$PI = \frac{PVC + NPV}{PVC}$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models*, Second Edition. Tulsa, Oklahoma: PennWell Corporation. Chapter 6, Page: 329.

## 7.36 Rate of growth per unit of exploration length

### Input(s)

$F$ : Total Meterage Drilled up to date and expected to be drilled at the end Discovery (ft)

$F_D$ : Total Meterage Drilled up to the date of Discovery (ft)

$r$ : The Ratio of Reserves Discovered in  $F$  Meterage to the  $F_D$  Exploration Metrage (fraction)

$r_u$ : Latest Rate of Growth for Discovered Resources (fraction)

$b$ : Ratio Constant (dimensionless)

### Output(s)

$\frac{dr}{d(F-F_D)}$ : Increase in Rate of Growth per Additional Unit of Length Drilled (bbl/ft)

### Formula(s)

$$\frac{dr}{d(F-F_D)} = b(r_u - r)$$

If There is no Extra Exploration ( $F - F_D = 0$ ;  $r = 1$ );

$$r = r_u - [(r_u - 1) \cdot e^{-b(F-F_D)}]$$

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 79.

## 7.37 Simple interest

### Input(s)

$P$ : Principal Amount (currency unit)

$t$ : Time (12 months or 360 days)

$r$ : Interest Rate per Period (fraction)

### Output(s)

SI: Simple interest (currency unit)

**Formula(s)**

$$SI = P * r * t$$

Reference: Mian, M. A. 2011. *Project Economics and Decision Analysis Volume 1: Deterministic Models, Second Edition*. Tulsa, Oklahoma: PennWell Corporation. Chapter 2, Page: 39.

### **7.38 Total expected additional production discovery**

**Input(s)**

- $K_{i\_add}$ : Number of Fields estimated to be discovered with additional number of Exploration Wells (dimensionless)  
 $R_{ui}$ : Total Production of an Average Size Field in Class "i" (bbl)

**Output(s)**

- $R_u$ : Total Expected Additional Production Discovery in Selected Field Class (acres)

**Formula(s)**

$$R_u = \sum K_{i\_add} \cdot R_{ui}$$

*Note: All Formulas need to be applied in each Field Class separately.*

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 79.

### **7.39 Total expected additional production discovery in constant production per unit area**

**Input(s)**

- $K_{i\_add}$ : Number of Fields estimated to be discovered with additional number of Exploration Wells (dimensionless)  
 $R_{ui}$ : Total Production of an Average Size Field in Class "i" (bbl)  
 $A_i$ : Total Area of Discovered Fields in Field Class "i" (acres)  
 $A_{i\_add}$ : Total Area Estimated to be discovered in Selected Field Class (acres)

**Output(s)**

- $R_u$ : Total Expected Additional Production Discovery in Selected Field Class (acres)

**Formula(s)**

$$R_u = \sum K_{i\_add} \cdot A_{i\_add} \cdot \frac{R_{ui}}{A_i}$$

*Note: All Formulas need to be applied in each Field Class separately.*

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008)* Page: 81.

## 7.40 Total new production area estimation expected to be discovered

### Input(s)

$K_{i\_add}$ : Number of Fields estimated to be discovered with additional number of Exploration Wells (dimensionless)  
 $A_i$ : Total Area of Discovered Fields in Field Class "i" (acres)

### Output(s)

$A_{i\_add}$ : Total Area Estimated to be discovered in Selected Field Class (acres)

### Formula(s)

$$A_{i\_add} = \sum (K_{i\_add}) \cdot (A_i)$$

*Note: All Formulas need to be applied in each Field Class separately.*

Reference: Serpen, U., *Petroleum Economics, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, (2008) Page: 81.*

## Chapter 8

# Phase behavior and thermodynamics formulas and calculations

### Chapter Outline

8.1 Amount of heat required to increase the temperature	377	8.27 Necessary inhibitor concentration required in liquid phase to reduce hydrate point	388
8.2 Benedict-Webb-Rubin PVT equation	378	8.28 Peng Robinson characterization factor	388
8.3 Critical pressure Cavett relation	378	8.29 Peng Robinson PVT equation	388
8.4 Critical temperature Cavett method	379	8.30 Pseudo-reduced conditions	389
8.5 Effective thermal conductivity of composite solids	379	8.31 Radiant heat transfer between disks	389
8.6 Einstein equation effective viscosity	379	8.32 Radiated energy flux	390
8.7 Equilibrium vaporization ratio	380	8.33 Radiation across an annular gap	390
8.8 Equilibrium vaporization ratio of heptane	380	8.34 Radiation shields	391
8.9 Evaporation loss from an oxygen tank	380	8.35 Rayleigh number	391
8.10 Expansion factor for diffuse layer	381	8.36 Redlich-Kwong PVT equation	391
8.11 Flat-plate boundary layer model	381	8.37 Reservoir gas density	392
8.12 Freezing of a spherical drop	382	8.38 Stefan-Boltzmann law	392
8.13 General thermal conductivity	382	8.39 Surface temperature of a heating coil	393
8.14 Heat conduction in a cooling fan	382	8.40 Temperature due to free convection	393
8.15 Heat flux distribution in a wall	383	8.41 Temperature increase due to forced convection	393
8.16 Heat loss by free convection from a horizontal pipe	383	8.42 Temperature profile after viscous heat transfer	394
8.17 Heat released during in-situ combustion given by Burger & Sahuquet	384	8.43 Temperature profile with a nuclear heat source	394
8.18 Heat transfer coefficient for condensation—Pure vapors on solid surface	384	8.44 Thermal conductivity for pure metals	395
8.19 Heat transfer in packed bed	385	8.45 Thermal conductivity of polyatomic gases	395
8.20 Heat transfer rate in laminar forced convection along heated flat plate	385	8.46 Thermal conductivity of liquids	395
8.21 Jacoby aromaticity factor	385	8.47 Thermal conductivity of solids with gas pockets	396
8.22 Joule Thompson expansion	386	8.48 Thermal diffusivity	396
8.23 Latent heat of hydrocarbon mixture	386	8.49 Thermal energy of a fissionable substance	397
8.24 Mixing fluids of different densities	387	8.50 Thermoelastic effect on stress	397
8.25 Mole fraction of a component in liquid phase	387	8.51 Unsteady evaporation of a liquid	397
8.26 Mole fraction of a component in vapor phase	387	8.52 Van Der Waals PVT equation	398
		8.53 Wien displacement law	398

### 8.1 Amount of heat required to increase the temperature

#### Input(s)

- $V_r$ : Volume of Reservoir (acre ft)  
 $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/ft<sup>3</sup> F)  
 $T_i$ : Initial Temperature (K)  
 $T_f$ : Final Temperature (K)

### Output(s)

$Q$ : Amount of Heat Required to Increase the Temperature (BTU)

### Formula(s)

$$Q = 43560 * V_r * M_r * (T_f - T_i)$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 5, Page: 41.*

## 8.2 Benedict-Webb-Rubin PVT equation

### Input(s)

- R: Gas Constant (BTU/mol psi K)
- T: Temperature (K)
- $\rho$ : Density (g/cc)
- A: Correlation Constant (dimensionless)
- B: Correlation Constant (dimensionless)
- C: Correlation Constant (dimensionless)
- a: Correlation Constant (dimensionless)
- b: Correlation Constant (dimensionless)
- c: Correlation Constant (dimensionless)
- $\alpha$ : Correlation Constant (dimensionless)
- $\gamma$ : Correlation Constant (dimensionless)

### Output(s)

P: Pressure (psi)

### Formula(s)

$$P = R * T * \rho + \left( \left( B * R * T - A - \frac{C}{T^2} \right) * (\rho^2) \right) + (\rho^3) * (b * R * T - a) + a * \alpha * (\rho^6) + \left( c * \frac{\rho^3}{T^2} \right) * (1 + \gamma * (\rho^2)) * \exp((-1) * \gamma * (\rho^2))$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. I, Page: 48.*

## 8.3 Critical pressure Cavett relation

### Input(s)

- $T_b$ : Boiling Point Temperature (°F)
- API: API Gravity of oil (API)

### Output(s)

$P_c$ : Critical Pressure (Psi)

### Formula(s)

$$\begin{aligned} P_c = & 10^{(2.829)} + (0.0009112 * T_b) - (0.0000030175 * T_b^2) + (0.0000000015141 * T_b^3) - (0.000020876 * T_b * API) \\ & + (0.000000011048 * T_b^2 * API) + (0.0000000001395 * T_b)^2 * A \end{aligned}$$

Reference: *Wikipedia.org*.

## 8.4 Critical temperature Cavett method

### Input(s)

$T_b$ : Boiling Point Temperature (°F)

API: API Gravity (API)

### Output(s)

$T_c$ : Critical Temperature (°F)

### Formula(s)

$$T_c = 768.071 + 1.7134 * T_b - (0.0010834 * T_b^2) + (0.0000003889 * T_b^3) - (0.00089213 * T_b * API) + (0.00000053095 * T_b^2 * API) + (0.000000032712 * T_b^2 * API^2)$$

Reference: *Wikipedia.org*.

## 8.5 Effective thermal conductivity of composite solids

### Input(s)

$k_o$ : Thermal Conductivity of Continuous Phase (W/m K)

$k_1$ : Thermal Conductivity of Embedded Phase (W/m K)

$\phi$ : Porosity (fraction)

### Output(s)

$k_{eff}$ : Effective Thermal Conductivity (W/m K)

### Formula(s)

$$\frac{k_{eff}}{k_o} = 1 + \left( \frac{3 * \phi}{\left( \frac{k_1 + 2 * k_o}{k_1 - k_o} \right) - \phi} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 281.*

## 8.6 Einstein equation effective viscosity

### Input(s)

$\phi$ : Porosity (volume fraction of the spheres)

$\mu_0$ : Viscosity of Suspending Medium (g/cm s)

### Output(s)

$\mu_{eff}$ : Effective Viscosity (g/cm s)

**Formula(s)**

$$\mu_{\text{eff}} = \mu_0 * \left( 1 + \frac{5}{2} * \phi \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 1, Page: 32.*

**8.7 Equilibrium vaporization ratio****Input(s)**

- y: Mole Fraction of Component in Vapor Phase (fraction)
- x: Mole Fraction of Component in Liquid Phase (fraction)

**Output(s)**

- K: Equilibrium Vaporization Ratio (dimensionless)

**Formula(s)**

$$K = \frac{y}{x}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 3, Page: 3-1.*

**8.8 Equilibrium vaporization ratio of heptane****Input(s)**

- $K_a$ : Heptane (fraction)
- $K_b$ : Ethane (fraction)
- b: Constant (dimensionless)

**Output(s)**

- K: Equilibrium Vaporization Ratio (fraction)

**Formula(s)**

$$K = \frac{K_a}{\left(\frac{K_b}{K_a}\right)^b}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 3, Page: 3-1.*

**8.9 Evaporation loss from an oxygen tank****Input(s)**

- $r_o$ : Inner Diameter (m)
- $r_1$ : Outer Diameter (m)
- $T_o$ : Inner Layer Temperature (K)
- $T_1$ : Outer Layer Temperature (K)

- $k_o$ : Thermal Conductivity of Inner Layer (W/m K)  
 $k_1$ : Thermal Conductivity of Outer Layer (W/m K)

**Output(s)**

$Q_o$ : Flow Rate of Evaporation (g/s)

**Formula(s)**

$$Q_o = 4 * \pi * r_o * r_1 * \left( \frac{k_o + k_1}{2} \right) * \left( \frac{T_o - T_1}{r_1 - r_o} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 327.*

**8.10 Expansion factor for diffuse layer****Input(s)**

- $n_c$ : Critical Salt Concentration (mol/L)  
 $n$ : Salt Concentration (mol/L)

**Output(s)**

$f_{dl}$ : Expansion Factor (dimensionless)

**Formula(s)**

$$f_{dl} = \left( \frac{n_c}{n} \right)^{0.5}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 1, Page: 15.*

**8.11 Flat-plate boundary layer model****Input(s)**

- $v_{y0}$ : Horizontal Velocity (cm/s)  
 $v_\infty$ : Velocity (cm/s)  
 $x$ : Distance (cm)  
 $\mu$ : Viscosity (cP)

**Output(s)**

$K$ : Net Mass Flux from the Plate (dimensionless)

**Formula(s)**

$$K = \left( \frac{v_{y0}}{v_\infty} \right) * \left( \left( \frac{2 * v_\infty * x}{\mu} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 708.*

## 8.12 Freezing of a spherical drop

### Input(s)

$v_{y0}$ : Horizontal Velocity (cm/s)

$v_\infty$ : Velocity (cm/s)

x: Distance (cm)

$\mu$ : Viscosity (cP)

### Output(s)

K: Net Mass Flux from the Plate (dimensionless)

### Formula(s)

$$K = \left( \frac{v_{y0}}{v_\infty} \right) * \left( \left( \frac{2 * v_\infty * x}{\mu} \right)^{0.5} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 22, Page: 708.

## 8.13 General thermal conductivity

### Input(s)

Q: Heat Flow Rate (BTU/s)

L: Length (ft)

A: Area ( $\text{ft}^2$ )

$\delta T$ : Temperature Difference (K)

### Output(s)

k: Thermal Conductivity (BTU/ft s K)

### Formula(s)

$$k = \frac{Q * L}{A * \delta T}$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 266.

## 8.14 Heat conduction in a cooling fan

### Input(s)

z: Distance from Start of Fan (cm)

L: Length of Fan (cm)

B: Breadth of Fan (cm)

h: Heat Transfer Coefficient ( $\text{J/s m}^2 \text{ K}$ )

k: Thermal Conductivity (W/cm K)

$T_w$ : Temperature of Wall (K)

$T_a$ : Ambient Temperature of Air (K)

**Output(s)**

- $\xi$ : Dimensionless Distance (dimensionless)
- N: Dimensionless Heat Transfer Coefficient (dimensionless)
- $\Theta$ : Dimensionless Temperature (dimensionless)
- T: Temperature at Distance Z from Wall at the Fan (K)

**Formula(s)**

$$\begin{aligned}\xi &= \frac{z}{L} \\ N &= \left( \frac{h * (L^2)}{k * B} \right)^{0.5} \\ \Theta &= \frac{\cosh(N * (1 - \xi))}{\cosh(N)} \\ T &= T_a + \Theta * (T_w - T_a)\end{aligned}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Pages: 308, 309.*

**8.15 Heat flux distribution in a wall****Input(s)**

- k: Thermal Conductivity (J/ft s K)
- x: Distance from Center of Wall (ft)
- b: Breadth of Wall (ft)
- $T_o$ : Initial Temperature of Wall (K)
- $T_1$ : Final Temperature of Wall (K)

**Output(s)**

- Q: Heat Flux (J)

**Formula(s)**

$$Q = \frac{2 * k * \sec\left(\frac{\pi * x}{b}\right) * (T_1 - T_o)}{b}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 12, Page: 387.*

**8.16 Heat loss by free convection from a horizontal pipe****Input(s)**

- $h_m$ : Heat Transfer Coefficient (W/m K)
- D: Diameter (m)
- L: Length (m)
- $T_o$ : Final Temperature (K)
- $T_b$ : Initial Temperature (K)

**Output(s)**

Q: Heat Loss Rate (J/s)

**Formula(s)**

$$Q = h_m * \pi * D * L * (T_o - T_b)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 14, Page: 445.*

**8.17 Heat released during in-situ combustion given by Burger & Sahuquet****Input(s)**

m: Molar Ratio of H/C (fraction)

x: Ratio of Carbon Monoxide to Carbon Emissions (fraction)

**Output(s)**

$dh_a$ : Heat Liberated (BTU/scf)

**Formula(s)**

$$dh_a = \frac{94 - 67.9 * m + 31.2 * x}{1 - 0.5 * m + 0.25 * x}$$

Reference: *Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 93.*

**8.18 Heat transfer coefficient for condensation—Pure vapors on solid surface****Input(s)**

k: Thermal Conductivity (J/m s K)

$\rho$ : Density (g/cc)

g: Acceleration Due to Gravity ( $m/s^2$ )

$\mu$ : Viscosity (cP)

D: Diameter (m)

$T_d$ : Dew Point Temperature (K)

$T_o$ : Original Temperature (K)

$\Delta H_{vap}$ : Heat of Vaporization ( $J ft^2/s^2$ )

**Output(s)**

$h_m$ : Heat Transfer Coefficient (W/m K)

**Formula(s)**

$$h_m = 0.725 * \left( \frac{(k^3) * (\rho^2) * g * \Delta H_{vap}}{\mu * D * (T_d - T_o)} \right)^{0.25}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 14, Page: 447.*

## 8.19 Heat transfer in packed bed

### Input(s)

- a: Number of Particles in Bed (dimensionless)
- $S_{dz}$ : Representative Volume for Each Particle ( $m^3$ )
- $T_o$ : Final Temperature (K)
- $T_b$ : Initial Temperature (K)
- $h_{loc}$ : Heat Transfer Coefficient (W/m K)

### Output(s)

- dQ: Heat Loss (J/s)

### Formula(s)

$$dQ = h_{loc} * a * (S_{dz}) * (T_o - T_b)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 14, Page: 441.*

## 8.20 Heat transfer rate in laminar forced convection along heated flat plate

### Input(s)

- W: Width (ft)
- L: Length (ft)
- $T_o$ : Final Temperature (K)
- $T_\infty$ : Temperature at X from Center of Temperature Profile (K)
- k: Thermal Conductivity (BTU/ft h K)
- Pr: Prandtl (dimensionless)
- Re: Reynolds (dimensionless)

### Output(s)

- Q: Heat Loss Rate (BTU/h)

### Formula(s)

$$Q = \left( \left( \frac{148}{315} \right)^{0.5} \right) * 2 * W * L * (T_o - T_\infty) * \left( \frac{k}{L} \right) * (Pr 0.334) * (Re^{0.5})$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 12, Page: 390.*

## 8.21 Jacoby aromaticity factor

### Input(s)

- $\gamma$ : Specific Gravity (unitless)
- M: Molecular Weight (lbm/lbm mol)

### Output(s)

- $J_a$ : Jacoby Aromaticity Factor (unitless)

### Formula(s)

$$J_a = \frac{(\gamma) + \left(\frac{15.8}{M}\right) - 0.8468}{0.2456 - \left(\frac{1.77}{M}\right)}$$

Reference: Whitson, C.H., Brule, M.R., 2000. *Phase Behavior. SPE Monograph Series.* Richardson, Texas, Chapter: 5, Page: 78.

## 8.22 Joule Thompson expansion

### Input(s)

R: Universal Gas Constant (psi ft<sup>3</sup>/K lbmol)

C<sub>p</sub>: Specific Heat Capacity at Constant Pressure (J/lb K)

$\left(\frac{\partial Z}{\partial T}\right)_p$  : Variation of Gas Compressibility Factor w.r.t. Temperature Change at Constant Pressure (1/K)

### Output(s)

$\frac{\partial T}{\partial p}$ : Joule-Thompson Expansion (K/psi)

### Formula(s)

$$\frac{\partial T}{\partial p} = \left( \frac{RT^2}{pC_p} \right) * \left( \frac{\partial Z}{\partial T} \right)_p$$

Reference: Ahmed, T. McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter:3, Page: 272.

## 8.23 Latent heat of hydrocarbon mixture

### Input(s)

T: Average Boiling Point (K)

M: Molecular Weight (dimensionless)

### Output(s)

$\Delta h_m$ : Latent Heat (BTU/lb)

### Formula(s)

$$\Delta h_m = \left( \frac{T}{M} \right) * (7.58 + 4.57 * \log(T))$$

Reference: John M. Campbell, *Gas Conditioning and Processing, Campbell Petroleum Series*, Oklahoma, 1992, Vol. 1, Page: 207.

## 8.24 Mixing fluids of different densities

### Input(s)

- $D_1$ : Density of fluid 1 (ppg)
- $D_2$ : Density of fluid 2 (ppg)
- $V_1$ : Volume of fluid 1 (bbl)
- $V_2$ : Volume of fluid 2 (bbl)

### Output(s)

- $D_f$ : Density of final fluid (ppg)
- $V_f$ : Volume of final fluid (bbl)

### Formula(s)

$$D_f = V_1 + V_2$$

$$V_f = \frac{V_1 * D_1 + V_2 * D_2}{D_f}$$

Reference: *Wikipedia.org.*

## 8.25 Mole fraction of a component in liquid phase

### Input(s)

- $z$ : Mole of Component in Feed Per Mole Total Feed (mol)
- $L$ : Moles of Liquid Leaving a Stream (mol)
- $V$ : Moles of Vapor Leaving a Stream (mol)
- $K$ : Equilibrium Vapor Factor (ratio)

### Output(s)

- $x$ : Mole Fraction of a Component in Liquid Phase (ratio)

### Formula(s)

$$x = \frac{z}{L + V * K}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 107.*

## 8.26 Mole fraction of a component in vapor phase

### Input(s)

- $z$ : Mole Fraction of Component in Feed (mol)
- $L$ : Moles of Liquid in Outlet Stream (mol)
- $V$ : Moles of Vapor in Outlet Stream (mol)
- $K$ : Equilibrium Vaporization Ratio (ratio)

### Output(s)

- $y$ : Mole Fraction of Component in Vapor Phase (fraction)

**Formula(s)**

$$y = \frac{z}{L + \left(\frac{V}{K}\right)}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 107.*

### **8.27 Necessary inhibitor concentration required in liquid phase to reduce hydrate point**

**Input(s)**

- d: Desired Depression of Hydrate Point (F)  
 M: Molecular Weight of Inhibitor (dimensionless)

**Output(s)**

- X: Interstitial Velocity (cm/s)

**Formula(s)**

$$X = \frac{d * M * 100}{d * M + 2335}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 180.*

### **8.28 Peng Robinson characterization factor**

**Input(s)**

- M: Molecular weight (g)

**Output(s)**

- $k_{Ci}$ : Characterization factor (unitless)

**Formula(s)**

$$k_{Ci} = 0.0289 + 0.0001633 * M$$

Reference: *Wikipedia.org.*

### **8.29 Peng Robinson PVT equation**

**Input(s)**

- R: Gas Constant (BTU/psi K mol)  
 T: Temperature (K)  
 V: Volume ( $\text{ft}^3$ )  
 a: Correlation Constant (dimensionless)  
 b: Correlation Constant (dimensionless)

**Output(s)**

P: Pressure (psi)

**Formula(s)**

$$P = \frac{R * T}{V - b} - \frac{a * T}{V * (V + b) + b * (V - b)}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 48.*

**8.30 Pseudo-reduced conditions****Input(s)**

P: Pressure (psi)

$P_{cA}$ : Critical Pressure at A (psi)

$P_{cB}$ : Critical Pressure at B (psi)

T: Temperature (K)

$T_{cA}$ : Critical Temperature at A (K)

$T_{cB}$ : Critical Temperature at B (K)

**Output(s)**

$P_r$ : Pseudo Reduced Pressure (dimensionless)

$T_r$ : Pseudo Reduced Temperature (dimensionless)

**Formula(s)**

$$P_r = \frac{P}{(P_{cA} * P_{cB})^{0.5}}$$

$$T_r = \frac{T}{(T_{cA} * T_{cB})^{0.5}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 17, Page: 524.*

**8.31 Radiant heat transfer between disks****Input(s)**

A: Area of Body ( $\text{ft}^2$ )

$\sigma$ : Stefan-Boltzmann Constant ( $\text{BTU/h ft}^2 {}^\circ\text{R}^4$ )

F: View Factor of Body (fraction)

$T_1$ : Initial Temperature of Emitting Body ( ${}^\circ\text{R}$ )

$T_2$ : Temperature of Absorbing Body ( ${}^\circ\text{R}$ )

**Output(s)**

Q: Heat Transfer Rate by Radiation (BTU/h)

### Formula(s)

$$Q = A * F * \sigma * ((T_1^4) - (T_2^4))$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 501.*

## 8.32 Radiated energy flux

### Input(s)

- c: Speed of Light (ft/s)
- h: Planck's Constant (lb ft<sup>2</sup>/s)
- $\lambda$ : Wavelength (ft)
- k: Stefan-Boltzmann Constant (BTU/h ft<sup>2</sup> °R<sup>4</sup>)
- T: Temperature (K)

### Output(s)

- q: Radiated Energy Flux from a Black Surface in the Wavelength Range (BTU/ft<sup>2</sup> s<sup>3</sup>)

### Formula(s)

$$q = \frac{2 * \pi * (c^2) * h}{(\lambda^5) * \left( \exp\left(\frac{c * h}{\lambda * k * T}\right) - 1 \right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 494.*

## 8.33 Radiation across an annular gap

### Input(s)

- $T_1$ : Temperature at Cylinder 1 (K)
- $T_2$ : Temperature at Cylinder 2 (K)
- $e_1$ : Emissivity at Cylinder 1 (fraction)
- $e_2$ : Emissivity at Cylinder 2 (fraction)
- $A_1$ : Area at Cylinder 1 (ft)
- $A_2$ : Area at Cylinder 2 (ft)
- k: Stefan-Boltzmann Constant (BTU/h ft<sup>2</sup> °R<sup>4</sup>)

### Output(s)

- Q: Heat Transfer Rate (BTU/h)

### Formula(s)

$$Q = \frac{k * ((T_1^4) - (T_2^4))}{\left(\frac{1}{A_1 * e_1}\right) + \left(\frac{1}{A_2}\right) * \left(\left(\frac{1}{e_2}\right) - 1\right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 494.*

## 8.34 Radiation shields

### Input(s)

$k$ : Stefan-Boltzmann Constant (BTU/s ft<sup>2</sup> °R<sup>4</sup>)

$T_i$ : First Sheet's Temperature (K)

$T_{i+1}$ : Next Sheet's Temperature (K)

$Q_{i, i+1}$ : Heat Transfer Rate (BTU/s)

### Output(s)

$R_{i, i+1}$ : Resistance to Radiation (ft<sup>2</sup> K/W)

### Formula(s)

$$R_{i,i+1} = \frac{\sigma * (T_i^4 - T_{i+1}^4)}{Q_{i,i+1}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 509.*

## 8.35 Rayleigh number

### Input(s)

$g$ : Acceleration due to Earth's gravity (m/s<sup>2</sup>)

$\beta$ : Volumetric thermal expansion coefficient (K)

$T_s$ : Surface temperature (K)

$T_i$ : Bulk temperature (K)

$x$ : Characteristic Length (m)

$\nu$ : Kinematic viscosity (m<sup>2</sup>/s)

$\alpha$ : Thermal Conductivity of fluid (K)

### Output(s)

$Ra$ : Rayleigh number (dimensionless)

### Formula(s)

$$Ra = \frac{g * \beta * (T_s - T_i) * x^3}{\nu * \alpha}$$

Reference: *Wikipedia.org.*

## 8.36 Redlich-Kwong PVT equation

### Input(s)

$R$ : Gas Constant (BTU/mol K psi)

$T$ : Temperature (K)

$V$ : Volume (ft<sup>3</sup>)

$a$ : Correlation Constant (dimensionless)

$b$ : Correlation Constant (dimensionless)

**Output(s)**

P: Pressure (psi)

**Formula(s)**

$$P = \frac{R * T}{V - b} - \frac{a}{(T^{0.5}) * V * (V + b)}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 48.*

**8.37 Reservoir gas density****Input(s)**

$\gamma_g$ : Specific Gravity of Gas (fraction)

T: Temperature ( $^{\circ}$ F)

p: Pressure (psi)

z: Gas deviation factor (fraction)

R: Gas Constant (fraction)

**Output(s)**

$\rho_g$ : Reservoir Gas Density ( $\text{lb}/\text{ft}^3$ )

**Formula(s)**

$$\rho_g = 28.97 * \gamma_g * \frac{p}{z * R * T}$$

Reference: *Applied Petroleum Reservoir Engineering, Second Edition, Craft & Hawkins, Page: 32.*

**8.38 Stefan-Boltzmann law****Input(s)**

$\sigma$ : Stefan Boltzmann Constant ( $1.713 \times 10^{-9}$ ) ( $\text{BTU}/\text{ft}^2 \text{ h } ^{\circ}\text{R}^4$ )

T: Temperature ( $^{\circ}$ F)

$\epsilon$ : Emissivity (fraction)

**Output(s)**

$u_i$ : Rate of Radiation Heat Transfer Per Unit Area ( $\text{BTU}/\text{h ft}^2$ )

**Formula(s)**

$$u_i = \sigma * \epsilon * (T + 460)^4$$

Reference: *Pratts, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 19.*

## 8.39 Surface temperature of a heating coil

### Input(s)

- $T_o$ : Initial Surface Temperature (K)
- $\mu$ : Viscosity (Pa s)
- $\rho$ : Density ( $\text{g}/\text{m}^3$ )
- $g$ : Gravitational Acceleration ( $\text{ft}/\text{s}^2$ )
- $\beta$ : Velocity Flux (1/s)
- $D$ : Diameter (m)
- $Q$ : Net Energy Input Rate (J/s)
- $k$ : Thermal Conductivity ( $\text{J}/(\text{m s K})$ )

### Output(s)

- $T_1 - T_o$ : Final Temperature of Surface (K)

### Formula(s)

$$T_1 - T_o = \frac{\mu^2}{(\rho^2) * g * \beta * (D^3)} * Q \left( \frac{Q * (\rho^2) * g * \beta * (D^2)}{k * (\mu^2)} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 11, Page: 361.*

## 8.40 Temperature due to free convection

### Input(s)

- $T_1$ : Temperature at Cooled Plate (K)
- $T_2$ : Temperature at Heated Plate (K)
- $y$ : Distance from Centre (cm)
- $B$ : Half Distance between the Plates (cm)

### Output(s)

- $T$ : Temperature (K)

### Formula(s)

$$T = 0.5 * (T_1 + T_2) - 0.5 * (T_2 - T_1) * \left( \frac{y}{B} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 317.*

## 8.41 Temperature increase due to forced convection

### Input(s)

- $q$ : Interfacial Heat Flux ( $\text{J}/(\text{m}^2 \text{s})$ )
- $D$ : Diameter (m)
- $k$ : Thermal Heat Conductivity ( $\text{W}/(\text{m K})$ )

**Output(s)**

T: Temperature Increase (K)

**Formula(s)**

$$T = 11 * q * \frac{D}{48 * k}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons.*  
*Chapter: 10, Page: 316.*

**8.42 Temperature profile after viscous heat transfer****Input(s)**

T<sub>b</sub>: Temperature of Outer Perimeter (K)  
 b: Distance of Outer Boundary (ft)  
 Br: Brinkman (dimensionless)  
 T<sub>o</sub>: Temperature of Inner Perimeter (K)  
 x: Distance from Centre of Pipe (ft)

**Output(s)**

T: Temperature at Distance X (K)

**Formula(s)**

$$\left( \frac{T - T_o}{T_b - T_o} \right) = \frac{1}{2} * Br * \left( \frac{x}{b} \right) * \left( 1 - \frac{x}{b} \right) + \frac{x}{b}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons,*  
*Chapter: 10, Page: 300.*

**8.43 Temperature profile with a nuclear heat source****Input(s)**

S<sub>no</sub>: Volume Rate Heat Production at the Center of the Source (cal/cm<sup>3</sup> s)  
 R<sup>F</sup>: Radius of Fissionable Substance (cm)  
 K<sup>F</sup>: Thermal Conductivity of Fissionable Substance (W/cm K)  
 r: Radius of Core (cm)  
 b: Constant of Fission (dimensionless)  
 k<sup>C</sup>: Thermal Conductivity of Casing Substance (W/cm K)  
 R<sup>C</sup>: Radius of Casing Substance (cm)  
 T<sub>o</sub>: Initial Temperature (K)

**Output(s)**

T<sub>f</sub>: Temperature of Fissionable Substance (K)

**Formula(s)**

$$T_f = \left( \frac{S_{no} * (R^F)^2}{6 * K^F} \right) * \left( \left( 1 - \left( \frac{r}{R^F} \right)^2 \right) + \frac{3}{10} * b * \left( 1 - \left( \frac{r}{R^F} \right)^4 \right) \right) + \left( \frac{S_{no} * (R^F)^2}{3 * k^c} \right) * \left( 1 + \frac{3}{5} * b \right) * \left( 1 - \frac{R^F}{R^c} \right) + T_o$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 298.*

**8.44 Thermal conductivity for pure metals****Input(s)**

- L: Lorenz Number ( $V^2/K^2$ )
- $k_e$ : Electrical Conductivity ( $S/m$ )
- T: Temperature (K)

**Output(s)**

- k: Thermal Conductivity ( $W/m\ K$ )

**Formula(s)**

$$k = L * k_e * T$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 280.*

**8.45 Thermal conductivity of polyatomic gases****Input(s)**

- R: Gas Constant ( $g\ cm^2/s^2\ K\ gmol$ )
- M: Molecular Weight ( $g/gmol$ )
- $\mu$ : Viscosity ( $g/cm\ s$ )
- $\hat{C}_p$ : Specific Heat ( $cal/g\ -mol\ K$ )

**Output(s)**

- k: Thermal Conductivity ( $W/m\ K$ )

**Formula(s)**

$$k = \left( \hat{C}_p + \frac{5}{4} * R \right) * \left( \frac{\mu}{M} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 278.*

**8.46 Thermal conductivity of liquids****Input(s)**

- $\tilde{N}$ : Avogadro's Number ( $1/gmol$ )
- k: Thermal Conductivity ( $J/m\ s\ K$ )
- K: Boltzmann Constant ( $g\ cm^2/s^2\ F$ )
- $v_s$ : Sonic Velocity (cm/s)
- $\tilde{v}$ : Volume ( $cm^3$ )

### Output(s)

$k$ : Thermal Conductivity ( $\text{g c/F s}^3$ )

### Formula(s)

$$k = \left( 2.80 * \left( \frac{\tilde{N}}{\tilde{V}} \right)^{\frac{2}{3}} \right) * K * v_s$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 279.*

## 8.47 Thermal conductivity of solids with gas pockets

### Input(s)

$k_o$ : Thermal Conductivity of Solid Phase ( $\text{W/m K}$ )

$k_1$ : Thermal Conductivity of the Gas ( $\text{W/m K}$ )

$\emptyset$ : Porosity (fraction)

$T$ : Temperature (K)

$L$ : Length (m)

$\sigma$ : Stefan-Boltzmann Constant ( $\text{kg/s}^3 \text{ K}^4$ )

### Output(s)

$k_{eff}$ : Effective Thermal Conductivity ( $\text{W/m K}$ )

### Formula(s)

$$\frac{k_{eff}}{k_o} = \frac{1}{1 - \emptyset + \left( \left( \frac{k_1}{k_o * \emptyset} \right) + \left( \frac{4 * \sigma * T^3 * L}{k_o} \right) \right)^{-1}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 282.*

## 8.48 Thermal diffusivity

### Input(s)

$k$ : Thermal Conductivity ( $\text{W/(m K)}$ )

$\hat{C}_p$ : Heat Capacity at Constant Pressure ( $\text{J/(kg K)}$ )

$\rho$ : Density ( $\text{kg/m}^3$ )

### Output(s)

$\alpha$ : Thermal Diffusivity ( $\text{m}^2/\text{s}$ )

### Formula(s)

$$\alpha = \frac{k}{\hat{C}_p * \rho}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 9, Page: 268.*

## 8.49 Thermal energy of a fissionable substance

### Input(s)

- $S_{no}$ : Rate of Heat Production at Center of Substance (cal/cm<sup>3</sup> s)  
**b**: Constant of Fission (dimensionless)  
**r**: Radius of Centre of Sphere (cm)  
 $R^F$ : Radius of Fissionable Material (cm)

### Output(s)

- $S_n$ : Thermal Energy Resulting from Volume Source (cal/cm<sup>3</sup> s)

### Formula(s)

$$S_n = S_{no} * \left( 1 + b * \left( \left( \frac{r}{R^F} \right)^2 \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 296.*

## 8.50 Thermoelastic effect on stress

### Input(s)

- $\lambda$ : Lame (psi)  
 $\delta_{ij}$ : Kronecker Delta (dimensionless)  
 $\varepsilon_{00}$ : Initial Strain (dimensionless)  
**G**: Modulus of Shear (psi)  
 $\varepsilon_{ij}$ : Final Strain (dimensionless)  
 $\alpha_T$ : Coefficient of Linear Thermal Expansion (1/K)  
**P<sub>0</sub>**: Pressure Applied (psi)  
**K**: Bulk Modulus (psi)  
 $\Delta T$ : Temperature Difference (K)

### Output(s)

- $S_{ij}$ : Stress (psi)

### Formula(s)

$$S_{ij} = \lambda * \delta_{ij} * \varepsilon_{00} + 2 * G * \varepsilon_{ij} - \alpha_T * \delta_{ij} * P_0 - K * \alpha_T * \delta_{ij} * \Delta T$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Express, UK, Page: 83.*

## 8.51 Unsteady evaporation of a liquid

### Input(s)

- S**: Surface Area (cm<sup>2</sup>)  
 $\emptyset$ : Location of Film (cm)  
 $D_{AB}$ : Difusivity (cm<sup>2</sup>/s)  
**t**: Time (s)

### Output(s)

$V_a$ : Rate of Evaporation (cc/s)

### Formula(s)

$$V_a = S * \phi * \left( \left( \frac{4 * D_{AB} * t}{\pi} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 20, Page: 615.*

## 8.52 Van Der Waals PVT equation

### Input(s)

- P: Pressure (psi)
- T: Temperature (K)
- a: Correlation Constant (dimensionless)
- b: Correlation Constant (dimensionless)
- V: Volume (ft<sup>3</sup>)

### Output(s)

R: Gas Constant (BTU/psi mol K)

### Formula(s)

$$R = \frac{\left( P + \frac{a}{V^2} \right) * (V - b)}{T}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 1, Page: 48, Campbell Petroleum Series, Oklahoma, 1992.*

## 8.53 Wien displacement law

### Input(s)

$\lambda_{max}$ : Maximum Wavelength (1/cm)

### Output(s)

T: Temperature (K)

### Formula(s)

$$T = \frac{0.2884}{\lambda_{max}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 16, Page: 495.*

## Chapter 9

# Petroleum engineering laboratory formulas and calculations

### Chapter Outline

9.1 Absolute viscosity for Saybolt viscosimeter measurements	399	9.20 Gas permeability measurement (lab measurement using Klinkenberg effect)	407
9.2 Absolute viscosity for Ubbelohde viscosimeter measurements	400	9.21 Kinematic viscosity for Saybolt viscosimeter measurements	408
9.3 Adhesion tension	400	9.22 Liquid permeability (permeameter lab measurement)	408
9.4 Amott-Harvey wettability index	400	9.23 Permeability determination using porosity data (Kozeny-Carman equation)	409
9.5 Apparent facial tension (De Nouy ring method)	401	9.24 Pycnometer volume correction	409
9.6 Average compressibility of oil	401	9.25 Relative centrifugal force	409
9.7 Average gas solubility	401	9.26 Relative permeability	410
9.8 Characterization factor for oil distillation	402	9.27 Reservoir wettability characterization (rise in core method)	410
9.9 Clasius-Clapeyron equation for water vapor	402	9.28 Resistance	411
9.10 Clay concentration of drilling mud (methylene blue test)	402	9.29 Resistivity	411
9.11 Contact angle	403	9.30 Resistivity index—Archie's law	412
9.12 Correction factor for facial tension (De Nouy ring method)	404	9.31 Solid content ratio of drilling mud	412
9.13 Drilling mud density (solid content analysis of drilling muds)	404	9.32 Specific gravity of air (upper phase) (De Nouy ring method)	412
9.14 Effective porosity	405	9.33 Standard discharge time for Saybolt viscosimeter measurements	413
9.15 Error percentage of porosity measurements	405	9.34 Total porosity	413
9.16 Facial tension (De Nouy ring method)	405	9.35 USBM wettability index	413
9.17 Filtration rate for API fluid loss measurement	406	9.36 Yield of clays as drilling fluids	414
9.18 Filtration volume without spurt loss	406		
9.19 Filtration volume with spurt loss	407		

### 9.1 Absolute viscosity for Saybolt viscosimeter measurements

#### Input(s)

- v: Kinematic Viscosity (Centistokes)  
ρ: Measured Density of Fluid (g/cm<sup>3</sup>)

#### Output(s)

- μ: Absolute Viscosity (cP)

#### Formula(s)

$$\mu = v \cdot \rho$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 2–7.

## 9.2 Absolute viscosity for Ubbelohde viscosimeter measurements

### Input(s)

$t$ : Measured time (s)  
 $C_u$ : Viscosimeter Constant (mPa)

### Output(s)

$\mu$ : Absolute Viscosity (cP)

### Formula(s)

$$\mu = C_u \cdot t$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 2–7.

## 9.3 Adhesion tension

### Input(s)

$\sigma_{so}$ : Interfacial Tension between the solid and oil (dyn/cm)  
 $\sigma_{sw}$ : Interfacial Tension between the solid and water (dyn/cm)  
 $\sigma_{wo}$ : Interfacial Tension between the water and oil (dyn/cm)  
 $\theta$ : Contact Angle (degree)

### Output(s)

$\tau$ : Adhesion Tension (dyn/cm)

### Formula(s)

$$\tau = \sigma_{so} - \sigma_{sw} = \sigma_{wo} \cos \theta$$

Reference: Tiab, D., & Donaldson, E. C. (2015). Petrophysics: Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties. Gulf Professional Publishing. Page: 363.

## 9.4 Amott-Harvey wettability index

### Input(s)

$I_o$ : Oil Spontaneous Imbibition Ratio (dyn/cm)  
 $I_w$ : Water Spontaneous Imbibition Ratio (dyn/cm)

### Output(s)

$I_{AH}$ : Amott-Harvey Index (dimensionless)

### Formula(s)

$$I_{AH} = I_w - I_o$$

Reference: Ghedan, S. G., Canbaz, C. H., Boyd, D. A., Mani, G. M., & Haggag, M. K. (2010, January). Wettability Profile of a Thick Carbonate Reservoir by the New Rise in Core Wettability Characterization Method. Abu Dhabi International Petroleum Exhibition and Conference. Society of Petroleum Engineers. Page: 3.

## 9.5 Apparent facial tension (De Nouy ring method)

### Input(s)

- $g$ : Acceleration of Gravity (980 cm/s<sup>2</sup>)
- $m$ : Measured Weight (g)
- $l$ : Perimeter of the Ring (cm)

### Output(s)

- $S$ : Apparent Facial Tension (dyn/cm)

### Formula(s)

$$S = \frac{m \cdot g}{2 \cdot l}$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 4–3.*

## 9.6 Average compressibility of oil

### Input(s)

- $V$ : Reference Volume (volume fraction relative to bubble point volume or higher pressure)
- $V_1$ : Volume Fraction at Higher Pressure (fraction)
- $V_2$ : Volume Fraction at Lower Pressure (fraction)
- $p_1$ : Pressure (Relative to  $V_1$ ) (psi)
- $p_2$ : Pressure (Relative to  $V_2$ ) (psi)

### Output(s)

- $C_o$ : Average Compressibility of Oil (psi<sup>-1</sup>)

### Formula(s)

$$C_o = -\frac{1}{V} * \left( \frac{V_1 - V_2}{p_1 - p_2} \right)$$

Reference: *Craft, B. C., Hawkins, M., & Terry, R. E. (1991). Applied Petroleum Reservoir Engineering. 2nd Edition, Page: 38.*

## 9.7 Average gas solubility

### Input(s)

- $s_1$ : Solubility at  $p_1$  (SCF/STB)
- $s_2$ : Solubility at  $p_2$  (SCF/STB)
- $p_1$ : Pressure1 (psi)
- $p_2$ : Pressure2 (psi)

### Output(s)

- $S_{avg}$ : Average Gas Solubility (SCF/STB/psi)

**Formula(s)**

$$S_{avg} = \frac{s_1 - s_2}{p_1 - p_2}$$

Reference: *Craft, B. C., Hawkins, M., & Terry, R. E. (1991). Applied Petroleum Reservoir Engineering. 2nd Edition, Page: 32.*

**9.8 Characterization factor for oil distillation****Input(s)**

$T_b$ : Average Boiling Point ( $^{\circ}\text{R}$  (or  $^{\circ}\text{F} + 460$ ))

$\gamma_{60/60^{\circ}\text{F}}$ : Specific Gravity (dimensionless)

**Output(s)**

$K$ : Characterization Factor (dimensionless)

**Formula(s)**

$$\frac{K = T_b^{1/3}}{\gamma_{60/60^{\circ}\text{F}}}$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 5–2.*

**9.9 Clasius-Clapeyron equation for water vapor****Input(s)**

$L_v$ : Heat of Vaporization of one mole of Liquid (J/mol)

$T_1$ : Absolute Temperature of Condition 1 ( $^{\circ}\text{F}$ )

$T_2$ : Absolute Temperature of Condition 2 ( $^{\circ}\text{F}$ )

$R$ : Gas Constant for Water Vapor (psi/mol  $^{\circ}\text{F}$  s<sup>2</sup>)

**Output(s)**

$p_{v1}$ : Vapor Pressure at the Temperature  $T_1$  (psi)

$p_{v2}$ : Vapor Pressure at the Temperature  $T_2$  (psi)

**Formula(s)**

$$\ln\left(\frac{p_{v1}}{p_{v2}}\right) = \frac{L_v}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

Reference: *McCain Jr, W. D. (1990). Properties of Petroleum Fluids. PennWell Corporation. Page: 54.*

**9.10 Clay concentration of drilling mud (methylene blue test)****Input(s)**

$V_{mb}$ : Volume of Methylene Blue used in Experiment, meq/100 mL (mL)

$V_{dm}$ : Volume of Drilling Mud used in Experiment, meq/100 mL (mL)

- $V_{sc}$ : Volume of Solid Content (Cuttings or Bentonite) in Drilling Mud, meq/100 mL (g)  
 $MBT_m$ : Methylene Blue Result of Drilling Mud (mL/mL)  
 $MBT_{ds}$ : Methylene Blue Result of Drilling Cuttings (mL/mL)  
 $MBT_c$ : Methylene Blue Result of Clay (mL/mL)

**Output(s)**

- $f_c$ : Clay Content Ratio in Drilling Mud (fraction)  
 $C_c$ : Clay Concentration in Drilling Mud (lb/bbl)

**Formula(s)**

$$MBT_m = \frac{V_{mb}}{V_{dm}}$$

$$MBT_{ds} = \frac{V_{mb}}{V_{sc}}$$

$$MBT_c = \frac{V_{mb}}{V_{sc}}$$

$$f_c = \frac{MBT_m - 2.6 \cdot f_{lg} \cdot MBT_{ds}}{2.6(MBT_c - MBT_{ds})}$$

$$C_c = f_c \cdot 21.7 \frac{\text{lb}}{\text{gal}} \cdot 42 \frac{\text{gal}}{\text{bbl}}$$

Reference: Altun, G., *Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 3, Page: 4.*

**9.11 Contact angle****Input(s)**

- $\sigma_{so}$ : Interfacial Tension between the solid and oil (dyn/cm)  
 $\sigma_{sw}$ : Interfacial Tension between the solid and water (dyn/cm)  
 $\sigma_{wo}$ : Interfacial Tension between the water and oil (dyn/cm)

**Output(s)**

- $\theta$ : Contact Angle (degree)

**Formula(s)**

$$\cos\theta = \frac{\sigma_{so} - \sigma_{sw}}{\sigma_{wo}}$$

Reference: Tiab, D., & Donaldson, E. C. (2015). *Petrophysics: Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties*. Gulf Professional Publishing. Page: 362.

## 9.12 Correction factor for facial tension (De Nouy ring method)

### Input(s)

- $R$ : Radius of the Ring (cm)
- $r$ : Radius of the Ring String (cm)
- $S$ : Apparent Facial/Interfacial Tension (dyn/cm)
- $D$ : Specific Gravity of Lower Phase ( $\text{g}/\text{cm}^3$ )
- $d$ : Specific Gravity of Upper Phase ( $\text{g}/\text{cm}^3$ )
- $l$ : Perimeter of the Ring (cm)

### Output(s)

- $C$ : Correction Factor (dimensionless)

### Formula(s)

$$C = 0.7250 + \sqrt{\frac{0.01452 \cdot S}{l^2(D-d)} + 0.04534 - \frac{1.679 \cdot r}{R}}$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., *Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 4–4.*

## 9.13 Drilling mud density (solid content analysis of drilling muds)

### Input(s)

- $\rho_w$ : Density of used Water ( $\text{lb}/\text{gal}$ )
- $f_w$ : Water Content Ratio of Drilling Mud (fraction)
- $\rho_{lg}$ : Density of Clay and Cuttings ( $\text{lb}/\text{gal}$ )
- $f_{lg}$ : Content Ratio of Clay and Cuttings in Drilling Mud (fraction)
- $\rho_B$ : Density of Barite ( $\text{lb}/\text{gal}$ )
- $f_B$ : Content Ratio of Barite (fraction)
- $\rho_o$ : Density of Oil that is used in Oil-based Muds ( $\text{lb}/\text{gal}$ )
- $f_o$ : Oil Content Ratio in Drilling Mud (fraction)

### Output(s)

- $\rho_m$ : Drilling Mud Density ( $\text{lb}/\text{gal}$ )
- $f_s$ : Total Solid Content Ratio in a Drilling Mud (fraction)

### Formula(s)

$$\rho_m = \rho_w f_w + \rho_{lg} f_{lg} + \rho_B f_B + \rho_o f_o$$

For water-based muds

$$f_w + f_{lg} + f_B = 1$$

$$f_s = f_{lg} + f_B$$

$$f_w = 1 - f_s$$

$$f_s = 0.3125 \left[ \frac{\rho_m}{8.33} - 1 \right] + 0.5f_{lg}$$

Reference: Altun, G., *Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 3, Page: 1.*

## 9.14 Effective porosity

### Input(s)

- $V_{P_e}$ : Effective Pore Volume ( $\text{cm}^3$ )
- $V_b$ : Bulk Volume ( $\text{cm}^3$ )
- $V_o$ : Total Oil Volume in Interconnected Pores ( $\text{cm}^3$ )
- $V_g$ : Total Gas Volume in Interconnected Pores ( $\text{cm}^3$ )
- $V_w$ : Total Water Volume in Interconnected Pores ( $\text{cm}^3$ )

### Output(s)

- $\phi_{eff}$ : Effective Porosity (fraction)

### Formula(s)

$$V_{P_e} = V_o + V_g + V_w$$

$$\phi_{eff} = \frac{V_{P_e}}{V_b}$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 1–3.

## 9.15 Error percentage of porosity measurements

### Input(s)

- $\phi_m$ : Measured Porosity (fraction)
- $\phi_{ps}$ : Calculated Porosity or Pseudo-Porosity (fraction)

### Output(s)

- $E_\phi$ : Effective Porosity (fraction)

### Formula(s)

$$E_\phi = \frac{\phi_m - \phi_{ps}}{\phi_m} \cdot 100$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 1–7.

## 9.16 Facial tension (De Nouy ring method)

### Input(s)

- $C$ : Correction Factor (dimensionless)
- $S$ : Apparent Facial Tension ( $\text{dyn}/\text{cm}$ )

**Output(s)**

$\sigma$ : Real Facial Tension (dyn/cm)

**Formula(s)**

$$\sigma = S \cdot C$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., *Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 4–4.*

**9.17 Filtration rate for API fluid loss measurement****Input(s)**

$h_{mc}$ : Filter Cake Thickness (cm)

$A$ : Filter Cake Area ( $\text{cm}^2$ )

$\Delta P$ : Pressure Difference (Atm)

$k$ : Permeability of the Filter Cake (D)

$\mu$ : Viscosity of the Filtration Fluid (cP)

**Output(s)**

$\frac{dV_f}{dt}$ : Filtration Rate (cc/s)

**Formula(s)**

$$\frac{dV_f}{dt} = \frac{k \cdot A \cdot \Delta P}{\mu \cdot h_{mc}}$$

Reference: Altun, G., *Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 2, Page: 4.*

**9.18 Filtration volume without spurt loss****Input(s)**

$V_{7.5}$ : Filtration Volume Measurement at 7.5 min (cc)

**Output(s)**

$V_{30}$ : Filtration Volume Measurement at 30 min (cc)

**Formula(s)**

$$V_{30} = 2 \cdot V_{7.5}$$

Reference: Altun, G., *Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 2, Page: 6.*

## 9.19 Filtration volume with spurt loss

### Input(s)

- $V_{sp}$ : Spurt Loss Volume (cc)  
 $V_{f1}$ : Filtration Volume at time "1" (cc)  
 $V_{f2}$ : Filtration Volume at time "2" (cc)  
 $t_1$ : Time value "1" (min)  
 $t_2$ : Time value "2" (min)  
 $t$ : Filtration time of the measured Filtration Volume " $V_f$ " (min)

### Output(s)

- $V_f$ : Filtration Volume (cc)

### Formula(s)

$$V_f = V_{sp} + \frac{V_{f2} - V_{f1}}{\sqrt{t_2} - \sqrt{t_1}} \sqrt{t}$$

Reference: Altun, G., Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 2, Page: 6.

## 9.20 Gas permeability measurement (lab measurement using Klinkenberg effect)

### Input(s)

- $\mu$ : Viscosity of Liquid Phase (cP)  
 $\mu_{air}$ : Viscosity of Air (cP)  
 $\mu_{CO_2}$ : Viscosity of CO<sub>2</sub> (cP)  
 $Q_{avg}$ : Average Rate in Porous Media (cm<sup>3</sup>/s)  
 $P_{avg}$ : Average Pressure in Porous Media (Atm)  
 $L$ : Plug Length (cm)  
 $A$ : Open-Flow Area of the Plug (cm<sup>2</sup>)  
 $P_1$ : Inlet Pressure (Atm)  
 $P_2$ : Outlet Pressure (Atm)  
 $V$ : Liquid Volume that flows in time  $t$  (cm<sup>3</sup>)  
 $V_1$ : Liquid Volume that flows at Inlet Part in Time  $t$  (cm<sup>3</sup>)  
 $V_2$ : Liquid Volume that flows at Outlet Part in Time  $t$  (cm<sup>3</sup>)  
 $Q_1$ : Liquid Rate that flows at Inlet Part (cm<sup>3</sup>)  
 $Q_2$ : Liquid Rate that flows at Outlet Part (cm<sup>3</sup>)

### Output(s)

- $k$ : Liquid Permeability (D)  
 $k_g$ : Gas Permeability (D)

### Formula(s)

$$k = \frac{2\mu \cdot Q_{avg} \cdot P_{avg} \cdot L}{A \cdot (P_1^2 - P_2^2)}$$

$$P_1 \cdot V_1 = P_2 \cdot V_2 = P_{avg} \cdot V_{avg}$$

$$P_1 \cdot Q_1 = P_2 \cdot Q_2 = P_{avg} \cdot Q_{avg}$$

$$k_g = k \left( 1 + \frac{b}{P_{avg}} \right)$$

$$\mu_{air} = 0.000044848 \cdot T + 0.0165$$

$$\mu_{CO_2} = 0.00005000 \cdot T + 0.0138$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 3–4.

## 9.21 Kinematic viscosity for Saybolt viscosimeter measurements

### Input(s)

$t_o$ : Measured Discharge Time (s)

### Output(s)

$v$ : Kinematic Viscosity (Centistokes)

### Formula(s)

$$v = 0.220 t_o - \frac{180}{t_o}$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 2–6.

## 9.22 Liquid permeability (permeameter lab measurement)

### Input(s)

$\mu$ : Viscosity of Liquid Phase (cP)

$t$ : Flowing Time (s)

$V$ : Liquid Volume that flows in time  $t$ —Burette Volume ( $cm^3$ )

$P$ : Pressure value that read from Pressure Gauge (Atm)

$L$ : Plug Length (cm)

$A$ : Open-Flow Area of the Plug ( $cm^2$ )

### Output(s)

$k$ : Permeability (D)

**Formula(s)**

$$k = \frac{\mu \cdot V \cdot L}{A \cdot P \cdot t}$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 2–6.*

## **9.23 Permeability determination using porosity data (Kozeny-Carman equation)**

**Input(s)**

- $\phi$ : Effective Porosity (fraction)
- $d$ : Average Diameter of Rock Particles (in.)

**Output(s)**

- $k$ : Permeability (mD)

**Formula(s)**

$$k = 3.631 \cdot 10^9 \cdot \frac{d^2 - \phi^3}{(1 - \phi)^2}$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 1–8.*

## **9.24 Pycnometer volume correction**

**Input(s)**

- $V_k$ : Calibration Volume of Specific Gravity bottle (Pycnometer) (mL or  $\text{cm}^3$ )
- $T_t$ : Test Temperature ( $^\circ\text{C}$ )
- $T_k$ : Calibration Temperature ( $^\circ\text{C}$ )
- $\Omega$ : Material Constant (for Ordinary Glass =  $0.276 \cdot 10^{-4}/^\circ\text{C}$ )
- $\Omega$ : Material Constant (for Pyrex Glass =  $0.156 \cdot 10^{-4}/^\circ\text{C}$ )

**Output(s)**

- $V_g$ : Real Volume of Specific Gravity bottle (Pycnometer) (mL or  $\text{cm}^3$ )

**Formula(s)**

$$V_g = V_k \left[ 1 + \Omega (T_g - T_k) \right]$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page 1–4.*

## **9.25 Relative centrifugal force**

**Input(s)**

- RPM: Revolutions per Minute (rpm)
- $d$ : Expansion Diameter between two confronted tubes (in.)

### Output(s)

*RCF:* Relative Centrifugal Force (cP)

### Formula(s)

$$RCF = \left( \frac{RPM}{265} \right)^2 \cdot d$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 3–2.*

## 9.26 Relative permeability

### Input(s)

- $q_o$ : Oil Flow Rate ( $\text{cm}^3/\text{s}$ )
- $q_w$ : Water Flow Rate ( $\text{cm}^3/\text{s}$ )
- $q_T$ : Total Flow Rate ( $\text{cm}^3/\text{s}$ )
- $k_o$ : Effective Permeability to Oil (mD)
- $k_w$ : Effective Permeability to Water (mD)
- $k$ : Absolute Permeability (mD)
- $\mu_o$ : Viscosity of Oil Phase (cP)
- $\mu_w$ : Viscosity of Water Phase (cP)
- $P$ : Pressure value that read from Pressure Gauge (Atm)
- L: Plug Length (cm)
- A: Open-Flow Area of the Plug ( $\text{cm}^2$ )

### Output(s)

- $k_{ro}$ : Relative Permeability to Oil (mD)
- $k_{rw}$ : Relative Permeability to Water (mD)

### Formula(s)

$$k_{ro} = \frac{k_o}{k}$$

$$k_{rw} = \frac{k_w}{k}$$

$$k_{ro} = -\frac{1}{0.001127} \cdot \frac{q_o \mu_o L}{k \cdot A \cdot \Delta P}$$

$$k_{rw} = -\frac{1}{0.001127} \cdot \frac{q_w \mu_w L}{k \cdot A \cdot \Delta P}$$

Reference: *Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, II-Properties of Porous Media, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 4–2.*

## 9.27 Reservoir wettability characterization (rise in core method)

### Input(s)

- $\mu_1$ : Viscosity of Oil Phase (cP)
- $\mu_2$ : Viscosity of Water Phase (cP)
- $\rho_1$ : Density of Oil Phase ( $\text{g}/\text{cm}^3$ )
- $\rho_2$ : Density of Water Phase ( $\text{g}/\text{cm}^3$ )
- $m$ : Mass of Fluid Penetrated into a Porous Rock (g)

*t*: Time (min)

*C*: Characteristic Constant of the Porous Rock (dimensionless)

$\gamma_{L2L1}$ : Surface Tension between oil and water phases (dyn/cm)

### Output(s)

$\theta_{12}$ : Contact Angle of Liquid/Liquid/Rock System (degree)

### Formula(s)

$$\cos \theta_{12} = \frac{(\mu_1 \rho_2^2) - (\mu_2 \rho_1^2)}{\rho_1^2 \rho_2^2 \cdot C \cdot \gamma_{L2L1}} \cdot \frac{m^2}{t}$$

Reference: *Ghedan, S., & Canbaz, C. H. (2014). U.S. Patent No. 8,768,628. Washington, DC: U.S. Patent and Trademark Office, Page: 5.*

## 9.28 Resistance

### Input(s)

*dv*: Potential drop (V)

*I*: Current (A)

### Output(s)

*r*: Resistance (ohm)

### Formula(s)

$$r = \frac{dv}{I}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

## 9.29 Resistivity

### Input(s)

*r*: Resistance in ohm (ohm)

*A*: Cross Section area ( $m^2$ )

*L*: Length (m)

### Output(s)

*R*: Resistivity (ohm m)

### Formula(s)

$$R = r * \frac{A}{L}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

### 9.30 Resistivity index—Archie's law

#### Input(s)

- $R_t$ : True Resistivity (ohm m)  
 $R_o$ : Resistivity of Rock Filled with Water (ohm m)

#### Output(s)

- $RI$ : Resistivity Index (unitless)

#### Formula(s)

$$RI = \frac{R_t}{R_o}$$

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics, Page: 42.*

### 9.31 Solid content ratio of drilling mud

#### Input(s)

- $V_m$ : Volume of Drilling Mud (cc)  
 $f_{sc}$ : Solid Content Ratio of Filter Cake (ratio)  
 $h_{mc}$ : Filter Cake Thickness (cm)  
 $A$ : Filter Cake Area ( $\text{cm}^2$ )

#### Output(s)

- $f_{sm}$ : Solid Content Ratio of Drilling Mud (ratio)

#### Formula(s)

$$f_{sm} = \frac{f_{sc} \cdot h_{mc} \cdot A}{V_m}$$

Reference: *Altun, G., Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 2, Page: 4.*

### 9.32 Specific gravity of air (upper phase) (De Nouy ring method)

#### Input(s)

- P: Pressure (psi)  
T: Temperature ( $^{\circ}\text{R}$ )

#### Output(s)

- $\rho_{air}$ : Specific Gravity of Air at P and T ( $\text{g}/\text{cm}^3$ )

**Formula(s)**

$$\rho_{\text{air}} = 4.324 \cdot 10^{-2} \left( \frac{P}{T} \right)$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 4–4.

### 9.33 Standard discharge time for Saybolt viscosimeter measurements

**Input(s)**

- $t_o$ : Measured Discharge Time (s)
- $T_s$ : Standard Temperature (°F)
- $T_o$ : Test Temperature (°F)

**Output(s)**

- $t_s$ : Standard Discharge Time (s)

**Formula(s)**

$$t_s = t_o [1 + 0.000064(T_o - T_s)]$$

Reference: Mihcakan, I.M., Alkan, K.H., Ugur, Z., Petroleum and Natural Gas Laboratory, Course Notes, I-Fluid Properties, ITU, Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2001. Page: 2–6.

### 9.34 Total porosity

**Input(s)**

- $V_i$ : Volume of Interconnected Pores (cm<sup>3</sup>)
- $V_d$ : Volume of Dead-end Pores (cm<sup>3</sup>)
- $V_b$ : Total or Bulk Volume (cm<sup>3</sup>)

**Output(s)**

- $\phi$ : Total Porosity (fraction)

**Formula(s)**

$$\phi = \frac{V_i + V_d}{V_b}$$

Reference: Dandekar, A. Y. 2006. Petroleum Reservoir Rock and Fluid Properties. Boca Raton, FL: CRC Press Taylor & Francis Group, chapter: 3, Page: 15.

### 9.35 USBM wettability index

**Input(s)**

- $A_1$ : Area Ratio of Oil Displacing Water (dimensionless)
- $A_2$ : Area Ratio of Water Displacing Oil (dimensionless)

**Output(s)**

$I_U$ : Amott-Harvey Index (dimensionless)

**Formula(s)**

$$I_U = \log \left( \frac{A_1}{A_2} \right)$$

Reference: Ghedan, S. G., Canbaz, C. H., Boyd, D. A., Mani, G. M., & Haggag, M. K. (2010, January). Wettability profile of a thick carbonate reservoir by the new rise in core wettability characterization method. In Abu Dhabi International Petroleum Exhibition and Conference. Society of Petroleum Engineers. Page: 3.

### **9.36 Yield of clays as drilling fluids**

**Input(s)**

$M_{clay}$ : Weight of Clay used per barrel of mud with 15 cP Apparent Viscosity (tonnes/bbl)

**Output(s)**

$E_{clay}$ : Efficiency of Selected Clay (bbl/tonnes)

**Formula(s)**

$$E_{clay} = \frac{1}{M_{clay}}$$

Reference: Altun, G., Drilling Fluids Lab, Course Notes, ITU Petroleum and Natural Gas Engineering, Istanbul, Turkey, 2013–2014. Experiment 1, Page: 9.

## Chapter 10

# Enhanced oil recovery and geothermal formulas and calculations

### Chapter Outline

10.1 Areal extent of heated zone	416	10.28 Fraction of injected heat remaining in reservoir	427
10.2 Average reservoir temperature in a cyclical steam injection process	416	10.29 Fractional flow of water in hot floods dependent on temperature and saturation in hot water flood	427
10.3 Bottomhole pressure in a static geothermal well	417	10.30 Growth of steam-heated area—Marx-Langenheim	428
10.4 Chromatographic lag in polymer flooding	417	10.31 Heat loss over an incremental length of a well (two-phase flow)	428
10.5 Cumulative heat injected for steam drive—Myhill and Stegemeier	417	10.32 Heat ratio of contents in a geothermal reservoir	428
10.6 Depth of carbon dioxide alteration front (Battlet-Gouedard, 2006)	417	10.33 Heat released during in-situ combustion—Burger & Sahuguet	429
10.7 Depth of carbon dioxide alteration front (Kutchko, 2008)	418	10.34 Heat remaining in reservoir—Marx and Langenheim	429
10.8 Dimensionless heat injection rate (Gringarten and Sauty)	418	10.35 Horizontal well breakthrough time in a bottom-water-drive reservoir	430
10.9 Dimensionless injection rate of air for in-situ combustion	418	10.36 Ignition delay time in in-situ combustion	430
10.10 Dimensionless ratio of effective volumetric heat capacity of injected steam to that of the steam zone	419	10.37 Injected air required to burn through unit bulk of reservoir for in-situ combustion by Nelson and McNeil	431
10.11 Dimensionless time for semi-steady state flow in coal bed methane reservoirs	419	10.38 Mass of fuel burned per unit bulk reservoir volume combustion—Nelson and McNeil	431
10.12 Dimensionless time in wet combustion by Kuo	420	10.39 Minimum air flux required for advance of fire front—Nelson and McNeil	432
10.13 Dykstra-Parsons coefficient	421	10.40 Oil breakthrough newly-swept zone	432
10.14 Effective (apparent) transmissivity	421	10.41 Oil recovery as a function of the fraction of oil displaced from heated zone	432
10.15 Effective oil transmissivity for thermal stimulation	421	10.42 Oil solubilization factor	433
10.16 Equivalent atomic H/C ratio of fuel for in-situ combustion	422	10.43 Oil volume at breakthrough by Craig, Geffen, and Morse	433
10.17 Equivalent volume of steam injected—Myhill and Stegemeier	422	10.44 Oil-steam ratio—Marx & Langenheim	434
10.18 Equivalent water saturation in burned zone in-situ combustion by Nelson	423	10.45 Proppant settlement in fracture	434
10.19 Estimates of cumulative oil displacement	423	10.46 Rate of advancement of combustion front (in-situ combustion)	434
10.20 Estimates of oil displacement rate	423	10.47 Rate of growth of heated zone in hot water heated reservoir	435
10.21 Estimating fraction of heat injected in latent form (steam-drive)	424	10.48 Rate of oxygen-reacted per unit mass of fuel	435
10.22 Estimating heat injection rate (steam-drive)	424	10.49 Relationship with real and dimensionless time in hot water floods	436
10.23 Estimating performance prediction of steam-drive reservoirs (cumulative oil produced)	425	10.50 Reservoir flow for gas flow in a formation	436
10.24 Estimating recovery steam drive (volume of steam in reservoir)	425	10.51 Reservoir flow through the wellbore of a geothermal well	436
10.25 Estimating steady-state five-spot injection rate (steam-drive)	425	10.52 Saturation of layer under hot water flood	437
10.26 Estimating volume of steam injection (steam-drive)	425	10.53 Slug size in polymer floods	437
10.27 Fraction of heat injected in latent form—Myhill and Stegemeier	426	10.54 Temperature increase with time during in-situ combustion process	438
	426	10.55 Temperature of a producing geothermal well	438

<b>10.56 Temperature of a single-phase liquid or gas injected geothermal well</b>	<b>439</b>	<b>10.60 Total water production from in-situ combustion—</b>	<b>440</b>
<b>10.57 Total heat loss of a geothermal well</b>	<b>439</b>	<b>Nelson &amp; McNeil</b>	
<b>10.58 Total oil production from in-situ combustion—</b>		<b>10.61 Volume of burned part of reservoir (in-situ combustion)</b>	<b>441</b>
<b>Nelson &amp; McNeil</b>		<b>10.62 Volume of reservoir burnt by wet combustion</b>	<b>441</b>
<b>10.59 Total oil production from wet in-situ combustion—</b>	<b>440</b>	<b>10.63 Volumetric heat capacity</b>	<b>441</b>
<b>Nelson &amp; McNeil</b>		<b>10.64 Wet combustion design (in-situ combustion)</b>	<b>442</b>

## 10.1 Areal extent of heated zone

### Input(s)

- $Q_i$ : Amount of Heat Injected (BTU)  
 $h$ : Height (ft)  
 $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/ft<sup>3</sup> F)  
 $G$ : Dimensionless Time Function (dimensionless)  
 $\Delta T$ : Temperature Differential (K)  
 $M_s$ : Volumetric Heat Capacity of Steam (BTU/ft<sup>3</sup> F)  
 $\alpha_s$ : Thermal Diffusivity (ft<sup>2</sup>/d)

### Output(s)

- A: Area (acres)

### Formula(s)

$$A = \frac{Q_i * h * M_r * G}{43560 * 4 * \Delta T * \alpha_s * M_s^2}$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 5, Page: 44.

## 10.2 Average reservoir temperature in a cyclical steam injection process

### Input(s)

- $T_i$ : Initial Temperature (K)  
 $T_s$ : Temperature of Steam (K)  
 $f_{HD}$ : Time-dependent Conduction Loss in Direction of Heated interval (dimensionless)  
 $f_{VD}$ : Time-dependent Conduction Loss Normal to the Direction of Heated interval (dimensionless)  
 $f_{pD}$ : Time-dependent Quantity for Heat Loss by Produced Fluid (dimensionless)

### Output(s)

- $T_a$ : Average Temperature within the Heated Zone (K)

### Formula(s)

$$T_a = T_i + (T_s - T_i) * \left( f_{VD} * f_{HD} * \left( 1 - f_{pD} \right) - f_{pD} \right)$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 9, Page: 115.

### 10.3 Bottomhole pressure in a static geothermal well

#### Input(s)

- $\rho$ : Density ( $\text{lb}/\text{ft}^3$ )
- $D$ : Vertical Depth (ft)
- $g$ : Acceleration of Gravity ( $\text{m}/\text{s}^2$ )
- $g_c$ : Units Conversion Factor ( $\text{m}/\text{s}^2$ )

#### Output(s)

- $\frac{dp}{dD}$ : Bottomhole Pressure in a Static Well (psi/ft)

#### Formula(s)

$$\frac{dp}{dD} = \frac{\rho}{144} \cdot \frac{g}{g_c}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 7.4.

### 10.4 Chromatographic lag in polymer flooding

#### Input(s)

- $A$ : Adsorption Rate (g polymer/g rock)
- $\rho$ : Rock Density (g/cc)
- $\phi$ : Porosity (fraction)
- $C$ : Polymer Concentration (g/cc)
- $S_w$ : Water Saturation (fraction)

#### Output(s)

- $CL$ : Chromatographic Lag (dimensionless)

#### Formula(s)

$$CL = \frac{1}{1 + \frac{A * \rho * (1 - \phi)}{C * \phi * S_w}}$$

Reference: Petrowiki.org.

### 10.5 Cumulative heat injected for steam drive—Myhill and Stegemeier

#### Input(s)

- $w_i$ : Mass Rate of Injection of Steam into Reservoir (lb/d)
- $c_w$ : Average Specific Heat (BTU/lb K)
- $\Delta T$ : Temperature Differential (K)
- $f_{sah}$ : Steam Quality (fraction)
- $L_{vdh}$ : Latent Heat of Steam (BTU/lb)

### Output(s)

$Q_i$ : Heat Injection Rate (BTU/d)

### Formula(s)

$$Q_i = w_i * (c_w * \Delta T + f_{sdh} * L_{vdh})$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 7, Page: 76.*

## 10.6 Depth of carbon dioxide alteration front (Battlet-Gouedard, 2006)

### Input(s)

t: Time in days (d)

### Output(s)

D: Depth of Alteration Front (mm)

### Formula(s)

$$D = 0.26 * (t^{0.5})$$

Reference: *Runar Nygaard. Waban Area Carbon-Dioxide Sequestration project. Energy and Environmental Group, University of Calgary, Calgary, Alberta.*

## 10.7 Depth of carbon dioxide alteration front (Kutchko, 2008)

### Input(s)

t: Time in days (d)

### Output(s)

D: Depth of Alteration Front (mm)

### Formula(s)

$$D = 0.016 * (t^{0.5})$$

Reference: *Runar Nygaard. Waban Area Carbon-Dioxide Sequestration project. Energy and Environmental Group, University of Calgary, Calgary, Alberta.*

## 10.8 Dimensionless heat injection rate (Gringarten and Sauty)

### Input(s)

$M_f$ : Volumetric Heat Capacity of the Injected Hot Fluid (BTU/ft<sup>3</sup> F)

$M_r$ : Volumetric Heat Capacity of the Reservoir (BTU/ft<sup>3</sup> F)

$h$ : Height (ft)

i: Injection Rate (ft<sup>3</sup>/d)

$\alpha_s$ : Thermal Diffusivity to Overburden (ft<sup>2</sup>/d)

$M_s$ : Volumetric Heat Capacity of Steam (BTU/ft<sup>3</sup> F)

L: Length (ft)

**Output(s)**

$Q_{ID}$ : Dimensionless Injection Rate (dimensionless)

**Formula(s)**

$$Q_{ID} = \frac{M_f * M_r * h_i * i}{4 * \alpha_s * (M_s^2) * (L^2)}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 5, Page: 51.*

**10.9 Dimensionless injection rate of air for in-situ combustion****Input(s)**

$i_a$ : Injection Rate ( $\text{ft}^3/\text{d}$ )  
 $L$ : Length Between Injector and Producer in Pattern (ft)  
 $h$ : Formation Thickness (ft)  
 $u_{min}$ : Minimum Air Flux ( $\text{ft}/\text{d}$ )

**Output(s)**

$i_D$ : Dimensionless Air Injection Rate (dimensionless)

**Formula(s)**

$$i_D = \frac{i_a}{L * h * u_{min}}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 100.*

**10.10 Dimensionless ratio of effective volumetric heat capacity of injected steam to that of the steam zone****Input(s)**

$\rho_w$ : Density of Water (g/cc)  
 $C_w$ : Specific Heat of Water (BTU/K mol psi)  
 $dT$ : Temperature Differential (K)  
 $f$ : Steam Quality (fraction)  
 $L_v$ : Latent Heat of Vaporization (BTU/lbm)  
 $M_r$ : Volumetric Heat Capacity of the Reservoir (BTU/ $\text{ft}^3$  K)

**Output(s)**

$F_{dh}$ : Dimensionless Ratio of Effective Volumetric Heat Capacity of injected Steam to that of the Steam Zone (dimensionless)

**Formula(s)**

$$F_{dh} = \frac{\rho_w * (C_w * dT + f_s * L_v)}{M_r * dT}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 12, Page: 164.*

**10.11 Dimensionless time for semi-steady state flow in coal bed methane reservoirs****Input(s)**

- $k_g$ : Effective Gas Compressibility (1/psi)
- $t$ : Time (h)
- $\phi$ : Porosity (fraction)
- $\mu_{gi}$ : Gas Viscosity at Initial Pressure (cP)
- $c_{ti}$ : Total Compressibility at Initial Pressure (1/psi)
- A: Drainage Area ( $\text{ft}^2$ )

**Output(s)**

- $t_{DA}$ : Dimensionless Time (dimensionless)

**Formula(s)**

$$t_{DA} = \frac{0.0002637 * k_g * t}{\phi * \mu_{gi} * c_{ti} * A}$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 221.*

**10.12 Dimensionless time in wet combustion by Kuo****Input(s)**

- $M_s$ : Volumetric Heat Capacity of Steam (BTU/ $\text{ft}^3$  K)
- $M_r$ : Volumetric Heat Capacity of the Reservoir (BTU/ $\text{ft}^3$  K)
- $\alpha_s$ : Thermal Diffusivity of Steam to Overburden ( $\text{ft}^2/\text{d}$ )
- $h_t$ : Thickness of Reservoir (ft)
- t: Time (d)

**Output(s)**

- $t_D$ : Dimensionless Time (dimensionless)

**Formula(s)**

$$t_D = \frac{4 * (M_s^2) * \alpha_s * t}{(M_r^2) * h_t^2}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 104.*

### 10.13 Dykstra-Parsons coefficient

#### Input(s)

- $k_{50}$ : Permeability of Core Samples (mD)  
 $k_{84.1}$ : Permeability of Core Samples (mD)

#### Output(s)

- V: Dykstra-Parsons Coefficient (dimensionless)

#### Formula(s)

$$V = \frac{k_{50} - k_{84.1}}{k_{50}}$$

Reference: Willhite, G. P., 1986, Waterflooding, Vol. 3, Richardson, Texas: Textbook Series, SPE, Chapter: 5, Page: 172.

### 10.14 Effective (apparent) transmissivity

#### Input(s)

- $k_{ai}$ : Effective Permeability to Steam (mD)  
 $h_a$ : Net Thickness of Steam Zone (ft)  
 $\mu_{ai}$ : Apparent Viscosity of Steam (cP)

#### Output(s)

- $T_{ai}$ : Transmissivity (mD ft/cP)

#### Formula(s)

$$T_{ai} = \frac{k_{ai} * h_a}{\mu_{ai}}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 12, Page: 167.

### 10.15 Effective oil transmissivity for thermal stimulation

#### Input(s)

- $F_G$ : Geometric Factor (dimensionless)  
 $dP, q_{max}$ : Maximum Flow Resistance (psi/bbl/d)

#### Output(s)

- $T_{ao}$ : Transmissivity (mD ft/cP)

**Formula(s)**

$$T_{ao} = 141.2 * \frac{F_G}{dP, q_{max}}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 12, Page: 167.*

**10.16 Equivalent atomic H/C ratio of fuel for in-situ combustion****Input(s)**

$m$ : Mole Ratio of Carbon Monoxide to Carbon Emissions (fraction)

$c_{N_2}$ : Concentration of Nitrogen (mole fraction)

$c_{O_2}$ : Concentration of Oxygen (mole fraction)

$c_{CO_2}$ : Concentration of Carbon Dioxide (mole fraction)

**Output(s)**

$x$ : Equivalent Atomic H/C Ratio of Fuel for in-situ Combustion (ratio)

**Formula(s)**

$$x = 4 * (1 - m) * \left( \frac{0.27 * c_{N_2} - c_{O_2}}{c_{CO_2}} \right) + 2 * m - 4$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 91.*

**10.17 Equivalent volume of steam injected—Myhill and Stegemeier****Input(s)**

$C_w$ : Specific Heat of Water Steam (BTU/lb K)

$T_{sb}$ : Steam Temperature at Boiler Outlet (K)

$T_a$ : Ambient Temperature (K)

$f_{sb}$ : Steam Quality (fraction)

$L_{vb}$ : Latent Heat of Vaporization (BTU/lb)

$T_i$ : Downhole Steam Injection Temperature (K)

$T_o$ : Input Temperature (K)

$f_{vdh}$ : Quality of Steam Downhole (fraction)

$L_{vdh}$ : Latent Heat of Vaporization Downhole (BTU/lb)

**Output(s)**

$W_{s, eq}$ : Equivalent Volume of Steam (bbl)

**Formula(s)**

$$W_{s, eq} = 2.853 * (10^{-6}) * \left( \frac{C_w * (T_{sb} - T_a) + f_{sb} * L_{vb}}{C_w * (T_i - T_o) + f_{vdh} * L_{vdh}} \right)$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 7, Page: 78.*

## 10.18 Equivalent water saturation in burned zone in-situ combustion by Nelson

### Input(s)

- x: Equivalent H/C Molar Ratio (ratio)
- $\phi$ : Volume of Air Required to Burn through a Unit Volume of Reservoir (Mscf/ft<sup>3</sup>)
- $a_r$ : Volume Required to Burn through Reservoir (ft<sup>3</sup>)
- m: Ratio of Carbon Monoxide to Carbon Emissions (fraction)

### Output(s)

- $S_{wF}$ : Water Saturation Resulting from Combustion (fraction)

### Formula(s)

$$S_{wF} = \frac{0.319 * x * a_r}{\phi * (4 - 2 * m + x)}$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 8, Page: 92.

## 10.19 Estimates of cumulative oil displacement

### Input(s)

- $V_p$ : Pore Volume (bbl)
- $S_w$ : Average Water Saturation (fraction)
- $S_{iw}$ : Interstitial Water Saturation (fraction)

### Output(s)

- $N_p$ : Cumulative Oil Displaced (bbl)

### Formula(s)

$$N_p = V_p * (S_w - S_{iw})$$

Notes: Where the FVF was assumed to be 1.0.

Reference: Willhite, G. P., 1986, *Waterflooding*, Vol. 3, Richardson, Texas: Textbook Series, SPE, Chapter: 3, Page: 65.

## 10.20 Estimates of oil displacement rate

### Input(s)

- $S_w$ : Average Water Saturation (fraction)
- $S_{iw}$ : Interstitial Water Saturation (fraction)
- $f$ : Fraction of Total Flowing Stream (fraction)

### Output(s)

- $Q_p$ : Oil Displacement Rate (dimensionless)

**Formula(s)**

$$Q_p = \frac{S_w - S_{iw}}{1 - f_s}$$

Reference: Willhite, G. P., 1986, *Waterflooding*, Vol. 3, Richardson, Texas: Textbook Series, SPE, Chapter: 3, Page: 65.

**10.21 Estimating fraction of heat injected in latent form (steam-drive)****Input(s)**

- $C_w$ : Specific Heat of Water (BTU/lbm F)
- $T_i$ : Injection Temperature (°F)
- $T_a$ : Ambient Temperature (°F)
- $f_{sdh}$ : Steam Quality (fraction)
- $L_{hc}$ : Latent Heat of Condensation (BTU/lbm)

**Output(s)**

- $f_{hv}$ : Fraction of Heat Injected in Latent Form (fraction)

**Formula(s)**

$$f_{hv} = \left( 1 + \left( (C_w) * \frac{T_i - T_a}{f_{sdh} * L_{hc}} \right) \right)^{-1}$$

Reference: Pratts, M. (1986). *Thermal Recovery Monograph* Vol. 7. Society of Petroleum Engineers, Houston, Page: 77.

**10.22 Estimating heat injection rate (steam-drive)****Input(s)**

- $w_i$ : Boiler Feed Water Rate (B/d)
- $C_w$ : Specific Heat Capacity of Water (BTU/lbm F)
- $T_i$ : Injection Temperature (°F)
- $T_a$ : Ambient Temperature (°F)
- $f_{sdh}$ : Steam Quality (fraction)
- $L_{hc}$ : Latent Heat of Condensation (BTU/lbm)

**Output(s)**

- $Q_i$ : Heat Injection Rate (BTU/d)

**Formula(s)**

$$Q_i = w_i * 62.4 * 5.615 * (C_w * (T_i - T_a) + f_{sdh} * L_{hc})$$

Reference: Pratts, M. (1986). *Thermal Recovery Monograph* Vol. 7. Society of Petroleum Engineers, Houston, Page: 76.

## 10.23 Estimating performance prediction of steam-drive reservoirs (cumulative oil produced)

### Input(s)

- $\phi$ : Porosity (fraction)
- $h_n$ : Net Thickness (ft)
- $h_t$ : Gross Thickness (ft)
- $S_{oi}$ : Initial Oil Saturation (fraction)
- $S_{or}$ : Residual Oil Saturation (fraction)
- $E_c$ : Capture Efficiency (fraction)
- $V_s$ : Volume of Steam in Reservoir (ac ft)

### Output(s)

- $N_p$ : Cumulative Oil Produced (BBL)

### Formula(s)

$$N_p = 7758 * \phi * \left( \frac{h_n}{h_t} \right) * (S_{oi} - S_{or}) * E_c * V_s$$

Reference: Pratts, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 75.

## 10.24 Estimating recovery steam drive (volume of steam in reservoir)

### Input(s)

- $Q_i$ : Heat Injection Rate (BTU/d)
- $t$ : Injection Time (d)
- $E_{hs}$ : Thermal Efficiency of Steam Zone (dimensionless)
- $T_i$ : Injection Temperature (°F)
- $T_a$ : Ambient Temperature (°F)

### Output(s)

- $V_s$ : Gross Volume of Steam in Reservoir (ac ft)

### Formula(s)

$$V_s = \frac{Q_i * t * E_{hs}}{38.1 * 43560 * (T_i - T_a)}$$

Reference: Pratts, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 76.

## 10.25 Estimating steady-state five-spot injection rate (steam-drive)

### Input(s)

- $k$ : Permeability (mD)
- $h$ : Pay zone Thickness (ft)
- $\mu$ : Viscosity (cP)
- $A$ : Area Per Pattern (acre)

- $r_w$ : Wellbore Radius (ft)  
 $P_i$ : Injection Pressure (psi)  
 $P_b$ : Borehole Pressure (psi)

**Output(s)**

i: Injection Rate (BBL/d)

**Formula(s)**

$$i = \left( 7.082 * \frac{10^{-3}}{2 * \pi} \right) * \left( \frac{\pi * k * \frac{h}{\mu}}{\ln \left( \frac{208.71 * A^{0.5}}{r_w} \right) - 0.964} \right) * (P_i - P_b)$$

Reference: Pratts, M. (1986). *Thermal Recovery Monograph Vol. 7*. Society of Petroleum Engineers, Houston, Page: 83.

**10.26 Estimating volume of steam injection (steam-drive)****Input(s)**

- $C_w$ : Specific Heat Capacity of Water (BTU/LBM F)  
 $T_{sb}$ : Temperature of Steam at Boiler Outlet (°F)  
 $T_a$ : Ambient Temperature (°F)  
 $f_{sb}$ : Fraction of Steam at Boiler Outlet (fraction)  
 $T_{idh}$ : Injection Temperature Down Hole (°F)  
 $T_i$ : Injection Temperature (°F)  
 $f_{sdh}$ : Fraction of Steam Down Hole (fraction)  
 $L_{vdh}$ : Latent Heat of Vaporization Down Hole (BTU/lbm)  
 $L_{vb}$ : Latent Heat of Vaporization at Boiler Outlet (BTU/lbm)

**Output(s)**

$W_{s, eq}$ : Volume of Steam injected, as Water Equivalent (BBL/d)

**Formula(s)**

$$W_{s, eq} = (2.853 * 10^{-6}) * \frac{C_w * (T_{sb} - T_a) + f_{sb} * L_{vb}}{C_w * (T_{idh} - T_i) + f_{sdh} * L_{vdh}}$$

Reference: Pratts, M. (1986). *Thermal Recovery Monograph Vol. 7*. Society of Petroleum Engineers, Houston, Page: 78.

**10.27 Fraction of heat injected in latent form—Myhill and Stegemeier****Input(s)**

- h: Height (ft)  
 $h_t$ : Cumulative Height (ft)  
k: Permeability (mD)  
 $k_t$ : Cumulative Permeability (mD)

**Output(s)**

FMO: FMO (dimensionless)

**Formula(s)**

$$FMO = \left( \frac{1}{h_t} \right) * \left( h + \frac{k_t * h_t - k * h}{k} \right)$$

Reference: Ehrlich, R., 2016. PT E 531 Enhanced Oil Recovery. University of Southern California Lecture Notes.

**10.28 Fraction of injected heat remaining in reservoir****Input(s)**

$Q_i$ : Total Heat Injected (BTU)

$Q$ : Total Heat Remaining (BTU)

**Output(s)**

$E_h$ : Fraction of the Injected Heat Remaining in the Reservoir (fraction)

**Formula(s)**

$$E_h = \frac{Q}{Q_i}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 5, Page: 44.

**10.29 Fractional flow of water in hot floods dependent on temperature and saturation in hot water flood****Input(s)**

$M(S, T)$ : Mobility Ratio of the Co-flowing Fluids (dimensionless)

**Output(s)**

$f_{w(S, T)}$ : Fractional Flow (dimensionless)

**Formula(s)**

$$f_{w(S, T)} = \frac{1}{1 + (M(S, T))^{-1}}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 6, Page: 60.

### 10.30 Growth of steam-heated area—Marx-Langenheim

#### Input(s)

- $Q_i$ : Injected Heat Content (BTU)
- $t_d$ : Dimensionless Time (dimensionless)
- $e_t$ : Error Function of square root of dimensionless function (dimensionless)
- T: Temperature Differential (K)
- $M_r$ : Volumetric Heat Capacity of the Reservoir (BTU/ft<sup>3</sup> F)
- h: Height (ft)

#### Output(s)

- A: Growth of Steam Zone (ac/d)

#### Formula(s)

$$A = \frac{Q_i * \exp(t_d) * e_t}{43560 * T * M_r * h}$$

Reference: Michael Prats. *Thermal recovery*. Society of Petroleum Engineers. New York. 1986, Page: 61.

### 10.31 Heat loss over an incremental length of a well (two-phase flow)

#### Input(s)

- $T_s$ : Temperature in the Well (Saturation Temperature) (°F)
- $T_e$ : Undisturbed Formation Temperature (°F)
- y: Distance from the Bottom of the Well (ft)
- k: Thermal Conductivity of Earth (=33.6 BTU/(ft d °F))
- $f(t)$ : Dimensionless Time Function that Represents the Transient Heat Transfer to the formation (dimensionless)

#### Output(s)

- $dq$ : Heat Loss over an Incremental Length of the Wellbore (BTU/h)

#### Formula(s)

$$dq = \frac{2\pi k(T_s - T_e)}{f(t)} dy$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 6.12.

### 10.32 Heat ratio of contents in a geothermal reservoir

#### Input(s)

- $\phi$ : Porosity (dimensionless)
- $\rho_w$ : Water Density (kg/m<sup>3</sup>)
- $C_w$ : Performance Coefficient, a Function of Mean Pressure Level (KJ/kg °C)

**Output(s)**

$H_w$ : Heat in Water Content (KJ/m<sup>3</sup> °C)  
 $H_T$ : Total Heat (KJ/m<sup>3</sup> °C)

**Formula(s)**

$$\frac{H_w}{H_T} = \frac{\rho_w C_w \phi}{\rho_w C_w \phi + \rho_r C_r (1 - \phi)}$$

Reference: Ramey Jr, H. J. (1981). Reservoir Engineering Assessment of Geothermal Systems. Department of Petroleum Engineering, Stanford University. Page: 9.6.

**10.33 Heat released during in-situ combustion—Burger & Sahuguet****Input(s)**

m: Molar Ratio of H/C Emission (fraction)  
x: Proportion of Carbon Monoxide to Carbon Emissions (fraction)

**Output(s)**

(dh)<sub>a</sub>: Heat Released (BTU/SCF)

**Formula(s)**

$$(dh)_a = \frac{94 - 67.9 * m + 31.2 * x}{1 - 0.5 * m + 0.25 * x}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 93.

**10.34 Heat remaining in reservoir—Marx and Langenheim****Input(s)**

$Q_i$ : Total Heat Injected (BTU)  
 $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/ft<sup>3</sup> F)  
h: Height (ft)  
G: Dimensionless Time Constant (dimensionless)  
 $\alpha_s$ : Steam Diffusivity (ft<sup>2</sup>/d)  
 $M_s$ : Volumetric Heat Capacity of Formations Adjacent to Surrounding Heated Zone (BTU/ft<sup>3</sup>F)

**Output(s)**

Q: Heat Remaining in Reservoir (BTU)

**Formula(s)**

$$Q = \frac{Q_i * (M_r^2) * (h^2) * G}{4 * \alpha_s * M_s^2}$$

Reference: Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 5, Page: 44.

### 10.35 Horizontal well breakthrough time in a bottom-water-drive reservoir

#### Input(s)

- $\phi$ : Porosity (fraction)
- $S_{wc}$ : Connate Water Saturation (fraction)
- $S_{oir}$ : Residual Oil Saturation (fraction)
- $E_s$ : Sweep Efficiency (dimensionless)
- $k_h$ : Horizontal Permeability (mD)
- $k_v$ : Vertical Permeability (mD)
- $h$ : Oil Column Thickness (ft)
- $q_o$ : Flow Rate (STB/d)
- $B_o$ : Oil Formation Volume Factor (RB/STB)

#### Output(s)

- $f_d$ : Saturation Constant (dimensionless)
- $t_{BT}$ : Water Breakthrough Time (d)

#### Formula(s)

$$f_d = \phi * (1 - S_{wc} - S_{oir})$$

$$t_{BT} = \left( f_d * h^3 * \frac{E_s}{5.615 * q_o * B_o} \right) * \frac{k_h}{k_v}$$

Reference: Joshi, S. D. 1991, *Horizontal Well Technology*. Tulsa, Oklahoma: PennWell Publishing Company. Chapter: 8, Page: 295.

### 10.36 Ignition delay time in in-situ combustion

#### Input(s)

- $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/ft<sup>3</sup> K)
- $T_a$ : Initial Absolute Temperature (K)
- $n$ : exponent
- $R$ : Gas Constant (BTU/mol K psi)
- $E$ : Activation Energy (BTU/K mol)
- $(dh)_a$ : Heat Generated by Oxygen (BTU)
- $\phi$ : Porosity (fraction)
- $S_o$ : Saturation of Oil (fraction)
- $\rho_o$ : Density (g/cc)
- $A_c$ : Pre Exponential Constant (1/psi K)
- $P_{o2}$ : Partial Pressure of Oxygen (psi)

#### Output(s)

- $t_{ig}$ : Ignition Delay (s)

**Formula(s)**

$$t_{ig} = \frac{2.04 * 10^{-7} * M_r * T_a^2 * \left(1 + \left(\frac{2 * R * T_a}{E}\right)\right) * R * \exp\left(\frac{E}{R * T_a}\right)}{E * (dh)_a * \phi * S_o * \rho_o * A_c * P_{O2}^n}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 95.*

### **10.37 Injected air required to burn through unit bulk of reservoir for in-situ combustion by Nelson and McNeil**

**Input(s)**

$a_r$ : Air Required to Burn through Reservoir (MSCF/ft<sup>3</sup>)

$T_{sc, ab}$ : Temperature at Standard Condition (K)

$T_{ab}$ : Temperature at Absolute Condition (K)

$P_{sc, ab}$ : Pressure at Standard Condition (psi)

$P_{inj, ab}$ : Pressure at Absolute Condition (psi)

$\phi$ : Porosity (fraction)

$E_{O2}$ : Utilization Efficiency of Oxygen (fraction)

**Output(s)**

$a_r$ : Injected Air Required to Burn through Unit Reservoir Bulk (MSCF/ft<sup>3</sup>)

**Formula(s)**

$$a_r = \frac{a_r + (10^{-3}) * \left(\frac{T_{sc,ab}}{T_{ab}}\right) * \left(\frac{P_{inj,ab}}{P_{sc,ab}}\right) * \phi}{E_{O2}}$$

Reference: *Michael Prats. Thermal recovery. Society of Petroleum Engineers. New York.1986, Page: 96.*

### **10.38 Mass of fuel burned per unit bulk reservoir volume combustion—Nelson and McNeil**

**Input(s)**

$\phi_E$ : Effective Porosity (fraction)

$\phi$ : Porosity (fraction)

$m_E$ : Mass of Fuel Burned per Unit Bulk Volume in the Laboratory Experiment (lbm/ft<sup>3</sup>)

**Output(s)**

$m_R$ : Mass of Fuel Burned per Unit Bulk Reservoir Volume (lbm/ft<sup>3</sup>)

**Formula(s)**

$$m_R = \left( \frac{1 - \phi}{1 - \phi_E} \right) * m_E$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 89.*

**10.39 Minimum air flux required for advance of fire front—Nelson and McNeil****Input(s)**

- $a_r$ : Air Required to Burn Unit Volume of Reservoir (MSCF/ft<sup>3</sup>)  
 $E_{O_2}$ : Oxygen Consumption Efficiency (fraction)

**Output(s)**

- $u_{min}$ : Minimum Air Flux (SCF/ft<sup>2</sup> d)

**Formula(s)**

$$u_{min} = \frac{0.125 * a_r}{E_{O_2}}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 100.*

**10.40 Oil breakthrough newly swept zone****Input(s)**

- $PV$ : Pore Volume (dimensionless)  
 $dE_{as}$ : Areal Sweep Efficiency from New Swept Zone (fraction)  
 $S_{wbt}$ : Water Saturation at Breakthrough in Swept Zone (fraction)  
 $S_{wi}$ : Initial Water Saturation (fraction)

**Output(s)**

- $O_{nsz}$ : Oil Volume at Breakthrough in New Swept Zones (bbl)

**Formula(s)**

$$O_{nsz} = PV * dE_{as} * (S_{wbt} - S_{wi})$$

Reference: *Ehrlich Enhanced Oil Recovery, PTE 531, University of Southern California Lecture Notes, 2016.*

**10.41 Oil recovery as a function of the fraction of oil displaced from heated zone****Input(s)**

- $F$ : Air Injected per Unit Oil Produced (Mscf/bbl)  
 $\phi$ : Porosity (fraction)  
 $a_r$ : Air Required to Burn a Unit Volume of Reservoir (MSCF/ft<sup>3</sup>)

- $S_{oi}$ : Initial Water Saturation (fraction)  
 $S_{of}$ : Oil Saturation Burned (fraction)  
 $E_{O2}$ : Oxygen Consumption Efficiency (fraction)

**Output(s)**

- $E_{cb}$ : Oil Recovery (fraction)

**Formula(s)**

$$E_{cb} = 5.615 \frac{a_r}{F * \varphi * E_{O2} * (S_{oi} - S_{of})}$$

Reference: *Prats, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Chapter: 8, Page: 104.*

**10.42 Oil solubilization factor****Input(s)**

- $C_o$ : Concentration of Oil in Solvent (g/cc)  
 $C_s$ : Concentration of Solvent in Solution (g/cc)

**Output(s)**

- S: Oil Solubilization Factor (dimensionless)

**Formula(s)**

$$S = \frac{C_o}{C_s}$$

Reference: *Ehrlich Enhanced, R 2016.PTE 531 Oil Recovery. University of Southern California Lecture Notes.*

**10.43 Oil volume at breakthrough by Craig, Geffen, and Morse****Input(s)**

- $PV$ : Pore Volume (bbl)  
 $(Eas)_{bt}$ : Areal Sweep Breakthrough (%)  
 $Swbt_{av}$ : Water Breakthrough (fraction)  
 $S_{wi}$ : Initial Water Saturation (fraction)

**Output(s)**

- O: Oil Volume at Breakthrough (bbl)

**Formula(s)**

$$O = PV * (Eas)_{bt} * (Swbt_{av} - S_{wi})$$

Reference: *Ehrlich. Enhanced Recovery, 2016.PTE 531 Oil Recovery. University of Southern California Lecture Notes.*

## 10.44 Oil-steam ratio—Marx & Langenheim

### Input(s)

$W_{s, eq}$ : Measure of Steam Used (bbl)  
 $N_p$ : Cumulative Oil Produced (bbl)

### Output(s)

$F_{so}$ : Oil-steam Ratio (bbl)

### Formula(s)

$$F_{so} = \frac{W_{s, eq}}{N_p}$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 7, Page: 77.

## 10.45 Proppant settlement in fracture

### Input(s)

$\rho_p$ : Proppant Density (lbm/ft<sup>3</sup>)  
 $\rho_f$ : Fluid Density (lbm/ft<sup>3</sup>)  
 $g$ : Gravity Acceleration (ft/s<sup>2</sup>)  
 $d_p$ : Particle Diameter (ft)  
 $v_t$ : Terminal Particle Settling Velocity (ft/s)

### Output(s)

$C_D$ : Coefficient of Drag (unitless)

### Formula(s)

$$C_D = 4 * (\rho_p - \rho_f) * g * \frac{d_p}{\rho_f * v_t^2}$$

Reference: Daneshy, A. 2013. *Fundamentals of Hydraulic Fracturing*, Daneshy Consultants International, Page: 74.

## 10.46 Rate of advancement of combustion front (in-situ combustion)

### Input(s)

$E_O$ : Oxygen Consumption Efficiency (fraction)  
 $u_a$ : Air Flux (SCF/d ft<sup>2</sup>)  
 $a_r$ : Air Requirement (Mscf/ft<sup>3</sup>)

### Output(s)

$v_b$ : Rate of Advancement (ft/d)

**Formula(s)**

$$v_b = (E_O) * \frac{u_a}{a_r}$$

Reference: *Pratts, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 100.*

**10.47 Rate of growth of heated zone in hot water heated reservoir****Input(s)**

- h: Height of Reservoir (ft)
- $\phi$ : Porosity (fraction)
- q: Flow Rate of Ambient Reservoir Temperature and Pressure (bbl/d)
- $T_j$ : Temperature of Specific Layer (K)
- $f_w$ : Fractional flow of water in hot floods dependent on Temperature and Saturation (dimensionless)

**Output(s)**

- A: Rate of Area Growth with time (ft ft/d)

**Formula(s)**

$$A = 1.289 * (10^{-4}) * \left( \frac{q * T_j * f_w}{\phi * h} \right)$$

Reference: *Pratts, M. 1986. Thermal Recovery. Society of Petroleum Engineers, New York, Page: 45.*

**10.48 Rate of oxygen-reacted per unit mass of fuel****Input(s)**

- $P_o$ : Partial pressure of Oxygen (psi)
- $A_c$ : Pre-exponential Constant (1/s psi)
- E: Activation Energy (BTU/lbm mol)
- R: Gas Constant (BTU/lbm K mol)
- $T_a$ : Absolute Temperature (K)

**Output(s)**

- m: Rate of Oxygen Reacted per unit mass of fuel (mol/s lbm)

**Formula(s)**

$$m = P_o * A_c * \exp \left( \frac{E}{R * T_a} \right)$$

Reference: *Enhanced Oil Recovery, Green & Willhite, Page: 386.*

## 10.49 Relationship with real and dimensionless time in hot water floods

### Input(s)

- $h$ : Thickness of Layer (ft)  
 $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/ft<sup>3</sup> F)  
 $t_D$ : Dimensionless Time (dimensionless)  
 $\alpha_s$ : Steam Diffusivity (ft<sup>2</sup>/d)  
 $M_s$ : Specific Volumetric Heat Capacity of Steam (BTU/ft<sup>3</sup> F)

### Output(s)

- $t$ : Time (d)

### Formula(s)

$$t = \frac{(h^2) * (M_r^2) * t_D}{4 * \alpha_s * (M_s^2)}$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 6, Page: 61.

## 10.50 Reservoir flow for gas flow in a formation

### Input(s)

- $n$ : The Performance Exponent (dimensionless)  
 $\bar{p}$ : The Mean Formation Pressure in the Drainage System of Well (psi)  
 $P_{wf}$ : Bottomhole Flowing Pressure (psi)  
 $C$ : Performance Coefficient, a Function of Mean Pressure Level (bbl/d psi)

### Output(s)

- $W$ : Flow Rate (bbl/d)

### Formula(s)

$$W = C (\bar{p}^2 - P_{wf}^2)^n$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 8.8.

## 10.51 Reservoir flow through the wellbore of a geothermal well

### Input(s)

- $n$ : Performance Exponent for Flow in the wellbore (dimensionless)  
 $P_{gf}$ : Wellhead Flowing Pressure (psi)  
 $P_{wf}$ : Bottomhole Flowing Pressure (psi)  
 $C_1$ : Performance Coefficient for Flow upwards in the Wellbore (bbl/d psi)

**Output(s)**

$W$ : Flow Rate (bbl/d)

**Formula(s)**

$$W = C_1 (P_{wf}^2 - P_{tf}^2)^n$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 8.8.

**10.52 Saturation of layer under hot water flood****Input(s)**

$T_j$ : Temperature of jth Zone (°F)

**Output(s)**

$S$ : Saturation (fraction)

**Formula(s)**

$$S = 0.698 - 0.1 * \left( \frac{T_j - 117}{275} \right)$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 6, Page: 64.

**10.53 Slug size in polymer floods****Input(s)**

$A$ : Adsorption (g polymer/g rock)

$\rho$ : Density (g/cc)

$\phi$ : Porosity (fraction)

$C$ : Concentration (g/cc)

**Output(s)**

$S$ : Slug Size (dimensionless)

**Formula(s)**

$$S = \left( A * \rho * \frac{1 - \phi}{C * \phi} \right)$$

Reference: Ehrlich., Enhanced R 2016.PTE 531 Oil Recovery. University of Southern California Lecture Notes.

## 10.54 Temperature increase with time during in-situ combustion process

### Input(s)

$S_o$ : Saturation of Oil (fraction)  
 $\phi$ : Porosity (fraction)  
 $n$ : exponent  
 $\rho$ : Density of Oil (lb/ft<sup>3</sup>)  
 $A_c$ : Pre-exponential Constant (1/F psi)  
 $P_{O_2}$ : Partial Pressure of Oxygen (psi)  
 $M_r$ : Volumetric Heat Capacity of Reservoir (BTU/F ft<sup>3</sup>)  
 $E$ : Activation Eergy (BTU/lbm mol)  
 $R$ : Gas Constant (BTU/lbm mol F)  
 $T_{ab}$ : Absolute Temperature (K)

### Output(s)

$\frac{dT}{dt}$ : Temperature Increase with Time (K/s)

### Formula(s)

$$\frac{dT}{dt} = 86,400 \left( \frac{S_o * \rho * \phi * A_c * P_{O_2}^n}{M_r} \right) * \exp \left( -\frac{E}{R * T_{ab}} \right)$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 8, Page: 95.

## 10.55 Temperature of a producing geothermal well

### Input(s)

$T_o$ : Inflowing Fluid Temperature (°F)  
 $T_{bh}$ : Downhole Reservoir Temperature (°F)  
 $a$ : Geothermal Gradient (°F/ft)  
 $A$ : Diffusion Depth (ft)  
 $y$ : Distance from the Bottom of the Well (ft)  
 $t$ : Function of Time (d)

### Output(s)

$T$ : Temperature of a Producing Geothermal Well (°F)

### Formula(s)

$$T = (T_{bh} - ay) + aA \left( 1 - e^{-y/A} \right) + (T_o - T_{bh})e^{-y/A}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 6.2.

## 10.56 Temperature of a single-phase liquid or gas injected geothermal well

### Input(s)

- $T_{surf}$ : Surface Temperature of the Earth (°F)
- $T_{inj}$ : Temperature of the Injected Fluid (°F)
- $a$ : Geothermal Gradient (°F/ft)
- $A$ : Diffusion Depth (ft)
- $z$ : Distance from the Bottom of the Well (ft)

### Output(s)

- $T$ : Temperature of a Single-Phase Liquid or Gas Injected Geothermal Well (°F)

### Formula(s)

$$T = T_{surf} + az - aA + \left( T_{inj} - T_{surf} + aA \right) e^{-z/A}$$

Reference: Ramey Jr, H. J. (1981). *Reservoir Engineering Assessment of Geothermal Systems*. Department of Petroleum Engineering, Stanford University. Page: 6.6.

## 10.57 Total heat loss of a geothermal well

### Input(s)

- $T_o$ : Inflowing Fluid Temperature (°F)
- $w$ : Mass Flow Rate (lb/h)
- $c$ : Thermal Heat Capacity of the Fluid (BTU/lbm °F)
- $a$ : Geothermal Gradient (°F/ft)
- $A$ : Diffusion Depth (ft)
- $H$ : Total Well Depth (ft)
- $b$ : Surface Temperature (°F)

### Output(s)

- $q$ : Total Heat Loss of a Geothermal Well (BTU/h)

### Formula(s)

$$q = -wc \left[ aH - (T_o + aA - b) \left( 1 - e^{-\frac{H}{A}} \right) \right]$$

Reference: Horne, R. N., & Shinohara, K. (1979). *Wellbore Heat Loss in Production and Injection Wells*. Journal of Petroleum Technology, 31(01), Page: 117.

## 10.58 Total oil production from in-situ combustion—Nelson & McNeil

### Input(s)

- $V_r$ : Volume of reservoir Burned (ft<sup>3</sup>)
- $S_i$ : Initial oil Saturation (fraction)
- $S_f$ : Oil Saturation left after Combustion (fraction)
- $V_p$ : Pattern Volume (ft<sup>3</sup>)
- $\phi$ : Porosity (fraction)

### Output(s)

$N_p$ : Cumulative Oil production (bbl)

### Formula(s)

$$N_p = 7758 * \phi * \left( V_r * (S_i - S_f) + 0.4 * (V_p - V_r) * S_i \right)$$

Reference: *Enhanced Oil Recovery, Green & Willhite, Page: 395.*

## 10.59 Total oil production from wet in-situ combustion—Nelson & McNeil

### Input(s)

- $V_r$ : Volume of Burnt reservoir (ft<sup>3</sup>)
- $\phi$ : Porosity (fraction)
- $S_i$ : Initial Oil Saturation (fraction)
- $S_f$ : Oil Saturation post fire flood (fraction)
- $V_s$ : Volume of Steam (ft<sup>3</sup>)
- $S_r$ : Residual oil Saturation (fraction)
- $h_n$ : Net thickness of Reservoir (ft)
- $h_t$ : Total thickness of reservoir (ft)
- E: Efficiency of Fire Flood (fraction)

### Output(s)

$N_p$ : Cumulative oil production (bbl)

### Formula(s)

$$N_p = \left( \frac{7758 * \phi * E * h_n}{h_t} \right) * (V_r * (S_i - S_f) + V_s * (S_i - S_r))$$

Reference: *Enhanced Oil Recovery, Green & Willhite, Page: 395.*

## 10.60 Total water production from in-situ combustion—Nelson & McNeil

### Input(s)

- $V_r$ : Volume of Reservoir burnt (ft<sup>3</sup>)
- $\phi$ : Porosity (fraction)
- $S_i$ : Initial Water Saturation (fraction)
- $S_f$ : Water Saturation after burning Reservoir (fraction)

### Output(s)

$W_p$ : Cumulative Water production (bbl)

**Formula(s)**

$$W_p = 7758 * V_r * \phi * (S_i - S_f)$$

Reference: *Enhanced Oil Recovery, Green & Willhite, Page: 395.*

**10.61 Volume of burned part of reservoir (in-situ combustion)****Input(s)**

- $G_a$ : Total Air Requirement (MMSCF)
- $E_O$ : Oxygen Consumption Efficiency (fraction)
- $a_R$ : Air Requirement (MSCF/ft<sup>3</sup>)

**Output(s)**

- $V_{rb}$ : Volume of Reservoir Burned (ac ft)

**Formula(s)**

$$V_{rb} = 0.0230 * (G_a) * \frac{E_O}{a_R}$$

Reference: *Prats, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 100.*

**10.62 Volume of reservoir burnt by wet combustion****Input(s)**

- $G$ : Total Amount of Gas Injected (MSCF)
- $a_r$ : Air required to burn through reservoir (MSCF)
- $E$ : Efficiency of fire flood (fraction)

**Output(s)**

- $V_r$ : Volume of Reservoir Burnt (ft<sup>3</sup>)

**Formula(s)**

$$V_r = \frac{0.023 * G * E}{a_r}$$

Reference: *Prats, M. (1986). Thermal Recovery Monograph Vol. 7. Society of Petroleum Engineers, Houston, Page: 106.*

**10.63 Volumetric heat capacity****Input(s)**

- $M_S \alpha^{0.5}$  Volumetric Heat Capacity of Formations Adjacent to Surrounding Heated Zone (BTU/ft<sup>3</sup> F)
- $C_w$ : Specific Heat of Water (BTU/lb F)
- T: Temperature Differential (F)

$L_v$ : Latent Heat of Vaporization (BTU/lb)

$dt$ : Time Differential (d)

$M_{Rse}$ : Effective Volumetric Heat Capacity of Steam Zone (BTU/ft<sup>3</sup> F)

### Output(s)

$dz_s$ : Steam Zone Growth (ft)

### Formula(s)

$$dz_s = \left( \frac{4 * M_S \alpha^{0.5} * C_w * T}{L_v * M_{Rse}} \right) * \left( \frac{dt}{\pi} \right)^{0.5}$$

Reference: Prats, M. 1986. *Thermal Recovery*. Society of Petroleum Engineers, New York, Chapter: 7, Page: 80.

## 10.64 Wet combustion design (in-situ combustion)

### Input(s)

$V_{rb}$ : Volume of Reservoir Burned (ac ft)

$S_{oi}$ : Initial Oil Saturation (fraction)

$S_{of}$ : Oil Saturation burned (fraction)

$\phi$ : Porosity (fraction)

$V_s$ : Maximum Steamzone Volume (ac ft)

$S_{or}$ : Residual Oil saturation of steam flood zone (fraction)

$E_c$ : Capture efficiency (fraction)

$h_n$ : Net thickness (ft)

$h_t$ : Gross thickness (ft)

### Output(s)

$N_p$ : Cumulative Oil Production (bbl)

### Formula(s)

$$N_p = 7758 * \left( \left( V_{rb} * (S_{oi} - S_{of}) * \phi \right) + \left( V_s * (S_{oi} - S_{or}) * phi \right) \right) * E_c * \left( \frac{h_n}{h_t} \right)$$

Reference: *Thermal Recovery Monograph Vol. 7*, Page: 106.

## Chapter 11

# Geomechanics and fracturing formulas and calculations

### Chapter Outline

11.1 Axial stress around vertical wellbore	444	11.35 Maximum anisotropic failure stress	456
11.2 Axis of a deviated borehole from an arbitrary origin	444	11.36 Maximum compression at vertical wellbore	456
11.3 Bulk modulus (using Lame)	445	11.37 Maximum normal stress in tangential direction at wellbore wall (hoop stress)	457
11.4 Bulk modulus (using Poisson's ratio and Lame's constant)	445	11.38 Maximum plane tangential stress acting on deviated wellbore	460
11.5 Bulk modulus (using Poisson's ratio and shear modulus)	445	11.39 Maximum principal stress failure (Hoek and Brown)	457
11.6 Change in pore volume due to initial water and rock expansion	446	11.40 Maximum principal stress in normal faulting	458
11.7 Cohesive strength of rocks	446	11.41 Maximum principal stress in reverse faulting	458
11.8 Compressibility of a coalbed methane formation	446	11.42 Maximum principal stress in strike-slip faulting	459
11.9 Effect of pore pressure on stress	447	11.43 Maximum principal stress calculation using breakout width	459
11.10 Effective stress on individual grains	447	11.44 Minimum compression at vertical wellbore	459
11.11 Failure criteria (Mohr-Coulomb)	447	11.45 Minimum normal stress in tangential direction at wellbore wall (hoop stress)	460
11.12 Formation compressibility by using hydrofrac data	448	11.46 Maximum plane tangential stress acting on deviated wellbore	460
11.13 Fracture conductivity	448	11.47 Modified lae criterion	460
11.14 Fracture gradient (Eaton)	448	11.48 Normal stress in radial direction near wellbore	461
11.15 Fracture gradient (Holbrook)	449	11.49 Normal stress in rock at failure	461
11.16 Fracture gradient (Matthews and Kelly)	449	11.50 Normal stress in tangential direction at wellbore wall (hoop stress)	462
11.17 Fracture gradient (Zoback and Healy)	449	11.51 Normal stress in tangential direction near wellbore (hoop stress)	462
11.18 Fracture pressure (Hubert & Willis)	450	11.52 Pore pressure increase due to fluid activity (Mody & Hale)	463
11.19 Fracture volume (GDK method)	450	11.53 Pore pressure increase due to given fluid activity contrast (Mody and Hale)	463
11.20 Fracture volume (Perkins and Kern method)	451	11.54 Pore pressure of shale (Flemings)	464
11.21 Fracture width (GDK method)	451	11.55 Pore pressure of shale (Traugott)	464
11.22 Fracture width (Perkins and Kern method)	451	11.56 Porosity irreversible plastic deformation occurs	464
11.23 Hoek and brown criteria for principal stress failure	452	11.57 Pressure required to induce a tensile fracture (breakdown pressure)	465
11.24 Horizontal effective stress (assuming no lateral strain as per Lorenz and Teufel)	452	11.58 Pressure to grow fractures (Abe, Mura, et al.)	465
11.25 Horizontal maximum stress (Bredehoeft)	452	11.59 Radial stress around vertical wellbore	466
11.26 Induced fracture dip	453	11.60 Ratio of pore pressure change to original due to depletion	466
11.27 Initial effective horizontal stress	453	11.61 Rotation of maximum principal stress near wellbore	467
11.28 Isothermal compressibility of limestones (Newman correlation)	454	11.62 Rotation of maximum principal stress near wellbore (Zoback & Day-Lewis)	467
11.29 Least principal stress as function of depth in Gulf of Mexico (Hubbert and Willis)	454	11.63 Shale compaction	467
11.30 Least principal stress as function of depth in Gulf of Mexico (Matthew and Kelly)	454	11.64 Shear modulus	468
11.31 Linearized Mohr failure line	455		
11.32 Linearized Mohr Coulomb criteria	455		
11.33 M modulus (using shear modulus and bulk modulus)	455		
11.34 M modulus (using Young's modulus and Poisson's ratio)	456		

<b>11.65 Shear modulus from Young's modulus</b>	<b>468</b>	<b>11.75 Stress perturbation (Segall and Fitzgerald)</b>	<b>472</b>
<b>11.66 Shear stress near vertical well</b>	<b>468</b>	<b>11.76 Subsidence due to uniform pore pressure reduction in free surfaces</b>	<b>473</b>
<b>11.67 Slowness of the formation</b>	<b>469</b>	<b>11.77 Unconfined compressive strength of rock</b>	<b>473</b>
<b>11.68 Storativity of fractures</b>	<b>469</b>	<b>11.78 Velocity of bulk compressional waves</b>	<b>474</b>
<b>11.69 Stress at edge of wellbore breakout</b>	<b>470</b>	<b>11.79 Velocity of compression waves</b>	<b>474</b>
<b>11.70 Stress component near normal faulting in reservoir</b>	<b>470</b>	<b>11.80 Velocity of shear waves</b>	<b>474</b>
<b>11.71 Stress components in original coordinate system in depletion drive</b>	<b>471</b>	<b>11.81 <math>V_p</math> and <math>V_s</math> calculation (Eberhart-Phillips)</b>	<b>475</b>
<b>11.72 Stress intensity at tip of mode I fracture</b>	<b>471</b>	<b>11.82 <math>V_p</math> and <math>V_s</math> calculation (geomechanical model)</b>	<b>475</b>
<b>11.73 Stress path (induced normal faulting)</b>	<b>472</b>	<b>11.83 Yield strength (Bingham plastic model)</b>	<b>476</b>
<b>11.74 Stress path of reservoir with changes in production</b>	<b>472</b>		

## 11.1 Axial stress around vertical wellbore

### Input(s)

R: Radius of Wellbore (ft)  
r: Position in Respect to Centre of Wellbore (ft)  
Θ: Azimuth of  $S_{hmax}$  (rad)  
 $S_{hmax}$ : Maximum Horizontal Stress (psi)  
 $S_{hmin}$ : Minimum Horizontal Stress (psi)

### Output(s)

τ: Twisting Stress (psi)

### Formula(s)

$$\tau = 0.5 * (S_{hmax} - S_{hmin}) * \left( 1 + \left( \frac{2 * R^2}{r^2} \right) - \left( \frac{3 * R^4}{r^4} \right) \right) * \sin(2 * \Theta)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 170.*

## 11.2 Axis of a deviated borehole from an arbitrary origin

### Input(s)

$P_o$ : Pore Pressure (psi)  
R: Radius of Wellbore (ft)  
r: Position in Respect to Centre of Wellbore (ft)  
Θ: Azimuth of  $S_{hmax}$  (rad)  
 $\sigma^{\Delta t}$ : Thermal Stress (psi)  
 $S_{hmax}$ : Maximum Horizontal Stress (psi)  
 $S_{hmin}$ : Minimum Horizontal Stress (psi)

### Output(s)

$\sigma_{aa}$ : Stress (psi)

**Formula(s)**

$$\sigma_{aa} = 0.5 * (S_{hmax} + S_{hmin} - 2 * P_o) * \left( 1 + \left( \frac{R^2}{r^2} \right) \right) - 0.5 * (S_{hmax} - S_{hmin}) * \left( 1 + \left( \frac{3 * R^4}{r^4} \right) \right) * \cos(2 * \Theta) - \left( \frac{P_o * R^2}{r^2} \right) - \sigma^{\Delta t}$$

Reference: *Mark D. Zoback., Reservoir Geomechanics, Cambridge University Press, UK, Page: 170.*

**11.3 Bulk modulus (using Lame)****Input(s)**

- p: Pressure (Pa)
- $\Delta V$ : Volume ( $m^3$ )
- v: Volume ( $m^3$ )

**Output(s)**

- K: Bulk Modulus (Pa)

**Formula(s)**

$$K = \frac{p}{\frac{\Delta V}{v}}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 3, Page: 45.*

**11.4 Bulk modulus (using Poisson's ratio and Lame's constant)****Input(s)**

- $\lambda$ : Lame (dimensionless)
- G: Shear Modulus ( $N/m^2$ )

**Output(s)**

- K: Bulk Modulus ( $N/m^2$ )

**Formula(s)**

$$K = \lambda + 2 * \frac{G}{3}$$

Reference: *PetroWiki.org.*

**11.5 Bulk modulus (using Poisson's ratio and shear modulus)****Input(s)**

- $\lambda$ : Lame (dimensionless)
- G: Shear Modulus ( $N/m^2$ )

**Output(s)**

- K: Bulk Modulus ( $N/m^2$ )

**Formula(s)**

$$K = \lambda + 2 * \frac{G}{3}$$

Reference: *PetroWiki.org*.

**11.6 Change in pore volume due to initial water and rock expansion****Input(s)**

- $\lambda$ : Lame (dimensionless)
- $v$ : Poisson (dimensionless)

**Output(s)**

- K: Bulk Modulus (N/m<sup>2</sup>)

**Formula(s)**

$$K = \frac{\lambda * (1 + v)}{3 * v}$$

Reference: *PetroWiki.org*.

**11.7 Cohesive strength of rocks****Input(s)**

- G: Shear Modulus (N/m<sup>2</sup>)
- v: Poisson (dimensionless)

**Output(s)**

- K: Bulk Modulus (N/m<sup>2</sup>)

**Formula(s)**

$$K = \frac{2 * G * (1 + v)}{3 * (1 - 2 * v)}$$

Reference: *PetroWiki.org*.

**11.8 Compressibility of a coalbed methane formation****Input(s)**

- $W_i$ : Total Volume of Water in the Reservoir (bbl)
- $W_p$ : Total Volume of Water Removed (bbl)
- $P_f$ : Initial Reservoir Pressure (psi)
- $P_d$ : Desorption Pressure (psi)

**Output(s)**

- $c_t$ : Total Compressibility (1/psi)

**Formula(s)**

$$c_t = \left( \frac{1}{W_i} \right) * \left( \frac{W_p}{P_i - P_d} \right)$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter:3, Page: 219.

**11.9 Effect of pore pressure on stress****Input(s)**

- $\lambda$ : Lames First Constant (psi)
- $\delta$ : Kronecker Delta (dimensionless)
- $\xi_0$ : Initial Strain (dimensionless)
- $\xi$ : Final Strain (dimensionless)
- G: Modulus of Shear (psi)
- $\alpha$ : Biot (dimensionless)
- P: Pressure Applied (psi)

**Output(s)**

- S: Stress (psi)

**Formula(s)**

$$S = \lambda * \xi_0 * \delta + 2 * G * \xi - \alpha * \delta * P$$

Reference: Mark D. Zoback, *Reservoir Geomechanics*, Cambridge University Press, UK, Page: 68.

**11.10 Effective stress on individual grains****Input(s)**

- S: Normal Stress (psi)
- $P_p$ : Pore Pressure (psi)

**Output(s)**

- $\sigma_g$ : Effective Stress (psi)

**Formula(s)**

$$\sigma_g = S - P_p$$

Reference: Mark D. Zoback, *Reservoir Geomechanics*, Cambridge University Press, UK, Page: 87.

**11.11 Failure criteria (Mohr-Coulomb)****Input(s)**

- $\sigma_a$ : Principle Stress (psi)
- $\sigma_b$ : Second Strongest Stress (psi)
- $\beta$ : Angle Between Fault and Direction of Principle Stress (degrees)

### Output(s)

$\tau$ : Shear Stress (psi)

$\sigma$ : Normal Stress (psi)

### Formula(s)

$$\begin{aligned}\tau &= 0.5 * (\sigma_a - \sigma_b) * \sin(2 * \beta) \\ \sigma &= 0.5 * (\sigma_a + \sigma_b) + 0.5 * (\sigma_a - \sigma_b) * \cos(2 * \beta)\end{aligned}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 89.*

## 11.12 Formation compressibility by using hydrofrac data

### Input(s)

$V_s$ : Volume Associated to Conduct a Hydrofrac (bbl)

$dV_s$ : Change in Volume (bbl)

$dP$ : Change in Pressure (psi)

### Output(s)

$\beta$ : Formation Compressibility (psi)

### Formula(s)

$$\beta = \frac{1}{V_s} * \left( \frac{dV_s}{dP} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 221.*

## 11.13 Fracture conductivity

### Input(s)

$k_f$ : Fracture Permeability (mD)

$w_f$ : Width of Fracture (ft)

### Output(s)

$F_C$ : Fracture Conductivity (mD ft)

### Formula(s)

$$F_C = k_f * w_f$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter:1, Page: 93.*

## 11.14 Fracture gradient (Eaton)

### Input(s)

$v$ : Poisson (dimensionless)

$S_v$ : Vertical Stress (psi)

$P_p$ : Pore Pressure (psi)

**Output(s)**

$S_{hmin}$ : Minimum Horizontal Stress (psi)

**Formula(s)**

$$S_{hmin} = \left( \frac{v}{1-v} \right) * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 282.*

**11.15 Fracture gradient (Holbrook)****Input(s)**

$v$ : Poisson (dimensionless)

$S_v$ : Vertical Stress (psi)

$P_p$ : Pore Pressure (psi)

**Output(s)**

$S_{hmin}$ : Minimum Horizontal Stress (psi)

**Formula(s)**

$$S_{hmin} = \left( \frac{v}{1-v} \right) * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 282.*

**11.16 Fracture gradient (Matthews and Kelly)****Input(s)**

$\sigma_{min}$ : Minimum Principle Horizontal Stress (psi)

$\sigma_v$ : Vertical Principle Stress (psi)

$S_v$ : Vertical Stress (psi)

$P_p$ : Pore Pressure (psi)

**Output(s)**

$S_{hmin}$ : Minimum Horizontal Stress (psi)

**Formula(s)**

$$S_{hmin} = \left( \frac{\sigma_{min}}{\sigma_v} \right) * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 281.*

**11.17 Fracture gradient (Zoback and Healy)****Input(s)**

$\mu$ : Viscosity (cP)

$S_v$ : Vertical Stress (psi)

$S_p$ : Pore Pressure (psi)

### Output(s)

$S_{hmin}$ : Minimum Horizontal Stress (psi)

### Formula(s)

$$S_{hmin} = \left( \left( (1 + \mu^2)^{0.5} \right) + \mu \right)^{-2} * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 281.*

## 11.18 Fracture pressure (Hubert & Willis)

### Input(s)

$S_v$ : Vertical Stress (psi)

$P_p$ : Pore Pressure (psi)

### Output(s)

$S_{hmin}$ : Fracture Pressure (psi)

### Formula(s)

$$S_{hmin} = 0.3 * (S_v - P_p) + P_p$$

$$\frac{\sigma_{hmin}}{\sigma_v} = 0.3$$

Notes: Fracture Pressure Is assumed to be Equal to Minimum Horizontal Stress.

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 281.*

## 11.19 Fracture volume (GDK method)

### Input(s)

$Q$ : Flow Rate (B/m)

$\mu$ : Viscosity (cP)

$L$ : Length (ft)

$H$ : Height (ft)

$G$ : Shear Modulus (psi)

### Output(s)

$V_f$ : Fracture Volume ( $\text{ft}^3$ )

### Formula(s)

$$V_f = 0.03561 * \left( \mu * Q * L^6 * \frac{H^3}{G} \right)^{0.25}$$

Reference: *Daneshy, A. 2013. Fundamentals of Hydraulic Fracturing, Daneshy Consultants International, Page: 70.*

## 11.20 Fracture volume (Perkins and Kern method)

### Input(s)

- H: Fracture Height (ft)
- v: Poisson (unitless)
- $\mu$ : Viscosity (cP)
- Q: Flowrate (B/m)
- E: Young (psi)
- L: Fracture Length (ft)

### Output(s)

$V_f$  Fracture Volume ( $\text{ft}^3$ )

### Formula(s)

$$V_f = 0.04 * H * \left( (1 - v^2) * \mu * \frac{Q}{E} \right)^{0.25} * L^{\frac{5}{4}}$$

Reference: Daneshy, A. 2013. *Fundamentals of Hydraulic Fracturing*, Daneshy Consultants International, Page: 57.

## 11.21 Fracture width (GDK method)

### Input(s)

- Q: Flow Rate (B/m)
- G: Shear Modulus (psi)
- $\mu$ : Viscosity (cP)
- L: Length (ft)
- H: Height (ft)

### Output(s)

w: Fracture Width (in.)

### Formula(s)

$$w = 0.272 * \left( \mu * Q * \frac{L^2}{G * H} \right)^{0.25}$$

Reference: Daneshy, A. 2013. *Fundamentals of Hydraulic Fracturing*, Daneshy Consultants International, Page: 70.

## 11.22 Fracture width (Perkins and Kern method)

### Input(s)

- v: Poisson (unitless)
- Q: Flowrate (B/m)
- $\mu$ : Viscosity (cP)
- L: Fracture Length (ft)
- E: Young's Modulus (psi)

**Output(s)**

$W_{max}$ : Fracture Width (in.)

**Formula(s)**

$$W_{max} = 0.389 * \left( (1 - v^2) * Q * \mu * \frac{L}{E} \right)^{0.25}$$

Reference: *Daneshy, A. 2013. Fundamentals of Hydraulic Fracturing, Daneshy Consultants International, Page: 58.*

**11.23 Hoek and Brown criteria for principal stress failure****Input(s)**

$C_o$ : Unconfined Compressive Strength of Rock (psi)

$m$ : Constant Depending on Property of Rock and Extent to Which It Is Broken (dimensionless)

$s$ : Constant Depending on Property of Rock and Extent to Which It Is Broken (dimensionless)

$\sigma_c$ : Minimum Effective Principal Stress (psi)

**Output(s)**

$\sigma_a$ : Maximum Effective Principal Stress (psi)

**Formula(s)**

$$\sigma_a = \sigma_c + C_o * \left( m * \frac{\sigma_c}{C_o} + s \right)^{0.5}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 98.*

**11.24 Horizontal effective stress (assuming no lateral strain as per Lorenz and Teufel)****Input(s)**

$v$ : Poisson Ratio (dimensionless)

$S_v$ : Vertical Overburden Stress (psi)

$\alpha$ : Biot Coefficient (dimensionless)

$P$ : Pore Pressure (psi)

**Output(s)**

$S_{hor}$ : Horizontal Stress (psi)

**Formula(s)**

$$S_{hor} = \left( \frac{v}{1-v} \right) * S_v + \alpha * P * \left( 1 - \frac{v}{1-v} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 381.*

**11.25 Horizontal maximum stress (Bredehoeft)****Input(s)**

$S_{hmin}$ : Minimum Horizontal Stress (psi)

- $P_b$ : Breakdown Pressure at initial Hydrofrac (psi)  
 $P_p$ : Pore Pressure (psi)

**Output(s)**

$S_{hmax}$ : Maximum Horizontal Stress (psi)

**Formula(s)**

$$S_{hmax} = 3 * S_{hmin} - P_b - P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 220.*

**11.26 Induced fracture dip****Input(s)**

- $h$ : Height of Fracture (ft)  
 $d$ : Diameter of Well (ft)

**Output(s)**

$Dip$ : Dip (degrees)

**Formula(s)**

$$\text{Dip} = \arctan\left(\frac{h}{d}\right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 146.*

**11.27 Initial effective horizontal stress****Input(s)**

- $v$ : Poisson (unitless)  
 $\rho$ : Overburden Density ( $\text{lb}/\text{ft}^3$ )  
 $H$ : Formation Depth (ft)  
 $\alpha$ : Biot Constant (unitless)  
 $p_p$ : Reservoir Pressure (psi)

**Output(s)**

$\sigma_h$ : Effective Horizontal Stress (psi)

**Formula(s)**

$$\sigma_h = \left( \frac{v}{1-v} \right) * \left( \left( \rho * \frac{H}{144} \right) - \left( \alpha * p_p \right) \right)$$

Reference: *Boyun, G., William, C., & Ali Ghalambar, G. (2007). Petroleum Production Engineering: A Computer-Assisted Approach Page: 259.*

## 11.28 Isothermal compressibility of limestones (Newman correlation)

### Input(s)

$\phi$ : Porosity (fraction)

### Output(s)

$C_t$ : Compressibility ( $\text{psi}^{-1}$ )

### Formula(s)

$$C_t = \frac{97.32 * 10^{-6}}{(1 + 55.8721 * \phi)^{1.42869}}$$

Notes: Check for  $0.02 < \phi < 0.23$ .

Reference: *Applied Petroleum Reservoir Engineering*, Craft & Hawkins, Page: 11.

## 11.29 Least principal stress as function of depth in Gulf of Mexico (Hubbert and Willis)

### Input(s)

$S_v$ : Vertical Overburden Stress (psi)

$P_p$ : Pore Pressure in Reservoir (psi)

### Output(s)

$S_{hmin}$ : Minimum Principal Stress in Reservoir (psi)

### Formula(s)

$$S_{hmin} = 0.3 * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics*, Cambridge University Press, UK, Page: 280.

## 11.30 Least principal stress as function of depth in Gulf of Mexico (Matthew and Kelly)

### Input(s)

$S_v$ : Vertical Overburden Stress (psi)

$K_i$ : Constant as Function of Depth (dimensionless)

$P_p$ : Pore Pressure in Reservoir (psi)

### Output(s)

$S_{hmin}$ : Minimum Principal Stress in Reservoir (psi)

### Formula(s)

$$S_{hmin} = K_i * (S_v - P_p) + P_p$$

Reference: *Mark D. Zoback, Reservoir Geomechanics*, Cambridge University Press, UK, Page: 280.

### 11.31 Linearized Mohr failure line

#### Input(s)

- $S_o$ : Stress (psi)
- $\sigma_n$ : Normal Stress (psi)
- $\mu_i$ : Coefficient of internal Friction (cP)

#### Output(s)

- $\tau$ : Shear Stress (psi)

#### Formula(s)

$$\tau = S_o + \sigma_n * \mu_i$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 89.*

### 11.32 Linearized Mohr coulomb criteria

#### Input(s)

- $C_o$ : Unconfined Compressive Strength of Rock (psi)
- $\mu$ : Slope of Failure Line (dimensionless)
- $\sigma_c$ : Minimum Effective Principal Stress (psi)

#### Output(s)

- $\sigma_1$ : Maximum Effective Principal Stress (psi)

#### Formula(s)

$$\sigma_1 = (C_o) + \left( (\mu^2 + 1)^{0.5} + \mu \right)^2 * \sigma_c$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 93.*

### 11.33 M Modulus (using shear modulus and bulk modulus)

#### Input(s)

- $G$ : Shear Modulus ( $N/m^2$ )
- $K$ : Bulk Modulus ( $N/m^2$ )

#### Output(s)

- $M$ : M Modulus ( $N/m^2$ )

#### Formula(s)

$$M = K + 4 * \frac{G}{3}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 64.*

### 11.34 M Modulus (using Young's modulus and Poisson's ratio)

#### Input(s)

$v$ : Poisson Ratio (dimensionless)

$E$ : Young Modulus ( $\text{N}/\text{m}^2$ )

#### Output(s)

$M$ : M Modulus ( $\text{N}/\text{m}^2$ )

#### Formula(s)

$$M = E * \frac{1 - v}{(1 + v) * (1 - 2 * v)}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 64.*

### 11.35 Maximum anisotropic failure stress

#### Input(s)

$\sigma_c$ : Minimum Principle Stress (psi)

$S_w$ : Intact Rock Strength (psi)

$\mu$ : Internal Friction of Weak Bedding (cP)

$\beta$ : Angle of Weak Plane to Maximum Principle Stress (degrees)

#### Output(s)

$\sigma$ : Stress (cm/s)

#### Formula(s)

$$\sigma = \frac{\sigma_c * 2 * (S_w + \mu * \sigma_c)}{(1 - \mu * \cot(\beta)) * \sin(2 * \beta)}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 107.*

### 11.36 Maximum compression at vertical wellbore

#### Input(s)

$\sigma_c$ : Minimum Principle Stress (psi)

$S_w$ : Intact Rock Strength (psi)

$\mu$ : Internal Friction of Weak Bedding (cP)

$\beta$ : Angle of Weak Plane to Maximum Principle Stress (degrees)

#### Output(s)

$\sigma$ : Stress (cm/s)

**Formula(s)**

$$\sigma = \frac{\sigma_c * 2 * (S_w + \mu * \sigma_c)}{(1 - \mu * \cot(\beta)) * \sin(2 * \beta)}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 107.*

**11.37 Maximum normal stress in tangential direction at wellbore wall (hoop stress)****Input(s)**

- $S_{hmax}$ : Maximum Principal Stress in Reservoir (psi)
- $S_{hmin}$ : Minimum Principal Stress in Reservoir (psi)
- $P_o$ : Pore Pressure (psi)
- $S_{dt}$ : Stress induced Due to Temperature (psi)
- $dP$ : Difference Between Wellbore Pressure and Mud Weight (psi)

**Output(s)**

- $\sigma_{th}$ : Maximum Hoop Stress in Tangential Direction at Wellbore Wall (psi)

**Formula(s)**

$$\sigma_{th} = 3 * S_{hmax} - S_{hmin} - 2 * P_o - dP - S_{dt}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 174.*

**11.38 Maximum plane tangential stress acting on deviated wellbore****Input(s)**

- $\sigma_{zz}$ : Stress in Radial Direction (psi)
- $\sigma_{aa}$ : Stress in Axial Direction (psi)
- $\tau$ : Shear Stress (psi)

**Output(s)**

- $\sigma_{tmax}$ : Maximum Tangential Stress (psi)

**Formula(s)**

$$\sigma_{tmax} = \frac{1}{2} * \left( \sigma_{zz} + \sigma_{aa} + \sqrt{(\sigma_{zz} - \sigma_{aa})^2 + 4 * \tau^2} \right)$$

$$\sigma_{tmax} = \frac{1}{2} * \left( \sigma_{zz} + \sigma_{aa} - \sqrt{(\sigma_{zz} - \sigma_{aa})^2 + 4 * \tau^2} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 239.*

**11.39 Maximum principal stress failure (Hoek and Brown)****Input(s)**

- $\sigma_b$ : Second Largest Stress (psi)
- C: Rock Compressive Strength (lbf)
- m: Hoek - Brown Constant Dependent on Rock Type (dimensionless)
- s: Hoek and Brown Constant Dependent on Shape (dimensionless)

### Output(s)

$\sigma_a$ : Principal Stress (psi)

### Formula(s)

$$\sigma_a = \sigma_b + \left( C * \left( m * \left( \frac{\sigma_b}{C} \right) + s \right)^{0.5} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 98.*

## 11.40 Maximum principal stress in normal faulting

### Input(s)

$\sigma_c$ : Least Principle Stress (psi)

$S_v$ : Vertical Stress (psi)

$P_p$ : Pore Pressure (psi)

$S_{hm}$ : Minimum Horizontal Stress (psi)

### Output(s)

$\sigma$ : Maximum Principle Stress (psi)

### Formula(s)

$$\sigma = \frac{\sigma_c * (S_v - P_p)}{S_{hm} - P_p}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 133.*

## 11.41 Maximum principal stress in reverse faulting

### Input(s)

$\sigma_c$ : Least Principle Stress (psi)

$P_p$ : Pore Pressure (psi)

$S_v$ : Vertical Stress (psi)

$S_{hmax}$ : Maximum Horizontal Stress (psi)

### Output(s)

$\sigma$ : Maximum Principle Stress (psi)

### Formula(s)

$$\sigma = \frac{\sigma_c * (S_{hmax} - P_p)}{S_v - P_p}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 133.*

## 11.42 Maximum principal stress in strike-slip faulting

### Input(s)

$\sigma_c$ : Least Principle Stress (psi)  
 $S_{hmax}$ : Maximum Horizontal Stress (psi)  
 $P_p$ : Pore Pressure (psi)  
 $S_{hmin}$ : Minimum Horizontal Stress (psi)

### Output(s)

$\sigma$ : Maximum Principle Stress (psi)

### Formula(s)

$$\sigma = \frac{\sigma_c * (S_{hmax} - P_p)}{S_{hmin} - P_p}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 133.*

## 11.43 Maximum principal stress calculation using breakout width

### Input(s)

$S_{hmin}$ : Minimum Principal Stress in Reservoir (psi)  
 $C_o$ : Distance from Wellbore (ft)  
 $P_p$ : Pore Pressure (psi)  
 $S_{dt}$ : Stress induced Due to Temperature (psi)  
 $dP$ : Difference Between Wellbore Pressure and Mud Weight (psi)  
 $\Theta$ : Angle from Wellbore Breakout Width (rad)

### Output(s)

$S_{hmin}$ : Maximum Principal Stress in Reservoir (psi)

### Formula(s)

$$S_{hmax} = \frac{((C_o) + 2 * P_p + dP + S_{dt}) - S_{hmin} * (1 + 2 * \cos(\Theta))}{1 - 2 * \cos(\Theta)}$$

Reference: *Mark D. Zoback., Reservoir Geomechanics, Cambridge University Press, UK, Page: 223.*

## 11.44 Minimum compression at vertical wellbore

### Input(s)

$S_{hmin}$ : Minimum Horizontal Stress (psi)  
 $S_{hmax}$ : Maximum Horizontal Stress (psi)  
 $P_o$ : Pore Pressure (psi)  
 $P$ : Pressure Drawdown (psi)  
 $\sigma_t$ : Thermal Stress (psi)

### Output(s)

$\sigma_{minaxial}$ : Minimum Axial Stress (psi)

### Formula(s)

$$\sigma_{minaxial} = 3 * S_{hmin} - S_{hmax} - 2 * P_o - P - \sigma_t$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 174.*

## 11.45 Minimum normal stress in tangential direction at wellbore wall (hoop stress)

### Input(s)

$S_{hmax}$ : Maximum Principal Stress in Reservoir (psi)

$S_{hmin}$ : Minimum Principal Stress in Reservoir (psi)

Po: Pore Pressure (psi)

$S_{dt}$ : Stress induced due to Temperature (psi)

dP: Difference Between Wellbore Pressure and Mud Weight (psi)

### Output(s)

$\sigma_{th}$ : Minimum Hoop Stress in Tangential Direction at Wellbore Wall (psi)

### Formula(s)

$$\sigma_{th} = 3 * S_{hmin} - S_{hmax} - 2 * Po - dP - S_{dt}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 238.*

## 11.46 Maximum plane tangential stress acting on deviated wellbore

### Input(s)

$\sigma_{zz}$ : Radial Stress (psi)

$\sigma_{aa}$ : Axial Stress (psi)

$\tau$ : Shear Stress (psi)

### Output(s)

$\sigma_{tmax}$ : Maximum Tangential Stress (psi)

### Formula(s)

$$\sigma_{tmax} = 0.5 * \left( \sigma_{zz} + \sigma_{aa} - \left( (\sigma_{zz} - \sigma_{aa})^2 + 4 * \tau^2 \right)^{0.5} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 239.*

## 11.47 Modified Lade criterion

### Input(s)

$S_a$ : Principle Stress (psi)

$S_b$ : Intermediate Stress (psi)

$S_c$ : Minimum Stress (psi)  
 $P_a$ : Pressure (psi)  
 $m$ : Material Strength Constant (dimensionless)

### Output(s)

$I_a$ : First Invariant of Stress Tensor (psi)  
 $I_c$ : Third Invariant of Stress Tensor (psi)  
 $\eta$ : Lades Coefficient (dimensionless)

### Formula(s)

$$\begin{aligned} I_a &= S_a + S_b + S_c \\ I_c &= S_a * S_b * S_c \\ \eta &= \left( \left( \frac{I_a^3}{I_c^3} \right) - 27 \right) * \left( \left( \frac{I_a}{P_a} \right)^m \right) \end{aligned}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 99.*

## 11.48 Normal stress in radial direction near wellbore

### Input(s)

$S_{hmax}$ : Maximum Principal Stress in reservoir (psi)  
 $S_{hmin}$ : Minimum Principal Stress in reservoir (psi)  
 $r$ : Distance from wellbore (ft)  
 $P_o$ : Pore Pressure (psi)  
 $R$ : Radius of wellbore (ft)  
 $\theta$ : Angle from  $S_{hmax}$  at which stress is measured (degrees)

### Output(s)

$\sigma_{rr}$ : Normal Stress in Radial Direction Near Wellbore (psi)

### Formula(s)

$$\sigma_{rr} = 0.5 * (S_{hmax} + S_{hmin} - 2 * P_o) * \left( 1 - \frac{R^2}{r^2} \right) + 0.5 * (S_{hmax} - S_{hmin}) * \left( 1 - 4 * \frac{R^2}{r^2} + 3 * \frac{R^4}{r^4} \right) * \cos \left( 2 * \theta * \frac{3.142}{180} \right) + P_o * \frac{R^2}{r^2}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 100.*

## 11.49 Normal stress in rock at failure

### Input(s)

$S_{hmax}$ : Maximum Principal Stress in reservoir (psi)  
 $S_{hmin}$ : Minimum Principal Stress in reservoir (psi)  
 $r$ : Distance from wellbore (ft)  
 $P_o$ : Pore Pressure (psi)  
 $R$ : Radius of wellbore (ft)  
 $\theta$ : Angle from  $S_{hmax}$  at which stress is measured (degrees)

**Output(s)**

$\sigma_{rr}$  : Normal Stress in Radial Direction Near Wellbore (psi)

**Formula(s)**

$$\sigma_{rr} = 0.5 * (\text{Shmax} + \text{Shmin} - 2 * \text{Po}) * \left(1 - \frac{R^2}{r^2}\right) + 0.5 * (\text{Shmax} - \text{Shmin}) * \left(1 - 4 * \frac{R^2}{r^2} + 3 * \frac{R^4}{r^4}\right) * \cos\left(2 * \theta * \frac{3.142}{180}\right) + \text{Po} * \frac{R^2}{r^2}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 101.*

**11.50 Normal stress in tangential direction at wellbore wall (hoop stress)****Input(s)**

$S_{hmax}$ : Maximum Principal Stress in reservoir (psi)

$S_{hmin}$ : Minimum Principal Stress in reservoir (psi)

$Po$ : Pore Pressure (psi)

$Sdt$ : Stress induced due to temperature (psi)

$dP$ : Difference between Wellbore Pressure and Mud Weight (psi)

$\theta$ : Angle from  $S_{hmax}$  at which stress is measured (degrees)

**Output(s)**

$\sigma_{th}$  : Hoop Stress in Tangential Direction at Wellbore Wall (psi)

**Formula(s)**

$$\sigma_{th} = (\text{Shmax} + \text{Shmin}) - 2 * (\text{Shmax} - \text{Shmin}) * \cos\left(2 * \theta * \frac{3.142}{180}\right) - 2 * \text{Po} - dP - Sdt$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 102.*

**11.51 Normal stress in tangential direction near wellbore (hoop stress)****Input(s)**

$S_{hmax}$ : Maximum Principal Stress in reservoir (psi)

$S_{hmin}$ : Minimum Principal Stress in reservoir (psi)

$Po$ : Pore Pressure (psi)

$Sdt$ : Stress induced due to temperature (psi)

$dP$ : Difference between Wellbore Pressure and Mud Weight (psi)

$\theta$ : Angle from  $S_{hmax}$  at which stress is measured (degrees)

$r$ : Distance from wellbore (ft)

$R$ : Radius of wellbore (ft)

**Output(s)**

$\sigma_{gt}$ : Normal Stress in Tangential Direction Near Wellbore (Hoop Stress) (psi)

**Formula(s)**

$$Sg_{th} = 0.5 * (Shmax + Shmin - 2 * Po) * \left(1 + \frac{R^2}{r^2}\right) - 0.5 * (Shmax - Shmin) * \left(1 + 3 * \frac{R^4}{r^4}\right) * \cos\left(2 * \theta * \frac{3.142}{180}\right) - Po * \frac{R^2}{r^2} - Sdt$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 104.*

**11.52 Pore pressure increase due to fluid activity (Mody & Hale)****Input(s)**

- $E_m$ : Membrane Efficiency (dimensionless)
- R: Gas Constant (psi/mol K)
- T: Temperature (K)
- V: Volume ( $\text{ft}^3$ )
- $A_p$ : Pore Fluid Activity (dimensionless)
- $A_m$ : Mud Activity (dimensionless)

**Output(s)**

- $\delta P$ : Pore Pressure Increase (psi)

**Formula(s)**

$$\delta P = E_m * \left(\frac{R * T}{V}\right) * \ln\left(\frac{A_p}{A_m}\right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 321.*

**11.53 Pore pressure increase due to given fluid activity contrast (Mody and Hale)****Input(s)**

- $E_m$ : Membrane Efficiency (dimensionless)
- R: Gas Constant (dimensionless)
- T: Temperature (K)
- V: Molar Volume of Water (L/mol)
- $A_p$ : Pore Fluid Activity (dimensionless)
- $A_m$ : Mud Activity (dimensionless)

**Output(s)**

- dP: Pore Pressure Increase (psi)

**Formula(s)**

$$dP = Em * \left(R * \frac{T}{V}\right) * \log\left(\frac{Ap}{Am}\right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 105.*

## 11.54 Pore pressure of shale (Flemings)

### Input(s)

- $S_v$ : Sonic Velocity (ft/s)
- $\beta_c$ : Compressibility (1/psi)
- $\phi_o$ : Initial Porosity (fraction)
- $t$ : Sonic Travel Time (s)
- $\phi$ : Porosity from Sonic Log (fraction)

### Output(s)

- $P_p$ : Pore Pressure (psi)

### Formula(s)

$$P_p = S_v - \left( \left( \frac{1}{\beta_c} \right) * \ln \left( \frac{\phi_o}{\phi} \right) \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 48.*

## 11.55 Pore pressure of shale (Traugott)

### Input(s)

- $z$ : Depth (ft)
- $S_v$ : Sonic Velocity (ft/s)
- $R_o$ : Resistivity of Shale (ohm ft)
- $R_n$ : Expected Resistivity (ohm ft)
- $(P_p)^{hydro}$ : Hydrostatic Pore Pressure (psi)

### Output(s)

- $P_{psh}$ : Pore Pressure of Shale (psi)

### Formula(s)

$$P_{psh} = z * \left( \left( \frac{S_v}{z} \right) - \left( \left( \frac{S_v}{z} \right) - \left( \frac{(P_p)^{hydro}}{z} \right) \right) * \left( \left( \frac{R_o}{R_n} \right)^{1.2} \right) \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 47.*

## 11.56 Porosity irreversible plastic deformation occurs

### Input(s)

- $\mu$ : Viscosity (cP)
- $S_v$ : Vertical Stress (psi)
- $S_H$ : Maximum Horizontal Stress (psi)
- $S_h$ : Minimum Horizontal Stress (psi)
- $P_p$ : Change in Pore Pressure (psi)

**Output(s)**

- M: Mobility (dimensionless)  
 p: Porosity (dimensionless)

**Formula(s)**

$$M = \frac{6 * \mu}{3 * ((\mu^2) + 1)^{0.5} - \mu}$$

$$p = \left( \frac{1}{3 * (S_v * S_H + S_h) - 9 * P_p} \right) * \left( 9 * P_p^2 + \left( 1 + \frac{9}{M^2} \right) * (S_v^2 + S_H^2 + S_h^2) + \left( 2 - \frac{9}{M^2} \right) * (S_v * S_H + S_v * S_h + S_H * S_h) - 6 * P_p * (S_v + S_H + S_h) \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 400.*

**11.57 Pressure required to induce a tensile fracture (breakdown pressure)****Input(s)**

- $S_{hmax}$ : Maximum Principal Stress in reservoir (psi)  
 $S_{hmin}$ : Minimum Principal Stress in reservoir (psi)  
 $P_p$ : Pore Pressure (psi)  
 $T_o$ : Minimum Hoop Stress for formation at which crack initiates (psi)

**Output(s)**

- $P_b$ : Breakdown Pressure (psi)

**Formula(s)**

$$P_b = 3 * S_{hmin} - S_{hmax} - P_p + T_o$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 56.*

**11.58 Pressure to grow fractures (Abe, Mura, et al.)****Input(s)**

- $S_c$ : Minimum Principle Stress (psi)  
 $P_p$ : Pore Pressure (psi)  
 $c_f$ : Radius of Fracture (in.)  
 $c_i$ : Radius of Invaded Zone (in.)

**Output(s)**

- $P_{grow}$ : Growth Pressure (psi)

### Formula(s)

$$P_{\text{grow}} = S_c * \left( \frac{1 - \left( \frac{P_p}{S_c} \right) * \left( 1 - \left( \frac{c_f}{c_i} \right)^2 \right)^{0.5}}{1 - \left( \left( 1 - \left( \frac{c_f}{c_i} \right)^2 \right)^{0.5} \right)} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 326.*

## 11.59 Radial stress around vertical wellbore

### Input(s)

$S_{h\max}$ : Maximum Horizontal Stress (psi)

$S_{h\min}$ : Minimum Horizontal Stress (psi)

$P_o$ : Pore Pressure (psi)

$R$ : Radius of Wellbore (ft)

$r$ : Relative Position to Centre (ft)

$\theta$ : Azimuth of  $S_{h\max}$  (rad)

### Output(s)

$\sigma_{rr}$ : Stress (psi)

### Formula(s)

$$\sigma_{rr} = 0.5 * (S_{h\max} + S_{h\min} - 2 * P_o) * \left( 1 - \left( \frac{R^2}{r^2} \right) \right) + 0.5 * (S_{h\max} - S_{h\min}) * \left( 1 + \left( \frac{3 * R^4}{r^4} \right) - \left( \frac{4 * R^2}{r^2} \right) \right) * \cos(2 * \theta) + \left( \frac{P_o * R^2}{r^2} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 170.*

## 11.60 Ratio of pore pressure change to original due to depletion

### Input(s)

$P$ : Change in Pore Pressure (psi)

$S_{h\max}$ : Maximum Horizontal Stress (psi)

$S_{h\min}$ : Minimum Horizontal Stress (psi)

### Output(s)

$q$ : Pore Pressure Ratio (fraction)

### Formula(s)

$$q = \frac{P_p}{S_{h\max} - S_{h\min}}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 393.*

## 11.61 Rotation of maximum principal stress near wellbore

### Input(s)

A: Stress Field Direction (dimensionless)  
 P<sub>p</sub>: Change in Pore Pressure (psi)  
 θ: Fault Orientation (degrees)  
 S<sub>hmax</sub>: Maximum Principle Stress (psi)  
 S<sub>hmin</sub>: Minimum Principle Stress (psi)

### Output(s)

γ: Rotation (degrees)

### Formula(s)

$$\gamma = 0.5 * \text{atan} \left( \frac{A * P_p * \sin(2 * \theta)}{S_{hmax} - S_{hmin} + A * P_p * \cos(2 * \theta)} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 393.*

## 11.62 Rotation of maximum principal stress near wellbore (Zoback & Day-Lewis)

### Input(s)

A: Constant a Value (dimensionless)  
 Δ: Fault Orientation (degrees)  
 q: Ratio of Pore Pressure to Differential Stress (dimensionless)

### Output(s)

γ: Stress Rotation (radian)

### Formula(s)

$$\gamma = 0.5 * \text{atan} \left( A * q * \frac{\sin(2 * \Delta)}{1 + A * q * \cos(2 * \Delta)} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 393.*

## 11.63 Shale compaction

### Input(s)

Ø: Porosity (fraction)  
 β: Second Empirical Constant from Porosity vs Vertical Stress Graph (1/MPa)  
 σ<sub>v</sub>: Vertical Effective Stress (MPa)

### Output(s)

Ø<sub>c</sub>: Shale Compaction (fraction)

### Formula(s)

$$\varnothing_c = \varnothing * e^{-\beta * \sigma_v}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 46.*

## 11.64 Shear modulus

### Input(s)

- F: Force (N)
- A: Area ( $m^2$ )
- $\theta$ : Deformation Angle (degrees)

### Output(s)

- G: Shear Modulus (Pa)

### Formula(s)

$$G = \frac{F/A}{\theta}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 3, Page: 45.*

## 11.65 Shear modulus from Young's modulus

### Input(s)

- v: Poisson's Ratio (dimensionless)
- E: Young's Modulus ( $N/m^2$ )

### Output(s)

- G: Modulus of Rigidity (psi)

### Formula(s)

$$G = \frac{E}{2*(1+v)}$$

Reference: *Samuel. E Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 356.*

## 11.66 Shear stress near vertical well

### Input(s)

- $S_{hmax}$ : Maximum Principal Stress in reservoir (psi)
- $S_{hmin}$ : Minimum Principal Stress in reservoir (psi)
- r: Distance from wellbore (ft)
- R: Radius of wellbore (ft)
- $\theta$ : Angle from  $S_{hmax}$  at which stress is measured (degrees)

**Output(s)**

$\text{sig}_{\text{rth}}$ : Shear Stress near Vertical Well (psi)

**Formula(s)**

$$\text{sig}_{\text{rth}} = 0.5 * (\text{S}_{\text{hmax}} + \text{S}_{\text{hmin}}) * \left( 1 + 2 * \frac{\text{R}^2}{\text{r}^2} - 3 * \frac{\text{R}^4}{\text{r}^4} \right) * \sin \left( 2 * \theta * \frac{3.142}{180} \right)$$

Reference: *petrowiki.org*.

**11.67 Slowness of the formation****Input(s)**

$d_h$ : Borehole Diameter (in.)  
 $\Delta t_m$ : Interval Travel Time ( $\mu\text{s}/\text{ft}$ )  
 $d_t$ : Tool Diameter (in.)  
 $L_s$ : Spacing of the Tool (ft)  
 $l_c$ : Eccentricity of the Tool (ft)  
 $t_l$ : Time Between Initiation of the Pulse and First Arrival Acoustic Energy at the Receiver ( $\mu\text{s}/\text{ft}$ )

**Output(s)**

$\Delta t$ : Slowness of Formation Observed by Sonic Log ( $\mu\text{s}/\text{ft}$ )  
 $t_m$ : Mud Path Correction Time ( $\mu\text{s}/\text{ft}$ )

**Formula(s)**

$$\Delta t = \frac{t_l - t_m}{L_s}$$

$$t_m = (\Delta t_m) * (d_h - (d_t + 2 * l_c)) * \left( 1 - \left( \frac{\Delta t}{\Delta t_m} \right)^2 \right)^{0.5}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*. SPE Textbook Series Vol. 4. Chapter 10, Page: 189.

**11.68 Storativity of fractures****Input(s)**

$\phi_f$ : Porosity of Fracture (fraction)  
 $\phi_m$ : Porosity of Matrix (fraction)  
 $h_f$ : Fracture Thickness (ft)  
 $h_m$ : Matrix Thickness (ft)  
 $c_{tf}$ : Total Fracture Compressibility (1/psi)  
 $c_{tm}$ : Total Matrix Compressibility (1/psi)

**Output(s)**

$\omega$ : Storativity of Fracture (dimensionless)

**Formula(s)**

$$\omega = \frac{\phi_f * h_f * c_{tf}}{\phi_f * h_f * c_{tf} + \phi_m * h_m * c_{tm}}$$

Reference: Ahmed, T., McKinney, P.D. 2005. *Advanced Reservoir Engineering*, Gulf Publishing of Elsevier, Chapter:1, Page: 82.

**11.69 Stress at edge of wellbore breakout****Input(s)**

- $C_o$ : Wellbore Strength (psi)
- $P_p$ : Pore Pressure (psi)
- $P$ : Drawdown Differential in Pressure (psi)
- $\sigma_t$ : Thermally Induced Stress (psi)
- $w_b$ : Wellbore Breakout (degrees)
- $S_{hmin}$ : Minimum Horizontal Stress (psi)

**Output(s)**

- $\theta_b$ : Breakout Angle (degrees)
- $S_{hmax}$ : Maximum Principle Stress (psi)

**Formula(s)**

$$2\theta_b = \pi - w_b$$

$$S_{hmax} = \frac{((C_o) + 2 * P_p + P + \sigma_t) - S_{hmin} * (1 + 2 * \cos(\theta_b))}{1 - 2 * \cos(\theta_b)}$$

Reference: Mark D. Zoback, *Reservoir Geomechanics*, Cambridge University Press, UK, Page: 223.

**11.70 Stress component near normal faulting in reservoir****Input(s)**

- $\alpha$ : Biot (dimensionless)
- $v$ : Poisson (dimensionless)
- $S_{hmax}$ : Maximum Principal Stress (psi)
- $S_{hmin}$ : Minimum Principal Stress (psi)
- $dP$ : Change in Pore Pressure (psi)
- $\theta$ : Fault Orientation (degrees)

**Output(s)**

- $A$ : Constant a Value (dimensionless)
- $S_x$ : Stress in X Direction (psi)
- $S_y$ : Stress in Y Direction (psi)
- $T_{xy}$ : Normal Stress in Y Direction (psi)

**Formula(s)**

$$A = \alpha * \frac{1 - 2 * v}{1 - v}$$

$$S_x = S_{h\max} - A * dP - A * \frac{dP}{2} * \left(1 - \cos\left(2 * \theta * \frac{\pi}{180}\right)\right)$$

$$S_y = S_{h\min} - A * dP - A * \frac{dP}{2} * \left(1 + \cos\left(2 * \theta * \frac{\pi}{180}\right)\right)$$

$$T_{xy} = A * \frac{dP}{2} * \sin\left(2 * \theta * \frac{\pi}{180}\right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 381.*

**11.71 Stress components in original coordinate system in depletion drive****Input(s)**

- $S_{h\max}$ : Maximum Stress in Horizontal Direction (psi)
- $S_{h\min}$ : Minimum Stress in Horizontal Direction (psi)
- $\delta P_p$ : Change in Pore Pressure due to Depletion (psi)
- $A$ : Stress Path (dimensionless)
- $\Delta$ : Fault Orientation (degrees)

**Output(s)**

- $S_x$ : Stress in X-direction (psi)
- $S_y$ : Stress in Y-direction (psi)
- $\tau$ : Stress Around Wellbore (psi)

**Formula(s)**

$$S_x = S_{h\max} - A * P_p - \left(\frac{A * \delta P_p}{2}\right) * (1 - \cos(2 * \Delta))$$

$$S_y = S_{h\min} - A * P_p - \left(\frac{A * \delta P_p}{2}\right) * (1 + \cos(2 * \Delta))$$

$$\tau = \left(\frac{A * \delta P_p}{2}\right) * \sin(2 * \Delta)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 393.*

**11.72 Stress intensity at tip of mode I fracture****Input(s)**

- $P_f$ : Fracture Pressure (psi)
- $S_c$ : Minimum Principle Stress (psi)
- $L$ : Length of Fracture (ft)

**Output(s)**

- $K$ : Stress Intensity (psi ft)

**Formula(s)**

$$K = (P_f - S_c) * \pi * (L^{0.5})$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 122.*

**11.73 Stress path (induced normal faulting)****Input(s)**

$\mu$ : Friction Coefficient (dimensionless)

**Output(s)**

A: Stress Path (dimensionless)

**Formula(s)**

$$A = 1 - \frac{1}{\left( \left( (\mu^2 + 1)^{0.5} \right) + \mu \right)^2}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 385.*

**11.74 Stress path of reservoir with changes in production****Input(s)**

$\alpha$ : Biots Coefficient (dimensionless)

$v$ : Poisson (dimensionless)

**Output(s)**

A: Stress Path (dimensionless)

**Formula(s)**

$$A = \frac{\alpha * (1 - 2 * v)}{1 - v}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 381.*

**11.75 Stress perturbation (Segall and Fitzgerald)****Input(s)**

$v$ : Poisson (dimensionless)

$H$ : Height of Reservoir (ft)

$R$ : Half the Lateral Extent (ft)

$\alpha$ : Constant of Stress Propagation (dimensionless)

**Output(s)**

M: Stress Perturbation (dimensionless)

**Formula(s)**

$$M = \alpha * \left( \frac{(1 - 2 * v) * \pi * H}{(1 - v) * 4 * 2 * R} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics. Cambridge University Press. Cambridge, UK, Page: 112.*

**11.76 Subsidence due to uniform pore pressure reduction in free surfaces****Input(s)**

- c<sub>m</sub>: Formation Compaction per Unit Change in Pore Pressure Reduction (ft<sup>3</sup>/psi)
- v: Poisson's (dimensionless)
- r: Radius of Area Involved (ft)
- D: Depth of Formation in Consideration (ft)
- ΔP<sub>p</sub>: Pore Pressure Change (psi)
- V: Volume of Reservoir (ft<sup>3</sup>)

**Output(s)**

- u<sub>z</sub>: Subsidence in Z Direction (ft)
- u<sub>r</sub>: Subsidence Along R (ft)

**Formula(s)**

$$u_z = (-1) * \left( \frac{c_m * (1 - v) * D * \Delta P_p * V}{\pi * ((r^2) + (D^2))^{1.5}} \right)$$

$$u_r = \left( \frac{c_m * (1 - v) * r * \Delta P_p * V}{\pi * ((r^2) + (D^2))^{1.5}} \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 412.*

**11.77 Unconfined compressive strength of rock****Input(s)**

- S<sub>o</sub>: Cohesive Strength (psi)
- μ: Slope of Failure Line (dimensionless)

**Output(s)**

- C<sub>o</sub>: Unconfined Compressive Strength of Rock (psi)

**Formula(s)**

$$C_o = 2 * S_o * \left( (\mu^2 + 1)^{0.5} + \mu \right)$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, Page: 89.*

## 11.78 Velocity of bulk compressional waves

### Input(s)

E: Young's Modulus (lbf/ft<sup>2</sup>)

$\rho$ : Density (lbm/ft<sup>3</sup>)

$\mu$ : Poisson's Ratio (dimensionless)

### Output(s)

$V_b$ : Velocity of Bulk Compressional Waves (ft/s)

### Formula(s)

$$V_b = \left( \frac{E}{\rho} * \frac{1 - \mu}{(1 + \mu) * (1 - 2 * \mu)} \right)^{0.5}$$

Reference: *Core Laboratories. 2005. Formation Evaluation and Petrophysics, Page: 23.*

## 11.79 Velocity of compression waves

### Input(s)

K: Bulk Modulus (Pa)

G: Shear Modulus (Pa)

$\rho$ : Density (kg/m<sup>3</sup>)

### Output(s)

$V_p$ : Velocity of Compression Waves (m/s)

### Formula(s)

$$V_p = \left( \left( K + \frac{4}{3} \right) * \frac{G}{\rho} \right)^{0.5}$$

Reference: Bassiouni, Z. 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 3, Page: 46.*

## 11.80 Velocity of shear waves

### Input(s)

G: Shear Modulus (Pa)

$\rho$ : Density (kg/m<sup>3</sup>)

### Output(s)

$V_s$ : Velocity of Shear Waves (m/s)

**Formula(s)**

$$V_s = \left( \frac{G}{\rho} \right)^{0.5}$$

Reference: Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 3, Page: 46.*

**11.81  $V_p$  and  $V_s$  calculation (Eberhart-Phillips)****Input(s)**

- $\phi$ : Porosity (fraction)
- C: Clay Content (fraction)
- $\sigma$ : Effective Stress (psi)

**Output(s)**

$V_p$ : Velocity of Compressional Waves (ft/s)

$V_s$ : Shear Waves (ft/s)

**Formula(s)**

$$V_p = 5.77 - 6.94 * \phi - 1.73 * (C^{0.5}) + 0.446 * \left( \sigma - (-1) * e^{(-1) * 16.7 * \sigma} \right)$$

$$V_s = 3.7 - 4.94 * \phi - 1.57 * (C^{0.5}) + 0.361 * \left( \sigma - (-1) * e^{(-1) * 16.7 * \sigma} \right)$$

Reference: Mark D. Zoback, *Reservoir Geomechanics*, Cambridge University Press, UK, Page: 53.

**11.82  $V_p$  and  $V_s$  calculation (geomechanical model)****Input(s)**

- K: Bulk Modulus (psi)
- G: Shear Modulus (psi)
- $\rho$ : Density (ppg)

**Output(s)**

$V_p$ : Velocity of Compressional Waves (ft/s)

$V_s$ : Shear Waves (ft/s)

**Formula(s)**

$$V_p = \left( \frac{K + \frac{4 * G}{3}}{\rho} \right)^{0.5}$$

$$V_s = \left( \frac{G}{\rho} \right)^{0.5}$$

Reference: Mark D. Zoback, *Reservoir Geomechanics*, Cambridge University Press, UK, Page: 63.

### 11.83 Yield strength (Bingham plastic model)

#### Input(s)

K: Bulk Modulus (psi)  
G: Shear Modulus (psi)  
 $\rho$ : Density (ppg)

#### Output(s)

$V_p$ : Velocity of Compressional Waves (ft/s)  
 $V_s$ : Shear Waves (ft/s)

#### Formula(s)

$$V_p = \left( \frac{K + \frac{4 * G}{3}}{\rho} \right)^{0.5}$$
$$V_s = \left( \frac{G}{\rho} \right)^{0.5}$$

Reference: *Mark D. Zoback, Reservoir Geomechanics, Cambridge University Press, UK, page 63.*

## Chapter 12

# Facilities and process engineering formulas and calculations

### Chapter Outline

12.1 Allowable gas velocity through gas separator	478	12.39 Pan-Maddox equation for molecular weight	492
12.2 Allowable velocity in downcomer for tray type tower	478	12.40 Photoelectric effect	493
12.3 Bed diameter of adsorption unit	478	12.41 Power requirement for pumping a compressible flow fluid through a long pipe	493
12.4 Bed length of adsorption unit	479	12.42 Pressure criteria for separator by ASME (external radius)	494
12.5 Block efficiency factor	479	12.43 Pressure criteria for separator by ASME (internal radius)	494
12.6 Bottom distillation column rate	480	12.44 Pressure storage	494
12.7 Breakthrough time in an adsorption unit	480	12.45 Proportional band in pressure controller	495
12.8 Breathing loss of natural gas	481	12.46 Raoult's law in glycol dehydration unit	495
12.9 Capacity coefficient of valves in gas processing	481	12.47 Refrigerator shaft speed	496
12.10 Column diameter of packed towers	481	12.48 Relative humidity	496
12.11 Cooling of an ideal gas	482	12.49 Required oil length in separator	496
12.12 Correction factor for foamless separation	482	12.50 Required separator liquid section	497
12.13 Correlation factor for Benedict-Webb-Rubin equation	482	12.51 Required water length in separator	497
12.14 Critical pressure values for pressure in Van Der Waals equation	482	12.52 Residence time of water in separator	497
12.15 Downcomer velocity in tray type tower	483	12.53 Residence time oil in separator	498
12.16 Electrical heating of a pipe	483	12.54 Retention time in a liquid-liquid vessel	498
12.17 Energy requirement of single-stage ideal compressor	484	12.55 Safety relief valves sizing in vapor services	499
12.18 Error in thermocouple temperature measurement	484	12.56 Steady-state temperature controller	499
12.19 Eyring molecular refraction	485	12.57 Still column diameter in glycol dehydration unit	499
12.20 Fenske's method for minimum theoretical plates	485	12.58 Stripping factor	500
12.21 Gas capacity of separator	485	12.59 Surface tension from density	500
12.22 Gas mass velocity in an adsorption unit	486	12.60 Tarnishing of metal surfaces	501
12.23 Gas mass velocity in separator	486	12.61 TEG weight percent in glycol dehydration unit	501
12.24 Gas originally adsorbed	487	12.62 Temperature after refrigeration	501
12.25 Gas pressure testing time for unsteady gas flow	487	12.63 Temperature dependent source rate for flow reactor	502
12.26 Gravitational attraction of a layer (Bouguer correction)	487	12.64 Temperature distribution in a hot-wire anemometer	502
12.27 Heating of a liquid in an agitated tank	488	12.65 Terminal velocity in a separator	503
12.28 Height of downcomer filling	488	12.66 Thickness criteria of spherical shells in separator (ASME)	503
12.29 Inhibitor injection rate required	489	12.67 Top distillation column rate	504
12.30 Instrumentation noise control	489	12.68 Vapor mass velocity of tray type tower	504
12.31 Internal diameter of gas separator	489	12.69 Wall thickness criteria for separator by ASME	
12.32 Isostacy—Airy hypothesis	490	(external radius)	505
12.33 Lift coefficient	490	12.70 Wall thickness criteria for separator by ASME (internal radius)	505
12.34 Mass of steel shell in adsorption unit	491	12.71 Water loading in adsorption unit	505
12.35 Mass transfer zone length of adsorption unit	491	12.72 Weight of rich TEG in glycol dehydration unit	506
12.36 Modified Clapeyron criteria	492	12.73 Wobbe index	506
12.37 Packed column actual height	492	12.74 Work done by expansion tube refrigerator	506
12.38 Pan-Maddox equation for density	492		

## 12.1 Allowable gas velocity through gas separator

### Input(s)

- $K_s$ : Empirical Gas Constant (ft/s)
- $\rho_l$ : Liquid Density (g/cc)
- $\rho_g$ : Gas Density (g/cc)

### Output(s)

- v: Allowable Gas Velocity (ft/s)

### Formula(s)

$$v = K_s * \left( \frac{\rho_l - \rho_g}{\rho_g} \right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 73.*

## 12.2 Allowable velocity in downcomer for tray type tower

### Input(s)

- h: Height of Liquid Downcomer (in.)
- t: Residence Time (s)

### Output(s)

- $v_d$ : Allowable Velocity in Downcomer (in./s)

### Formula(s)

$$v_d = \frac{h}{t}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 73.*

## 12.3 Bed diameter of adsorption unit

### Input(s)

- q: Flow Rate (bbl/m)
- $\gamma$ : Fluid Relative Density (dimensionless)
- P: Pressure Drop (psi)

### Output(s)

- $C_v$ : Capacity Coefficient (dimensionless)

**Formula(s)**

$$C_v = \left(\frac{q}{A}\right) * \left(\frac{\gamma}{P}\right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 265.*

**12.4 Bed length of adsorption unit****Input(s)**

- x: Maximum Desiccant Useful Capacity (kg water/100 kg desiccant)
- $x_s$ : Dynamic Capacity at Saturation (kg water/100 kg desiccant)
- $h_z$ : MTZ Length (ft)

**Output(s)**

- $h_b$ : Bed Length (ft)

**Formula(s)**

$$h_b = \frac{0.45 * h_z * x_s}{x_s - x}$$

Reference: *John M. Campbell, Gas Conditioning and Processing Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 388.*

**12.5 Block efficiency factor****Input(s)**

- F: Friction Factor (unitless)
- n: No. of Rolling Sheaves (unitless)

**Output(s)**

- E: Block Efficiency Factor (unitless)

**Formula(s)**

$$E = \frac{F^n - 1}{F^n * n * (F - 1)}$$

Reference: *Samuel E. Robello. 501 Solved Problems and Calculations for Drilling Operations. Sigma Quadrant. 2015. Houston, Texas, Page: 11.*

**12.6 Bottom distillation column rate****Input(s)**

- L: Liquid Mass Velocity ( $\text{lbm}/\text{ft}^2 \text{ h}$ )
- G: Gas Mass Velocity ( $\text{lbm}/\text{dt}^2 \text{ h}$ )
- $\sigma_g$ : Gas Density (g/cc)
- $\sigma_l$ : Liquid Density (g/cc)

**Output(s)**

X: Bottom Distillation Column Rate (dimensionless)

**Formula(s)**

$$X = \frac{L * \sigma_g}{G * \sigma_l}$$

Reference: *Campbell, J. M., (1992, Houston, TX (United States)), Gas Conditioning and Processing, Vol. 2, Campbell Petroleum Series, Page: 74.*

**12.7 Breakthrough time in an adsorption unit****Input(s)**

- x: Height of Unit (ft)
- $\rho_b$ : Bulk Density of Desicant (lb/ft<sup>3</sup>)
- $h_b$ : Bed Length of Unit (ft)
- q: Water Loading (lb/ft<sup>2</sup> h)

**Output(s)**

$\Theta$ : Breakthrough Time (h)

**Formula(s)**

$$\Theta = \frac{0.01 * x * \rho_b * h_b}{q}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 391.*

**12.8 Breathing loss of natural gas****Input(s)**

- P: Pressure (psi)
- D: Tank Diameter (ft)
- $F_p$ : Paint Factor (dimensionless)
- $F_o$ : Outage Factor (dimensionless)

**Output(s)**

B: Breathing Loss (API bbl)

**Formula(s)**

$$B = \left( \frac{P * D^{1.8}}{14.5} \right) * F_p * F_o$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 123.*

## 12.9 Capacity coefficient of valves in gas processing

### Input(s)

- q: Flow Rate (bbl/m)
- $\gamma$ : Fluid Relative Density (dimensionless)
- P: Pressure Drop (psi)

### Output(s)

- $C_v$ : Capacity Coefficient (dimensionless)

### Formula(s)

$$C_v = \left(\frac{q}{A}\right) * \left(\frac{\gamma}{P}\right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 265.*

## 12.10 Column diameter of packed towers

### Input(s)

- m: Mass Flow Rate (lb/s)
- G: Gas Mass Flow Rate (lb/ft<sup>2</sup> h)

### Output(s)

- d: Column Diameter (ft)

### Formula(s)

$$d = \left(\frac{4 * m}{\pi * G}\right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 319.*

## 12.11 Cooling of an ideal gas

### Input(s)

- $H_1$ : Initial Enthalpy Per Unit Mass (ft<sup>2</sup>/s<sup>2</sup>)
- $H_2$ : Final Enthalpy Per Unit Mass (ft<sup>2</sup>/s<sup>2</sup>)
- $v_1$ : Initial Velocity (ft/s)
- $v_2$ : Final Velocity (ft/s)
- g: Acceleration Due to Gravity (ft/s<sup>2</sup>)
- $h_1$ : Initial Height (ft)
- $h_2$ : Final Height (ft)

### Output(s)

- Q: Energy Rate (Btu/s)

**Formula(s)**

$$Q = (H_2 - H_1) + 0.5 * ((v_2^2) - (v_1^2)) + g * (h_2 - h_1)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 576.*

**12.12 Correction factor for foamless separation****Input(s)**

- L: Length of Tank (ft)
- D: Diameter of Tank (ft)

**Output(s)**

- K: Correction Factor (dimensionless)

**Formula(s)**

$$K = \left( \frac{L/D}{5} \right)^{0.56}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 73.*

**12.13 Correlation factor for Benedict-Webb-Rubin equation****Input(s)**

- A: Mole Fraction of Hydrogen Sulfide and Carbon Dioxide in Gas Phase (fraction)
- B: Mole Fraction of Hydrogen Sulfide in Gas Phase (fraction)

**Output(s)**

- $\varepsilon$ : Correlation Factor (R)

**Formula(s)**

$$\varepsilon = 120 * ((A^{0.9}) - (A^{1.6})) + 15 * ((B^{0.5}) - (B^4))$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 54.*

**12.14 Critical pressure values for pressure in Van Der Waals equation****Input(s)**

- P: Pseudocritical Pressure (psi)
- T: Pseudocritical Temperature (K)
- $T_c$ : Critical Temperature (K)
- B: Mole Fraction of Hydrogen Sulfide (fraction)
- $\varepsilon$ : Correlation Constant (dimensionless)

**Output(s)**

$P_c$ : Critical Pressure (psi)

**Formula(s)**

$$P_c = \frac{P * T_c}{T + B * (1 - B) * \varepsilon}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 54.*

**12.15 Downcomer velocity in tray type tower****Input(s)**

$h$ : Height of Liquid Downcomer (in.)

$t$ : Residence Time (s)

**Output(s)**

$v_d$ : Allowable Velocity in Downcomer (in./s)

**Formula(s)**

$$v_d = \frac{h}{t}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 73.*

**12.16 Electrical heating of a pipe****Input(s)**

$R$ : Radius (m)

$\kappa$ : Ratio of Inner Radius to Outer Radius ( $m^2 \text{ kg s}^{-2} \text{ K}^{-1}$ )

$L$ : Length (m)

$T_k$ : Desired Temperature (K)

$T_a$ : Ambient Air Temperature (K)

$k$ : Thermal Conductivity (W/m K)

$h$ : Heat Transfer Coefficient (W/(m<sup>2</sup> K))

**Output(s)**

$P$ : Electrical Power (Watt)

**Formula(s)**

$$P = \frac{\pi * R^2 * (1 - \kappa^2) * L * (T_k - T_a)}{(1 - \kappa^2) * \frac{R}{2 * h} - \frac{(\kappa * R)^2}{4 * k} * \left(1 - \frac{1}{\kappa^2} - 2 * \ln(\kappa)\right)}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 325.*

## 12.17 Energy requirement of single-stage ideal compressor

### Input(s)

$v_1$ : Velocity (ft/s)  
 $P_1$ : Initial Pressure (psi)  
 $P_2$ : Final Pressure (psi)  
 $R$ : Ideal Gas Constant ( $\text{ft}^3 \text{ lb/mol K}$ )  
 $T$ : Temperature (K)  
 $M$ : Mass (lbs)  
 $\gamma$ : Adiabatic Constant (dimensionless)

### Output(s)

$W$ : Energy (ft lbf/lbm)

### Formula(s)

$$W = \left( \frac{v_1^2}{2} \right) * \left( 1 - \left( \left( \frac{P_1}{P_2} \right)^2 \right) \right) + \left( \frac{R * T * \gamma}{M * (\gamma - 1)} \right) * \left( \left( \left( \frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \right) - 1 \right)$$

Reference: *Bird R. Byron, Stewart E. Warren, Lightfoot N. Edward.*

## 12.18 Error in thermocouple temperature measurement

### Input(s)

$T$ : Temperature indicated by Thermocouple (K)  
 $T_w$ : Temperature of Wall (K)  
 $h$ : Heat Conduction Constant (dimensionless)  
 $L$ : Length (cm)  
 $k$ : Thermal Conductivity (W/m K)  
 $B$ : Breadth (cm)

### Output(s)

$n$ : Constant (K)  
 $T_a$ : Actual Thermocouple Temperature (K)

### Formula(s)

$$n = \left( \cosh \left( \left( h * \frac{L^2}{k} * B \right)^{0.5} \right) \right)^{-1}$$

$$T_a = \frac{T - n * T_w}{1 - n}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 310.*

## 12.19 Eykman molecular refraction

### Input(s)

- P<sub>v</sub>: Vapor Pressure of Water (psi)
- P: Pressure (psi)
- f: Fugacity of Water at Vapor Pressure (dimensionless)
- f<sub>w</sub>: Fugacity of Water at Pressure P (dimensionless)

### Output(s)

- k: Eykman Constant (dimensionless)

### Formula(s)

$$k = \left( \frac{P_v}{P} \right) * \left( \frac{f/P_v}{f_w/P_v} \right) * \left( \frac{P}{P_v} \right)^{0.049}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 50.*

## 12.20 Fenske's method for minimum theoretical plates

### Input(s)

- X<sub>lkd</sub>: Distillate Mole Fraction of Light Component (fraction)
- X<sub>hkd</sub>: Distillate Mole Fraction of Heavy Component (fraction)
- X<sub>lkb</sub>: Bottom Mole Fraction of Light Component (fraction)
- X<sub>hkb</sub>: Bottom Mole Fraction of Heavy Component (fraction)
- $\alpha_a$ : Relative Volatility (fraction)

### Output(s)

- S<sub>m</sub>: Number of Minimum Theoretical Stages (dimensionless)

### Formula(s)

$$S_m = \log \left( \left( \frac{X_{lkd}}{X_{hkd}} \right) * \left( \frac{X_{lkb}}{X_{hkb}} \right) \right) / \log (\alpha_a)$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 288.*

## 12.21 Gas capacity of separator

### Input(s)

- K<sub>s</sub>: Separator Coefficient (ft/s)
- d: Total Internal Diameter of Separator (ft)
- F: Fraction of Total Area Available to Gas (fraction)
- z: Compressibility Factor (dimensionless)
- P: Separation Pressure (psi)
- P<sub>s</sub>: Base Pressure (psi)
- T: Absolute Separation Temperature (K)

$T_s$ : Base Temperature (K)  
 $\rho_l$ : Liquid Density (g/cc)  
 $\rho_g$ : Gas Density (g/cc)

**Output(s)**

$q_s$ : Gas Rate ( $\text{ft}^3/\text{d}$ )

**Formula(s)**

$$q_s = 67824 * K_s * (d^2) * F * \left(\frac{1}{z}\right) * \left(\frac{P}{P_s}\right) * \left(\frac{T_s}{T}\right) * \left(\frac{\rho_l - \rho_g}{\rho_g}\right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 74.*

**12.22 Gas mass velocity in an adsorption unit****Input(s)**

$v_g$ : Gas Velocity ( $\text{ft}/\text{m}$ )  
 $\gamma_g$ : Specific Gravity of Gas (dimensionless)  
 $P$ : Pressure (psi)  
 $T$ : Inlet Gas Temperature (K)  
 $z$ : Compressibility Factor (dimensionless)

**Output(s)**

$w$ : Gas Mass Velocity ( $\text{lb}/\text{h ft}^2$ )

**Formula(s)**

$$w = \frac{162 * v_g * \gamma_g * P}{T * z}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 391.*

**12.23 Gas mass velocity in separator****Input(s)**

$w$ : Mass Mass Flow Velocity ( $\text{lb}/\text{h ft}^2$ )  
 $d$ : Internal Diameter of Separator (ft)  
 $F$ : Fraction of Area Available for Gas (fraction)

**Output(s)**

$m$ : Mass Rate ( $\text{lb}/\text{h}$ )

**Formula(s)**

$$m = 0.785 * w * (d^2) * F$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 75.*

**12.24 Gas originally adsorbed****Input(s)**

- A: Drainage Area (acres)
- h: Thickness (ft)
- $\rho_B$ : Bulk Density of Coal (g/cc)
- $G_c$ : Gas Content (scf/ton)

**Output(s)**

- G: Gas Initially in Place (scf)

**Formula(s)**

$$G = 1359.7 * A * h * \rho_B * G_c$$

Reference: *Ahmed, T., McKinney, P.D. 2005. Advanced Reservoir Engineering, Gulf Publishing of Elsevier, Chapter: 3, Page: 227.*

**12.25 Gas pressure testing time for unsteady gas flow****Input(s)**

- d: Internal Pipe Diameter (in.)
- L: Length of Pipe (miles)
- P: Initial Pressure (psi)

**Output(s)**

- $t_m$ : Minimum Time Needed for Testing (h)

**Formula(s)**

$$t_m = \frac{3 * (d^2) * L}{P}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 30.*

**12.26 Gravitational attraction of a layer (Bouguer correction)****Input(s)**

- G: Gravitational Constant ( $N m^2/kg^2$ )
- $\Delta z$ : Thickness (m)
- $\Delta p$ : Density Contrast ( $kg/m^3$ )

**Output(s)**

$\Delta g_z$ : Bouguer Gravity ( $m/s^2$ )

**Formula(s)**

$$\Delta g_z = 2 * \pi * G * \Delta z * \Delta \rho$$

Reference: <https://sites.ualberta.ca/~unsworth/UA-classes/210/exams210/210-final-2008-formula-sheet.pdf>.

**12.27 Heating of a liquid in an agitated tank****Input(s)**

$T_1$ : Initial Temperature (K)  
 $T_s$ : Steam Temperature (K)  
 $U_0$ : Heat Coefficient (W/ft K)  
 $A_0$ : Area ( $ft^2$ )  
 $w_1$ : Weight (lbm)  
 $C_p$ : Specific Heat of Mass (lbf/lbs K)

**Output(s)**

$T_0$ : Final Temperature (K)

**Formula(s)**

$$\frac{T_0 - T_1}{T_s - T_1} = 1 - \left( \frac{1 - \left( \exp \left( \frac{-U_0 * A_0}{w_1 * C_p} \right) \right)}{\frac{U_0 * A_0}{w_1 * C_p}} \right)$$

Reference: Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena* (Second Ed.). John Wiley & Sons, Chapter: 15, Page: 468.

**12.28 Height of downcomer filling****Input(s)**

$h_c$ : Clear Liquid Height (in.)  
 $h_d$ : Dry Tray Height (in.)  
 $h_u$ : Head Loss Under Downcomer (in.)  
 $\rho_l$ : Liquid Density (g/cc)  
 $\rho_g$ : Gas Density (g/cc)  
 $h_i$ : Tray Inlet Head (in.)

**Output(s)**

$h_d$ : Height of Downcomer Filling (in.)

**Formula(s)**

$$h_d = (h_c + h_e + h_u) * \left( \frac{\rho_l}{\rho_l - \rho_g} \right) + h_i + 1$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 316.*

**12.29 Inhibitor injection rate required****Input(s)**

- $m_w$ : Mass of Water (lb)
- $X_r$ : Rich Inhibitor Concentration (wt%)
- $X_l$ : Lean Inhibitor Concentration (wt%)

**Output(s)**

- $m$ : Mass of Inhibitor (lb)

**Formula(s)**

$$m = m_w * (X_r / (X_l - X_r))$$

$$A = \pi r^2$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 181.*

**12.30 Instrumentation noise control****Input(s)**

- $P_a$ : Pressure of Sound Measured (psi)
- $P_o$ : Reference Pressure (psi)

**Output(s)**

- $dB$ : Decibel (dB)

**Formula(s)**

$$dB = 20 * \log \left( \frac{P_a}{P_o} \right)$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 297.*

**12.31 Internal diameter of gas separator****Input(s)**

- $m$ : Mass Flow Rate (lb/h)
- $K_s$ : Separator Coefficient (ft/h)

F: Fraction of Separator Available for Gas (fraction)

$\rho_l$ : Liquid Density (g/cc)

$\rho_g$ : Gas Density (g/cc)

### Output(s)

d: Internal Diameter (ft)

### Formula(s)

$$d = \frac{0.0188 * \left( \frac{m}{F * K_s} \right)^{0.5}}{\left( \frac{\rho_l - \rho_g}{\rho_g} \right)^{0.25}}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 75.*

## 12.32 Isostacy—Airy hypothesis

### Input(s)

h: Mountain Height (m)

$\rho_c$ : Crustal Density ( $\text{kg}/\text{m}^3$ )

$\rho_m$ : Mantle Density ( $\text{kg}/\text{m}^3$ )

### Output(s)

r: Root Depth (m)

### Formula(s)

$$r = h * \frac{\rho_c}{\rho_m - \rho_c}$$

Reference: <https://sites.ualberta.ca/~unsworth/UA-classes/210/exams210/210-final-2008-formula-sheet.pdf>.

## 12.33 Lift coefficient

### Input(s)

$\rho$ : Density ( $\text{kg}/\text{m}^3$ )

v: True Air Speed (m/s).

L: Lift Force (Newton)

S: Planform Area ( $\text{m}^2$ )

### Output(s)

$C_L$ : Lift Coefficient (dimensionless)

**Formula(s)**

$$C_L = 2 * \frac{L}{\rho * v^2 * S}$$

Reference: [Wikipedia.org](https://en.wikipedia.org).

**12.34 Mass of steel Shell in adsorption unit****Input(s)**

- h: Vessel Length (ft)
- d: Vessel Internal Diameter (in.)
- t: Shell Thickness (in.)

**Output(s)**

- m: Mass of Steel Shell (lb)

**Formula(s)**

$$m = 15 * h * d * t$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 2, Page: 398, Campbell Petroleum Series, Oklahoma, 1992.*

**12.35 Mass transfer zone length of adsorption unit****Input(s)**

- q: Water Loading ( $\text{lb}/\text{ft}^2 \text{ h}$ )
- $v_g$ : Velocity (ft/min)
- RS: Relative Saturation of Inlet Gas (%)

**Output(s)**

- $h_z$ : Mass Transfer Zone Length (ft)

**Formula(s)**

$$h_z = \frac{375 * (q^{0.7895})}{(v_g^{0.5506}) * (RS^{0.2646})}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 390.*

**12.36 Modified Clapeyron criteria****Input(s)**

- C: Component Constant for Hydrocarbon Levels to Pressure (K)

### Output(s)

T: Hydrate Forming Temperature (R)

### Formula(s)

$$T = 3.89 * (C^{0.5})$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 178.*

## 12.37 Packed column actual height

### Input(s)

HTU: Height of a Transfer Unit (ft)

NTU: Number of Transfer Units (dimensionless)

### Output(s)

h: Height of Column (ft)

### Formula(s)

$$h = HTU * NTU$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 280.*

## 12.38 Pan-Maddox equation for density

### Input(s)

HTU: Height of a Transfer Unit (ft)

NTU: Number of Transfer Units (dimensionless)

### Output(s)

h: Height of Column (ft)

### Formula(s)

$$h = HTU * NTU$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 280.*

## 12.39 Pan-Maddox equation for molecular weight

### Input(s)

T<sub>b</sub>: Boiling Temperature (K)

$\gamma$ : Relative Density at 15.5 Degrees (g/cc)

**Output(s)**

M: Molecular Weight (g)

**Formula(s)**

$$M = 1.66 * (10^{-4}) * (T_b^{2.2}) * (\gamma^{-1.02})$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 1, Page: 76, Campbell Petroleum Series, Oklahoma, 1992.*

**12.40 Photoelectric effect****Input(s)**

m: Mass of Body at Rest (kg)

E<sub>k</sub>: Kinetic Energy (Joule)

C: Velocity of Light ( $3 \times 10^8$  m/s)

**Output(s)**

v: Velocity of Particle (m/s)

**Formula(s)**

$$v = C * \left( 1 - \left( 1 + \left( \frac{E_k}{m * C^2} \right) \right)^{-2} \right)^{0.5}$$

Reference: *Bassiouni, Z., 1994, Theory, Measurement, and interpretation of Well Logs. SPE Textbook Series Vol. 4. Chapter 2, Page: 33.*

**12.41 Power requirement for pumping a compressible flow fluid through a long pipe****Input(s)**

D: Diameter of Pipe (ft)

$\rho_1$ : Pressure (psi)

M: Mass (lbs)

R: Constant ( $\text{ft}^3 \text{ psi/lbmol K}$ )

T: Temperature (K)

W<sub>m</sub>: Energy Required by Compressor (ft lbf/lbm)

v: Velocity (ft/s)

**Output(s)**

P: Power (hp)

**Formula(s)**

$$P = \frac{(v_1) * \pi * (D^2) * \rho_1 * M * W_m}{4 * R * T}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 15, Page: 465.*

## 12.42 Pressure criteria for separator by ASME (external radius)

### Input(s)

S: Maximum Allowable Stress (psi)  
 E: Joint Efficiency (fraction)  
 t: Shell Plate Thickness (ft)  
 R<sub>o</sub>: Outer Radius (ft)

### Output(s)

P: Pressure (psi)

### Formula(s)

$$P = \frac{S * E * t}{R_o - 0.4 * t}$$

Reference: *John M. Campbell., Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 64.*

## 12.43 Pressure criteria for separator by ASME (internal radius)

### Input(s)

S: Maximum Allowable Stress (psi)  
 E: Joint Efficiency (fraction)  
 t: Thickness of Shell (ft)  
 R<sub>i</sub>: Internal Radius of Shell (ft)

### Output(s)

P: Pressure (psi)

### Formula(s)

$$P = \frac{S * E * t}{R_i + 0.6 * t}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 64.*

## 12.44 Pressure storage

### Input(s)

P<sub>max</sub>: Pressure at Maximum Liquid Temperature (psi)  
 P<sub>v</sub>: Pressure at Which Vacuum Vent Opens (psi)  
 P<sub>min</sub>: Pressure at Minimum Liquid Temperature (psi)  
 T<sub>max</sub>: Maximum Average Temperature of Vapor (K)  
 T<sub>min</sub>: Minimum Average Temperature of Vapor (K)  
 P<sub>a</sub>: Atmospheric Pressure (psi)

**Output(s)**

$P_s$ : Pressure Storage (psi)

**Formula(s)**

$$P_s = P_{\max} + \left( (P_v - P_{\min}) * \left( \frac{T_{\max}}{T_{\min}} \right) \right) - P_a$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 206.*

**12.45 Proportional band in pressure controller****Input(s)**

O: Tolerable Overshoot (psi)

Span: Transmitter Range (psi)

**Output(s)**

PB: Proportional Band (%)

**Formula(s)**

$$PB = \frac{200 * O}{Span}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 1, Page: 276, Campbell Petroleum Series, Oklahoma, 1992.*

**12.46 Raoult's law in glycol dehydration unit****Input(s)**

P: System Pressure (psi)

$P_v$ : Water Vapor Pressure at Reboiler Temperature (psi)

$y_w$ : Mol Fraction of Water in Reboiler Vapor (fraction)

**Output(s)**

$x_w$ : Mole Fraction of Water in Lean Glycol (fraction)

**Formula(s)**

$$x_w = \left( \frac{P}{P_v} \right) * y_w$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 334.*

## 12.47 Refrigerator shaft speed

### Input(s)

- N: Shaft Speed (rpm)  
 q<sub>a</sub>: Exhaust Turbine Volume (ft<sup>3</sup>/s)  
 h: Isentropic Heat Change (btu/lb)

### Output(s)

- $N_s$ : Speed (rpm)

### Formula(s)

$$N_s = \frac{N * (q_a^{0.5})}{(A * h)^{0.75}}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 157.*

## 12.48 Relative humidity

### Input(s)

- $P_p$ : Partial Pressure of Water Vapor in Air Mixture (psi)  
 $P_v$ : Saturation Pressure (psi)

### Output(s)

- $\phi$ : Relative Humidity (dimensionless)

### Formula(s)

$$\phi = \frac{P_p}{P_v}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 2, Page: 157, Campbell Petroleum Series, Oklahoma, 1992.*

## 12.49 Required oil length in separator

### Input(s)

- $t_o$ : Residence Oil Time (min)  
 $q_o$ : Oil Rate (ft<sup>3</sup>/m)  
 $A_o$ : Area of Oil (ft<sup>2</sup>)

### Output(s)

- $L_o$ : Length Required by Oil Flow (ft)

**Formula(s)**

$$L_o = \frac{t_o * q_o}{A_o}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 80.*

**12.50 Required separator liquid section****Input(s)**

- $q_l$ : Liquid Throughput (bbl/d)  
 $t$ : Design Residence Time (min)

**Output(s)**

- $V_l$ : Required Liquid Section Capacity of Separator (bbl)

**Formula(s)**

$$V_l = \frac{q_l * t}{1440}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 2, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 76.*

**12.51 Required water length in separator****Input(s)**

- $t_w$ : Time of Water Residence (min)  
 $q_w$ : Rate of Water Flow ( $\text{ft}^3/\text{m}$ )  
 $A_w$ : Area of Water ( $\text{ft}^2$ )

**Output(s)**

- $L_w$ : Length of Water Column (ft)

**Formula(s)**

$$L_w = \frac{t_w * q_w}{A_w}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 80.*

**12.52 Residence time of water in separator****Input(s)**

- $h_o$ : Height of Oil Column in Separator (ft)  
 $v_w$ : Terminal Velocity of Water (ft/h)

**Output(s)**

$t_w$ : Residence Time of Water (min)

**Formula(s)**

$$t_w = \left( \frac{h_o}{60 * v_w} \right)$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 2, Page: 80, Campbell Petroleum Series, Oklahoma, 1992.*

**12.53 Residence time oil in separator****Input(s)**

$h_w$ : Height of Water Column in Separator (ft)

$v_o$ : Terminal Velocity of Water (ft/min)

**Output(s)**

$t_o$ : Oil Residence Time (min)

**Formula(s)**

$$t_o = \left( \frac{h_w}{60 * v_o} \right)$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Vol. 2, Page: 80, Campbell Petroleum Series, Oklahoma, 1992.*

**12.54 Retention time in a liquid-liquid vessel****Input(s)**

$\mu$ : Viscosity of Predominant Phase (cP)

A: Separator Constant—varies from 0.05 to 1.0 (dimensionless)

$\gamma_b$ : Specific Gravity of Bottom Phase (dimensionless)

$\gamma_t$ : Specific Gravity of Top Phase (dimensionless)

**Output(s)**

T: Retention Time (h)

**Formula(s)**

$$T = \frac{A * \mu}{\gamma_b - \gamma_t}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 87.*

## 12.55 Safety relief valves sizing in vapor services

### Input(s)

- C: Specific Heat Ratio (dimensionless)
- $K_o$ : Valve Discharge Coefficient (dimensionless)
- A: Effective Discharge Area ( $\text{in.}^2$ )
- B: Conversion constant (dimensionless)
- P: Upstream Relieving Pressure (psi)
- M: Gas Molecular Weight (dimensionless)
- Z: Compressibility Factor (dimensionless)
- T: Inlet Temperature (R)

### Output(s)

- w: Weight Flow Through Valve (lb/h)

### Formula(s)

$$w = (B) * C * K_o * A * P * \left( \frac{M}{Z * T} \right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 296.*

## 12.56 Steady-state temperature controller

### Input(s)

- $T_o$ : Outlet Temperature (K)
- $T_m$ : Maximum Controller Temperature (K)
- b: Constant (dimensionless)
- U: Overall Heat Coefficient (W/ft K)
- A: Area (ft)

### Output(s)

- $T_i$ : Initial Temperature (K)

### Formula(s)

$$T_i = T_o * \left( 1 - \left( \frac{b}{U * A} \right) \right) + \frac{b * T_m}{U * A}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 15, Page: 470.*

## 12.57 Still column diameter in glycol dehydration unit

### Input(s)

- m: Glycol circulation rate (gal/m)

### Output(s)

d: Diameter (in.)

### Formula(s)

$$d = 9.1 * m^{0.5}$$

Reference: *John M. Campbell. Gas Conditioning & Processing Vol. 2. Campbell Petroleum Series. OK. 1992.*

## 12.58 Stripping factor

### Input(s)

K: Stripping Coefficient (dimensionless)  
V: Moles of Stripping Medium Entered (moles)  
L: Moles of Lean Oil Leaving Stripper (moles)

### Output(s)

S: Stripping Factor (dimensionless)

### Formula(s)

$$S = \frac{K * V}{L}$$

Reference: *Campbell, J. M., (1992, Oklahoma (United States)) Gas Conditioning and Processing Volume 2, Campbell Petroleum Series, Page: 313.*

## 12.59 Surface tension from density

### Input(s)

P: Pressure (dyn/cm<sup>2</sup>)  
M: Molecular Wt (gms)  
 $\rho_l$ : Liquid Density (g/cc)  
 $\rho_v$ : Vapor Density (g/cc)

### Output(s)

$\omega$ : Surface Tension (dyn/cm)

### Formula(s)

$$\omega^{0.25} = \left( \frac{P}{M} \right) * (\rho_l - \rho_v)$$

Reference: *Campbell, J. M., (1992, Houston, TX (United States)), Gas Conditioning and Processing, Vol. 1, Campbell Petroleum Series, Page: 75.*

## 12.60 Tarnishing of metal surfaces

### Input(s)

$D_{O_2\text{MO}_x}$ : Diffusivity ( $\text{cm}^2/\text{s}$ )  
 $t$ : Time (s)  
 $x$ : Mole Fraction of Oxygen (dimensionless)  
 $c_o$ : Solubility of Oxygen ( $\text{g}/\text{cc}$ )  
 $c_f$ : Molar Density of the Film ( $\text{g mol}/\text{cc}$ )

### Output(s)

$z_f$ : Thickness of Film (cm)

### Formula(s)

$$z_f = \left( \left( \frac{2 * D_{O_2\text{MO}_x} * t * c_o}{x * c_f} \right)^{0.5} \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 18, Page: 576.*

## 12.61 TEG weight percent in glycol dehydration unit

### Input(s)

$m$ : Weight of Lean TEG (g)  
 $w$ : Weight of Water Absorbed (g)  
 $l$ : Weight of Water in Lean TEG (g)

### Output(s)

$wt$ : Weight Percent (%)

### Formula(s)

$$wt = \frac{m * 100}{m + w + l}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 341.*

## 12.62 Temperature after refrigeration

### Input(s)

$T_i$ : Input Temperature (K)  
 $P_i$ : Input Pressure (psi)  
 $P_o$ : Output Pressure (psi)  
 $E$ : Isentropic Efficiency (dimensionless)  
 $m$ : Cycle Efficiency (dimensionless)

**Output(s)**

$T_o$ : Output Temperature (K)

**Formula(s)**

$$T_o = T_i + T_i * \left( \left( \left( \frac{P_o}{P_i} \right)^m \right) - 1 \right) * E$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 254.*

**12.63 Temperature dependent source rate for flow reactor****Input(s)**

K: Constant (dimensionless)

E: Constant of Time-dependence (dimensionless)

R: Gas Constant ( $\text{m cm}^2/\text{s}^2 \text{ K}$ )

T: Temperature (K)

**Output(s)**

$S_e$ : Entropy ( $\text{g cm}^2/\text{s}^2 \text{ K}$ )

**Formula(s)**

$$S_e = K * e^{\frac{-E}{R*T}}$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 326.*

**12.64 Temperature distribution in a hot-wire anemometer****Input(s)**

D: Diameter (ft)

L: Length (ft)

I: Current (Amp)

h: Heat Transfer Coefficient ( $\text{btu/h ft}^2 \text{ F}$ )

$k_c$ : Thermal Conductivity of Ambiance ( $1/\Omega \text{ ft}$ )

z: Distance (ft)

**Output(s)**

T: Temperature increase (F)

**Formula(s)**

$$T = \left( \frac{D * (I^2)}{4 * h * k_e} \right) * \left( 1 - \left( \frac{\cosh \left( \left( \frac{4 * h}{k * D * z} \right)^{0.5} \right)}{\cosh \left( \left( \frac{4 * h}{k * D * L} \right)^{0.5} \right)} \right) \right)$$

Reference: *Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). Transport Phenomena (Second Ed.). John Wiley & Sons, Chapter: 10, Page: 328.*

**12.65 Terminal velocity in a separator****Input(s)**

- g: Acceleration Due to Gravity (ft/s<sup>2</sup>)
- D<sub>p</sub>: Particle Diameter (ft)
- N: Drag Coefficient (fraction)
- ρ<sub>p</sub>: Particle Density (g/cc)
- ρ<sub>f</sub>: Fluid Density (g/cc)
- A: Flow Regime Constant (dimensionless) (dimensionless)
- μ: Viscosity (cP)

**Output(s)**

- v<sub>t</sub>: Terminal Velocity of a Particle falling through a fluid by the pull of Gravity (ft/s)

**Formula(s)**

$$v_t = \left( \frac{4 * g * (D_p^{N+1}) * (\rho_p - \rho_f)}{3 * A * (\mu^N) * (\rho_f^{1-N})} \right)^{\frac{1}{2-N}}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 71.*

**12.66 Thickness criteria of spherical shells in separator (ASME)****Input(s)**

- P: Pressure (psi)
- R: Radius (ft)
- E: Joint Efficiency (fraction)
- S: Maximum Allowable Stress (psi)

**Output(s)**

- t: Thickness of the Spherical Shell (ft)

**Formula(s)**

$$t = \frac{P * R}{2 * S * E - 0.2 * P}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 64.*

**12.67 Top distillation column rate****Input(s)**

- G: Gas Mass Velocity ( $\text{lb}/\text{ft}^2 \text{s}$ )
- F: Packing Factor (dimensionless)
- $\mu_l$ : Liquid Viscosity ( $\text{cP}$ )
- $\rho_g$ : Gas Density ( $\text{lbm}/\text{ft}^3$ )
- $\rho_l$ : Liquid Density ( $\text{lbm}/\text{ft}^3$ )

**Output(s)**

- Y: Top Distillation Column Rate (dimensionless)

**Formula(s)**

$$Y = \frac{(G^2) * F * (\mu_l^{0.1})}{\rho_g * (\rho_l - \rho_g)}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 317.*

**12.68 Vapor mass velocity of tray type tower****Input(s)**

- $K_s$ : Sizing Constant ( $\text{ft}/\text{s}$ )
- $\rho_l$ : Liquid Density ( $\text{lbm}/\text{ft}^3$ )
- $\rho_g$ : Gas Density ( $\text{lbm}/\text{ft}^3$ )

**Output(s)**

- w: Vapor Mass Velocity ( $\text{lbm}/\text{h ft}^2$ )

**Formula(s)**

$$w = 3600 * K_s * \left( (\rho_l - \rho_g) * \rho_g \right)^{0.5}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 315.*

## 12.69 Wall thickness criteria for separator by ASME (external radius)

### Input(s)

- P: Pressure (psi)
- R<sub>o</sub>: External Radius of Separator (ft)
- S: Maximum Allowable Stress (psi)
- E: Joint Efficiency (fraction)

### Output(s)

- t: Thickness (ft)

### Formula(s)

$$t = \frac{P * R_o}{S * E + 0.4 * P}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 64.*

## 12.70 Wall thickness criteria for separator by ASME (internal radius)

### Input(s)

- P: Pressure (psi)
- R<sub>i</sub>: Internal Radius (ft)
- S: Maximum Allowable Stress (psi)
- E: Joint Efficiency (fraction)

### Output(s)

- t: Thickness (ft)

### Formula(s)

$$t = \frac{P * R_i}{S * E - 0.6 * P}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 64.*

## 12.71 Water loading in adsorption unit

### Input(s)

- q: Flow Rate (mmscf/d)
- w: Water Content (lb/mmscf)
- d: Bed Diameter (ft)

### Output(s)

- $q_i$ : Water Loading (lb/ft<sup>2</sup> h)

**Formula(s)**

$$q_l = \frac{0.053 * q * w}{d^2}$$

Reference: *Campbell, J. M., (1992, Oklahoma (United States)) Gas Conditioning and Processing Volume 2, Campbell Petroleum Series, Page: 391.*

**12.72 Weight of rich TEG in glycol dehydration unit****Input(s)**

- $\rho$ : Liquid Density (g/cc)  
 TEG: Weight Percent of TEG in Lean TEG Solution (%)  
 m: Lean TEG Rate (gal/lb)

**Output(s)**

- R: Weight of Rich TEG in TEG Solution (%)

**Formula(s)**

$$R = \frac{\rho * \text{TEG}}{\rho + \left(\frac{1}{m}\right)}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 2, Page: 341.*

**12.73 Wobbe index****Input(s)**

- GHV: Gross Heating Value (btu/ft<sup>3</sup>)  
 $\gamma$ : Specific Gravity (dimensionless)

**Output(s)**

- W: Wobbe Number (dimensionless)

**Formula(s)**

$$W = \frac{GHV}{\gamma^{0.5}}$$

Reference: *John M. Campbell, Gas Conditioning and Processing, Campbell Petroleum Series, Oklahoma, 1992, Vol. 1, Page: 291.*

**12.74 Work done by expansion tube refrigerator****Input(s)**

- E: Efficiency (dimensionless)  
 m: Mass Flow Rate (lb/ft)

$h_o$ : Outlet Enthalpy (btu/lb)  
 $h_i$ : Inlet Enthalpy (btu/lb)

### **Output(s)**

w: Work Done (btu)

### **Formula(s)**

$$w = E * m * (h_o - h_i)$$

Reference: *John M. Campbell, Gas Conditioning & Processing Vol. 2. Campbell Petroleum Series. OK. 1992, Page: 89.*

# Index

## A

Abnormally pressured gas reservoirs  
Roach plot, 52  
rock expansion term, 52

Absolute viscosity  
for Saybolt viscosimeter measurements, 399  
for Ubbelohde viscosimeter  
measurements, 400

Acceptable reliability level, 359–360

Accumulator  
capacity, 73–74  
precharge pressure, 74

Acid dissolving power, 251

Acid penetration distance, 204–205

Acoustic transit time, 324

Additional production estimation with new wells, 360

Additional salt need to raise density  
single-salt systems, 205–206  
two-salt systems, 206–207

Additives, 74–75, 172

Adhesion tension, 400

Adsorption unit  
bed diameter, 478–479  
bed length, 479  
breakthrough time, 480  
gas mass velocity, 486  
mass of steel shell, 491  
mass transfer zone length, 491  
water loading, 505–506

Agitated tank, heating of liquid in, 488

Airy hypothesis, 490

Allowable gas velocity, 478

Allowable velocity in downcomer, 478

Amott-Harvey wettability index, 400

Amount of cement, 75

Amount of heat required, 377–378

Amount of mud required, 75

Amount of solids generated during drilling, 165

Amplitude transmission coefficient, 324

Angle of twist, 75–76

Anisotropic reservoirs, effective wellbore radius for, 14–15

Annual average rate of return method, 361–362

Annual gross revenue after royalties and wellhead taxes, 360

Annual rate of return prediction, 366–367

Annuity  
from future value, 360–361  
from present value, 361

Annular capacity, 76–77  
using circulation rate, 77–78  
using pump output, 77–78

Annular volume capacity of pipe, 78–79

Annulus pressure loss due to friction  
laminar flow, 207  
turbulence flow, 207–208

API gravity, 3, 62

API water loss calculations, 79

Apparent intensity reflected by recorder, 325

Apparent transmissivity, 421

Approximate ideal counterbalanced load, 208

Archie's equation, 333, 337

Archie's law, 412

Archimedes number, 258–259

Area below casing shoe, 79

Areal extent of heated zone, 416

Area of pipe subject to uniform axial force, 108

Arithmetic average temperature, 5

Atlas wireline neutron lifetime log, 326

Average book rate of return method, 362

Average downstroke load, 208

Average fracture permeability, 178

Average fracture width, 208–209

Average specific weight of formation, 209–210

Average upstroke load, 210

Average wellbore fluid density, 210

Axial dispersion coefficient, 316

Axial loads in slips, 79–80

Axial stress around vertical wellbore, 444

Axis of a deviated borehole from an arbitrary origin, 444–445

## B

Barenblatt-Chorin universal velocity distribution, 326

Beam force, 80

Bed diameter, 478–479

Bed length, 479

Beggs/Robinson correlation, 63

Beggs-standing correlation, 41–42

Benedict-Webb-Rubin equation, 378, 482

Bingham plastic model  
plastic viscosity, 144  
yield strength, 476

Bit nozzle pressure loss, 80–81

Bit nozzle selection, 81–82

Block efficiency factor, 479

Blowdown time, 262

Boltzmann equation, 262

Borehole torsion, 82–83

Bottom distillation column rate, 479–480

Bottomhole annulus pressure, 83

Bottomhole assembly length, 83

Bottomhole pressure, in static geothermal well, 417

Bottom-water-drive reservoir, 430

Bouguer correction, 487–488

Bournazel and Jeanson water breakthrough correlation, 65–66

Boussinesq approximation, 262–263

Breakdown pressure, 465

Breakthrough time, 480

Breathing loss, 480

Brine saturated rock conductivity, 70

Brinkman number, 263

Brons and Marting method, 234–235

Buckingham Reiner equation, 263–264

Buildup pressure  
for field calculations, 180  
for liquid flow, 180–181  
for steam/gas flow, 181

Buildup time, 181–182

Bulk density of cuttings, 83–84

Bulk modulus, 446  
using Lame, 445  
using Poisson's ratio and Lame's constant, 445  
using Poisson's ratio and shear modulus, 445–446  
using Poisson's ratio and Young's modulus, 84

Buoyancy factor, 85

Buoyancy weight, 84

## C

Capacity coefficient of valves, 481

Capacity formulas  
bbl/ft, 87–88  
gal/ft, 88

Capacity of tubulars and open-hole, 88

Capacity ratio, of hydraulically fractured surface, 210–211

Capillary number, 6

Capillary pressure, 6

- Casing  
 amount of cement to be left in, 75  
 burst pressure, 162–163  
 downward force, 99  
 pressure by each barrel of mud in, 147  
 pressure required to open valve, 124  
 Cavett method, 378–379  
 Cementing  
   lateral load imposed on a casing centralizer, 129  
   lateral load imposed on a casing centralizer with a dogleg, 129  
   load to break cement bond, 130  
   radial force related to axial load, 152  
 Centrifuge, evaluation of, 110–111  
 Chaperon correlation, 28  
 Characteristic time, 6–7  
 Characterization factor, for oil distillation, 402  
 Chevron formula, 337  
 Choke discharge coefficient, 211  
 Choke line pressure loss, 162  
 Chromatographic lag, in polymer flooding, 417  
 Clasius-Clapeyron equation, 402  
 Clay concentration of drilling mud, 402–403  
 Clean sands, 70  
 Close-ended displacement volume of pipe, 211  
 Coal bed methane reservoirs  
   diffusion through cleat spacing, 273–274  
   formation permeability, 54  
   fractional gas recovery, 18  
   gas adsorbed in, 19  
   gas solubility, 24–25  
   remaining gas in place, 51  
   time for semi-steady state flow, 420  
   volume of gas adsorbed, 64–65  
 Coefficient of reflection, 327  
 Cohesive strength of rocks, 446  
 Cole plot, 7  
 Collapse pressure, 179  
 Column diameter of packed towers, 481  
 Combined momentum flux tensor, 265  
 Combined radiation and convection, 265  
 Combined solubility of hydrocarbon gas, CO<sub>2</sub> and H<sub>2</sub>S, 89  
 Compaction correction factor, 327  
 Composite capture cross section of formation, 327–328  
 Compound interest, 363  
 Compressibility  
   of coalbed methane formation, 446–447  
   drive in gas reservoirs, 8  
   isothermal, 35  
   of oil, 401  
   total, 56  
   total pore volume, 56–57  
 Compressibility drive index, 8  
 Compressible flow, in horizontal circular tube, 265–266  
 Compton scattering, 266  
 Conductivity of oil, 329  
 Constant-rate flow test  
   permeability, 195  
   skin factor, 195  
 Constant-rate production  
   dimensionless radius, 12  
   dimensionless wellbore storage coefficient, 13–14  
 Contact angle, 403  
 Controlling resistance, 269  
 Convective mass transfer  
   for laminar flow, 212  
   for turbulent flow, 212  
 Corey exponents, 51  
 Correct counterbalance, 213  
 Correction factor  
   for facial tension, 404  
   for foamless separation, 482  
   Hammerlindl, 8  
   for stagnant film, 266  
 Correlation factor for Benedict-Webb-Rubin equation, 482  
 Correlation of mud cake resistivity to mud resistivity, 328  
 Correlation of mud filtrate resistivity to mud resistivity, 328  
 CO<sub>2</sub> solubility, in oil and oil-mud emulsifiers, 88–89  
 Cost depletion, 363  
 Cost per foot  
   coring, 91  
   drilling, 90–91  
 Craft and Hawkins method, 59  
 Critical flow rate  
   critical annular velocity, 91–92  
   for flow regime change, 92  
 Critical pressure, 378–379, 482–483  
 Critical temperature, 379  
 Critical velocity for change in flow regime, 92–93  
 Crossflow index, 9–10  
 Crown block capacity, 93  
 Cumulative effective compressibility, 10  
 Cumulative gas production, 10  
 Cumulative heat injected for steam drive, 417–418  
 Cumulative interest on operational expenses, 364  
 Cumulative oil displacement, 423  
 Cumulative oil production, 11, 425, 442  
 Cumulative pore volume compressibility, 56–57  
 Cumulative production, 182  
 Current drag force, 93  
 Curvature radius of borehole, 94  
 Cuttings  
   bulk density of, using the mud balance, 83–84  
   produced per foot of hole drilled  
     bbls, 94–95  
     lbs, 95  
     slip velocity, 94  
 Cylindrical helical method, 82–83  
 Cylindrical tank draining, 276
- D**  
 Damaged/undamaged zone productivity, 214  
 Darcy Weisbach equation  
   head loss form, 267  
   pressure loss form, 267  
 Dean number, 267–268  
 Deborah number, 268  
 Decay of thermal neutrons, 268  
 De Nouy ring method, 401, 404–406, 412–413  
 Density of brine, 214  
 Depth of carbon dioxide alteration front, 418  
 Depth of washout, 96  
 Derrick efficiency factor, 96–97  
 D exponent, 95  
 Diameter of falling sphere, 269  
 Dietz method, 198  
 Differential hydrostatic pressure, 97–98  
 Diffuse-layer thickness, 328–329  
 Diffusion  
   in colloidal suspensions, 317  
   convection and chemical reaction, 275  
   into a falling liquid film, 271  
   with heterogeneous chemical reaction, 274  
   with homogeneous chemical reaction, 274  
   from instantaneous point source, 270  
   of low-density gases with equal mass, 271  
   in moving film, 270  
   in polymers, 270–271  
   through cleat spacing, 273–274  
   through non-isothermal spherical film, 272  
   through stagnant film, 273  
   through stagnant gas film, 273  
 Diffusion depth, for geothermal well, 180  
 Diffusion potential, 272  
 Diffusivity of liquids, 279–280  
 Dilution water/mud required, 98  
 Dimensionless  
   air injection rate, 419  
   heat injection rate, 418–419  
   length, 183  
   pressure, 11–12, 183–185  
   pressure drop, 186–187  
   radius of radial flow, 12  
   ratio of effective volumetric heat capacity, 419–420  
   vertical well critical rate, 13  
   wellbore storage coefficient, 13–14  
 Dimensionless time  
   Earlougher method, 13  
   Myhill and Stegemeier's method, 12  
   for semi-steady state flow, 420  
   in wet combustion, 420  
 Dip direction, 98  
 Directional curvature, 99  
 Downcomer velocity, 483  
 Downhole operating pressure, 215  
 Downward force of casing, 99  
 Drag coefficient, 275  
 Drag force, 275–276  
 Draining  
   cylindrical tank, 276  
   spherical tank, 276–277  
 Drawdown correlating parameter, 182–183  
 Drill collar capacity, 99–100  
 Drilled gas entry rate, 101  
 Drilling cost per foot, 101–102  
 Drilling mud density, 404  
 Drilling ton miles  
   core operation, 102  
   drilling/connection, 102  
   round trip, 102–103  
   setting casing, 103–104  
   short trip, 103

- Drill pipe/drill collar displacement, 100  
 Drill pipe/drill collar weight, 100  
 Drill pipe, friction factor in, 113  
 Drill pipe length, bottomhole assembly, 100–101  
 Drill-rate model  
     alternative equation, 143  
     basic equation, 143–144  
 Drill string design, 100–101  
 Dual water model, 332–333  
 Duplex pump factor, 104  
 Duplex pump output  
     liner and rod diameters, 105  
     liner diameter, 104  
     rod diameter, 104–105  
 Dykstra-Parsons coefficient, 421  
 Dynamically coupled linear flow, 105–106
- E**  
 Earlougher shape factor, 52–53  
 Eckert number, 277  
 Edge-water drive reservoirs, 8–9  
 Effective compressibility, 14  
 Effective interest rate for periodic compounding, 364  
 Effective oil transmissivity, 421–422  
 Effective photoelectric absorption cross section index, 329  
 Effective porosity, 405  
 Effective thermal conductivity, 379  
 Effective transmissivity, 421  
 Effective weight during drilling, 106  
 Effective wellbore radius  
     for finite-conductivity fractures, 106–107  
     in infinite-conductivity fractures, 107  
     of horizontal well  
         anisotropic reservoirs, 14–15  
         isotropic reservoirs, 15  
         uniform-flux fractures, 16  
         van der Vlis et al. method, 16–17  
 Efficiency, of block and tackle system, 107–108  
 Eflux time, 278  
 Egbogah correlation, 63  
 Einstein equation effective viscosity, 379–380  
 Ekman number, 278–279  
 Electrical power, 483  
 Electric resistance, 330  
 Electrochemical potential, 330  
 Electrokinetic potential, 330–331  
 Electron density index, 331  
 Elimination of circulation, 279  
 Emissivity of hole, 277  
 Energy emitted from surface of black body, 279  
 Energy, of single-stage ideal compressor, 484  
 Energy rate, 481–482  
 Entrance hole size, 215–216  
 Epithermal neutron diffusion coefficient, 331  
 Epithermal neutron distribution, 332  
 Equilibrium vaporization ratio, 380  
 Equivalent atomic H/C ratio, 422  
 Equivalent circulating density, 108–109  
 Equivalent formation water resistivity, 109  
 Equivalent mud weight  
     deviated well, 109–110  
     vertical well, 110  
 Equivalent skin factor, 216
- Equivalent volume of steam injected, 422  
 Equivalent water saturation, 423  
 Error, in thermocouple temperature measurement, 484  
 Error percentage of porosity measurements, 405  
 Euler number, 281  
 Evaporation loss, from an oxygen tank, 380–381  
 Expansion factor, for diffuse layer, 381  
 Expected value, 366  
 Exploration efficiency, 364–365  
 Eykman molecular refraction, 485
- F**  
 Facial tension, 401, 405–406  
 Failure criteria, 447–448  
 Falling-cylinder viscometer  
     velocity profile, 247  
     viscosity by, 320  
 Fanning friction factor  
     laminar flow, 281  
     turbulent flow, 282  
 Fenske's method for minimum theoretical plates, 485  
 Fertl and Hammack equation, 332  
 Fetkovich cumulative effective compressibility, 10  
 Fick's law  
     of binary diffusion, 282  
     mass transfer in acid solutions, 225–226  
 Film condensation  
     on vertical pipes, 282–283  
     on vertical tubes, 283  
 Film thickness, 284  
 Filter cake on the fracture, 216  
 Filtration rate, 406  
 Filtration volume  
     with spurt loss, 407  
     without spurt loss, 406  
 Flat-plate boundary layer model, 381  
 Flow  
     liquid-liquid ejector pump, 284  
     near a corner, 285  
     of power law fluid through a narrow slit, 286  
     slit with uniform cross flow, 285  
 Flow coefficient during drawdown, 217  
 Flow period duration, 191  
 Flow rate through orifice, 217  
 Flow through fracture, 217–218  
 Fluid kinetic force  
     in conduits, 286  
     in flow around submerged objects, 286–287  
 Fluid loss velocity, 216  
 Fluid volume required, 111–112  
 Force  
     applied to stretch material, 112  
     exerted by the fluid on solid surface, 112  
 Formation compressibility, 448  
 Formation conductivity, 332–333  
 Formation factor, 333  
 Formation fluid compressibility, 218  
 Formation invasion  
     dynamically coupled linear flow, 105–106  
     matching conditions at cake-to-rock interface, 131  
     mudcake growth equation, 138
- mudcake permeability, 139  
 Formation permeability, 54  
 Formation pressure with well shut-in on kick, 126  
 Formation resistivity and permeability (Carothers) relation  
     for limestones, 334  
     for sandstones, 334  
 Formation resistivity and porosity relations  
     for carbonate rocks, 334–335  
     from well log data, 335  
 Formation resistivity factor and porosity, relationship between, 337, 340  
 Formation temperature, 17  
 Formation volume factor  
     gas, 22  
     oil  
         Beggs-standing correlation, 41–42  
         Standing's correlation, 41  
     two-phase, 58  
     water, 68–69  
     water two-phase, 70  
 Form drag, 287  
 Fractional flow  
     of water in hot floods, 427  
     Welge extension, 70  
 Fractional flow curve, 5  
 Fraction of heat injected in latent form, 424, 426–427  
 Fraction of injected heat remaining in reservoir, 427  
 Fraction of total porosity occupied by clays, 335  
 Fracture  
     area in time, 218–219  
     coefficient, 219  
     conductivity, 191–192, 448  
     pressure, 450  
 Fracture fluid coefficient  
     for reservoir-controlled liquids, 219  
     for viscosity-controlled liquids, 220  
 Fracture-fluid invasion of formation, 221  
 Fracture geometry, 220  
 Fracture gradient, 220–221  
     Eaton, 448–449  
     Holbrook, 449  
     Matthews and Kelly, 449  
     Zoback and Healy, 449–450  
 Fracture volume  
     GDK method, 450  
     Perkins and Kern method, 451  
 Fracture width  
     GDK method, 451  
     Perkins and Kern method, 451–452  
 Free air correction, 287  
 Free batch expansion of compressible fluid, 288  
 Free convection heat transfer, 288  
 Free gas in place, 18  
 Freezing of spherical drop, 382  
 Frictional pressure drop, 221–222  
 Friction drag, 288–289  
 Friction factor  
     for creeping flow around a sphere, 289  
     in drill pipe, 113  
     in flow around submerged objects, 289  
     in flow through conduits, 290

Friction factor (*Continued*)

in packed column, 290–291

Front displacement of particle in reservoir, 113

Future value

of an annuity, 365

of present sum, 365

Fxo/Fs approach, 336

**G**

Galilei number, 291

Gamma ray log shale index, 336–337

Gas absorption

with rapid reaction, 293

from rising bubbles for creeping flow, 291

Gas bubble radius, 19

Gas capacity of separator, 485–486

Gas cap ratio, 19

Gas cap shrinkage, 20

Gas drive index, 20

Gas expansion factor, 20–21

Gas flow

under laminar viscous conditions, 22

rate into wellbore, 21

Gas formation volume factor, 22

Gas hydrate dissociation pressure, 22–23

Gas in place

free, 18

Hammerlindl method, 26–27

initial, for water-drive gas reservoirs, 32–33

original, 44

remaining, 51

Gas mass rate flow, 293

Gas mass velocity

adsorption unit, 486

separator, 486–487

Gas material balance equation, 23

Gas migration velocity, 114

Gas/mud ratio, 114–115

Gas originally adsorbed, 487

Gas permeability, 407–408

Gas pressure testing time, 487

Gas produced by gas expansion, 23–24

Gas reservoirs

compressibility drive, 8

gas drive index, 20

gas expansion term, 21

water-drive index, 67

water expansion term, 68

Gas saturation, of water-drive gas reservoirs, 23–24

Gas separator

allowable gas velocity, 478

internal diameter, 489–490

Gas solubility, 401–402

coal bed methane reservoirs, 24–25

in mud system, 114

Gas velocity under sonic flow conditions, 222

Geertsma's model, 25

Gel strength

optimal solid removal efficiency, 115

solid control efficiency, 115

solids build-up in system, 116

Geometric coefficient

for electrode, 338

for lateral device, 338

for normal sonde, 338–339

Geothermal gradient, 25

Geothermal well

bottomhole pressure, 417

diffusion depth, 180

flow efficiency, 202

total heat loss, 439

Glycol dehydration unit

still column diameter, 499–500

TEG weight percent, 501

weight of rich TEG, 506

Graetz number, 293–294

Graham equation viscosity ratio, 294

Grashof number, 294–295

Gravitational attraction of layer, 487–488

Growth of steam-heated area, 428

Growth pressure, 465–466

Growth rate of return for continuous

compounding, 366

**H**

Hagen number, 295

Hagen-Poiseuille equation, 26, 295

Hagoort and Hoogstra gas flow, 26

Half thickness value, 339

Hammerlindl correction factor, 8

Havlena and Odeh method, 38, 59

Hawkins method, 242

Head loss due to friction, 267

Heat conduction in cooling fan, 382–383

Heat flux distribution in wall, 383

Heating of liquid in agitated tank, 488

Heat injection rate, 424

Heat loss

by free convection from horizontal pipe, 383–384

of geothermal well, 439

over incremental length of well, 428

Heat ratio of contents, in geothermal reservoir, 428

Heat released during in-situ combustion, 384, 429

Heat remaining in reservoir, 429

Heat transfer

coefficient for condensation, 384

in packed bed, 385

rate in laminar forced convection, 385

Height of cement in annulus, 116

Height of downcomer filling, 488–489

Height of influx, 127

Herschel and Buckley law, 135–136

High-pressure region gas flow rate, 27

Hingle nonlinear-resistivity/linear-porosity

crossplot, 339

Hoek and Brown criteria, 452

Horizontal effective stress, 452

Horizontal maximum stress, 452–453

Horizontal stress, 453

Horizontal wells

breakthrough time, 27–28

critical rate correlations

Chaperon, 28

Efros, 28–29

Giger and Karcher, 29

Joshi method, 29–30

inflow performance relationship, 192–193

pseudo-steady state productivity, 48–50

Horner equation, 196, 202

Hoskold method, 366–367

Hot water heated reservoir, 435

Hot-wire anemometer, 502–503

Hoyland, Papatzacos and Skjaeveland method, 60

Humble equation, 340

Hydraulically fractured wells

flow period duration, 191

lifetime, 225

Hydraulic fracturing

efficiency, 222

fracture gradient, 220–221

injection pressure, 224

maximum treatment pressure, 226

pressure loss due to perforations, 232

Hydraulic horsepower, 116–117, 223

Hydraulics analysis, 117

Hydrocarbon pore volume occupied

by evolved solution gas, 30

by gas cap, 30

by remaining oil, 31

Hydroclone evaluation, 111

Hydromechanical specific energy, 118

Hydrostatic pressure, 31, 119

Hydrostatic pulling, 118

wet pipe out of hole, 119

**I**

Ideal fracture conductivity, 223

Ignition delay time, 430–431

Impact force, 120

Impringing jet, 120–121

Incremental cumulative oil production, 31–32

Incremental density, 223–224

Induced fracture dip, 453

Ineffective porosity, 32

Inflow performance relationship, 192–193

Influx, 125

Inhibitor injection rate, 489

Initial capital needed to survive in minimum chance scenario, 367–368

Initial gas cap, 32

Initial gas in place, 32–33

Initial volume required

with barite, 123

with calcium carbonate, 123

with hematite, 124

Injected air required to burn through unit bulk of reservoir, 431

Injection pressure

hydraulic fracturing, 224

required to open valve, 124

Injection rate, 425–426

Injectivity index, 33

Input power of pump, 124–125

In-situ combustion process

ignition delay time, 430–431

temperature increase with time, 438

Instantaneous gas-oil ratio, 33

- Instrumentation noise control, 489  
 Integrated radial geometric factor, 340  
 Interest rate, 362–363  
 Internal diameter, of gas separator, 489–490  
 Interporosity flow coefficient, 34, 193–194  
 Interstitial velocity, 34, 388  
 Isostacy, 490  
 Isothermal compressibility  
   of limestones, 454  
   of oil, 35  
   of water, 35  
 Isotropic reservoirs  
   effective wellbore radius of a horizontal well, 15  
   vertical well critical rate correlations, 60
- J**  
 Jacoby aromaticity factor, 385–386  
 Jet velocity, 125  
 Joule Thompson expansion, 386
- K**  
 Kamal and Brigham dimensionless pressure, 11–12  
 Kerns method, 35–36  
 Kick analysis  
   formation pressure with well shut-in on a kick, 126  
   height of influx, 127  
   influx, 125  
   maximum pit gain from a gas kick in water-based mud, 126  
   maximum surface pressure from a gas kick in water-based mud, 126–127  
   shut-in drill pipe pressure, 127  
 Kill weight mud determination, 127–128  
 Kinematic viscosity, 408  
 Kinetic friction, 128  
 Klinkenberg gas effect, 36, 407–408  
 Knudsen flow, 296  
 Kozeny-Carman relationship, 36–37, 409  
 Kozeny equation, 36  
 Krieger Dougherty equation viscosity ratio, 296–297
- L**  
 Laminar flow  
   along a flat plate, 297  
   of an incompressible power-law fluid, 297–298  
 Laplace number, 298  
 Laser specific energy, 128  
 Latent heat of hydrocarbon mixture, 386  
 Lateral load imposed on a casing  
   centralizer, 129  
   with dogleg, 129  
 Least principal stress, 454  
 Length of slant wellbore, 133  
 Lennard Jones potential, 340–341  
 Leverett J-function, 37  
 Lewis number, 298  
 Lifetime of hydraulically fractured well, 225  
 Lift coefficient, 490–491
- Linear absorption/attenuation coefficient, 341  
 Linear annular capacity of pipe, 129–130  
 Linear capacity of pipe, 130  
 Linearized Mohr coulomb criteria, 455  
 Linearized Mohr failure line, 455  
 Line-source solution for damaged/stimulated wells, 37–38  
 Liquid permeability, 408–409  
 Logarithmic energy decrement, 342–343  
 Low-gravity solids, 133–134  
 Low-pressure region gas flow rate, 38
- M**  
 Mach number, 298–299  
 Manning formula, 299  
 Marangoni number, 299  
 Mass absorption/attenuation coefficient, 300  
 Mass flow rate, 264, 301, 308  
   circular tube, 302  
   falling film, 302  
   network of tubes, 300  
   rotating cone pump, 300–301  
   squared duct, 301–302  
 Mass flux, 305  
 Mass of fuel burned per unit bulk reservoir, 431–432  
 Mass of rock dissolved per unit mass of acid, 225  
 Mass rate of flow through annulus, 131  
 Mass transfer  
   in acid solutions, 225–226  
   for creeping flow around a gas bubble, 303  
   to drops and bubbles, 303  
 Matching conditions at cake-to-rock interface, 131  
 Material balance for cumulative water influx, 38  
 Maximum allowable mud weight, 131–132  
   subsea considerations, 163–164  
 Maximum allowed tubing pressure, 255  
 Maximum anisotropic failure stress, 456  
 Maximum compression at vertical wellbore, 456–457  
 Maximum drilling rate  
   control drilling, 89–90  
   larger holes, 132  
 Maximum equivalent derrick load, 132  
 Maximum flow rate, 303–304  
 Maximum height of oil column, 39  
 Maximum hoop stress, 457  
 Maximum pit gain, 126  
 Maximum plane tangential stress, 457, 460  
 Maximum potential for self-potential log, 341  
 Maximum principal stress  
   breakout width, 459  
   failure, 457–458  
   normal faulting, 458  
   reverse faulting, 458  
   strike-slip faulting, 459  
 Maximum surface pressure, 126–127  
 Maximum velocity  
   falling film, 304–305  
   flow through a circular tube, 304–305  
 Maximum weight on bit, 134  
 McCain correlation, 64, 68–69
- MDH method  
   average reservoir pressure, 17  
   dimensionless shut-in time, 188  
 Mean free path, 342  
 Mechanical energy balance, for wellbore fluids, 134–135  
 Mechanical resistant torque, 226  
 Mechanical specific energy, 135  
 Membrane potential, 342  
 Meterage model, 368  
 Methylene blue test, 402–403  
 Meyer, Gardner and Pirson method, 60–61  
 Minimum air flux required, 432  
 Minimum compression at vertical wellbore, 459–460  
 Minimum hoop stress, 460  
 Minimum number of jobs to survive in minimum chance scenario, 368–369  
 Minimum polished rod load, 226–227  
 Minimum profit ratio per risky job, 369  
 Minimum theoretical plates, 485  
 Mixing fluids of different densities, 387  
 M modulus, 455–456  
 Modified capillary number, 306  
 Modified Clapeyron criteria, 491–492  
 Modified Cole plot, 39  
 Modified Kozeny-Carman relationship, 39–40  
 Modified laide criterion, 460–461  
 Modified Van Driest equation, 306  
 Modulus of rigidity, 468  
 Molar rate, 296  
 Mole fraction of component  
   in liquid phase, 387  
   in vapor phase, 387–388  
 Momentum flux distribution  
   calculation, 264  
   of flow through a circular tube, 306–307  
   of flow through an annulus, 307  
   of fluids in flow of two adjacent immiscible fluids, 307–308  
 Mooney equation viscosity, 308–309  
 Moore equation, 127–128  
 Mudcake growth equation, 138  
 Mudcake permeability, 139  
 Mud density increase  
   by barite, 121  
   by calcium carbonate, 121  
   by hematite, 121–122  
 Mud rheology  
   Bingham plastic model, 137  
   Herschel and Buckley law, 135–136  
   power-law, 136–137  
   power-law model  
     consistency index, 136  
     power-law index, 136  
 Mud volume increase  
   by barite, 122  
   by calcium carbonate, 122  
   by hematite, 122–123  
 Mud weight, 90  
   increase required, 137  
   reduction by dilution, 137–138  
 Myhill and Stegemeier's method, 12

**N**

Net cash flow, 369  
 Net present value, 369–370  
 Neutron lethargy, 342–343  
 Neutron porosity of shale zone, 343  
 Newman correlation, 454  
 New pump circulating pressure, 139  
 Non-Newtonian flow in annulus, 309  
 Normalized saturation, 40  
 Normal stress  
     in radial direction near wellbore, 461  
     in rock at failure, 461–462  
     in tangential direction at wellbore wall, 462  
     in tangential direction near wellbore, 462–463  
 Nozzle area calculation, 139–140  
 Nozzle hydraulic analysis  
     impact force, 120  
     jet velocity, 125  
 Number of collisions to reduce neutron energy, 259  
 Number of feet to be cemented, 85–86  
 Number of sacks of cement required, 140–141  
 Number of sacks of lead cement required for annulus, 141  
 Number of sacks of tail cement required for casing, 141–142  
 Nusselt number, 309–310

**O**

Odeh correlation, 236  
 Ohnesorge number, 310  
 Oil breakthrough newly swept zone, 432  
 Oil bubble radius, 40  
 Oil density, 41  
 Oil displacement rate, 423–424  
 Oil formation volume factor  
     Beggs-standing correlation, 41–42  
     Standing's correlation, 41  
 Oil in place  
     saturated oil reservoirs, 43  
     undersaturated oil reservoirs without fluid injection, 42–43  
 Oil length in separator, 496–497  
 Oil lost in migration, 43  
 Oil production from in-situ combustion, 440  
 Oil recovery, 432–433  
 Oil residence time, 498  
 Oil saturation, 44, 343  
 Oil solubilization factor, 433  
 Oil-steam ratio, 434  
 Oil volume at breakthrough, 433  
 Open-ended displacement volume of pipe, 142  
 Operating cash income, 370  
 Optimized hydraulics for two and three jets, 81–82  
 Original gas in place, 44  
 Osif correlation, 35  
 Overall efficiency  
     diesel engines to mud pump, 142  
     power system, 143

**P**

Packed column actual height, 492  
 Packed towers, column diameter, 481

**P**air production, 343–344

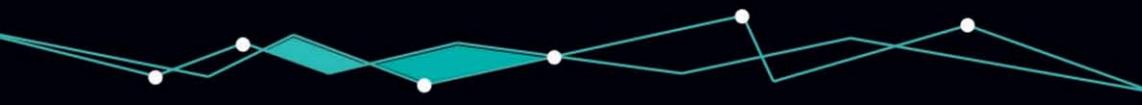
Pan-Maddox equation  
     for density, 492  
     for molecular weight, 492–493  
 Pan-Maddox equation for density, 492  
 Papatzacos correlation, 236–237  
 Payback period, 370  
 Payne method, 44–45  
 Peak polished rod load, 244–245  
 Peclét number for fluid loss, 227  
 Penetration model, 266  
 Penetration rate, 143–144  
 Peng Robinson equation, 388–389  
 Percentage of bit nozzle pressure loss, 144  
 Perforation  
     friction factor, 227  
     friction pressure, 228  
     hole size, 228  
     length in formation, 228–229  
     penetration ratio, 229  
     skin factor, 229  
 Performance coefficient, for shallow gas reservoirs, 45  
 Permeability  
     average  
         linear flow, 3–4  
         parallel-layered systems, 4  
         radial systems, 4–5  
     determination using porosity data, 409  
     formation, 54  
     of hydraulically fractured formation, 209  
 Kozeny equation, 36  
 modified Kozeny-Carman relationship, 39–40  
 relative  
     Corey exponents, 51  
     Torcaso and Wyllie's correlation, 55–56  
     and reservoir pressure, 194–195  
 Permeameter lab measurement, 408–409  
 Phillips equation, 344  
 Photoelectric absorption cross sectional area, 344  
 Photoelectric effect, 493  
 Photon absorption, 342  
 Pickett crossplot, 345  
 Plastic viscosity, 144  
 Plug length, 145  
 Poisson's ratio, 45, 345  
 Polar moment of inertia, 145  
 Polished rod horsepower, 145–146  
 Pore growth function, 230  
 Pore pressure change, 466  
 Pore-pressure gradient  
     Rehm and McClendon, 146  
     Zamora, 146  
 Pore pressure increase  
     due to fluid activity, 463  
     due to fluid activity contrast, 463  
 Pore pressure of shale  
     Flemings, 464  
     Traugott, 464  
 Pore throat sorting, 46  
 Pore volume occupied, 46  
 Porosity  
     corrected for gas effect, 346  
     IES and FDC logs, 47  
     ineffective, 32  
     by using density log data, 345–346  
 Porosity irreversible plastic deformation, 464–465  
 Porosity-neutron flux relationship, 346  
 Porosity/transit time relationships, 25, 197–198, 353–354  
 Pot aquifer model, 69  
 Potential flow around a cylinder, 310  
 Power requirement for pumping, 493  
 Prandtl number, 310–311  
 Present value  
     of an annuity, 370–371  
     of deferred annuity, 371  
     of future sum, 371  
     of profit/investment ratio for an oil well, 372  
     of uniform gradient series, 372  
 Present worth expectation for a risky job, 373  
 Pressure  
     average reservoir, 17  
     capillary, 6  
     in creeping flow around sphere, 311  
     dimensionless, 11–12  
     gas hydrate dissociation, 22–23  
     hydrostatic, 31  
     storage, 494–495  
 Pressure analysis  
     by each barrel of mud in casing, 147  
     surface pressure during drill stem test, 147  
 Pressure buildup equation, 196  
 Pressure build-up test  
     interporosity flow coefficient, 193–194  
     shut-in time, 198  
     skin factor, 199  
     slope of Horner plot, 200  
     slope of pseudo-steady state flow, 200  
     true wellbore storage coefficient, 201  
 Pressure controller, 495  
 Pressure criteria for separator, 494  
 Pressure drop  
     across perforations  
         gas wells, 230–231  
         oil wells, 231  
     due to skin, 205  
 Pressure drop per length, 311–312  
 Pressure gradient, 147  
     between cement and mud, 97  
 Pressure loss  
     due to perforations, 232  
     due to sudden enlargement, 312  
 Pressure required to break circulation, 148  
     annulus, 148  
     drill string, 148  
 Pressure required to overcome gel strength  
     in annulus, 149  
     in drill string, 148–149  
 Principal stress  
     due to petro-static pressure, 232–233  
     failure, 452  
 Probability of an oilfield discovery, 373  
 Produced gas-oil ratio, 47  
 Production  
     constant-rate  
         dimensionless radius of radial flow, 12

- dimensionless wellbore storage coefficient  
of radial flow, 13–14
- cumulative gas, 10
- cumulative oil, 11
- incremental cumulative oil, 31–32
- time, dimensionless, 187
- Productivity index, 233  
for gas well, 48
- Productivity ratio, 233–234
- Profitability index, 373–374
- Proportional band, 495
- Proppant settlement in fracture, 434
- Pseudo-reduced conditions, 389
- Pseudo skin factor due to partial penetration  
Brons and Marting method, 234–235
- Odeh correlation, 236
- Papatzacos correlation, 236–237
- Yeh and Reynolds correlation, 235–236
- Pseudo-skin factor due to perforations, 237–238
- Pseudo-steady state  
productivity, 48–50
- radial flow equation, 50
- Pump calculations  
power required, 149–150
- pressure, 149
- Pump displacement, 150
- Pump flow rate, 150
- Pump head rating, 151
- Pump output gpm, 151
- Pump output triplex pump, 151
- Pump pressure/pump stroke relationship, 152
- Pycnometer volume correction, 409
- Q**
- Quantifying formation damage and improvement, 238
- R**
- Radial flow constant-pressure production, 182
- Radial force related to axial load, 152
- Radial stress around vertical wellbore, 466
- Radiant heat transfer between disks, 389–390
- Radiated energy flux, 390
- Radiation across an annular gap, 390
- Radiation shields, 391
- Radius of investigation  
at the end of injection time, 196
- flow time, 196–197
- shut-in time, 197
- Ramey type curves, 177
- Range of load, 152–153
- Raoult's law in glycol dehydration unit, 495
- Rate  
of advancement of combustion front, 434–435
- dimensionless, 187
- of fuel consumption by pump, 153
- of gas portion that enters the mud, 153
- of growth of heated zone in hot water heated reservoir, 435
- of growth per unit of exploration length, 374
- of oxygen-reacted per unit mass of fuel, 435
- of radioactive decay, 346–347
- Rayleigh number, 391
- Raymer Hunt Gardner method, 351–352
- Raymer Hunt transform, 197–198
- Recovery steam drive, 425
- Redlich-Kwong PVT equation, 391–392
- Reduction in bottomhole pressure, 120
- Refrigerator shaft speed, 496
- Relative centrifugal force, 409–410
- Relative humidity, 496
- Relative permeability, 51, 410
- Relative permeability ratio, 55–56
- Remaining gas in place, 51
- Required separator liquid section, 497
- Reservoir flow  
for gas flow in a formation, 436
- through the wellbore of a geothermal well, 436–437
- Reservoir gas density, 392
- Reservoir permeability, 198
- Reservoir pressure, 17
- Reservoir temperature, in cyclical steam injection process, 416
- Reservoir wettability characterization, 410–411
- Residence time of water in separator, 497–498
- Resistance, 411
- Resistivity, 325, 411  
of partially saturated shaly sand with hydrocarbons, 349
- of water-saturated shaly sand, 349–350
- Resistivity index, 412
- Retained gas volume of total gas, 17–18
- Retention time, 498
- Reynolds number, 312  
for acid flow into the fracture, 239
- for fluid loss, 239–240
- Rise in core method, 410–411
- Roach plot, 52
- Rock conductivity, 350
- Rock expansion term, in abnormally pressurized gas reservoirs, 52
- Rock removal rate, 154
- Rotating horsepower, 154
- Rotation of maximum principal stress, 467
- S**
- Safety relief valves sizing, 499
- Sand weight needed to refill hydraulically fractured reservoir volume, 240
- Saturated oil reservoirs, oil in place, 43
- Saturation of layer under hot water flood, 437
- Saybolt viscosimeter measurements  
absolute viscosity, 399
- kinematic viscosity, 408
- standard discharge time, 413
- Schlumberger thermal decay time tool, 327–328
- Schmidt number, 313
- Seismic arrival time method, 345
- Self diffusivity at high density, 280
- Separator  
gas capacity, 485–486
- pressure criteria, 494
- residence time of water in, 497–498
- Shale compaction, 467–468
- Shale index from gamma ray spectrometry, 350–351
- Shallow gas reservoirs  
deliverability equation, 11
- performance coefficient, 45
- Shape factor  
Earlougher, 52–53
- skin factor, 240
- Shear modulus, 468
- Shear stress near vertical well, 468–469
- Sherwood number, 313
- Shut-in drill pipe pressure, 127
- Shut-in pressure, 202
- Shut-in time  
dimensionless, 188
- minimum, 194
- Side force on bit, 155
- Simandoux equation, 351
- Simple interest, 374–375
- Single-phase gas flow, 240–241
- Single-phase liquid flow through choke, 241
- Single-stage ideal compressor, energy requirement of, 484
- Single well-circular reservoir, 200–201
- Sinusoidal buckling, 155
- Skin, in partially depleted wells, 246–247
- Skin effect, 186
- Skin factor, 241–242  
for deviated well, 243–244
- due to partial penetration, 242–243
- due to reduced crushed-zone permeability, 243
- Hawkins method, 242
- Slant well productivity, 16–17
- Slit flow in Bingham fluid, 313–314
- Slope  
of Horner plot, 200
- of pseudo-steady state flow, 200
- of semilog plot for bottom-hole flowing pressure vs time for drawdown test, 244
- Slowness of formation, 469
- Slug size in polymer floods, 437
- Slurry density for cementing calculations, 155–156
- Smoluchowski equation, 314
- Sobociński and Cornelius water breakthrough correlation, 66
- Solid control efficiency, 115
- Solid content analysis, of drilling muds, 404
- Solid content ratio, of drilling mud, 412
- Solid removal efficiency, 115
- Solids analysis  
high-salt content muds, 156
- low-salt content muds, 157
- Solids fractions, 134
- Solution gas-drive reservoirs, 192–193
- Solution gas oil ratio  
Beggs-standing correlation, 53
- Standing's correlation, 53
- Solution gas water ratio, 54
- Somerton method for formation permeability, 54
- Sonic porosity, 351–352
- Sorption compressibility, 325–326
- Spacer volume, 157–158

- Specific gravity  
of air, 412–413  
of cuttings by using mud balance, 158  
of gas hydrate forming components, 54
- Spherical tank draining, 276–277
- Spotting pills, 86–87
- Standard discharge time, 413
- Standing's correlation, 41, 62
- Stanton number, 314–315
- Starting volume of liquid, 170
- Static self potential, 352
- $R_w$   
NaCl predominant, 348  
non-ideal shale membrane, 348  
water containing salts, 348–349
- Steady-state five-spot injection rate, 425–426
- Steady-state temperature controller, 499
- Steam-drive reservoirs, performance prediction, 425
- Steam zone growth, 442
- Stefan-Boltzmann law, 392
- Stefan number, 315
- Stiles water cut, 67
- Still column diameter, 499–500
- Stokes number, 315
- Storage constant  
for gases, 188  
for liquids, 188–189
- Storativity of fractures, 469–470
- Stress  
at edge of wellbore breakout, 470  
individual grains, 447  
pore pressure, 447
- Stress components  
near normal faulting in reservoir, 470–471  
in original coordinate system in depletion drive, 471
- Stress intensity at tip of mode I fracture, 471–472
- Stress path, 472
- Stress perturbation, 472–473
- Stripping factor, 500
- Stripping/snubbing  
breakover point, 158–159  
constant BHP with a gas bubble rising, 161  
height gain and casing pressure, 159  
maximum allowable surface pressure governed by casing burst pressure, 159–160  
maximum allowable surface pressure governed by formation, 160  
minimum surface pressure before stripping, 160
- Stroke per minute required, 161
- Strouhal number, 315–316
- Stuck pipe calculations, 161–162
- Subsea considerations  
adjusting choke line pressure loss for higher mud weight, 162  
casing burst pressure-subsea stack, 162–163  
casing pressure decrease when bringing well on choke, 164  
choke line pressure loss, 163  
maximum allowable mud weight, 163–164  
velocity through choke line, 164
- Subsidence, 473
- Sucker rod, 244–245
- Sucker-rod pump  
average downstroke load, 208  
average upstroke load, 210  
correct counterbalance, 213  
minimum polished rod load, 226–227  
range of load, 152–153
- Surface injection pressure, 226
- Surface pressure during drill stem test, 147
- Surface temperature of heating coil, 393
- Surface tension from density, 500
- Surface test pressure, 164–165
- Suspension property of static fluids, 245
- T**
- Tarner's method, 10
- Tarnishing of metal surfaces, 501
- Taylor dispersion, 316
- Taylor equation viscosity, 316
- Taylor number, 317
- Temperature  
after refrigeration, 501–502  
at choke outlet, 246  
in hot-wire anemometer, 502–503  
of producing geothermal well, 438  
of single-phase liquid/gas injected geothermal well, 439
- Temperature dependent source rate for flow reactor, 502
- Temperature increase  
due to forced convection, 393–394  
due to free convection, 393  
with time, 438
- Temperature profile  
after viscous heat transfer, 394  
with nuclear heat source, 394–395
- Terminal velocity, 503
- Thermal conductivity, 382  
for liquids, 395–396  
for polyatomic gases, 395  
for pure metals, 395  
for solids with gas pockets, 396  
for solid with spherical inclusions, 277–278
- Thermal diffusivity, 396
- Thermal energy, of fissionable substance, 397
- Thermoelastic effect on stress, 397
- Thickness of spherical shells, 503–504
- Tight gas reservoirs  
communication between compartments in, 7  
communication factor in a compartment, 7–8  
Hagoort and Hoogstra gas flow, 26  
Payne method for intercompartmental flow, 44–45  
pore volume through squared method, 46–47
- Time-average relation  
in compacted formations, 353  
in uncompact formations, 353–354
- Time between initiation of pulse and first arrival acoustic energy, 353
- Time, dimensionless, 189–190
- Time function, dimensionless, 190
- Time in hot water floods, 436
- Time to end of infinite-acting period, 55
- Time to reach semi-steady state, 55
- Ton miles  
coreing operation, 102  
drilling/connection, 102  
round trip, 102–103  
setting casing, 103–104  
short trip, 103
- Top distillation column rate, 504
- Torcaso and Wyllie's correlation, 55–56
- Toricelli equation, 317–318
- Torque exerted, 245
- Tortuosity, 354
- Total compressibility, 56
- Total expected additional production discovery, 375  
constant production per unit area, 375
- Total force of fluid, 318
- Total gamma ray, 347
- Total heat energy consumed by engine, 165
- Total new production area, 376
- Total number of sacks of tail cement required, 166
- Total porosity, 413
- Total rock conductivity, 354–355
- Total shale equation, 351
- Total water requirement per sack of cement, 166
- Transmissibility  
of compartment, 57  
between compartments, 57
- Transmissivity, 57–58
- Transmitter-to-receiver spacing, 352
- Trapped gas volume in water-invaded zones, 58
- Traveling block velocity, 154
- Tray type tower  
downcomer velocity, 478, 483  
vapor mass velocity, 504
- Triplex pump factor, 166
- True porosity, 355
- True resistivity, 355
- True wellbore storage coefficient, 201
- Two-phase formation volume factor, 58
- U**
- Ubbelohde viscosimeter measurements, 400
- Unconfined compressive strength of rock, 473
- Underbalanced environment for perforation, 239
- Underground fluid withdrawal, 59
- Undersaturated oil reservoirs  
cumulative oil production, 11  
effective compressibility, 14  
incremental oil production, 31–32  
oil in place without fluid injection, 42–43
- Unsteady evaporation of liquid, 397–398
- Upward force, 167
- USBM wettability index, 413–414
- V**
- Van Der Waals equation, 398, 482–483
- van Everdingen and Hurst unsteady-state model, 69
- Vapor mass velocity, 504

- Vasquez/Beggs correlation, 63–64
- Velocity**
- of bulk compressional waves, 474
  - of compression waves, 474–475
  - of falling film with variable viscosity, 259
  - of flow through a circular tube, 260
  - of flow through an annulus, 260
  - over cross section of falling film, 261
  - of shear waves, 474–475
  - of two adjacent immiscible fluids, 261
- Velocity distribution**
- in the annular slit of a falling-cylinder viscometer, 247
  - in creeping flow around a sphere, 318
  - of falling film with variable viscosity, 319
  - flow through an annulus, 248
  - of flow through circular tube, 319
  - of fluids in flow of two adjacent immiscible fluids, 319–320
  - narrow annular region in annular flow with inner cylinder moving axially, 247–248
- Velocity of fluid**
- in annulus, 248–249
  - in pipe, 249
- Vertical curvature for deviated wells, 167
- Vertical wells
- critical rate correlations
    - Craft and Hawkins method, 59
    - gas coning, 61
    - isotropic reservoirs, 60
    - simultaneous gas and water coning, 60
    - water coning, 61
    - skin, 199
    - water breakthrough correlation, 65–66
- Villena-Lanzi correlation, 35
- Viscosity, 62
- Viscosity**
- crude oil, 62
  - dead oil, 62–63
  - by falling-cylinder viscometer, 320
  - live oil, 63
  - of oil, 63–64
  - of pure liquid, 280–281
  - water at atmospheric pressure, 64
  - water at reservoir pressure, 64
- Viscous force, 249
- Viscous shear stress at outer mudcake boundary, 167–168
- Vogel's equation, 303–304
- Volume
- of cuttings, 168
  - of dilution water/mud required, 168
  - of gas adsorbed, 64–65
  - of reservoir burned, 441
  - of reservoir burnt, 441
  - of slurry per sack of cement, 170
  - of steam injection, 426
  - of water diluting the original brine, 251
- Volume capacity of pipe, 250
- Volume of fluid displaced
- for duplex pumps, 168–169
  - for single-acting pump, 169
  - for triplex pump, 169–170
- Volume of fluid loss per unit area
- dynamic test, 250
  - static test, 250–251
- Volumes and strokes
- annular volume, 171
  - drill string volume, 171
  - total strokes, 171–172
- Volumetric heat capacity, 65, 441–442
- Volumetric photoelectric absorption cross section, 355–356
- W**
- Wall thickness criteria, 505
- Water breakthrough correlation, 65–66
- Water breakthrough time, 430
- Water content of sour gas, 66–67
- Water cut, 67
- Water-drive gas reservoirs
- gas saturation, 23–24
  - initial gas in place, 32–33
- Water-drive index, 67
- Water-drive recovery, 68
- Water expansion term, 68
- Water-filled porosity, 336
- Water formation volume factor, 68–69
- Water influx, 69
- Water influx constant, 69
- Water length in separator, 497
- Water loading, in adsorption unit, 505–506
- Water production from in-situ combustion, 440–441
- Water salinity index ratio, 356
- Water saturation
- determination, 356
  - from neutron tools, 356–357
  - from resistivity logs, 337, 357
  - rock resistivity and, 347
- Water two-phase formation volume factor, 70
- Wavelength equation, 357
- Waxman and Smits model, 70
- Weight**
- of additive per sack of cement, 172
  - of crystalline  $\text{CaCl}_2$  and  $\text{CaBr}_2$  salt addition to brine, 252
- Weighted cementing calculations, 172
- Weight flow through valve, 499
- Welge extension, 70
- Well**
- bottomhole flowing pressure, 178
  - diffusion depth, 180
  - flow efficiency, 202
  - flowing pressure, 252–253
  - flow test with smoothly varying rates, 174
  - PI test, 176–177
  - post-fracture
    - constant-rate flow test, 174–175
    - pressure buildup test, 175–176
  - shut-in pressure, 202
  - storage coefficient, 201
  - time to reach, 201
    - pseudo-steady state, 200–201
    - semi-steady state, 201
- Wellbore electric voltage generation, 358
- Wellbore pressure loss due to friction
- laminar flow, 253
  - turbulence flow, 254
- Wellbore storage coefficient, 254
- dimensionless, 191
  - due to fluid level, 254–255
- Wellhead pressure, 255
- Wet combustion design, 442
- Wien displacement law, 398
- Winsauer equation, 320–321
- Wobbe index, 506
- Work done by expansion tube refrigerator, 506–507
- Workover operations, 255
- Y**
- Yeh and Reynolds correlation, 235–236
- Yield of clays, 414
- Yield strength, 476
- Young modulus, 256
- Z**
- z component of force, 246

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