# Description Logics State Features for Planning (dlplan)

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January 25, 2022

## 1 Introduction

This document aims at providing a better understanding of the different types of elements and different elements itself that can be used with this library. There are four different types of elements: concepts, roles, boolean, and numericals. The main difference is that each type returns different types of objects when evaluated on a given planning state. Evaluating a concept returns a set of unary relations over objects, evaluating a role returns a set of binary relations over objects, evaluating a boolean returns a Boolean, and evaluating a numerical returns a natural number. The following section shows how elements can be composed to obtain more complex elements.

## 2 Available Elements

In this section, we describe the elements that are available to construct more complex elements. We make use of description logics [1] and include Boolean and numerical elements as described in [2].

Consider concepts $C, D$, roles $R, S, T$, the universe $\Delta$ containing all objects, and $X$ to be either a concept or a role. Also consider some predicate $p(c_0, \ldots, c_{n-1})$ with arity $n$, integers $0 \leq i, j \leq n-1$, integer $k \in \{0, 1\}$, and some constant $a$ that occurs in every instance.

## 2.1 Syntax Overview

### 2.1.1 Concepts

| Name | Abstract syntax | Concrete syntax |
|---|---|---|
| Primitive concept | $p[i]$ | c_primitive(p,i) |
| Top | $\top$ | c_top |
| Bottom | $\bot$ | c_bot |
| Intersection | $C \sqcap D$ | c_and(C, D) |
| Union | $C \sqcup D$ | c_or(C, D) |
| Negation | $\neg C$ | c_not(C) |
| Difference | $C \setminus D$ | c_diff(C, D) |
| Value restriction | $\forall R.C$ | c_all(R, C) |
| Existential quantification | $\exists R.C$ | c_some(R, C) |
| Role-value-map | $R \subseteq S$ | c_subset(R, S) |
| | $R = S$ | c_equal(R, S) |
| One-of | $a$ | c_one_of(a) |
| Projection | $R[k]$ | c_projection(R, k) |

Table 1: Concepts

### 2.1.2 Roles

| Name | Abstract syntax | Concrete syntax |
|---|---|---|
| Primitive role | $p[i, j]$ | r_primitive(p,i,j) |
| Universal role | $\top$ | r_top |
| Intersection | $R \sqcap S$ | r_and(R, S) |
| Union | $R \sqcup S$ | r_or(R, S) |
| Complement | $\neg R$ | r_not(R) |
| Difference | $R \setminus S$ | r_diff(R, S) |
| Inverse | $R^{-1}$ | r_inverse(R) |
| Composition | $R \circ S$ | r_compose(R, S) |
| Transitive closure | $R^{+}$ | r_transitive_closure(R) |
| Transitive reflexive closure | $R^{*}$ | r_transitive_reflexive_closure(R) |
| Role restriction | $R\|_{C}$ | r_restrict(R, C) |
| Identity | $id(C)$ | r_identity(C) |

Table 2: Roles

### 2.1.3 Booleans

| Name | Abstract syntax | Concrete syntax |
|---|---|---|
| Empty | $\mathit{Empty}(X)$ | b_empty(X) |
| Concept inclusion | $C \sqsubseteq D$ | b_inclusion(C, D) |
| Role inclusion | $R \sqsubseteq S$ | b_inclusion(R, S) |
| Nullary | $\mathit{Nullary}(p)$ | b_nullary(p) |

Table 3: Booleans

### 2.1.4 Numericals

| Name | Abstract syntax | Concrete syntax |
|---|---|---|
| Count | $\mathit{Count}(X)$ | n_count(X) |
| Concept distance | $\mathit{ConceptDistance}(C, R, D)$ | n_concept_distance(C, R, D) |
| Sum concept distance | $\mathit{SumConceptDistance}(C, R, D)$ | n_sum_concept_distance(C, R, D) |
| Role distance | $\mathit{RoleDistance}(R, S, T)$ | n_role_distance(R, S, T) |
| Sum role distance | $\mathit{SumRoleDistance}(R, S, T)$ | n_sum_role_distance(R, S, T) |

Table 4: Numericals

## 2.2 Semantics

Our *interpretation* is a state $s$ that consists of ground atoms over a set of predicates.

### 2.2.1 Concepts

- $(p[i])^s = \{c_i \in \Delta \mid p(c_0, \ldots, c_i, \ldots, c_{arity(p)}) \in s\}$
- $\top^s = \Delta$
- $\bot^s = \emptyset$
- $(C \sqcap D)^s = C^s \cap D^s$
- $(C \sqcup D)^s = C^s \cup D^s$
- $(\neg C)^s = \Delta \setminus C^s$
- $(C \setminus D)^s = (C^s \setminus D^s)$
- $(\forall R.C)^s = \{a \mid \forall b : (a, b) \in R^s \rightarrow b \in C^s\}$
- $(\exists R.C)^s = \{a \mid \exists b : (a, b) \in R^s \wedge b \in C^s\}$
- $(R \subseteq S)^s = \{a \mid \forall b : (a, b) \in R^s \rightarrow (a, b) \in S^s\}$
- $(R = S)^s = \{a \mid \forall b : (a, b) \in R^s \leftrightarrow (a, b) \in S^s\}$

- $a^s = \{a\}$
- $(R[k])^s = \begin{cases} (\exists R.\top)^s & \text{if } k = 0 \\ (\exists R^{-1}.\top)^s & \text{if } k = 1 \end{cases}$

### 2.2.2 Roles

- $(p[i,j])^s = \{(c_i, c_j) \in \Delta \times \Delta \mid p(c_0, \ldots, c_i, \ldots, c_j, \ldots, c_{arity(p)}) \in s\}$
- $\top^s = \Delta \times \Delta$
- $(R \sqcap S)^s = R^s \cap S^s$
- $(R \sqcup S)^s = R^s \cup S^s$
- $(\neg R)^s = \top^s \setminus R^s$
- $(R \setminus S)^s = (R^s \setminus S^s)$
- $(R^{-1})^s = \{(b,a) \in \Delta \times \Delta \mid (a,b) \in R^s\}$
- $(R \circ S)^s = \{(a,c) \in \Delta \times \Delta \mid (a,b) \in R^s \wedge (b,c) \in S^s\}$
- $(R^+)^s = \bigcup_{n \geq 1} (R^s)^n$
- $(R^*)^s = \bigcup_{n \geq 0} (R^s)^n$
- $(R|_C)^s = R^s \sqcap (\Delta \times C^s)$
- $(id(C))^s = \{(d,d) \mid d \in C^s\}$

where the iterated composition $(R^s)^n$ is inductively defined as $(R^s)^0 = \{(d,d) \mid d \in \Delta\}$ and $(R^s)^{n+1} = (R^s)^n \circ R^s$.

### 2.2.3 Booleans

- $Empty(X)^s$ is true iff $|X^s| = 0$
- $(C \sqsubseteq D)^s$ is true iff $C^s \subseteq D^s$
- $(R \sqsubseteq S)^s$ is true iff $R^s \subseteq S^s$
- $Nullary(p)^s$ is true iff $p() \in s$

### 2.2.4 Numericals

- $\mathit{Count}(X)^s \equiv |X^s|$
- $\mathit{ConceptDistance}(C, R, D)^s$ is the smallest $n \in \mathbb{N}_0$ s.t. there are objects $x_0, \ldots, x_n$ with $x_0 \in C^s$, $x_n \in D^s$ and $(x_i, x_{i+1}) \in R^s$ for $i = 0, \ldots, n-1$. Special cases: If $C^s$ is empty or no such $n$ exists then it evaluates to $\infty$.
- $\mathit{SumConceptDistance}(C, R, D)^s := \sum_{x \in C^s} \mathit{ConceptDistance}(\{x\}, R, D)^s$ where the sum evaluates to $\infty$ if any term is $\infty$ or the sum is empty.
- $\mathit{RoleDistance}(R, S, T)^s$ is the smallest $n \in \mathbb{N}_0$ s.t. there are objects $x_0, \ldots, x_n$, there exists some $(a, x_0) \in R^s$, $(a, x_n) \in T^s$, and $(x_i, x_{i+1}) \in S^s$ for $i = 0, \ldots, n-1$. Special cases: If $R^s$ is emtpy or no such $n$ exists then it evaluates to $\infty$.
- $\mathit{SumRoleDistance}(R, S, T)^s := \sum_{r \in R^s} \mathit{RoleDistance}(\{r\}, S, T)^s$, where the sum evaluates to $\infty$ if any term is $\infty$ or the sum is empty.

# 3 Feature Generation

In the feature generation, we place the following additional restrictions on the above elements in order to decrease the combinatorial blowup.

- r_restrict(R, C), r_inverse(R), r_transitive_closure(R), c_equal(R, S), r_transitive_reflexive_closure(R) where R, S are a primitive roles and C is a primitive concept
- $\mathit{ConceptDistance}(C, R, D)$ where R is restricted to complexity at most 2

# References


[1] Franz Baader, Diego Calvanese, Deborah L. McGuinness, Daniele Nardi, and Peter F. Patel-Schneider, editors. *The Description Logic Handbook: Theory, Implementation and Applications.* Cambridge University Press, 2003.

[2] Dominik Drexler, Jendrik Seipp, and Hector Geffner. Expressing and exploiting the common subgoal structure of classical planning domains using sketches: Extended version. arXiv:2105.04250 [cs.AI], 2021.