

Math Sec 3.1

Rex McArthur
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Exercise. 3.1

(i)

$$\begin{aligned}\langle \mathbf{x}, \mathbf{y} \rangle &= \frac{1}{2}(\langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{x}, \mathbf{y} \rangle) \\ &= \frac{1}{2}(\frac{1}{2}(\langle \mathbf{x}, \mathbf{x} \rangle + 2\langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{y}, \mathbf{y} \rangle - (\langle \mathbf{x}, \mathbf{x} \rangle - 2\langle \mathbf{x}, \mathbf{y} \rangle - \langle \mathbf{y}, \mathbf{y} \rangle))) \\ &= \frac{1}{4}(\langle \mathbf{x} + \mathbf{y}, \mathbf{x} + \mathbf{y} \rangle - \langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle) \\ &= \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)\end{aligned}$$

(ii)

$$\begin{aligned}\|x\|^2 + \|y\|^2 &= \frac{1}{2}(\langle \mathbf{x}, \mathbf{x} \rangle + \langle \mathbf{x}, \mathbf{y} \rangle) \\ &= \frac{1}{2}(\langle \mathbf{x}, \mathbf{x} \rangle + \langle \mathbf{y}, \mathbf{y} \rangle \langle \mathbf{x}, \mathbf{x} \rangle + \langle \mathbf{y}, \mathbf{y} \rangle) \\ &= \frac{1}{2}(\frac{1}{2}(\langle \mathbf{x}, \mathbf{x} \rangle + 2\langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{y}, \mathbf{y} \rangle - (\langle \mathbf{x}, \mathbf{y} \rangle - \langle \mathbf{y}, \mathbf{y} \rangle))) \\ &= \frac{1}{4}(\langle \mathbf{x} + \mathbf{y}, \mathbf{x} + \mathbf{y} \rangle - \langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle) \\ &= \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)\end{aligned}$$