

Math Sec 1.6

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Math 320

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Exercise. 1.19

(i)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{N}(A) = \text{span} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Thus,

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(ii) Proof: Given an integer $a = \sum_{k=0}^{n-1} a_k 10^k = a_0 10^0 + \cdots + a_{n-1} 10^{n-1}$, Note that

$$\begin{aligned} a &= \sum a_k 10^k \\ &= a_0 + \cdots + a_{n-1} 10^{n-1} \\ &= (9a_1 + 99a_2 + \cdots + (10^{n-1} - 1)a_{n-1}) + (a_0 + a_1 + \cdots + a_{n-1}) \\ &= 9(1a_1 + 11a_2 + \cdots + (10^{n-1} - 1)/3a_{n-1}) + (a_0 + \cdots + a_{n-1}) \\ &= 9(1a_1 + 11a_2 + \cdots + (10^{n-1} - 1)/3a_{n-1}) + \sum_{k=0}^{n-1} a_k \end{aligned}$$

Note that $9|a$ if and only if $9|\sum_{k=0}^{n-1} a_k$, by definition of divisibility, because it is apparent that 9 divides the first term.

(iii) Proof: Given an integer $a = \sum_{k=0}^{n-1} a_k 10^k = a_0 10^0 + \cdots + a_{n-1} 10^{n-1}$, Note that

$$\begin{aligned}
a &= \sum_{k=0}^{n-1} a_k 10^k \\
&= a_0 10^0 + a_1 10^1 + \cdots + a_{n-1} 10^{n-1} \\
&= (11a_1 + 99a_2 + 1001a_3 + \cdots + (10^{n-1} - 1)a_{n-1}) + (a_0 - a_1 + a_2 - \cdots + (-1)^{n-1}a_{n-1}) \\
&\quad \text{(Where the coefficients of } a_i^{th} \text{ term given by } 10^i - (-1)^{i-1}) \\
&= 11(a_1 + 9a_2 + 91a_3 + 909a_4 \cdots + ((10^{n-1} - 1)/11)a_{n-1}) + (a_0 - a_1 + a_2 \cdots + (-1)^{n-1}a_{n-1})
\end{aligned}$$

Note that a is divisible by 11 if and only if the second expression is divisible by 11, by definition of divisibility, since the entire first part is divisible by 11.

Exercise. 1.20