

# Math 320

## Sec 3.2

Rex McArthur

October 15, 2015

### Exercise 1. 3.7

To prove that the probability space has a probability of 1, we note that  $p'$  is the probability metric on  $\mathcal{F}'$  such that  $p'(F) = 1$ , since  $\mathcal{F}'$  is the power set of  $F$ . In order to prove additivity, we note  $\Omega$  fulfilled this when it was our sample space. Since  $E' \subset E$ , we know that the same condition will apply when  $F$  is our new sample space.

### Exercise 2. 3.8

By DeMorgan's Law and definition 3.2.3:

$$P\left(\bigcup_{k=1}^n E_k\right) = 1 - P\left(\left(\bigcup_{k=1}^n E_k\right)^c\right) = 1 - P\left(\bigcap_{k=1}^n E_k^c\right) = 1 - \prod_{k=1}^n P(E_k^c) = 1 - \prod_{i=1}^n (1 - P(E_k))$$

### Exercise 3. 3.9

Using Bayes formula, we have that for .004 prevelance, .95 sensitivity, and .95 specificity, probability is .070896

Using Bayes formula, we have that for .004 prevelance, .95 sensitivity, and .90 specificity, probability is .03675

Using Bayes formula, we have that for .004 prevelance, .95 sensitivity, and .999 specificity, probability is .79233

Using Bayes formula, we have that for .004 prevelance, .90 sensitivity, and .95 specificity, probability is .07116

Using Bayes formula, we have that for .004 prevelance, .999 sensitivity, and .95 specificity, probability is .07063

Using Bayes formula, we have that for .001 prevelance, .95 sensitivity, and .95 specificity, probability is .01866

Using Bayes formula, we have that for .05 prevelance, .95 sensitivity, and .95 specificity, probability is .5

### Exercise 4. 3.10

Let R be the probability the witness was correct, and W the probability the witness

was wrong, we find that:

$$\begin{aligned}
 P(\text{Blue}) &= P(R|\text{Blue}) - P(W|\text{Red}) \\
 &= \frac{P(\text{Blue}|R)P(R)}{P(\text{Blue}|W)P(W) + P(\text{Blue}|R)P(R)} - \frac{P(\text{Red}|W)P(W)}{P(\text{Red}|W)P(W) + P(\text{Red}|R)P(R)} \\
 &= \frac{(.1)(.8)}{(.1)(.2) + (.1)(.8)} - \frac{(.9)(.2)}{(.9)(.2) + (.9)(.1)} = .133333
 \end{aligned}$$

**Exercise 5. 3.11**

We want to find the probability that the DNA is his, given that he tested positive. So we use Bayes rule.

Note,

$P(\text{Test positive}|\text{DNA his}): 1$

$P(\text{Test Positive}|\text{DNA NOT his}): \frac{1}{3 \text{ million}}$

$P(\text{DNA his}): \frac{1}{250 \text{ million}}$

$P(\text{DNA not his}): 1 - \frac{1}{250 \text{ million}}$

**Exercise 6. 3.12**

For three doors 1, 2, and 3 the probability that the car is behind one of them in particular is obviously .333, and the probability that goat is behind one is .667. Without Loss of Generality, suppose we pick door number 1. Thus, there is .333 chance that the door picked has the car, and a .667 chance that the other doors have the car. Now, Monty opens a door with a goat, and you know that the other remaining door still has a .667 chance of having the car. Thus, if you switch to that door, your probability increases from .333, to .667.

Similarly, if you have 10 doors, and you pick door number one, your chance of getting the car is .100, The chances that the car is behind any of the other doors is obviously .900. Now, if Monty kindly opens up 8 of the doors with it, there are only two left. Your door, and the door that you didn't pick, which now has a .900 chance of having the car. I think you should switch.