Math Sec 1.6

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Exercise. 2.39 The 7 inversions are

$$(4,3),(4,2),(3,2),(6,5),(9,8),(9,7),(8,7)$$

Exercise. 2.40 By Thm. 2.7.22,

$$\det(A) = \det(A^T) \implies \det(\bar{A}) = \det(\bar{A}^T) = \det(A^H)$$

Thus, it is sufficent to show that $\det(\bar{A}) = \overline{\det(A)}$

$$\det(\overline{A}) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma)\overline{a_{1\sigma(1)}}\,\overline{a_{2\sigma(2)}}\cdots\overline{a_{n\sigma(n)}}$$

Note, $\operatorname{sign}(\sigma) \in \mathbb{R}$, thus the conjugate is equal to itself.

$$\det(\overline{A}) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma)\overline{a_{1\sigma(1)}}\,\overline{a_{2\sigma(2)}}\cdots\overline{a_{n\sigma(n)}} = \sum_{\sigma \in S_n} \overline{\operatorname{sign}(\sigma)}\overline{a_{1\sigma(1)}}\,\overline{a_{2\sigma(2)}}\cdots\overline{a_{n\sigma(n)}} = \overline{\det(A)}$$

$$\implies \overline{\det(A)} = \det(\bar{A}) = \det(\bar{A}^T) = \det(A^H)$$

Exercise. 2.41

$$(1,2,3,4),(1,2,4,3),(1,3,4,2),(1,3,2,4),(1,4,3,2),(1,4,2,3)$$

$$(2,1,3,4),(2,1,4,3),(2,3,1,4),(2,3,4,1),(2,4,3,1),(2,4,1,3)$$

$$(3,1,2,4),(3,1,4,2),(3,2,1,4),(3,2,4,1),(3,4,2,1),(3,4,1,2)$$

$$(4,1,2,3),(4,1,3,2),(4,2,1,3),(4,2,3,1),(4,3,2,1),(4,3,1,2)$$

Exercise. 2.42

$$\begin{aligned}
\det(A) = & a_{11}a_{22}a_{33}a_{44}+ \\
& -a_{11}a_{22}a_{34}a_{43}+ \\
& a_{11}a_{23}a_{34}a_{42}+ \\
& -a_{11}a_{23}a_{32}a_{44}+ \\
& a_{11}a_{24}a_{32}a_{43}+ \\
& -a_{11}a_{24}a_{33}a_{42}+ \\
\\
& a_{12}a_{21}a_{34}a_{43}+ \\
& -a_{12}a_{21}a_{33}a_{44}+ \\
& -a_{12}a_{23}a_{34}a_{41}+ \\
& a_{12}a_{23}a_{31}a_{44}+ \\
& -a_{12}a_{24}a_{33}a_{41}+ \\
& a_{12}a_{24}a_{31}a_{43}+ \\
\\
& a_{13}a_{21}a_{32}a_{44}+ \\
& -a_{13}a_{21}a_{34}a_{42}+ \\
& -a_{13}a_{22}a_{31}a_{44}+ \\
& a_{13}a_{22}a_{34}a_{41}+ \\
& -a_{13}a_{24}a_{32}a_{41}+ \\
& a_{13}a_{24}a_{31}a_{42}+ \\
\\
& -a_{14}a_{21}a_{32}a_{43}+ \\
& a_{14}a_{21}a_{33}a_{42}+ \\
& a_{14}a_{22}a_{31}a_{43}+ \\
& -a_{14}a_{22}a_{33}a_{41}+ \\
& a_{14}a_{23}a_{32}a_{41}+ \\
& -a_{14}a_{23}a_{31}a_{42}+
\end{aligned}$$

$$
\begin{aligned}
&= 0 \cdot 0 \cdot 7 \cdot 0 + \\
&- 0 \cdot 0 \cdot 1 \cdot 9 + \\
&0 \cdot 0 \cdot 1 \cdot 0 + \\
&- 0 \cdot 0 \cdot 6 \cdot 0 + \\
&0 \cdot 5 \cdot 6 \cdot 9 + \\
&- 0 \cdot 5 \cdot 7 \cdot 0 + \\
\\
&2 \cdot 4 \cdot 1 \cdot 9 + \\
&- 2 \cdot 4 \cdot 7 \cdot 0 + \\
&- 2 \cdot 0 \cdot 1 \cdot 8 + \\
&2 \cdot 0 \cdot 0 \cdot 0 + \\
&2 \cdot 5 \cdot 7 \cdot 8 + \\
&2 \cdot 5 \cdot 0 \cdot 9 + \\
\\
&3 \cdot 4 \cdot 6 \cdot 0 + \\
&- 3 \cdot 4 \cdot 1 \cdot 0 + \\
&- 3 \cdot 0 \cdot 0 \cdot 0 + \\
&3 \cdot 0 \cdot 1 \cdot 8 + \\
&- 3 \cdot 5 \cdot 6 \cdot 8 + \\
&3 \cdot 5 \cdot 0 \cdot 0 + \\
\\
&- 0 \cdot 4 \cdot 6 \cdot 9 + \\
&0 \cdot 4 \cdot 7 \cdot 0 + \\
&0 \cdot 0 \cdot 0 \cdot 9 + \\
&- 0 \cdot 0 \cdot 7 \cdot 8 + \\
&0 \cdot 0 \cdot 6 \cdot 8 + \\
&- 0 \cdot 0 \cdot 0 \cdot 0 + \\
\\
&= -88
\end{aligned}
$$

Exercise. 2.43 Proof: Suppose $A_{ij} = 0 \ \forall i$

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)}$$

Thus, for everysingle elementary product with i, it will equal zero. And since the determinent is equal to the summation of all elemantary products, the sum of j zeros is zeros. Supposing for a column of zeros is identical. Therefore,

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) 0 \cdot 0 \cdot \cdots \cdot 0 = 0$$