Math Sec 3.3

Rex McArthur
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Exercise. 3.12
The Gram Schmidt would yield zero vectors, because they are linearly dependent, and are just linear combinations of one another.

Exercise. 3.13
Let $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Thus, the normalized x_1 is $v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ Now applying the Gram-Schmidt, we find that

$$\mathbf{p}_1 = \mathrm{proj}_{v_1}(x_2) = \langle \mathbf{v_1}, \mathbf{x_2} \rangle \mathbf{v_1} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \tag{1}$$

Thus,

$$q_2 = \frac{\mathbf{x_2} - \mathbf{p_1}}{\|\mathbf{x_2} - \mathbf{p_1}\|} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

The set of orothonomral vectors are $\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \}$

Exercise. 3.14
consider the set $\{1, x, x^2, x^3\}$. Using the Chebyshev inner product, we have that $u_1 = \frac{1}{\sqrt{\pi}}$. Note, $1, x$ are orthoganal to eachoter, because their inner product is zero, thus we can just normalize x to find
$u_2 = \frac{x}{\sqrt{\pi/2}}$ By applying the Gram-Shmidt process on u_2 and x^2 we find that
$v_3 = x^2 - \frac{1}{2}$, and we can normalize that to get $u_3 = \frac{x^2 - \frac{1}{2}}{\sqrt{\frac{\pi}{8}}}$ By applying the Gram-Shmidt process on u_3 and x^3 we find that, $v_4 = x^3 - \frac{3}{4}x$, and by normalizing we obtain $u_4 = \frac{x^3 - \frac{3}{4}x}{\sqrt{\frac{7\pi}{8}}}$.
Thus, the set of orthonormal basis vectors for this set is
$(\frac{1}{\sqrt{\pi}}, \frac{x}{\sqrt{\pi/2}}, \frac{x^2 - \frac{1}{2}}{\sqrt{\pi/8}}, \frac{x^3 - \frac{3}{4}x}{\sqrt{\frac{7\pi}{8}}})$