IRT

firdaus

# Prepare Environment

## Load Libraries

library(psych) # For basic psychometrics and scale reliability analysis  
library(foreign) # For reading and writing data in foreign statistical formats  
library(ltm) # To fit 2PL IRT models and other latent trait models

Loading required package: MASS

Loading required package: msm

Loading required package: polycor

Attaching package: 'polycor'

The following object is masked from 'package:psych':  
  
 polyserial

Attaching package: 'ltm'

The following object is masked from 'package:psych':  
  
 factor.scores

library(irtoys) # For IRT utilities

Loading required package: sm

Package 'sm', version 2.2-6.0: type help(sm) for summary information

Attaching package: 'sm'

The following object is masked from 'package:MASS':  
  
 muscle

Attaching package: 'irtoys'

The following object is masked from 'package:psych':  
  
 sim

library(mirt) # Modern IRT package for multi-item response theory

Loading required package: stats4

Loading required package: lattice

Attaching package: 'mirt'

The following object is masked from 'package:ltm':  
  
 Science

library(latticeExtra) # For enhanced plotting in lattice-based plots  
library(tidyverse) # For data manipulation, cleaning, and visualization

── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──  
✔ dplyr 1.1.4 ✔ readr 2.1.5  
✔ forcats 1.0.0 ✔ stringr 1.5.1  
✔ ggplot2 4.0.0 ✔ tibble 3.3.0  
✔ lubridate 1.9.4 ✔ tidyr 1.3.1  
✔ purrr 1.1.0

── Conflicts ────────────────────────────────────────── tidyverse\_conflicts() ──  
✖ ggplot2::%+%() masks psych::%+%()  
✖ ggplot2::alpha() masks psych::alpha()  
✖ dplyr::filter() masks stats::filter()  
✖ dplyr::lag() masks stats::lag()  
✖ ggplot2::layer() masks latticeExtra::layer()  
✖ dplyr::select() masks MASS::select()  
ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors

library(haven) # For importing and exporting SPSS, Stata, and SAS files  
library(writexl) # For exporting data frames to Excel files  
library(readxl) # For reading data from Excel files

## Load Data

data1=read\_xlsx("IRT\_knowledge\_V1.xlsx") ##read data from Excel   
names(data1) # List down variables in the data set

[1] "K1" "K2" "K3" "K4" "K5" "K6" "K7" "K8" "K9" "K10" "K11" "K12"  
[13] "K13" "K14" "K15" "K16" "K17" "K18" "K19" "K20" "K21" "K22" "K23" "K24"  
[25] "K25" "K26" "K27" "K28" "K29" "K30" "K31" "K32" "K33" "K34" "K35" "K36"  
[37] "K37"

dim(data1) # Data set consists of 37 variables and 177 parents

[1] 204 37

### Recode Data

# Define reverse-coded items  
reverse\_items <- c("K2", "K3", "K4", "K5", "K8", "K10", "K35")  
  
# Recode  
data2 <- data1 %>%  
 mutate(across(  
 -all\_of(reverse\_items),   
 ~ case\_when(  
 tolower(.) == "ya" ~ 1,  
 tolower(.) == "tidak" ~ 0,  
 tolower(.) == "tidak pasti" ~ 2,  
 TRUE ~ NA\_real\_  
 )  
 )) %>%  
 mutate(across(  
 all\_of(reverse\_items),  
 ~ case\_when(  
 tolower(.) == "ya" ~ 0,  
 tolower(.) == "tidak" ~ 1,  
 tolower(.) == "tidak pasti" ~ 2,  
 TRUE ~ NA\_real\_  
 )  
 ))

#Recode 1 = 1 (correct answer), 2 and 0 = 0 (incorrect answer)  
  
data3 <- data2 %>%  
 mutate(across(  
 everything(),  
 ~ case\_when(  
 . == 1 ~ 1,  
 . %in% c(0, 2) ~ 0,  
 TRUE ~ NA\_real\_  
 )  
 ))

# Descriptive Statistics

## Response Frequencies

response.frequencies(data3)

0 1 miss  
K1 0.21078431 0.7892157 0  
K2 0.60784314 0.3921569 0  
K3 0.53921569 0.4607843 0  
K4 0.61764706 0.3823529 0  
K5 0.32352941 0.6764706 0  
K6 0.83823529 0.1617647 0  
K7 0.55882353 0.4411765 0  
K8 0.57352941 0.4264706 0  
K9 0.47549020 0.5245098 0  
K10 0.50490196 0.4950980 0  
K11 0.62254902 0.3774510 0  
K12 0.75980392 0.2401961 0  
K13 0.19117647 0.8088235 0  
K14 0.33823529 0.6617647 0  
K15 0.60294118 0.3970588 0  
K16 0.62254902 0.3774510 0  
K17 0.16666667 0.8333333 0  
K18 0.39705882 0.6029412 0  
K19 0.47058824 0.5294118 0  
K20 0.45098039 0.5490196 0  
K21 0.42647059 0.5735294 0  
K22 0.40196078 0.5980392 0  
K23 0.30392157 0.6960784 0  
K24 0.27941176 0.7205882 0  
K25 0.25490196 0.7450980 0  
K26 0.40686275 0.5931373 0  
K27 0.71078431 0.2892157 0  
K28 0.55392157 0.4460784 0  
K29 0.44117647 0.5588235 0  
K30 0.81372549 0.1862745 0  
K31 0.19607843 0.8039216 0  
K32 0.30392157 0.6960784 0  
K33 0.36764706 0.6323529 0  
K34 0.18627451 0.8137255 0  
K35 0.58333333 0.4166667 0  
K36 0.06862745 0.9313725 0  
K37 0.15686275 0.8431373 0

### Descriptive Statistics

descript(data3)

Descriptive statistics for the 'data3' data-set  
  
Sample:  
 37 items and 204 sample units; 0 missing values  
  
Proportions for each level of response:  
 0 1 logit  
K1 0.2108 0.7892 1.3202  
K2 0.6078 0.3922 -0.4383  
K3 0.5392 0.4608 -0.1572  
K4 0.6176 0.3824 -0.4796  
K5 0.3235 0.6765 0.7376  
K6 0.8382 0.1618 -1.6452  
K7 0.5588 0.4412 -0.2364  
K8 0.5735 0.4265 -0.2963  
K9 0.4755 0.5245 0.0981  
K10 0.5049 0.4951 -0.0196  
K11 0.6225 0.3775 -0.5004  
K12 0.7598 0.2402 -1.1516  
K13 0.1912 0.8088 1.4424  
K14 0.3382 0.6618 0.6712  
K15 0.6029 0.3971 -0.4177  
K16 0.6225 0.3775 -0.5004  
K17 0.1667 0.8333 1.6094  
K18 0.3971 0.6029 0.4177  
K19 0.4706 0.5294 0.1178  
K20 0.4510 0.5490 0.1967  
K21 0.4265 0.5735 0.2963  
K22 0.4020 0.5980 0.3973  
K23 0.3039 0.6961 0.8287  
K24 0.2794 0.7206 0.9474  
K25 0.2549 0.7451 1.0726  
K26 0.4069 0.5931 0.3769  
K27 0.7108 0.2892 -0.8992  
K28 0.5539 0.4461 -0.2165  
K29 0.4412 0.5588 0.2364  
K30 0.8137 0.1863 -1.4744  
K31 0.1961 0.8039 1.4110  
K32 0.3039 0.6961 0.8287  
K33 0.3676 0.6324 0.5423  
K34 0.1863 0.8137 1.4744  
K35 0.5833 0.4167 -0.3365  
K36 0.0686 0.9314 2.6080  
K37 0.1569 0.8431 1.6818  
  
  
Frequencies of total scores:  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27  
Freq 1 0 1 1 2 1 2 1 4 5 6 5 3 9 4 7 10 4 10 7 8 16 10 8 12 11 5 9  
 28 29 30 31 32 33 34 35 36 37  
Freq 10 6 9 3 3 3 2 1 3 2  
  
  
Point Biserial correlation with Total Score:  
 Included Excluded  
K1 0.3742 0.3274  
K2 0.2793 0.2194  
K3 0.2500 0.1880  
K4 0.3596 0.3028  
K5 0.0997 0.0392  
K6 0.3816 0.3397  
K7 0.2545 0.1928  
K8 0.2645 0.2034  
K9 0.3174 0.2574  
K10 0.3082 0.2478  
K11 0.2970 0.2381  
K12 0.4038 0.3558  
K13 0.4129 0.3693  
K14 0.3313 0.2750  
K15 0.5002 0.4503  
K16 0.4716 0.4205  
K17 0.4539 0.4142  
K18 0.5610 0.5151  
K19 0.6431 0.6027  
K20 0.6762 0.6387  
K21 0.6889 0.6528  
K22 0.6503 0.6112  
K23 0.6857 0.6520  
K24 0.7137 0.6833  
K25 0.6905 0.6591  
K26 0.4828 0.4316  
K27 0.5352 0.4912  
K28 0.4978 0.4469  
K29 0.5478 0.5002  
K30 0.5352 0.4977  
K31 0.6280 0.5953  
K32 0.6193 0.5804  
K33 0.5917 0.5487  
K34 0.3898 0.3457  
K35 0.2232 0.1612  
K36 0.4188 0.3912  
K37 0.4330 0.3935  
  
  
Cronbach's alpha:  
 value  
All Items 0.8935  
Excluding K1 0.8921  
Excluding K2 0.8941  
Excluding K3 0.8948  
Excluding K4 0.8927  
Excluding K5 0.8969  
Excluding K6 0.8919  
Excluding K7 0.8947  
Excluding K8 0.8945  
Excluding K9 0.8936  
Excluding K10 0.8937  
Excluding K11 0.8938  
Excluding K12 0.8917  
Excluding K13 0.8915  
Excluding K14 0.8931  
Excluding K15 0.8901  
Excluding K16 0.8907  
Excluding K17 0.8909  
Excluding K18 0.8890  
Excluding K19 0.8874  
Excluding K20 0.8867  
Excluding K21 0.8865  
Excluding K22 0.8873  
Excluding K23 0.8868  
Excluding K24 0.8864  
Excluding K25 0.8869  
Excluding K26 0.8905  
Excluding K27 0.8895  
Excluding K28 0.8902  
Excluding K29 0.8892  
Excluding K30 0.8897  
Excluding K31 0.8882  
Excluding K32 0.8880  
Excluding K33 0.8884  
Excluding K34 0.8918  
Excluding K35 0.8952  
Excluding K36 0.8917  
Excluding K37 0.8912  
  
  
Pairwise Associations:  
 Item i Item j p.value  
1 2 28 1.000  
2 4 26 1.000  
3 5 22 1.000  
4 5 35 1.000  
5 7 17 1.000  
6 7 19 1.000  
7 7 20 1.000  
8 7 35 1.000  
9 8 35 1.000  
10 10 21 1.000

# Fitting 2PL IRT Model with ltm Package

## Fit 2PL Model (ltm)

irt.data3 <- ltm(data3 ~ z1, IRT.param = TRUE)

## Item Parameter Estimates

# Obtain difficulty and discrimination parameter estimates  
item\_parms <- coef(irt.data3)

# Tidy view: Item | a (Discrimination) | b (Difficulty)  
  
item\_parms\_tbl <- item\_parms |>  
 as.data.frame() |>  
 transform(Item = rownames(item\_parms),  
 Difficulty = Dffclt,  
 Discrimination = Dscrmn) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination")])() |>  
 (\(d) within(d, {   
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 }))()  
  
item\_parms\_tbl

Item Difficulty Discrimination  
K1 K1 -1.426 0.873  
K2 K2 1.275 0.473  
K3 K3 0.719 0.390  
K4 K4 1.094 0.660  
K5 K5 -9.376 0.076  
K6 K6 2.247 0.994  
K7 K7 1.228 0.260  
K8 K8 1.778 0.203  
K9 K9 0.001 0.333  
K10 K10 0.368 0.298  
K11 K11 1.779 0.349  
K12 K12 2.059 0.724  
K13 K13 -1.250 1.135  
K14 K14 -0.559 0.908  
K15 K15 0.676 1.475  
K16 K16 0.800 1.268  
K17 K17 -1.078 1.618  
K18 K18 -0.106 1.362  
K19 K19 0.210 4.683  
K20 K20 0.158 5.578  
K21 K21 0.096 6.871  
K22 K22 0.053 4.676  
K23 K23 -0.163 5.311  
K24 K24 -0.082 16.964  
K25 K25 -0.266 6.626  
K26 K26 -0.159 0.976  
K27 K27 1.073 1.614  
K28 K28 0.531 1.132  
K29 K29 0.050 1.214  
K30 K30 1.382 2.216  
K31 K31 -0.564 3.755  
K32 K32 -0.265 2.718  
K33 K33 -0.113 2.130  
K34 K34 -1.202 1.225  
K35 K35 1.416 0.309  
K36 K36 -1.530 2.379  
K37 K37 -1.220 1.471

## Model Summary

# Includes log-likelihood, AIC/BIC, SEs, and Wald z-values  
summary(irt.data3)

Call:  
ltm(formula = data3 ~ z1, IRT.param = TRUE)  
  
Model Summary:  
 log.Lik AIC BIC  
 -3748.378 7644.756 7890.297  
  
Coefficients:  
 value std.err z.vals  
Dffclt.K1 -1.4264 0.4021 -3.5474  
Dffclt.K2 1.2750 0.4395 2.9011  
Dffclt.K3 0.7195 0.4000 1.7987  
Dffclt.K4 1.0943 0.2948 3.7118  
Dffclt.K5 -9.3762 19.8814 -0.4716  
Dffclt.K6 2.2471 0.4202 5.3475  
Dffclt.K7 1.2275 0.7566 1.6224  
Dffclt.K8 1.7782 1.2819 1.3871  
Dffclt.K9 0.0009 0.4468 0.0021  
Dffclt.K10 0.3675 0.4764 0.7714  
Dffclt.K11 1.7786 0.7647 2.3260  
Dffclt.K12 2.0587 0.4714 4.3673  
Dffclt.K13 -1.2502 0.2985 -4.1882  
Dffclt.K14 -0.5586 0.2302 -2.4270  
Dffclt.K15 0.6763 0.1252 5.4015  
Dffclt.K16 0.7998 0.1486 5.3817  
Dffclt.K17 -1.0782 0.2101 -5.1321  
Dffclt.K18 -0.1062 0.1297 -0.8185  
Dffclt.K19 0.2102 0.0539 3.8974  
Dffclt.K20 0.1584 0.0500 3.1693  
Dffclt.K21 0.0956 0.0422 2.2641  
Dffclt.K22 0.0527 0.0515 1.0217  
Dffclt.K23 -0.1631 0.0568 -2.8727  
Dffclt.K24 -0.0817 0.3221 -0.2537  
Dffclt.K25 -0.2657 0.0658 -4.0389  
Dffclt.K26 -0.1590 0.1747 -0.9101  
Dffclt.K27 1.0730 0.1472 7.2883  
Dffclt.K28 0.5313 0.1449 3.6679  
Dffclt.K29 0.0504 0.1353 0.3729  
Dffclt.K30 1.3822 0.1532 9.0193  
Dffclt.K31 -0.5639 0.0873 -6.4600  
Dffclt.K32 -0.2649 0.0864 -3.0675  
Dffclt.K33 -0.1135 0.0932 -1.2167  
Dffclt.K34 -1.2024 0.2740 -4.3881  
Dffclt.K35 1.4162 0.7078 2.0009  
Dffclt.K36 -1.5304 0.2380 -6.4304  
Dffclt.K37 -1.2203 0.2437 -5.0064  
Dscrmn.K1 0.8727 0.2206 3.9555  
Dscrmn.K2 0.4727 0.1642 2.8785  
Dscrmn.K3 0.3899 0.1564 2.4939  
Dscrmn.K4 0.6598 0.1775 3.7173  
Dscrmn.K5 0.0763 0.1565 0.4879  
Dscrmn.K6 0.9942 0.2469 4.0263  
Dscrmn.K7 0.2602 0.1516 1.7159  
Dscrmn.K8 0.2027 0.1501 1.3505  
Dscrmn.K9 0.3331 0.1547 2.1530  
Dscrmn.K10 0.2983 0.1529 1.9504  
Dscrmn.K11 0.3486 0.1586 2.1979  
Dscrmn.K12 0.7243 0.1992 3.6356  
Dscrmn.K13 1.1346 0.2524 4.4945  
Dscrmn.K14 0.9076 0.2045 4.4370  
Dscrmn.K15 1.4748 0.2662 5.5409  
Dscrmn.K16 1.2676 0.2407 5.2662  
Dscrmn.K17 1.6182 0.3240 4.9951  
Dscrmn.K18 1.3621 0.2489 5.4718  
Dscrmn.K19 4.6826 0.8496 5.5116  
Dscrmn.K20 5.5780 1.2893 4.3264  
Dscrmn.K21 6.8712 1.5567 4.4140  
Dscrmn.K22 4.6763 0.8492 5.5065  
Dscrmn.K23 5.3111 1.1319 4.6924  
Dscrmn.K24 16.9641 66.8082 0.2539  
Dscrmn.K25 6.6261 1.4010 4.7297  
Dscrmn.K26 0.9758 0.2055 4.7490  
Dscrmn.K27 1.6137 0.2974 5.4255  
Dscrmn.K28 1.1320 0.2235 5.0660  
Dscrmn.K29 1.2142 0.2308 5.2616  
Dscrmn.K30 2.2162 0.4249 5.2159  
Dscrmn.K31 3.7552 0.6691 5.6120  
Dscrmn.K32 2.7180 0.4797 5.6656  
Dscrmn.K33 2.1299 0.3664 5.8124  
Dscrmn.K34 1.2254 0.2628 4.6628  
Dscrmn.K35 0.3093 0.1548 1.9986  
Dscrmn.K36 2.3787 0.5652 4.2086  
Dscrmn.K37 1.4710 0.3023 4.8656  
  
Integration:  
method: Gauss-Hermite  
quadrature points: 21   
  
Optimization:  
Convergence: 0   
max(|grad|): 0.088   
quasi-Newton: BFGS

## Items Removal Plan 1

**Selection criteria a > 0.64 (moderate discrimination) (Baker, 2001) ; -3 < b > +3**

K2 - a = 0.47

K3 - a = 0.39

K5 - a = 0.08 , b = -9.3762

K7 - a = 0.26

K8 - a = 0.20

K9 - a = 0.33

K10 - a = 0.30

K11 - a = 0.35

K24 - a = 16.9641 (?Overfitting)

K35 - a = 0.31

### 2PL Model - Remove Items

# Remove the items  
irt\_removed\_items <- c("K2", "K3", "K5", "K7", "K8", "K9", "K10", "K11","K35","K24")  
  
# Create new dataset with only included items  
data4 <- data3 %>% dplyr::select(-any\_of(irt\_removed\_items))

### Descriptive Statistics

descript(data4)

Descriptive statistics for the 'data4' data-set  
  
Sample:  
 27 items and 204 sample units; 0 missing values  
  
Proportions for each level of response:  
 0 1 logit  
K1 0.2108 0.7892 1.3202  
K4 0.6176 0.3824 -0.4796  
K6 0.8382 0.1618 -1.6452  
K12 0.7598 0.2402 -1.1516  
K13 0.1912 0.8088 1.4424  
K14 0.3382 0.6618 0.6712  
K15 0.6029 0.3971 -0.4177  
K16 0.6225 0.3775 -0.5004  
K17 0.1667 0.8333 1.6094  
K18 0.3971 0.6029 0.4177  
K19 0.4706 0.5294 0.1178  
K20 0.4510 0.5490 0.1967  
K21 0.4265 0.5735 0.2963  
K22 0.4020 0.5980 0.3973  
K23 0.3039 0.6961 0.8287  
K25 0.2549 0.7451 1.0726  
K26 0.4069 0.5931 0.3769  
K27 0.7108 0.2892 -0.8992  
K28 0.5539 0.4461 -0.2165  
K29 0.4412 0.5588 0.2364  
K30 0.8137 0.1863 -1.4744  
K31 0.1961 0.8039 1.4110  
K32 0.3039 0.6961 0.8287  
K33 0.3676 0.6324 0.5423  
K34 0.1863 0.8137 1.4744  
K36 0.0686 0.9314 2.6080  
K37 0.1569 0.8431 1.6818  
  
  
Frequencies of total scores:  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27  
Freq 1 2 5 1 3 4 5 5 5 8 9 11 8 7 5 9 9 12 13 19 13 7 9 10 8 8 2 6  
  
  
Point Biserial correlation with Total Score:  
 Included Excluded  
K1 0.3583 0.3021  
K4 0.3060 0.2365  
K6 0.3679 0.3176  
K12 0.3244 0.2641  
K13 0.4843 0.4362  
K14 0.4266 0.3647  
K15 0.5811 0.5282  
K16 0.5401 0.4843  
K17 0.5056 0.4613  
K18 0.5350 0.4783  
K19 0.7215 0.6817  
K20 0.7268 0.6877  
K21 0.7317 0.6934  
K22 0.7003 0.6589  
K23 0.6881 0.6482  
K25 0.7066 0.6707  
K26 0.4435 0.3801  
K27 0.5680 0.5182  
K28 0.5095 0.4499  
K29 0.5101 0.4505  
K30 0.5508 0.5072  
K31 0.6681 0.6323  
K32 0.6636 0.6213  
K33 0.6340 0.5867  
K34 0.4376 0.3876  
K36 0.4432 0.4113  
K37 0.4696 0.4246  
  
  
Cronbach's alpha:  
 value  
All Items 0.9077  
Excluding K1 0.9077  
Excluding K4 0.9096  
Excluding K6 0.9073  
Excluding K12 0.9085  
Excluding K13 0.9055  
Excluding K14 0.9070  
Excluding K15 0.9038  
Excluding K16 0.9047  
Excluding K17 0.9051  
Excluding K18 0.9048  
Excluding K19 0.9006  
Excluding K20 0.9005  
Excluding K21 0.9003  
Excluding K22 0.9011  
Excluding K23 0.9015  
Excluding K25 0.9012  
Excluding K26 0.9068  
Excluding K27 0.9040  
Excluding K28 0.9054  
Excluding K29 0.9054  
Excluding K30 0.9043  
Excluding K31 0.9022  
Excluding K32 0.9020  
Excluding K33 0.9026  
Excluding K34 0.9062  
Excluding K36 0.9062  
Excluding K37 0.9057  
  
  
Pairwise Associations:  
 Item i Item j p.value  
1 2 17 1.000  
2 4 5 1.000  
3 1 19 1.000  
4 2 5 0.880  
5 1 18 0.723  
6 17 25 0.704  
7 5 20 0.643  
8 2 26 0.627  
9 4 27 0.593  
10 8 17 0.591

### Refit 2PL Model

irt.data4 <- ltm(data4 ~ z1, IRT.param = TRUE)

### Item Parameter Estimates

# Obtain difficulty and discrimination parameter estimates  
item\_parms\_refined <- coef(irt.data4)  
  
# Tidy view: Item | a (Discrimination) | b (Difficulty)  
  
item\_parms\_refined\_tbl <- item\_parms\_refined |>  
 as.data.frame() |>  
 transform(Item = rownames(item\_parms\_refined),  
 Difficulty = Dffclt,  
 Discrimination = Dscrmn) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination")])() |>  
 (\(d) within(d, {   
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 }))()  
  
item\_parms\_refined\_tbl

Item Difficulty Discrimination  
K1 K1 -1.516 0.840  
K4 K4 1.242 0.533  
K6 K6 2.324 0.939  
K12 K12 2.289 0.621  
K13 K13 -1.235 1.201  
K14 K14 -0.565 0.936  
K15 K15 0.670 1.508  
K16 K16 0.788 1.300  
K17 K17 -1.166 1.546  
K18 K18 -0.138 1.294  
K19 K19 0.215 6.013  
K20 K20 0.160 6.561  
K21 K21 0.099 7.075  
K22 K22 0.051 5.670  
K23 K23 -0.283 2.875  
K25 K25 -0.394 3.904  
K26 K26 -0.200 0.920  
K27 K27 1.090 1.571  
K28 K28 0.530 1.087  
K29 K29 0.011 1.063  
K30 K30 1.404 2.165  
K31 K31 -0.640 3.489  
K32 K32 -0.311 2.520  
K33 K33 -0.138 2.058  
K34 K34 -1.359 1.102  
K36 K36 -1.708 2.117  
K37 K37 -1.373 1.324

### Model Summary

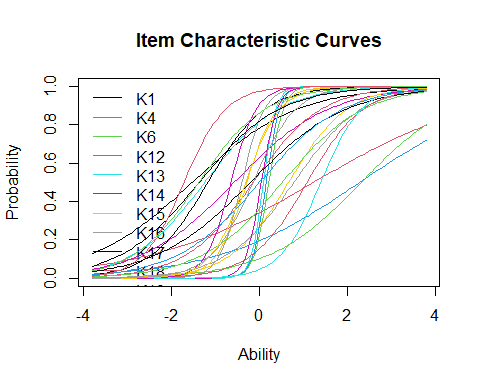
# Includes log-likelihood, AIC/BIC, SEs, and Wald z-values  
summary(irt.data4)

Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Model Summary:  
 log.Lik AIC BIC  
 -2476.419 5060.839 5240.017  
  
Coefficients:  
 value std.err z.vals  
Dffclt.K1 -1.5157 0.4169 -3.6361  
Dffclt.K4 1.2418 0.3868 3.2108  
Dffclt.K6 2.3242 0.4602 5.0507  
Dffclt.K12 2.2892 0.6018 3.8037  
Dffclt.K13 -1.2345 0.2731 -4.5200  
Dffclt.K14 -0.5647 0.2238 -2.5238  
Dffclt.K15 0.6699 0.1256 5.3336  
Dffclt.K16 0.7883 0.1472 5.3546  
Dffclt.K17 -1.1663 0.2227 -5.2379  
Dffclt.K18 -0.1382 0.1417 -0.9755  
Dffclt.K19 0.2150 0.0603 3.5667  
Dffclt.K20 0.1600 0.0571 2.8001  
Dffclt.K21 0.0991 0.0501 1.9787  
Dffclt.K22 0.0513 0.0563 0.9121  
Dffclt.K23 -0.2827 0.1000 -2.8258  
Dffclt.K25 -0.3936 0.0992 -3.9665  
Dffclt.K26 -0.2001 0.1893 -1.0573  
Dffclt.K27 1.0903 0.1508 7.2290  
Dffclt.K28 0.5296 0.1532 3.4570  
Dffclt.K29 0.0107 0.1573 0.0679  
Dffclt.K30 1.4037 0.1563 8.9788  
Dffclt.K31 -0.6401 0.1106 -5.7882  
Dffclt.K32 -0.3107 0.1058 -2.9359  
Dffclt.K33 -0.1385 0.1063 -1.3032  
Dffclt.K34 -1.3595 0.3177 -4.2786  
Dffclt.K36 -1.7079 0.2693 -6.3419  
Dffclt.K37 -1.3732 0.2804 -4.8982  
Dscrmn.K1 0.8402 0.2093 4.0135  
Dscrmn.K4 0.5326 0.1664 3.2013  
Dscrmn.K6 0.9389 0.2439 3.8489  
Dscrmn.K12 0.6215 0.1915 3.2446  
Dscrmn.K13 1.2006 0.2516 4.7721  
Dscrmn.K14 0.9360 0.1997 4.6869  
Dscrmn.K15 1.5084 0.2697 5.5918  
Dscrmn.K16 1.3000 0.2434 5.3414  
Dscrmn.K17 1.5463 0.3062 5.0508  
Dscrmn.K18 1.2938 0.2351 5.5031  
Dscrmn.K19 6.0135 1.0683 5.6291  
Dscrmn.K20 6.5612 1.3057 5.0251  
Dscrmn.K21 7.0752 1.5854 4.4628  
Dscrmn.K22 5.6704 1.2451 4.5543  
Dscrmn.K23 2.8749 0.5396 5.3279  
Dscrmn.K25 3.9044 0.8070 4.8379  
Dscrmn.K26 0.9197 0.1952 4.7110  
Dscrmn.K27 1.5705 0.2858 5.4947  
Dscrmn.K28 1.0871 0.2135 5.0915  
Dscrmn.K29 1.0627 0.2102 5.0562  
Dscrmn.K30 2.1649 0.4152 5.2139  
Dscrmn.K31 3.4887 0.6991 4.9903  
Dscrmn.K32 2.5203 0.4525 5.5697  
Dscrmn.K33 2.0583 0.3603 5.7122  
Dscrmn.K34 1.1020 0.2425 4.5449  
Dscrmn.K36 2.1173 0.4878 4.3408  
Dscrmn.K37 1.3244 0.2746 4.8230  
  
Integration:  
method: Gauss-Hermite  
quadrature points: 21   
  
Optimization:  
Convergence: 0   
max(|grad|): 0.0049   
quasi-Newton: BFGS

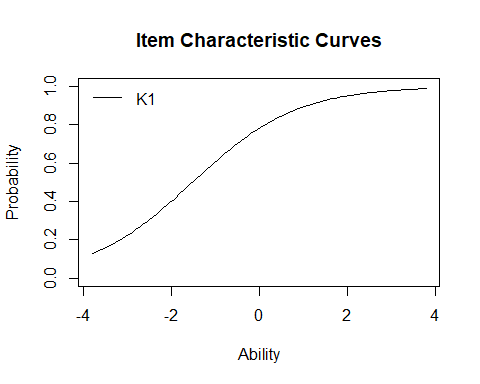
## Graphical Presentation

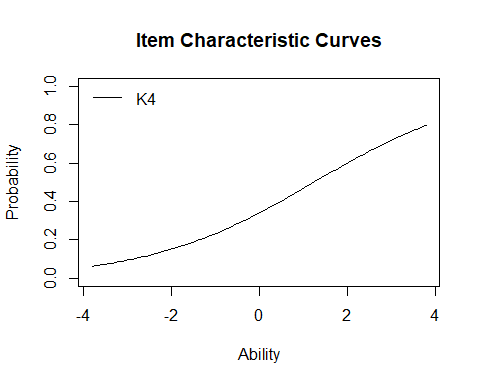
### Item Characteristic Curves (ICC)

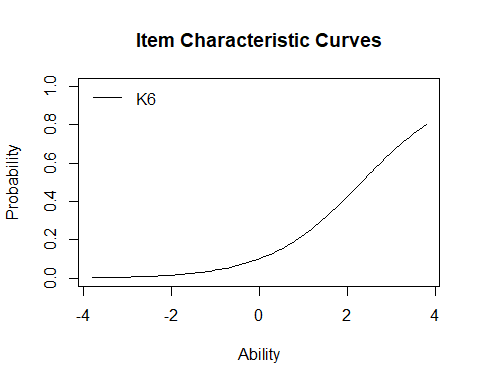
# ICC for All Items  
# Plot ICC for all items  
plot(irt.data4, type = "ICC", legend = TRUE)

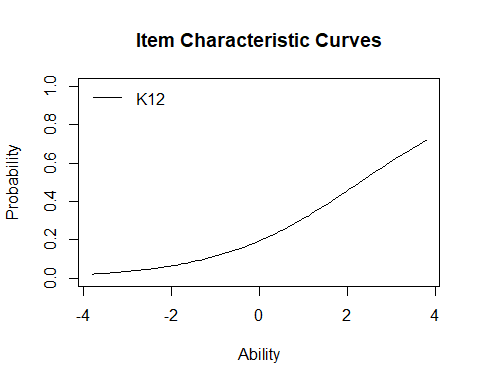


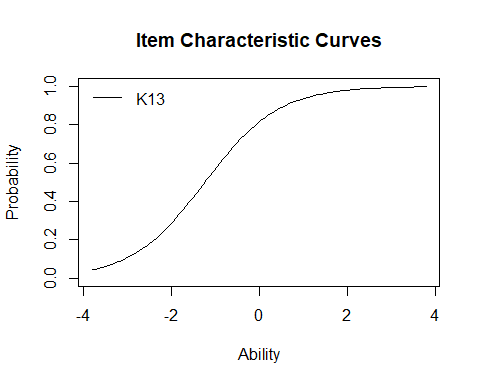
# ICC for Individual Items  
  
# Get total number of items  
ICC\_items <- nrow(coef(irt.data4))  
  
# Plot ICC for each item  
for (i in 1:ICC\_items) {  
 plot(irt.data4, type = "ICC", legend = TRUE, items = i)  
}

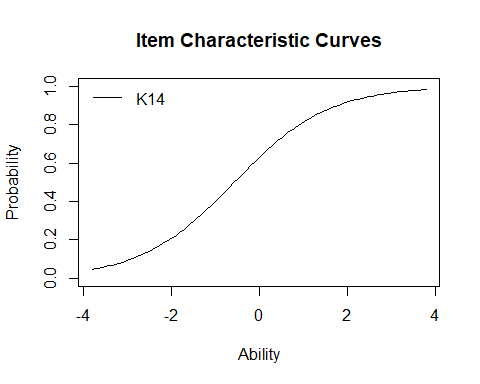


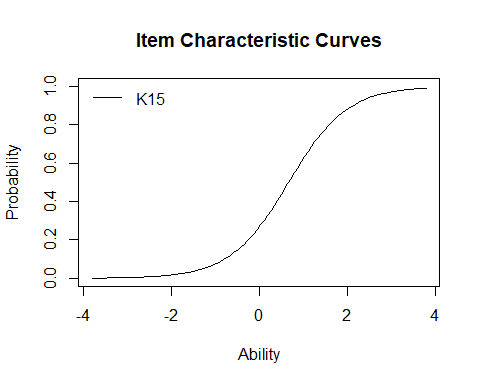


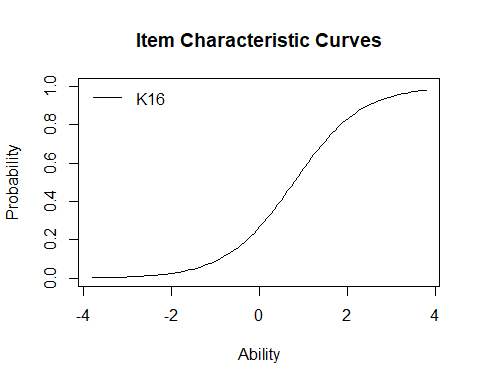


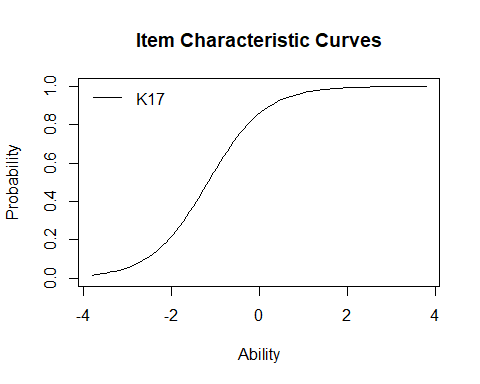


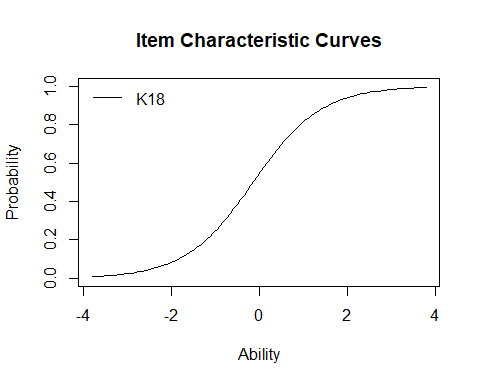


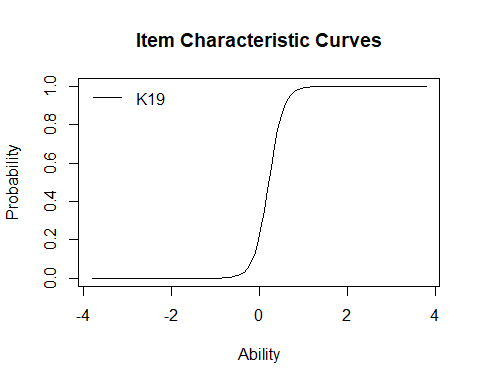


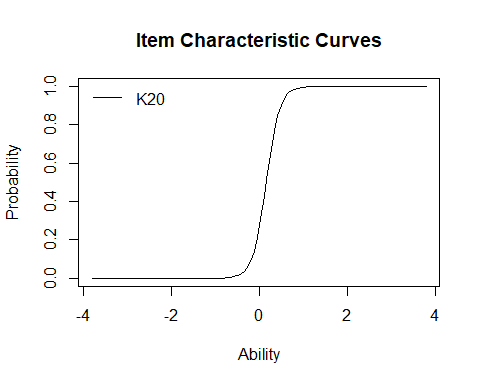


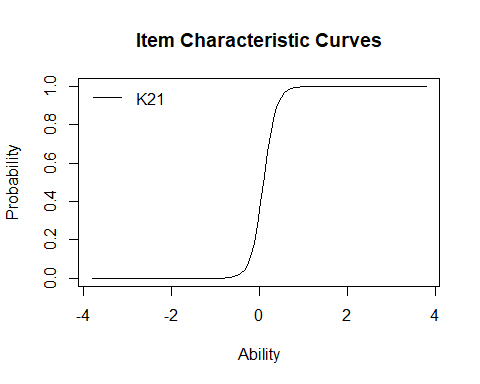


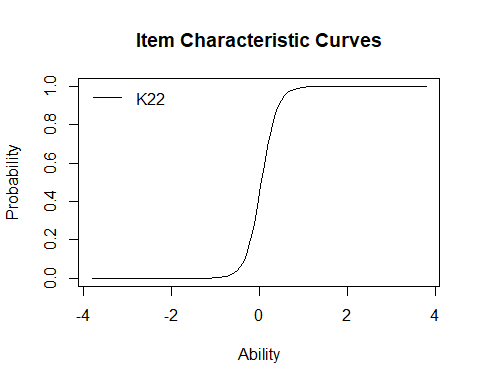


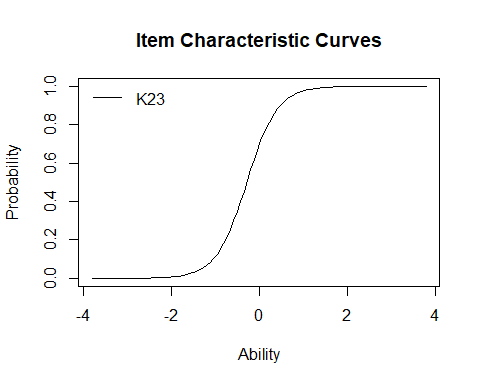


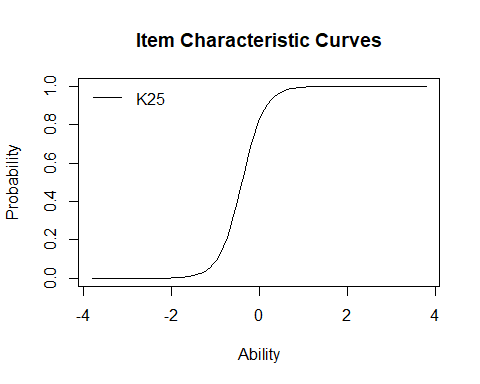


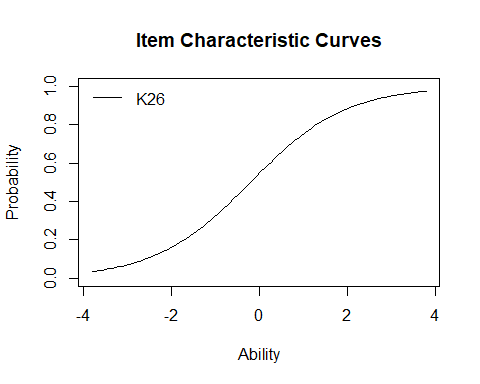


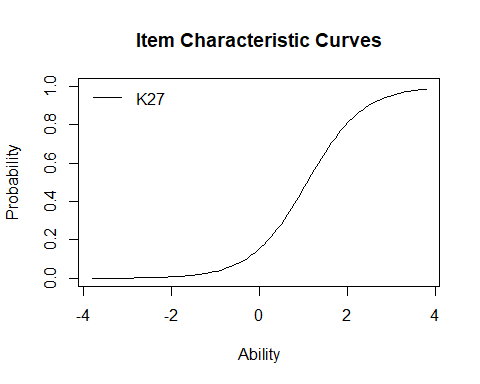


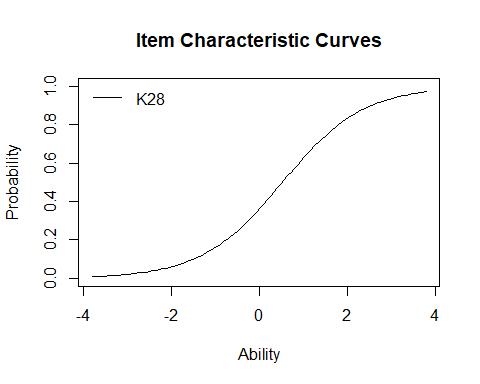


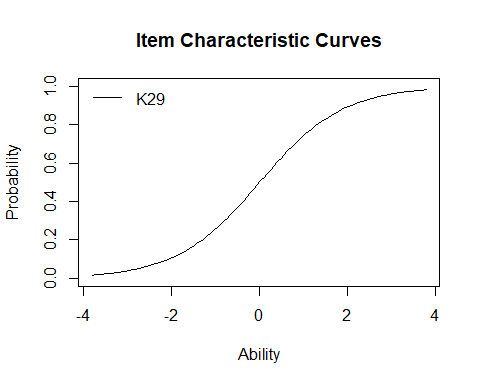


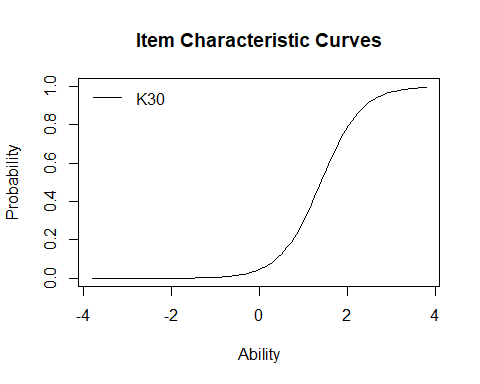


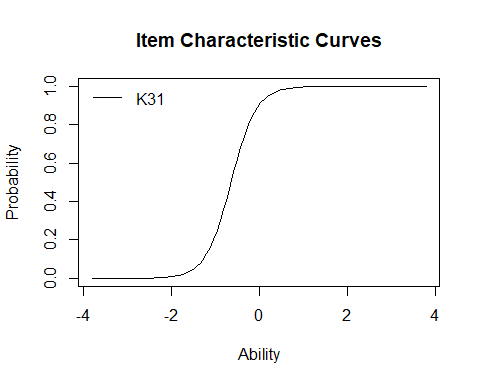


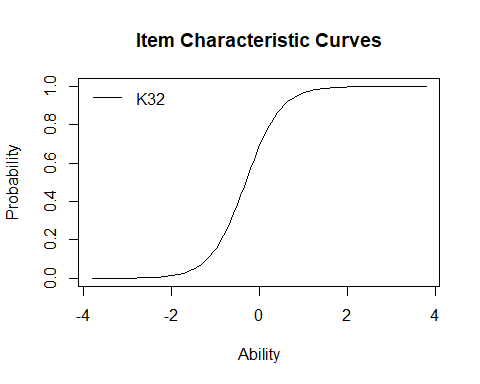


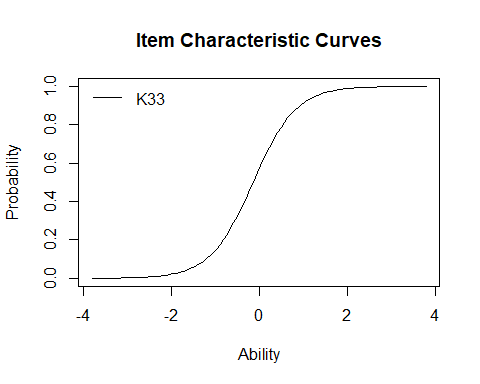


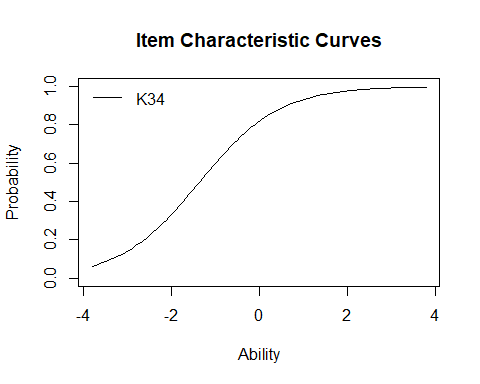


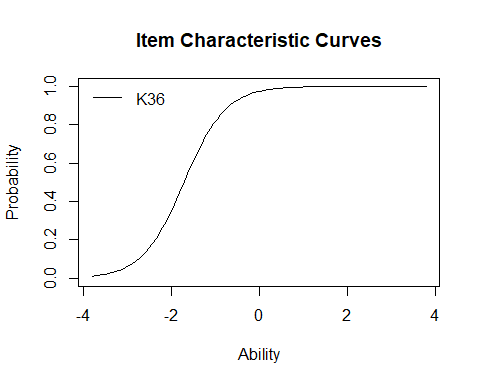


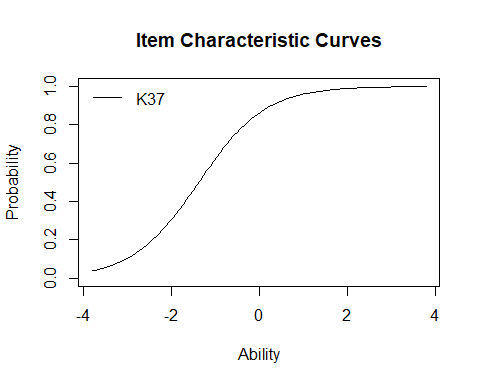












## Goodness-of-Fit Tests

### Item Fit Statistics

item\_fit <- item.fit(irt.data4)  
item\_fit

Item-Fit Statistics and P-values  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Alternative: Items do not fit the model  
Ability Categories: 10  
  
 X^2 Pr(>X^2)  
K1 4.9206 0.766  
K4 15.3684 0.0524  
K6 7.2905 0.5056  
K12 4.2084 0.8378  
K13 11.3718 0.1815  
K14 14.7225 0.0648  
K15 12.7751 0.1198  
K16 13.6769 0.0906  
K17 16.4946 0.0358  
K18 11.3648 0.1819  
K19 3.1158 0.9269  
K20 17.5518 0.0248  
K21 7.7055 0.4628  
K22 9.0532 0.3378  
K23 7.4196 0.4921  
K25 5.7966 0.67  
K26 17.6430 0.0241  
K27 28.2902 0.0004  
K28 24.5023 0.0019  
K29 14.4974 0.0697  
K30 25.8528 0.0011  
K31 7.4821 0.4856  
K32 20.0696 0.0101  
K33 18.4095 0.0184  
K34 9.3176 0.3162  
K36 13.5752 0.0935  
K37 8.5269 0.3838

### Fit on the Two-Way Margins

margins\_output <- margins(irt.data4)  
margins\_output

Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Fit on the Two-Way Margins  
  
Response: (0,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 12 16 43 70.16 10.51 \*\*\*  
2 14 23 43 69.90 10.35 \*\*\*  
3 14 24 48 74.96 9.70 \*\*\*  
  
Response: (1,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 9 34.51 18.86 \*\*\*  
2 11 26 4 0.63 17.99 \*\*\*  
3 18 19 2 19.69 15.90 \*\*\*  
  
Response: (0,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 5 31.84 22.63 \*\*\*  
2 23 24 5 22.51 13.62 \*\*\*  
3 17 20 17 38.18 11.75 \*\*\*  
  
Response: (1,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 72 32.41 48.37 \*\*\*  
2 18 19 57 27.32 32.25 \*\*\*  
3 18 21 34 15.23 23.14 \*\*\*  
  
'\*\*\*' denotes a chi-squared residual greater than 3.5

### Person Fit Statistics

person\_fit <- person.fit(irt.data4)  
person\_fit

Person-Fit Statistics and P-values  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Alternative: Inconsistent response pattern under the estimated model  
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K19 K20 K21 K22 K23 K25 K26 K27 K28  
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1  
4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0  
5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0  
6 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 1 0 0  
7 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
8 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1  
9 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0  
10 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 1 1 0 1  
11 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0  
12 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
13 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 1 0 0  
14 0 0 0 0 1 0 0 0 1 0 0 1 1 1 1 0 1 0 1  
15 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0  
16 0 0 0 0 1 0 0 0 1 1 0 0 1 1 1 1 1 0 1  
17 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0  
18 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0  
19 0 0 0 0 1 1 0 0 0 0 1 0 1 1 1 1 1 0 0  
20 0 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0 0 0 1  
21 0 0 0 0 1 1 0 0 1 1 1 0 1 1 1 1 0 1 1  
22 0 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
23 0 0 0 0 1 1 0 1 1 1 1 1 1 1 1 1 0 0 0  
24 0 0 0 0 1 1 1 1 1 0 1 0 0 0 1 1 1 1 1  
25 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0  
26 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1  
27 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 1 1 1  
28 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 1  
29 0 0 0 1 1 1 1 0 1 1 0 1 1 1 1 1 1 0 0  
30 0 0 0 1 1 1 1 1 1 0 1 0 0 0 1 1 0 1 1  
31 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
32 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
33 0 0 1 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0  
34 0 0 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1  
35 0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 1 1  
36 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
37 0 1 0 0 1 0 0 0 1 0 0 0 0 0 1 1 0 0 0  
38 0 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
39 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
40 0 1 0 1 0 0 0 0 1 1 0 0 0 0 1 1 0 0 0  
41 0 1 0 1 0 1 1 1 1 0 1 1 1 1 0 0 0 1 1  
42 0 1 0 1 1 1 1 1 1 1 0 1 1 0 0 0 0 1 1  
43 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
44 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 1  
45 1 0 0 0 0 0 0 0 1 0 0 0 0 0 1 1 1 0 0  
46 1 0 0 0 0 0 0 0 1 0 1 1 1 1 1 1 1 0 0  
47 1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
48 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1  
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68 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
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74 1 0 0 0 1 1 0 0 1 0 1 1 1 1 1 1 0 0 0  
75 1 0 0 0 1 1 0 0 1 1 0 0 0 0 0 1 0 0 0  
76 1 0 0 0 1 1 0 0 1 1 0 0 0 1 0 1 1 0 1  
77 1 0 0 0 1 1 0 0 1 1 0 0 0 1 1 1 0 0 0  
78 1 0 0 0 1 1 0 0 1 1 0 0 1 1 0 1 1 0 1  
79 1 0 0 0 1 1 0 0 1 1 0 0 1 1 1 1 0 0 0  
80 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0 1 0 0 0  
81 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 0 0 0  
82 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 0 0 0  
83 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0  
84 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0  
85 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
86 1 0 0 0 1 1 0 1 1 1 0 0 0 0 1 1 0 0 0  
87 1 0 0 0 1 1 0 1 1 1 0 0 0 0 1 1 1 1 1  
88 1 0 0 0 1 1 1 0 1 0 1 1 1 1 1 1 0 0 1  
89 1 0 0 0 1 1 1 0 1 1 0 0 0 0 1 1 1 0 0  
90 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 0  
91 1 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 0 1  
92 1 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1  
93 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0  
94 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1  
95 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1  
96 1 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 1 0 0  
97 1 0 0 0 1 1 1 1 1 1 1 0 0 1 0 0 0 0 0  
98 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0  
99 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0  
100 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0  
101 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0  
102 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
103 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
104 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
105 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
106 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
107 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0  
108 1 0 0 1 0 0 0 0 0 1 0 1 0 1 0 0 1 0 1  
109 1 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 1 0 0  
110 1 0 0 1 0 0 0 0 1 1 1 1 1 1 1 1 1 0 1  
111 1 0 0 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
112 1 0 0 1 1 0 0 0 0 1 0 0 1 1 1 1 1 1 1  
113 1 0 0 1 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0  
114 1 0 0 1 1 0 0 0 1 1 0 0 0 0 1 1 1 0 0  
115 1 0 0 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0  
116 1 0 0 1 1 0 1 1 1 1 1 1 1 1 0 1 1 0 1  
117 1 0 0 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 1  
118 1 0 0 1 1 1 0 1 1 1 1 0 0 0 1 1 0 0 0  
119 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1  
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124 1 0 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 0 0  
125 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
126 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0  
127 1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0  
128 1 1 0 0 0 0 0 0 1 0 1 1 1 1 1 1 0 0 0  
129 1 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
130 1 1 0 0 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0  
131 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1  
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177 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 1 0 0  
178 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 0 0 0  
179 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1  
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181 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
182 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 K29 K30 K31 K32 K33 K34 K36 K37 L0 Lz Pr(<Lz)  
1 0 0 0 0 0 0 0 0 -2.2459 1.3406 0.91  
2 0 0 0 0 0 1 1 0 -4.9404 1.2702 0.898  
3 0 0 0 1 0 0 0 0 -10.5886 -0.9929 0.1604  
4 0 0 0 0 0 0 0 0 -4.6553 0.6934 0.756  
5 0 0 1 1 1 1 1 1 -12.3064 0.3408 0.6334  
6 1 0 0 0 0 0 1 0 -14.7805 -1.2331 0.1088  
7 0 0 1 1 1 1 1 1 -10.4111 0.7987 0.7878  
8 1 0 0 0 0 1 1 1 -9.3434 0.4830 0.6855  
9 0 0 1 1 0 1 1 1 -10.6107 1.0013 0.8417  
10 1 0 1 1 1 0 1 1 -14.2719 -0.0812 0.4676  
11 0 0 0 0 0 1 1 1 -13.7618 -0.7563 0.2247  
12 0 0 0 0 0 1 1 1 -6.4688 1.1368 0.8722  
13 0 0 1 0 0 1 1 1 -8.8410 1.1693 0.8789  
14 1 0 1 0 0 0 1 1 -16.9485 -1.1656 0.1219  
15 0 0 1 1 1 1 1 1 -10.0965 1.1031 0.865  
16 1 0 1 0 0 0 1 1 -14.2973 -0.0470 0.4813  
17 0 0 0 0 0 0 0 0 -12.0806 -1.5098 0.0656  
18 0 0 1 1 1 1 1 0 -12.4471 0.3689 0.6439  
19 1 0 1 1 0 1 1 1 -13.2705 0.2067 0.5819  
20 0 0 0 0 0 1 1 1 -9.4779 0.7932 0.7862  
21 0 0 1 1 1 1 1 1 -11.9475 0.3427 0.6341  
22 0 0 1 1 1 1 1 0 -11.8017 -0.3019 0.3814  
23 0 0 1 1 1 1 1 1 -9.4608 0.6846 0.7532  
24 1 1 1 1 1 1 1 1 -17.8407 -1.6096 0.0537  
25 0 0 1 1 1 1 1 1 -10.5298 0.2666 0.6051  
26 1 0 1 1 1 1 1 1 -8.8145 0.4520 0.6744  
27 1 0 1 1 1 1 1 1 -16.0919 -0.9060 0.1825  
28 1 0 1 1 1 1 1 1 -9.9138 0.3180 0.6248  
29 1 1 1 1 1 1 1 1 -12.9876 -0.4683 0.3198  
30 1 1 1 1 1 1 1 1 -19.4987 -2.2866 0.0111  
31 1 1 1 1 1 1 1 1 -7.0489 0.0384 0.5153  
32 0 0 0 0 0 0 0 0 -8.4967 -0.5214 0.3011  
33 0 0 1 1 1 1 1 0 -14.7450 -1.2173 0.1118  
34 1 1 1 1 1 1 1 1 -12.5373 -1.5559 0.0599  
35 1 0 1 1 1 0 0 0 -28.0811 -5.6484 <0.0001  
36 0 0 0 0 0 0 0 0 -6.5188 0.2917 0.6147  
37 0 0 1 1 0 1 1 1 -10.3445 1.1830 0.8816  
38 0 0 0 0 0 0 1 1 -14.0499 -0.9703 0.1659  
39 1 1 1 1 0 1 1 1 -10.3359 -0.8544 0.1965  
40 1 0 1 0 0 0 1 0 -16.0638 -1.3115 0.0948  
41 1 0 1 0 0 0 0 1 -28.8190 -6.1119 <0.0001  
42 1 0 1 1 1 1 1 1 -19.6620 -2.3637 0.009  
43 1 0 0 0 0 1 1 1 -7.1074 1.0652 0.8566  
44 1 0 1 1 1 1 1 1 -11.9429 -0.0832 0.4668  
45 1 0 1 1 1 0 1 1 -11.4320 0.8399 0.7995  
46 1 0 1 1 1 1 1 1 -10.1412 0.6734 0.7496  
47 0 0 0 0 0 1 1 1 -6.5318 1.5587 0.9405  
48 1 0 0 0 0 1 1 0 -9.2482 0.3769 0.6469  
49 1 0 0 0 0 0 0 0 -10.5770 -0.5763 0.2822  
50 0 0 1 1 1 1 1 1 -10.2629 1.3048 0.904  
51 1 0 1 0 0 0 1 0 -16.9137 -1.1668 0.1216  
52 1 1 1 1 0 1 1 1 -13.8229 -1.1837 0.1183  
53 0 0 1 0 0 1 1 0 -9.9376 1.2353 0.8916  
54 0 0 1 1 1 0 1 1 -9.9667 1.4248 0.9229  
55 1 0 1 1 1 1 1 1 -9.5478 1.7308 0.9583  
56 0 0 1 1 1 0 1 1 -11.5895 -0.0626 0.4751  
57 1 1 1 1 1 1 1 1 -14.3263 -0.0646 0.4742  
58 1 0 1 1 1 1 1 1 -12.4676 0.2284 0.5903  
59 0 0 1 1 1 1 1 1 -7.7325 1.2346 0.8915  
60 1 0 1 1 1 1 1 1 -7.0721 1.3373 0.9094  
61 0 0 0 0 0 1 1 1 -12.7749 0.1319 0.5525  
62 0 0 0 0 0 0 1 0 -6.4393 1.0707 0.8579  
63 0 0 0 0 0 0 1 1 -6.5457 1.2877 0.9011  
64 0 0 1 1 1 1 1 1 -10.4183 1.0295 0.8484  
65 1 0 1 0 0 0 0 0 -23.9903 -4.0931 <0.0001  
66 1 0 1 0 0 1 1 1 -12.4266 0.1885 0.5748  
67 0 0 0 0 0 0 0 0 -7.3273 0.5833 0.7201  
68 0 0 0 0 0 1 1 1 -6.3516 1.8409 0.9672  
69 0 0 1 0 0 1 1 1 -7.5766 1.9331 0.9734  
70 1 0 0 0 0 1 1 1 -9.4526 0.9996 0.8413  
71 0 0 1 0 0 1 1 1 -10.5064 1.0137 0.8446  
72 0 0 1 1 0 0 1 1 -13.0084 0.1371 0.5545  
73 0 0 0 0 0 1 1 1 -14.2242 -0.2262 0.4105  
74 0 0 1 0 1 1 1 1 -10.0647 0.9622 0.832  
75 0 0 1 1 1 1 1 1 -8.6605 1.9216 0.9727  
76 1 0 1 1 1 1 1 1 -10.1502 1.6548 0.951  
77 0 0 1 1 1 0 1 1 -10.4363 1.5072 0.9341  
78 0 0 1 0 0 0 1 0 -15.2308 -0.4462 0.3277  
79 0 0 0 0 0 1 1 1 -12.9413 0.4867 0.6868  
80 0 0 0 1 1 1 1 1 -13.0021 0.0710 0.5283  
81 0 0 1 1 0 1 1 1 -8.6817 1.2801 0.8997  
82 0 0 1 1 1 1 1 1 -7.4114 1.4902 0.9319  
83 0 0 1 0 0 0 1 0 -14.0728 -0.3628 0.3584  
84 0 0 1 1 1 1 1 1 -6.7170 1.6251 0.9479  
85 1 0 1 1 1 1 1 1 -6.2222 1.3381 0.9096  
86 0 0 1 1 0 1 1 1 -9.5453 1.6626 0.9518  
87 1 1 1 1 1 1 1 1 -14.7303 -0.2262 0.4105  
88 1 0 1 1 1 1 1 1 -7.7587 1.0518 0.8536  
89 1 0 0 1 0 1 1 1 -11.3711 0.8854 0.812  
90 1 0 1 1 1 1 1 1 -5.9722 1.5776 0.9427  
91 1 0 1 0 0 1 1 1 -12.0262 0.7121 0.7618  
92 1 1 1 1 1 1 1 1 -15.9053 -0.7165 0.2369  
93 0 0 1 1 1 1 1 1 -8.7562 0.8004 0.7883  
94 1 0 1 1 1 1 1 1 -6.7785 1.1243 0.8696  
95 1 0 1 1 1 1 1 1 -6.7520 0.9024 0.8166  
96 0 0 0 0 0 1 1 1 -12.0579 0.0982 0.5391  
97 0 0 0 0 0 1 1 0 -19.1732 -2.1798 0.0146  
98 0 0 1 1 1 1 1 1 -7.6994 1.0130 0.8445  
99 0 0 1 1 1 1 1 0 -9.6249 0.3525 0.6378  
100 1 0 1 1 0 1 1 1 -7.8675 0.9605 0.8316  
101 1 0 1 1 1 1 1 1 -5.8199 1.4798 0.9305  
102 0 0 1 1 1 1 1 1 -6.3597 1.2604 0.8962  
103 1 0 1 1 1 1 1 1 -5.2005 1.5951 0.9447  
104 1 0 1 1 1 1 1 1 -4.7733 1.5213 0.9359  
105 1 1 1 0 1 1 1 1 -9.1258 -0.2949 0.384  
106 1 1 1 1 1 1 1 1 -3.8975 1.3476 0.9111  
107 1 0 0 0 0 1 1 1 -10.2531 0.0711 0.5283  
108 0 0 1 1 1 0 1 0 -21.1737 -2.8821 0.002  
109 0 0 1 0 0 1 1 1 -10.6838 0.6009 0.726  
110 1 0 1 1 1 1 1 0 -12.9438 -0.5536 0.2899  
111 1 1 1 1 1 1 1 1 -10.3920 -0.6478 0.2585  
112 1 0 1 1 1 1 1 0 -16.4577 -1.0616 0.1442  
113 1 0 1 1 1 1 1 1 -10.6973 1.1324 0.8713  
114 1 0 1 1 1 1 1 1 -10.2927 1.4674 0.9289  
115 0 0 1 1 1 1 1 1 -8.8485 0.8886 0.8129  
116 1 0 1 1 1 1 1 1 -10.9543 -0.1881 0.4254  
117 1 0 1 1 1 0 1 1 -10.1800 0.3091 0.6214  
118 0 0 1 1 1 1 0 0 -17.8213 -1.5079 0.0658  
119 1 0 1 1 1 1 1 1 -11.2376 -0.0555 0.4779  
120 1 0 1 1 1 1 1 1 -5.1973 1.2011 0.8852  
121 1 1 1 1 1 1 1 1 -3.9389 1.0187 0.8458  
122 1 1 1 0 0 0 1 1 -20.5661 -2.7493 0.003  
123 0 0 1 1 1 1 1 1 -13.4256 -0.4439 0.3285  
124 1 0 1 1 0 1 1 1 -11.0053 0.1080 0.543  
125 0 0 0 0 0 0 0 0 -5.3828 0.5453 0.7072  
126 1 0 1 0 1 1 1 1 -12.5693 0.2771 0.6091  
127 1 0 1 0 0 0 1 0 -11.3029 0.0235 0.5094  
128 1 0 1 1 1 1 1 1 -11.1612 0.3440 0.6346  
129 0 0 1 0 0 1 1 1 -8.7705 1.2559 0.8954  
130 0 0 0 1 1 1 1 1 -14.2274 -0.2088 0.4173  
131 0 0 0 0 0 1 1 1 -11.8885 -0.1170 0.4534  
132 0 0 1 1 1 1 1 1 -9.4504 1.6593 0.9515  
133 0 0 0 0 0 1 1 1 -12.0165 0.1367 0.5544  
134 1 0 1 1 1 1 1 1 -7.3612 1.1590 0.8768  
135 1 0 1 1 1 1 1 1 -7.7946 0.7728 0.7802  
136 1 0 1 1 1 0 1 1 -13.4591 -0.1992 0.4211  
137 1 0 1 1 0 1 1 1 -11.8874 0.6267 0.7346  
138 0 0 1 0 0 1 1 1 -8.5797 1.5584 0.9404  
139 0 0 0 0 0 1 1 1 -9.9562 1.1461 0.8741  
140 1 0 1 1 1 1 1 1 -11.4269 1.1163 0.8679  
141 1 0 1 1 1 1 1 1 -11.6387 0.6821 0.7524  
142 1 1 1 1 1 1 1 1 -6.6069 0.7458 0.7721  
143 0 0 0 0 0 1 1 1 -11.9480 -0.0183 0.4927  
144 0 0 1 1 1 1 1 1 -11.4187 1.0305 0.8486  
145 0 0 1 1 1 1 1 1 -11.5193 0.0969 0.5386  
146 0 0 1 1 1 1 1 1 -9.0738 0.6089 0.7287  
147 1 0 1 1 1 1 1 1 -7.5033 0.6795 0.7516  
148 0 0 0 0 0 1 1 1 -12.6590 -0.1901 0.4246  
149 0 0 1 0 1 1 1 1 -10.4381 0.2803 0.6104  
150 0 0 1 1 1 1 1 1 -7.9519 0.8427 0.8003  
151 1 1 1 1 1 0 1 1 -7.2112 0.0667 0.5266  
152 1 1 1 1 1 1 1 1 -3.3839 1.3976 0.9189  
153 1 0 0 0 0 1 1 1 -16.0020 -1.2681 0.1024  
154 1 1 1 1 1 1 1 1 -11.0613 -1.1893 0.1172  
155 1 0 1 1 1 1 1 1 -8.5351 0.7865 0.7842  
156 0 0 1 1 1 0 1 1 -14.1792 -0.1239 0.4507  
157 1 0 1 1 1 1 1 1 -11.6506 0.6032 0.7268  
158 0 0 1 1 1 1 1 1 -10.4453 1.4219 0.9225  
159 1 0 1 1 1 1 1 1 -6.6872 1.0283 0.8481  
160 1 1 1 1 0 1 1 1 -8.0553 -0.2133 0.4156  
161 1 1 1 1 1 1 1 1 -3.3022 1.2174 0.8883  
162 1 0 1 1 1 1 1 1 -11.9063 -0.1513 0.4399  
163 1 0 1 1 1 1 1 1 -11.9432 0.2537 0.6001  
164 1 0 1 1 0 1 1 1 -14.3787 -0.1299 0.4483  
165 1 0 1 0 0 1 1 1 -17.9101 -1.5872 0.0562  
166 1 1 1 1 1 1 1 1 -7.2691 0.2424 0.5958  
167 1 1 1 1 1 1 1 1 -9.7146 -0.2810 0.3893  
168 0 0 1 1 0 0 1 1 -13.1431 -0.5735 0.2831  
169 1 0 1 1 1 1 1 1 -6.3257 0.8086 0.7906  
170 0 0 1 1 1 1 1 1 -10.5889 -0.1003 0.4601  
171 0 0 1 1 1 1 1 1 -13.5217 0.2422 0.5957  
172 1 0 1 1 1 1 1 1 -11.8455 -0.1595 0.4366  
173 1 0 1 1 1 1 1 1 -6.9923 0.7827 0.7831  
174 1 1 1 1 1 1 1 1 -3.2300 1.1343 0.8717  
175 1 1 1 1 0 1 1 1 -12.9220 -1.2587 0.1041  
176 1 1 1 0 0 1 1 0 -19.1310 -2.9147 0.0018  
177 1 0 1 0 1 1 1 1 -15.7928 -0.8666 0.1931  
178 0 0 1 1 1 0 1 1 -13.6854 -0.9607 0.1684  
179 1 0 1 1 1 1 1 1 -16.8540 -1.1238 0.1305  
180 1 1 1 1 1 1 1 1 -7.0207 0.1488 0.5592  
181 0 0 1 1 0 1 1 1 -10.8760 -0.5568 0.2888  
182 1 1 1 1 1 1 1 1 -2.8786 1.0557 0.8545

# Checking Assumptions

## Unidimensionality

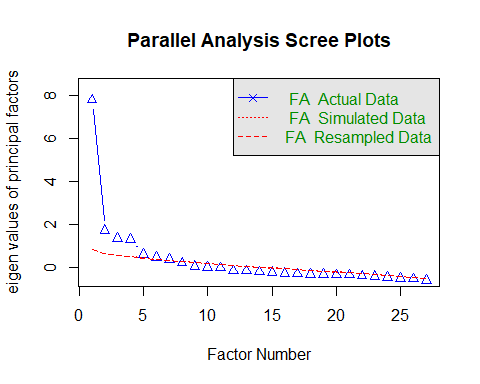
set.seed(2025)  
unidimTest(irt.data4) #Take A long time, insert # if want to skip and avoid long time

Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value

Unidimensionality Check using Modified Parallel Analysis  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Matrix of tertachoric correlations  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.0000 0.3928 0.3148 0.1320 0.5059 0.3255 0.1825 0.1469 0.3468 0.4518  
K4 0.3928 1.0000 0.6331 0.2507 0.0433 0.2261 0.3858 0.3139 0.3245 0.1661  
K6 0.3148 0.6331 1.0000 0.4902 0.3832 0.2398 0.3888 0.2308 0.4773 0.3316  
K12 0.1320 0.2507 0.4902 1.0000 0.0214 -0.1444 0.1427 0.1429 0.2160 0.4846  
K13 0.5059 0.0433 0.3832 0.0214 1.0000 0.6660 0.7007 0.7606 0.5424 0.5789  
K14 0.3255 0.2261 0.2398 -0.1444 0.6660 1.0000 0.7623 0.7745 0.5468 0.2252  
K15 0.1825 0.3858 0.3888 0.1427 0.7007 0.7623 1.0000 0.9762 0.7444 0.3013  
K16 0.1469 0.3139 0.2308 0.1429 0.7606 0.7745 0.9762 1.0000 0.7282 0.3529  
K17 0.3468 0.3245 0.4773 0.2160 0.5424 0.5468 0.7444 0.7282 1.0000 0.3172  
K18 0.4518 0.1661 0.3316 0.4846 0.5789 0.2252 0.3013 0.3529 0.3172 1.0000  
K19 0.3259 0.1517 0.3846 0.2008 0.5128 0.4997 0.6328 0.5927 0.5793 0.4284  
K20 0.3594 0.2639 0.4642 0.2442 0.4151 0.4505 0.5965 0.4979 0.5575 0.5022  
K21 0.3617 0.1726 0.4310 0.2808 0.5356 0.4428 0.5497 0.4479 0.5450 0.5584  
K22 0.4047 0.0783 0.4565 0.2361 0.5326 0.4062 0.5311 0.4282 0.5353 0.5628  
K23 0.3136 0.2121 0.4624 0.2789 0.5047 0.2601 0.4998 0.4659 0.5210 0.5196  
K25 0.4635 0.1576 0.4843 0.3465 0.5639 0.2948 0.4741 0.4425 0.5706 0.5902  
K26 0.2759 -0.0087 0.2348 0.4139 0.1877 -0.1414 0.1298 0.0776 0.1082 0.4965  
K27 0.0725 0.3402 0.3435 0.3352 0.4309 0.3632 0.4736 0.4820 0.4527 0.5069  
K28 0.0078 0.1038 0.1641 0.3543 0.2957 0.0944 0.2803 0.3083 0.2609 0.4440  
K29 0.2130 0.2084 0.3416 0.5093 0.0814 -0.0831 0.2481 0.1625 0.0987 0.4122  
K30 0.1337 0.3832 0.5244 0.4499 0.4308 0.3686 0.5640 0.5149 0.5174 0.5107  
K31 0.3073 0.2538 0.5238 0.5265 0.5015 0.2962 0.3774 0.3986 0.5742 0.5170  
K32 0.2278 0.1747 0.3138 0.3293 0.4650 0.3298 0.4634 0.4282 0.4784 0.4583  
K33 0.3087 0.1603 0.1712 0.3029 0.3877 0.2972 0.4935 0.4881 0.5002 0.3650  
K34 0.3850 0.1232 0.1736 0.1304 0.3888 0.4196 0.4559 0.5432 0.5987 0.3579  
K36 0.5792 0.1361 0.2306 0.1801 0.5359 0.4805 0.3828 0.2388 0.7232 0.5003  
K37 0.4313 0.3541 0.2018 0.1154 0.4287 0.5080 0.5003 0.4096 0.7176 0.2208  
 K19 K20 K21 K22 K23 K25 K26 K27 K28 K29  
K1 0.3259 0.3594 0.3617 0.4047 0.3136 0.4635 0.2759 0.0725 0.0078 0.2130  
K4 0.1517 0.2639 0.1726 0.0783 0.2121 0.1576 -0.0087 0.3402 0.1038 0.2084  
K6 0.3846 0.4642 0.4310 0.4565 0.4624 0.4843 0.2348 0.3435 0.1641 0.3416  
K12 0.2008 0.2442 0.2808 0.2361 0.2789 0.3465 0.4139 0.3352 0.3543 0.5093  
K13 0.5128 0.4151 0.5356 0.5326 0.5047 0.5639 0.1877 0.4309 0.2957 0.0814  
K14 0.4997 0.4505 0.4428 0.4062 0.2601 0.2948 -0.1414 0.3632 0.0944 -0.0831  
K15 0.6328 0.5965 0.5497 0.5311 0.4998 0.4741 0.1298 0.4736 0.2803 0.2481  
K16 0.5927 0.4979 0.4479 0.4282 0.4659 0.4425 0.0776 0.4820 0.3083 0.1625  
K17 0.5793 0.5575 0.5450 0.5353 0.5210 0.5706 0.1082 0.4527 0.2609 0.0987  
K18 0.4284 0.5022 0.5584 0.5628 0.5196 0.5902 0.4965 0.5069 0.4440 0.4122  
K19 1.0000 0.9707 0.9535 0.9573 0.7310 0.7921 0.2797 0.4877 0.3902 0.3552  
K20 0.9707 1.0000 0.9747 0.9592 0.6793 0.6933 0.2996 0.4885 0.4278 0.4357  
K21 0.9535 0.9747 1.0000 0.9870 0.7417 0.8356 0.3028 0.4822 0.3967 0.4447  
K22 0.9573 0.9592 0.9870 1.0000 0.7046 0.7841 0.3378 0.3648 0.3960 0.3982  
K23 0.7310 0.6793 0.7417 0.7046 1.0000 0.9879 0.4691 0.5236 0.4112 0.4936  
K25 0.7921 0.6933 0.8356 0.7841 0.9879 1.0000 0.4410 0.4376 0.3571 0.4470  
K26 0.2797 0.2996 0.3028 0.3378 0.4691 0.4410 1.0000 0.6339 0.7336 0.8035  
K27 0.4877 0.4885 0.4822 0.3648 0.5236 0.4376 0.6339 1.0000 0.9278 0.7821  
K28 0.3902 0.4278 0.3967 0.3960 0.4112 0.3571 0.7336 0.9278 1.0000 0.7858  
K29 0.3552 0.4357 0.4447 0.3982 0.4936 0.4470 0.8035 0.7821 0.7858 1.0000  
K30 0.6034 0.5293 0.5489 0.5172 0.9842 0.9803 0.7777 0.8927 0.7413 0.9903  
K31 0.7767 0.7943 0.7782 0.7672 0.8178 0.8320 0.6168 0.6878 0.4983 0.5685  
K32 0.7310 0.7059 0.6373 0.5975 0.7605 0.8422 0.3736 0.5236 0.4795 0.3989  
K33 0.6524 0.6383 0.5838 0.5566 0.6877 0.7769 0.4017 0.4974 0.4350 0.3803  
K34 0.4549 0.3980 0.3512 0.2622 0.4834 0.5013 0.0720 0.3522 0.1376 0.2383  
K36 0.3192 0.4397 0.5677 0.5923 0.7833 0.8243 0.3960 0.2507 0.3220 0.3546  
K37 0.3516 0.4302 0.4178 0.3597 0.5210 0.5234 0.1029 0.3502 0.3323 0.3468  
 K30 K31 K32 K33 K34 K36 K37  
K1 0.1337 0.3073 0.2278 0.3087 0.3850 0.5792 0.4313  
K4 0.3832 0.2538 0.1747 0.1603 0.1232 0.1361 0.3541  
K6 0.5244 0.5238 0.3138 0.1712 0.1736 0.2306 0.2018  
K12 0.4499 0.5265 0.3293 0.3029 0.1304 0.1801 0.1154  
K13 0.4308 0.5015 0.4650 0.3877 0.3888 0.5359 0.4287  
K14 0.3686 0.2962 0.3298 0.2972 0.4196 0.4805 0.5080  
K15 0.5640 0.3774 0.4634 0.4935 0.4559 0.3828 0.5003  
K16 0.5149 0.3986 0.4282 0.4881 0.5432 0.2388 0.4096  
K17 0.5174 0.5742 0.4784 0.5002 0.5987 0.7232 0.7176  
K18 0.5107 0.5170 0.4583 0.3650 0.3579 0.5003 0.2208  
K19 0.6034 0.7767 0.7310 0.6524 0.4549 0.3192 0.3516  
K20 0.5293 0.7943 0.7059 0.6383 0.3980 0.4397 0.4302  
K21 0.5489 0.7782 0.6373 0.5838 0.3512 0.5677 0.4178  
K22 0.5172 0.7672 0.5975 0.5566 0.2622 0.5923 0.3597  
K23 0.9842 0.8178 0.7605 0.6877 0.4834 0.7833 0.5210  
K25 0.9803 0.8320 0.8422 0.7769 0.5013 0.8243 0.5234  
K26 0.7777 0.6168 0.3736 0.4017 0.0720 0.3960 0.1029  
K27 0.8927 0.6878 0.5236 0.4974 0.3522 0.2507 0.3502  
K28 0.7413 0.4983 0.4795 0.4350 0.1376 0.3220 0.3323  
K29 0.9903 0.5685 0.3989 0.3803 0.2383 0.3546 0.3468  
K30 1.0000 0.9778 0.5139 0.4134 0.4220 0.9558 0.5005  
K31 0.9778 1.0000 0.8931 0.8963 0.3753 0.6756 0.5157  
K32 0.5139 0.8931 1.0000 0.9411 0.5231 0.6072 0.6057  
K33 0.4134 0.8963 0.9411 1.0000 0.5306 0.6268 0.5512  
K34 0.4220 0.3753 0.5231 0.5306 1.0000 0.8841 0.7444  
K36 0.9558 0.6756 0.6072 0.6268 0.8841 1.0000 0.9111  
K37 0.5005 0.5157 0.6057 0.5512 0.7444 0.9111 1.0000  
  
Alternative hypothesis: the second eigenvalue of the observed data is substantially larger   
 than the second eigenvalue of data under the assumed IRT model  
  
Second eigenvalue in the observed data: 3.3002  
Average of second eigenvalues in Monte Carlo samples: 1.8546  
Monte Carlo samples: 100  
p-value: 0.0099

### Checking Dominant Factor (Essential Unidimensionality)

# Extract the response data from the fitted model  
irt\_mat <- as.matrix(irt.data4$X)  
  
# Parallel analysis  
library(psych)  
fa.parallel(irt\_mat, fa="fa")



Parallel analysis suggests that the number of factors = 7 and the number of components = NA

# Eigenvalues  
ev <- eigen(cor(irt\_mat, use = "pairwise.complete.obs"))$values  
  
# First and second eigenvalues  
first\_ev <- ev[1]  
second\_ev <- ev[2]  
  
# Ratio  
dominance\_ratio <- first\_ev / second\_ev  
  
# Print  
first\_ev

[1] 8.423431

second\_ev

[1] 2.509731

dominance\_ratio

[1] 3.356309

Parallel analysis suggested up to seven factors, as seven eigenvalues from the actual data exceeded those from randomly simulated data. However, the scree plot demonstrated a sharp drop between the first (9.02) and second (2.51) eigenvalues, yielding a ratio of 3.59. This indicates a single dominant factor underlying item responses, with additional weaker factors. Consistent with the concept of **essential unidimensionality** (Reckase, 1979; Hambleton, Swaminathan, & Rogers, 1991; Embretson & Reise, 2000), the scale was considered suitable for unidimensional IRT modeling despite the presence of minor secondary dimensions.

class(irt.data4)

[1] "ltm"

str(irt.data4, max.level = 1)

List of 14  
 $ coefficients: num [1:27, 1:2] 1.274 -0.661 -2.182 -1.423 1.482 ...  
 ..- attr(\*, "dimnames")=List of 2  
 $ log.Lik : num -2476  
 $ convergence : int 0  
 $ hessian : num [1:54, 1:54] 29.894 -0.195 -0.172 -0.167 -0.325 ...  
 $ counts : Named int [1:2] 3 1  
 ..- attr(\*, "names")= chr [1:2] "function" "gradient"  
 $ patterns :List of 2  
 $ GH :List of 2  
 $ max.sc : num 0.00487  
 $ ltst :List of 5  
 $ X : tibble [204 × 27] (S3: tbl\_df/tbl/data.frame)  
 $ control :List of 6  
 $ IRT.param : logi TRUE  
 $ formula :Class 'formula' language data4 ~ z1  
 .. ..- attr(\*, ".Environment")=<environment: R\_GlobalEnv>   
 $ call : language ltm(formula = data4 ~ z1, IRT.param = TRUE)  
 - attr(\*, "class")= chr "ltm"

## Local Independence

### Yen’s Q3 residual cYen’s Q3 residual correlationsorrelations

# ==========================================  
# Load required package  
# ==========================================  
library(mirt)  
library(mokken)

Loading required package: poLCA

Loading required package: scatterplot3d

Attaching package: 'mokken'

The following object is masked from 'package:dplyr':  
  
 recode

The following object is masked from 'package:psych':  
  
 ICC

# Extract the raw item response matrix from the ltm object  
irt\_mat <- as.data.frame(irt.data4$X) # now it's a dataframe  
irt\_mat <- as.matrix(irt\_mat) # convert to numeric matrix  
  
  
  
# ==========================================  
# 1. Fit a unidimensional 2PL model  
# ==========================================  
  
mod2pl <- mirt(irt\_mat, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

mod1pl <- mirt(irt\_mat, 1, itemtype = "Rasch")

# ==========================================  
# 2. Assumption: Local Independence  
# ==========================================  
# (a) Yen’s Q3 residual correlations  
# Q3\_resid <- resid(mod2pl, type = "Q3")  
# Assumption 2: Local Independence  
Q3\_resid <- residuals(mod2pl, type = "Q3")

Q3 summary statistics:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.480 -0.094 -0.025 -0.007 0.055 0.800   
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.000 0.151 0.052 -0.015 0.170 0.069 -0.065 -0.073 0.012 0.133  
K4 0.151 1.000 0.295 0.077 -0.078 0.046 0.129 0.074 0.081 -0.013  
K6 0.052 0.295 1.000 0.209 0.050 -0.005 0.038 -0.064 0.064 0.026  
K12 -0.015 0.077 0.209 1.000 -0.110 -0.211 -0.070 -0.059 -0.024 0.176  
K13 0.170 -0.078 0.050 -0.110 1.000 0.301 0.203 0.230 0.101 0.160  
K14 0.069 0.046 -0.005 -0.211 0.301 1.000 0.350 0.356 0.170 -0.067  
K15 -0.065 0.129 0.038 -0.070 0.203 0.350 1.000 0.800 0.183 -0.088  
K16 -0.073 0.074 -0.064 -0.059 0.230 0.356 0.800 1.000 0.180 -0.015  
K17 0.012 0.081 0.064 -0.024 0.101 0.170 0.183 0.180 1.000 -0.094  
K18 0.133 -0.013 0.026 0.176 0.160 -0.067 -0.088 -0.015 -0.094 1.000  
K19 -0.083 -0.090 -0.113 -0.114 -0.042 0.066 -0.002 0.079 -0.053 -0.200  
K20 -0.027 0.100 -0.024 -0.066 -0.217 -0.025 -0.075 -0.106 -0.078 -0.078  
K21 -0.004 -0.041 -0.109 -0.038 -0.051 -0.074 -0.205 -0.237 -0.123 0.021  
K22 0.052 -0.162 -0.046 -0.097 -0.050 -0.099 -0.163 -0.206 -0.095 0.006  
K23 -0.052 0.008 0.027 -0.021 -0.031 -0.141 -0.034 -0.020 -0.048 -0.014  
K25 0.083 -0.047 0.013 -0.004 -0.017 -0.137 -0.083 -0.068 -0.025 0.009  
K26 0.042 -0.121 -0.016 0.140 -0.068 -0.285 -0.188 -0.202 -0.149 0.168  
K27 -0.137 0.063 -0.013 0.069 0.029 -0.002 -0.032 0.008 0.008 0.084  
K28 -0.154 -0.068 -0.087 0.104 -0.014 -0.144 -0.114 -0.061 -0.062 0.093  
K29 -0.020 0.022 0.037 0.202 -0.176 -0.273 -0.119 -0.162 -0.194 0.067  
K30 -0.118 0.031 0.109 0.142 0.001 -0.059 -0.043 -0.063 0.003 0.019  
K31 -0.108 0.018 0.040 0.088 -0.104 -0.123 -0.150 -0.091 -0.059 -0.099  
K32 -0.110 -0.023 -0.038 0.030 -0.038 -0.050 -0.043 -0.033 -0.068 -0.049  
K33 -0.018 -0.032 -0.106 0.037 -0.074 -0.062 0.021 0.052 -0.022 -0.095  
K34 0.082 -0.019 -0.026 -0.037 0.025 0.106 0.078 0.139 0.177 0.013  
K36 0.116 -0.044 -0.007 -0.065 -0.001 0.036 -0.025 -0.090 0.122 -0.022  
K37 0.089 0.104 -0.022 -0.060 0.014 0.150 0.085 0.042 0.266 -0.127  
 K19 K20 K21 K22 K23 K25 K26 K27 K28 K29  
K1 -0.083 -0.027 -0.004 0.052 -0.052 0.083 0.042 -0.137 -0.154 -0.020  
K4 -0.090 0.100 -0.041 -0.162 0.008 -0.047 -0.121 0.063 -0.068 0.022  
K6 -0.113 -0.024 -0.109 -0.046 0.027 0.013 -0.016 -0.013 -0.087 0.037  
K12 -0.114 -0.066 -0.038 -0.097 -0.021 -0.004 0.140 0.069 0.104 0.202  
K13 -0.042 -0.217 -0.051 -0.050 -0.031 -0.017 -0.068 0.029 -0.014 -0.176  
K14 0.066 -0.025 -0.074 -0.099 -0.141 -0.137 -0.285 -0.002 -0.144 -0.273  
K15 -0.002 -0.075 -0.205 -0.163 -0.034 -0.083 -0.188 -0.032 -0.114 -0.119  
K16 0.079 -0.106 -0.237 -0.206 -0.020 -0.068 -0.202 0.008 -0.061 -0.162  
K17 -0.053 -0.078 -0.123 -0.095 -0.048 -0.025 -0.149 0.008 -0.062 -0.194  
K18 -0.200 -0.078 0.021 0.006 -0.014 0.009 0.168 0.084 0.093 0.067  
K19 1.000 0.087 -0.270 -0.026 -0.187 -0.181 -0.144 -0.063 -0.105 -0.171  
K20 0.087 1.000 -0.018 -0.027 -0.343 -0.480 -0.111 -0.083 -0.049 -0.045  
K21 -0.270 -0.018 1.000 0.297 -0.243 -0.100 -0.104 -0.141 -0.149 -0.081  
K22 -0.026 -0.027 0.297 1.000 -0.272 -0.226 -0.053 -0.326 -0.144 -0.157  
K23 -0.187 -0.343 -0.243 -0.272 1.000 0.697 0.090 -0.004 -0.030 0.050  
K25 -0.181 -0.480 -0.100 -0.226 0.697 1.000 0.034 -0.126 -0.127 -0.048  
K26 -0.144 -0.111 -0.104 -0.053 0.090 0.034 1.000 0.197 0.390 0.502  
K27 -0.063 -0.083 -0.141 -0.326 -0.004 -0.126 0.197 1.000 0.556 0.325  
K28 -0.105 -0.049 -0.149 -0.144 -0.030 -0.127 0.390 0.556 1.000 0.448  
K29 -0.171 -0.045 -0.081 -0.157 0.050 -0.048 0.502 0.325 0.448 1.000  
K30 -0.043 -0.182 -0.179 -0.189 0.104 0.062 0.176 0.399 0.198 0.243  
K31 -0.119 -0.069 -0.210 -0.180 0.036 -0.055 0.215 0.061 0.035 0.097  
K32 -0.042 -0.098 -0.345 -0.376 0.094 0.174 0.002 0.020 0.058 -0.027  
K33 -0.073 -0.111 -0.299 -0.313 0.037 0.090 0.045 0.036 0.040 -0.017  
K34 -0.032 -0.102 -0.195 -0.250 0.062 0.079 -0.127 0.002 -0.099 -0.040  
K36 -0.342 -0.194 -0.029 0.023 0.147 0.223 0.026 -0.093 -0.040 -0.055  
K37 -0.198 -0.082 -0.137 -0.171 0.037 0.011 -0.127 -0.016 0.012 0.012  
 K30 K31 K32 K33 K34 K36 K37  
K1 -0.118 -0.108 -0.110 -0.018 0.082 0.116 0.089  
K4 0.031 0.018 -0.023 -0.032 -0.019 -0.044 0.104  
K6 0.109 0.040 -0.038 -0.106 -0.026 -0.007 -0.022  
K12 0.142 0.088 0.030 0.037 -0.037 -0.065 -0.060  
K13 0.001 -0.104 -0.038 -0.074 0.025 -0.001 0.014  
K14 -0.059 -0.123 -0.050 -0.062 0.106 0.036 0.150  
K15 -0.043 -0.150 -0.043 0.021 0.078 -0.025 0.085  
K16 -0.063 -0.091 -0.033 0.052 0.139 -0.090 0.042  
K17 0.003 -0.059 -0.068 -0.022 0.177 0.122 0.266  
K18 0.019 -0.099 -0.049 -0.095 0.013 -0.022 -0.127  
K19 -0.043 -0.119 -0.042 -0.073 -0.032 -0.342 -0.198  
K20 -0.182 -0.069 -0.098 -0.111 -0.102 -0.194 -0.082  
K21 -0.179 -0.210 -0.345 -0.299 -0.195 -0.029 -0.137  
K22 -0.189 -0.180 -0.376 -0.313 -0.250 0.023 -0.171  
K23 0.104 0.036 0.094 0.037 0.062 0.147 0.037  
K25 0.062 -0.055 0.174 0.090 0.079 0.223 0.011  
K26 0.176 0.215 0.002 0.045 -0.127 0.026 -0.127  
K27 0.399 0.061 0.020 0.036 0.002 -0.093 -0.016  
K28 0.198 0.035 0.058 0.040 -0.099 -0.040 0.012  
K29 0.243 0.097 -0.027 -0.017 -0.040 -0.055 0.012  
K30 1.000 0.037 -0.035 -0.081 0.007 0.021 0.019  
K31 0.037 1.000 0.269 0.254 -0.121 -0.051 -0.063  
K32 -0.035 0.269 1.000 0.581 0.108 -0.001 0.137  
K33 -0.081 0.254 0.581 1.000 0.116 0.012 0.084  
K34 0.007 -0.121 0.108 0.116 1.000 0.371 0.351  
K36 0.021 -0.051 -0.001 0.012 0.371 1.000 0.388  
K37 0.019 -0.063 0.137 0.084 0.351 0.388 1.000

Q3\_resid # inspect residual correlations (Q3 > .20 may indicate local dependence)

K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.000 0.151 0.052 -0.015 0.170 0.069 -0.065 -0.073 0.012 0.133  
K4 0.151 1.000 0.295 0.077 -0.078 0.046 0.129 0.074 0.081 -0.013  
K6 0.052 0.295 1.000 0.209 0.050 -0.005 0.038 -0.064 0.064 0.026  
K12 -0.015 0.077 0.209 1.000 -0.110 -0.211 -0.070 -0.059 -0.024 0.176  
K13 0.170 -0.078 0.050 -0.110 1.000 0.301 0.203 0.230 0.101 0.160  
K14 0.069 0.046 -0.005 -0.211 0.301 1.000 0.350 0.356 0.170 -0.067  
K15 -0.065 0.129 0.038 -0.070 0.203 0.350 1.000 0.800 0.183 -0.088  
K16 -0.073 0.074 -0.064 -0.059 0.230 0.356 0.800 1.000 0.180 -0.015  
K17 0.012 0.081 0.064 -0.024 0.101 0.170 0.183 0.180 1.000 -0.094  
K18 0.133 -0.013 0.026 0.176 0.160 -0.067 -0.088 -0.015 -0.094 1.000  
K19 -0.083 -0.090 -0.113 -0.114 -0.042 0.066 -0.002 0.079 -0.053 -0.200  
K20 -0.027 0.100 -0.024 -0.066 -0.217 -0.025 -0.075 -0.106 -0.078 -0.078  
K21 -0.004 -0.041 -0.109 -0.038 -0.051 -0.074 -0.205 -0.237 -0.123 0.021  
K22 0.052 -0.162 -0.046 -0.097 -0.050 -0.099 -0.163 -0.206 -0.095 0.006  
K23 -0.052 0.008 0.027 -0.021 -0.031 -0.141 -0.034 -0.020 -0.048 -0.014  
K25 0.083 -0.047 0.013 -0.004 -0.017 -0.137 -0.083 -0.068 -0.025 0.009  
K26 0.042 -0.121 -0.016 0.140 -0.068 -0.285 -0.188 -0.202 -0.149 0.168  
K27 -0.137 0.063 -0.013 0.069 0.029 -0.002 -0.032 0.008 0.008 0.084  
K28 -0.154 -0.068 -0.087 0.104 -0.014 -0.144 -0.114 -0.061 -0.062 0.093  
K29 -0.020 0.022 0.037 0.202 -0.176 -0.273 -0.119 -0.162 -0.194 0.067  
K30 -0.118 0.031 0.109 0.142 0.001 -0.059 -0.043 -0.063 0.003 0.019  
K31 -0.108 0.018 0.040 0.088 -0.104 -0.123 -0.150 -0.091 -0.059 -0.099  
K32 -0.110 -0.023 -0.038 0.030 -0.038 -0.050 -0.043 -0.033 -0.068 -0.049  
K33 -0.018 -0.032 -0.106 0.037 -0.074 -0.062 0.021 0.052 -0.022 -0.095  
K34 0.082 -0.019 -0.026 -0.037 0.025 0.106 0.078 0.139 0.177 0.013  
K36 0.116 -0.044 -0.007 -0.065 -0.001 0.036 -0.025 -0.090 0.122 -0.022  
K37 0.089 0.104 -0.022 -0.060 0.014 0.150 0.085 0.042 0.266 -0.127  
 K19 K20 K21 K22 K23 K25 K26 K27 K28 K29  
K1 -0.083 -0.027 -0.004 0.052 -0.052 0.083 0.042 -0.137 -0.154 -0.020  
K4 -0.090 0.100 -0.041 -0.162 0.008 -0.047 -0.121 0.063 -0.068 0.022  
K6 -0.113 -0.024 -0.109 -0.046 0.027 0.013 -0.016 -0.013 -0.087 0.037  
K12 -0.114 -0.066 -0.038 -0.097 -0.021 -0.004 0.140 0.069 0.104 0.202  
K13 -0.042 -0.217 -0.051 -0.050 -0.031 -0.017 -0.068 0.029 -0.014 -0.176  
K14 0.066 -0.025 -0.074 -0.099 -0.141 -0.137 -0.285 -0.002 -0.144 -0.273  
K15 -0.002 -0.075 -0.205 -0.163 -0.034 -0.083 -0.188 -0.032 -0.114 -0.119  
K16 0.079 -0.106 -0.237 -0.206 -0.020 -0.068 -0.202 0.008 -0.061 -0.162  
K17 -0.053 -0.078 -0.123 -0.095 -0.048 -0.025 -0.149 0.008 -0.062 -0.194  
K18 -0.200 -0.078 0.021 0.006 -0.014 0.009 0.168 0.084 0.093 0.067  
K19 1.000 0.087 -0.270 -0.026 -0.187 -0.181 -0.144 -0.063 -0.105 -0.171  
K20 0.087 1.000 -0.018 -0.027 -0.343 -0.480 -0.111 -0.083 -0.049 -0.045  
K21 -0.270 -0.018 1.000 0.297 -0.243 -0.100 -0.104 -0.141 -0.149 -0.081  
K22 -0.026 -0.027 0.297 1.000 -0.272 -0.226 -0.053 -0.326 -0.144 -0.157  
K23 -0.187 -0.343 -0.243 -0.272 1.000 0.697 0.090 -0.004 -0.030 0.050  
K25 -0.181 -0.480 -0.100 -0.226 0.697 1.000 0.034 -0.126 -0.127 -0.048  
K26 -0.144 -0.111 -0.104 -0.053 0.090 0.034 1.000 0.197 0.390 0.502  
K27 -0.063 -0.083 -0.141 -0.326 -0.004 -0.126 0.197 1.000 0.556 0.325  
K28 -0.105 -0.049 -0.149 -0.144 -0.030 -0.127 0.390 0.556 1.000 0.448  
K29 -0.171 -0.045 -0.081 -0.157 0.050 -0.048 0.502 0.325 0.448 1.000  
K30 -0.043 -0.182 -0.179 -0.189 0.104 0.062 0.176 0.399 0.198 0.243  
K31 -0.119 -0.069 -0.210 -0.180 0.036 -0.055 0.215 0.061 0.035 0.097  
K32 -0.042 -0.098 -0.345 -0.376 0.094 0.174 0.002 0.020 0.058 -0.027  
K33 -0.073 -0.111 -0.299 -0.313 0.037 0.090 0.045 0.036 0.040 -0.017  
K34 -0.032 -0.102 -0.195 -0.250 0.062 0.079 -0.127 0.002 -0.099 -0.040  
K36 -0.342 -0.194 -0.029 0.023 0.147 0.223 0.026 -0.093 -0.040 -0.055  
K37 -0.198 -0.082 -0.137 -0.171 0.037 0.011 -0.127 -0.016 0.012 0.012  
 K30 K31 K32 K33 K34 K36 K37  
K1 -0.118 -0.108 -0.110 -0.018 0.082 0.116 0.089  
K4 0.031 0.018 -0.023 -0.032 -0.019 -0.044 0.104  
K6 0.109 0.040 -0.038 -0.106 -0.026 -0.007 -0.022  
K12 0.142 0.088 0.030 0.037 -0.037 -0.065 -0.060  
K13 0.001 -0.104 -0.038 -0.074 0.025 -0.001 0.014  
K14 -0.059 -0.123 -0.050 -0.062 0.106 0.036 0.150  
K15 -0.043 -0.150 -0.043 0.021 0.078 -0.025 0.085  
K16 -0.063 -0.091 -0.033 0.052 0.139 -0.090 0.042  
K17 0.003 -0.059 -0.068 -0.022 0.177 0.122 0.266  
K18 0.019 -0.099 -0.049 -0.095 0.013 -0.022 -0.127  
K19 -0.043 -0.119 -0.042 -0.073 -0.032 -0.342 -0.198  
K20 -0.182 -0.069 -0.098 -0.111 -0.102 -0.194 -0.082  
K21 -0.179 -0.210 -0.345 -0.299 -0.195 -0.029 -0.137  
K22 -0.189 -0.180 -0.376 -0.313 -0.250 0.023 -0.171  
K23 0.104 0.036 0.094 0.037 0.062 0.147 0.037  
K25 0.062 -0.055 0.174 0.090 0.079 0.223 0.011  
K26 0.176 0.215 0.002 0.045 -0.127 0.026 -0.127  
K27 0.399 0.061 0.020 0.036 0.002 -0.093 -0.016  
K28 0.198 0.035 0.058 0.040 -0.099 -0.040 0.012  
K29 0.243 0.097 -0.027 -0.017 -0.040 -0.055 0.012  
K30 1.000 0.037 -0.035 -0.081 0.007 0.021 0.019  
K31 0.037 1.000 0.269 0.254 -0.121 -0.051 -0.063  
K32 -0.035 0.269 1.000 0.581 0.108 -0.001 0.137  
K33 -0.081 0.254 0.581 1.000 0.116 0.012 0.084  
K34 0.007 -0.121 0.108 0.116 1.000 0.371 0.351  
K36 0.021 -0.051 -0.001 0.012 0.371 1.000 0.388  
K37 0.019 -0.063 0.137 0.084 0.351 0.388 1.000

# Chen & Thissen’s LD χ² statistic  
LD\_resid <- residuals(mod2pl, type = "LD")

LD matrix (lower triangle) and standardized residual correlations (upper triangle)  
  
Upper triangle summary:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.313 -0.101 -0.041 -0.018 0.052 0.588   
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K19  
K1 0.134 0.053 -0.012 0.137 0.055 -0.070 -0.076 0.030 0.104 -0.070  
K4 3.664 0.283 0.081 -0.085 0.035 0.107 0.069 0.058 -0.027 -0.114  
K6 0.584 16.307 0.190 0.055 0.010 0.050 -0.029 0.071 0.031 -0.030  
K12 0.031 1.329 7.370 -0.096 -0.191 -0.057 -0.046 -0.023 0.155 -0.087  
K13 3.843 1.476 0.607 1.894 0.228 0.161 0.192 0.080 0.128 -0.050  
K14 0.612 0.255 0.021 7.463 10.606 0.280 0.296 0.118 -0.068 -0.019  
K15 1.000 2.321 0.503 0.661 5.295 16.005 0.588 0.145 -0.090 -0.037  
K16 1.187 0.964 0.176 0.426 7.488 17.859 70.602 0.148 -0.032 -0.026  
K17 0.187 0.693 1.040 0.105 1.298 2.822 4.286 4.463 -0.069 -0.060  
K18 2.217 0.147 0.194 4.889 3.359 0.940 1.656 0.208 0.981 -0.158  
K19 1.014 2.650 0.180 1.527 0.520 0.077 0.282 0.140 0.734 5.116   
K20 0.495 0.318 0.110 0.687 2.732 0.422 0.928 2.078 1.140 2.081 0.715  
K21 0.533 2.073 0.004 0.271 0.312 0.618 2.642 4.304 1.673 0.763 0.712  
K22 0.142 5.130 0.135 0.789 0.321 1.236 2.923 4.673 1.802 0.561 0.489  
K23 0.768 0.712 0.683 0.321 0.342 3.719 1.363 1.026 0.858 0.456 5.740  
K25 0.960 2.175 1.143 0.853 0.778 3.705 3.202 2.472 1.126 0.832 6.016  
K26 0.156 2.565 0.010 3.812 1.033 13.065 4.409 5.305 3.784 2.934 5.659  
K27 3.572 1.038 0.106 0.664 0.157 0.064 0.043 0.216 0.169 0.557 5.344  
K28 4.354 0.630 0.549 1.780 0.199 3.580 1.345 0.336 1.183 0.622 4.162  
K29 0.147 0.035 0.476 6.919 4.760 11.853 2.016 3.654 5.721 0.242 5.516  
K30 2.285 1.185 2.094 2.224 0.126 0.110 0.191 0.119 0.216 0.171 3.078  
K31 1.478 0.947 1.463 2.746 1.023 3.101 5.737 2.967 1.044 1.167 3.979  
K32 1.598 0.865 0.274 0.345 0.275 1.088 1.208 0.965 0.829 0.635 1.821  
K33 0.130 0.702 1.683 0.165 0.464 0.978 0.157 0.077 0.140 1.546 2.190  
K34 1.143 0.220 0.194 0.191 0.233 1.246 0.543 2.414 3.531 0.040 0.638  
K36 2.427 0.469 0.217 0.201 0.313 0.359 0.640 2.359 2.394 0.088 17.779  
K37 1.357 1.237 0.208 0.488 0.225 2.333 0.518 0.117 6.915 2.038 5.895  
 K20 K21 K22 K23 K25 K26 K27 K28 K29 K30  
K1 -0.049 -0.051 -0.026 -0.061 0.069 0.028 -0.132 -0.146 -0.027 -0.106  
K4 -0.040 -0.101 -0.159 -0.059 -0.103 -0.112 0.071 -0.056 0.013 0.076  
K6 0.023 0.004 0.026 0.058 0.075 0.007 0.023 -0.052 0.048 0.101  
K12 -0.058 -0.036 -0.062 -0.040 0.065 0.137 0.057 0.093 0.184 0.104  
K13 -0.116 -0.039 -0.040 -0.041 0.062 -0.071 0.028 -0.031 -0.153 -0.025  
K14 -0.045 -0.055 -0.078 -0.135 -0.135 -0.253 0.018 -0.132 -0.241 -0.023  
K15 -0.067 -0.114 -0.120 -0.082 -0.125 -0.147 -0.014 -0.081 -0.099 0.031  
K16 -0.101 -0.145 -0.151 -0.071 -0.110 -0.161 0.033 -0.041 -0.134 0.024  
K17 -0.075 -0.091 -0.094 -0.065 -0.074 -0.136 0.029 -0.076 -0.167 0.033  
K18 -0.101 -0.061 -0.052 -0.047 0.064 0.120 0.052 0.055 0.034 0.029  
K19 0.059 -0.059 0.049 -0.168 -0.172 -0.167 -0.162 -0.143 -0.164 -0.123  
K20 0.037 0.038 -0.230 -0.313 -0.153 -0.159 -0.114 -0.104 -0.191  
K21 0.281 0.113 -0.176 -0.138 -0.154 -0.169 -0.141 -0.102 -0.176  
K22 0.299 2.604 -0.201 -0.188 -0.125 -0.254 -0.135 -0.131 -0.189  
K23 10.816 6.332 8.234 0.318 0.040 -0.066 -0.077 -0.040 0.095  
K25 20.021 3.906 7.203 20.571 -0.067 -0.143 -0.133 -0.086 0.090  
K26 4.748 4.855 3.175 0.334 0.915 0.181 0.322 0.405 0.196  
K27 5.177 5.826 13.194 0.899 4.151 6.652 0.429 0.264 0.297  
K28 2.634 4.077 3.691 1.221 3.603 21.195 37.564 0.356 0.191  
K29 2.226 2.138 3.504 0.328 1.503 33.470 14.183 25.848 0.227  
K30 7.453 6.291 7.299 1.824 1.668 7.816 18.045 7.451 10.520   
K31 3.130 5.203 5.056 1.027 1.010 2.867 1.493 0.961 1.150 1.486  
K32 2.959 8.523 10.656 0.603 2.431 0.450 0.408 0.212 0.775 1.078  
K33 2.824 6.528 7.466 0.346 1.782 0.048 0.171 0.054 0.421 2.460  
K34 1.744 3.346 6.691 0.299 0.786 2.941 0.098 2.217 0.457 0.117  
K36 7.677 2.148 1.430 1.314 2.080 0.060 2.944 0.528 0.322 0.304  
K37 2.855 3.624 5.786 0.368 1.049 2.962 0.347 0.098 0.066 0.218  
 K31 K32 K33 K34 K36 K37  
K1 -0.085 -0.088 -0.025 0.075 0.109 0.082  
K4 -0.068 -0.065 -0.059 -0.033 -0.048 0.078  
K6 0.085 -0.037 -0.091 -0.031 -0.033 -0.032  
K12 0.116 0.041 0.028 -0.031 -0.031 -0.049  
K13 -0.071 -0.037 -0.048 0.034 0.039 0.033  
K14 -0.123 -0.073 -0.069 0.078 0.042 0.107  
K15 -0.168 -0.077 -0.028 0.052 -0.056 0.050  
K16 -0.121 -0.069 0.019 0.109 -0.108 0.024  
K17 -0.072 -0.064 -0.026 0.132 0.108 0.184  
K18 -0.076 -0.056 -0.087 0.014 0.021 -0.100  
K19 -0.140 -0.094 -0.104 -0.056 -0.295 -0.170  
K20 -0.124 -0.120 -0.118 -0.092 -0.194 -0.118  
K21 -0.160 -0.204 -0.179 -0.128 -0.103 -0.133  
K22 -0.157 -0.229 -0.191 -0.181 -0.084 -0.168  
K23 0.071 0.054 0.041 -0.038 0.080 -0.042  
K25 -0.070 0.109 0.093 -0.062 0.101 -0.072  
K26 0.119 -0.047 0.015 -0.120 0.017 -0.121  
K27 0.086 -0.045 -0.029 -0.022 -0.120 -0.041  
K28 -0.069 0.032 -0.016 -0.104 -0.051 -0.022  
K29 0.075 -0.062 -0.045 -0.047 -0.040 -0.018  
K30 0.085 -0.073 -0.110 0.024 0.039 0.033  
K31 0.169 0.195 -0.099 -0.066 -0.073  
K32 5.793 0.353 0.052 -0.033 0.067  
K33 7.729 25.415 0.071 0.034 0.051  
K34 1.984 0.556 1.038 0.248 0.254  
K36 0.884 0.217 0.232 12.558 0.253  
K37 1.096 0.911 0.533 13.137 13.053

LD\_resid # values > 10 indicate local dependence

K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K19  
K1 NA 0.134 0.053 -0.012 0.137 0.055 -0.070 -0.076 0.030 0.104 -0.070  
K4 3.664 NA 0.283 0.081 -0.085 0.035 0.107 0.069 0.058 -0.027 -0.114  
K6 0.584 16.307 NA 0.190 0.055 0.010 0.050 -0.029 0.071 0.031 -0.030  
K12 0.031 1.329 7.370 NA -0.096 -0.191 -0.057 -0.046 -0.023 0.155 -0.087  
K13 3.843 1.476 0.607 1.894 NA 0.228 0.161 0.192 0.080 0.128 -0.050  
K14 0.612 0.255 0.021 7.463 10.606 NA 0.280 0.296 0.118 -0.068 -0.019  
K15 1.000 2.321 0.503 0.661 5.295 16.005 NA 0.588 0.145 -0.090 -0.037  
K16 1.187 0.964 0.176 0.426 7.488 17.859 70.602 NA 0.148 -0.032 -0.026  
K17 0.187 0.693 1.040 0.105 1.298 2.822 4.286 4.463 NA -0.069 -0.060  
K18 2.217 0.147 0.194 4.889 3.359 0.940 1.656 0.208 0.981 NA -0.158  
K19 1.014 2.650 0.180 1.527 0.520 0.077 0.282 0.140 0.734 5.116 NA  
K20 0.495 0.318 0.110 0.687 2.732 0.422 0.928 2.078 1.140 2.081 0.715  
K21 0.533 2.073 0.004 0.271 0.312 0.618 2.642 4.304 1.673 0.763 0.712  
K22 0.142 5.130 0.135 0.789 0.321 1.236 2.923 4.673 1.802 0.561 0.489  
K23 0.768 0.712 0.683 0.321 0.342 3.719 1.363 1.026 0.858 0.456 5.740  
K25 0.960 2.175 1.143 0.853 0.778 3.705 3.202 2.472 1.126 0.832 6.016  
K26 0.156 2.565 0.010 3.812 1.033 13.065 4.409 5.305 3.784 2.934 5.659  
K27 3.572 1.038 0.106 0.664 0.157 0.064 0.043 0.216 0.169 0.557 5.344  
K28 4.354 0.630 0.549 1.780 0.199 3.580 1.345 0.336 1.183 0.622 4.162  
K29 0.147 0.035 0.476 6.919 4.760 11.853 2.016 3.654 5.721 0.242 5.516  
K30 2.285 1.185 2.094 2.224 0.126 0.110 0.191 0.119 0.216 0.171 3.078  
K31 1.478 0.947 1.463 2.746 1.023 3.101 5.737 2.967 1.044 1.167 3.979  
K32 1.598 0.865 0.274 0.345 0.275 1.088 1.208 0.965 0.829 0.635 1.821  
K33 0.130 0.702 1.683 0.165 0.464 0.978 0.157 0.077 0.140 1.546 2.190  
K34 1.143 0.220 0.194 0.191 0.233 1.246 0.543 2.414 3.531 0.040 0.638  
K36 2.427 0.469 0.217 0.201 0.313 0.359 0.640 2.359 2.394 0.088 17.779  
K37 1.357 1.237 0.208 0.488 0.225 2.333 0.518 0.117 6.915 2.038 5.895  
 K20 K21 K22 K23 K25 K26 K27 K28 K29 K30  
K1 -0.049 -0.051 -0.026 -0.061 0.069 0.028 -0.132 -0.146 -0.027 -0.106  
K4 -0.040 -0.101 -0.159 -0.059 -0.103 -0.112 0.071 -0.056 0.013 0.076  
K6 0.023 0.004 0.026 0.058 0.075 0.007 0.023 -0.052 0.048 0.101  
K12 -0.058 -0.036 -0.062 -0.040 0.065 0.137 0.057 0.093 0.184 0.104  
K13 -0.116 -0.039 -0.040 -0.041 0.062 -0.071 0.028 -0.031 -0.153 -0.025  
K14 -0.045 -0.055 -0.078 -0.135 -0.135 -0.253 0.018 -0.132 -0.241 -0.023  
K15 -0.067 -0.114 -0.120 -0.082 -0.125 -0.147 -0.014 -0.081 -0.099 0.031  
K16 -0.101 -0.145 -0.151 -0.071 -0.110 -0.161 0.033 -0.041 -0.134 0.024  
K17 -0.075 -0.091 -0.094 -0.065 -0.074 -0.136 0.029 -0.076 -0.167 0.033  
K18 -0.101 -0.061 -0.052 -0.047 0.064 0.120 0.052 0.055 0.034 0.029  
K19 0.059 -0.059 0.049 -0.168 -0.172 -0.167 -0.162 -0.143 -0.164 -0.123  
K20 NA 0.037 0.038 -0.230 -0.313 -0.153 -0.159 -0.114 -0.104 -0.191  
K21 0.281 NA 0.113 -0.176 -0.138 -0.154 -0.169 -0.141 -0.102 -0.176  
K22 0.299 2.604 NA -0.201 -0.188 -0.125 -0.254 -0.135 -0.131 -0.189  
K23 10.816 6.332 8.234 NA 0.318 0.040 -0.066 -0.077 -0.040 0.095  
K25 20.021 3.906 7.203 20.571 NA -0.067 -0.143 -0.133 -0.086 0.090  
K26 4.748 4.855 3.175 0.334 0.915 NA 0.181 0.322 0.405 0.196  
K27 5.177 5.826 13.194 0.899 4.151 6.652 NA 0.429 0.264 0.297  
K28 2.634 4.077 3.691 1.221 3.603 21.195 37.564 NA 0.356 0.191  
K29 2.226 2.138 3.504 0.328 1.503 33.470 14.183 25.848 NA 0.227  
K30 7.453 6.291 7.299 1.824 1.668 7.816 18.045 7.451 10.520 NA  
K31 3.130 5.203 5.056 1.027 1.010 2.867 1.493 0.961 1.150 1.486  
K32 2.959 8.523 10.656 0.603 2.431 0.450 0.408 0.212 0.775 1.078  
K33 2.824 6.528 7.466 0.346 1.782 0.048 0.171 0.054 0.421 2.460  
K34 1.744 3.346 6.691 0.299 0.786 2.941 0.098 2.217 0.457 0.117  
K36 7.677 2.148 1.430 1.314 2.080 0.060 2.944 0.528 0.322 0.304  
K37 2.855 3.624 5.786 0.368 1.049 2.962 0.347 0.098 0.066 0.218  
 K31 K32 K33 K34 K36 K37  
K1 -0.085 -0.088 -0.025 0.075 0.109 0.082  
K4 -0.068 -0.065 -0.059 -0.033 -0.048 0.078  
K6 0.085 -0.037 -0.091 -0.031 -0.033 -0.032  
K12 0.116 0.041 0.028 -0.031 -0.031 -0.049  
K13 -0.071 -0.037 -0.048 0.034 0.039 0.033  
K14 -0.123 -0.073 -0.069 0.078 0.042 0.107  
K15 -0.168 -0.077 -0.028 0.052 -0.056 0.050  
K16 -0.121 -0.069 0.019 0.109 -0.108 0.024  
K17 -0.072 -0.064 -0.026 0.132 0.108 0.184  
K18 -0.076 -0.056 -0.087 0.014 0.021 -0.100  
K19 -0.140 -0.094 -0.104 -0.056 -0.295 -0.170  
K20 -0.124 -0.120 -0.118 -0.092 -0.194 -0.118  
K21 -0.160 -0.204 -0.179 -0.128 -0.103 -0.133  
K22 -0.157 -0.229 -0.191 -0.181 -0.084 -0.168  
K23 0.071 0.054 0.041 -0.038 0.080 -0.042  
K25 -0.070 0.109 0.093 -0.062 0.101 -0.072  
K26 0.119 -0.047 0.015 -0.120 0.017 -0.121  
K27 0.086 -0.045 -0.029 -0.022 -0.120 -0.041  
K28 -0.069 0.032 -0.016 -0.104 -0.051 -0.022  
K29 0.075 -0.062 -0.045 -0.047 -0.040 -0.018  
K30 0.085 -0.073 -0.110 0.024 0.039 0.033  
K31 NA 0.169 0.195 -0.099 -0.066 -0.073  
K32 5.793 NA 0.353 0.052 -0.033 0.067  
K33 7.729 25.415 NA 0.071 0.034 0.051  
K34 1.984 0.556 1.038 NA 0.248 0.254  
K36 0.884 0.217 0.232 12.558 NA 0.253  
K37 1.096 0.911 0.533 13.137 13.053 NA

Priority clusters to address (strongest first):

**K15–K16** (Q3 = 0.800; LD = 70.6) → almost certainly redundant; keep one.

**K26–K27–K28–K29** (multiple LD > 14; Q3 up to 0.556) → reduce to 1–2 items or reword to separate cues.

**K32–K33 (and K31)** (LD 25.4; Q3 0.581, 0.254) → likely very similar content; trim or rephrase.

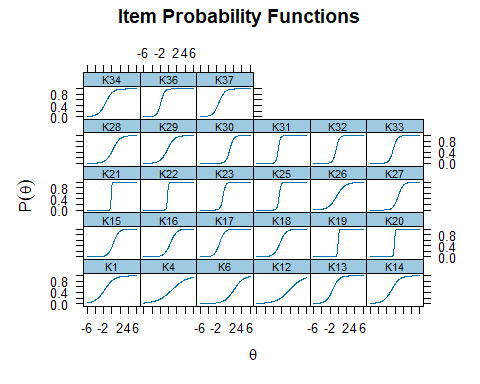
**K34–K36–K37** (LD ≈ 12–13; Q3 ≈ 0.35–0.39) → consider trimming one.

**K23–K25** (LD 20.6; Q3 0.697) → choose one or reword to target different facets (e.g., medical consequence vs psychological).

**K4–K6** (LD 16.3; Q3 0.295) → consider keeping one or differentiating wording.

## Monotonicity

# ==========================================  
# 3. Assumption: Monotonicity  
# ==========================================  
  
# (a) Plot Item Characteristic Curves (ICCs) to visually check monotonicity  
plot(mod2pl, type = "trace") # S-shaped, increasing curves are expected



# (b) Optional: Use Mokken scale analysis for monotonicity check  
  
# Run monotonicity check  
check.monotonicity(irt\_mat) # flags items with non-monotonic patterns

$results  
$results[[1]]  
$results[[1]][[1]]  
[1] "K1"  
  
$results[[1]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 73 25 48 0.6575342 0.6575342  
[2,] 2 13 26 131 18 113 0.8625954 0.8625954  
  
$results[[1]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[1]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[2]]  
$results[[2]][[1]]  
[1] "K4"  
  
$results[[2]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 68 48 20 0.2941176 0.2941176  
[2,] 2 13 26 136 78 58 0.4264706 0.4264706  
  
$results[[2]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[2]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[3]]  
$results[[3]][[1]]  
[1] "K6"  
  
$results[[3]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 13 74 73 1 0.01351351 0.01351351  
[2,] 2 14 26 130 98 32 0.24615385 0.24615385  
  
$results[[3]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[3]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[4]]  
$results[[4]][[1]]  
[1] "K12"  
  
$results[[4]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 69 62 7 0.1014493 0.1014493  
[2,] 2 13 26 135 93 42 0.3111111 0.3111111  
  
$results[[4]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[4]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[5]]  
$results[[5]][[1]]  
[1] "K13"  
  
$results[[5]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 30 44 0.5945946 0.5945946  
[2,] 2 13 26 130 9 121 0.9307692 0.9307692  
  
$results[[5]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[5]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[6]]  
$results[[6]][[1]]  
[1] "K14"  
  
$results[[6]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 72 39 33 0.4583333 0.4583333  
[2,] 2 13 26 132 30 102 0.7727273 0.7727273  
  
$results[[6]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[6]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[7]]  
$results[[7]][[1]]  
[1] "K15"  
  
$results[[7]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 13 75 67 8 0.1066667 0.1066667  
[2,] 2 14 26 129 56 73 0.5658915 0.5658915  
  
$results[[7]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[7]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[8]]  
$results[[8]][[1]]  
[1] "K16"  
  
$results[[8]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 68 60 8 0.1176471 0.1176471  
[2,] 2 13 26 136 67 69 0.5073529 0.5073529  
  
$results[[8]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[8]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[9]]  
$results[[9]][[1]]  
[1] "K17"  
  
$results[[9]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 27 47 0.6351351 0.6351351  
[2,] 2 13 26 130 7 123 0.9461538 0.9461538  
  
$results[[9]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[9]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[10]]  
$results[[10]][[1]]  
[1] "K18"  
  
$results[[10]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 70 47 23 0.3285714 0.3285714  
[2,] 2 13 26 134 34 100 0.7462687 0.7462687  
  
$results[[10]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[10]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[11]]  
$results[[11]][[1]]  
[1] "K19"  
  
$results[[11]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 69 64 5 0.07246377 0.07246377  
[2,] 2 13 26 135 32 103 0.76296296 0.76296296  
  
$results[[11]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[11]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[12]]  
$results[[12]][[1]]  
[1] "K20"  
  
$results[[12]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 69 62 7 0.1014493 0.1014493  
[2,] 2 13 26 135 30 105 0.7777778 0.7777778  
  
$results[[12]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[12]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[13]]  
$results[[13]][[1]]  
[1] "K21"  
  
$results[[13]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 70 61 9 0.1285714 0.1285714  
[2,] 2 13 26 134 26 108 0.8059701 0.8059701  
  
$results[[13]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[13]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[14]]  
$results[[14]][[1]]  
[1] "K22"  
  
$results[[14]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 71 57 14 0.1971831 0.1971831  
[2,] 2 13 26 133 25 108 0.8120301 0.8120301  
  
$results[[14]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[14]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[15]]  
$results[[15]][[1]]  
[1] "K23"  
  
$results[[15]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 71 49 22 0.3098592 0.3098592  
[2,] 2 13 26 133 13 120 0.9022556 0.9022556  
  
$results[[15]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[15]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[16]]  
$results[[16]][[1]]  
[1] "K25"  
  
$results[[16]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 73 46 27 0.3698630 0.3698630  
[2,] 2 13 26 131 6 125 0.9541985 0.9541985  
  
$results[[16]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[16]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[17]]  
$results[[17]][[1]]  
[1] "K26"  
  
$results[[17]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 69 41 28 0.4057971 0.4057971  
[2,] 2 13 26 135 42 93 0.6888889 0.6888889  
  
$results[[17]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[17]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[18]]  
$results[[18]][[1]]  
[1] "K27"  
  
$results[[18]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 13 74 72 2 0.02702703 0.02702703  
[2,] 2 14 26 130 73 57 0.43846154 0.43846154  
  
$results[[18]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[18]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[19]]  
$results[[19]][[1]]  
[1] "K28"  
  
$results[[19]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 68 54 14 0.2058824 0.2058824  
[2,] 2 13 26 136 59 77 0.5661765 0.5661765  
  
$results[[19]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[19]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[20]]  
$results[[20]][[1]]  
[1] "K29"  
  
$results[[20]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 70 49 21 0.3000000 0.3000000  
[2,] 2 13 26 134 41 93 0.6940299 0.6940299  
  
$results[[20]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[20]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[21]]  
$results[[21]][[1]]  
[1] "K30"  
  
$results[[21]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 13 74 74 0 0.0000000 0.0000000  
[2,] 2 14 26 130 92 38 0.2923077 0.2923077  
  
$results[[21]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 0 0 NaN 0 0 NaN 0 0 0 0  
Total 0 0 NaN 0 0 NaN 0 0 0 0  
  
$results[[21]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[22]]  
$results[[22]][[1]]  
[1] "K31"  
  
$results[[22]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 73 37 36 0.4931507 0.4931507  
[2,] 2 13 26 131 3 128 0.9770992 0.9770992  
  
$results[[22]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[22]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[23]]  
$results[[23]][[1]]  
[1] "K32"  
  
$results[[23]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 72 49 23 0.3194444 0.3194444  
[2,] 2 13 26 132 13 119 0.9015152 0.9015152  
  
$results[[23]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[23]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[24]]  
$results[[24]][[1]]  
[1] "K33"  
  
$results[[24]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 70 51 19 0.2714286 0.2714286  
[2,] 2 13 26 134 24 110 0.8208955 0.8208955  
  
$results[[24]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[24]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[25]]  
$results[[25]][[1]]  
[1] "K34"  
  
$results[[25]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 71 26 45 0.6338028 0.6338028  
[2,] 2 13 26 133 12 121 0.9097744 0.9097744  
  
$results[[25]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[25]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[26]]  
$results[[26]][[1]]  
[1] "K36"  
  
$results[[26]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 12 62 0.8378378 0.8378378  
[2,] 2 13 26 130 2 128 0.9846154 0.9846154  
  
$results[[26]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[26]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[27]]  
$results[[27]][[1]]  
[1] "K37"  
  
$results[[27]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 24 50 0.6756757 0.6756757  
[2,] 2 13 26 130 8 122 0.9384615 0.9384615  
  
$results[[27]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[27]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
  
$I.labels  
 [1] "K1" "K4" "K6" "K12" "K13" "K14" "K15" "K16" "K17" "K18" "K19" "K20"  
[13] "K21" "K22" "K23" "K25" "K26" "K27" "K28" "K29" "K30" "K31" "K32" "K33"  
[25] "K34" "K36" "K37"  
  
$Hi  
 K1 K4 K6 K12 K13 K14 K15 K16   
0.2764982 0.2156167 0.4446250 0.3025641 0.4084400 0.2915036 0.4642901 0.4362880   
 K17 K18 K19 K20 K21 K22 K23 K25   
0.4596081 0.3686416 0.5317350 0.5285261 0.5271176 0.5004022 0.5197159 0.5669049   
 K26 K27 K28 K29 K30 K31 K32 K33   
0.2947985 0.5330675 0.3818424 0.3506838 0.6504665 0.5797645 0.4991323 0.4540330   
 K34 K36 K37   
0.3679740 0.6575401 0.4375481   
  
$m  
[1] 2  
  
$X  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K19 K20 K21 K22 K23 K25 K26 K27 K28  
 [1,] 1 0 0 1 1 0 0 0 1 1 0 0 0 0 1 1 1 0 0  
 [2,] 1 0 0 1 1 0 0 0 1 1 0 0 0 0 1 1 1 0 0  
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 [65,] 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1  
 [66,] 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 0 1  
 [67,] 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
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 [69,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
 [70,] 1 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1  
 [71,] 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0  
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 [79,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0  
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 [89,] 1 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 0 1  
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 [93,] 1 1 0 0 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0  
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 [95,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
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attr(,"class")  
[1] "monotonicity.class"

## Model Fitness

### Global fit statistics

# ==========================================  
# 4. Assumption: Model Fit  
# ==========================================  
# (a) Global fit statistics (M2, RMSEA, SRMSR)  
M2(mod2pl) # RMSEA < 0.08 and SRMSR < 0.05 = good fit

M2 df p RMSEA RMSEA\_5 RMSEA\_95 SRMSR TLI CFI  
stats 1295.559 324 0 0.1215386 0.1143697 0.1281896 0.1107436 0.8387448 0.851149

### Item-level fit

# (b) Item-level fit (S-X² or G² statistics)  
itemfit\_stats <- itemfit(mod2pl)  
itemfit\_stats # significant misfit items should be reviewed

item S\_X2 df.S\_X2 RMSEA.S\_X2 p.S\_X2  
1 K1 11.970 18 0.000 0.849  
2 K4 24.900 19 0.039 0.164  
3 K6 19.540 12 0.056 0.076  
4 K12 16.999 16 0.018 0.386  
5 K13 17.768 16 0.023 0.338  
6 K14 8.226 18 0.000 0.975  
7 K15 13.625 15 0.000 0.554  
8 K16 9.260 15 0.000 0.864  
9 K17 13.202 13 0.009 0.432  
10 K18 18.775 17 0.023 0.342  
11 K19 25.800 5 0.143 0.000  
12 K20 20.065 6 0.107 0.003  
13 K21 29.530 4 0.177 0.000  
14 K22 38.942 6 0.164 0.000  
15 K23 8.013 11 0.000 0.712  
16 K25 5.544 7 0.000 0.594  
17 K26 19.194 18 0.018 0.380  
18 K27 15.104 12 0.036 0.236  
19 K28 23.698 16 0.049 0.096  
20 K29 25.531 18 0.045 0.111  
21 K30 10.893 8 0.042 0.208  
22 K31 9.587 7 0.043 0.213  
23 K32 10.418 13 0.000 0.659  
24 K33 13.726 13 0.017 0.393  
25 K34 14.995 16 0.000 0.525  
26 K36 5.159 5 0.013 0.397  
27 K37 12.242 15 0.000 0.661

### Compare 1PL vs 2PL with likelihood ratio test

# (c) Compare 1PL vs 2PL with likelihood ratio test  
# Fit 1PL and 2PL models  
mod1pl <- mirt(irt\_mat, 1, itemtype = "Rasch")

mod2pl <- mirt(irt\_mat, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

# Likelihood ratio test: does 2PL fit better than Rasch?  
anova(mod1pl, mod2pl)

AIC SABIC HQ BIC logLik X2 df p  
mod1pl 5257.663 5261.858 5295.245 5350.57 -2600.831   
mod2pl 5035.831 5043.922 5108.312 5215.01 -2463.916 273.832 26 0

## Items Removal Plan 2 (Based on discrimination, difficulty and local dependence)

**Selection criteria a > 0.64 (moderate discrimination) (Baker, 2001) ; -3 < b > +3**

K2 - a = 0.47

K3 - a = 0.39

K5 - a = 0.08 , b = -9.3762

K7 - a = 0.26

K8 - a = 0.20

K9 - a = 0.33

K10 - a = 0.30

K11 - a = 0.35

K24 - a = 16.9641 (?Overfitting)

K35 - a = 0.31

**Based on Local Dependence**

K4 : K4–K6 (LD 16.3; Q3 0.295)

K15 : K15–K16 (Q3 = 0.800; LD = 70.6)

K27 : K26–K27–K28–K29 (multiple LD > 14; Q3 up to 0.556)

K34 : K34–K36–K37 (LD ≈ 12–13; Q3 ≈ 0.35–0.39)

### 2PL Model - Remove Items 2

# Remove the items   
irt\_removed\_items2 <-c("K2", "K3", "K5", "K7", "K8", "K9", "K10", "K11","K35","K24","K4","K15","K27","K34")   
  
# Create new dataset with only included items  
data5 <- data3 %>% dplyr::select(-any\_of(irt\_removed\_items2))

### Descriptive Statistics

descript(data5)

Descriptive statistics for the 'data5' data-set  
  
Sample:  
 23 items and 204 sample units; 0 missing values  
  
Proportions for each level of response:  
 0 1 logit  
K1 0.2108 0.7892 1.3202  
K6 0.8382 0.1618 -1.6452  
K12 0.7598 0.2402 -1.1516  
K13 0.1912 0.8088 1.4424  
K14 0.3382 0.6618 0.6712  
K16 0.6225 0.3775 -0.5004  
K17 0.1667 0.8333 1.6094  
K18 0.3971 0.6029 0.4177  
K19 0.4706 0.5294 0.1178  
K20 0.4510 0.5490 0.1967  
K21 0.4265 0.5735 0.2963  
K22 0.4020 0.5980 0.3973  
K23 0.3039 0.6961 0.8287  
K25 0.2549 0.7451 1.0726  
K26 0.4069 0.5931 0.3769  
K28 0.5539 0.4461 -0.2165  
K29 0.4412 0.5588 0.2364  
K30 0.8137 0.1863 -1.4744  
K31 0.1961 0.8039 1.4110  
K32 0.3039 0.6961 0.8287  
K33 0.3676 0.6324 0.5423  
K36 0.0686 0.9314 2.6080  
K37 0.1569 0.8431 1.6818  
  
  
Frequencies of total scores:  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23  
Freq 1 5 3 1 4 6 9 5 11 7 6 7 9 9 12 12 17 19 13 12 12 14 4 6  
  
  
Point Biserial correlation with Total Score:  
 Included Excluded  
K1 0.3649 0.3008  
K6 0.3503 0.2920  
K12 0.3305 0.2617  
K13 0.4908 0.4360  
K14 0.4020 0.3293  
K16 0.4837 0.4149  
K17 0.4942 0.4426  
K18 0.5484 0.4844  
K19 0.7380 0.6942  
K20 0.7430 0.7000  
K21 0.7591 0.7187  
K22 0.7391 0.6963  
K23 0.7075 0.6638  
K25 0.7377 0.6999  
K26 0.4673 0.3962  
K28 0.5027 0.4335  
K29 0.5106 0.4422  
K30 0.5197 0.4674  
K31 0.6970 0.6587  
K32 0.6815 0.6347  
K33 0.6450 0.5916  
K36 0.4469 0.4106  
K37 0.4553 0.4028  
  
  
Cronbach's alpha:  
 value  
All Items 0.9006  
Excluding K1 0.9009  
Excluding K6 0.9008  
Excluding K12 0.9019  
Excluding K13 0.8981  
Excluding K14 0.9009  
Excluding K16 0.8989  
Excluding K17 0.8979  
Excluding K18 0.8971  
Excluding K19 0.8916  
Excluding K20 0.8914  
Excluding K21 0.8909  
Excluding K22 0.8916  
Excluding K23 0.8926  
Excluding K25 0.8920  
Excluding K26 0.8994  
Excluding K28 0.8985  
Excluding K29 0.8983  
Excluding K30 0.8974  
Excluding K31 0.8934  
Excluding K32 0.8934  
Excluding K33 0.8944  
Excluding K36 0.8990  
Excluding K37 0.8987  
  
  
Pairwise Associations:  
 Item i Item j p.value  
1 3 4 1.000  
2 1 16 1.000  
3 4 17 0.643  
4 3 23 0.593  
5 6 15 0.591  
6 7 17 0.570  
7 5 17 0.563  
8 15 23 0.562  
9 3 22 0.546  
10 7 15 0.524

### Refit 2PL Model

irt.data5 <- ltm(data5 ~ z1, IRT.param = TRUE)

### Item Parameter Estimates

# Obtain difficulty and discrimination parameter estimates   
item\_parms\_refined2 <- coef(irt.data5)  
  
# Tidy view: Item | a (Discrimination) | b (Difficulty)  
  
pp <- coef(irt.data5) # matrix with columns "Dffclt", "Dscrmn"  
  
item\_parms\_refined\_tbl2 <- data.frame(  
 Item = rownames(pp),  
 Difficulty = round(pp[,"Dffclt"], 3),  
 Discrimination = round(pp[,"Dscrmn"], 3),  
 row.names = NULL  
)  
  
item\_parms\_refined\_tbl2

Item Difficulty Discrimination  
1 K1 -1.506 0.840  
2 K6 2.240 0.989  
3 K12 2.231 0.643  
4 K13 -1.238 1.184  
5 K14 -0.580 0.900  
6 K16 0.836 1.155  
7 K17 -1.204 1.460  
8 K18 -0.131 1.265  
9 K19 0.243 6.361  
10 K20 0.185 6.810  
11 K21 0.120 7.424  
12 K22 0.074 6.133  
13 K23 -0.271 2.772  
14 K25 -0.376 3.860  
15 K26 -0.194 0.909  
16 K28 0.550 1.019  
17 K29 0.015 1.027  
18 K30 1.425 2.060  
19 K31 -0.632 3.419  
20 K32 -0.292 2.524  
21 K33 -0.123 2.031  
22 K36 -1.742 2.025  
23 K37 -1.414 1.262

### Model Summary

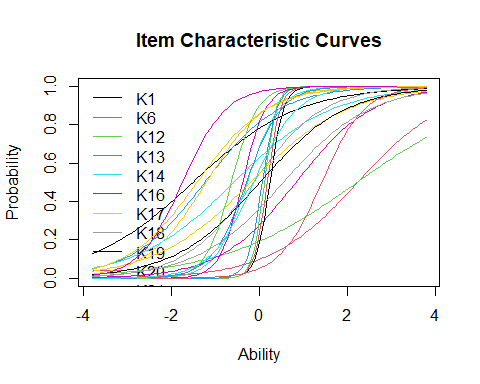
# Includes log-likelihood, AIC/BIC, SEs, and Wald z-values   
summary(irt.data5)

Call:  
ltm(formula = data5 ~ z1, IRT.param = TRUE)  
  
Model Summary:  
 log.Lik AIC BIC  
 -2059.606 4211.212 4363.845  
  
Coefficients:  
 value std.err z.vals  
Dffclt.K1 -1.5064 0.4192 -3.5936  
Dffclt.K6 2.2396 0.4420 5.0667  
Dffclt.K12 2.2311 0.5863 3.8051  
Dffclt.K13 -1.2380 0.2815 -4.3974  
Dffclt.K14 -0.5797 0.2375 -2.4409  
Dffclt.K16 0.8362 0.1618 5.1679  
Dffclt.K17 -1.2043 0.2397 -5.0249  
Dffclt.K18 -0.1308 0.1446 -0.9048  
Dffclt.K19 0.2426 0.0559 4.3383  
Dffclt.K20 0.1854 0.0544 3.4091  
Dffclt.K21 0.1197 0.0472 2.5352  
Dffclt.K22 0.0736 0.0501 1.4685  
Dffclt.K23 -0.2705 0.1009 -2.6814  
Dffclt.K25 -0.3757 0.0986 -3.8098  
Dffclt.K26 -0.1941 0.1922 -1.0095  
Dffclt.K28 0.5499 0.1596 3.4453  
Dffclt.K29 0.0152 0.1618 0.0938  
Dffclt.K30 1.4253 0.1632 8.7319  
Dffclt.K31 -0.6318 0.1153 -5.4777  
Dffclt.K32 -0.2917 0.1056 -2.7625  
Dffclt.K33 -0.1230 0.1060 -1.1601  
Dffclt.K36 -1.7421 0.2829 -6.1585  
Dffclt.K37 -1.4140 0.2993 -4.7241  
Dscrmn.K1 0.8399 0.2102 3.9965  
Dscrmn.K6 0.9895 0.2627 3.7665  
Dscrmn.K12 0.6430 0.2013 3.1939  
Dscrmn.K13 1.1838 0.2500 4.7356  
Dscrmn.K14 0.8996 0.2000 4.4986  
Dscrmn.K16 1.1555 0.2327 4.9652  
Dscrmn.K17 1.4596 0.2920 4.9982  
Dscrmn.K18 1.2652 0.2342 5.4026  
Dscrmn.K19 6.3615 1.0689 5.9516  
Dscrmn.K20 6.8099 1.3156 5.1762  
Dscrmn.K21 7.4237 1.5930 4.6603  
Dscrmn.K22 6.1331 1.4524 4.2226  
Dscrmn.K23 2.7723 0.4991 5.5545  
Dscrmn.K25 3.8596 0.7758 4.9749  
Dscrmn.K26 0.9088 0.1948 4.6657  
Dscrmn.K28 1.0190 0.2091 4.8732  
Dscrmn.K29 1.0274 0.2079 4.9413  
Dscrmn.K30 2.0603 0.4160 4.9523  
Dscrmn.K31 3.4192 0.6879 4.9702  
Dscrmn.K32 2.5241 0.4491 5.6198  
Dscrmn.K33 2.0312 0.3549 5.7234  
Dscrmn.K36 2.0245 0.4708 4.3005  
Dscrmn.K37 1.2623 0.2663 4.7409  
  
Integration:  
method: Gauss-Hermite  
quadrature points: 21   
  
Optimization:  
Convergence: 0   
max(|grad|): 8.1e-05   
quasi-Newton: BFGS

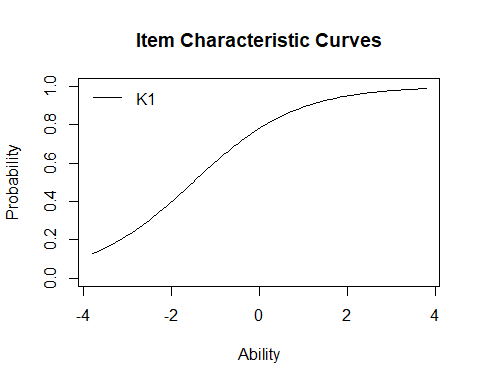
## Graphical Presentation

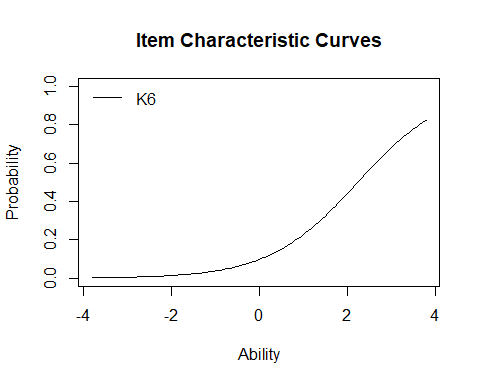
### Item Characteristic Curves (ICC)

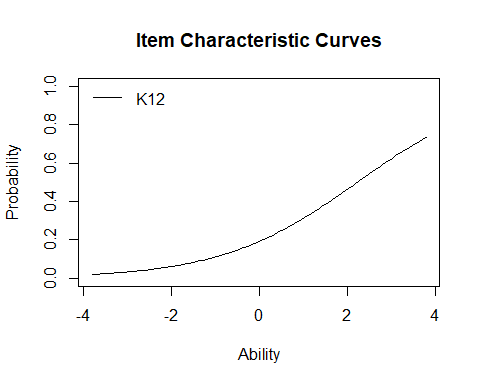
# ICC for All Items # Plot ICC for all items   
plot(irt.data5, type = "ICC", legend = TRUE)

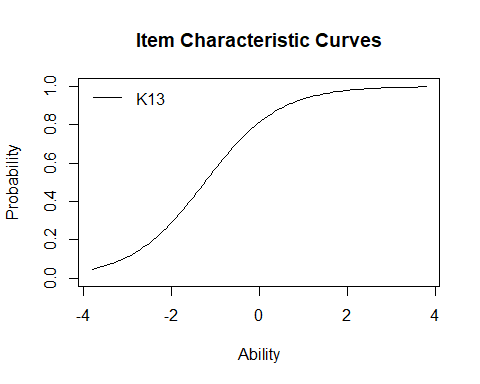


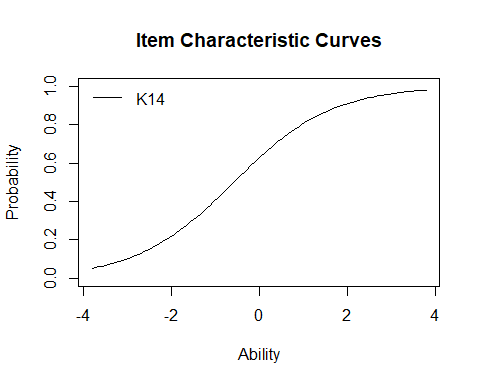
# ICC for Individual Items   
  
# Get total number of items   
ICC\_items2 <- nrow(coef(irt.data5))   
  
# Plot ICC for each item   
for (i in 1:ICC\_items2)   
plot(irt.data5, type = "ICC", legend = TRUE, items = i)

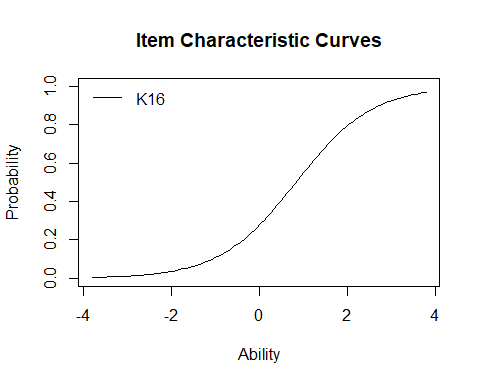


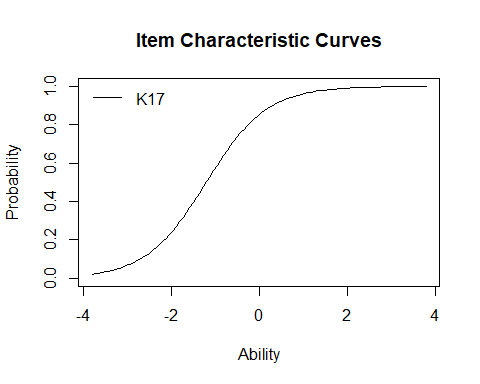


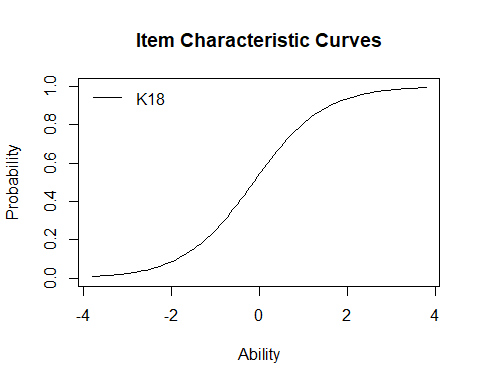


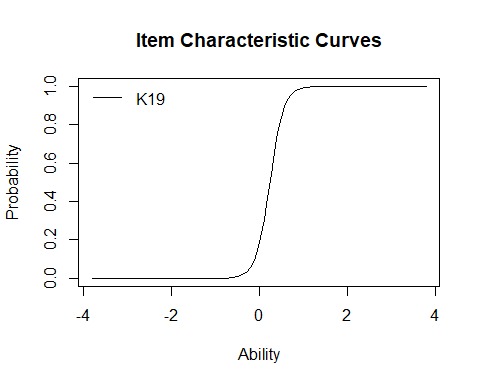


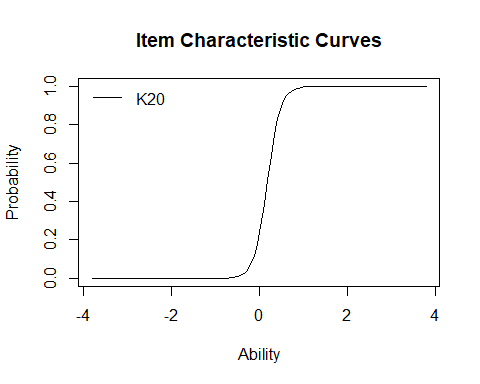


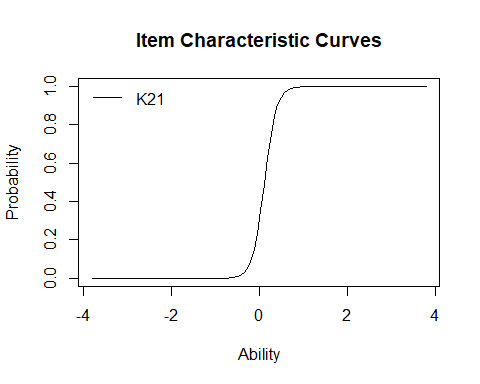


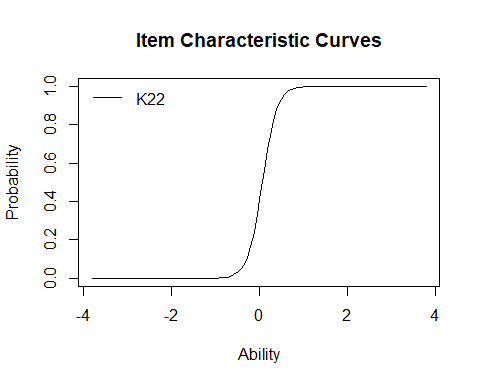


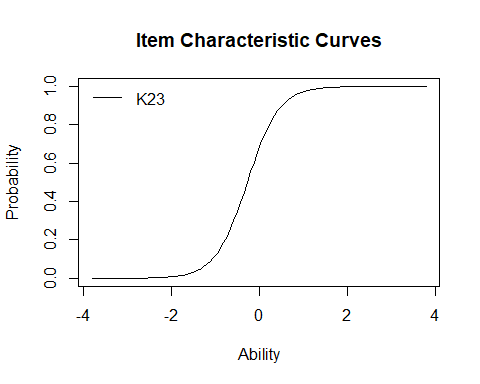


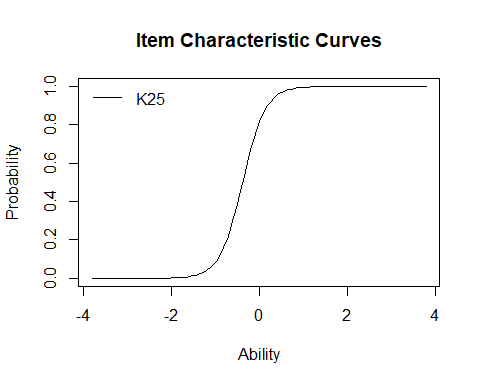


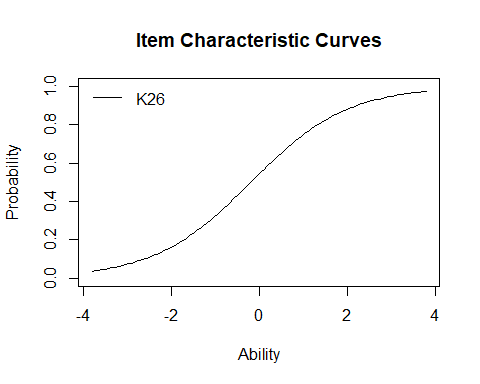


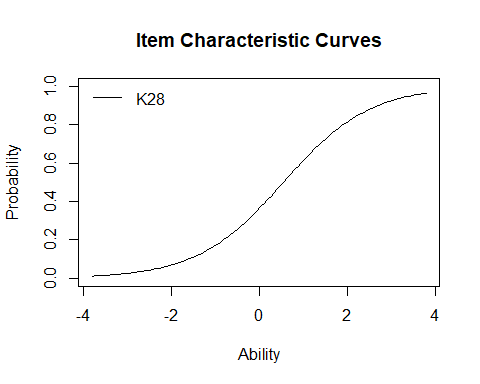


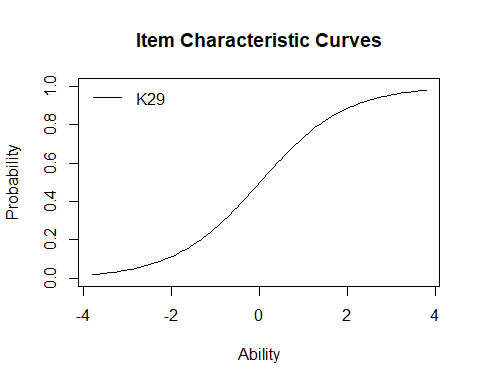


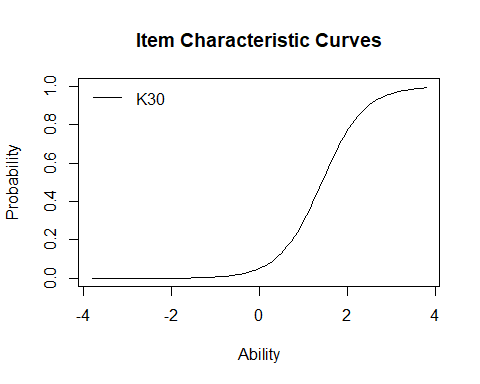


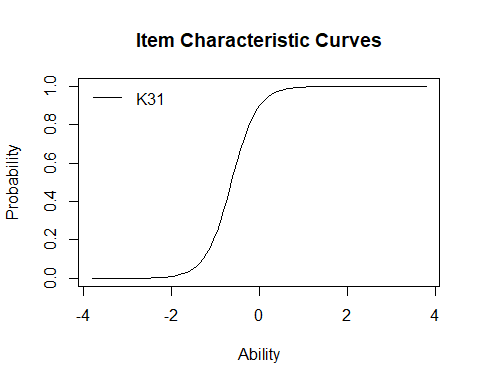


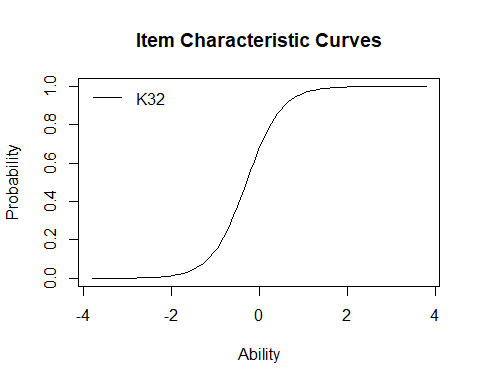


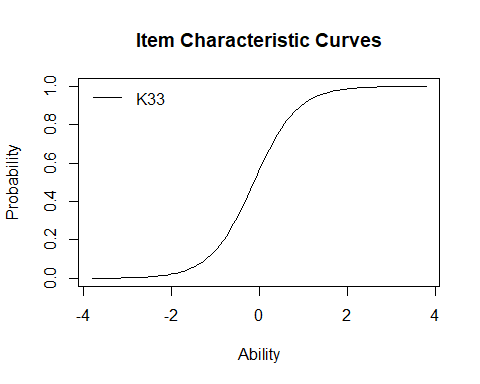


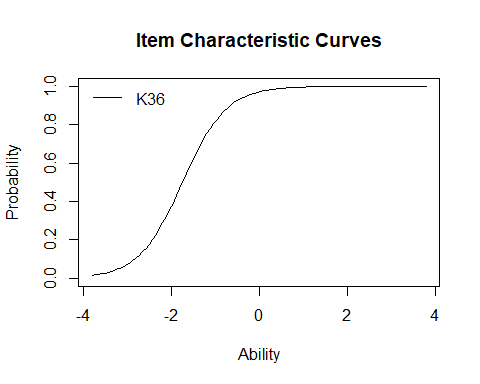


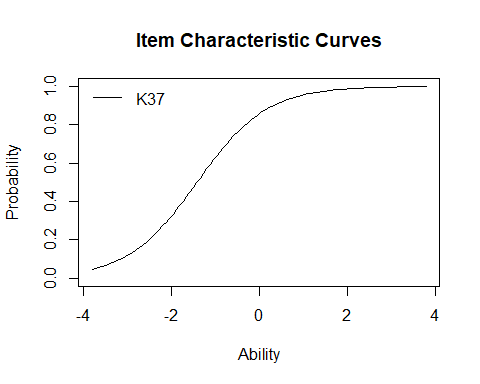












## Goodness-of-Fit Tests

### Item Fit Statistics

item\_fit2 <- item.fit(irt.data5)   
item\_fit2

Item-Fit Statistics and P-values  
  
Call:  
ltm(formula = data5 ~ z1, IRT.param = TRUE)  
  
Alternative: Items do not fit the model  
Ability Categories: 10  
  
 X^2 Pr(>X^2)  
K1 8.6691 0.371  
K6 13.7265 0.0892  
K12 5.4669 0.7067  
K13 6.1832 0.6267  
K14 6.2519 0.619  
K16 5.3303 0.7218  
K17 19.5198 0.0123  
K18 12.1064 0.1465  
K19 4.7808 0.7807  
K20 33.2116 0.0001  
K21 7.4125 0.4929  
K22 11.5120 0.1743  
K23 11.5594 0.172  
K25 17.4826 0.0255  
K26 22.3525 0.0043  
K28 22.2056 0.0045  
K29 26.4975 0.0009  
K30 19.2043 0.0138  
K31 4.9370 0.7643  
K32 17.5674 0.0247  
K33 16.4049 0.0369  
K36 17.4161 0.0261  
K37 12.5563 0.1281

### Fit on the Two-Way Margins

margins\_output2 <- margins(irt.data5)   
margins\_output2

Call:  
ltm(formula = data5 ~ z1, IRT.param = TRUE)  
  
Fit on the Two-Way Margins  
  
Response: (0,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 10 14 43 71.60 11.43 \*\*\*  
2 12 20 43 71.60 11.42 \*\*\*  
3 12 21 48 76.48 10.60 \*\*\*  
  
Response: (1,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 9 22 4 0.64 17.61 \*\*\*  
2 13 14 1 17.07 15.13 \*\*\*  
3 16 17 12 30.98 11.63 \*\*\*  
  
Response: (0,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 20 21 5 22.80 13.89 \*\*\*  
2 15 17 17 38.52 12.03 \*\*\*  
3 5 6 4 17.88 10.78 \*\*\*  
  
Response: (1,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 16 17 79 47.90 20.19 \*\*\*  
2 15 17 97 62.82 18.59 \*\*\*  
3 15 16 79 50.27 16.42 \*\*\*  
  
'\*\*\*' denotes a chi-squared residual greater than 3.5

### Person Fit Statistics

person\_fit2 <- person.fit(irt.data5)   
person\_fit2

Person-Fit Statistics and P-values  
  
Call:  
ltm(formula = data5 ~ z1, IRT.param = TRUE)  
  
Alternative: Inconsistent response pattern under the estimated model  
  
 K1 K6 K12 K13 K14 K16 K17 K18 K19 K20 K21 K22 K23 K25 K26 K28 K29 K30 K31  
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0  
4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0  
5 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1  
6 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 1 1 0 1  
7 0 0 0 0 0 0 0 1 0 0 0 0 1 1 1 0 1 0 0  
8 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0  
9 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1  
10 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 0  
11 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0 1  
12 0 0 0 0 0 0 1 0 0 0 0 1 1 1 1 1 1 0 1  
13 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0  
14 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
15 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 0 0 0 1  
16 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0 0 0 0 1  
17 0 0 0 1 0 0 1 0 0 1 1 1 1 0 1 1 1 0 1  
18 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1  
19 0 0 0 1 0 0 1 1 0 0 1 1 1 1 1 1 1 0 1  
20 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
21 0 0 0 1 1 0 0 0 0 0 0 0 1 1 1 0 0 0 1  
22 0 0 0 1 1 0 0 0 1 0 1 1 1 1 1 0 1 0 1  
23 0 0 0 1 1 0 1 1 0 0 0 0 0 0 0 1 0 0 0  
24 0 0 0 1 1 0 1 1 1 0 1 1 1 1 0 1 0 0 1  
25 0 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 0 1  
26 0 0 0 1 1 1 1 0 1 0 0 0 1 1 1 1 1 1 1  
27 0 0 0 1 1 1 1 0 1 1 1 1 1 1 0 0 0 0 1  
28 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1  
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34 0 0 1 0 1 1 1 0 1 1 1 1 0 0 0 1 1 0 1  
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36 0 0 1 1 1 0 1 1 0 1 1 1 1 1 1 0 1 1 1  
37 0 0 1 1 1 1 1 0 1 0 0 0 1 1 0 1 1 1 1  
38 0 0 1 1 1 1 1 1 0 1 1 0 0 0 0 1 1 0 1  
39 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
40 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
41 0 1 0 1 1 1 1 0 1 1 1 1 1 1 0 0 0 0 1  
42 0 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1  
43 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
44 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0  
45 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 1  
46 1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 1  
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50 1 0 0 0 0 0 1 0 1 1 1 1 1 1 1 0 1 0 1  
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54 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0  
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83 1 0 0 1 1 0 1 0 0 0 0 1 0 0 0 0 0 0 1  
84 1 0 0 1 1 0 1 0 1 1 1 1 0 1 0 0 0 0 1  
85 1 0 0 1 1 0 1 0 1 1 1 1 1 1 0 0 0 0 0  
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88 1 0 0 1 1 0 1 1 0 0 0 0 0 1 0 0 0 0 1  
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92 1 0 0 1 1 0 1 1 0 0 0 1 1 1 0 0 0 0 1  
93 1 0 0 1 1 0 1 1 0 0 1 1 0 1 1 1 0 0 1  
94 1 0 0 1 1 0 1 1 0 0 1 1 1 1 0 0 0 0 0  
95 1 0 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 1  
96 1 0 0 1 1 0 1 1 1 1 1 1 0 1 0 0 0 0 0  
97 1 0 0 1 1 0 1 1 1 1 1 1 1 1 0 0 0 0 1  
98 1 0 0 1 1 0 1 1 1 1 1 1 1 1 0 0 0 0 1  
99 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1  
100 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1  
101 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1  
102 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1  
103 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1  
104 1 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0  
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106 1 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 1  
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114 1 0 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1  
115 1 0 0 1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1  
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117 1 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1  
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119 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1  
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121 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1  
122 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1  
123 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
124 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
125 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
126 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0  
127 1 0 1 0 0 0 0 1 0 0 0 0 1 1 1 1 1 0 0  
128 1 0 1 0 0 0 0 1 0 1 0 1 0 0 1 1 0 0 1  
129 1 0 1 0 0 0 1 1 0 0 0 0 0 0 1 0 0 0 1  
130 1 0 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1  
131 1 0 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1  
132 1 0 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1  
133 1 0 1 1 0 0 0 1 0 0 1 1 1 1 1 1 1 0 1  
134 1 0 1 1 0 0 1 0 0 0 0 0 0 1 1 0 1 0 1  
135 1 0 1 1 0 0 1 1 0 0 0 0 1 1 1 0 1 0 1  
136 1 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0 0 1  
137 1 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1 0 1  
138 1 0 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1  
139 1 0 1 1 1 0 0 1 0 0 0 0 1 1 1 1 0 0 1  
140 1 0 1 1 1 0 1 0 0 0 0 0 0 0 0 0 1 0 1  
141 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 0 1  
142 1 0 1 1 1 0 1 1 0 0 0 0 1 1 1 0 0 0 1  
143 1 0 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0 1  
144 1 0 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 0 1  
145 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1  
146 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
147 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
148 1 1 0 0 1 0 1 0 1 1 1 1 1 1 0 0 1 0 1  
149 1 1 0 0 1 0 1 1 0 1 1 1 1 1 1 0 1 0 1  
150 1 1 0 1 0 0 1 1 0 0 0 0 1 1 1 1 1 0 1  
151 1 1 0 1 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1  
152 1 1 0 1 1 0 1 1 0 1 0 0 0 0 1 1 1 0 1  
153 1 1 0 1 1 0 1 1 1 1 1 1 1 1 0 0 0 0 1  
154 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1  
155 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1  
156 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 0 0 0 1  
157 1 1 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1  
158 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1  
159 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1  
160 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
161 1 1 1 1 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1  
162 1 1 1 1 0 0 1 1 1 1 1 1 0 1 0 0 0 0 1  
163 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1 1 1  
164 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1  
165 1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 0 1 0 1  
166 1 1 1 1 1 0 1 0 1 1 1 1 1 1 0 0 0 0 1  
167 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 0 1  
168 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1 0 1  
169 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1  
170 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1  
171 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 K32 K33 K36 K37 L0 Lz Pr(<Lz)  
1 0 0 0 0 -2.1103 1.2697 0.8979  
2 0 0 1 0 -3.6756 1.2549 0.8952  
3 1 0 0 0 -9.9820 -1.2348 0.1085  
4 0 0 0 0 -4.4143 0.5574 0.7114  
5 1 1 1 1 -11.4032 -0.1022 0.4593  
6 1 1 0 0 -23.2207 -4.9673 <0.0001  
7 0 0 1 0 -13.2142 -1.3975 0.0811  
8 0 0 0 0 -4.1754 0.8513 0.8027  
9 1 1 1 1 -9.5917 0.4318 0.667  
10 0 0 1 1 -8.3041 0.2314 0.5915  
11 1 0 1 1 -9.7180 0.6285 0.7352  
12 1 1 1 1 -11.7006 0.1444 0.5574  
13 0 0 1 1 -13.2590 -1.4468 0.074  
14 0 0 1 1 -5.4039 0.9987 0.841  
15 0 0 1 1 -7.9616 0.9127 0.8193  
16 1 0 1 1 -8.6028 1.1700 0.879  
17 0 0 1 1 -14.1473 -1.0186 0.1542  
18 1 1 1 1 -9.2206 0.7659 0.7781  
19 0 0 1 1 -11.5191 0.2131 0.5844  
20 0 0 0 0 -7.3677 -0.3635 0.3581  
21 1 1 1 0 -11.4723 -0.0467 0.4814  
22 1 0 1 1 -12.0117 -0.2436 0.4038  
23 0 0 1 1 -8.4969 0.5348 0.7036  
24 1 1 1 1 -9.4215 0.5471 0.7079  
25 1 1 1 0 -9.3390 -0.2152 0.4148  
26 1 1 1 1 -14.2554 -1.0361 0.1501  
27 1 1 1 1 -8.7599 0.1269 0.5505  
28 1 1 1 1 -7.1200 0.2520 0.5995  
29 0 0 1 1 -9.0234 0.3085 0.6212  
30 1 1 1 1 -11.6246 -0.0869 0.4654  
31 1 1 1 1 -7.8236 0.2843 0.6119  
32 1 0 1 1 -8.8614 -0.7658 0.2219  
33 0 0 1 0 -13.2303 -1.0185 0.1542  
34 0 0 0 1 -23.0852 -5.0472 <0.0001  
35 1 1 1 1 -8.0694 -0.0989 0.4606  
36 1 1 1 1 -11.1353 -0.5609 0.2874  
37 1 1 1 1 -15.9019 -1.7809 0.0375  
38 1 1 1 1 -15.5475 -1.6269 0.0519  
39 1 1 1 1 -5.9653 -0.1438 0.4428  
40 0 0 0 0 -8.2708 -0.8292 0.2035  
41 1 1 1 0 -12.9129 -1.4159 0.0784  
42 1 1 1 1 -9.8435 -1.5597 0.0594  
43 0 0 0 0 -3.1497 1.1153 0.8676  
44 0 0 1 1 -6.1362 0.8773 0.8099  
45 0 1 1 1 -10.7868 0.1854 0.5736  
46 1 1 1 1 -10.2988 -0.5152 0.3032  
47 0 0 1 0 -8.7758 0.4860 0.6865  
48 1 1 1 1 -9.1113 1.0847 0.861  
49 1 1 1 1 -9.2540 0.1298 0.5516  
50 1 1 1 1 -8.6101 0.2515 0.5993  
51 0 0 1 1 -5.5794 1.3986 0.919  
52 0 0 1 1 -6.8168 1.4453 0.9258  
53 1 1 1 1 -12.5749 -0.4048 0.3428  
54 0 0 1 0 -8.2725 0.1122 0.5447  
55 0 0 0 0 -9.7407 -0.7234 0.2347  
56 1 1 1 1 -9.2566 0.9836 0.8373  
57 0 0 1 0 -14.4187 -1.0745 0.1413  
58 1 0 1 1 -11.2492 -1.0924 0.1373  
59 0 0 1 1 -6.6476 1.1664 0.8783  
60 1 1 1 1 -7.6959 1.6668 0.9522  
61 0 0 1 0 -9.0373 0.9052 0.8173  
62 1 1 1 1 -7.6508 1.7224 0.9575  
63 1 1 1 1 -8.4315 1.4689 0.9291  
64 0 0 1 1 -10.5659 -0.0240 0.4904  
65 1 1 1 1 -7.0145 0.6838 0.753  
66 1 1 1 1 -11.3030 0.3091 0.6214  
67 1 1 1 1 -11.0487 -0.2055 0.4186  
68 1 1 1 1 -6.0039 0.8687 0.8075  
69 1 1 1 1 -5.2012 1.0149 0.8449  
70 0 0 1 1 -10.1420 0.3670 0.6432  
71 0 0 1 0 -5.6223 1.0645 0.8564  
72 0 0 1 1 -5.5347 1.3633 0.9136  
73 1 1 1 1 -10.5138 0.2604 0.6027  
74 1 1 1 1 -9.4970 0.6897 0.7548  
75 1 0 1 1 -10.0316 0.4883 0.6873  
76 0 0 0 0 -21.1872 -4.2002 <0.0001  
77 0 0 1 1 -11.0149 -0.2500 0.4013  
78 0 0 0 0 -6.5388 0.5299 0.7019  
79 0 0 1 1 -5.4416 1.6943 0.9549  
80 0 0 1 1 -6.7002 1.7656 0.9613  
81 0 0 1 1 -8.4762 0.7540 0.7746  
82 0 0 1 1 -8.1423 1.2348 0.8915  
83 0 0 1 1 -9.8208 0.5779 0.7183  
84 1 0 1 1 -9.8264 0.3290 0.6289  
85 0 0 1 1 -12.9814 -0.7281 0.2333  
86 0 1 1 1 -8.6643 0.5402 0.7055  
87 1 1 1 1 -6.0169 0.9043 0.8171  
88 1 1 1 1 -7.6558 1.6613 0.9517  
89 1 0 1 1 -9.0116 1.0446 0.8519  
90 1 1 1 1 -7.8399 1.8118 0.965  
91 1 1 1 1 -8.9754 1.3890 0.9176  
92 1 1 1 1 -7.8355 1.9009 0.9713  
93 0 0 1 0 -12.5835 -0.2398 0.4052  
94 0 0 1 1 -11.8953 0.0649 0.5259  
95 1 1 1 1 -8.5728 1.2384 0.8922  
96 1 1 1 1 -11.6153 -0.3794 0.3522  
97 1 0 1 1 -7.1909 0.8800 0.8106  
98 1 1 1 1 -5.7958 1.1134 0.8672  
99 0 0 1 0 -10.8191 -0.1805 0.4284  
100 1 1 1 1 -5.0084 1.2753 0.8989  
101 1 1 1 1 -4.2223 1.4269 0.9232  
102 1 1 1 1 -3.8195 1.4427 0.9254  
103 1 1 1 1 -4.0255 1.0097 0.8437  
104 0 0 1 1 -7.5417 0.9347 0.825  
105 1 1 1 1 -8.4013 1.4452 0.9258  
106 0 0 1 1 -9.5952 0.8634 0.806  
107 1 1 1 1 -11.8997 0.0554 0.5221  
108 1 1 1 1 -9.2366 0.2081 0.5824  
109 1 1 1 1 -6.9965 0.6708 0.7488  
110 1 1 1 1 -5.4466 0.9546 0.8301  
111 1 1 1 1 -5.0651 0.9380 0.8259  
112 0 0 1 1 -8.4621 0.6912 0.7553  
113 0 0 1 1 -8.9776 0.5586 0.7118  
114 1 0 1 1 -8.4035 1.4126 0.9211  
115 1 1 1 1 -11.6846 0.1644 0.5653  
116 0 0 1 0 -17.0923 -2.4020 0.0082  
117 0 1 1 1 -8.2970 0.3646 0.6423  
118 1 1 1 1 -5.9707 0.8605 0.8052  
119 1 1 1 0 -7.8355 0.1957 0.5776  
120 1 0 1 1 -6.1301 0.8142 0.7922  
121 1 1 1 1 -4.0800 1.3128 0.9054  
122 1 1 1 1 -4.6338 1.0797 0.8599  
123 1 1 1 1 -3.4046 1.4545 0.9271  
124 0 1 1 1 -7.6742 -0.3411 0.3665  
125 1 1 1 1 -2.8693 1.2908 0.9016  
126 0 0 1 1 -9.3935 -0.2755 0.3915  
127 0 0 1 1 -14.1517 -1.5129 0.0652  
128 1 1 1 0 -18.7083 -2.9873 0.0014  
129 0 0 1 1 -9.8420 0.2292 0.5907  
130 1 1 1 0 -11.2399 -1.0178 0.1544  
131 1 1 1 1 -9.4807 -1.1090 0.1337  
132 1 1 1 1 -7.7628 -0.5902 0.2775  
133 1 1 1 0 -13.7005 -0.8647 0.1936  
134 1 1 1 1 -9.6884 0.7966 0.7872  
135 1 1 1 1 -9.1936 1.1706 0.8791  
136 1 1 1 1 -7.1451 0.4891 0.6876  
137 1 1 1 1 -6.3922 0.6039 0.727  
138 1 1 1 1 -9.1311 -0.3603 0.3593  
139 1 1 1 1 -10.9430 0.3871 0.6507  
140 1 1 1 1 -9.8641 0.5622 0.713  
141 1 1 1 1 -6.0313 0.6303 0.7357  
142 1 1 1 1 -8.6501 1.4035 0.9198  
143 1 1 1 1 -8.6147 0.4003 0.6555  
144 1 1 0 0 -16.6886 -2.0885 0.0184  
145 1 1 1 1 -3.9067 1.1165 0.8679  
146 1 0 1 1 -7.0478 -0.3412 0.3665  
147 1 1 1 1 -2.8982 1.0921 0.8626  
148 1 1 1 1 -9.9246 -0.3970 0.3457  
149 1 1 1 1 -10.0869 0.0781 0.5311  
150 1 0 1 1 -10.7472 0.4691 0.6805  
151 1 1 1 1 -7.7021 -0.3714 0.3552  
152 0 0 1 1 -14.3015 -1.1224 0.1309  
153 1 0 1 1 -8.8807 0.0946 0.5377  
154 1 1 1 1 -4.8751 0.8271 0.7959  
155 1 1 1 1 -4.4542 0.5729 0.7167  
156 1 1 1 1 -8.5533 -0.0969 0.4614  
157 1 1 1 1 -10.6004 0.5701 0.7157  
158 1 1 1 1 -9.7806 -0.0716 0.4715  
159 1 1 1 1 -5.1002 0.7013 0.7584  
160 1 1 1 1 -2.8445 1.0078 0.8432  
161 0 0 1 1 -16.0423 -1.9395 0.0262  
162 1 1 1 1 -11.8675 -0.9065 0.1823  
163 0 0 1 0 -16.1597 -2.6730 0.0038  
164 1 0 1 1 -10.3073 -1.5121 0.0653  
165 0 1 1 1 -13.2380 -0.7093 0.2391  
166 1 1 1 1 -9.3743 -0.3212 0.374  
167 1 0 1 1 -9.2976 -0.3388 0.3674  
168 1 1 1 1 -12.4194 -0.1696 0.4326  
169 1 1 1 1 -4.6224 0.2660 0.6049  
170 1 0 1 1 -8.9465 -0.6431 0.2601  
171 1 1 1 1 -2.5620 0.9241 0.8223

# Checking Assumptions

## Unidimensionality

set.seed(2025)   
unidimTest(irt.data5) #Take A long time, insert # if want to skip and avoid long time}

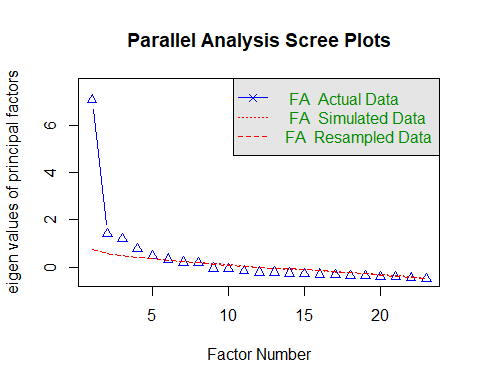
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value

Unidimensionality Check using Modified Parallel Analysis  
  
Call:  
ltm(formula = data5 ~ z1, IRT.param = TRUE)  
  
Matrix of tertachoric correlations  
 K1 K6 K12 K13 K14 K16 K17 K18 K19 K20  
K1 1.0000 0.3148 0.1320 0.5059 0.3255 0.1469 0.3468 0.4518 0.3259 0.3594  
K6 0.3148 1.0000 0.4902 0.3832 0.2398 0.2308 0.4773 0.3316 0.3846 0.4642  
K12 0.1320 0.4902 1.0000 0.0214 -0.1444 0.1429 0.2160 0.4846 0.2008 0.2442  
K13 0.5059 0.3832 0.0214 1.0000 0.6660 0.7606 0.5424 0.5789 0.5128 0.4151  
K14 0.3255 0.2398 -0.1444 0.6660 1.0000 0.7745 0.5468 0.2252 0.4997 0.4505  
K16 0.1469 0.2308 0.1429 0.7606 0.7745 1.0000 0.7282 0.3529 0.5927 0.4979  
K17 0.3468 0.4773 0.2160 0.5424 0.5468 0.7282 1.0000 0.3172 0.5793 0.5575  
K18 0.4518 0.3316 0.4846 0.5789 0.2252 0.3529 0.3172 1.0000 0.4284 0.5022  
K19 0.3259 0.3846 0.2008 0.5128 0.4997 0.5927 0.5793 0.4284 1.0000 0.9707  
K20 0.3594 0.4642 0.2442 0.4151 0.4505 0.4979 0.5575 0.5022 0.9707 1.0000  
K21 0.3617 0.4310 0.2808 0.5356 0.4428 0.4479 0.5450 0.5584 0.9535 0.9747  
K22 0.4047 0.4565 0.2361 0.5326 0.4062 0.4282 0.5353 0.5628 0.9573 0.9592  
K23 0.3136 0.4624 0.2789 0.5047 0.2601 0.4659 0.5210 0.5196 0.7310 0.6793  
K25 0.4635 0.4843 0.3465 0.5639 0.2948 0.4425 0.5706 0.5902 0.7921 0.6933  
K26 0.2759 0.2348 0.4139 0.1877 -0.1414 0.0776 0.1082 0.4965 0.2797 0.2996  
K28 0.0078 0.1641 0.3543 0.2957 0.0944 0.3083 0.2609 0.4440 0.3902 0.4278  
K29 0.2130 0.3416 0.5093 0.0814 -0.0831 0.1625 0.0987 0.4122 0.3552 0.4357  
K30 0.1337 0.5244 0.4499 0.4308 0.3686 0.5149 0.5174 0.5107 0.6034 0.5293  
K31 0.3073 0.5238 0.5265 0.5015 0.2962 0.3986 0.5742 0.5170 0.7767 0.7943  
K32 0.2278 0.3138 0.3293 0.4650 0.3298 0.4282 0.4784 0.4583 0.7310 0.7059  
K33 0.3087 0.1712 0.3029 0.3877 0.2972 0.4881 0.5002 0.3650 0.6524 0.6383  
K36 0.5792 0.2306 0.1801 0.5359 0.4805 0.2388 0.7232 0.5003 0.3192 0.4397  
K37 0.4313 0.2018 0.1154 0.4287 0.5080 0.4096 0.7176 0.2208 0.3516 0.4302  
 K21 K22 K23 K25 K26 K28 K29 K30 K31 K32  
K1 0.3617 0.4047 0.3136 0.4635 0.2759 0.0078 0.2130 0.1337 0.3073 0.2278  
K6 0.4310 0.4565 0.4624 0.4843 0.2348 0.1641 0.3416 0.5244 0.5238 0.3138  
K12 0.2808 0.2361 0.2789 0.3465 0.4139 0.3543 0.5093 0.4499 0.5265 0.3293  
K13 0.5356 0.5326 0.5047 0.5639 0.1877 0.2957 0.0814 0.4308 0.5015 0.4650  
K14 0.4428 0.4062 0.2601 0.2948 -0.1414 0.0944 -0.0831 0.3686 0.2962 0.3298  
K16 0.4479 0.4282 0.4659 0.4425 0.0776 0.3083 0.1625 0.5149 0.3986 0.4282  
K17 0.5450 0.5353 0.5210 0.5706 0.1082 0.2609 0.0987 0.5174 0.5742 0.4784  
K18 0.5584 0.5628 0.5196 0.5902 0.4965 0.4440 0.4122 0.5107 0.5170 0.4583  
K19 0.9535 0.9573 0.7310 0.7921 0.2797 0.3902 0.3552 0.6034 0.7767 0.7310  
K20 0.9747 0.9592 0.6793 0.6933 0.2996 0.4278 0.4357 0.5293 0.7943 0.7059  
K21 1.0000 0.9870 0.7417 0.8356 0.3028 0.3967 0.4447 0.5489 0.7782 0.6373  
K22 0.9870 1.0000 0.7046 0.7841 0.3378 0.3960 0.3982 0.5172 0.7672 0.5975  
K23 0.7417 0.7046 1.0000 0.9879 0.4691 0.4112 0.4936 0.9842 0.8178 0.7605  
K25 0.8356 0.7841 0.9879 1.0000 0.4410 0.3571 0.4470 0.9803 0.8320 0.8422  
K26 0.3028 0.3378 0.4691 0.4410 1.0000 0.7336 0.8035 0.7777 0.6168 0.3736  
K28 0.3967 0.3960 0.4112 0.3571 0.7336 1.0000 0.7858 0.7413 0.4983 0.4795  
K29 0.4447 0.3982 0.4936 0.4470 0.8035 0.7858 1.0000 0.9903 0.5685 0.3989  
K30 0.5489 0.5172 0.9842 0.9803 0.7777 0.7413 0.9903 1.0000 0.9778 0.5139  
K31 0.7782 0.7672 0.8178 0.8320 0.6168 0.4983 0.5685 0.9778 1.0000 0.8931  
K32 0.6373 0.5975 0.7605 0.8422 0.3736 0.4795 0.3989 0.5139 0.8931 1.0000  
K33 0.5838 0.5566 0.6877 0.7769 0.4017 0.4350 0.3803 0.4134 0.8963 0.9411  
K36 0.5677 0.5923 0.7833 0.8243 0.3960 0.3220 0.3546 0.9558 0.6756 0.6072  
K37 0.4178 0.3597 0.5210 0.5234 0.1029 0.3323 0.3468 0.5005 0.5157 0.6057  
 K33 K36 K37  
K1 0.3087 0.5792 0.4313  
K6 0.1712 0.2306 0.2018  
K12 0.3029 0.1801 0.1154  
K13 0.3877 0.5359 0.4287  
K14 0.2972 0.4805 0.5080  
K16 0.4881 0.2388 0.4096  
K17 0.5002 0.7232 0.7176  
K18 0.3650 0.5003 0.2208  
K19 0.6524 0.3192 0.3516  
K20 0.6383 0.4397 0.4302  
K21 0.5838 0.5677 0.4178  
K22 0.5566 0.5923 0.3597  
K23 0.6877 0.7833 0.5210  
K25 0.7769 0.8243 0.5234  
K26 0.4017 0.3960 0.1029  
K28 0.4350 0.3220 0.3323  
K29 0.3803 0.3546 0.3468  
K30 0.4134 0.9558 0.5005  
K31 0.8963 0.6756 0.5157  
K32 0.9411 0.6072 0.6057  
K33 1.0000 0.6268 0.5512  
K36 0.6268 1.0000 0.9111  
K37 0.5512 0.9111 1.0000  
  
Alternative hypothesis: the second eigenvalue of the observed data is substantially larger   
 than the second eigenvalue of data under the assumed IRT model  
  
Second eigenvalue in the observed data: 2.8789  
Average of second eigenvalues in Monte Carlo samples: 1.7275  
Monte Carlo samples: 100  
p-value: 0.0099

### Checking Dominant Factor (Essential Unidimensionality)

# Extract the response data from the fitted model   
irt\_mat2 <- as.matrix(irt.data5$X)   
# Parallel analysis   
library(psych)   
fa.parallel(irt\_mat2, fa="fa")

Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :  
The estimated weights for the factor scores are probably incorrect. Try a  
different factor score estimation method.



Parallel analysis suggests that the number of factors = 5 and the number of components = NA

# Eigenvalues   
ev2 <- eigen(cor(irt\_mat2, use = "pairwise.complete.obs"))$values   
# First and second eigenvalues   
first\_ev2 <- ev[1]   
second\_ev2 <- ev[2]   
# Ratio   
dominance\_ratio <- first\_ev2 / second\_ev2   
# Print first\_ev second\_ev dominance\_ratio}

Parallel analysis suggested up to seven factors, as seven eigenvalues from the actual data exceeded those from randomly simulated data. However, the scree plot demonstrated a sharp drop between the first (9.02) and second (2.51) eigenvalues, yielding a ratio of 3.59. This indicates a single dominant factor underlying item responses, with additional weaker factors. Consistent with the concept of **essential unidimensionality** (Reckase, 1979; Hambleton, Swaminathan, & Rogers, 1991; Embretson & Reise, 2000), the scale was considered suitable for unidimensional IRT modeling despite the presence of minor secondary dimensions.

class(irt.data5)

[1] "ltm"

str(irt.data5, max.level = 1)

List of 14  
 $ coefficients: num [1:23, 1:2] 1.265 -2.216 -1.435 1.466 0.521 ...  
 ..- attr(\*, "dimnames")=List of 2  
 $ log.Lik : num -2060  
 $ convergence : int 0  
 $ hessian : num [1:46, 1:46] 29.833 -0.215 -0.202 -0.366 -0.348 ...  
 $ counts : Named int [1:2] 3 1  
 ..- attr(\*, "names")= chr [1:2] "function" "gradient"  
 $ patterns :List of 2  
 $ GH :List of 2  
 $ max.sc : num 8.06e-05  
 $ ltst :List of 5  
 $ X : tibble [204 × 23] (S3: tbl\_df/tbl/data.frame)  
 $ control :List of 6  
 $ IRT.param : logi TRUE  
 $ formula :Class 'formula' language data5 ~ z1  
 .. ..- attr(\*, ".Environment")=<environment: R\_GlobalEnv>   
 $ call : language ltm(formula = data5 ~ z1, IRT.param = TRUE)  
 - attr(\*, "class")= chr "ltm"

## Local Independence

### Yen’s Q3 residual cYen’s Q3 residual correlationsorrelations

library(mirt)   
library(mokken)   
  
# Extract the raw item response matrix from the ltm object   
irt\_mat2 <- as.data.frame(irt.data5$X)   
  
# now it's a dataframe   
irt\_mat2 <- as.matrix(irt\_mat2)   
  
# convert to numeric matrix   
  
# 1. Fit a unidimensional 2PL model   
  
mod2pl2 <- mirt(irt\_mat2, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

mod1pl2 <- mirt(irt\_mat2, 1, itemtype = "Rasch")

# ==========================================

# 2. Assumption: Local Independence # ==========================================   
  
# (a) Yen’s Q3 residual correlations #   
  
Q3\_resid2 <- residuals(mod2pl2, type = "Q3")

Q3 summary statistics:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.481 -0.095 -0.020 -0.011 0.046 0.697   
  
 K1 K6 K12 K13 K14 K16 K17 K18 K19 K20  
K1 1.000 0.050 -0.018 0.173 0.075 -0.059 0.018 0.133 -0.095 -0.039  
K6 0.050 1.000 0.203 0.051 0.002 -0.050 0.064 0.026 -0.098 -0.006  
K12 -0.018 0.203 1.000 -0.110 -0.207 -0.055 -0.023 0.171 -0.099 -0.051  
K13 0.173 0.051 -0.110 1.000 0.309 0.242 0.112 0.166 -0.029 -0.204  
K14 0.075 0.002 -0.207 0.309 1.000 0.375 0.181 -0.053 0.090 0.001  
K16 -0.059 -0.050 -0.055 0.242 0.375 1.000 0.195 0.008 0.134 -0.043  
K17 0.018 0.064 -0.023 0.112 0.181 0.195 1.000 -0.084 -0.030 -0.053  
K18 0.133 0.026 0.171 0.166 -0.053 0.008 -0.084 1.000 -0.200 -0.079  
K19 -0.095 -0.098 -0.099 -0.029 0.090 0.134 -0.030 -0.200 1.000 0.082  
K20 -0.039 -0.006 -0.051 -0.204 0.001 -0.043 -0.053 -0.079 0.082 1.000  
K21 -0.011 -0.103 -0.015 -0.044 -0.050 -0.177 -0.089 0.011 -0.338 -0.121  
K22 0.052 -0.034 -0.097 -0.039 -0.088 -0.163 -0.077 0.001 -0.079 -0.124  
K23 -0.051 0.033 -0.023 -0.017 -0.121 0.013 -0.032 -0.010 -0.162 -0.319  
K25 0.088 0.019 -0.008 0.001 -0.116 -0.032 -0.005 0.015 -0.173 -0.481  
K26 0.039 -0.019 0.135 -0.070 -0.276 -0.177 -0.149 0.164 -0.143 -0.110  
K28 -0.151 -0.085 0.100 -0.007 -0.127 -0.028 -0.053 0.097 -0.070 -0.012  
K29 -0.021 0.037 0.198 -0.171 -0.258 -0.132 -0.187 0.068 -0.147 -0.019  
K30 -0.113 0.096 0.125 0.008 -0.029 -0.002 0.010 0.028 -0.011 -0.143  
K31 -0.105 0.043 0.084 -0.090 -0.105 -0.058 -0.039 -0.099 -0.107 -0.057  
K32 -0.111 -0.037 0.025 -0.029 -0.033 -0.004 -0.056 -0.050 -0.009 -0.061  
K33 -0.019 -0.107 0.031 -0.066 -0.044 0.077 -0.010 -0.095 -0.036 -0.072  
K36 0.121 -0.011 -0.061 0.011 0.046 -0.068 0.142 -0.016 -0.329 -0.186  
K37 0.096 -0.020 -0.058 0.029 0.160 0.059 0.283 -0.116 -0.172 -0.053  
 K21 K22 K23 K25 K26 K28 K29 K30 K31 K32  
K1 -0.011 0.052 -0.051 0.088 0.039 -0.151 -0.021 -0.113 -0.105 -0.111  
K6 -0.103 -0.034 0.033 0.019 -0.019 -0.085 0.037 0.096 0.043 -0.037  
K12 -0.015 -0.097 -0.023 -0.008 0.135 0.100 0.198 0.125 0.084 0.025  
K13 -0.044 -0.039 -0.017 0.001 -0.070 -0.007 -0.171 0.008 -0.090 -0.029  
K14 -0.050 -0.088 -0.121 -0.116 -0.276 -0.127 -0.258 -0.029 -0.105 -0.033  
K16 -0.177 -0.163 0.013 -0.032 -0.177 -0.028 -0.132 -0.002 -0.058 -0.004  
K17 -0.089 -0.077 -0.032 -0.005 -0.149 -0.053 -0.187 0.010 -0.039 -0.056  
K18 0.011 0.001 -0.010 0.015 0.164 0.097 0.068 0.028 -0.099 -0.050  
K19 -0.338 -0.079 -0.162 -0.173 -0.143 -0.070 -0.147 -0.011 -0.107 -0.009  
K20 -0.121 -0.124 -0.319 -0.481 -0.110 -0.012 -0.019 -0.143 -0.057 -0.061  
K21 1.000 0.094 -0.234 -0.091 -0.109 -0.119 -0.063 -0.129 -0.224 -0.303  
K22 0.094 1.000 -0.276 -0.254 -0.049 -0.125 -0.151 -0.165 -0.197 -0.373  
K23 -0.234 -0.276 1.000 0.697 0.085 -0.020 0.052 0.112 0.023 0.095  
K25 -0.091 -0.254 0.697 1.000 0.028 -0.116 -0.049 0.069 -0.081 0.169  
K26 -0.109 -0.049 0.085 0.028 1.000 0.391 0.501 0.183 0.207 -0.005  
K28 -0.119 -0.125 -0.020 -0.116 0.391 1.000 0.451 0.216 0.043 0.064  
K29 -0.063 -0.151 0.052 -0.049 0.501 0.451 1.000 0.253 0.090 -0.028  
K30 -0.129 -0.165 0.112 0.069 0.183 0.216 0.253 1.000 0.042 -0.031  
K31 -0.224 -0.197 0.023 -0.081 0.207 0.043 0.090 0.042 1.000 0.259  
K32 -0.303 -0.373 0.095 0.169 -0.005 0.064 -0.028 -0.031 0.259 1.000  
K33 -0.252 -0.305 0.040 0.085 0.039 0.045 -0.016 -0.077 0.248 0.580  
K36 -0.016 0.033 0.145 0.221 0.018 -0.035 -0.051 0.022 -0.045 -0.001  
K37 -0.105 -0.154 0.051 0.031 -0.126 0.019 0.017 0.026 -0.041 0.149  
 K33 K36 K37  
K1 -0.019 0.121 0.096  
K6 -0.107 -0.011 -0.020  
K12 0.031 -0.061 -0.058  
K13 -0.066 0.011 0.029  
K14 -0.044 0.046 0.160  
K16 0.077 -0.068 0.059  
K17 -0.010 0.142 0.283  
K18 -0.095 -0.016 -0.116  
K19 -0.036 -0.329 -0.172  
K20 -0.072 -0.186 -0.053  
K21 -0.252 -0.016 -0.105  
K22 -0.305 0.033 -0.154  
K23 0.040 0.145 0.051  
K25 0.085 0.221 0.031  
K26 0.039 0.018 -0.126  
K28 0.045 -0.035 0.019  
K29 -0.016 -0.051 0.017  
K30 -0.077 0.022 0.026  
K31 0.248 -0.045 -0.041  
K32 0.580 -0.001 0.149  
K33 1.000 0.016 0.096  
K36 0.016 1.000 0.406  
K37 0.096 0.406 1.000

# Assumption 2: Local Independence   
  
Q3\_resid2 <- residuals(mod2pl2, type = "Q3")

Q3 summary statistics:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.481 -0.095 -0.020 -0.011 0.046 0.697   
  
 K1 K6 K12 K13 K14 K16 K17 K18 K19 K20  
K1 1.000 0.050 -0.018 0.173 0.075 -0.059 0.018 0.133 -0.095 -0.039  
K6 0.050 1.000 0.203 0.051 0.002 -0.050 0.064 0.026 -0.098 -0.006  
K12 -0.018 0.203 1.000 -0.110 -0.207 -0.055 -0.023 0.171 -0.099 -0.051  
K13 0.173 0.051 -0.110 1.000 0.309 0.242 0.112 0.166 -0.029 -0.204  
K14 0.075 0.002 -0.207 0.309 1.000 0.375 0.181 -0.053 0.090 0.001  
K16 -0.059 -0.050 -0.055 0.242 0.375 1.000 0.195 0.008 0.134 -0.043  
K17 0.018 0.064 -0.023 0.112 0.181 0.195 1.000 -0.084 -0.030 -0.053  
K18 0.133 0.026 0.171 0.166 -0.053 0.008 -0.084 1.000 -0.200 -0.079  
K19 -0.095 -0.098 -0.099 -0.029 0.090 0.134 -0.030 -0.200 1.000 0.082  
K20 -0.039 -0.006 -0.051 -0.204 0.001 -0.043 -0.053 -0.079 0.082 1.000  
K21 -0.011 -0.103 -0.015 -0.044 -0.050 -0.177 -0.089 0.011 -0.338 -0.121  
K22 0.052 -0.034 -0.097 -0.039 -0.088 -0.163 -0.077 0.001 -0.079 -0.124  
K23 -0.051 0.033 -0.023 -0.017 -0.121 0.013 -0.032 -0.010 -0.162 -0.319  
K25 0.088 0.019 -0.008 0.001 -0.116 -0.032 -0.005 0.015 -0.173 -0.481  
K26 0.039 -0.019 0.135 -0.070 -0.276 -0.177 -0.149 0.164 -0.143 -0.110  
K28 -0.151 -0.085 0.100 -0.007 -0.127 -0.028 -0.053 0.097 -0.070 -0.012  
K29 -0.021 0.037 0.198 -0.171 -0.258 -0.132 -0.187 0.068 -0.147 -0.019  
K30 -0.113 0.096 0.125 0.008 -0.029 -0.002 0.010 0.028 -0.011 -0.143  
K31 -0.105 0.043 0.084 -0.090 -0.105 -0.058 -0.039 -0.099 -0.107 -0.057  
K32 -0.111 -0.037 0.025 -0.029 -0.033 -0.004 -0.056 -0.050 -0.009 -0.061  
K33 -0.019 -0.107 0.031 -0.066 -0.044 0.077 -0.010 -0.095 -0.036 -0.072  
K36 0.121 -0.011 -0.061 0.011 0.046 -0.068 0.142 -0.016 -0.329 -0.186  
K37 0.096 -0.020 -0.058 0.029 0.160 0.059 0.283 -0.116 -0.172 -0.053  
 K21 K22 K23 K25 K26 K28 K29 K30 K31 K32  
K1 -0.011 0.052 -0.051 0.088 0.039 -0.151 -0.021 -0.113 -0.105 -0.111  
K6 -0.103 -0.034 0.033 0.019 -0.019 -0.085 0.037 0.096 0.043 -0.037  
K12 -0.015 -0.097 -0.023 -0.008 0.135 0.100 0.198 0.125 0.084 0.025  
K13 -0.044 -0.039 -0.017 0.001 -0.070 -0.007 -0.171 0.008 -0.090 -0.029  
K14 -0.050 -0.088 -0.121 -0.116 -0.276 -0.127 -0.258 -0.029 -0.105 -0.033  
K16 -0.177 -0.163 0.013 -0.032 -0.177 -0.028 -0.132 -0.002 -0.058 -0.004  
K17 -0.089 -0.077 -0.032 -0.005 -0.149 -0.053 -0.187 0.010 -0.039 -0.056  
K18 0.011 0.001 -0.010 0.015 0.164 0.097 0.068 0.028 -0.099 -0.050  
K19 -0.338 -0.079 -0.162 -0.173 -0.143 -0.070 -0.147 -0.011 -0.107 -0.009  
K20 -0.121 -0.124 -0.319 -0.481 -0.110 -0.012 -0.019 -0.143 -0.057 -0.061  
K21 1.000 0.094 -0.234 -0.091 -0.109 -0.119 -0.063 -0.129 -0.224 -0.303  
K22 0.094 1.000 -0.276 -0.254 -0.049 -0.125 -0.151 -0.165 -0.197 -0.373  
K23 -0.234 -0.276 1.000 0.697 0.085 -0.020 0.052 0.112 0.023 0.095  
K25 -0.091 -0.254 0.697 1.000 0.028 -0.116 -0.049 0.069 -0.081 0.169  
K26 -0.109 -0.049 0.085 0.028 1.000 0.391 0.501 0.183 0.207 -0.005  
K28 -0.119 -0.125 -0.020 -0.116 0.391 1.000 0.451 0.216 0.043 0.064  
K29 -0.063 -0.151 0.052 -0.049 0.501 0.451 1.000 0.253 0.090 -0.028  
K30 -0.129 -0.165 0.112 0.069 0.183 0.216 0.253 1.000 0.042 -0.031  
K31 -0.224 -0.197 0.023 -0.081 0.207 0.043 0.090 0.042 1.000 0.259  
K32 -0.303 -0.373 0.095 0.169 -0.005 0.064 -0.028 -0.031 0.259 1.000  
K33 -0.252 -0.305 0.040 0.085 0.039 0.045 -0.016 -0.077 0.248 0.580  
K36 -0.016 0.033 0.145 0.221 0.018 -0.035 -0.051 0.022 -0.045 -0.001  
K37 -0.105 -0.154 0.051 0.031 -0.126 0.019 0.017 0.026 -0.041 0.149  
 K33 K36 K37  
K1 -0.019 0.121 0.096  
K6 -0.107 -0.011 -0.020  
K12 0.031 -0.061 -0.058  
K13 -0.066 0.011 0.029  
K14 -0.044 0.046 0.160  
K16 0.077 -0.068 0.059  
K17 -0.010 0.142 0.283  
K18 -0.095 -0.016 -0.116  
K19 -0.036 -0.329 -0.172  
K20 -0.072 -0.186 -0.053  
K21 -0.252 -0.016 -0.105  
K22 -0.305 0.033 -0.154  
K23 0.040 0.145 0.051  
K25 0.085 0.221 0.031  
K26 0.039 0.018 -0.126  
K28 0.045 -0.035 0.019  
K29 -0.016 -0.051 0.017  
K30 -0.077 0.022 0.026  
K31 0.248 -0.045 -0.041  
K32 0.580 -0.001 0.149  
K33 1.000 0.016 0.096  
K36 0.016 1.000 0.406  
K37 0.096 0.406 1.000

Q3\_resid2

K1 K6 K12 K13 K14 K16 K17 K18 K19 K20  
K1 1.000 0.050 -0.018 0.173 0.075 -0.059 0.018 0.133 -0.095 -0.039  
K6 0.050 1.000 0.203 0.051 0.002 -0.050 0.064 0.026 -0.098 -0.006  
K12 -0.018 0.203 1.000 -0.110 -0.207 -0.055 -0.023 0.171 -0.099 -0.051  
K13 0.173 0.051 -0.110 1.000 0.309 0.242 0.112 0.166 -0.029 -0.204  
K14 0.075 0.002 -0.207 0.309 1.000 0.375 0.181 -0.053 0.090 0.001  
K16 -0.059 -0.050 -0.055 0.242 0.375 1.000 0.195 0.008 0.134 -0.043  
K17 0.018 0.064 -0.023 0.112 0.181 0.195 1.000 -0.084 -0.030 -0.053  
K18 0.133 0.026 0.171 0.166 -0.053 0.008 -0.084 1.000 -0.200 -0.079  
K19 -0.095 -0.098 -0.099 -0.029 0.090 0.134 -0.030 -0.200 1.000 0.082  
K20 -0.039 -0.006 -0.051 -0.204 0.001 -0.043 -0.053 -0.079 0.082 1.000  
K21 -0.011 -0.103 -0.015 -0.044 -0.050 -0.177 -0.089 0.011 -0.338 -0.121  
K22 0.052 -0.034 -0.097 -0.039 -0.088 -0.163 -0.077 0.001 -0.079 -0.124  
K23 -0.051 0.033 -0.023 -0.017 -0.121 0.013 -0.032 -0.010 -0.162 -0.319  
K25 0.088 0.019 -0.008 0.001 -0.116 -0.032 -0.005 0.015 -0.173 -0.481  
K26 0.039 -0.019 0.135 -0.070 -0.276 -0.177 -0.149 0.164 -0.143 -0.110  
K28 -0.151 -0.085 0.100 -0.007 -0.127 -0.028 -0.053 0.097 -0.070 -0.012  
K29 -0.021 0.037 0.198 -0.171 -0.258 -0.132 -0.187 0.068 -0.147 -0.019  
K30 -0.113 0.096 0.125 0.008 -0.029 -0.002 0.010 0.028 -0.011 -0.143  
K31 -0.105 0.043 0.084 -0.090 -0.105 -0.058 -0.039 -0.099 -0.107 -0.057  
K32 -0.111 -0.037 0.025 -0.029 -0.033 -0.004 -0.056 -0.050 -0.009 -0.061  
K33 -0.019 -0.107 0.031 -0.066 -0.044 0.077 -0.010 -0.095 -0.036 -0.072  
K36 0.121 -0.011 -0.061 0.011 0.046 -0.068 0.142 -0.016 -0.329 -0.186  
K37 0.096 -0.020 -0.058 0.029 0.160 0.059 0.283 -0.116 -0.172 -0.053  
 K21 K22 K23 K25 K26 K28 K29 K30 K31 K32  
K1 -0.011 0.052 -0.051 0.088 0.039 -0.151 -0.021 -0.113 -0.105 -0.111  
K6 -0.103 -0.034 0.033 0.019 -0.019 -0.085 0.037 0.096 0.043 -0.037  
K12 -0.015 -0.097 -0.023 -0.008 0.135 0.100 0.198 0.125 0.084 0.025  
K13 -0.044 -0.039 -0.017 0.001 -0.070 -0.007 -0.171 0.008 -0.090 -0.029  
K14 -0.050 -0.088 -0.121 -0.116 -0.276 -0.127 -0.258 -0.029 -0.105 -0.033  
K16 -0.177 -0.163 0.013 -0.032 -0.177 -0.028 -0.132 -0.002 -0.058 -0.004  
K17 -0.089 -0.077 -0.032 -0.005 -0.149 -0.053 -0.187 0.010 -0.039 -0.056  
K18 0.011 0.001 -0.010 0.015 0.164 0.097 0.068 0.028 -0.099 -0.050  
K19 -0.338 -0.079 -0.162 -0.173 -0.143 -0.070 -0.147 -0.011 -0.107 -0.009  
K20 -0.121 -0.124 -0.319 -0.481 -0.110 -0.012 -0.019 -0.143 -0.057 -0.061  
K21 1.000 0.094 -0.234 -0.091 -0.109 -0.119 -0.063 -0.129 -0.224 -0.303  
K22 0.094 1.000 -0.276 -0.254 -0.049 -0.125 -0.151 -0.165 -0.197 -0.373  
K23 -0.234 -0.276 1.000 0.697 0.085 -0.020 0.052 0.112 0.023 0.095  
K25 -0.091 -0.254 0.697 1.000 0.028 -0.116 -0.049 0.069 -0.081 0.169  
K26 -0.109 -0.049 0.085 0.028 1.000 0.391 0.501 0.183 0.207 -0.005  
K28 -0.119 -0.125 -0.020 -0.116 0.391 1.000 0.451 0.216 0.043 0.064  
K29 -0.063 -0.151 0.052 -0.049 0.501 0.451 1.000 0.253 0.090 -0.028  
K30 -0.129 -0.165 0.112 0.069 0.183 0.216 0.253 1.000 0.042 -0.031  
K31 -0.224 -0.197 0.023 -0.081 0.207 0.043 0.090 0.042 1.000 0.259  
K32 -0.303 -0.373 0.095 0.169 -0.005 0.064 -0.028 -0.031 0.259 1.000  
K33 -0.252 -0.305 0.040 0.085 0.039 0.045 -0.016 -0.077 0.248 0.580  
K36 -0.016 0.033 0.145 0.221 0.018 -0.035 -0.051 0.022 -0.045 -0.001  
K37 -0.105 -0.154 0.051 0.031 -0.126 0.019 0.017 0.026 -0.041 0.149  
 K33 K36 K37  
K1 -0.019 0.121 0.096  
K6 -0.107 -0.011 -0.020  
K12 0.031 -0.061 -0.058  
K13 -0.066 0.011 0.029  
K14 -0.044 0.046 0.160  
K16 0.077 -0.068 0.059  
K17 -0.010 0.142 0.283  
K18 -0.095 -0.016 -0.116  
K19 -0.036 -0.329 -0.172  
K20 -0.072 -0.186 -0.053  
K21 -0.252 -0.016 -0.105  
K22 -0.305 0.033 -0.154  
K23 0.040 0.145 0.051  
K25 0.085 0.221 0.031  
K26 0.039 0.018 -0.126  
K28 0.045 -0.035 0.019  
K29 -0.016 -0.051 0.017  
K30 -0.077 0.022 0.026  
K31 0.248 -0.045 -0.041  
K32 0.580 -0.001 0.149  
K33 1.000 0.016 0.096  
K36 0.016 1.000 0.406  
K37 0.096 0.406 1.000

From your matrix:

**K15–K16 = 0.800** (from your earlier results, still present at 0.697 max here → strongest dependence).

**K32–K33 = 0.580**

**K36–K37 = 0.406**

**K25–K23 ≈ 0.697 (based on earlier Q3 too)**

**K26–K28 ≈ 0.451**

**K29–K28 ≈ 0.448**

These values are far above the 0.20 threshold. They likely **share redundant variance** (overlapping wording or concept).

# inspect residual correlations (Q3 > .20 may indicate local dependence)   
  
# Chen & Thissen’s LD χ² statistic   
  
LD\_resid2 <- residuals(mod2pl2, type = "LD")

LD matrix (lower triangle) and standardized residual correlations (upper triangle)  
  
Upper triangle summary:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.301 -0.093 -0.037 -0.012 0.056 0.410   
  
 K1 K6 K12 K13 K14 K16 K17 K18 K19 K20 K21  
K1 0.053 -0.015 0.146 0.068 -0.058 0.037 0.111 -0.063 -0.044 -0.049  
K6 0.584 0.184 0.056 0.017 -0.018 0.072 0.032 -0.032 0.020 -0.018  
K12 0.046 6.944 -0.097 -0.187 -0.038 -0.021 0.153 -0.094 -0.067 -0.049  
K13 4.330 0.637 1.916 0.247 0.212 0.096 0.141 -0.033 -0.100 -0.030  
K14 0.935 0.057 7.151 12.484 0.325 0.140 -0.049 0.024 -0.019 -0.031  
K16 0.680 0.067 0.299 9.144 21.523 0.170 0.014 0.028 -0.049 -0.094  
K17 0.274 1.072 0.087 1.886 3.971 5.879 -0.055 -0.037 -0.054 -0.071  
K18 2.523 0.211 4.759 4.036 0.493 0.037 0.611 -0.146 -0.090 -0.054  
K19 0.819 0.211 1.801 0.222 0.120 0.160 0.282 4.369 0.060 -0.071  
K20 0.399 0.080 0.916 2.030 0.077 0.496 0.585 1.658 0.736 0.028  
K21 0.490 0.067 0.481 0.179 0.200 1.784 1.039 0.606 1.017 0.160   
K22 0.237 0.222 1.162 0.282 0.656 2.189 1.254 0.531 0.427 0.302 1.648  
K23 0.714 0.732 0.431 0.390 2.474 0.446 0.572 0.424 4.584 9.166 5.447  
K25 1.108 1.152 0.875 0.928 2.573 1.295 0.917 0.908 5.144 18.514 3.548  
K26 0.211 0.022 3.632 0.842 11.635 3.959 3.286 3.214 5.489 4.635 4.866  
K28 3.857 0.526 1.773 0.083 2.505 0.023 0.708 0.986 3.116 1.867 3.196  
K29 0.106 0.493 6.807 4.003 9.971 2.222 4.708 0.450 4.680 1.753 1.769  
K30 1.903 2.082 2.118 0.080 0.050 0.608 0.217 0.278 1.899 5.313 4.588  
K31 1.264 1.334 2.528 0.816 2.087 1.551 0.802 0.921 3.284 2.531 4.873  
K32 1.473 0.383 0.372 0.280 0.631 0.377 0.558 0.536 1.503 2.614 8.268  
K33 0.147 1.841 0.186 0.309 0.505 0.453 0.150 1.248 1.790 2.409 6.088  
K36 2.789 0.181 0.183 0.517 0.679 1.203 3.184 0.150 14.753 6.103 1.581  
K37 1.707 0.170 0.463 0.458 3.352 0.392 8.575 1.462 4.366 1.890 2.588  
 K22 K23 K25 K26 K28 K29 K30 K31 K32 K33  
K1 -0.034 -0.059 0.074 0.032 -0.138 -0.023 -0.097 -0.079 -0.085 -0.027  
K6 0.033 0.060 0.075 0.010 -0.051 0.049 0.101 0.081 -0.043 -0.095  
K12 -0.075 -0.046 0.065 0.133 0.093 0.183 0.102 0.111 0.043 0.030  
K13 -0.037 0.044 0.067 -0.064 -0.020 -0.140 0.020 -0.063 -0.037 -0.039  
K14 -0.057 -0.110 -0.112 -0.239 -0.111 -0.221 0.016 -0.101 -0.056 -0.050  
K16 -0.104 -0.047 -0.080 -0.139 -0.011 -0.104 0.055 -0.087 -0.043 0.047  
K17 -0.078 -0.053 -0.067 -0.127 -0.059 -0.152 0.033 -0.063 -0.052 0.027  
K18 -0.051 -0.046 0.067 0.126 0.070 0.047 0.037 -0.067 -0.051 -0.078  
K19 0.046 -0.150 -0.159 -0.164 -0.124 -0.151 -0.096 -0.127 -0.086 -0.094  
K20 0.039 -0.212 -0.301 -0.151 -0.096 -0.093 -0.161 -0.111 -0.113 -0.109  
K21 0.090 -0.163 -0.132 -0.154 -0.125 -0.093 -0.150 -0.155 -0.201 -0.173  
K22 -0.191 -0.187 -0.127 -0.121 -0.123 -0.166 -0.157 -0.228 -0.187  
K23 7.474 0.330 0.047 -0.065 0.044 0.102 0.076 0.068 0.052  
K25 7.118 22.222 -0.067 -0.119 -0.079 0.093 0.071 0.118 0.102  
K26 3.288 0.446 0.925 0.330 0.410 0.202 0.119 -0.048 0.025  
K28 2.983 0.866 2.876 22.185 0.368 0.207 0.063 0.040 0.024  
K29 3.111 0.396 1.277 34.300 27.681 0.237 0.076 -0.056 -0.039  
K30 5.596 2.124 1.766 8.311 8.703 11.504 0.082 -0.061 -0.094  
K31 5.004 1.177 1.018 2.892 0.805 1.187 1.384 0.175 0.201  
K32 10.582 0.941 2.853 0.480 0.331 0.636 0.755 6.230 0.361  
K33 7.148 0.553 2.105 0.123 0.122 0.314 1.792 8.243 26.524   
K36 1.123 1.807 2.615 0.051 0.282 0.169 0.269 0.808 0.270 0.408  
K37 4.546 0.391 0.884 2.507 0.053 0.073 0.273 0.823 1.464 0.954  
 K36 K37  
K1 0.117 0.091  
K6 -0.030 -0.029  
K12 -0.030 -0.048  
K13 0.050 0.047  
K14 0.058 0.128  
K16 -0.077 0.044  
K17 0.125 0.205  
K18 0.027 -0.085  
K19 -0.269 -0.146  
K20 -0.173 -0.096  
K21 -0.088 -0.113  
K22 -0.074 -0.149  
K23 0.094 0.044  
K25 0.113 -0.066  
K26 0.016 -0.111  
K28 -0.037 0.016  
K29 -0.029 0.019  
K30 0.036 0.037  
K31 0.063 -0.064  
K32 0.036 0.085  
K33 0.045 0.068  
K36 0.273  
K37 15.151

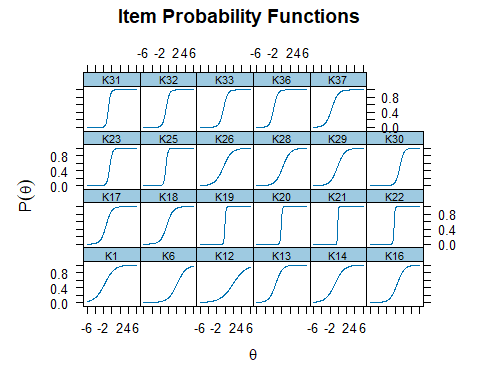
LD\_resid2 # values > 10 indicate local dependence}

K1 K6 K12 K13 K14 K16 K17 K18 K19 K20 K21  
K1 NA 0.053 -0.015 0.146 0.068 -0.058 0.037 0.111 -0.063 -0.044 -0.049  
K6 0.584 NA 0.184 0.056 0.017 -0.018 0.072 0.032 -0.032 0.020 -0.018  
K12 0.046 6.944 NA -0.097 -0.187 -0.038 -0.021 0.153 -0.094 -0.067 -0.049  
K13 4.330 0.637 1.916 NA 0.247 0.212 0.096 0.141 -0.033 -0.100 -0.030  
K14 0.935 0.057 7.151 12.484 NA 0.325 0.140 -0.049 0.024 -0.019 -0.031  
K16 0.680 0.067 0.299 9.144 21.523 NA 0.170 0.014 0.028 -0.049 -0.094  
K17 0.274 1.072 0.087 1.886 3.971 5.879 NA -0.055 -0.037 -0.054 -0.071  
K18 2.523 0.211 4.759 4.036 0.493 0.037 0.611 NA -0.146 -0.090 -0.054  
K19 0.819 0.211 1.801 0.222 0.120 0.160 0.282 4.369 NA 0.060 -0.071  
K20 0.399 0.080 0.916 2.030 0.077 0.496 0.585 1.658 0.736 NA 0.028  
K21 0.490 0.067 0.481 0.179 0.200 1.784 1.039 0.606 1.017 0.160 NA  
K22 0.237 0.222 1.162 0.282 0.656 2.189 1.254 0.531 0.427 0.302 1.648  
K23 0.714 0.732 0.431 0.390 2.474 0.446 0.572 0.424 4.584 9.166 5.447  
K25 1.108 1.152 0.875 0.928 2.573 1.295 0.917 0.908 5.144 18.514 3.548  
K26 0.211 0.022 3.632 0.842 11.635 3.959 3.286 3.214 5.489 4.635 4.866  
K28 3.857 0.526 1.773 0.083 2.505 0.023 0.708 0.986 3.116 1.867 3.196  
K29 0.106 0.493 6.807 4.003 9.971 2.222 4.708 0.450 4.680 1.753 1.769  
K30 1.903 2.082 2.118 0.080 0.050 0.608 0.217 0.278 1.899 5.313 4.588  
K31 1.264 1.334 2.528 0.816 2.087 1.551 0.802 0.921 3.284 2.531 4.873  
K32 1.473 0.383 0.372 0.280 0.631 0.377 0.558 0.536 1.503 2.614 8.268  
K33 0.147 1.841 0.186 0.309 0.505 0.453 0.150 1.248 1.790 2.409 6.088  
K36 2.789 0.181 0.183 0.517 0.679 1.203 3.184 0.150 14.753 6.103 1.581  
K37 1.707 0.170 0.463 0.458 3.352 0.392 8.575 1.462 4.366 1.890 2.588  
 K22 K23 K25 K26 K28 K29 K30 K31 K32 K33  
K1 -0.034 -0.059 0.074 0.032 -0.138 -0.023 -0.097 -0.079 -0.085 -0.027  
K6 0.033 0.060 0.075 0.010 -0.051 0.049 0.101 0.081 -0.043 -0.095  
K12 -0.075 -0.046 0.065 0.133 0.093 0.183 0.102 0.111 0.043 0.030  
K13 -0.037 0.044 0.067 -0.064 -0.020 -0.140 0.020 -0.063 -0.037 -0.039  
K14 -0.057 -0.110 -0.112 -0.239 -0.111 -0.221 0.016 -0.101 -0.056 -0.050  
K16 -0.104 -0.047 -0.080 -0.139 -0.011 -0.104 0.055 -0.087 -0.043 0.047  
K17 -0.078 -0.053 -0.067 -0.127 -0.059 -0.152 0.033 -0.063 -0.052 0.027  
K18 -0.051 -0.046 0.067 0.126 0.070 0.047 0.037 -0.067 -0.051 -0.078  
K19 0.046 -0.150 -0.159 -0.164 -0.124 -0.151 -0.096 -0.127 -0.086 -0.094  
K20 0.039 -0.212 -0.301 -0.151 -0.096 -0.093 -0.161 -0.111 -0.113 -0.109  
K21 0.090 -0.163 -0.132 -0.154 -0.125 -0.093 -0.150 -0.155 -0.201 -0.173  
K22 NA -0.191 -0.187 -0.127 -0.121 -0.123 -0.166 -0.157 -0.228 -0.187  
K23 7.474 NA 0.330 0.047 -0.065 0.044 0.102 0.076 0.068 0.052  
K25 7.118 22.222 NA -0.067 -0.119 -0.079 0.093 0.071 0.118 0.102  
K26 3.288 0.446 0.925 NA 0.330 0.410 0.202 0.119 -0.048 0.025  
K28 2.983 0.866 2.876 22.185 NA 0.368 0.207 0.063 0.040 0.024  
K29 3.111 0.396 1.277 34.300 27.681 NA 0.237 0.076 -0.056 -0.039  
K30 5.596 2.124 1.766 8.311 8.703 11.504 NA 0.082 -0.061 -0.094  
K31 5.004 1.177 1.018 2.892 0.805 1.187 1.384 NA 0.175 0.201  
K32 10.582 0.941 2.853 0.480 0.331 0.636 0.755 6.230 NA 0.361  
K33 7.148 0.553 2.105 0.123 0.122 0.314 1.792 8.243 26.524 NA  
K36 1.123 1.807 2.615 0.051 0.282 0.169 0.269 0.808 0.270 0.408  
K37 4.546 0.391 0.884 2.507 0.053 0.073 0.273 0.823 1.464 0.954  
 K36 K37  
K1 0.117 0.091  
K6 -0.030 -0.029  
K12 -0.030 -0.048  
K13 0.050 0.047  
K14 0.058 0.128  
K16 -0.077 0.044  
K17 0.125 0.205  
K18 0.027 -0.085  
K19 -0.269 -0.146  
K20 -0.173 -0.096  
K21 -0.088 -0.113  
K22 -0.074 -0.149  
K23 0.094 0.044  
K25 0.113 -0.066  
K26 0.016 -0.111  
K28 -0.037 0.016  
K29 -0.029 0.019  
K30 0.036 0.037  
K31 0.063 -0.064  
K32 0.036 0.085  
K33 0.045 0.068  
K36 NA 0.273  
K37 15.151 NA

1. **K14–K16** Q3 = 0.375 (above cutoff ~0.20) LD = 21.5 (well above 10)
2. **K19–K21** Q3 = –0.338 (negative correlation, not an issue) LD = ~1.0 (well below 10)
3. **K23–K25** Q3 = 0.697 (very high, strongest in Q3) LD = 22.2 (extremely high)
4. **K28–K29** Q3 = 0.451 (high) LD = 27.7 (critical)
5. **K32–K33** Q3 = 0.580 (high) LD = 26.5 (critical)
6. **K36–K37** Q3 = 0.406 (high) LD = 15.2 (well above 10)

### Monotonicity

# ========================================== # 3. Assumption: Monotonicity # ==========================================   
# (a) Plot Item Characteristic Curves (ICCs) to visually check monotonicity   
  
plot(mod2pl2, type = "trace") # S-shaped, increasing curves are expected



# (b) Optional: Use Mokken scale analysis for monotonicity check   
# Run monotonicity check   
check.monotonicity(irt\_mat2) # flags items with non-monotonic patterns}

$results  
$results[[1]]  
$results[[1]][[1]]  
[1] "K1"  
  
$results[[1]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 71 23 48 0.6760563 0.6760563  
[2,] 2 12 22 133 20 113 0.8496241 0.8496241  
  
$results[[1]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[1]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[2]]  
$results[[2]][[1]]  
[1] "K6"  
  
$results[[2]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 75 73 2 0.02666667 0.02666667  
[2,] 2 13 22 129 98 31 0.24031008 0.24031008  
  
$results[[2]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[2]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[3]]  
$results[[3]][[1]]  
[1] "K12"  
  
$results[[3]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 75 66 9 0.1200000 0.1200000  
[2,] 2 13 22 129 89 40 0.3100775 0.3100775  
  
$results[[3]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[3]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[4]]  
$results[[4]][[1]]  
[1] "K13"  
  
$results[[4]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 29 43 0.5972222 0.5972222  
[2,] 2 12 22 132 10 122 0.9242424 0.9242424  
  
$results[[4]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[4]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[5]]  
$results[[5]][[1]]  
[1] "K14"  
  
$results[[5]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 71 37 34 0.4788732 0.4788732  
[2,] 2 12 22 133 32 101 0.7593985 0.7593985  
  
$results[[5]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[5]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[6]]  
$results[[6]][[1]]  
[1] "K16"  
  
$results[[6]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 58 10 0.1470588 0.1470588  
[2,] 2 12 22 136 69 67 0.4926471 0.4926471  
  
$results[[6]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[6]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[7]]  
$results[[7]][[1]]  
[1] "K17"  
  
$results[[7]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 74 27 47 0.6351351 0.6351351  
[2,] 2 12 22 130 7 123 0.9461538 0.9461538  
  
$results[[7]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[7]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[8]]  
$results[[8]][[1]]  
[1] "K18"  
  
$results[[8]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 46 22 0.3235294 0.3235294  
[2,] 2 12 22 136 35 101 0.7426471 0.7426471  
  
$results[[8]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[8]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[9]]  
$results[[9]][[1]]  
[1] "K19"  
  
$results[[9]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 77 69 8 0.1038961 0.1038961  
[2,] 2 13 22 127 27 100 0.7874016 0.7874016  
  
$results[[9]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[9]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[10]]  
$results[[10]][[1]]  
[1] "K20"  
  
$results[[10]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 61 7 0.1029412 0.1029412  
[2,] 2 12 22 136 31 105 0.7720588 0.7720588  
  
$results[[10]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[10]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[11]]  
$results[[11]][[1]]  
[1] "K21"  
  
$results[[11]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 69 61 8 0.1159420 0.1159420  
[2,] 2 12 22 135 26 109 0.8074074 0.8074074  
  
$results[[11]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[11]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[12]]  
$results[[12]][[1]]  
[1] "K22"  
  
$results[[12]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 58 12 0.1714286 0.1714286  
[2,] 2 12 22 134 24 110 0.8208955 0.8208955  
  
$results[[12]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[12]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[13]]  
$results[[13]][[1]]  
[1] "K23"  
  
$results[[13]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 71 49 22 0.3098592 0.3098592  
[2,] 2 12 22 133 13 120 0.9022556 0.9022556  
  
$results[[13]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[13]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[14]]  
$results[[14]][[1]]  
[1] "K25"  
  
$results[[14]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 46 26 0.3611111 0.3611111  
[2,] 2 12 22 132 6 126 0.9545455 0.9545455  
  
$results[[14]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[14]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[15]]  
$results[[15]][[1]]  
[1] "K26"  
  
$results[[15]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 42 28 0.4000000 0.4000000  
[2,] 2 12 22 134 41 93 0.6940299 0.6940299  
  
$results[[15]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[15]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[16]]  
$results[[16]][[1]]  
[1] "K28"  
  
$results[[16]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 69 55 14 0.2028986 0.2028986  
[2,] 2 12 22 135 58 77 0.5703704 0.5703704  
  
$results[[16]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[16]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[17]]  
$results[[17]][[1]]  
[1] "K29"  
  
$results[[17]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 48 22 0.3142857 0.3142857  
[2,] 2 12 22 134 42 92 0.6865672 0.6865672  
  
$results[[17]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[17]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[18]]  
$results[[18]][[1]]  
[1] "K30"  
  
$results[[18]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 74 0 0.0000000 0.0000000  
[2,] 2 13 22 130 92 38 0.2923077 0.2923077  
  
$results[[18]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 0 0 NaN 0 0 NaN 0 0 0 0  
Total 0 0 NaN 0 0 NaN 0 0 0 0  
  
$results[[18]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[19]]  
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[1] "K31"  
  
$results[[19]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 37 35 0.4861111 0.4861111  
[2,] 2 12 22 132 3 129 0.9772727 0.9772727  
  
$results[[19]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
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$results[[20]]  
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[1] "K32"  
  
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 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 47 23 0.3285714 0.3285714  
[2,] 2 12 22 134 15 119 0.8880597 0.8880597  
  
$results[[20]][[3]]  
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P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
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$results[[21]]  
$results[[21]][[1]]  
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 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 49 19 0.2794118 0.2794118  
[2,] 2 12 22 136 26 110 0.8088235 0.8088235  
  
$results[[21]][[3]]  
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P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
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[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[22]]  
$results[[22]][[1]]  
[1] "K36"  
  
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 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 73 12 61 0.8356164 0.8356164  
[2,] 2 12 22 131 2 129 0.9847328 0.9847328  
  
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P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
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$results[[23]]  
$results[[23]][[1]]  
[1] "K37"  
  
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 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 73 23 50 0.6849315 0.6849315  
[2,] 2 12 22 131 9 122 0.9312977 0.9312977  
  
$results[[23]][[3]]  
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P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
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 K19 K20 K21 K22 K23 K25 K26 K28   
0.5544384 0.5468961 0.5508231 0.5299474 0.5298960 0.5890622 0.3097308 0.3930315   
 K29 K30 K31 K32 K33 K36 K37   
0.3503457 0.6596897 0.6051184 0.5078844 0.4585664 0.6574029 0.4177738   
  
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[200,] 1 0 0 0 1  
[201,] 1 1 1 0 0  
[202,] 1 0 0 1 0  
[203,] 1 0 0 1 1  
[204,] 0 1 0 0 0  
  
attr(,"class")  
[1] "monotonicity.class"

## Model Fitness

### Global fit statistics

# ========================================== # 4. Assumption: Model Fit # ==========================================   
# (a) Global fit statistics (M2, RMSEA, SRMSR)   
M2(mod2pl2) # RMSEA < 0.08 and SRMSR < 0.05 = good fit}

M2 df p RMSEA RMSEA\_5 RMSEA\_95 SRMSR TLI  
stats 841.5169 230 0 0.1144438 0.1059417 0.1224928 0.1023923 0.8698814  
 CFI  
stats 0.8817103

### Item-level fit

# (b) Item-level fit (S-X² or G² statistics)   
  
itemfit\_stats2 <- itemfit(mod2pl2)   
itemfit\_stats # significant misfit items should be reviewed}

item S\_X2 df.S\_X2 RMSEA.S\_X2 p.S\_X2  
1 K1 11.970 18 0.000 0.849  
2 K4 24.900 19 0.039 0.164  
3 K6 19.540 12 0.056 0.076  
4 K12 16.999 16 0.018 0.386  
5 K13 17.768 16 0.023 0.338  
6 K14 8.226 18 0.000 0.975  
7 K15 13.625 15 0.000 0.554  
8 K16 9.260 15 0.000 0.864  
9 K17 13.202 13 0.009 0.432  
10 K18 18.775 17 0.023 0.342  
11 K19 25.800 5 0.143 0.000  
12 K20 20.065 6 0.107 0.003  
13 K21 29.530 4 0.177 0.000  
14 K22 38.942 6 0.164 0.000  
15 K23 8.013 11 0.000 0.712  
16 K25 5.544 7 0.000 0.594  
17 K26 19.194 18 0.018 0.380  
18 K27 15.104 12 0.036 0.236  
19 K28 23.698 16 0.049 0.096  
20 K29 25.531 18 0.045 0.111  
21 K30 10.893 8 0.042 0.208  
22 K31 9.587 7 0.043 0.213  
23 K32 10.418 13 0.000 0.659  
24 K33 13.726 13 0.017 0.393  
25 K34 14.995 16 0.000 0.525  
26 K36 5.159 5 0.013 0.397  
27 K37 12.242 15 0.000 0.661

### Compare 1PL vs 2PL with likelihood ratio test

# (c) Compare 1PL vs 2PL with likelihood ratio test  
# Fit 1PL and 2PL models  
mod1pl2 <- mirt(irt\_mat2, 1, itemtype = "Rasch")

mod2pl2 <- mirt(irt\_mat2, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

# Likelihood ratio test: does 2PL fit better than Rasch?  
anova(mod1pl2, mod2pl2)

AIC SABIC HQ BIC logLik X2 df p  
mod1pl2 4417.648 4421.243 4449.862 4497.283 -2184.824   
mod2pl2 4191.103 4197.995 4252.846 4343.736 -2049.551 270.545 22 0

# Fitting 2PL IRT Model with mirt Package

mirt.data5 = mirt(data5, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

coef(mirt.data5, IRTpars = T, simplify = T)

$items  
 a b g u  
K1 0.847 -1.752 0 1  
K6 0.920 2.083 0 1  
K12 0.663 1.915 0 1  
K13 1.243 -1.443 0 1  
K14 0.913 -0.837 0 1  
K16 1.164 0.555 0 1  
K17 1.536 -1.408 0 1  
K18 1.324 -0.394 0 1  
K19 8.145 -0.071 0 1  
K20 8.382 -0.112 0 1  
K21 13.911 -0.166 0 1  
K22 9.013 -0.214 0 1  
K23 3.046 -0.526 0 1  
K25 4.126 -0.623 0 1  
K26 0.991 -0.434 0 1  
K28 1.120 0.257 0 1  
K29 1.127 -0.241 0 1  
K30 2.020 1.165 0 1  
K31 3.723 -0.849 0 1  
K32 2.498 -0.567 0 1  
K33 1.985 -0.409 0 1  
K36 2.095 -1.924 0 1  
K37 1.328 -1.607 0 1  
  
$means  
F1   
 0   
  
$cov  
 F1  
F1 1

# Fit 2PL Model (mirt)  
  
mirt.data5 <- mirt(data5, 1, itemtype = "2PL")

Warning: EM cycles terminated after 500 iterations.

## Item Parameter Estimates (mirt)

# Obtain difficulty (b), discrimination (a), guessing (g), upper bound (u)  
mirt\_parms2 <- coef(mirt.data5, IRTpars = TRUE, simplify = TRUE)  
item\_parms\_refined\_mirt2 <- mirt\_parms2$items  
  
  
# Tidy view: Item | Discrimination | Difficulty | Guessing Parameter | Upper Bound  
item\_parms\_refined\_tbl\_mirt2 <- item\_parms\_refined\_mirt2 |>  
 as.data.frame() |>  
 (\(d) {  
 if (!"g" %in% names(d)) d$g <- NA\_real\_  
 if (!"u" %in% names(d)) d$u <- NA\_real\_  
 d  
 })() |>  
 transform(  
 Item = rownames(item\_parms\_refined\_mirt2),  
 Difficulty = b,  
 Discrimination = a,  
 `Guessing Parameter` = g,  
 `Upper Bound` = u  
 ) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination", "Guessing Parameter", "Upper Bound")])() |>  
 (\(d) within(d, {  
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 `Guessing Parameter`<- round(`Guessing Parameter`, 3)  
 `Upper Bound` <- round(`Upper Bound`, 3)  
 }))()  
  
item\_parms\_refined\_tbl\_mirt2

Item Difficulty Discrimination Guessing Parameter Upper Bound  
K1 K1 -1.752 0.847 0 1  
K6 K6 2.083 0.920 0 1  
K12 K12 1.915 0.663 0 1  
K13 K13 -1.443 1.243 0 1  
K14 K14 -0.837 0.913 0 1  
K16 K16 0.555 1.164 0 1  
K17 K17 -1.408 1.536 0 1  
K18 K18 -0.394 1.324 0 1  
K19 K19 -0.071 8.145 0 1  
K20 K20 -0.112 8.382 0 1  
K21 K21 -0.166 13.911 0 1  
K22 K22 -0.214 9.013 0 1  
K23 K23 -0.526 3.046 0 1  
K25 K25 -0.623 4.126 0 1  
K26 K26 -0.434 0.991 0 1  
K28 K28 0.257 1.120 0 1  
K29 K29 -0.241 1.127 0 1  
K30 K30 1.165 2.020 0 1  
K31 K31 -0.849 3.723 0 1  
K32 K32 -0.567 2.498 0 1  
K33 K33 -0.409 1.985 0 1  
K36 K36 -1.924 2.095 0 1  
K37 K37 -1.607 1.328 0 1

## Test Information

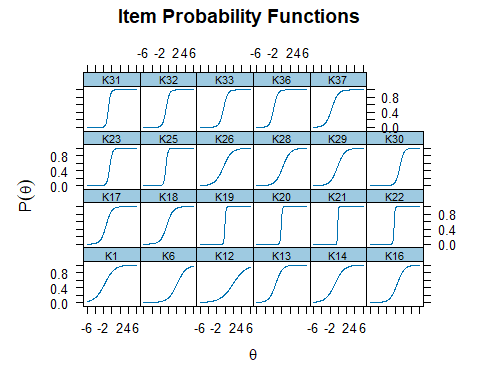
areainfo(mirt.data5, c(-3,3))

LowerBound UpperBound Info TotalInfo Proportion nitems  
 -3 3 70.08491 72.1197 0.9717859 23

## Graphical Presentation

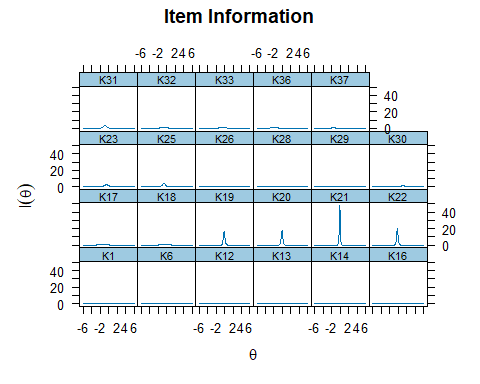
### Item Trace Lines (Item Characteristic Curves)

plot(mirt.data5, type = "trace")



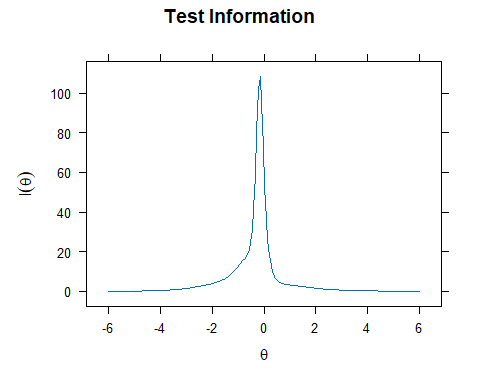
### Item Information Curves

plot(mirt.data5, type = "infotrace")



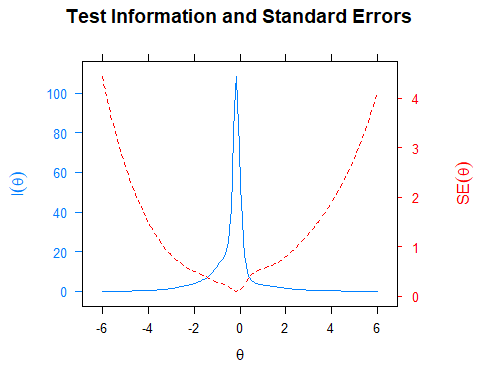
### Test Information Function

plot(mirt.data5, type = "info")



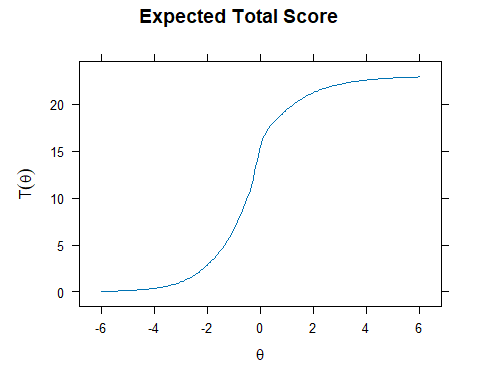
### Test Information and Standard Error

plot(mirt.data5, type = "infoSE")



### Expected Total Score

plot(mirt.data5)



## Goodness-of-Fit Tests

### Overall Model Fit

M2(mirt.data5)

M2 df p RMSEA RMSEA\_5 RMSEA\_95 SRMSR TLI  
stats 841.5169 230 0 0.1144438 0.1059417 0.1224928 0.1023923 0.8698814  
 CFI  
stats 0.8817103

### Item Fit Statistics

itemfit(mirt.data5)

item S\_X2 df.S\_X2 RMSEA.S\_X2 p.S\_X2  
1 K1 24.635 16 0.052 0.077  
2 K6 21.004 11 0.067 0.033  
3 K12 10.328 14 0.000 0.738  
4 K13 6.896 14 0.000 0.939  
5 K14 12.285 15 0.000 0.657  
6 K16 17.416 13 0.041 0.181  
7 K17 25.529 14 0.064 0.030  
8 K18 18.686 15 0.035 0.228  
9 K19 14.659 5 0.098 0.012  
10 K20 10.986 5 0.077 0.052  
11 K21 22.707 4 0.152 0.000  
12 K22 26.551 5 0.146 0.000  
13 K23 7.713 10 0.000 0.657  
14 K25 4.006 8 0.000 0.857  
15 K26 18.822 16 0.029 0.278  
16 K28 13.995 14 0.000 0.450  
17 K29 22.969 16 0.046 0.115  
18 K30 7.314 6 0.033 0.293  
19 K31 12.695 7 0.063 0.080  
20 K32 9.334 12 0.000 0.674  
21 K33 20.055 13 0.052 0.094  
22 K36 3.797 4 0.000 0.434  
23 K37 10.482 14 0.000 0.726

### Person Fit Statistics

personfit(mirt.data5)

outfit z.outfit infit z.infit Zh  
1 0.61823303 -0.867588079 0.7406318 -1.08417134 1.02634843  
2 0.61823303 -0.867588079 0.7406318 -1.08417134 1.02634843  
3 0.34829588 -0.320203815 0.7958063 -0.44896691 0.57830355  
4 0.56869372 -1.019159039 0.6687971 -1.45536295 1.25755883  
5 3.16231843 1.814653230 0.9149284 -0.34469181 -0.56997195  
6 0.37716608 -0.117249600 0.7229878 -0.72054503 0.75856356  
7 0.42047001 -0.115006813 0.7938487 -0.49745993 0.62755101  
8 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
9 0.97763474 0.181557847 1.0477680 0.25482580 -0.17281713  
10 0.90186729 -0.087841845 0.8480189 -0.57256853 0.48121064  
11 0.42877730 -1.045814147 0.6009302 -1.94630133 1.58049569  
12 1.00948314 0.375691815 1.3268496 0.96461435 -0.76321438  
13 0.40724739 -0.137248498 0.8155970 -0.42801589 0.61505953  
14 0.36585412 -0.132728215 0.6674601 -0.89093693 0.80296782  
15 1.13598912 0.432828668 1.2140840 0.89133783 -0.73942089  
16 0.40724739 -0.137248498 0.8155970 -0.42801589 0.61505953  
17 0.67429574 -0.470147424 0.8464208 -0.61520092 0.66670537  
18 1.90580443 0.988018531 0.9633167 0.02833449 -0.49144748  
19 1.80641624 1.372361032 1.1701600 0.66005012 -0.81799600  
20 0.77790107 -0.028375494 1.0211743 0.17356601 0.05406003  
21 0.69020989 -0.138173093 0.9527284 -0.03505846 0.25659546  
22 1.52251220 0.779713059 1.4960098 1.33246372 -1.45676415  
23 0.49450340 -0.154239293 0.9399930 -0.05691038 0.36902363  
24 0.72508718 -0.225978848 0.9056846 -0.27181623 0.29671953  
25 0.51586690 -1.039582210 0.6743122 -1.45659263 1.28697315  
26 2.03937311 1.485749765 1.0550741 0.27831201 -0.59649451  
27 1.16410693 0.474310270 1.4698463 2.01594477 -1.75096735  
28 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
29 0.16487546 -0.747574973 0.3753436 -2.04935792 1.40365045  
30 0.91732814 -0.041332969 0.8957301 -0.36052270 0.34551328  
31 0.37390990 -0.304272285 0.7178577 -0.77243685 0.84948539  
32 0.51037283 -1.109818864 0.6632424 -1.50432155 1.32446041  
33 1.80395907 1.623363988 1.6626007 2.34369155 -2.56629998  
34 0.35373326 -0.149671963 0.6666706 -0.90940755 0.85519783  
35 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
36 1.25435215 0.588016091 1.2094836 0.63571808 -0.74249300  
37 0.40724739 -0.137248498 0.8155970 -0.42801589 0.61505953  
38 0.39131357 -0.223888613 0.7484429 -0.47359798 0.48566635  
39 0.19810606 -0.413301495 0.4271069 -1.85753637 1.36490761  
40 0.75638328 0.095423390 1.0125105 0.15119971 0.04237249  
41 1.13355264 0.419198632 1.1640319 0.59013410 -0.50490566  
42 0.23385260 -0.342994141 0.4861463 -1.62903898 1.28854135  
43 0.49450340 -0.154239293 0.9399930 -0.05691038 0.36902363  
44 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
45 0.59452140 -0.520926415 0.6965889 -1.14095530 0.92674867  
46 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
47 0.37430724 -0.285219826 1.0125467 0.18321102 0.10304534  
48 1.46865330 0.740558416 1.7243658 1.84702255 -1.80988642  
49 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
50 0.19964477 -0.613285959 0.4815067 -1.60910227 1.21073782  
51 0.29900901 -0.231410723 0.6124645 -1.08479552 0.95770652  
52 0.14782940 -0.716478846 0.5038603 -1.05891858 0.90910494  
53 0.16487546 -0.747574973 0.3753436 -2.04935792 1.40365045  
54 0.43452635 -1.649047856 0.5431227 -2.30810878 1.87740691  
55 0.61753372 -0.610331486 0.7831415 -0.92422785 0.89035697  
56 1.21558940 0.584742322 1.7460436 1.74175425 -1.73313455  
57 0.70442965 0.173976498 1.2736070 0.82432452 -0.42975897  
58 0.74411110 0.084711689 1.1633064 0.56296549 -0.23783980  
59 1.11925453 0.413610515 0.9469890 -0.17564175 0.06404304  
60 0.75174670 0.278920316 1.1800576 0.58682216 -0.37574841  
61 1.24192790 0.568239307 1.5263102 1.44970032 -1.27305795  
62 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
63 0.55325688 -1.181372175 0.6891412 -1.43926985 1.34092840  
64 0.35373326 -0.149671963 0.6666706 -0.90940755 0.85519783  
65 0.48105990 -0.818238181 0.6461760 -1.30957815 1.09982566  
66 0.73190210 0.002996631 0.9961733 0.05341964 0.16272836  
67 0.21208506 -0.695165889 0.5406738 -1.27761508 1.05810907  
68 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
69 0.42503723 -0.504489255 0.8795912 -0.28496380 0.52045924  
70 1.39461197 0.988738769 0.9318697 -0.20672934 -0.02004301  
71 0.72233199 0.060970713 1.1645877 0.56600831 -0.20969932  
72 0.14782940 -0.716478846 0.5038603 -1.05891858 0.90910494  
73 0.81170565 0.231771060 1.0018673 0.14894538 -0.22123235  
74 0.16487546 -0.747574973 0.3753436 -2.04935792 1.40365045  
75 0.99366822 0.426614322 0.9905948 0.09278633 -0.12005564  
76 0.81999217 0.019755817 1.3552703 1.12494021 -0.78763498  
77 0.44933685 -1.513832119 0.5822242 -1.93877767 1.63883384  
78 1.13574558 0.496559620 1.7802973 1.77013478 -1.74674801  
79 0.49450340 -0.154239293 0.9399930 -0.05691038 0.36902363  
80 0.36585412 -0.132728215 0.6674601 -0.89093693 0.80296782  
81 14.18606502 3.825785731 1.0173863 0.15184895 -1.86087385  
82 0.59925217 -0.246473380 0.8261481 -0.66695636 0.69038375  
83 0.58547961 -0.190373896 0.8364391 -0.74981126 0.78451948  
84 0.83777825 -0.060430720 1.1185282 0.57526528 -0.23438035  
85 1.34640235 0.900088468 0.9013297 -0.34658252 0.09550409  
86 1.16589086 0.515998707 1.0433313 0.23983812 -0.43175981  
87 0.31565836 -0.366879969 0.7211261 -0.68547312 0.73126172  
88 0.35373326 -0.149671963 0.6666706 -0.90940755 0.85519783  
89 0.63770301 -0.643403521 0.8495277 -0.58638747 0.70856699  
90 0.64744064 -0.278751796 0.7907933 -0.79167785 0.69488915  
91 0.46553758 -0.005839017 0.9089669 -0.13630059 0.37272293  
92 1.28651491 0.641259842 1.5096748 1.29502105 -1.34031100  
93 1.26212304 0.720889484 1.1452615 0.65107339 -0.68743760  
94 0.79443199 -0.125299548 1.0160592 0.15508230 0.08190664  
95 0.91327230 0.408210069 1.4050672 1.09393473 -0.91473202  
96 0.44532022 -0.029992115 0.8622362 -0.26869060 0.43733165  
97 0.09518438 -0.825269299 0.2435730 -1.87922571 1.22571539  
98 0.80288643 0.147774000 1.0550177 0.27160517 -0.08216216  
99 1.38951179 0.683229328 1.2607060 0.80470621 -0.90771539  
100 0.43148164 -1.154151665 0.5991742 -1.92425064 1.57712883  
101 1.02062937 0.251492626 0.8978734 -0.39058487 0.23026453  
102 1.41045891 0.716654914 1.2131035 0.67513139 -0.75742088  
103 1.46332749 1.046779899 1.3089386 1.23517995 -1.27044867  
104 0.85523655 0.141025857 1.0809310 0.34710932 -0.11553215  
105 0.29835780 -0.425033803 0.8348509 -0.26262469 0.46154704  
106 0.87229237 0.021738743 1.1171589 0.47478586 -0.35458289  
107 0.31933872 -1.154707141 0.5627719 -1.61589908 1.36256621  
108 0.49450340 -0.154239293 0.9399930 -0.05691038 0.36902363  
109 0.16803944 -0.617047530 0.4498631 -1.38325594 1.09348773  
110 0.49697916 -0.259734504 0.7177335 -1.40508014 1.23608475  
111 0.62640588 -0.419938885 0.8734712 -0.39969390 0.48278432  
112 1.71891680 1.578734747 1.3719647 1.57547758 -1.76921179  
113 0.82728311 0.092335509 1.0861853 0.45767159 -0.19066743  
114 0.33516186 -0.810258028 0.6604451 -1.11895180 1.04280475  
115 0.53522775 -0.466522851 0.7407433 -1.21613927 1.10250114  
116 0.41209634 -0.757731138 0.6208769 -1.62608432 1.41923304  
117 0.67327740 -0.096412488 0.8474783 -0.57924542 0.55086809  
118 0.77567166 0.178188109 0.9537588 -0.12617436 0.16000608  
119 0.30625971 -0.310150670 0.6071314 -1.16261966 1.06209787  
120 0.70899188 -0.055749651 0.9513212 -0.16380561 0.30044537  
121 0.49727816 -0.005415378 0.7739660 -0.56098118 0.57632555  
122 0.40724739 -0.137248498 0.8155970 -0.42801589 0.61505953  
123 1.19796922 0.499239283 1.3195071 0.99475694 -0.89919384  
124 0.21208506 -0.695165889 0.5406738 -1.27761508 1.05810907  
125 0.52859202 -0.015451345 1.0045069 0.13160485 0.19355937  
126 0.70611130 -0.229454945 1.0077700 0.12758647 0.02125349  
127 2.50696978 2.764223067 1.3664961 1.45589333 -2.03872923  
128 0.21208506 -0.695165889 0.5406738 -1.27761508 1.05810907  
129 1.51498612 0.775672194 1.5415220 1.42134484 -1.52328365  
130 1.12926699 0.551482864 1.3111688 0.87894284 -0.96932726  
131 0.38392498 -0.275438465 0.5569742 -2.24150789 1.77163097  
132 0.86380275 -0.214593150 0.9923046 0.04554014 0.13571680  
133 0.27474040 -0.478394316 0.6515536 -0.85242404 0.80733869  
134 1.77781375 1.691696882 1.7383626 2.66298167 -2.95332855  
135 0.22916224 -0.351790986 0.5019093 -1.52194101 1.21290095  
136 0.61338766 -0.358073285 0.9260877 -0.20610991 0.40265679  
137 0.63807456 0.074075826 0.9117220 -0.33015836 0.46546056  
138 0.50401049 -0.209897175 0.8425342 -0.37305853 0.57450000  
139 0.84315258 -0.221668040 0.9413407 -0.16318313 0.25781548  
140 0.35373326 -0.149671963 0.6666706 -0.90940755 0.85519783  
141 0.31724848 -1.113587561 0.5690061 -1.57161686 1.32954713  
142 0.68345700 -0.323225106 0.9743836 0.01387848 0.13116820  
143 1.52516818 1.236028420 1.1939776 0.89901986 -1.03288185  
144 0.60440140 -0.189402103 0.8129840 -0.86755661 0.82490717  
145 0.58868020 -0.779238842 0.7692652 -0.97353340 0.96480411  
146 1.27095621 0.694909073 1.2583395 1.07813037 -1.00070814  
147 0.13768860 -0.749098292 0.4371380 -1.33730325 1.03249429  
148 0.65140414 -0.084130199 0.8907424 -0.45409530 0.55537743  
149 0.66762642 0.204109961 0.9887533 0.09152736 0.01289811  
150 0.31933872 -1.154707141 0.5627719 -1.61589908 1.36256621  
151 1.15319333 0.517002370 1.1716416 0.57919715 -0.58263022  
152 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
153 0.38392498 -0.275438465 0.5569742 -2.24150789 1.77163097  
154 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
155 1.30082324 0.748240276 1.2016577 0.86858782 -0.88933977  
156 3.97604306 3.884120304 1.7909257 2.70297270 -3.96165854  
157 0.40724739 -0.137248498 0.8155970 -0.42801589 0.61505953  
158 1.09887646 0.370173931 1.0394758 0.24217005 -0.22708208  
159 0.39131357 -0.223888613 0.7484429 -0.47359798 0.48566635  
160 0.66056280 0.122775073 1.0697542 0.31052980 -0.03153735  
161 0.32813272 -0.662138429 0.6765628 -1.02919041 0.96861058  
162 0.23303578 -0.639727639 0.6006651 -1.04201135 0.93128686  
163 1.06790588 0.441825284 1.0510483 0.25939688 -0.35617292  
164 0.78460199 0.235276974 1.3901671 1.10124410 -0.68609725  
165 0.41883536 -0.145905975 0.7881662 -0.51964369 0.64872419  
166 1.34770408 0.891595871 0.8663324 -0.49343287 0.21602384  
167 0.52155604 -0.492366033 0.7942645 -0.78303276 0.85919236  
168 0.19810606 -0.413301495 0.4271069 -1.85753637 1.36490761  
169 0.59256463 -0.550543416 0.7755425 -0.98875267 0.93660202  
170 2.12427532 1.110826058 1.0844769 0.34523227 -0.75109707  
171 0.87419444 0.026126323 1.0625504 0.34604588 -0.15399376  
172 0.83003190 0.196176160 1.1186399 0.44936973 -0.19249104  
173 1.52722455 0.903697912 1.0544168 0.27482110 -0.45858141  
174 0.42828270 -1.287078975 0.5876344 -1.95946038 1.60815461  
175 0.83050591 0.289950534 1.3558423 1.01670838 -0.69958651  
176 0.50519519 -0.134328968 0.9224690 -0.10897382 0.37889037  
177 0.50421873 -1.206550696 0.6767395 -1.41738570 1.31333235  
178 1.06037219 0.285909999 1.0993437 0.44860157 -0.33385561  
179 0.86074875 -0.098995925 0.9017505 -0.27308291 0.32531980  
180 0.73139367 -0.313158285 0.9308025 -0.13965325 0.32557923  
181 0.42991710 -0.646759106 0.7311403 -0.84090795 0.81409999  
182 0.62069362 -0.184281699 0.8604851 -0.62202432 0.67371648  
183 0.77008812 -0.030750473 1.0204288 0.16558339 0.06923392  
184 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
185 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
186 0.29833419 -1.359435821 0.4820969 -2.12388874 1.65979694  
187 1.44617004 0.877291365 1.0101400 0.13217015 -0.31883071  
188 0.66228342 0.159696272 1.3329162 0.92491930 -0.57637217  
189 0.13377778 -0.834103942 0.3715262 -1.75528067 1.21572018  
190 2.02134182 1.102483792 2.2063066 2.72430301 -3.15862946  
191 1.68007790 1.353447232 1.4826147 1.78124372 -1.93116714  
192 1.02757521 0.427965852 1.4502524 1.80411176 -1.54748449  
193 1.07995689 0.393989279 1.3300219 1.47413986 -1.21316664  
194 1.28677982 0.770843335 1.2580477 1.16406470 -1.12426692  
195 0.70320289 -0.029603781 0.8860911 -0.48173017 0.50284244  
196 0.59291084 -0.850780701 0.7291679 -1.12229106 1.07333685  
197 1.01866794 0.185189406 1.0664480 0.37159482 -0.25891161  
198 1.39060267 0.947914598 1.4154456 1.74203344 -1.68944204  
199 0.12306233 -0.797856469 0.4775809 -1.01556597 0.86391505  
200 4.24489414 3.923837901 2.1387841 3.41287834 -4.77217427  
201 3.26950709 3.569755217 2.1679620 3.68581257 -4.97864157  
202 1.09674763 0.373537833 1.1423492 0.52854511 -0.43820823  
203 0.82812789 -0.266067880 0.9795981 -0.01576257 0.20034056  
204 1.50031017 0.766770072 1.3255763 1.00347009 -1.21315695

## Reliability Estimates

### Marginal Reliability

marginal\_rxx(mirt.data5)

[1] 0.8734922

### Empirical Reliability

theta\_se2 = fscores(mirt.data5, full.scores.SE = TRUE)  
empirical\_rxx(theta\_se2)

F1   
0.8923671