IRT

firdaus

# Prepare Environment

## Load Libraries

library(psych) # For basic psychometrics and scale reliability analysis  
library(foreign) # For reading and writing data in foreign statistical formats  
library(ltm) # To fit 2PL IRT models and other latent trait models

Loading required package: MASS

Loading required package: msm

Loading required package: polycor

Attaching package: 'polycor'

The following object is masked from 'package:psych':  
  
 polyserial

Attaching package: 'ltm'

The following object is masked from 'package:psych':  
  
 factor.scores

library(irtoys) # For IRT utilities

Loading required package: sm

Package 'sm', version 2.2-6.0: type help(sm) for summary information

Attaching package: 'sm'

The following object is masked from 'package:MASS':  
  
 muscle

Attaching package: 'irtoys'

The following object is masked from 'package:psych':  
  
 sim

library(mirt) # Modern IRT package for multi-item response theory

Loading required package: stats4

Loading required package: lattice

Attaching package: 'mirt'

The following object is masked from 'package:ltm':  
  
 Science

library(latticeExtra) # For enhanced plotting in lattice-based plots  
library(tidyverse) # For data manipulation, cleaning, and visualization

── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──  
✔ dplyr 1.1.4 ✔ readr 2.1.5  
✔ forcats 1.0.0 ✔ stringr 1.5.1  
✔ ggplot2 4.0.0 ✔ tibble 3.3.0  
✔ lubridate 1.9.4 ✔ tidyr 1.3.1  
✔ purrr 1.1.0

── Conflicts ────────────────────────────────────────── tidyverse\_conflicts() ──  
✖ ggplot2::%+%() masks psych::%+%()  
✖ ggplot2::alpha() masks psych::alpha()  
✖ dplyr::filter() masks stats::filter()  
✖ dplyr::lag() masks stats::lag()  
✖ ggplot2::layer() masks latticeExtra::layer()  
✖ dplyr::select() masks MASS::select()  
ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors

library(haven) # For importing and exporting SPSS, Stata, and SAS files  
library(writexl) # For exporting data frames to Excel files  
library(readxl) # For reading data from Excel files

## Load Data

data1=read\_xlsx("IRT\_knowledge\_V1.xlsx") ##read data from Excel   
names(data1) # List down variables in the data set

[1] "K1" "K2" "K3" "K4" "K5" "K6" "K7" "K8" "K9" "K10" "K11" "K12"  
[13] "K13" "K14" "K15" "K16" "K17" "K18" "K19" "K20" "K21" "K22" "K23" "K24"  
[25] "K25" "K26" "K27" "K28" "K29" "K30" "K31" "K32" "K33" "K34" "K35" "K36"  
[37] "K37"

dim(data1) # Data set consists of 37 variables and 177 parents

[1] 204 37

### Recode Data

# Define reverse-coded items  
reverse\_items <- c("K2", "K3", "K4", "K5", "K8", "K10", "K35")  
  
# Recode  
data2 <- data1 %>%  
 mutate(across(  
 -all\_of(reverse\_items),   
 ~ case\_when(  
 tolower(.) == "ya" ~ 1,  
 tolower(.) == "tidak" ~ 0,  
 tolower(.) == "tidak pasti" ~ 2,  
 TRUE ~ NA\_real\_  
 )  
 )) %>%  
 mutate(across(  
 all\_of(reverse\_items),  
 ~ case\_when(  
 tolower(.) == "ya" ~ 0,  
 tolower(.) == "tidak" ~ 1,  
 tolower(.) == "tidak pasti" ~ 2,  
 TRUE ~ NA\_real\_  
 )  
 ))

#Recode 1 = 1 (correct answer), 2 and 0 = 0 (incorrect answer)  
  
data3 <- data2 %>%  
 mutate(across(  
 everything(),  
 ~ case\_when(  
 . == 1 ~ 1,  
 . %in% c(0, 2) ~ 0,  
 TRUE ~ NA\_real\_  
 )  
 ))

# Descriptive Statistics

## Response Frequencies

response.frequencies(data3)

0 1 miss  
K1 0.21078431 0.7892157 0  
K2 0.60784314 0.3921569 0  
K3 0.53921569 0.4607843 0  
K4 0.61764706 0.3823529 0  
K5 0.32352941 0.6764706 0  
K6 0.83823529 0.1617647 0  
K7 0.55882353 0.4411765 0  
K8 0.57352941 0.4264706 0  
K9 0.47549020 0.5245098 0  
K10 0.50490196 0.4950980 0  
K11 0.62254902 0.3774510 0  
K12 0.75980392 0.2401961 0  
K13 0.19117647 0.8088235 0  
K14 0.33823529 0.6617647 0  
K15 0.60294118 0.3970588 0  
K16 0.62254902 0.3774510 0  
K17 0.16666667 0.8333333 0  
K18 0.39705882 0.6029412 0  
K19 0.47058824 0.5294118 0  
K20 0.45098039 0.5490196 0  
K21 0.42647059 0.5735294 0  
K22 0.40196078 0.5980392 0  
K23 0.30392157 0.6960784 0  
K24 0.27941176 0.7205882 0  
K25 0.25490196 0.7450980 0  
K26 0.40686275 0.5931373 0  
K27 0.71078431 0.2892157 0  
K28 0.55392157 0.4460784 0  
K29 0.44117647 0.5588235 0  
K30 0.81372549 0.1862745 0  
K31 0.19607843 0.8039216 0  
K32 0.30392157 0.6960784 0  
K33 0.36764706 0.6323529 0  
K34 0.18627451 0.8137255 0  
K35 0.58333333 0.4166667 0  
K36 0.06862745 0.9313725 0  
K37 0.15686275 0.8431373 0

### Descriptive Statistics

descript(data3)

Descriptive statistics for the 'data3' data-set  
  
Sample:  
 37 items and 204 sample units; 0 missing values  
  
Proportions for each level of response:  
 0 1 logit  
K1 0.2108 0.7892 1.3202  
K2 0.6078 0.3922 -0.4383  
K3 0.5392 0.4608 -0.1572  
K4 0.6176 0.3824 -0.4796  
K5 0.3235 0.6765 0.7376  
K6 0.8382 0.1618 -1.6452  
K7 0.5588 0.4412 -0.2364  
K8 0.5735 0.4265 -0.2963  
K9 0.4755 0.5245 0.0981  
K10 0.5049 0.4951 -0.0196  
K11 0.6225 0.3775 -0.5004  
K12 0.7598 0.2402 -1.1516  
K13 0.1912 0.8088 1.4424  
K14 0.3382 0.6618 0.6712  
K15 0.6029 0.3971 -0.4177  
K16 0.6225 0.3775 -0.5004  
K17 0.1667 0.8333 1.6094  
K18 0.3971 0.6029 0.4177  
K19 0.4706 0.5294 0.1178  
K20 0.4510 0.5490 0.1967  
K21 0.4265 0.5735 0.2963  
K22 0.4020 0.5980 0.3973  
K23 0.3039 0.6961 0.8287  
K24 0.2794 0.7206 0.9474  
K25 0.2549 0.7451 1.0726  
K26 0.4069 0.5931 0.3769  
K27 0.7108 0.2892 -0.8992  
K28 0.5539 0.4461 -0.2165  
K29 0.4412 0.5588 0.2364  
K30 0.8137 0.1863 -1.4744  
K31 0.1961 0.8039 1.4110  
K32 0.3039 0.6961 0.8287  
K33 0.3676 0.6324 0.5423  
K34 0.1863 0.8137 1.4744  
K35 0.5833 0.4167 -0.3365  
K36 0.0686 0.9314 2.6080  
K37 0.1569 0.8431 1.6818  
  
  
Frequencies of total scores:  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27  
Freq 1 0 1 1 2 1 2 1 4 5 6 5 3 9 4 7 10 4 10 7 8 16 10 8 12 11 5 9  
 28 29 30 31 32 33 34 35 36 37  
Freq 10 6 9 3 3 3 2 1 3 2  
  
  
Point Biserial correlation with Total Score:  
 Included Excluded  
K1 0.3742 0.3274  
K2 0.2793 0.2194  
K3 0.2500 0.1880  
K4 0.3596 0.3028  
K5 0.0997 0.0392  
K6 0.3816 0.3397  
K7 0.2545 0.1928  
K8 0.2645 0.2034  
K9 0.3174 0.2574  
K10 0.3082 0.2478  
K11 0.2970 0.2381  
K12 0.4038 0.3558  
K13 0.4129 0.3693  
K14 0.3313 0.2750  
K15 0.5002 0.4503  
K16 0.4716 0.4205  
K17 0.4539 0.4142  
K18 0.5610 0.5151  
K19 0.6431 0.6027  
K20 0.6762 0.6387  
K21 0.6889 0.6528  
K22 0.6503 0.6112  
K23 0.6857 0.6520  
K24 0.7137 0.6833  
K25 0.6905 0.6591  
K26 0.4828 0.4316  
K27 0.5352 0.4912  
K28 0.4978 0.4469  
K29 0.5478 0.5002  
K30 0.5352 0.4977  
K31 0.6280 0.5953  
K32 0.6193 0.5804  
K33 0.5917 0.5487  
K34 0.3898 0.3457  
K35 0.2232 0.1612  
K36 0.4188 0.3912  
K37 0.4330 0.3935  
  
  
Cronbach's alpha:  
 value  
All Items 0.8935  
Excluding K1 0.8921  
Excluding K2 0.8941  
Excluding K3 0.8948  
Excluding K4 0.8927  
Excluding K5 0.8969  
Excluding K6 0.8919  
Excluding K7 0.8947  
Excluding K8 0.8945  
Excluding K9 0.8936  
Excluding K10 0.8937  
Excluding K11 0.8938  
Excluding K12 0.8917  
Excluding K13 0.8915  
Excluding K14 0.8931  
Excluding K15 0.8901  
Excluding K16 0.8907  
Excluding K17 0.8909  
Excluding K18 0.8890  
Excluding K19 0.8874  
Excluding K20 0.8867  
Excluding K21 0.8865  
Excluding K22 0.8873  
Excluding K23 0.8868  
Excluding K24 0.8864  
Excluding K25 0.8869  
Excluding K26 0.8905  
Excluding K27 0.8895  
Excluding K28 0.8902  
Excluding K29 0.8892  
Excluding K30 0.8897  
Excluding K31 0.8882  
Excluding K32 0.8880  
Excluding K33 0.8884  
Excluding K34 0.8918  
Excluding K35 0.8952  
Excluding K36 0.8917  
Excluding K37 0.8912  
  
  
Pairwise Associations:  
 Item i Item j p.value  
1 2 28 1.000  
2 4 26 1.000  
3 5 22 1.000  
4 5 35 1.000  
5 7 17 1.000  
6 7 19 1.000  
7 7 20 1.000  
8 7 35 1.000  
9 8 35 1.000  
10 10 21 1.000

# Fitting 2PL IRT Model with ltm Package

## Fit 2PL Model (ltm)

irt.data3 <- ltm(data3 ~ z1, IRT.param = TRUE)

## Item Parameter Estimates

# Obtain difficulty and discrimination parameter estimates  
item\_parms <- coef(irt.data3)

# Tidy view: Item | a (Discrimination) | b (Difficulty)  
  
item\_parms\_tbl <- item\_parms |>  
 as.data.frame() |>  
 transform(Item = rownames(item\_parms),  
 Difficulty = Dffclt,  
 Discrimination = Dscrmn) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination")])() |>  
 (\(d) within(d, {   
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 }))()  
  
item\_parms\_tbl

Item Difficulty Discrimination  
K1 K1 -1.426 0.873  
K2 K2 1.275 0.473  
K3 K3 0.719 0.390  
K4 K4 1.094 0.660  
K5 K5 -9.376 0.076  
K6 K6 2.247 0.994  
K7 K7 1.228 0.260  
K8 K8 1.778 0.203  
K9 K9 0.001 0.333  
K10 K10 0.368 0.298  
K11 K11 1.779 0.349  
K12 K12 2.059 0.724  
K13 K13 -1.250 1.135  
K14 K14 -0.559 0.908  
K15 K15 0.676 1.475  
K16 K16 0.800 1.268  
K17 K17 -1.078 1.618  
K18 K18 -0.106 1.362  
K19 K19 0.210 4.683  
K20 K20 0.158 5.578  
K21 K21 0.096 6.871  
K22 K22 0.053 4.676  
K23 K23 -0.163 5.311  
K24 K24 -0.082 16.964  
K25 K25 -0.266 6.626  
K26 K26 -0.159 0.976  
K27 K27 1.073 1.614  
K28 K28 0.531 1.132  
K29 K29 0.050 1.214  
K30 K30 1.382 2.216  
K31 K31 -0.564 3.755  
K32 K32 -0.265 2.718  
K33 K33 -0.113 2.130  
K34 K34 -1.202 1.225  
K35 K35 1.416 0.309  
K36 K36 -1.530 2.379  
K37 K37 -1.220 1.471

## Model Summary

# Includes log-likelihood, AIC/BIC, SEs, and Wald z-values  
summary(irt.data3)

Call:  
ltm(formula = data3 ~ z1, IRT.param = TRUE)  
  
Model Summary:  
 log.Lik AIC BIC  
 -3748.378 7644.756 7890.297  
  
Coefficients:  
 value std.err z.vals  
Dffclt.K1 -1.4264 0.4021 -3.5474  
Dffclt.K2 1.2750 0.4395 2.9011  
Dffclt.K3 0.7195 0.4000 1.7987  
Dffclt.K4 1.0943 0.2948 3.7118  
Dffclt.K5 -9.3762 19.8814 -0.4716  
Dffclt.K6 2.2471 0.4202 5.3475  
Dffclt.K7 1.2275 0.7566 1.6224  
Dffclt.K8 1.7782 1.2819 1.3871  
Dffclt.K9 0.0009 0.4468 0.0021  
Dffclt.K10 0.3675 0.4764 0.7714  
Dffclt.K11 1.7786 0.7647 2.3260  
Dffclt.K12 2.0587 0.4714 4.3673  
Dffclt.K13 -1.2502 0.2985 -4.1882  
Dffclt.K14 -0.5586 0.2302 -2.4270  
Dffclt.K15 0.6763 0.1252 5.4015  
Dffclt.K16 0.7998 0.1486 5.3817  
Dffclt.K17 -1.0782 0.2101 -5.1321  
Dffclt.K18 -0.1062 0.1297 -0.8185  
Dffclt.K19 0.2102 0.0539 3.8974  
Dffclt.K20 0.1584 0.0500 3.1693  
Dffclt.K21 0.0956 0.0422 2.2641  
Dffclt.K22 0.0527 0.0515 1.0217  
Dffclt.K23 -0.1631 0.0568 -2.8727  
Dffclt.K24 -0.0817 0.3221 -0.2537  
Dffclt.K25 -0.2657 0.0658 -4.0389  
Dffclt.K26 -0.1590 0.1747 -0.9101  
Dffclt.K27 1.0730 0.1472 7.2883  
Dffclt.K28 0.5313 0.1449 3.6679  
Dffclt.K29 0.0504 0.1353 0.3729  
Dffclt.K30 1.3822 0.1532 9.0193  
Dffclt.K31 -0.5639 0.0873 -6.4600  
Dffclt.K32 -0.2649 0.0864 -3.0675  
Dffclt.K33 -0.1135 0.0932 -1.2167  
Dffclt.K34 -1.2024 0.2740 -4.3881  
Dffclt.K35 1.4162 0.7078 2.0009  
Dffclt.K36 -1.5304 0.2380 -6.4304  
Dffclt.K37 -1.2203 0.2437 -5.0064  
Dscrmn.K1 0.8727 0.2206 3.9555  
Dscrmn.K2 0.4727 0.1642 2.8785  
Dscrmn.K3 0.3899 0.1564 2.4939  
Dscrmn.K4 0.6598 0.1775 3.7173  
Dscrmn.K5 0.0763 0.1565 0.4879  
Dscrmn.K6 0.9942 0.2469 4.0263  
Dscrmn.K7 0.2602 0.1516 1.7159  
Dscrmn.K8 0.2027 0.1501 1.3505  
Dscrmn.K9 0.3331 0.1547 2.1530  
Dscrmn.K10 0.2983 0.1529 1.9504  
Dscrmn.K11 0.3486 0.1586 2.1979  
Dscrmn.K12 0.7243 0.1992 3.6356  
Dscrmn.K13 1.1346 0.2524 4.4945  
Dscrmn.K14 0.9076 0.2045 4.4370  
Dscrmn.K15 1.4748 0.2662 5.5409  
Dscrmn.K16 1.2676 0.2407 5.2662  
Dscrmn.K17 1.6182 0.3240 4.9951  
Dscrmn.K18 1.3621 0.2489 5.4718  
Dscrmn.K19 4.6826 0.8496 5.5116  
Dscrmn.K20 5.5780 1.2893 4.3264  
Dscrmn.K21 6.8712 1.5567 4.4140  
Dscrmn.K22 4.6763 0.8492 5.5065  
Dscrmn.K23 5.3111 1.1319 4.6924  
Dscrmn.K24 16.9641 66.8082 0.2539  
Dscrmn.K25 6.6261 1.4010 4.7297  
Dscrmn.K26 0.9758 0.2055 4.7490  
Dscrmn.K27 1.6137 0.2974 5.4255  
Dscrmn.K28 1.1320 0.2235 5.0660  
Dscrmn.K29 1.2142 0.2308 5.2616  
Dscrmn.K30 2.2162 0.4249 5.2159  
Dscrmn.K31 3.7552 0.6691 5.6120  
Dscrmn.K32 2.7180 0.4797 5.6656  
Dscrmn.K33 2.1299 0.3664 5.8124  
Dscrmn.K34 1.2254 0.2628 4.6628  
Dscrmn.K35 0.3093 0.1548 1.9986  
Dscrmn.K36 2.3787 0.5652 4.2086  
Dscrmn.K37 1.4710 0.3023 4.8656  
  
Integration:  
method: Gauss-Hermite  
quadrature points: 21   
  
Optimization:  
Convergence: 0   
max(|grad|): 0.088   
quasi-Newton: BFGS

## Items Removal Plan 1

**Selection criteria a > 0.64 (moderate discrimination) (Baker, 2001) ; -3 < b > +3**

K2 - a = 0.47

K3 - a = 0.39

K5 - a = 0.08 , b = -9.3762

K7 - a = 0.26

K8 - a = 0.20

K9 - a = 0.33

K10 - a = 0.30

K11 - a = 0.35

K24 - a = 16.9641 (?Overfitting)

K35 - a = 0.31

**Based on Local Dependence**

K4 : K4–K6 (LD 16.3; Q3 0.295)

K15 : K15–K16 (Q3 = 0.800; LD = 70.6)

K27 : K26–K27–K28–K29 (multiple LD > 14; Q3 up to 0.556)

K34 : K34–K36–K37 (LD ≈ 12–13; Q3 ≈ 0.35–0.39)

**Based on Item Fit**

K19 : p =0.000

K22 : p = 0.000

### 2PL Model - Remove Items

# Remove the items  
irt\_removed\_items <- c("K2", "K3", "K5", "K7", "K8", "K9", "K10", "K11","K35","K24","K19","K22")  
  
# Create new dataset with only included items  
data4 <- data3 %>% dplyr::select(-any\_of(irt\_removed\_items))

### Descriptive Statistics

descript(data4)

Descriptive statistics for the 'data4' data-set  
  
Sample:  
 25 items and 204 sample units; 0 missing values  
  
Proportions for each level of response:  
 0 1 logit  
K1 0.2108 0.7892 1.3202  
K4 0.6176 0.3824 -0.4796  
K6 0.8382 0.1618 -1.6452  
K12 0.7598 0.2402 -1.1516  
K13 0.1912 0.8088 1.4424  
K14 0.3382 0.6618 0.6712  
K15 0.6029 0.3971 -0.4177  
K16 0.6225 0.3775 -0.5004  
K17 0.1667 0.8333 1.6094  
K18 0.3971 0.6029 0.4177  
K20 0.4510 0.5490 0.1967  
K21 0.4265 0.5735 0.2963  
K23 0.3039 0.6961 0.8287  
K25 0.2549 0.7451 1.0726  
K26 0.4069 0.5931 0.3769  
K27 0.7108 0.2892 -0.8992  
K28 0.5539 0.4461 -0.2165  
K29 0.4412 0.5588 0.2364  
K30 0.8137 0.1863 -1.4744  
K31 0.1961 0.8039 1.4110  
K32 0.3039 0.6961 0.8287  
K33 0.3676 0.6324 0.5423  
K34 0.1863 0.8137 1.4744  
K36 0.0686 0.9314 2.6080  
K37 0.1569 0.8431 1.6818  
  
  
Frequencies of total scores:  
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25  
Freq 1 2 5 1 3 4 5 6 6 10 7 15 8 8 6 15 15 16 15 13 7 11 9 8 2 6  
  
  
Point Biserial correlation with Total Score:  
 Included Excluded  
K1 0.3635 0.3009  
K4 0.3289 0.2521  
K6 0.3741 0.3182  
K12 0.3404 0.2737  
K13 0.4887 0.4352  
K14 0.4263 0.3569  
K15 0.5832 0.5240  
K16 0.5464 0.4844  
K17 0.5111 0.4618  
K18 0.5410 0.4779  
K20 0.6710 0.6201  
K21 0.6733 0.6230  
K23 0.6852 0.6400  
K25 0.6999 0.6589  
K26 0.4609 0.3911  
K27 0.5897 0.5357  
K28 0.5251 0.4595  
K29 0.5272 0.4619  
K30 0.5641 0.5162  
K31 0.6664 0.6261  
K32 0.6652 0.6179  
K33 0.6380 0.5852  
K34 0.4537 0.3986  
K36 0.4615 0.4266  
K37 0.4910 0.4418  
  
  
Cronbach's alpha:  
 value  
All Items 0.8936  
Excluding K1 0.8934  
Excluding K4 0.8953  
Excluding K6 0.8928  
Excluding K12 0.8941  
Excluding K13 0.8905  
Excluding K14 0.8926  
Excluding K15 0.8884  
Excluding K16 0.8894  
Excluding K17 0.8900  
Excluding K18 0.8896  
Excluding K20 0.8859  
Excluding K21 0.8858  
Excluding K23 0.8856  
Excluding K25 0.8854  
Excluding K26 0.8918  
Excluding K27 0.8882  
Excluding K28 0.8901  
Excluding K29 0.8901  
Excluding K30 0.8889  
Excluding K31 0.8865  
Excluding K32 0.8861  
Excluding K33 0.8869  
Excluding K34 0.8913  
Excluding K36 0.8913  
Excluding K37 0.8905  
  
  
Pairwise Associations:  
 Item i Item j p.value  
1 2 15 1.000  
2 4 5 1.000  
3 1 17 1.000  
4 2 5 0.880  
5 1 16 0.723  
6 15 23 0.704  
7 5 18 0.643  
8 2 24 0.627  
9 4 25 0.593  
10 8 15 0.591

### Refit 2PL Model

irt.data4 <- ltm(data4 ~ z1, IRT.param = TRUE)

### Item Parameter Estimates

# Obtain difficulty and discrimination parameter estimates  
item\_parms\_refined <- coef(irt.data4)  
  
# Tidy view: Item | a (Discrimination) | b (Difficulty)  
  
item\_parms\_refined\_tbl <- item\_parms\_refined |>  
 as.data.frame() |>  
 transform(Item = rownames(item\_parms\_refined),  
 Difficulty = Dffclt,  
 Discrimination = Dscrmn) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination")])() |>  
 (\(d) within(d, {   
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 }))()  
  
item\_parms\_refined\_tbl

Item Difficulty Discrimination  
K1 K1 -2.039 0.780  
K4 K4 0.751 0.591  
K6 K6 1.903 0.956  
K12 K12 1.656 0.719  
K13 K13 -1.610 1.260  
K14 K14 -1.008 0.903  
K15 K15 0.256 1.507  
K16 K16 0.367 1.341  
K17 K17 -1.585 1.554  
K18 K18 -0.568 1.278  
K20 K20 -0.292 2.523  
K21 K21 -0.363 2.633  
K23 K23 -0.697 3.951  
K25 K25 -0.813 5.400  
K26 K26 -0.572 1.067  
K27 K27 0.594 2.026  
K28 K28 0.083 1.325  
K29 K29 -0.378 1.310  
K30 K30 0.919 2.882  
K31 K31 -1.064 4.007  
K32 K32 -0.737 2.857  
K33 K33 -0.568 2.213  
K34 K34 -1.619 1.290  
K36 K36 -1.923 2.746  
K37 K37 -1.645 1.558

### Model Summary

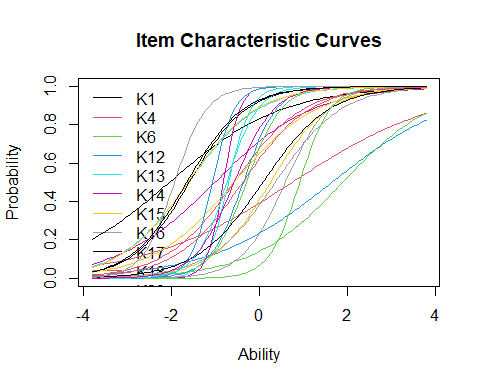
# Includes log-likelihood, AIC/BIC, SEs, and Wald z-values  
summary(irt.data4)

Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Model Summary:  
 log.Lik AIC BIC  
 -2337.231 4774.461 4940.367  
  
Coefficients:  
 value std.err z.vals  
Dffclt.K1 -2.0390 0.4735 -4.3060  
Dffclt.K4 0.7508 0.3435 2.1860  
Dffclt.K6 1.9035 0.4434 4.2926  
Dffclt.K12 1.6562 0.4725 3.5052  
Dffclt.K13 -1.6098 0.2616 -6.1528  
Dffclt.K14 -1.0076 0.2360 -4.2685  
Dffclt.K15 0.2556 0.1393 1.8353  
Dffclt.K16 0.3671 0.1545 2.3755  
Dffclt.K17 -1.5853 0.2207 -7.1842  
Dffclt.K18 -0.5682 0.1489 -3.8149  
Dffclt.K20 -0.2915 0.0980 -2.9743  
Dffclt.K21 -0.3634 0.0955 -3.8046  
Dffclt.K23 -0.6967 0.0831 -8.3862  
Dffclt.K25 -0.8134 0.0779 -10.4419  
Dffclt.K26 -0.5723 0.1707 -3.3524  
Dffclt.K27 0.5936 0.1404 4.2291  
Dffclt.K28 0.0826 0.1429 0.5780  
Dffclt.K29 -0.3780 0.1402 -2.6962  
Dffclt.K30 0.9193 0.1454 6.3228  
Dffclt.K31 -1.0643 0.1059 -10.0520  
Dffclt.K32 -0.7375 0.0996 -7.4070  
Dffclt.K33 -0.5677 0.1075 -5.2825  
Dffclt.K34 -1.6185 0.2560 -6.3235  
Dffclt.K36 -1.9233 0.2094 -9.1870  
Dffclt.K37 -1.6449 0.2303 -7.1429  
Dscrmn.K1 0.7799 0.2107 3.7024  
Dscrmn.K4 0.5908 0.1681 3.5157  
Dscrmn.K6 0.9561 0.2401 3.9831  
Dscrmn.K12 0.7191 0.1953 3.6829  
Dscrmn.K13 1.2599 0.2675 4.7095  
Dscrmn.K14 0.9028 0.2016 4.4780  
Dscrmn.K15 1.5071 0.2740 5.5009  
Dscrmn.K16 1.3411 0.2527 5.3077  
Dscrmn.K17 1.5545 0.3152 4.9322  
Dscrmn.K18 1.2777 0.2393 5.3401  
Dscrmn.K20 2.5235 0.4749 5.3137  
Dscrmn.K21 2.6328 0.5024 5.2409  
Dscrmn.K23 3.9508 0.8563 4.6139  
Dscrmn.K25 5.3998 1.5543 3.4740  
Dscrmn.K26 1.0669 0.2144 4.9770  
Dscrmn.K27 2.0262 0.3734 5.4266  
Dscrmn.K28 1.3246 0.2430 5.4502  
Dscrmn.K29 1.3098 0.2407 5.4406  
Dscrmn.K30 2.8819 0.6935 4.1557  
Dscrmn.K31 4.0074 0.8216 4.8775  
Dscrmn.K32 2.8568 0.5170 5.5257  
Dscrmn.K33 2.2133 0.3862 5.7317  
Dscrmn.K34 1.2904 0.2725 4.7360  
Dscrmn.K36 2.7464 0.7133 3.8505  
Dscrmn.K37 1.5585 0.3205 4.8630  
  
Integration:  
method: Gauss-Hermite  
quadrature points: 21   
  
Optimization:  
Convergence: 0   
max(|grad|): 0.017   
quasi-Newton: BFGS

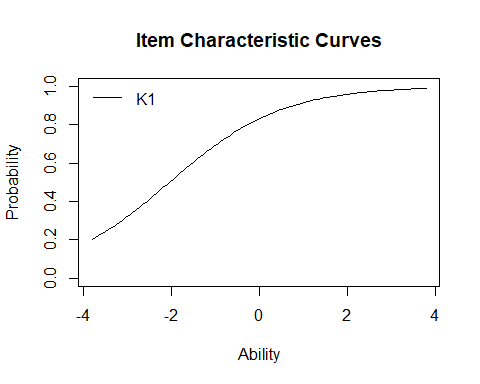
## Graphical Presentation

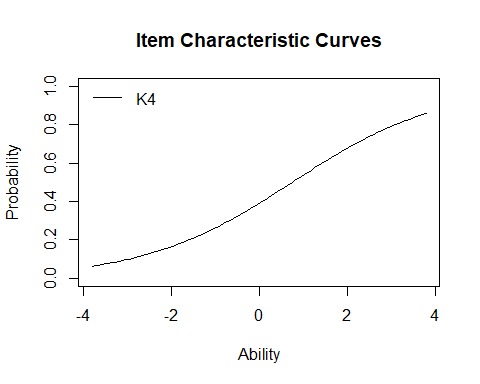
### Item Characteristic Curves (ICC)

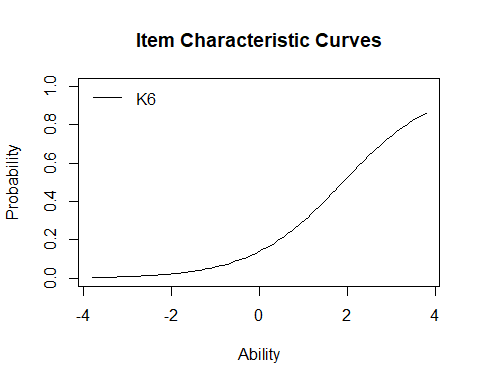
# ICC for All Items  
# Plot ICC for all items  
plot(irt.data4, type = "ICC", legend = TRUE)

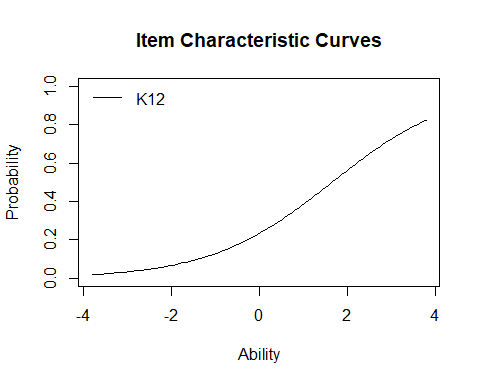


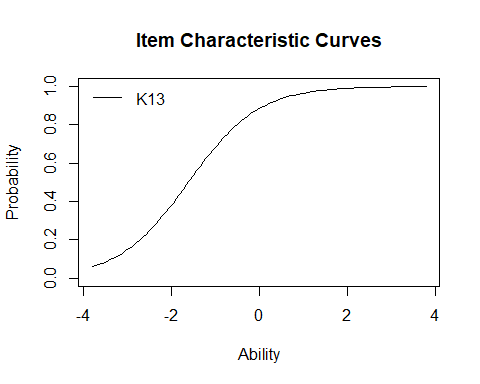
# ICC for Individual Items  
  
# Get total number of items  
ICC\_items <- nrow(coef(irt.data4))  
  
# Plot ICC for each item  
for (i in 1:ICC\_items) {  
 plot(irt.data4, type = "ICC", legend = TRUE, items = i)  
}

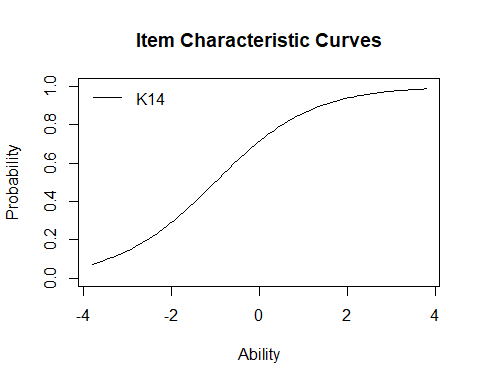


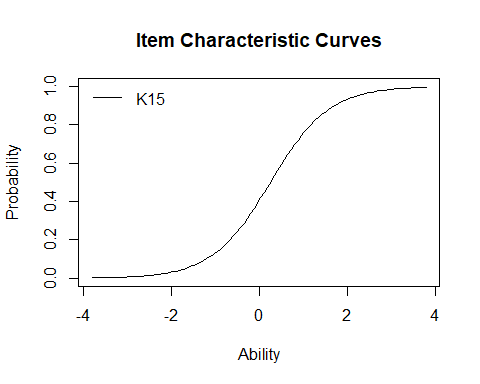


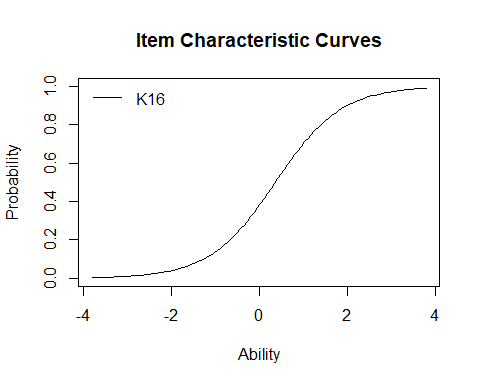


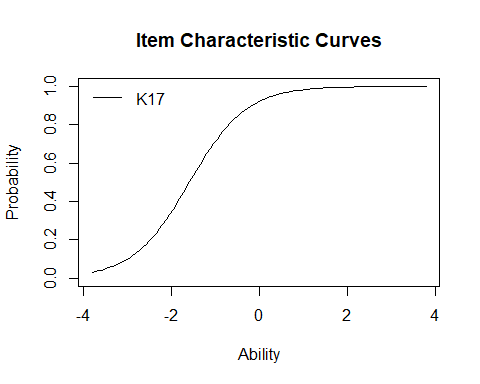


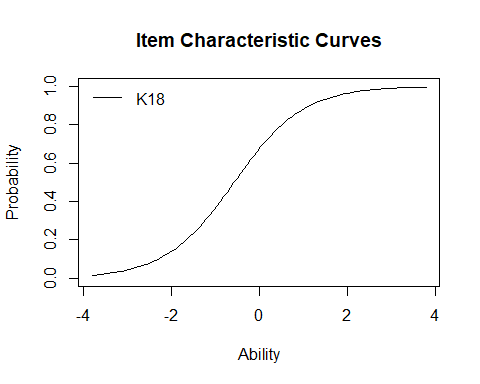


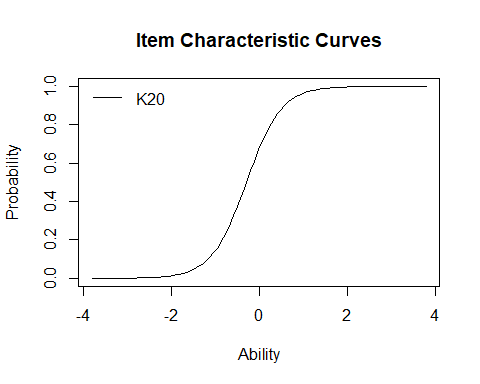


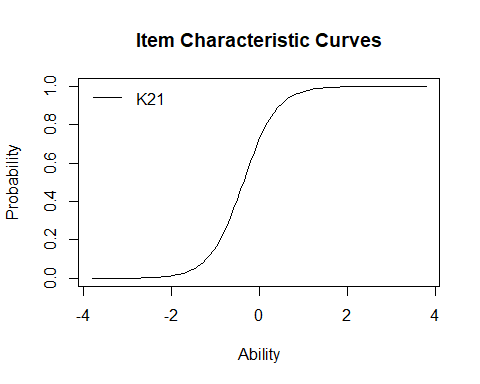


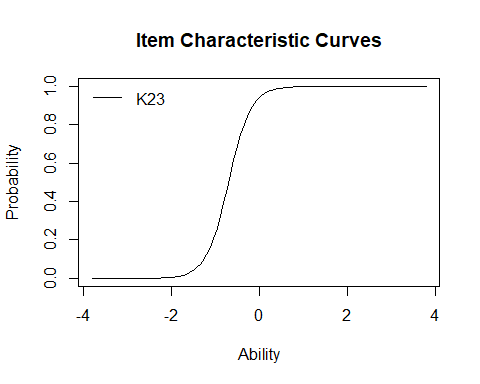


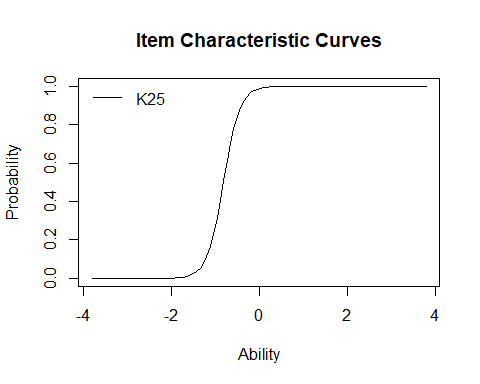


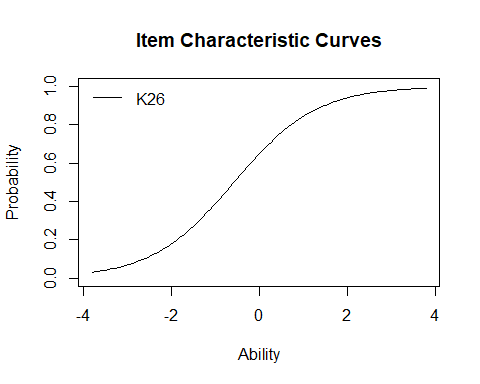


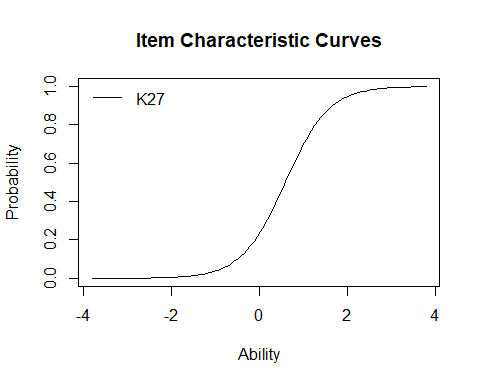


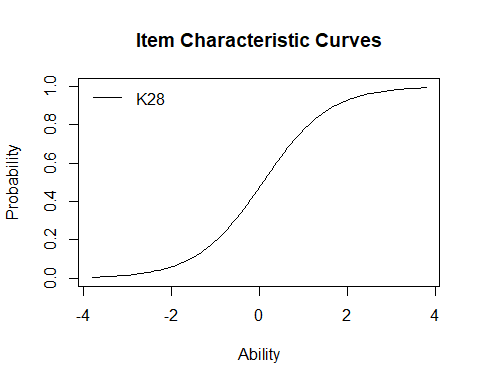


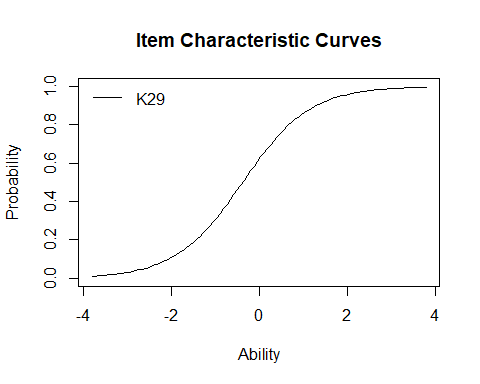


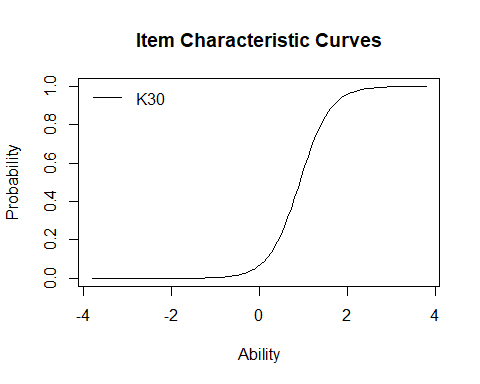


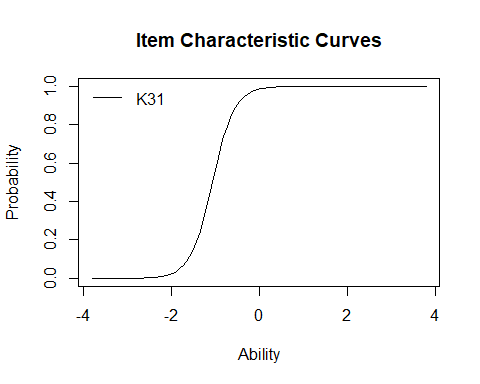


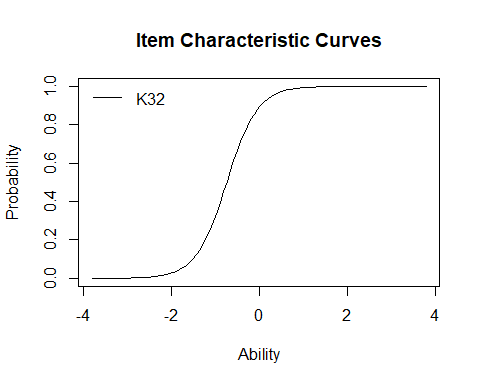


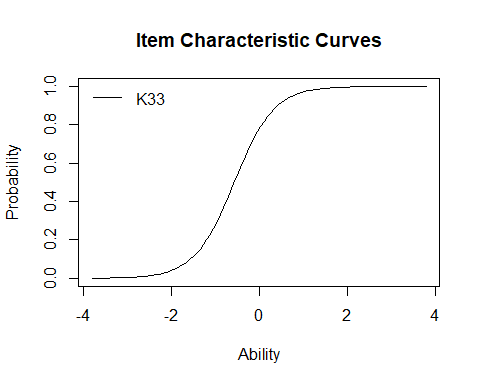


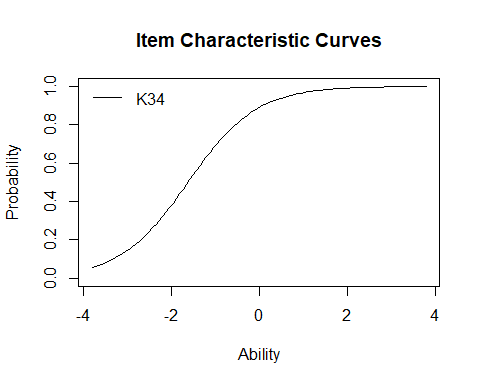


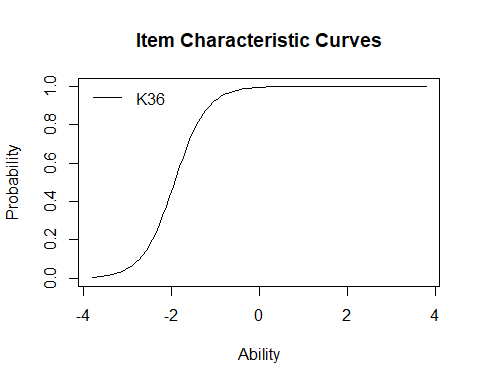


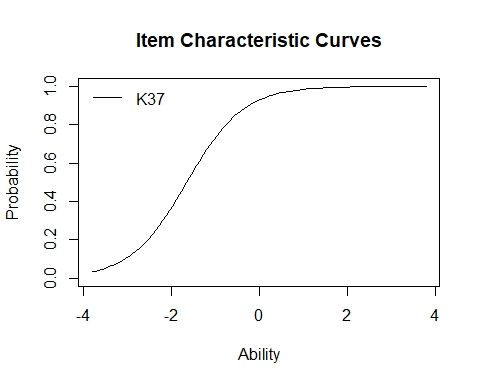












## Goodness-of-Fit Tests

### Item Fit Statistics

item\_fit <- item.fit(irt.data4)  
item\_fit

Item-Fit Statistics and P-values  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Alternative: Items do not fit the model  
Ability Categories: 10  
  
 X^2 Pr(>X^2)  
K1 1.6293 0.9903  
K4 12.8019 0.1189  
K6 13.6876 0.0903  
K12 9.8948 0.2725  
K13 7.1639 0.5191  
K14 11.4451 0.1777  
K15 13.1728 0.106  
K16 10.3996 0.2381  
K17 11.9806 0.1521  
K18 10.0589 0.2609  
K20 8.1621 0.4178  
K21 9.8064 0.2789  
K23 4.2761 0.8314  
K25 4.5488 0.8045  
K26 17.1120 0.029  
K27 8.8232 0.3574  
K28 11.7628 0.1621  
K29 11.8035 0.1602  
K30 2.4431 0.9643  
K31 4.4152 0.8179  
K32 21.2643 0.0065  
K33 16.5085 0.0357  
K34 8.1048 0.4233  
K36 5.4764 0.7057  
K37 13.2840 0.1024

### Fit on the Two-Way Margins

margins\_output <- margins(irt.data4)  
margins\_output

Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Fit on the Two-Way Margins  
  
Response: (0,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 118 81.90 15.92 \*\*\*  
2 15 18 66 41.84 13.96 \*\*\*  
3 11 12 82 54.92 13.36 \*\*\*  
  
Response: (1,0)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 9 38.59 22.69 \*\*\*  
2 16 17 2 20.16 16.36 \*\*\*  
3 11 12 5 22.79 13.89 \*\*\*  
  
Response: (0,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 5 33.89 24.63 \*\*\*  
2 14 16 6 1.60 12.10 \*\*\*  
3 11 12 10 27.73 11.33 \*\*\*  
  
Response: (1,1)  
 Item i Item j Obs Exp (O-E)^2/E   
1 7 8 72 49.63 10.09 \*\*\*  
2 2 3 26 17.54 4.08 \*\*\*  
3 6 18 73 91.22 3.64 \*\*\*  
  
'\*\*\*' denotes a chi-squared residual greater than 3.5

### Person Fit Statistics

person\_fit <- person.fit(irt.data4)  
person\_fit

Person-Fit Statistics and P-values  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Alternative: Inconsistent response pattern under the estimated model  
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K20 K21 K23 K25 K26 K27 K28 K29 K30  
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0  
4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0  
5 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0  
6 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 0 0 1 0  
7 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
8 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 0  
9 0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
10 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 1 1 0  
11 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0  
12 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
13 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
14 0 0 0 0 1 0 0 0 1 0 1 1 1 0 1 0 1 1 0  
15 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0  
16 0 0 0 0 1 0 0 0 1 1 0 1 1 1 1 0 1 1 0  
17 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0  
18 0 0 0 0 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0  
19 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 1 0  
20 0 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0 1 0 0  
21 0 0 0 0 1 1 0 0 1 1 0 1 1 1 0 1 1 0 0  
22 0 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0  
23 0 0 0 0 1 1 0 1 1 1 1 1 1 1 0 0 0 0 0  
24 0 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1  
25 0 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
26 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0  
27 0 0 0 0 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0  
28 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
29 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 0 0 1 1  
30 0 0 0 1 1 1 1 1 1 0 0 0 1 1 0 1 1 1 1  
31 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
32 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
33 0 0 1 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
34 0 0 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1  
35 0 1 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 0  
36 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
37 0 1 0 0 1 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
38 0 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
39 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
40 0 1 0 1 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0  
41 0 1 0 1 0 1 1 1 1 0 1 1 0 0 0 1 1 1 0  
42 0 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 0  
43 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0  
44 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 1 1 0  
45 1 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 1 0  
46 1 0 0 0 0 0 0 0 1 0 1 1 1 1 1 0 0 1 0  
47 1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
48 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0  
49 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 1 0  
50 1 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 0 0  
51 1 0 0 0 1 0 0 0 0 1 0 1 0 1 1 0 0 1 0  
52 1 0 0 0 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1  
53 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 0 0 0  
54 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 0 0 0  
55 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 1 1 0  
56 1 0 0 0 1 0 0 0 1 0 1 1 1 1 1 1 1 0 0  
57 1 0 0 0 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1  
58 1 0 0 0 1 0 0 0 1 1 1 1 0 0 1 0 0 1 0  
59 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 0 0  
60 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
61 1 0 0 0 1 0 1 1 1 1 0 0 1 1 0 0 0 0 0  
62 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
63 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
64 1 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 0 0 0  
65 1 0 0 0 1 1 0 0 0 1 1 1 0 0 0 0 0 1 0  
66 1 0 0 0 1 1 0 0 0 1 1 1 1 1 0 0 0 1 0  
67 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
68 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
69 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
70 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
71 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 0 1 1 0  
72 1 0 0 0 1 1 0 0 1 0 1 1 0 1 0 0 0 0 0  
73 1 0 0 0 1 1 0 0 1 0 1 1 1 1 0 0 0 0 0  
74 1 0 0 0 1 1 0 0 1 0 1 1 1 1 0 0 0 0 0  
75 1 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0  
76 1 0 0 0 1 1 0 0 1 1 0 0 0 1 1 0 1 1 0  
77 1 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 0 0  
78 1 0 0 0 1 1 0 0 1 1 0 1 0 1 1 0 1 0 0  
79 1 0 0 0 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0  
80 1 0 0 0 1 1 0 0 1 1 1 1 0 1 0 0 0 0 0  
81 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0  
82 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0  
83 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 0 0 0 0  
84 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 0 0 0 0  
85 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0  
86 1 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0  
87 1 0 0 0 1 1 0 1 1 1 0 0 1 1 1 1 1 1 1  
88 1 0 0 0 1 1 1 0 1 0 1 1 1 1 0 0 1 1 0  
89 1 0 0 0 1 1 1 0 1 1 0 0 1 1 1 0 0 1 0  
90 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1 0 0 1 0  
91 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 0 1 1 0  
92 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1  
93 1 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
94 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0  
95 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0  
96 1 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
97 1 0 0 0 1 1 1 1 1 1 0 0 0 0 1 0 0 0 0  
98 1 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
99 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0  
100 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
101 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
102 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0  
103 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0  
104 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
105 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
106 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
107 1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0  
108 1 0 0 1 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0  
109 1 0 0 1 0 0 0 0 1 1 0 0 0 0 1 0 0 0 0  
110 1 0 0 1 0 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
111 1 0 0 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
112 1 0 0 1 1 0 0 0 0 1 0 1 1 1 1 1 1 1 0  
113 1 0 0 1 1 0 0 0 1 0 0 0 0 1 1 0 0 1 0  
114 1 0 0 1 1 0 0 0 1 1 0 0 1 1 1 0 0 1 0  
115 1 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 0  
116 1 0 0 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 0  
117 1 0 0 1 1 1 0 0 1 0 1 1 1 1 1 0 1 1 0  
118 1 0 0 1 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0  
119 1 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0  
120 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
121 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
122 1 0 1 1 1 0 0 0 1 1 0 1 1 1 1 1 1 1 1  
123 1 0 1 1 1 0 0 0 1 1 1 1 0 1 0 0 0 0 0  
124 1 0 1 1 1 1 0 0 1 1 1 1 1 1 0 0 0 1 0  
125 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
126 1 1 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 0  
127 1 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0  
128 1 1 0 0 0 0 0 0 1 0 1 1 1 1 0 0 0 1 0  
129 1 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
130 1 1 0 0 0 1 0 0 1 0 0 1 0 1 0 0 0 0 0  
131 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
132 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0  
133 1 1 0 0 1 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
134 1 1 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
135 1 1 0 0 1 0 0 0 1 1 1 1 1 1 1 1 1 1 0  
136 1 1 0 0 1 1 0 0 0 0 0 0 0 0 1 0 1 1 0  
137 1 1 0 0 1 1 0 0 0 1 1 1 1 1 0 0 0 1 0  
138 1 1 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
139 1 1 0 0 1 1 0 0 1 0 0 0 1 1 0 0 0 0 0  
140 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0  
141 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0 1 1 1 0  
142 1 1 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
143 1 1 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0  
144 1 1 0 0 1 1 1 1 1 0 0 0 1 1 0 0 0 0 0  
145 1 1 0 0 1 1 1 1 1 0 1 1 0 1 0 0 0 0 0  
146 1 1 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
147 1 1 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 0  
148 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
149 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
150 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
151 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
152 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
153 1 1 0 1 0 0 0 0 0 1 0 0 1 1 1 0 1 1 0  
154 1 1 0 1 0 0 1 0 1 1 1 1 1 1 1 1 1 1 1  
155 1 1 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 1 0  
156 1 1 0 1 1 1 0 0 0 1 0 0 1 1 1 0 1 0 0  
157 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 1 0  
158 1 1 0 1 1 1 0 0 1 1 0 0 1 1 1 0 0 0 0  
159 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 0  
160 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
161 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
162 1 1 1 0 0 1 0 0 1 0 1 1 1 1 0 0 0 1 0  
163 1 1 1 0 0 1 0 0 1 1 1 1 1 1 1 0 0 1 0  
164 1 1 1 0 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0  
165 1 1 1 0 1 1 0 0 1 1 1 0 0 0 1 1 1 1 0  
166 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
167 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 0 0 1 1  
168 1 1 1 0 1 1 1 0 1 1 1 1 1 1 0 0 0 0 0  
169 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0  
170 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
171 1 1 1 0 1 1 1 1 1 1 0 0 1 1 0 0 0 0 0  
172 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0  
173 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
174 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
175 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 1 1 1  
176 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 0 1 1  
177 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 0 1 0  
178 1 1 1 1 1 1 1 0 1 0 1 1 1 1 0 0 0 0 0  
179 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0  
180 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1  
181 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0  
182 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 K31 K32 K33 K34 K36 K37 L0 Lz Pr(<Lz)  
1 0 0 0 0 0 0 -2.3186 1.3046 0.904  
2 0 0 0 1 1 0 -5.3505 1.1940 0.8838  
3 0 1 0 0 0 0 -11.0298 -1.1025 0.1351  
4 0 0 0 0 0 0 -4.8915 0.6164 0.7312  
5 1 1 1 1 1 1 -12.3666 0.3080 0.621  
6 0 0 0 0 1 0 -15.6665 -1.3466 0.0891  
7 1 1 1 1 1 1 -10.3897 0.9087 0.8183  
8 0 0 0 1 1 1 -9.8904 0.3576 0.6397  
9 1 1 0 1 1 1 -10.6948 1.0616 0.8558  
10 1 1 1 0 1 1 -13.3774 -0.2788 0.3902  
11 0 0 0 1 1 1 -9.3827 0.5385 0.7049  
12 0 0 0 1 1 1 -6.6437 1.1099 0.8665  
13 1 0 0 1 1 1 -8.8195 1.2236 0.8895  
14 1 0 0 0 1 1 -14.9682 -0.8465 0.1986  
15 1 1 1 1 1 1 -10.0852 1.2277 0.8902  
16 1 0 0 0 1 1 -13.0804 -0.1568 0.4377  
17 0 0 0 0 0 0 -11.7219 -1.4033 0.0803  
18 1 1 1 1 1 0 -12.4744 0.2161 0.5855  
19 1 1 0 1 1 1 -11.4827 0.4697 0.6807  
20 0 0 0 1 1 1 -9.5693 0.7718 0.7799  
21 1 1 1 1 1 1 -11.0160 0.0976 0.5389  
22 1 1 1 1 1 0 -12.1008 -0.5918 0.277  
23 1 1 1 1 1 1 -9.4645 0.6674 0.7477  
24 1 1 1 1 1 1 -14.3633 -1.8116 0.035  
25 1 1 1 1 1 1 -10.5696 0.1904 0.5755  
26 1 1 1 1 1 1 -8.7116 0.4520 0.6744  
27 1 1 1 1 1 1 -16.9769 -1.8353 0.0332  
28 1 1 1 1 1 1 -9.8064 0.2491 0.5984  
29 1 1 1 1 1 1 -9.9152 -0.3839 0.3505  
30 1 1 1 1 1 1 -16.3021 -2.5461 0.0054  
31 1 1 1 1 1 1 -6.4281 0.0325 0.513  
32 0 0 0 0 0 0 -8.3243 -0.4705 0.319  
33 1 1 1 1 1 0 -14.9587 -1.5239 0.0638  
34 1 1 1 1 1 1 -12.0886 -1.5757 0.0575  
35 1 1 1 0 0 0 -24.3678 -5.0159 <0.0001  
36 0 0 0 0 0 0 -6.3349 0.3378 0.6322  
37 1 1 0 1 1 1 -10.3661 1.1608 0.8771  
38 0 0 0 0 1 1 -13.8864 -0.8500 0.1977  
39 1 1 0 1 1 1 -10.0705 -0.8985 0.1845  
40 1 0 0 0 1 0 -16.5001 -1.5308 0.0629  
41 1 0 0 0 0 1 -27.0234 -6.2258 <0.0001  
42 1 1 1 1 1 1 -19.3201 -3.1584 0.0008  
43 0 0 0 1 1 1 -7.4738 0.9755 0.8353  
44 1 1 1 1 1 1 -11.7817 -0.2070 0.418  
45 1 1 1 0 1 1 -11.3946 0.6053 0.7275  
46 1 1 1 1 1 1 -9.8480 0.7015 0.7585  
47 0 0 0 1 1 1 -6.3863 1.5737 0.9422  
48 0 0 0 1 1 0 -9.7266 0.2621 0.6034  
49 0 0 0 0 0 0 -10.5780 -0.5932 0.2765  
50 1 1 1 1 1 1 -10.0666 1.1852 0.882  
51 1 0 0 0 1 0 -14.0191 -0.4420 0.3293  
52 1 1 0 1 1 1 -14.2290 -1.7108 0.0436  
53 1 0 0 1 1 0 -10.1605 1.3037 0.9038  
54 1 1 1 0 1 1 -9.8349 1.2820 0.9001  
55 1 1 1 1 1 1 -9.0479 1.2997 0.9031  
56 1 1 1 0 1 1 -11.7946 -0.3259 0.3722  
57 1 1 1 1 1 1 -12.6127 -0.8609 0.1946  
58 1 1 1 1 1 1 -11.8051 0.4476 0.6728  
59 1 1 1 1 1 1 -7.7344 1.2523 0.8948  
60 1 1 1 1 1 1 -7.0104 1.3726 0.9151  
61 0 0 0 1 1 1 -13.0880 -0.0015 0.4994  
62 0 0 0 0 1 0 -6.2082 1.1483 0.8746  
63 0 0 0 0 1 1 -6.3998 1.3250 0.9074  
64 1 1 1 1 1 1 -10.4962 1.1051 0.8654  
65 1 0 0 0 0 0 -17.0844 -1.9954 0.023  
66 1 0 0 1 1 1 -11.4342 0.4424 0.6709  
67 0 0 0 0 0 0 -6.6908 0.7373 0.7695  
68 0 0 0 1 1 1 -6.1816 1.8425 0.9673  
69 1 0 0 1 1 1 -6.9375 2.1093 0.9825  
70 1 0 0 1 1 1 -7.5026 2.0090 0.9777  
71 0 0 0 1 1 1 -9.8579 0.8834 0.8115  
72 1 1 0 0 1 1 -11.1909 0.7567 0.7754  
73 0 0 0 1 1 1 -11.7008 0.5908 0.7227  
74 1 0 1 1 1 1 -9.3847 1.2059 0.8861  
75 1 1 1 1 1 1 -8.5868 1.9145 0.9722  
76 1 1 1 1 1 1 -9.5118 1.2630 0.8967  
77 1 1 1 0 1 1 -9.3753 1.4028 0.9197  
78 1 0 0 0 1 0 -13.1946 -0.0497 0.4802  
79 0 0 0 1 1 1 -10.8481 0.9934 0.8397  
80 0 1 1 1 1 1 -11.8293 0.4413 0.6705  
81 1 1 0 1 1 1 -8.3499 1.4438 0.9256  
82 1 1 1 1 1 1 -7.3764 1.5861 0.9436  
83 1 0 0 0 1 0 -12.8008 -0.0376 0.485  
84 1 1 1 1 1 1 -6.7512 1.7053 0.9559  
85 1 1 1 1 1 1 -6.0706 1.3580 0.9128  
86 1 1 0 1 1 1 -9.2312 1.4322 0.924  
87 1 1 1 1 1 1 -11.4140 -0.6943 0.2438  
88 1 1 1 1 1 1 -7.8063 1.0417 0.8512  
89 0 1 0 1 1 1 -11.5584 0.5336 0.7032  
90 1 1 1 1 1 1 -6.0424 1.6206 0.9475  
91 1 0 0 1 1 1 -11.9107 0.2138 0.5846  
92 1 1 1 1 1 1 -12.4105 -1.1370 0.1278  
93 1 1 1 1 1 1 -8.9204 0.7613 0.7768  
94 1 1 1 1 1 1 -6.7341 1.1497 0.8749  
95 1 1 1 1 1 1 -6.5495 0.9182 0.8207  
96 0 0 0 1 1 0 -12.2319 -0.1973 0.4218  
97 0 0 0 1 1 1 -12.0463 0.1325 0.5527  
98 1 1 1 1 1 1 -7.9622 0.9807 0.8366  
99 1 1 1 1 1 0 -10.0253 0.2117 0.5838  
100 1 1 0 1 1 1 -8.0705 0.9149 0.8199  
101 1 1 1 1 1 1 -5.9242 1.4881 0.9316  
102 1 1 1 1 1 1 -6.5208 1.2416 0.8928  
103 1 1 1 1 1 1 -5.1998 1.5987 0.9451  
104 1 1 1 1 1 1 -4.6416 1.4846 0.9312  
105 1 0 1 1 1 1 -9.3721 -0.5095 0.3052  
106 1 1 1 1 1 1 -3.5484 1.2084 0.8866  
107 0 0 0 1 1 1 -10.7871 -0.0352 0.486  
108 1 1 1 0 1 0 -18.2648 -2.3414 0.0096  
109 1 0 0 1 1 1 -10.7264 0.6544 0.7436  
110 1 1 1 1 1 0 -13.0748 -0.8161 0.2072  
111 1 1 1 1 1 1 -10.1322 -0.7228 0.2349  
112 1 1 1 1 1 0 -15.4433 -1.6909 0.0454  
113 1 1 1 1 1 1 -10.8463 0.8803 0.8106  
114 1 1 1 1 1 1 -9.5992 1.0046 0.8424  
115 1 1 1 1 1 1 -8.8620 0.8722 0.8084  
116 1 1 1 1 1 1 -11.9173 -0.5616 0.2872  
117 1 1 1 0 1 1 -10.2898 0.1705 0.5677  
118 1 1 1 1 0 0 -16.3350 -1.5322 0.0627  
119 1 1 1 1 1 1 -6.7901 0.6499 0.7421  
120 1 1 1 1 1 1 -5.0514 1.1586 0.8767  
121 1 1 1 1 1 1 -3.6695 0.8528 0.8031  
122 1 0 0 0 1 1 -20.3469 -3.7490 0.0001  
123 1 1 1 1 1 1 -13.5256 -0.6060 0.2722  
124 1 1 0 1 1 1 -11.1489 -0.0426 0.483  
125 0 0 0 0 0 0 -5.1917 0.5876 0.7216  
126 1 0 1 1 1 1 -12.8187 0.0884 0.5352  
127 1 0 0 0 1 0 -11.4352 0.0510 0.5204  
128 1 1 1 1 1 1 -10.8332 0.3491 0.6365  
129 1 0 0 1 1 1 -8.6407 1.3333 0.9088  
130 0 1 1 1 1 1 -12.8257 0.1155 0.546  
131 0 0 0 1 1 1 -7.0961 1.4411 0.9252  
132 0 0 0 1 1 1 -12.7474 -0.3545 0.3615  
133 1 1 1 1 1 1 -9.2203 1.4912 0.932  
134 1 1 1 1 1 1 -7.2892 1.1813 0.8813  
135 1 1 1 1 1 1 -7.6067 0.7714 0.7798  
136 1 1 1 0 1 1 -13.9244 -0.3969 0.3457  
137 1 1 0 1 1 1 -10.8326 0.3836 0.6494  
138 1 0 0 1 1 1 -8.5724 1.6285 0.9483  
139 0 0 0 1 1 1 -10.3595 1.1685 0.8787  
140 1 1 1 1 1 1 -9.6977 0.4982 0.6908  
141 1 1 1 1 1 1 -7.3988 0.8803 0.8106  
142 1 1 1 1 1 1 -6.1839 0.7038 0.7592  
143 0 0 0 1 1 1 -11.8726 0.0388 0.5155  
144 1 1 1 1 1 1 -10.6428 0.5746 0.7172  
145 1 1 1 1 1 1 -11.9771 -0.0956 0.4619  
146 1 1 1 1 1 1 -9.2784 0.5398 0.7053  
147 1 1 1 1 1 1 -7.4521 0.6619 0.746  
148 0 0 0 1 1 1 -12.5860 -0.1321 0.4475  
149 1 0 1 1 1 1 -10.8095 0.1182 0.547  
150 1 1 1 1 1 1 -8.2376 0.7903 0.7853  
151 1 1 1 0 1 1 -7.1534 -0.0867 0.4655  
152 1 1 1 1 1 1 -2.9783 1.2592 0.896  
153 0 0 0 1 1 1 -16.8456 -1.6815 0.0463  
154 1 1 1 1 1 1 -10.6256 -1.1770 0.1196  
155 1 1 1 1 1 1 -8.5392 0.7558 0.7751  
156 1 1 1 0 1 1 -13.8726 -0.6607 0.2544  
157 1 1 1 1 1 1 -12.0110 0.4751 0.6827  
158 1 1 1 1 1 1 -9.6765 0.9542 0.83  
159 1 1 1 1 1 1 -6.5885 1.0527 0.8538  
160 1 1 0 1 1 1 -7.8024 -0.3026 0.3811  
161 1 1 1 1 1 1 -2.7707 1.1086 0.8662  
162 1 1 1 1 1 1 -11.9057 -0.2963 0.3835  
163 1 1 1 1 1 1 -10.3153 0.0536 0.5214  
164 1 1 0 1 1 1 -13.6266 -0.8538 0.1966  
165 1 0 0 1 1 1 -17.6568 -2.0511 0.0201  
166 1 1 1 1 1 1 -6.8369 0.2031 0.5805  
167 1 1 1 1 1 1 -9.8707 -0.4224 0.3364  
168 1 1 0 0 1 1 -13.3455 -0.7967 0.2128  
169 1 1 1 1 1 1 -6.1982 0.7879 0.7846  
170 1 1 1 1 1 1 -10.8802 -0.2363 0.4066  
171 1 1 1 1 1 1 -12.1577 -0.2774 0.3907  
172 1 1 1 1 1 1 -9.1211 0.1427 0.5567  
173 1 1 1 1 1 1 -7.1903 0.7376 0.7696  
174 1 1 1 1 1 1 -2.8910 0.9776 0.8359  
175 1 1 0 1 1 1 -13.1346 -1.5502 0.0605  
176 1 0 0 1 1 0 -20.2929 -3.9001 <0.0001  
177 1 0 1 1 1 1 -14.3301 -1.3860 0.0829  
178 1 1 1 0 1 1 -14.0063 -1.2543 0.1049  
179 1 1 1 1 1 1 -12.1203 -1.1249 0.1303  
180 1 1 1 1 1 1 -7.3036 -0.0473 0.4812  
181 1 1 0 1 1 1 -11.2031 -0.6850 0.2467  
182 1 1 1 1 1 1 -2.3575 0.9546 0.8301

# Checking Assumptions

## Unidimensionality

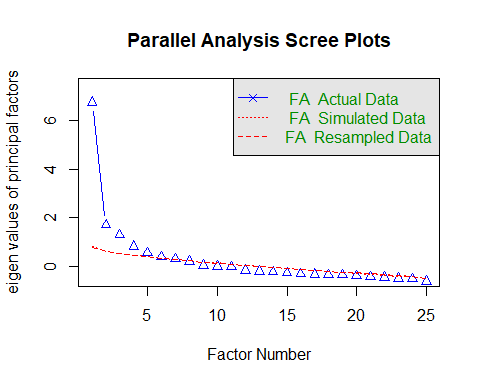
set.seed(2025)  
unidimTest(irt.data4) #Take A long time, insert # if want to skip and avoid long time

Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value  
Warning in optimise(f, interval = c(-maxcor, maxcor)): NA/NaN replaced by  
maximum positive value

Unidimensionality Check using Modified Parallel Analysis  
  
Call:  
ltm(formula = data4 ~ z1, IRT.param = TRUE)  
  
Matrix of tertachoric correlations  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.0000 0.3928 0.3148 0.1320 0.5059 0.3255 0.1825 0.1469 0.3468 0.4518  
K4 0.3928 1.0000 0.6331 0.2507 0.0433 0.2261 0.3858 0.3139 0.3245 0.1661  
K6 0.3148 0.6331 1.0000 0.4902 0.3832 0.2398 0.3888 0.2308 0.4773 0.3316  
K12 0.1320 0.2507 0.4902 1.0000 0.0214 -0.1444 0.1427 0.1429 0.2160 0.4846  
K13 0.5059 0.0433 0.3832 0.0214 1.0000 0.6660 0.7007 0.7606 0.5424 0.5789  
K14 0.3255 0.2261 0.2398 -0.1444 0.6660 1.0000 0.7623 0.7745 0.5468 0.2252  
K15 0.1825 0.3858 0.3888 0.1427 0.7007 0.7623 1.0000 0.9762 0.7444 0.3013  
K16 0.1469 0.3139 0.2308 0.1429 0.7606 0.7745 0.9762 1.0000 0.7282 0.3529  
K17 0.3468 0.3245 0.4773 0.2160 0.5424 0.5468 0.7444 0.7282 1.0000 0.3172  
K18 0.4518 0.1661 0.3316 0.4846 0.5789 0.2252 0.3013 0.3529 0.3172 1.0000  
K20 0.3594 0.2639 0.4642 0.2442 0.4151 0.4505 0.5965 0.4979 0.5575 0.5022  
K21 0.3617 0.1726 0.4310 0.2808 0.5356 0.4428 0.5497 0.4479 0.5450 0.5584  
K23 0.3136 0.2121 0.4624 0.2789 0.5047 0.2601 0.4998 0.4659 0.5210 0.5196  
K25 0.4635 0.1576 0.4843 0.3465 0.5639 0.2948 0.4741 0.4425 0.5706 0.5902  
K26 0.2759 -0.0087 0.2348 0.4139 0.1877 -0.1414 0.1298 0.0776 0.1082 0.4965  
K27 0.0725 0.3402 0.3435 0.3352 0.4309 0.3632 0.4736 0.4820 0.4527 0.5069  
K28 0.0078 0.1038 0.1641 0.3543 0.2957 0.0944 0.2803 0.3083 0.2609 0.4440  
K29 0.2130 0.2084 0.3416 0.5093 0.0814 -0.0831 0.2481 0.1625 0.0987 0.4122  
K30 0.1337 0.3832 0.5244 0.4499 0.4308 0.3686 0.5640 0.5149 0.5174 0.5107  
K31 0.3073 0.2538 0.5238 0.5265 0.5015 0.2962 0.3774 0.3986 0.5742 0.5170  
K32 0.2278 0.1747 0.3138 0.3293 0.4650 0.3298 0.4634 0.4282 0.4784 0.4583  
K33 0.3087 0.1603 0.1712 0.3029 0.3877 0.2972 0.4935 0.4881 0.5002 0.3650  
K34 0.3850 0.1232 0.1736 0.1304 0.3888 0.4196 0.4559 0.5432 0.5987 0.3579  
K36 0.5792 0.1361 0.2306 0.1801 0.5359 0.4805 0.3828 0.2388 0.7232 0.5003  
K37 0.4313 0.3541 0.2018 0.1154 0.4287 0.5080 0.5003 0.4096 0.7176 0.2208  
 K20 K21 K23 K25 K26 K27 K28 K29 K30 K31  
K1 0.3594 0.3617 0.3136 0.4635 0.2759 0.0725 0.0078 0.2130 0.1337 0.3073  
K4 0.2639 0.1726 0.2121 0.1576 -0.0087 0.3402 0.1038 0.2084 0.3832 0.2538  
K6 0.4642 0.4310 0.4624 0.4843 0.2348 0.3435 0.1641 0.3416 0.5244 0.5238  
K12 0.2442 0.2808 0.2789 0.3465 0.4139 0.3352 0.3543 0.5093 0.4499 0.5265  
K13 0.4151 0.5356 0.5047 0.5639 0.1877 0.4309 0.2957 0.0814 0.4308 0.5015  
K14 0.4505 0.4428 0.2601 0.2948 -0.1414 0.3632 0.0944 -0.0831 0.3686 0.2962  
K15 0.5965 0.5497 0.4998 0.4741 0.1298 0.4736 0.2803 0.2481 0.5640 0.3774  
K16 0.4979 0.4479 0.4659 0.4425 0.0776 0.4820 0.3083 0.1625 0.5149 0.3986  
K17 0.5575 0.5450 0.5210 0.5706 0.1082 0.4527 0.2609 0.0987 0.5174 0.5742  
K18 0.5022 0.5584 0.5196 0.5902 0.4965 0.5069 0.4440 0.4122 0.5107 0.5170  
K20 1.0000 0.9747 0.6793 0.6933 0.2996 0.4885 0.4278 0.4357 0.5293 0.7943  
K21 0.9747 1.0000 0.7417 0.8356 0.3028 0.4822 0.3967 0.4447 0.5489 0.7782  
K23 0.6793 0.7417 1.0000 0.9879 0.4691 0.5236 0.4112 0.4936 0.9842 0.8178  
K25 0.6933 0.8356 0.9879 1.0000 0.4410 0.4376 0.3571 0.4470 0.9803 0.8320  
K26 0.2996 0.3028 0.4691 0.4410 1.0000 0.6339 0.7336 0.8035 0.7777 0.6168  
K27 0.4885 0.4822 0.5236 0.4376 0.6339 1.0000 0.9278 0.7821 0.8927 0.6878  
K28 0.4278 0.3967 0.4112 0.3571 0.7336 0.9278 1.0000 0.7858 0.7413 0.4983  
K29 0.4357 0.4447 0.4936 0.4470 0.8035 0.7821 0.7858 1.0000 0.9903 0.5685  
K30 0.5293 0.5489 0.9842 0.9803 0.7777 0.8927 0.7413 0.9903 1.0000 0.9778  
K31 0.7943 0.7782 0.8178 0.8320 0.6168 0.6878 0.4983 0.5685 0.9778 1.0000  
K32 0.7059 0.6373 0.7605 0.8422 0.3736 0.5236 0.4795 0.3989 0.5139 0.8931  
K33 0.6383 0.5838 0.6877 0.7769 0.4017 0.4974 0.4350 0.3803 0.4134 0.8963  
K34 0.3980 0.3512 0.4834 0.5013 0.0720 0.3522 0.1376 0.2383 0.4220 0.3753  
K36 0.4397 0.5677 0.7833 0.8243 0.3960 0.2507 0.3220 0.3546 0.9558 0.6756  
K37 0.4302 0.4178 0.5210 0.5234 0.1029 0.3502 0.3323 0.3468 0.5005 0.5157  
 K32 K33 K34 K36 K37  
K1 0.2278 0.3087 0.3850 0.5792 0.4313  
K4 0.1747 0.1603 0.1232 0.1361 0.3541  
K6 0.3138 0.1712 0.1736 0.2306 0.2018  
K12 0.3293 0.3029 0.1304 0.1801 0.1154  
K13 0.4650 0.3877 0.3888 0.5359 0.4287  
K14 0.3298 0.2972 0.4196 0.4805 0.5080  
K15 0.4634 0.4935 0.4559 0.3828 0.5003  
K16 0.4282 0.4881 0.5432 0.2388 0.4096  
K17 0.4784 0.5002 0.5987 0.7232 0.7176  
K18 0.4583 0.3650 0.3579 0.5003 0.2208  
K20 0.7059 0.6383 0.3980 0.4397 0.4302  
K21 0.6373 0.5838 0.3512 0.5677 0.4178  
K23 0.7605 0.6877 0.4834 0.7833 0.5210  
K25 0.8422 0.7769 0.5013 0.8243 0.5234  
K26 0.3736 0.4017 0.0720 0.3960 0.1029  
K27 0.5236 0.4974 0.3522 0.2507 0.3502  
K28 0.4795 0.4350 0.1376 0.3220 0.3323  
K29 0.3989 0.3803 0.2383 0.3546 0.3468  
K30 0.5139 0.4134 0.4220 0.9558 0.5005  
K31 0.8931 0.8963 0.3753 0.6756 0.5157  
K32 1.0000 0.9411 0.5231 0.6072 0.6057  
K33 0.9411 1.0000 0.5306 0.6268 0.5512  
K34 0.5231 0.5306 1.0000 0.8841 0.7444  
K36 0.6072 0.6268 0.8841 1.0000 0.9111  
K37 0.6057 0.5512 0.7444 0.9111 1.0000  
  
Alternative hypothesis: the second eigenvalue of the observed data is substantially larger   
 than the second eigenvalue of data under the assumed IRT model  
  
Second eigenvalue in the observed data: 3.2819  
Average of second eigenvalues in Monte Carlo samples: 1.7635  
Monte Carlo samples: 100  
p-value: 0.0099

### Checking Dominant Factor (Essential Unidimensionality)

# Extract the response data from the fitted model  
irt\_mat <- as.matrix(irt.data4$X)  
  
# Parallel analysis  
library(psych)  
fa.parallel(irt\_mat, fa="fa")



Parallel analysis suggests that the number of factors = 6 and the number of components = NA

# Eigenvalues  
ev <- eigen(cor(irt\_mat, use = "pairwise.complete.obs"))$values  
  
# First and second eigenvalues  
first\_ev <- ev[1]  
second\_ev <- ev[2]  
  
# Ratio  
dominance\_ratio <- first\_ev / second\_ev  
  
# Print  
first\_ev

[1] 7.430122

second\_ev

[1] 2.499912

dominance\_ratio

[1] 2.972154

Parallel analysis suggested up to seven factors, as seven eigenvalues from the actual data exceeded those from randomly simulated data. However, the scree plot demonstrated a sharp drop between the first (9.02) and second (2.51) eigenvalues, yielding a ratio of 3.59. This indicates a single dominant factor underlying item responses, with additional weaker factors. Consistent with the concept of **essential unidimensionality** (Reckase, 1979; Hambleton, Swaminathan, & Rogers, 1991; Embretson & Reise, 2000), the scale was considered suitable for unidimensional IRT modeling despite the presence of minor secondary dimensions.

class(irt.data4)

[1] "ltm"

str(irt.data4, max.level = 1)

List of 14  
 $ coefficients: num [1:25, 1:2] 1.59 -0.444 -1.82 -1.191 2.028 ...  
 ..- attr(\*, "dimnames")=List of 2  
 $ log.Lik : num -2337  
 $ convergence : int 0  
 $ hessian : num [1:50, 1:50] 30.512 -0.247 -0.211 -0.224 -0.339 ...  
 $ counts : Named int [1:2] 112 38  
 ..- attr(\*, "names")= chr [1:2] "function" "gradient"  
 $ patterns :List of 2  
 $ GH :List of 2  
 $ max.sc : num 0.0167  
 $ ltst :List of 5  
 $ X : tibble [204 × 25] (S3: tbl\_df/tbl/data.frame)  
 $ control :List of 6  
 $ IRT.param : logi TRUE  
 $ formula :Class 'formula' language data4 ~ z1  
 .. ..- attr(\*, ".Environment")=<environment: R\_GlobalEnv>   
 $ call : language ltm(formula = data4 ~ z1, IRT.param = TRUE)  
 - attr(\*, "class")= chr "ltm"

## Local Independence

### Yen’s Q3 residual cYen’s Q3 residual correlationsorrelations

# ==========================================  
# Load required package  
# ==========================================  
library(mirt)  
library(mokken)

Loading required package: poLCA

Loading required package: scatterplot3d

Attaching package: 'mokken'

The following object is masked from 'package:dplyr':  
  
 recode

The following object is masked from 'package:psych':  
  
 ICC

# Extract the raw item response matrix from the ltm object  
irt\_mat <- as.data.frame(irt.data4$X) # now it's a dataframe  
irt\_mat <- as.matrix(irt\_mat) # convert to numeric matrix  
  
  
  
# ==========================================  
# 1. Fit a unidimensional 2PL model  
# ==========================================  
  
mod2pl <- mirt(irt\_mat, 1, itemtype = "2PL")

mod1pl <- mirt(irt\_mat, 1, itemtype = "Rasch")

# ==========================================  
# 2. Assumption: Local Independence  
# ==========================================  
# (a) Yen’s Q3 residual correlations  
# Q3\_resid <- resid(mod2pl, type = "Q3")  
# Assumption 2: Local Independence  
Q3\_resid <- residuals(mod2pl, type = "Q3")

Q3 summary statistics:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.340 -0.128 -0.053 -0.029 0.029 0.790   
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.000 0.149 0.049 -0.021 0.171 0.076 -0.063 -0.074 0.013 0.137  
K4 0.149 1.000 0.284 0.061 -0.090 0.039 0.111 0.055 0.071 -0.029  
K6 0.049 0.284 1.000 0.192 0.035 -0.018 0.010 -0.094 0.047 0.005  
K12 -0.021 0.061 0.192 1.000 -0.131 -0.229 -0.111 -0.097 -0.044 0.152  
K13 0.171 -0.090 0.035 -0.131 1.000 0.302 0.187 0.214 0.093 0.150  
K14 0.076 0.039 -0.018 -0.229 0.302 1.000 0.347 0.349 0.172 -0.066  
K15 -0.063 0.111 0.010 -0.111 0.187 0.347 1.000 0.790 0.167 -0.116  
K16 -0.074 0.055 -0.094 -0.097 0.214 0.349 0.790 1.000 0.163 -0.044  
K17 0.013 0.071 0.047 -0.044 0.093 0.172 0.167 0.163 1.000 -0.107  
K18 0.137 -0.029 0.005 0.152 0.150 -0.066 -0.116 -0.044 -0.107 1.000  
K20 0.014 0.000 0.010 -0.077 -0.108 0.054 -0.002 -0.074 -0.017 -0.017  
K21 0.008 -0.085 -0.019 -0.049 -0.008 0.046 -0.059 -0.129 -0.050 0.042  
K23 -0.096 -0.037 -0.043 -0.101 -0.132 -0.168 -0.128 -0.117 -0.149 -0.092  
K25 0.085 -0.101 -0.040 -0.075 -0.102 -0.141 -0.151 -0.142 -0.116 -0.036  
K26 0.032 -0.146 -0.046 0.112 -0.102 -0.309 -0.246 -0.258 -0.186 0.135  
K27 -0.160 0.025 -0.068 0.017 -0.011 -0.053 -0.149 -0.094 -0.037 0.024  
K28 -0.170 -0.100 -0.133 0.066 -0.049 -0.176 -0.196 -0.135 -0.099 0.051  
K29 -0.033 -0.005 -0.003 0.170 -0.218 -0.303 -0.196 -0.237 -0.236 0.025  
K30 -0.136 -0.014 0.081 0.098 -0.034 -0.130 -0.160 -0.169 -0.041 -0.053  
K31 -0.116 -0.004 0.017 0.055 -0.130 -0.111 -0.178 -0.118 -0.092 -0.125  
K32 -0.143 -0.065 -0.097 -0.024 -0.107 -0.060 -0.117 -0.109 -0.143 -0.112  
K33 -0.034 -0.071 -0.166 -0.013 -0.137 -0.080 -0.050 -0.016 -0.081 -0.158  
K34 0.071 -0.038 -0.051 -0.067 -0.005 0.093 0.039 0.105 0.150 -0.026  
K36 0.101 -0.058 -0.016 -0.093 -0.023 0.016 -0.053 -0.120 0.093 -0.063  
K37 0.079 0.090 -0.046 -0.091 -0.015 0.140 0.052 0.008 0.242 -0.169  
 K20 K21 K23 K25 K26 K27 K28 K29 K30 K31  
K1 0.014 0.008 -0.096 0.085 0.032 -0.160 -0.170 -0.033 -0.136 -0.116  
K4 0.000 -0.085 -0.037 -0.101 -0.146 0.025 -0.100 -0.005 -0.014 -0.004  
K6 0.010 -0.019 -0.043 -0.040 -0.046 -0.068 -0.133 -0.003 0.081 0.017  
K12 -0.077 -0.049 -0.101 -0.075 0.112 0.017 0.066 0.170 0.098 0.055  
K13 -0.108 -0.008 -0.132 -0.102 -0.102 -0.011 -0.049 -0.218 -0.034 -0.130  
K14 0.054 0.046 -0.168 -0.141 -0.309 -0.053 -0.176 -0.303 -0.130 -0.111  
K15 -0.002 -0.059 -0.128 -0.151 -0.246 -0.149 -0.196 -0.196 -0.160 -0.178  
K16 -0.074 -0.129 -0.117 -0.142 -0.258 -0.094 -0.135 -0.237 -0.169 -0.118  
K17 -0.017 -0.050 -0.149 -0.116 -0.186 -0.037 -0.099 -0.236 -0.041 -0.092  
K18 -0.017 0.042 -0.092 -0.036 0.135 0.024 0.051 0.025 -0.053 -0.125  
K20 1.000 0.691 -0.228 -0.263 -0.182 -0.205 -0.127 -0.102 -0.249 0.024  
K21 0.691 1.000 -0.170 -0.029 -0.183 -0.194 -0.159 -0.094 -0.199 -0.076  
K23 -0.228 -0.170 1.000 0.548 -0.009 -0.145 -0.147 -0.057 0.012 -0.202  
K25 -0.263 -0.029 0.548 1.000 -0.079 -0.261 -0.238 -0.160 0.001 -0.340  
K26 -0.182 -0.183 -0.009 -0.079 1.000 0.129 0.354 0.472 0.106 0.157  
K27 -0.205 -0.194 -0.145 -0.261 0.129 1.000 0.499 0.244 0.262 0.028  
K28 -0.127 -0.159 -0.147 -0.238 0.354 0.499 1.000 0.403 0.082 -0.001  
K29 -0.102 -0.094 -0.057 -0.160 0.472 0.244 0.403 1.000 0.143 0.052  
K30 -0.249 -0.199 0.012 0.001 0.106 0.262 0.082 0.143 1.000 -0.001  
K31 0.024 -0.076 -0.202 -0.340 0.157 0.028 -0.001 0.052 -0.001 1.000  
K32 -0.065 -0.240 -0.222 -0.127 -0.095 -0.090 -0.025 -0.131 -0.145 0.145  
K33 -0.087 -0.210 -0.232 -0.162 -0.038 -0.078 -0.053 -0.119 -0.203 0.155  
K34 -0.105 -0.168 -0.072 -0.054 -0.179 -0.064 -0.157 -0.095 -0.046 -0.206  
K36 -0.174 -0.104 0.034 0.100 -0.010 -0.143 -0.080 -0.104 0.001 -0.173  
K37 -0.100 -0.136 -0.116 -0.186 -0.179 -0.076 -0.033 -0.037 -0.033 -0.172  
 K32 K33 K34 K36 K37  
K1 -0.143 -0.034 0.071 0.101 0.079  
K4 -0.065 -0.071 -0.038 -0.058 0.090  
K6 -0.097 -0.166 -0.051 -0.016 -0.046  
K12 -0.024 -0.013 -0.067 -0.093 -0.091  
K13 -0.107 -0.137 -0.005 -0.023 -0.015  
K14 -0.060 -0.080 0.093 0.016 0.140  
K15 -0.117 -0.050 0.039 -0.053 0.052  
K16 -0.109 -0.016 0.105 -0.120 0.008  
K17 -0.143 -0.081 0.150 0.093 0.242  
K18 -0.112 -0.158 -0.026 -0.063 -0.169  
K20 -0.065 -0.087 -0.105 -0.174 -0.100  
K21 -0.240 -0.210 -0.168 -0.104 -0.136  
K23 -0.222 -0.232 -0.072 0.034 -0.116  
K25 -0.127 -0.162 -0.054 0.100 -0.186  
K26 -0.095 -0.038 -0.179 -0.010 -0.179  
K27 -0.090 -0.078 -0.064 -0.143 -0.076  
K28 -0.025 -0.053 -0.157 -0.080 -0.033  
K29 -0.131 -0.119 -0.095 -0.104 -0.037  
K30 -0.145 -0.203 -0.046 0.001 -0.033  
K31 0.145 0.155 -0.206 -0.173 -0.172  
K32 1.000 0.500 0.012 -0.109 0.041  
K33 0.500 1.000 0.032 -0.073 -0.001  
K34 0.012 0.032 1.000 0.340 0.319  
K36 -0.109 -0.073 0.340 1.000 0.348  
K37 0.041 -0.001 0.319 0.348 1.000

Q3\_resid # inspect residual correlations (Q3 > .20 may indicate local dependence)

K1 K4 K6 K12 K13 K14 K15 K16 K17 K18  
K1 1.000 0.149 0.049 -0.021 0.171 0.076 -0.063 -0.074 0.013 0.137  
K4 0.149 1.000 0.284 0.061 -0.090 0.039 0.111 0.055 0.071 -0.029  
K6 0.049 0.284 1.000 0.192 0.035 -0.018 0.010 -0.094 0.047 0.005  
K12 -0.021 0.061 0.192 1.000 -0.131 -0.229 -0.111 -0.097 -0.044 0.152  
K13 0.171 -0.090 0.035 -0.131 1.000 0.302 0.187 0.214 0.093 0.150  
K14 0.076 0.039 -0.018 -0.229 0.302 1.000 0.347 0.349 0.172 -0.066  
K15 -0.063 0.111 0.010 -0.111 0.187 0.347 1.000 0.790 0.167 -0.116  
K16 -0.074 0.055 -0.094 -0.097 0.214 0.349 0.790 1.000 0.163 -0.044  
K17 0.013 0.071 0.047 -0.044 0.093 0.172 0.167 0.163 1.000 -0.107  
K18 0.137 -0.029 0.005 0.152 0.150 -0.066 -0.116 -0.044 -0.107 1.000  
K20 0.014 0.000 0.010 -0.077 -0.108 0.054 -0.002 -0.074 -0.017 -0.017  
K21 0.008 -0.085 -0.019 -0.049 -0.008 0.046 -0.059 -0.129 -0.050 0.042  
K23 -0.096 -0.037 -0.043 -0.101 -0.132 -0.168 -0.128 -0.117 -0.149 -0.092  
K25 0.085 -0.101 -0.040 -0.075 -0.102 -0.141 -0.151 -0.142 -0.116 -0.036  
K26 0.032 -0.146 -0.046 0.112 -0.102 -0.309 -0.246 -0.258 -0.186 0.135  
K27 -0.160 0.025 -0.068 0.017 -0.011 -0.053 -0.149 -0.094 -0.037 0.024  
K28 -0.170 -0.100 -0.133 0.066 -0.049 -0.176 -0.196 -0.135 -0.099 0.051  
K29 -0.033 -0.005 -0.003 0.170 -0.218 -0.303 -0.196 -0.237 -0.236 0.025  
K30 -0.136 -0.014 0.081 0.098 -0.034 -0.130 -0.160 -0.169 -0.041 -0.053  
K31 -0.116 -0.004 0.017 0.055 -0.130 -0.111 -0.178 -0.118 -0.092 -0.125  
K32 -0.143 -0.065 -0.097 -0.024 -0.107 -0.060 -0.117 -0.109 -0.143 -0.112  
K33 -0.034 -0.071 -0.166 -0.013 -0.137 -0.080 -0.050 -0.016 -0.081 -0.158  
K34 0.071 -0.038 -0.051 -0.067 -0.005 0.093 0.039 0.105 0.150 -0.026  
K36 0.101 -0.058 -0.016 -0.093 -0.023 0.016 -0.053 -0.120 0.093 -0.063  
K37 0.079 0.090 -0.046 -0.091 -0.015 0.140 0.052 0.008 0.242 -0.169  
 K20 K21 K23 K25 K26 K27 K28 K29 K30 K31  
K1 0.014 0.008 -0.096 0.085 0.032 -0.160 -0.170 -0.033 -0.136 -0.116  
K4 0.000 -0.085 -0.037 -0.101 -0.146 0.025 -0.100 -0.005 -0.014 -0.004  
K6 0.010 -0.019 -0.043 -0.040 -0.046 -0.068 -0.133 -0.003 0.081 0.017  
K12 -0.077 -0.049 -0.101 -0.075 0.112 0.017 0.066 0.170 0.098 0.055  
K13 -0.108 -0.008 -0.132 -0.102 -0.102 -0.011 -0.049 -0.218 -0.034 -0.130  
K14 0.054 0.046 -0.168 -0.141 -0.309 -0.053 -0.176 -0.303 -0.130 -0.111  
K15 -0.002 -0.059 -0.128 -0.151 -0.246 -0.149 -0.196 -0.196 -0.160 -0.178  
K16 -0.074 -0.129 -0.117 -0.142 -0.258 -0.094 -0.135 -0.237 -0.169 -0.118  
K17 -0.017 -0.050 -0.149 -0.116 -0.186 -0.037 -0.099 -0.236 -0.041 -0.092  
K18 -0.017 0.042 -0.092 -0.036 0.135 0.024 0.051 0.025 -0.053 -0.125  
K20 1.000 0.691 -0.228 -0.263 -0.182 -0.205 -0.127 -0.102 -0.249 0.024  
K21 0.691 1.000 -0.170 -0.029 -0.183 -0.194 -0.159 -0.094 -0.199 -0.076  
K23 -0.228 -0.170 1.000 0.548 -0.009 -0.145 -0.147 -0.057 0.012 -0.202  
K25 -0.263 -0.029 0.548 1.000 -0.079 -0.261 -0.238 -0.160 0.001 -0.340  
K26 -0.182 -0.183 -0.009 -0.079 1.000 0.129 0.354 0.472 0.106 0.157  
K27 -0.205 -0.194 -0.145 -0.261 0.129 1.000 0.499 0.244 0.262 0.028  
K28 -0.127 -0.159 -0.147 -0.238 0.354 0.499 1.000 0.403 0.082 -0.001  
K29 -0.102 -0.094 -0.057 -0.160 0.472 0.244 0.403 1.000 0.143 0.052  
K30 -0.249 -0.199 0.012 0.001 0.106 0.262 0.082 0.143 1.000 -0.001  
K31 0.024 -0.076 -0.202 -0.340 0.157 0.028 -0.001 0.052 -0.001 1.000  
K32 -0.065 -0.240 -0.222 -0.127 -0.095 -0.090 -0.025 -0.131 -0.145 0.145  
K33 -0.087 -0.210 -0.232 -0.162 -0.038 -0.078 -0.053 -0.119 -0.203 0.155  
K34 -0.105 -0.168 -0.072 -0.054 -0.179 -0.064 -0.157 -0.095 -0.046 -0.206  
K36 -0.174 -0.104 0.034 0.100 -0.010 -0.143 -0.080 -0.104 0.001 -0.173  
K37 -0.100 -0.136 -0.116 -0.186 -0.179 -0.076 -0.033 -0.037 -0.033 -0.172  
 K32 K33 K34 K36 K37  
K1 -0.143 -0.034 0.071 0.101 0.079  
K4 -0.065 -0.071 -0.038 -0.058 0.090  
K6 -0.097 -0.166 -0.051 -0.016 -0.046  
K12 -0.024 -0.013 -0.067 -0.093 -0.091  
K13 -0.107 -0.137 -0.005 -0.023 -0.015  
K14 -0.060 -0.080 0.093 0.016 0.140  
K15 -0.117 -0.050 0.039 -0.053 0.052  
K16 -0.109 -0.016 0.105 -0.120 0.008  
K17 -0.143 -0.081 0.150 0.093 0.242  
K18 -0.112 -0.158 -0.026 -0.063 -0.169  
K20 -0.065 -0.087 -0.105 -0.174 -0.100  
K21 -0.240 -0.210 -0.168 -0.104 -0.136  
K23 -0.222 -0.232 -0.072 0.034 -0.116  
K25 -0.127 -0.162 -0.054 0.100 -0.186  
K26 -0.095 -0.038 -0.179 -0.010 -0.179  
K27 -0.090 -0.078 -0.064 -0.143 -0.076  
K28 -0.025 -0.053 -0.157 -0.080 -0.033  
K29 -0.131 -0.119 -0.095 -0.104 -0.037  
K30 -0.145 -0.203 -0.046 0.001 -0.033  
K31 0.145 0.155 -0.206 -0.173 -0.172  
K32 1.000 0.500 0.012 -0.109 0.041  
K33 0.500 1.000 0.032 -0.073 -0.001  
K34 0.012 0.032 1.000 0.340 0.319  
K36 -0.109 -0.073 0.340 1.000 0.348  
K37 0.041 -0.001 0.319 0.348 1.000

# Chen & Thissen’s LD χ² statistic  
LD\_resid <- residuals(mod2pl, type = "LD")

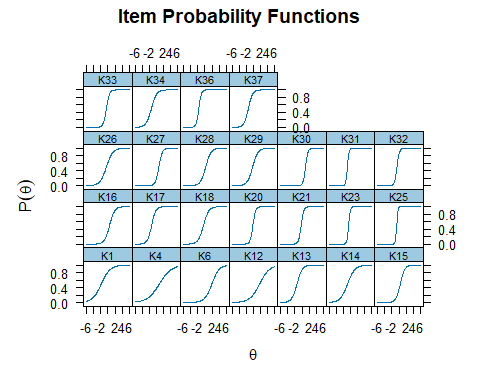
LD matrix (lower triangle) and standardized residual correlations (upper triangle)  
  
Upper triangle summary:  
 Min. 1st Qu. Median Mean 3rd Qu. Max.   
 -0.261 -0.082 -0.037 -0.014 0.035 0.580   
  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K20  
K1 0.135 0.051 -0.016 0.143 0.066 -0.062 -0.072 0.028 0.115 0.008  
K4 3.705 0.271 0.068 -0.092 0.034 0.099 0.059 0.048 -0.031 -0.017  
K6 0.536 14.996 0.170 0.044 0.003 0.035 -0.046 0.061 0.020 0.041  
K12 0.051 0.947 5.874 -0.113 -0.202 -0.077 -0.066 -0.024 0.141 -0.050  
K13 4.196 1.717 0.395 2.612 0.233 0.158 0.187 0.070 0.128 -0.059  
K14 0.880 0.229 0.002 8.307 11.068 0.286 0.299 0.121 -0.057 0.023  
K15 0.780 1.985 0.249 1.202 5.088 16.711 0.580 0.141 -0.089 0.020  
K16 1.044 0.716 0.435 0.901 7.117 18.184 68.530 0.142 -0.035 -0.034  
K17 0.162 0.476 0.747 0.115 1.009 2.983 4.064 4.104 -0.068 -0.015  
K18 2.680 0.192 0.084 4.077 3.327 0.668 1.602 0.244 0.950 -0.018  
K20 0.012 0.058 0.335 0.516 0.703 0.108 0.083 0.239 0.045 0.063   
K21 0.004 1.238 0.072 0.180 0.069 0.033 0.168 1.207 0.132 0.101 33.146  
K23 0.504 0.895 0.117 0.421 0.351 3.820 2.217 1.850 1.438 0.472 1.994  
K25 0.367 2.226 0.233 0.133 0.151 3.134 3.893 3.157 1.009 0.141 2.628  
K26 0.111 3.271 0.050 2.715 1.590 13.920 5.883 7.083 5.069 2.254 2.962  
K27 4.215 0.500 0.128 0.131 0.104 0.091 0.466 0.107 0.293 0.157 3.261  
K28 4.738 1.110 1.265 0.874 0.528 4.227 2.514 1.060 2.115 0.252 1.319  
K29 0.201 0.007 0.120 5.147 6.442 13.035 3.392 5.616 7.941 0.037 1.043  
K30 2.811 0.619 0.862 0.928 0.303 0.271 0.183 0.277 0.194 0.122 4.617  
K31 0.655 0.390 0.531 1.492 0.527 2.356 6.711 3.434 0.625 0.545 0.425  
K32 1.366 1.157 0.450 0.024 0.337 0.997 1.832 1.654 1.296 0.761 0.083  
K33 0.071 1.088 2.817 0.004 0.813 1.016 0.371 0.092 0.241 1.999 0.189  
K34 0.965 0.435 0.509 0.551 0.022 0.936 0.188 1.600 2.315 0.039 0.976  
K36 2.199 0.793 0.531 0.551 0.051 0.199 1.427 4.311 1.302 0.041 4.792  
K37 1.207 0.840 0.485 1.031 0.024 1.984 0.191 0.050 5.271 2.784 1.472  
 K21 K23 K25 K26 K27 K28 K29 K30 K31 K32  
K1 0.004 -0.050 0.042 0.023 -0.144 -0.152 -0.031 -0.117 -0.057 -0.082  
K4 -0.078 -0.066 -0.104 -0.127 0.049 -0.074 -0.006 0.055 -0.044 -0.075  
K6 0.019 0.024 0.034 -0.016 -0.025 -0.079 0.024 0.065 0.051 -0.047  
K12 -0.030 -0.045 -0.026 0.115 0.025 0.065 0.159 0.067 0.086 0.011  
K13 0.018 -0.041 -0.027 -0.088 -0.023 -0.051 -0.178 -0.039 -0.051 -0.041  
K14 0.013 -0.137 -0.124 -0.261 -0.021 -0.144 -0.253 -0.036 -0.107 -0.070  
K15 -0.029 -0.104 -0.138 -0.170 -0.048 -0.111 -0.129 -0.030 -0.181 -0.095  
K16 -0.077 -0.095 -0.124 -0.186 -0.023 -0.072 -0.166 -0.037 -0.130 -0.090  
K17 -0.025 -0.084 -0.070 -0.158 -0.038 -0.102 -0.197 -0.031 -0.055 -0.080  
K18 0.022 -0.048 -0.026 0.105 0.028 0.035 0.014 -0.024 -0.052 -0.061  
K20 0.403 -0.099 -0.113 -0.121 -0.126 -0.080 -0.072 -0.150 0.046 -0.020  
K21 -0.049 0.047 -0.123 -0.138 -0.109 -0.070 -0.141 0.034 -0.091  
K23 0.487 0.247 -0.037 -0.147 -0.131 -0.070 0.056 -0.050 -0.043  
K25 0.453 12.420 -0.070 -0.249 -0.183 -0.119 0.049 -0.076 0.035  
K26 3.089 0.286 1.006 0.140 0.289 0.372 0.163 0.076 -0.076  
K27 3.906 4.418 12.615 4.015 0.378 0.216 0.217 0.039 -0.100  
K28 2.423 3.516 6.837 17.073 29.220 0.315 0.139 -0.062 -0.049  
K29 1.013 1.010 2.872 28.185 9.556 20.277 0.189 -0.032 -0.106  
K30 4.058 0.642 0.484 5.431 9.566 3.932 7.274 0.048 -0.164  
K31 0.240 0.512 1.179 1.190 0.317 0.777 0.205 0.478 0.121  
K32 1.678 0.373 0.252 1.189 2.032 0.488 2.312 5.469 2.985   
K33 1.578 0.608 0.182 0.216 1.536 0.584 1.717 8.571 5.134 19.852  
K34 2.241 0.611 0.742 4.755 0.721 4.326 1.485 0.356 2.892 0.023  
K36 1.091 0.143 0.331 0.220 10.003 1.853 1.372 0.194 1.038 0.881  
K37 2.027 1.087 1.666 4.816 1.571 0.567 0.431 0.203 1.321 0.084  
 K33 K34 K36 K37  
K1 -0.019 0.069 0.104 0.077  
K4 -0.073 -0.046 -0.062 0.064  
K6 -0.118 -0.050 -0.051 -0.049  
K12 -0.004 -0.052 -0.052 -0.071  
K13 -0.063 0.010 0.016 0.011  
K14 -0.071 0.068 0.031 0.099  
K15 -0.043 0.030 -0.084 0.031  
K16 -0.021 0.089 -0.145 -0.016  
K17 -0.034 0.107 0.080 0.161  
K18 -0.099 -0.014 -0.014 -0.117  
K20 -0.030 -0.069 -0.153 -0.085  
K21 -0.088 -0.105 -0.073 -0.100  
K23 -0.055 -0.055 0.026 -0.073  
K25 0.030 -0.060 0.040 -0.090  
K26 -0.033 -0.153 -0.033 -0.154  
K27 -0.087 -0.059 -0.221 -0.088  
K28 -0.054 -0.146 -0.095 -0.053  
K29 -0.092 -0.085 -0.082 -0.046  
K30 -0.205 -0.042 0.031 -0.032  
K31 0.159 -0.119 -0.071 -0.080  
K32 0.312 -0.011 -0.066 0.020  
K33 0.031 -0.019 0.012  
K34 0.196 0.211 0.218  
K36 0.076 9.107 0.214  
K37 0.029 9.683 9.338

LD\_resid # values > 10 indicate local dependence

K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K20  
K1 NA 0.135 0.051 -0.016 0.143 0.066 -0.062 -0.072 0.028 0.115 0.008  
K4 3.705 NA 0.271 0.068 -0.092 0.034 0.099 0.059 0.048 -0.031 -0.017  
K6 0.536 14.996 NA 0.170 0.044 0.003 0.035 -0.046 0.061 0.020 0.041  
K12 0.051 0.947 5.874 NA -0.113 -0.202 -0.077 -0.066 -0.024 0.141 -0.050  
K13 4.196 1.717 0.395 2.612 NA 0.233 0.158 0.187 0.070 0.128 -0.059  
K14 0.880 0.229 0.002 8.307 11.068 NA 0.286 0.299 0.121 -0.057 0.023  
K15 0.780 1.985 0.249 1.202 5.088 16.711 NA 0.580 0.141 -0.089 0.020  
K16 1.044 0.716 0.435 0.901 7.117 18.184 68.530 NA 0.142 -0.035 -0.034  
K17 0.162 0.476 0.747 0.115 1.009 2.983 4.064 4.104 NA -0.068 -0.015  
K18 2.680 0.192 0.084 4.077 3.327 0.668 1.602 0.244 0.950 NA -0.018  
K20 0.012 0.058 0.335 0.516 0.703 0.108 0.083 0.239 0.045 0.063 NA  
K21 0.004 1.238 0.072 0.180 0.069 0.033 0.168 1.207 0.132 0.101 33.146  
K23 0.504 0.895 0.117 0.421 0.351 3.820 2.217 1.850 1.438 0.472 1.994  
K25 0.367 2.226 0.233 0.133 0.151 3.134 3.893 3.157 1.009 0.141 2.628  
K26 0.111 3.271 0.050 2.715 1.590 13.920 5.883 7.083 5.069 2.254 2.962  
K27 4.215 0.500 0.128 0.131 0.104 0.091 0.466 0.107 0.293 0.157 3.261  
K28 4.738 1.110 1.265 0.874 0.528 4.227 2.514 1.060 2.115 0.252 1.319  
K29 0.201 0.007 0.120 5.147 6.442 13.035 3.392 5.616 7.941 0.037 1.043  
K30 2.811 0.619 0.862 0.928 0.303 0.271 0.183 0.277 0.194 0.122 4.617  
K31 0.655 0.390 0.531 1.492 0.527 2.356 6.711 3.434 0.625 0.545 0.425  
K32 1.366 1.157 0.450 0.024 0.337 0.997 1.832 1.654 1.296 0.761 0.083  
K33 0.071 1.088 2.817 0.004 0.813 1.016 0.371 0.092 0.241 1.999 0.189  
K34 0.965 0.435 0.509 0.551 0.022 0.936 0.188 1.600 2.315 0.039 0.976  
K36 2.199 0.793 0.531 0.551 0.051 0.199 1.427 4.311 1.302 0.041 4.792  
K37 1.207 0.840 0.485 1.031 0.024 1.984 0.191 0.050 5.271 2.784 1.472  
 K21 K23 K25 K26 K27 K28 K29 K30 K31 K32  
K1 0.004 -0.050 0.042 0.023 -0.144 -0.152 -0.031 -0.117 -0.057 -0.082  
K4 -0.078 -0.066 -0.104 -0.127 0.049 -0.074 -0.006 0.055 -0.044 -0.075  
K6 0.019 0.024 0.034 -0.016 -0.025 -0.079 0.024 0.065 0.051 -0.047  
K12 -0.030 -0.045 -0.026 0.115 0.025 0.065 0.159 0.067 0.086 0.011  
K13 0.018 -0.041 -0.027 -0.088 -0.023 -0.051 -0.178 -0.039 -0.051 -0.041  
K14 0.013 -0.137 -0.124 -0.261 -0.021 -0.144 -0.253 -0.036 -0.107 -0.070  
K15 -0.029 -0.104 -0.138 -0.170 -0.048 -0.111 -0.129 -0.030 -0.181 -0.095  
K16 -0.077 -0.095 -0.124 -0.186 -0.023 -0.072 -0.166 -0.037 -0.130 -0.090  
K17 -0.025 -0.084 -0.070 -0.158 -0.038 -0.102 -0.197 -0.031 -0.055 -0.080  
K18 0.022 -0.048 -0.026 0.105 0.028 0.035 0.014 -0.024 -0.052 -0.061  
K20 0.403 -0.099 -0.113 -0.121 -0.126 -0.080 -0.072 -0.150 0.046 -0.020  
K21 NA -0.049 0.047 -0.123 -0.138 -0.109 -0.070 -0.141 0.034 -0.091  
K23 0.487 NA 0.247 -0.037 -0.147 -0.131 -0.070 0.056 -0.050 -0.043  
K25 0.453 12.420 NA -0.070 -0.249 -0.183 -0.119 0.049 -0.076 0.035  
K26 3.089 0.286 1.006 NA 0.140 0.289 0.372 0.163 0.076 -0.076  
K27 3.906 4.418 12.615 4.015 NA 0.378 0.216 0.217 0.039 -0.100  
K28 2.423 3.516 6.837 17.073 29.220 NA 0.315 0.139 -0.062 -0.049  
K29 1.013 1.010 2.872 28.185 9.556 20.277 NA 0.189 -0.032 -0.106  
K30 4.058 0.642 0.484 5.431 9.566 3.932 7.274 NA 0.048 -0.164  
K31 0.240 0.512 1.179 1.190 0.317 0.777 0.205 0.478 NA 0.121  
K32 1.678 0.373 0.252 1.189 2.032 0.488 2.312 5.469 2.985 NA  
K33 1.578 0.608 0.182 0.216 1.536 0.584 1.717 8.571 5.134 19.852  
K34 2.241 0.611 0.742 4.755 0.721 4.326 1.485 0.356 2.892 0.023  
K36 1.091 0.143 0.331 0.220 10.003 1.853 1.372 0.194 1.038 0.881  
K37 2.027 1.087 1.666 4.816 1.571 0.567 0.431 0.203 1.321 0.084  
 K33 K34 K36 K37  
K1 -0.019 0.069 0.104 0.077  
K4 -0.073 -0.046 -0.062 0.064  
K6 -0.118 -0.050 -0.051 -0.049  
K12 -0.004 -0.052 -0.052 -0.071  
K13 -0.063 0.010 0.016 0.011  
K14 -0.071 0.068 0.031 0.099  
K15 -0.043 0.030 -0.084 0.031  
K16 -0.021 0.089 -0.145 -0.016  
K17 -0.034 0.107 0.080 0.161  
K18 -0.099 -0.014 -0.014 -0.117  
K20 -0.030 -0.069 -0.153 -0.085  
K21 -0.088 -0.105 -0.073 -0.100  
K23 -0.055 -0.055 0.026 -0.073  
K25 0.030 -0.060 0.040 -0.090  
K26 -0.033 -0.153 -0.033 -0.154  
K27 -0.087 -0.059 -0.221 -0.088  
K28 -0.054 -0.146 -0.095 -0.053  
K29 -0.092 -0.085 -0.082 -0.046  
K30 -0.205 -0.042 0.031 -0.032  
K31 0.159 -0.119 -0.071 -0.080  
K32 0.312 -0.011 -0.066 0.020  
K33 NA 0.031 -0.019 0.012  
K34 0.196 NA 0.211 0.218  
K36 0.076 9.107 NA 0.214  
K37 0.029 9.683 9.338 NA

## Monotonicity

# ==========================================  
# 3. Assumption: Monotonicity  
# ==========================================  
  
# (a) Plot Item Characteristic Curves (ICCs) to visually check monotonicity  
plot(mod2pl, type = "trace") # S-shaped, increasing curves are expected



# (b) Optional: Use Mokken scale analysis for monotonicity check  
  
# Run monotonicity check  
check.monotonicity(irt\_mat) # flags items with non-monotonic patterns

$results  
$results[[1]]  
$results[[1]][[1]]  
[1] "K1"  
  
$results[[1]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 71 25 46 0.6478873 0.6478873  
[2,] 2 12 24 133 18 115 0.8646617 0.8646617  
  
$results[[1]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[1]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[2]]  
$results[[2]][[1]]  
[1] "K4"  
  
$results[[2]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 49 19 0.2794118 0.2794118  
[2,] 2 12 24 136 77 59 0.4338235 0.4338235  
  
$results[[2]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[2]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[3]]  
$results[[3]][[1]]  
[1] "K6"  
  
$results[[3]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 73 72 1 0.01369863 0.01369863  
[2,] 2 13 24 131 99 32 0.24427481 0.24427481  
  
$results[[3]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[3]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[4]]  
$results[[4]][[1]]  
[1] "K12"  
  
$results[[4]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 77 68 9 0.1168831 0.1168831  
[2,] 2 13 24 127 87 40 0.3149606 0.3149606  
  
$results[[4]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[4]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[5]]  
$results[[5]][[1]]  
[1] "K13"  
  
$results[[5]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 29 43 0.5972222 0.5972222  
[2,] 2 12 24 132 10 122 0.9242424 0.9242424  
  
$results[[5]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[5]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[6]]  
$results[[6]][[1]]  
[1] "K14"  
  
$results[[6]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 37 33 0.4714286 0.4714286  
[2,] 2 12 24 134 32 102 0.7611940 0.7611940  
  
$results[[6]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[6]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[7]]  
$results[[7]][[1]]  
[1] "K15"  
  
$results[[7]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 66 8 0.1081081 0.1081081  
[2,] 2 13 24 130 57 73 0.5615385 0.5615385  
  
$results[[7]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[7]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[8]]  
$results[[8]][[1]]  
[1] "K16"  
  
$results[[8]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 76 66 10 0.1315789 0.1315789  
[2,] 2 13 24 128 61 67 0.5234375 0.5234375  
  
$results[[8]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[8]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[9]]  
$results[[9]][[1]]  
[1] "K17"  
  
$results[[9]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 25 45 0.6428571 0.6428571  
[2,] 2 12 24 134 9 125 0.9328358 0.9328358  
  
$results[[9]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[9]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[10]]  
$results[[10]][[1]]  
[1] "K18"  
  
$results[[10]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 47 23 0.3285714 0.3285714  
[2,] 2 12 24 134 34 100 0.7462687 0.7462687  
  
$results[[10]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[10]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[11]]  
$results[[11]][[1]]  
[1] "K20"  
  
$results[[11]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 77 65 12 0.1558442 0.1558442  
[2,] 2 13 24 127 27 100 0.7874016 0.7874016  
  
$results[[11]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[11]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[12]]  
$results[[12]][[1]]  
[1] "K21"  
  
$results[[12]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 57 11 0.1617647 0.1617647  
[2,] 2 12 24 136 30 106 0.7794118 0.7794118  
  
$results[[12]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[12]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[13]]  
$results[[13]][[1]]  
[1] "K23"  
  
$results[[13]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 71 48 23 0.3239437 0.3239437  
[2,] 2 12 24 133 14 119 0.8947368 0.8947368  
  
$results[[13]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[13]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[14]]  
$results[[14]][[1]]  
[1] "K25"  
  
$results[[14]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 45 27 0.3750000 0.3750000  
[2,] 2 12 24 132 7 125 0.9469697 0.9469697  
  
$results[[14]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[14]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[15]]  
$results[[15]][[1]]  
[1] "K26"  
  
$results[[15]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 41 29 0.4142857 0.4142857  
[2,] 2 12 24 134 42 92 0.6865672 0.6865672  
  
$results[[15]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[15]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[16]]  
$results[[16]][[1]]  
[1] "K27"  
  
$results[[16]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 74 71 3 0.04054054 0.04054054  
[2,] 2 13 24 130 74 56 0.43076923 0.43076923  
  
$results[[16]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[16]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[17]]  
$results[[17]][[1]]  
[1] "K28"  
  
$results[[17]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 68 54 14 0.2058824 0.2058824  
[2,] 2 12 24 136 59 77 0.5661765 0.5661765  
  
$results[[17]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[17]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[18]]  
$results[[18]][[1]]  
[1] "K29"  
  
$results[[18]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 69 49 20 0.2898551 0.2898551  
[2,] 2 12 24 135 41 94 0.6962963 0.6962963  
  
$results[[18]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[18]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[19]]  
$results[[19]][[1]]  
[1] "K30"  
  
$results[[19]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 12 73 73 0 0.0000000 0.0000000  
[2,] 2 13 24 131 93 38 0.2900763 0.2900763  
  
$results[[19]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 0 0 NaN 0 0 NaN 0 0 0 0  
Total 0 0 NaN 0 0 NaN 0 0 0 0  
  
$results[[19]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[20]]  
$results[[20]][[1]]  
[1] "K31"  
  
$results[[20]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 37 35 0.4861111 0.4861111  
[2,] 2 12 24 132 3 129 0.9772727 0.9772727  
  
$results[[20]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[20]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[21]]  
$results[[21]][[1]]  
[1] "K32"  
  
$results[[21]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 70 49 21 0.3000000 0.3000000  
[2,] 2 12 24 134 13 121 0.9029851 0.9029851  
  
$results[[21]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[21]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[22]]  
$results[[22]][[1]]  
[1] "K33"  
  
$results[[22]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 69 52 17 0.2463768 0.2463768  
[2,] 2 12 24 135 23 112 0.8296296 0.8296296  
  
$results[[22]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[22]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[23]]  
$results[[23]][[1]]  
[1] "K34"  
  
$results[[23]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 69 26 43 0.6231884 0.6231884  
[2,] 2 12 24 135 12 123 0.9111111 0.9111111  
  
$results[[23]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[23]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[24]]  
$results[[24]][[1]]  
[1] "K36"  
  
$results[[24]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 73 12 61 0.8356164 0.8356164  
[2,] 2 12 24 131 2 129 0.9847328 0.9847328  
  
$results[[24]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[24]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
$results[[25]]  
$results[[25]][[1]]  
[1] "K37"  
  
$results[[25]][[2]]  
 Group Lo Score Hi Score N F 0 F 1 Mean P(X >=1)  
[1,] 1 0 11 72 24 48 0.6666667 0.6666667  
[2,] 2 12 24 132 8 124 0.9393939 0.9393939  
  
$results[[25]][[3]]  
 #ac #vi #vi/#ac maxvi sum sum/#ac zmax group group #zsig  
P(X >=1) 1 0 0 0 0 0 0 0 0 0  
Total 1 0 0 0 0 0 0 0 0 0  
  
$results[[25]][[4]]  
[1] "Minsize = 68 Minvi = 0.03"  
  
  
  
$I.labels  
 [1] "K1" "K4" "K6" "K12" "K13" "K14" "K15" "K16" "K17" "K18" "K20" "K21"  
[13] "K23" "K25" "K26" "K27" "K28" "K29" "K30" "K31" "K32" "K33" "K34" "K36"  
[25] "K37"  
  
$Hi  
 K1 K4 K6 K12 K13 K14 K15 K16   
0.2688575 0.2277432 0.4351316 0.3074685 0.3964261 0.2840730 0.4566994 0.4316744   
 K17 K18 K20 K21 K23 K25 K26 K27   
0.4468085 0.3704985 0.4826127 0.4789746 0.5067038 0.5461531 0.3054287 0.5407772   
 K28 K29 K30 K31 K32 K33 K34 K36   
0.3891504 0.3621297 0.6460366 0.5582631 0.4900076 0.4523869 0.3678149 0.6606439   
 K37   
0.4415886   
  
$m  
[1] 2  
  
$X  
 K1 K4 K6 K12 K13 K14 K15 K16 K17 K18 K20 K21 K23 K25 K26 K27 K28 K29 K30  
 [1,] 1 0 0 1 1 0 0 0 1 1 0 0 1 1 1 0 0 1 0  
 [2,] 1 0 0 1 1 0 0 0 1 1 0 0 1 1 1 0 0 1 0  
 [3,] 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
 [4,] 1 1 0 1 1 1 0 0 1 1 0 0 1 1 1 0 0 0 0  
 [5,] 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
 [6,] 1 1 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 0  
 [7,] 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 0 0  
 [8,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [9,] 1 1 1 0 0 1 0 0 1 1 1 1 1 1 1 0 0 1 0  
 [10,] 1 1 1 0 1 1 1 1 1 1 0 0 1 1 0 0 0 0 0  
 [11,] 1 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0  
 [12,] 1 1 1 0 0 1 0 0 1 0 1 1 1 1 0 0 0 1 0  
 [13,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
 [14,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0  
 [15,] 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 0 1 0  
 [16,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
 [17,] 1 0 0 1 1 0 0 0 1 0 0 0 0 1 1 0 0 1 0  
 [18,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [19,] 1 0 0 0 1 1 0 0 1 0 1 1 1 1 0 0 0 0 0  
 [20,] 1 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0  
 [21,] 1 0 0 0 1 1 0 0 1 0 1 1 1 1 0 0 0 0 0  
 [22,] 1 0 0 1 0 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
 [23,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [24,] 0 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
 [25,] 1 0 0 0 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0  
 [26,] 1 0 0 0 1 1 0 0 1 1 1 1 0 1 0 0 0 0 0  
 [27,] 1 1 0 1 0 0 0 0 0 1 0 0 1 1 1 0 1 1 0  
 [28,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [29,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
 [30,] 1 1 1 0 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0  
 [31,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0  
 [32,] 1 1 0 0 1 1 1 1 1 0 0 0 1 1 0 0 0 0 0  
 [33,] 1 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
 [34,] 1 1 0 0 1 0 0 0 1 1 1 1 1 1 1 1 1 1 0  
 [35,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [36,] 1 0 0 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
 [37,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
 [38,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0  
 [39,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0  
 [40,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
 [41,] 1 0 0 0 1 1 0 0 0 1 1 1 1 1 0 0 0 1 0  
 [42,] 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1 0 0 1 0  
 [43,] 1 1 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [44,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [45,] 1 1 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0  
 [46,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [47,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1  
 [48,] 0 0 1 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [49,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [50,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
 [51,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0  
 [52,] 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [53,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
 [54,] 1 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 0 0  
 [55,] 1 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 0 0  
 [56,] 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 1 1 1  
 [57,] 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [58,] 1 1 1 0 1 1 1 0 1 1 1 1 1 1 0 0 0 0 0  
 [59,] 1 0 0 0 1 1 0 0 1 1 0 1 1 1 0 0 0 0 0  
 [60,] 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
 [61,] 1 0 1 1 1 0 0 0 1 1 1 1 0 1 0 0 0 0 0  
 [62,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [63,] 1 0 0 0 1 1 0 0 1 1 0 0 0 1 1 0 1 1 0  
 [64,] 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
 [65,] 1 1 0 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0  
 [66,] 1 1 0 0 1 1 0 0 0 0 0 0 0 0 1 0 1 1 0  
 [67,] 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
 [68,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [69,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
 [70,] 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1  
 [71,] 0 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [72,] 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [73,] 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [74,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0  
 [75,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0  
 [76,] 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 1 0  
 [77,] 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0  
 [78,] 0 0 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1  
 [79,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
 [80,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0  
 [81,] 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0  
 [82,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 0 1 1 0  
 [83,] 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0  
 [84,] 0 0 0 0 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0  
 [85,] 1 0 0 0 1 1 0 1 1 1 0 0 1 1 1 1 1 1 1  
 [86,] 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 [87,] 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0  
 [88,] 1 1 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
 [89,] 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 0 1 1 0  
 [90,] 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0  
 [91,] 1 1 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 1 0  
 [92,] 1 1 0 1 0 0 1 0 1 1 1 1 1 1 1 1 1 1 1  
 [93,] 1 1 0 0 0 1 0 0 1 0 0 1 0 1 0 0 0 0 0  
 [94,] 1 0 0 0 1 1 0 0 1 0 1 1 0 1 0 0 0 0 0  
 [95,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0  
 [96,] 1 0 0 1 1 1 0 0 1 0 1 1 1 1 1 0 1 1 0  
 [97,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
 [98,] 1 0 0 0 0 0 0 0 1 0 1 1 1 1 1 0 0 1 0  
 [99,] 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 1 1 0  
[100,] 1 1 0 0 1 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
[101,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
[102,] 1 0 0 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 0  
[103,] 1 1 1 0 1 1 0 0 1 1 1 0 0 0 1 1 1 1 0  
[104,] 1 1 0 0 1 1 1 1 1 0 1 1 0 1 0 0 0 0 0  
[105,] 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
[106,] 1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0  
[107,] 1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
[108,] 1 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0  
[109,] 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
[110,] 1 1 0 0 1 1 0 0 1 0 0 0 1 1 0 0 0 0 0  
[111,] 0 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0 1 0 0  
[112,] 0 0 0 1 1 1 1 1 1 0 0 0 1 1 0 1 1 1 1  
[113,] 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0  
[114,] 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
[115,] 0 1 0 0 1 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
[116,] 1 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
[117,] 1 0 0 0 1 1 1 1 1 1 0 0 0 0 1 0 0 0 0  
[118,] 1 0 0 1 0 0 0 0 1 1 0 0 0 0 1 0 0 0 0  
[119,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 0 0 0 0  
[120,] 1 0 0 0 1 0 1 1 1 1 0 0 1 1 0 0 0 0 0  
[121,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
[122,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
[123,] 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 0 0 1 1  
[124,] 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0  
[125,] 1 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 0  
[126,] 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0  
[127,] 1 0 0 1 1 1 0 1 1 1 0 0 1 1 0 0 0 0 0  
[128,] 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 0  
[129,] 1 0 0 0 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1  
[130,] 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[131,] 1 1 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
[132,] 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 1 1 0  
[133,] 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
[134,] 1 0 0 1 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0  
[135,] 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0  
[136,] 1 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0  
[137,] 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0  
[138,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0  
[139,] 1 1 0 1 1 1 0 0 0 1 0 0 1 1 1 0 1 0 0  
[140,] 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1 0 1 1 0  
[141,] 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
[142,] 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 0  
[143,] 0 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1  
[144,] 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 0 0 0  
[145,] 1 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 1 0  
[146,] 0 0 0 0 1 0 0 0 1 0 1 1 1 0 1 0 1 1 0  
[147,] 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[148,] 0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0  
[149,] 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0  
[150,] 1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
[151,] 0 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0  
[152,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[153,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
[154,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[155,] 1 0 0 1 1 0 0 0 0 1 0 1 1 1 1 1 1 1 0  
[156,] 1 0 0 0 1 1 0 0 0 1 1 1 0 0 0 0 0 1 0  
[157,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0  
[158,] 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0  
[159,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0  
[160,] 0 0 0 0 1 1 0 1 1 1 1 1 1 1 0 0 0 0 0  
[161,] 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
[162,] 1 1 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1  
[163,] 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0  
[164,] 1 1 1 1 1 1 1 0 1 0 1 1 1 1 0 0 0 0 0  
[165,] 1 0 0 0 1 1 1 0 1 0 1 1 1 1 0 0 1 1 0  
[166,] 1 0 0 0 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1  
[167,] 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0  
[168,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0  
[169,] 1 0 0 0 1 1 1 0 1 1 0 0 1 1 1 0 0 1 0  
[170,] 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
[171,] 0 0 0 0 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0  
[172,] 1 1 0 0 0 0 0 0 1 0 1 1 1 1 0 0 0 1 0  
[173,] 1 0 0 0 1 0 0 0 1 1 1 1 0 0 1 0 0 1 0  
[174,] 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 0 0 0  
[175,] 1 0 1 1 1 1 0 0 1 1 1 1 1 1 0 0 0 1 0  
[176,] 1 0 0 0 1 0 0 0 1 0 1 1 1 1 1 1 1 0 0  
[177,] 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 0 1 1 0  
[178,] 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 1 0  
[179,] 1 1 0 0 1 1 0 0 0 1 1 1 1 1 0 0 0 1 0  
[180,] 0 0 0 0 1 1 0 0 1 1 0 1 1 1 0 1 1 0 0  
[181,] 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0  
[182,] 1 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 0 0 0  
[183,] 1 1 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 0  
[184,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[185,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[186,] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0  
[187,] 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0  
[188,] 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 0 0 1 1  
[189,] 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[190,] 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 0 1 1  
[191,] 1 0 1 1 1 0 0 0 1 1 0 1 1 1 1 1 1 1 1  
[192,] 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 0 0 1 0  
[193,] 0 1 0 1 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0  
[194,] 1 0 0 0 1 0 0 0 0 1 0 1 0 1 1 0 0 1 0  
[195,] 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 1 0  
[196,] 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0 1 1 1 0  
[197,] 1 0 0 0 1 1 0 0 1 1 0 1 0 1 1 0 1 0 0  
[198,] 0 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 0  
[199,] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
[200,] 0 1 0 1 0 1 1 1 1 0 1 1 0 0 0 1 1 1 0  
[201,] 0 1 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 0  
[202,] 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 0 0 0 0  
[203,] 0 0 0 0 1 0 0 0 1 1 0 1 1 1 1 0 1 1 0  
[204,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0  
 K31 K32 K33 K34 K36 K37  
 [1,] 1 1 1 1 1 1  
 [2,] 1 1 1 1 1 1  
 [3,] 1 1 1 1 1 1  
 [4,] 1 1 1 1 1 1  
 [5,] 0 0 0 1 1 1  
 [6,] 1 1 1 1 1 1  
 [7,] 1 1 1 1 1 1  
 [8,] 1 1 1 1 1 1  
 [9,] 1 1 1 1 1 1  
 [10,] 1 1 1 1 1 1  
 [11,] 1 1 1 1 1 1  
 [12,] 1 1 1 1 1 1  
 [13,] 1 1 1 1 1 1  
 [14,] 1 1 1 1 1 1  
 [15,] 1 0 1 1 1 1  
 [16,] 1 1 1 1 1 1  
 [17,] 1 1 1 1 1 1  
 [18,] 1 0 1 1 1 1  
 [19,] 0 0 0 1 1 1  
 [20,] 1 1 1 1 1 1  
 [21,] 1 0 1 1 1 1  
 [22,] 1 1 1 1 1 0  
 [23,] 1 1 1 1 1 1  
 [24,] 0 0 0 0 1 1  
 [25,] 1 1 0 1 1 1  
 [26,] 0 1 1 1 1 1  
 [27,] 0 0 0 1 1 1  
 [28,] 1 1 1 1 1 1  
 [29,] 1 1 1 1 1 1  
 [30,] 1 1 0 1 1 1  
 [31,] 1 1 1 1 1 1  
 [32,] 1 1 1 1 1 1  
 [33,] 0 0 0 1 1 0  
 [34,] 1 1 1 1 1 1  
 [35,] 1 1 1 1 1 1  
 [36,] 1 1 1 1 1 1  
 [37,] 1 1 1 1 1 1  
 [38,] 0 0 0 0 0 0  
 [39,] 1 1 1 1 1 1  
 [40,] 1 0 1 1 1 1  
 [41,] 1 0 0 1 1 1  
 [42,] 1 1 1 1 1 1  
 [43,] 1 1 1 1 1 1  
 [44,] 1 1 1 1 1 1  
 [45,] 0 0 0 1 1 1  
 [46,] 1 1 1 1 1 1  
 [47,] 1 1 1 1 1 1  
 [48,] 1 1 1 1 1 0  
 [49,] 1 1 1 1 1 1  
 [50,] 0 0 0 1 1 0  
 [51,] 1 1 1 1 1 1  
 [52,] 1 1 1 1 1 1  
 [53,] 1 1 1 1 1 1  
 [54,] 1 1 1 0 1 1  
 [55,] 1 1 1 1 1 1  
 [56,] 1 1 0 1 1 1  
 [57,] 1 1 1 1 1 1  
 [58,] 1 1 0 0 1 1  
 [59,] 0 0 0 1 1 1  
 [60,] 1 1 1 1 1 1  
 [61,] 1 1 1 1 1 1  
 [62,] 1 1 1 1 1 1  
 [63,] 1 1 1 1 1 1  
 [64,] 1 1 1 1 1 1  
 [65,] 0 0 0 1 1 1  
 [66,] 1 1 1 0 1 1  
 [67,] 1 1 1 1 1 1  
 [68,] 1 1 1 1 1 1  
 [69,] 0 0 0 0 0 0  
 [70,] 1 1 1 1 1 1  
 [71,] 1 1 1 1 1 1  
 [72,] 1 1 1 1 1 1  
 [73,] 1 1 1 1 1 1  
 [74,] 1 1 1 1 1 1  
 [75,] 1 1 1 1 1 0  
 [76,] 0 0 0 0 0 0  
 [77,] 1 1 1 1 1 1  
 [78,] 1 1 1 1 1 1  
 [79,] 1 1 1 1 1 1  
 [80,] 1 1 1 1 1 1  
 [81,] 0 0 0 1 1 1  
 [82,] 0 0 0 1 1 1  
 [83,] 1 1 1 1 1 1  
 [84,] 1 1 1 1 1 1  
 [85,] 1 1 1 1 1 1  
 [86,] 1 1 0 1 1 1  
 [87,] 1 1 1 1 1 1  
 [88,] 1 1 1 1 1 1  
 [89,] 1 0 0 1 1 1  
 [90,] 0 0 0 1 1 1  
 [91,] 1 1 1 1 1 1  
 [92,] 1 1 1 1 1 1  
 [93,] 0 1 1 1 1 1  
 [94,] 1 1 0 0 1 1  
 [95,] 1 1 0 1 1 1  
 [96,] 1 1 1 0 1 1  
 [97,] 0 0 0 0 0 0  
 [98,] 1 1 1 1 1 1  
 [99,] 1 1 1 1 1 1  
[100,] 1 1 1 1 1 1  
[101,] 1 0 0 1 1 1  
[102,] 1 1 1 1 1 1  
[103,] 1 0 0 1 1 1  
[104,] 1 1 1 1 1 1  
[105,] 1 1 1 1 1 1  
[106,] 0 0 0 1 1 1  
[107,] 0 0 0 1 1 1  
[108,] 1 1 1 1 1 1  
[109,] 0 0 0 0 0 0  
[110,] 0 0 0 1 1 1  
[111,] 0 0 0 1 1 1  
[112,] 1 1 1 1 1 1  
[113,] 1 1 1 1 1 1  
[114,] 0 0 0 0 1 0  
[115,] 1 1 0 1 1 1  
[116,] 1 0 0 1 1 1  
[117,] 0 0 0 1 1 1  
[118,] 1 0 0 1 1 1  
[119,] 1 1 1 1 1 1  
[120,] 0 0 0 1 1 1  
[121,] 1 1 0 1 1 1  
[122,] 1 1 1 1 1 1  
[123,] 1 1 1 1 1 1  
[124,] 1 1 1 1 1 1  
[125,] 1 1 1 1 1 1  
[126,] 0 0 0 1 1 0  
[127,] 1 1 1 1 0 0  
[128,] 1 1 1 1 1 1  
[129,] 1 1 0 1 1 1  
[130,] 1 1 0 1 1 1  
[131,] 1 0 0 1 1 1  
[132,] 1 1 1 0 1 1  
[133,] 0 0 0 0 0 0  
[134,] 1 1 1 0 1 0  
[135,] 1 1 1 1 1 1  
[136,] 1 0 0 0 1 0  
[137,] 1 1 1 1 1 1  
[138,] 1 1 0 1 1 1  
[139,] 1 1 1 0 1 1  
[140,] 1 1 1 1 1 1  
[141,] 0 0 0 0 1 1  
[142,] 0 0 0 1 1 1  
[143,] 1 1 1 1 1 1  
[144,] 1 0 0 1 1 0  
[145,] 1 1 1 0 1 1  
[146,] 1 0 0 0 1 1  
[147,] 1 1 1 1 1 1  
[148,] 1 1 0 1 1 1  
[149,] 1 1 1 1 1 1  
[150,] 0 0 0 1 1 1  
[151,] 1 1 1 1 1 0  
[152,] 1 1 1 1 1 1  
[153,] 1 0 0 1 1 1  
[154,] 1 1 1 1 1 1  
[155,] 1 1 1 1 1 0  
[156,] 1 0 0 0 0 0  
[157,] 1 1 1 1 1 1  
[158,] 1 1 1 1 1 1  
[159,] 0 0 0 0 0 0  
[160,] 1 1 1 1 1 1  
[161,] 0 0 0 1 1 1  
[162,] 1 1 1 1 1 1  
[163,] 0 0 0 0 0 0  
[164,] 1 1 1 0 1 1  
[165,] 1 1 1 1 1 1  
[166,] 1 1 1 1 1 1  
[167,] 1 0 0 1 1 1  
[168,] 1 1 1 1 1 1  
[169,] 0 1 0 1 1 1  
[170,] 0 0 0 0 0 0  
[171,] 1 1 1 1 1 0  
[172,] 1 1 1 1 1 1  
[173,] 1 1 1 1 1 1  
[174,] 1 1 1 0 1 1  
[175,] 1 1 0 1 1 1  
[176,] 1 1 1 0 1 1  
[177,] 1 1 1 1 1 1  
[178,] 1 1 0 1 1 1  
[179,] 1 1 0 1 1 1  
[180,] 1 1 1 1 1 1  
[181,] 0 0 0 1 1 1  
[182,] 1 1 1 1 1 1  
[183,] 1 0 1 1 1 1  
[184,] 1 1 1 0 1 1  
[185,] 1 1 1 1 1 1  
[186,] 0 0 0 1 1 1  
[187,] 1 1 1 1 1 1  
[188,] 1 1 1 1 1 1  
[189,] 1 1 1 1 1 1  
[190,] 1 0 0 1 1 0  
[191,] 1 0 0 0 1 1  
[192,] 0 0 0 0 1 0  
[193,] 1 0 0 0 1 0  
[194,] 1 0 0 0 1 0  
[195,] 1 1 1 1 1 1  
[196,] 1 1 1 1 1 1  
[197,] 1 0 0 0 1 0  
[198,] 1 1 1 1 1 1  
[199,] 1 1 1 1 1 1  
[200,] 1 0 0 0 0 1  
[201,] 1 1 1 0 0 0  
[202,] 1 0 0 0 1 0  
[203,] 1 0 0 0 1 1  
[204,] 0 1 0 0 0 0  
  
attr(,"class")  
[1] "monotonicity.class"

## Model Fitness

### Global fit statistics

# ==========================================  
# 4. Assumption: Model Fit  
# ==========================================  
# (a) Global fit statistics (M2, RMSEA, SRMSR)  
M2(mod2pl) # RMSEA < 0.08 and SRMSR < 0.05 = good fit

M2 df p RMSEA RMSEA\_5 RMSEA\_95 SRMSR TLI CFI  
stats 1126.864 275 0 0.1235295 0.1157898 0.1307558 0.1057892 0.8124837 0.82811

### Item-level fit

# (b) Item-level fit (S-X² or G² statistics)  
itemfit\_stats <- itemfit(mod2pl)  
itemfit\_stats # significant misfit items should be reviewed

item S\_X2 df.S\_X2 RMSEA.S\_X2 p.S\_X2  
1 K1 9.452 16 0.000 0.894  
2 K4 18.807 17 0.023 0.340  
3 K6 12.789 10 0.037 0.236  
4 K12 17.862 14 0.037 0.213  
5 K13 11.180 14 0.000 0.672  
6 K14 13.678 17 0.000 0.690  
7 K15 5.421 12 0.000 0.942  
8 K16 9.055 13 0.000 0.769  
9 K17 8.427 11 0.000 0.675  
10 K18 8.762 14 0.000 0.846  
11 K20 6.552 11 0.000 0.834  
12 K21 8.049 10 0.000 0.624  
13 K23 6.677 7 0.000 0.463  
14 K25 4.886 5 0.000 0.430  
15 K26 19.137 16 0.031 0.262  
16 K27 4.849 9 0.000 0.847  
17 K28 12.006 14 0.000 0.606  
18 K29 20.161 15 0.041 0.166  
19 K30 6.880 6 0.027 0.332  
20 K31 10.799 6 0.063 0.095  
21 K32 7.283 11 0.000 0.776  
22 K33 8.482 12 0.000 0.746  
23 K34 15.131 14 0.020 0.369  
24 K36 1.367 3 0.000 0.713  
25 K37 11.850 12 0.000 0.458

### Compare 1PL vs 2PL with likelihood ratio test

# (c) Compare 1PL vs 2PL with likelihood ratio test  
# Fit 1PL and 2PL models  
mod1pl <- mirt(irt\_mat, 1, itemtype = "Rasch")

mod2pl <- mirt(irt\_mat, 1, itemtype = "2PL")

# Likelihood ratio test: does 2PL fit better than Rasch?  
anova(mod1pl, mod2pl)

AIC SABIC HQ BIC logLik X2 df p  
mod1pl 4894.093 4897.988 4928.991 4980.364 -2421.047   
mod2pl 4771.532 4779.023 4838.644 4937.438 -2335.766 170.561 24 0

# Fitting 2PL IRT Model with mirt Package

mirt.data4 = mirt(data4, 1, itemtype = "2PL")

coef(mirt.data4, IRTpars = T, simplify = T)

$items  
 a b g u  
K1 0.813 -1.835 0 1  
K4 0.601 0.860 0 1  
K6 0.983 1.966 0 1  
K12 0.729 1.748 0 1  
K13 1.346 -1.399 0 1  
K14 0.967 -0.827 0 1  
K15 1.608 0.361 0 1  
K16 1.433 0.466 0 1  
K17 1.701 -1.361 0 1  
K18 1.341 -0.420 0 1  
K20 2.638 -0.156 0 1  
K21 2.773 -0.224 0 1  
K23 4.204 -0.538 0 1  
K25 5.635 -0.658 0 1  
K26 1.116 -0.425 0 1  
K27 2.117 0.687 0 1  
K28 1.375 0.202 0 1  
K29 1.355 -0.240 0 1  
K30 3.017 0.995 0 1  
K31 4.321 -0.882 0 1  
K32 2.952 -0.581 0 1  
K33 2.297 -0.420 0 1  
K34 1.349 -1.427 0 1  
K36 2.821 -1.733 0 1  
K37 1.647 -1.442 0 1  
  
$means  
F1   
 0   
  
$cov  
 F1  
F1 1

# Fit 2PL Model (mirt)  
  
mirt.data4 <- mirt(data4, 1, itemtype = "2PL")

## Item Parameter Estimates (mirt)

# Obtain difficulty (b), discrimination (a), guessing (g), upper bound (u)  
mirt\_parms <- coef(mirt.data4, IRTpars = TRUE, simplify = TRUE)  
item\_parms\_refined\_mirt <- mirt\_parms$items  
  
  
# Tidy view: Item | Discrimination | Difficulty | Guessing Parameter | Upper Bound  
item\_parms\_refined\_tbl\_mirt <- item\_parms\_refined\_mirt |>  
 as.data.frame() |>  
 (\(d) {  
 if (!"g" %in% names(d)) d$g <- NA\_real\_  
 if (!"u" %in% names(d)) d$u <- NA\_real\_  
 d  
 })() |>  
 transform(  
 Item = rownames(item\_parms\_refined\_mirt),  
 Difficulty = b,  
 Discrimination = a,  
 `Guessing Parameter` = g,  
 `Upper Bound` = u  
 ) |>  
 (\(d) d[, c("Item", "Difficulty", "Discrimination", "Guessing Parameter", "Upper Bound")])() |>  
 (\(d) within(d, {  
 Difficulty <- round(Difficulty, 3)  
 Discrimination <- round(Discrimination, 3)  
 `Guessing Parameter`<- round(`Guessing Parameter`, 3)  
 `Upper Bound` <- round(`Upper Bound`, 3)  
 }))()  
  
item\_parms\_refined\_tbl\_mirt

Item Difficulty Discrimination Guessing Parameter Upper Bound  
K1 K1 -1.835 0.813 0 1  
K4 K4 0.860 0.601 0 1  
K6 K6 1.966 0.983 0 1  
K12 K12 1.748 0.729 0 1  
K13 K13 -1.399 1.346 0 1  
K14 K14 -0.827 0.967 0 1  
K15 K15 0.361 1.608 0 1  
K16 K16 0.466 1.433 0 1  
K17 K17 -1.361 1.701 0 1  
K18 K18 -0.420 1.341 0 1  
K20 K20 -0.156 2.638 0 1  
K21 K21 -0.224 2.773 0 1  
K23 K23 -0.538 4.204 0 1  
K25 K25 -0.658 5.635 0 1  
K26 K26 -0.425 1.116 0 1  
K27 K27 0.687 2.117 0 1  
K28 K28 0.202 1.375 0 1  
K29 K29 -0.240 1.355 0 1  
K30 K30 0.995 3.017 0 1  
K31 K31 -0.882 4.321 0 1  
K32 K32 -0.581 2.952 0 1  
K33 K33 -0.420 2.297 0 1  
K34 K34 -1.427 1.349 0 1  
K36 K36 -1.733 2.821 0 1  
K37 K37 -1.442 1.647 0 1

## Test Information

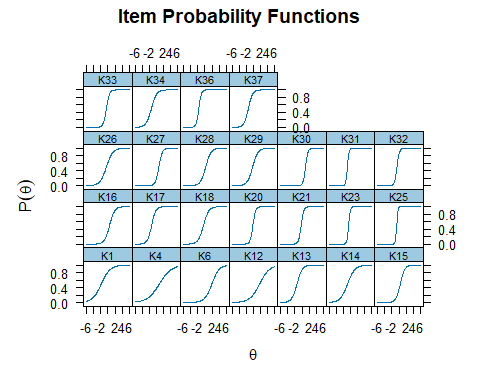
areainfo(mirt.data4, c(-3,3))

LowerBound UpperBound Info TotalInfo Proportion nitems  
 -3 3 49.15004 51.13713 0.9611419 25

## Graphical Presentation

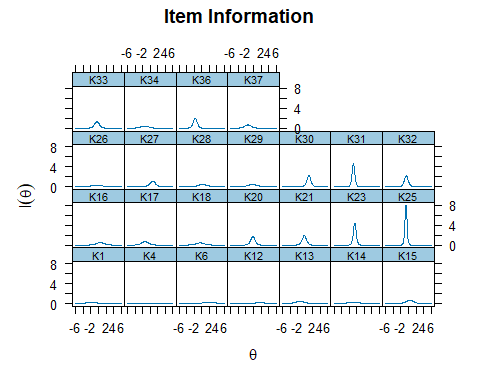
### Item Trace Lines (Item Characteristic Curves)

plot(mirt.data4, type = "trace")



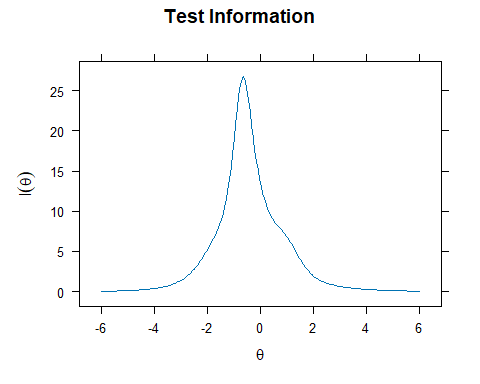
### Item Information Curves

plot(mirt.data4, type = "infotrace")



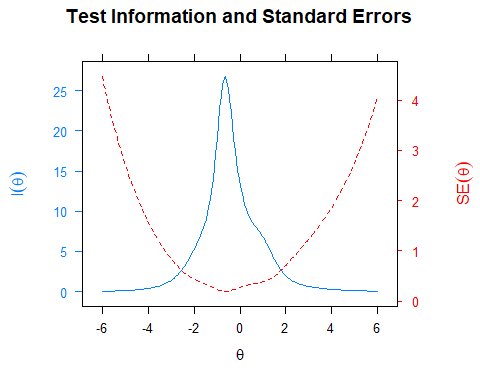
### Test Information Function

plot(mirt.data4, type = "info")



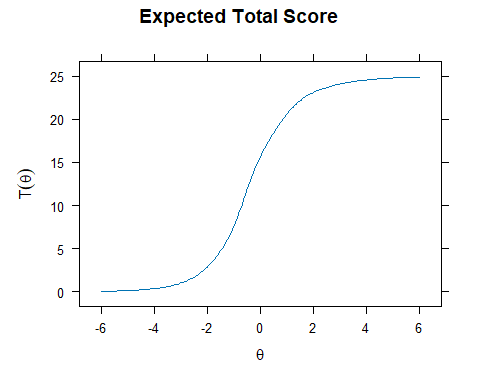
### Test Information and Standard Error

plot(mirt.data4, type = "infoSE")



### Expected Total Score

plot(mirt.data4)



## Goodness-of-Fit Tests

### Overall Model Fit

M2(mirt.data4)

M2 df p RMSEA RMSEA\_5 RMSEA\_95 SRMSR TLI CFI  
stats 1126.864 275 0 0.1235295 0.1157898 0.1307558 0.1057892 0.8124837 0.82811

### Item Fit Statistics

itemfit(mirt.data4)

item S\_X2 df.S\_X2 RMSEA.S\_X2 p.S\_X2  
1 K1 9.452 16 0.000 0.894  
2 K4 18.807 17 0.023 0.340  
3 K6 12.789 10 0.037 0.236  
4 K12 17.862 14 0.037 0.213  
5 K13 11.180 14 0.000 0.672  
6 K14 13.678 17 0.000 0.690  
7 K15 5.421 12 0.000 0.942  
8 K16 9.055 13 0.000 0.769  
9 K17 8.427 11 0.000 0.675  
10 K18 8.762 14 0.000 0.846  
11 K20 6.552 11 0.000 0.834  
12 K21 8.049 10 0.000 0.624  
13 K23 6.677 7 0.000 0.463  
14 K25 4.886 5 0.000 0.430  
15 K26 19.137 16 0.031 0.262  
16 K27 4.849 9 0.000 0.847  
17 K28 12.006 14 0.000 0.606  
18 K29 20.161 15 0.041 0.166  
19 K30 6.880 6 0.027 0.332  
20 K31 10.799 6 0.063 0.095  
21 K32 7.283 11 0.000 0.776  
22 K33 8.482 12 0.000 0.746  
23 K34 15.131 14 0.020 0.369  
24 K36 1.367 3 0.000 0.713  
25 K37 11.850 12 0.000 0.458

### Person Fit Statistics

personfit(mirt.data4)

outfit z.outfit infit z.infit Zh  
1 0.59614105 -0.603746628 0.7702708 -0.977665142 0.96712283  
2 0.59614105 -0.603746628 0.7702708 -0.977665142 0.96712283  
3 0.45136808 -0.336084151 0.7998695 -0.613800563 0.71046864  
4 0.60009153 -0.578518850 0.7861210 -0.893719017 0.91774192  
5 0.41084085 -0.846233290 0.5933688 -1.589553045 1.33843170  
6 0.50669480 -0.261071814 0.8077026 -0.587614205 0.63683041  
7 0.45245202 -0.356009379 0.6656376 -1.333418999 1.21547045  
8 0.13825733 -0.657183643 0.3935536 -1.550010559 1.16020309  
9 0.93408060 0.338010596 0.9643497 -0.038638061 0.01807830  
10 0.89544562 0.100836098 1.1368285 0.606225120 -0.32040132  
11 0.45223067 -1.233962298 0.5704466 -2.321223985 1.89881552  
12 1.00693501 0.288698783 1.0878506 0.416939480 -0.33495913  
13 0.50612247 -0.114563698 0.8107430 -0.624143283 0.76286489  
14 0.41371736 -0.417480421 0.6420635 -1.311649064 1.12720817  
15 1.31212625 0.616435925 1.4194235 1.487156520 -1.41214075  
16 0.50612247 -0.114563698 0.8107430 -0.624143283 0.76286489  
17 0.69319329 -0.531754219 0.8072190 -0.882711619 0.87718299  
18 4.90048789 2.479085173 0.8358306 -0.420416673 -0.52977921  
19 0.79334747 -0.268672522 0.8729482 -0.559783233 0.58058275  
20 0.46112208 -0.370819473 0.8076272 -0.539354284 0.60666612  
21 0.56024101 -0.771208401 0.7257673 -1.235853933 1.16159398  
22 1.39626675 0.694171895 1.1737297 0.717963869 -0.86104129  
23 0.54837619 -0.180909343 0.8375127 -0.550795490 0.72103006  
24 1.29046961 0.605763813 1.2443323 0.949875836 -0.93826625  
25 0.53104996 -0.951840009 0.6802636 -1.538630915 1.38191585  
26 0.89905694 -0.046264450 0.8875034 -0.466722143 0.42377901  
27 1.28603243 0.658575901 1.3912377 1.745138418 -1.65314719  
28 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
29 0.23221379 -0.864794769 0.4682436 -1.940905347 1.43774773  
30 1.06754010 0.330501861 1.2809147 1.121775038 -0.88066911  
31 0.39985901 -0.698179652 0.5805184 -1.855895871 1.54308153  
32 0.65794228 -0.457075340 0.9198605 -0.268701473 0.52120984  
33 1.14145169 0.429601604 1.0433430 0.249955247 -0.29719626  
34 0.49062539 -0.290819322 0.7944211 -0.659681484 0.73896640  
35 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
36 1.36744756 0.663183851 1.2156771 0.745456385 -0.78223981  
37 0.50612247 -0.114563698 0.8107430 -0.624143283 0.76286489  
38 0.42446813 -0.055682029 0.7207832 -0.476062024 0.48728121  
39 0.32828911 -0.600194075 0.5828594 -1.579838127 1.33259580  
40 0.86516875 0.139008222 0.9853554 0.030777885 0.08217227  
41 0.82234827 -0.191091395 0.9072888 -0.347213116 0.39857763  
42 0.31215388 -0.439552454 0.5087490 -2.030397271 1.59549167  
43 0.58604171 -0.049082929 0.9088235 -0.253269898 0.50412001  
44 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
45 1.07447877 0.350265217 0.9632295 -0.059523192 -0.04480574  
46 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
47 0.55598364 -0.101185426 1.1442536 0.484648521 -0.13421696  
48 1.49608802 0.793775018 1.4171096 1.519696446 -1.57093038  
49 0.17722851 -0.659377595 0.4738308 -1.348710725 1.11145883  
50 0.27759148 -0.441467045 0.6034044 -1.334166990 1.11901083  
51 0.37840908 -0.481289815 0.6160266 -1.433378619 1.22280248  
52 0.14017869 -0.547967346 0.5051810 -1.025741704 0.90182463  
53 0.23221379 -0.864794769 0.4682436 -1.940905347 1.43774773  
54 0.57704630 -0.830834493 0.6708546 -1.605752253 1.36151822  
55 0.65420750 -0.633762410 0.7253984 -1.321443581 1.15065030  
56 1.25881534 0.572440813 1.5477353 1.696475004 -1.57710171  
57 0.78664598 0.208713772 1.1460821 0.591637032 -0.26470040  
58 1.13591472 0.425791500 1.2369522 0.955413327 -0.83412729  
59 0.68821127 -0.493993521 0.7844961 -1.047578044 0.98732035  
60 0.72977140 0.148006998 0.9541223 -0.076162284 0.21730560  
61 1.13805966 0.421115539 1.1495993 0.685027508 -0.63262083  
62 0.17722851 -0.659377595 0.4738308 -1.348710725 1.11145883  
63 0.56731908 -0.827647631 0.7119469 -1.346934987 1.23242670  
64 0.40532019 -0.245334017 0.6049425 -1.576571321 1.34024308  
65 2.02762070 1.311023015 0.9664524 -0.043288512 -0.40715819  
66 0.95300885 0.112999630 1.1216654 0.625926604 -0.38838875  
67 0.27659362 -0.719020224 0.6029731 -1.228656589 1.09273189  
68 0.13825733 -0.657183643 0.3935536 -1.550010559 1.16020309  
69 0.40589954 -0.249810167 0.8491713 -0.410338271 0.60832787  
70 1.20080282 0.526533673 1.3551421 1.196287836 -1.15926121  
71 0.73843904 -0.008566279 1.0032889 0.100094890 0.14481860  
72 0.19806267 -0.414549658 0.6530645 -0.654587994 0.76135031  
73 0.79154149 0.223201463 0.9820657 0.104826400 -0.06819426  
74 0.26043906 -0.730125158 0.4685601 -2.134548517 1.57543054  
75 1.12279976 0.489354328 0.8819245 -0.356330596 0.17121217  
76 0.86249230 0.188236030 1.3157515 1.062042146 -0.69909980  
77 0.63988259 -0.117585713 0.9062866 -0.277364674 0.46619310  
78 1.16472097 0.486553490 1.6336971 1.783179099 -1.62593072  
79 0.54837619 -0.180909343 0.8375127 -0.550795490 0.72103006  
80 0.44995106 -0.321150669 0.6878540 -1.046516239 0.88892597  
81 0.99262234 0.226912555 0.8075281 -0.628906028 0.43373203  
82 0.65699899 -0.250213115 0.7908724 -0.799834715 0.81497551  
83 0.59149904 -0.547366294 0.7387494 -1.249493329 1.21015641  
84 1.49396750 1.025161197 1.4300850 1.788864398 -1.86532223  
85 1.05584432 0.402835788 1.2166000 0.796148835 -0.71667147  
86 2.44674364 1.285253018 0.8892659 -0.157440064 -0.38225787  
87 0.38105096 -0.523289888 0.7763570 -0.624280843 0.72687518  
88 0.43516558 -0.210140921 0.6664537 -1.256220403 1.14969135  
89 0.77792431 -0.278220186 1.0067822 0.105730561 0.16866120  
90 1.07826536 0.362128925 1.0189990 0.158751747 -0.19700119  
91 0.54270752 -0.061074374 0.8193422 -0.603696557 0.72533585  
92 1.73962734 0.944882456 1.3558105 1.049594538 -1.26522860  
93 0.85840585 -0.093533564 0.9907940 0.024990419 0.13129314  
94 0.70689647 -0.494790242 0.8513044 -0.656865281 0.73794421  
95 1.02998366 0.366834948 1.2318092 0.835988388 -0.70276096  
96 0.87400255 0.231728153 0.9446695 -0.118572966 0.13831144  
97 0.08151351 -0.730154353 0.2122651 -1.828659412 1.19189515  
98 0.71384813 -0.212726034 0.8208509 -0.687595042 0.65341876  
99 1.17972436 0.478904706 1.0084841 0.118178199 -0.26686588  
100 0.51525046 -1.023344740 0.6699050 -1.620497844 1.45968717  
101 0.35011407 -0.862453574 0.4604086 -2.635607794 2.01539926  
102 1.96252189 1.038984050 1.0561703 0.292499864 -0.60399361  
103 1.70758295 1.332517693 1.4058807 1.770850333 -2.03184717  
104 0.94625026 0.137090093 1.0548922 0.302566299 -0.14555507  
105 0.50176155 -0.211453802 1.0256731 0.193064183 0.10769308  
106 0.89361673 0.084677540 1.0527204 0.276351761 -0.11628089  
107 0.33545692 -1.014991248 0.5491983 -1.785033992 1.46392097  
108 0.54837619 -0.180909343 0.8375127 -0.550795490 0.72103006  
109 0.39602740 -0.092115104 0.7752084 -0.375118929 0.47537795  
110 0.61386026 -0.554498912 0.7510454 -1.215962754 1.17012098  
111 0.66428733 -0.286588689 0.8204011 -0.619814703 0.67625144  
112 1.52416683 0.776735325 1.8430983 2.477492054 -2.57179839  
113 0.83952808 -0.156991181 0.9441137 -0.201916299 0.29722636  
114 0.34125599 -0.567478631 0.6656158 -1.174872580 1.06037552  
115 0.61958947 -0.713756606 0.7548255 -1.193218170 1.14775497  
116 0.51110965 -0.532282168 0.6808093 -1.317541896 1.24351546  
117 1.00216548 0.261051794 0.9524181 -0.118924589 0.06588061  
118 0.76819093 -0.056484100 0.8464490 -0.581595056 0.60112224  
119 0.35453318 -0.608851129 0.5243537 -2.107950016 1.66748271  
120 0.94452085 0.073424181 1.0011838 0.075073291 -0.01585658  
121 0.56631004 -0.028018621 0.7262381 -0.992467642 0.88855337  
122 0.47557819 -0.136399195 0.7490230 -0.896957555 0.95023445  
123 0.81462906 0.147200318 1.1840681 0.678259254 -0.40986194  
124 0.27659362 -0.719020224 0.6029731 -1.228656589 1.09273189  
125 0.55636200 -0.275919398 0.7901162 -0.774846057 0.83606824  
126 0.78021337 -0.078986201 0.9549352 -0.061474426 0.17146163  
127 2.97424541 2.813078076 1.1810764 0.852899594 -1.54870613  
128 0.38148268 -0.488631715 0.6943008 -1.053445975 1.02173333  
129 1.83227479 0.962687275 1.4500083 1.485910588 -1.75817734  
130 1.92251259 1.070516874 1.1268146 0.471716391 -0.94037660  
131 0.48553267 -0.606500744 0.6121599 -1.822058315 1.57107671  
132 0.96874883 0.102643618 1.0915542 0.476624628 -0.30323811  
133 0.49457679 0.027658979 0.9446519 -0.008048544 0.20232300  
134 1.45987042 0.859844101 1.5546402 2.283973741 -2.32569176  
135 0.31160855 -0.614146637 0.5347839 -1.836278379 1.46803118  
136 0.77690952 -0.090231978 1.0821392 0.387349957 -0.03674957  
137 0.64233332 -0.318192755 0.8121581 -0.776721758 0.86176550  
138 0.47032316 -0.835327493 0.6492714 -1.586883024 1.39563913  
139 1.14214301 0.431445935 1.1685605 0.771557191 -0.69553300  
140 0.40532019 -0.245334017 0.6049425 -1.576571321 1.34024308  
141 0.34484467 -0.832257151 0.6198103 -1.409593243 1.22802232  
142 0.75626682 -0.138194144 0.9363779 -0.133634448 0.26196982  
143 1.37716626 0.674399157 1.5759327 1.808232509 -1.83662173  
144 0.61500854 -0.637627068 0.7146229 -1.457831026 1.29997871  
145 0.76941160 -0.341203457 0.8710473 -0.543816216 0.57872455  
146 1.04517381 0.258160413 1.2383445 1.126509449 -0.85630163  
147 0.13007739 -0.575099342 0.4326743 -1.292912972 1.01974669  
148 0.65611024 -0.574546729 0.7759891 -1.095448332 1.05354537  
149 0.67139191 0.014070666 0.8620772 -0.406292733 0.42616342  
150 0.33545692 -1.014991248 0.5491983 -1.785033992 1.46392097  
151 1.34129246 0.648068373 1.1229757 0.525236020 -0.62868436  
152 0.13825733 -0.657183643 0.3935536 -1.550010559 1.16020309  
153 0.38974690 -0.780519638 0.5053974 -2.434322594 1.93343981  
154 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
155 1.68699847 0.987344004 1.4159435 1.534642302 -1.74200529  
156 1.64689799 0.943026288 1.4993264 1.839630755 -2.04045415  
157 0.50612247 -0.114563698 0.8107430 -0.624143283 0.76286489  
158 1.26348245 0.576457349 1.3380125 1.137672440 -1.15272666  
159 0.42446813 -0.055682029 0.7207832 -0.476062024 0.48728121  
160 0.63817062 -0.177357677 0.8465256 -0.534570546 0.62339061  
161 0.36850125 -0.660068434 0.6960463 -1.057740744 1.02115498  
162 0.40206946 -0.467823378 0.7972392 -0.524666670 0.63356194  
163 2.42474036 1.254543854 1.3287086 1.044400660 -1.56168588  
164 1.23222061 0.545695983 1.3961121 1.418540694 -1.28612103  
165 0.47265287 -0.138044226 0.7206724 -1.023271568 1.01109828  
166 1.07209751 0.451024436 1.2884086 1.052016427 -0.88910353  
167 0.51835602 -0.528930686 0.7236478 -1.095230495 1.12430019  
168 0.32828911 -0.600194075 0.5828594 -1.579838127 1.33259580  
169 0.92855974 0.017491102 0.8501854 -0.649288344 0.50002957  
170 2.30638822 1.152832102 1.0121816 0.163103721 -0.58122348  
171 0.91524319 -0.008483986 0.9450083 -0.191282127 0.19178995  
172 0.78197625 -0.126317294 0.9367580 -0.182410137 0.30401133  
173 0.78954701 -0.294104244 0.9215397 -0.297899372 0.42390390  
174 0.60288866 -0.774320472 0.7106207 -1.403111619 1.25057270  
175 0.84791418 0.122930436 1.0379629 0.230455114 -0.07081936  
176 0.96960489 0.275406080 1.1194002 0.524363527 -0.36270289  
177 0.51434823 -0.866013027 0.7070806 -1.325050145 1.25856635  
178 0.82653593 -0.193778255 0.8969341 -0.402315224 0.42474962  
179 0.87857090 0.022707406 0.8972636 -0.357735728 0.33156397  
180 0.78139298 -0.016283339 1.0214000 0.168342763 0.05964628  
181 0.47528052 -0.577061956 0.7272278 -0.935617468 0.89179527  
182 0.62975286 -0.513407627 0.7694574 -1.109455519 1.09444417  
183 0.87611466 -0.087202530 1.0003893 0.072315959 0.08494985  
184 1.90771452 1.001589314 0.8249802 -0.322161783 -0.17680216  
185 0.13825733 -0.657183643 0.3935536 -1.550010559 1.16020309  
186 0.31219709 -1.125631485 0.4713527 -2.261244669 1.72682334  
187 0.83365694 0.159661264 0.9326336 -0.137532624 0.11523598  
188 0.74063322 0.078195727 1.2344023 0.820453680 -0.45262862  
189 0.13825733 -0.657183643 0.3935536 -1.550010559 1.16020309  
190 2.53894382 1.362800501 2.0405365 3.056324412 -3.91619726  
191 2.24858876 1.360866253 1.9980942 3.083324339 -3.76036308  
192 1.14552179 0.436138663 1.4208925 1.634784891 -1.36955214  
193 1.25594894 0.609781718 1.3601134 1.622528765 -1.51636216  
194 0.98207486 0.164727745 1.1242999 0.636919501 -0.43833195  
195 0.80786742 -0.185787392 0.8978811 -0.438330791 0.49241211  
196 0.45252258 -0.344958571 0.7630470 -0.788870352 0.85389010  
197 0.91149965 0.005301352 1.0263131 0.194652823 -0.04684697  
198 3.16914016 2.265039501 1.7062412 2.455324724 -3.19867813  
199 0.10532225 -0.648134306 0.4273664 -1.196862603 0.91071329  
200 3.34831284 2.898149338 2.2259721 4.475206064 -6.21541811  
201 2.91423107 2.148027429 2.0675219 3.750670765 -4.98008843  
202 0.95806109 0.081516524 1.0195972 0.161748651 -0.07198736  
203 0.90860389 -0.022313445 1.0779324 0.417298117 -0.18235114  
204 2.11361390 1.104721398 1.2185741 0.747068812 -1.19606613

## Reliability Estimates

### Marginal Reliability

marginal\_rxx(mirt.data4)

[1] 0.8979498

### Empirical Reliability

theta\_se = fscores(mirt.data4, full.scores.SE = TRUE)  
empirical\_rxx(theta\_se)

F1   
0.9025618