Lab 5: RC, RL, and RLC circuits: Frequency Domain Response

Reference Reading: Chapter 3, Sections 3.2 - 3.7.

Time: Two lab periods will be devoted to this lab.

Goals:

1 Gain familiarity with the AC frequency response of simple *RC*, *RL*, and *RLC* circuits. In particular, with the amplitude and phase response, both in terms of measuring and calculating these responses.
2 Understand the meaning of a "characteristic frequency" for these three types of circuits.
3 Understand the meaning of the terms "low-pass filter" and "high-pass filter" and be able to identify them in a circuit.
4 Understand the requirements for coupling circuit units together in a modular fashion.

Expectations:

1. You will measure the frequency response of five filters in this lab which means that you should have a Bode plot and phase-difference plot for each of these measurements.
2. All plots should be generated in a program like Excel or your Jupyter Notebook (JN). If you use Excel to generate these files, they need to be included in a JN as images for the report. As with all plots, make sure that your data points are clearly visible on these plots, make sure that the axes are clearly labeled and that all axes have units.
3. You have excellent theoretical predictions for each of these filters. You are expected to overlay calculated expectations on top of your measured data. If the two do not agree, you need to provide some discussion in your lab report.
4. Following the directions in Appendix C of the lab manual on fitting curves in Excel, you should fit your gain, |G|, data for the high-pass and low-pass filters to determine the best characteristic frequency and then compare the fit values with your expected values.

5.1 Introduction

We now re-examine the circuits of Lab 4 in a different way. Here, we apply sinusoidal waves rather than step inputs and measure the amplitude and phase (relative to the source) of the output signal. This lab is an excellent place to compare theory to your measurements and such comparisons are expected in your lab book. Your measurements should be compared to quantitative calculations of the expected behavior of the circuits and the results plotted on top of your data.

5.1.1 The Generic Filter

The *RC* and *RL* circuits in this lab can be modelled as AC voltage dividers. To understand this, we consider the very general voltage divider network shown in Figure 1. The components with impedances Z_1 and Z_2 can be any combination of elements, and we can model the behavior

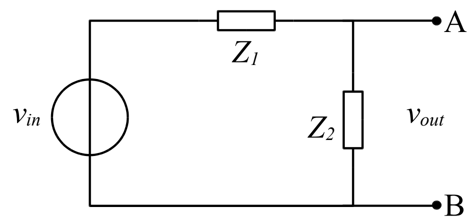

Figure 1 A generalized voltage divider constructed of two components with impedances of Z_1 and Z_2. The output is between the terminals A and B.

of this circuit as discussed in chapter 3 of our textbook. For a generic input voltage, v_{in}, the output voltage, v_{out}, is given by our voltage-divider equation.

$$v_{out} = \frac{Z_2}{Z_1 + Z_2} v_{in}$$

Since the impedances Z_1 and Z_2 may depend on the input frequency of our signal, it is helpful to write that explicitly. We can view the circuit as a "filter", which could amplify or attenuate an input signal, depending on its frequency. Thus, the gain of our filter is given as

$$G(\omega) = \frac{Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)}$$

Equation 1

We also recall that the output impedance, or Thèvenin impedance of a divider is given as

$$Z_{out}(\omega) = \frac{Z_1(\omega) Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)},$$

Equation 2

and the input impedance of our unloaded circuit is just

$$Z_{in}(\omega) = Z_1(\omega) + Z_2(\omega).$$

Equation 3.

As noted in chapter 3 of our textbook, these three quantities characterize the behavior of our filter and allow us to view the circuit in terms of the equivalent circuit shown in Figure 5.2.

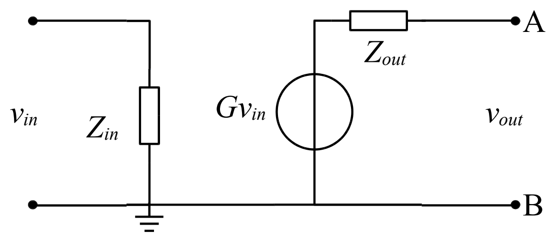

Figure 2 The equivalent circuit for our filter in Figure 1, shown explicitly in terms of the three characteristic properties of a filter, the gain, G, the input impedance, Z_{in}, and the output impedance, Z_{out}.

We now write the input, $v_{in}(t)$, and output, $v_{out}(t)$, explicitly as sinusoidal signals

$$v_{in}(t) \; = \; V_{in} \, e^{j(\omega t+\varphi_{in})} \, ,$$

$$v_{out}(t) \; = \; V_{out} \, e^{j(\omega t+\varphi_{out})} \, .$$

The quantities V_{in} and V_{out} are the amplitudes of the signals, while the angular frequency ω is given as $2\pi f$. The two phases, φ_{in} and φ_{out}, essentially set the value of the signals at time $t = 0$. The magnitude of the gain of our circuit is then

$$|G| = \frac{V_{out}}{V_{in}} ,$$

and our measured phase difference is

$$\Delta\varphi \; = \; \varphi_{out} - \varphi_{in} \, .$$

This allows us to write the complex gain as

$$G \; = \; |G|e^{j\Delta\varphi}.$$

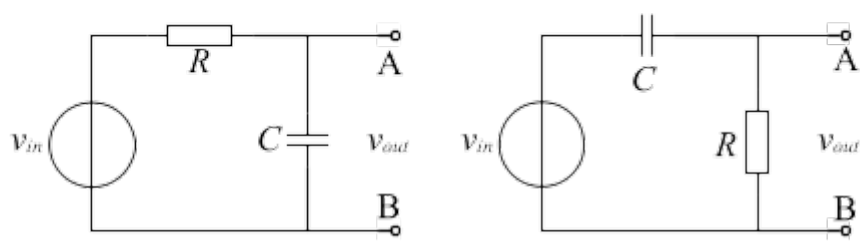

Figure 3 The two RC configurations we will consider in this lab. The left-hand circuit is the low-pass filter, while the right-hand circuit is the high-pass filter. We will see that the low-pass filter is also known as an integrating circuit, while the high-pass is known as a differentiating circuit.

5.1.2 RC Filters

We can now apply what we saw in Section 5.1.1 to the specific case of *RC* filters. For us, we will consider the two simple filters shown in Figure 3. To analyze these, we simply identify Z_1 in Figure 1 with the resistor or the capacitor, and Z_2 with the opposite component. We also recall that, as we found in our textbook, the characteristic frequency of these circuits is given as

$$\omega_{RC} = \frac{1}{RC}.$$

Equation 4

In our textbook, we saw that the left-hand circuit is known as a low-pass filter, while the right-hand circuit is known as the high-pass filter. When applying Equation 1 to these, we find that the gain for the low-pass and the high-pass filters are given as

$$Z_1 = R; \qquad Z_2(\omega) = \frac{1}{j\omega C},$$

$$G_{lp} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}},$$

with a little algebra this can be written as

$$G_{lp} = \frac{1}{1 + j\frac{\omega}{\omega_{RC}}}.$$

Equation 5

In the same manner we can identify the gain for a high pass filter.

$$Z_1(\omega) = \frac{1}{j\omega C}; \qquad Z_2(\omega) = R,$$

$$G_{lp} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{\frac{1}{j\omega C} + R},$$

$$G_{hp} = \frac{1}{1 - j\frac{\omega_{RC}}{\omega}}$$

Equation 6

We also found that the two filters have the same input impedances and have the same output impedances. We found these in the textbook to be

$$Z_{in} \; = \; Z_1(\omega) + Z_2(\omega) = R\;(1 - \frac{j\;\omega_{RC}}{\omega})$$

Equation 7

$$Z_{out} \; = \frac{Z_1(\omega)Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)} = \frac{R}{1 + j\dfrac{\omega}{\omega_{RC}}}.$$

Equation 8

For frequencies small compared to ω_{RC}, the gain of the low-pass filter is

$$G_{lp}(\omega \; \ll \; \omega_{RC}) \; = \; 1$$

Equation 9

and for frequencies large compared to ω_{RC} the gain of the high-pass filter is

$$G_{hp}(\omega \gg \omega_{RC}) \; = \; 1.$$

Equation 10

As discussed in our text, in the region where the frequency is much larger than the characteristic frequency, the gain of the low-pass filter becomes

$$G_{lp}(\omega \; \gg \; \omega_{RC}) = \; -j\frac{\omega_{RC}}{\omega},$$

Equation 11

which we showed yields a circuit that integrates the input voltage. Similarly, for the case of frequencies small compared to the characteristic frequency, the gain of the high-pass filter is

$$G_{hp}(\omega \; \ll \; \omega_{RC}) = \; j\frac{\omega}{\omega_{RC}}$$

Equation 12

This yields a circuit that differentiates the input voltage.

5.1.3 RL Filters

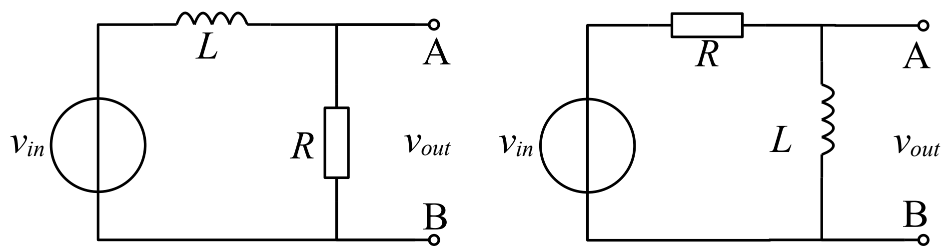

Figure 4 The two RL configurations we will consider in this lab. The left-hand circuit is the low-pass filter, while the right-hand circuit is the high-pass filter.

Similar to the *RC* filter from the previous section, we can also build high-pass and low-pass filters using resistors and inductors. These circuits are shown in Figure 4, where the left-hand circuit is a low-pass filter, and the right-hand circuit is a high-pass filter. Before continuing, we need to point out that for most "physical" inductors, there is an internal resistance, R_L, that may well be of similar size as the explicit R in our circuit. Thus, in analyzing these circuits, we need to be sure to consider that

$$Z_L = R_L + j\omega L.$$

Accounting for the R_L of the inductor, we define a characteristic frequency for our R_L circuit as

$$\omega_{RL} = \frac{R + R_L}{L}.$$

Equation 13

Using this, we can express the gain of these filters in terms of R, R_L and ω_{LR}. For the low-pass configuration in Figure 4, it is easy to show that the gain is

$$G_{lp} = \left(\frac{R}{R + R_L}\right)\frac{1}{1 + j\frac{\omega}{\omega_{RL}}}.$$

Equation 14

For the high-pass filter, the gain is

$$G_{hp} = \frac{\left(\frac{R_L}{R + R_L}\right) + j\frac{\omega}{\omega_{RL}}}{1 + j\frac{\omega}{\omega_{RL}}} = .$$

Equation 15

5.1.4 RLC Filters

The voltage-divider result can also be applied to *RLC* circuits, with Z_1 being replaced by R and Z_L combined in the appropriate way, and Z_2 being the capacitor impedance, Z_C. Such a circuit is shown in Figure 5. The resulting behavior is more complex than the *RC* and *RL* circuits, as the *RLC* circuit exhibits resonant behavior at a characteristic frequency,

$$\omega_{LC} = \frac{1}{\sqrt{LC}}.$$

Equation 16

These circuits are discussed in detail in the textbook (Section 3.6). Here, we only give the gain as measured across the capacitor

$$G_C(\omega) = \frac{1}{\left(1 - \frac{\omega}{\omega_{LC}}\right)^2 + j\frac{\omega}{\omega_{RC}}},$$

Equation 17

where $\omega_{RC} = \frac{1}{RC}$, as defined above.

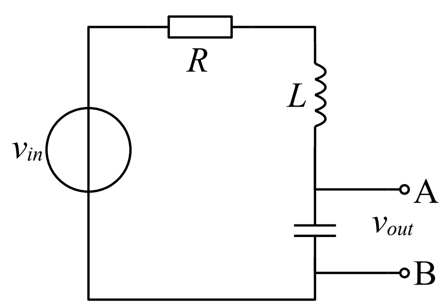

Figure 5 The RLC circuit configured to measure the voltage across the capacitor.

5.1.5 Band-pass Filters

As discussed in Sections 3.7 and 3.8 of the textbook, there are many occasions when we need to couple one functional block of circuitry to another. In fact, it's hard to think of a situation where this is not necessary! In the case of a high-pass filter connected to a low-pass filter, we create a band-pass filter. Such a circuit will attenuate the input signal both above and below some characteristic frequency. The new feature is that we need to worry about the *input impedance* of the second filter relative to the *output impedance* of the first filter. We show the equivalent circuit for this in Figure 6. In order for the overall gain of the combined circuit to be the product of the two individual gains, we must have that

$$|(Z_{in})_2| \gg |(Z_{out})_1|.$$

See your textbook for a more detailed discussion of this circuit.

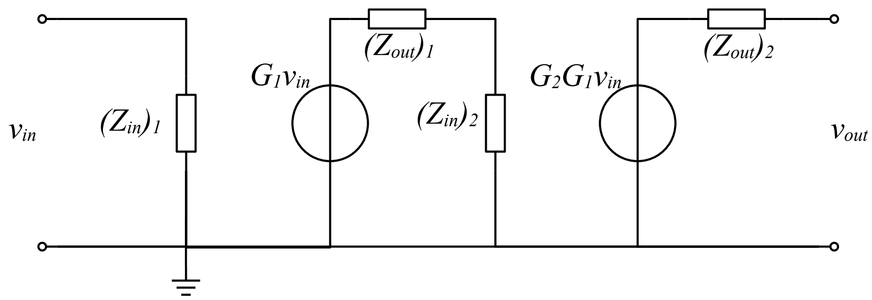

Figure 6: The equivalent circuit for the output of one filter used as the input to a second filter.

5.2 Preliminary Lab Questions

The work in this section must be completed and signed off by an instructor before you start working on the lab. Do this work in your lab book.

1. You are given the following power series expansions:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\sin(\mathrm{x}) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Show that $e^{jx} = cos(x) + j\,sin(x)$, where $j = \sqrt{-1}$.

2. Consider a voltage source, $v(t) = V_0\,cos(\omega t)$, where the frequency, *f*, varies from 10 Hz up to 100 Hz. Plot the numerical magnitude of the impedance, |*Z*|, as a function of frequency for each of the following components: a 100 Ω resistor, a 1.0 μF capacitor, and a 1 mH inductor.

3. A voltage source of $v(t) = (5.0V)cos(\omega t)$ is separately applied to each of the three components given in problem 2. The frequency *f* is 100Hz. Sketch the voltage across each component as a function of time over one cycle of the wave. Sketch the current as a function of time through each component over one cycle of the wave. Show the maximum value of the current for each component on your plot.

4. Show that the gain for the low pass *RL* circuit is given by Equation 14 above. Be sure to explain what you are doing, do not simply write out the math.

5. Show that the gain for the high pass *RL* circuit is given by Equation 15 above. Be sure to explain what you are doing, do not simply write out the math.

5.3 Equipment and Parts

In this lab, we will utilize the following equipment. This equipment is located at your lab station.

1. A Rigol DS1054Z digital oscilloscope.
2. Two probes for the oscilloscope.
3. One USB memory stick (optional).
4. A Rigol DG1022Z signal generator.
5. One BNC-to-alligator-clip cable.
6. An Amprobe 37XR-A digital meter.
7. A Jameco proto-board.

You will also need the following components in order to carry out this lab. It makes more sense to get them as you need them, rather than all at once before the start of the lab.

1. One each of the following resistors: 2.2 kΩ and 22 kΩ.
2. One 0.008 μF capacitor.
3. One 500 mH inductor.
4. Additional resistors and capacitors you choose to match your circuit design.

5.4 Procedure

Reminder: At the beginning of each section below, enter into your lab notebook a summary of what you are setting out to do and what the relevant equations are expected to be. Derivations and great lengths of verbiage are not necessary, but some orienting explanation is. This should be standard practice in any lab notebook!

5.4.1 Gotcha!

1. Is the DG1022Z set to be “High-Z”?
2. Is the voltage offset of the DG1022Z set to 0 V?
3. Do you want to use AC or DC coupling on your oscilloscope? If you are interested in seeing a DC offset, then you need to DC couple. If you want to only see the time-varying part of a signal with a large DC offset, then you want to AC couple.
4. Recall from Lab 2 that at very-low and very-high frequencies, AC coupling may attenuate your signals.
5. Are all of your grounds connected to the same point? Are you grounding your circuit in the wrong place?

5.4.2 Data Collection

We recall from Lab 2 what we need to measure to be able to make Bode and phase plots for our filters. This is shown in Table 1. We note that the units listed in the table may not be the best choice—milliseconds might be better than seconds. We also recall from Lab 2 that the phase difference, $\Delta\varphi$, that the scope will measure the time difference (Δt) between when the output signal rises and when the input signal rises. Make sure you understand how to do this. Using Δt and the frequency, f, of the signal, the phase difference, $\Delta\varphi$, is

$$\Delta\varphi = (2\pi) f \Delta t.$$

As noted previously, the convention is that if v_{out} is earlier than v_{in}, then Δt, and the corresponding $\Delta\varphi$ are positive, v_{out} "leads" v_{in}. Use this convention and be sure to record when Δt and $\Delta\varphi$ are negative.

Remember that it is important to measure both v_{in} and v_{out} using your oscilloscope, and we need to choose a consistent form for v. It can be RMS, amplitude, or peak-to-peak. However, whatever we choose needs to be consistent throughout our measurements. Finally, in using your scope, it is best to put the larger signal on the channel that you use for the trigger. In this case, whichever channel is measuring v_{in} will be the channel to trigger the scope from.

Table 1: The data needed to make Bode and phase-shift plots of a circuit.

Measured Quantities				Computed Quantities		
Frequency f (Hz)	Input Voltage v_{in} (V)	Output Volteage v_{out} (V)	Time Shift Δt (s)	Gain $\lvert v_{out}/v_{in}\rvert$	Attenuation $20\ dB \log\lvert G\rvert$	Phase Shift $\Delta\phi$ (rad)

5.4.3 Frequency response of the RC voltage divider

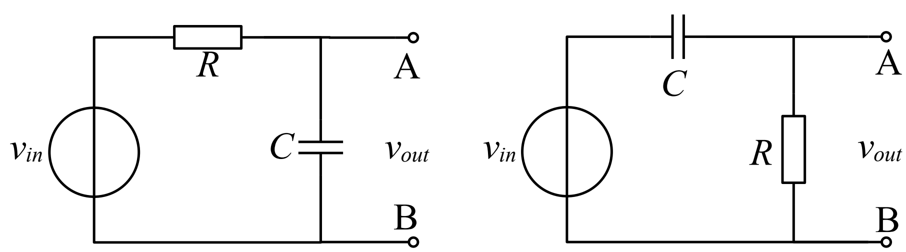

Figure 7: The low-pass (left) and high-pass (right) filter configurations of the RC circuit to be studied in this lab.

1 Set up the RC circuit using $C = 0.008\mu F$ and $R = 22k\Omega$ and a sine wave of reasonable amplitude (say, 5 V). Calculate the expected characteristic frequency in radians per second and in cycles per second (Hz).

2 For both the high-pass (differentiator) and low-pass (integrator) configurations, make careful measurements of v_{in} and v_{out} over a frequency range that extends at least two decades below and above the calculated characteristic frequency. Recall that your lab equipment reads frequency, f, in Hertz, while we often work in angular frequency, ω ($\omega = 2\pi f$). Be careful about factors of 2π. As you take the data, plot the "gain", $|G(f)| = |v_{out}|/|v_{in}|$, on a Bode plot and the phase shift between v_{out} and v_{in} using semi-log scales. [Recall that a Bode plot is $20dB\times\log_{10}(|G|)$ versus $\log_{10}(f)$.]

3 Set up the scope to display the input and output signals at the same time. The scope can be set to measure the amplitudes of each channel.
4 Set the scope to measure, Δt, between the rise of input and output signals. If the output signal peaks before the input signal, treat the time as positive. In the case where the output peaks after the input, treat the time as negative.
5 Using your measured time difference, Δt, and the known frequency, *f*, of the signal, calculate the phase difference, $\phi_{out} - \phi_{in}$.
6 In one configuration, v_{out} is the voltage across *R*; in the other, v_{out} is the voltage across *C*. You must determine the necessary wiring for each case. Be careful with this.
7 Choose your frequency steps so that your measurements will be roughly equally spaced on a logarithmic frequency axis. Three points per decade is suggested.

Question 5.1 *Should you be using the scope's AC- or DC-coupling input mode for this measurement?*

8 Determine the slope of the Bode plot (dB per decade) in the high- or low-frequency limit (wherever *G*(*f*) is varying). Make a plot of your data, together with the expected theoretical function going through (or near) the data.

Question 5.2 *Determine the frequency at which* $|v_C(f)| = |v_R(f)|$. *Compare your measured value to the calculated value.*

Question 5.3 *Over what range of frequencies do you expect the circuit to integrate or differentiate the input signal? To figure this out, you can use the analysis in the Lab 4 write-up or use the frequency domain logic given in Section 3.4 of the textbook.*

9. Use the different waveforms available from the signal generator to see that the proper mathematical operation is performed. Choose an appropriate period for the waves so the integrator or differentiator should work well.
10. Use screen captures of the output voltages in your lab report, and explain why they are the integral or the derivative of the input voltage.

5.4.4 Frequency response of the RL voltage divider

1 Build the "low-pass" configuration of an RL circuit, as shown in left-hand diagram of Figure 8. Use a 2.2 kΩ resistor for *R* and a 500 mH inductor for *L*. Measure the resistance, *R*, the inductance, *L*, and the resistance, R_L, of your inductor.
2 Repeat the procedure from the *RC* filters for the low-pass configuration the *RL* filter, with frequency measurements from a few Hz up to 50kHz
3 For extra credit, carefully map out the frequency response of the circuit from 50kHz up to the highest frequency that you can measure. Explain the response that you observe.

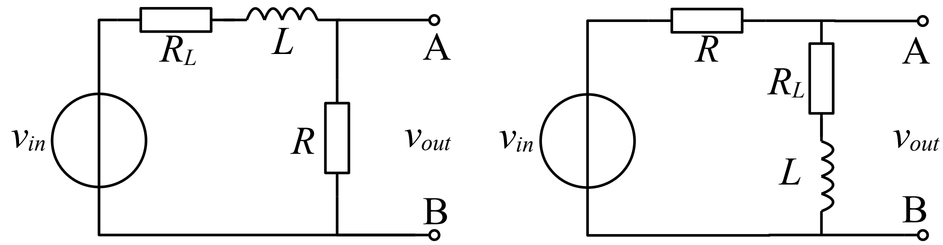

Figure 8: The low-pass configuration (left) and the high-pass configuration (right) of the RL circuit to be studied in this lab. The resistance of the inductor, R_L, has been shown explicitly. You only need to measure the low-pass configuration, but you may optionally measure both.

Question 5.4 *In the low-frequency limit, what do we expect the gain of the RL low-pass filter to be? Be sure to include the effect of R_L.*

Question 5.5 *In the high-frequency limit, what do we expect the gain of the RL high-pass filter to be? Be sure to include the effect of R_L.*

Question 5.6 *Which circuit, RL or RC, works better as a low-pass filter? Why?*

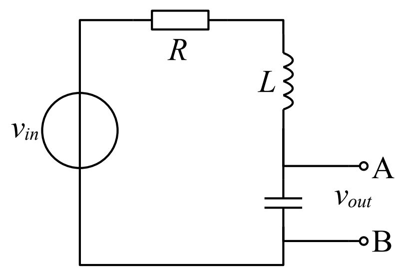

Figure 9: The RLC circuit configured to measure the voltage across the capacitor. The resistor is from the input impedance, r_s, and the inductor resistance, R_L. Don't add any additional resistance.

5.4.5 *RLC* Resonant Circuit

1. Study the discussion of *RLC* circuits given in Section 3.6 of the textbook and calculate a predicted resonant frequency, ω_{LC}.
2. Construct a series *RLC* circuit as shown in Figure 5.9. Use your 0.008 μF capacitor and the inductor from the previous part of the lab. Do not include any extra resistance in this circuit. The source resistance, r_s, and the internal resistance of the inductor, R_L, will provide the resistance for this circuit.
3. Measure the frequency response of the voltage across the capacitor over an appropriate frequency range. Again, choose frequency steps that will be equally spaced on a logarithmic frequency axis. But choose more points near ω_{LC} to accurately map the behavior. Near resonance, ω_{LC}, the voltage across the capacitor can exceed the maximum voltage the scope can display.
4. Make a Bode plot and a phase-shift plot, as in the above procedures.
5. Compare your results to the expected behavior of your circuit. Use your measurements of the amplitude as a function of frequency to determine the value of the internal resistance

of the inductor, R_L. Comment on the result and compare with what you expect, drawing on the appropriate mathematical relations in your textbook. Overlay your expectations for the phase on your data and comment on the agreement.

5.4.6 Coupling Circuits Together: The Bandpass RC Filter

As discussed in Sections 3.7 and 3.8 of the textbook, there are many occasions when we need to couple one functional block of circuitry to another – in fact, it's hard to think of a situation where this is not necessary! Here, you will design a "band-pass" filter circuit by taking the output of a high-pass *RC* filter and putting it into a low-pass *RC* filter with the same characteristic frequency of $f_{RC} \approx 1$kHz. We reproduce Figure 3.19 of the textbook here in Figure 10. The input voltage with $r_s = 50\Omega$ represents the function generator. The first stage is a high-pass filter with characteristic frequency $\omega_{RC} = 1/R_1C_1$, while the second stage is a low-pass filter with characteristic frequency $\omega_{RC} = 1/R_2C_2$. The combination of filters is used to drive some load, represented as Z_L.

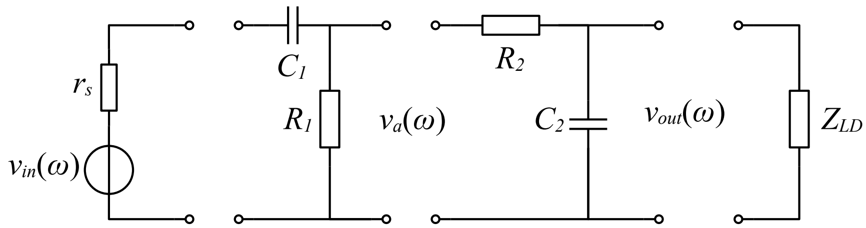

Figure 10: Connecting a low-pass filter on the right to a high-pass filter on the left to achieve a band-pass filter. We use the Thévenin equivalent circuit for the source on the left. The next stage is the high-pass filter, whose output is labeled v_a. This output is connected to the low-pass filter which, in turn, will be connected to some other circuitry represented by the complex load Z_{LD}. The voltage at the output of the low-pass filter is $v_{out}(\omega)$.

1 Repeat the design logic given in the first example in Section 3.8 of the textbook, but use $R_1 \approx 20r_S$ instead of the $100r_s$ as used in the text. Recall that r_s is the 50Ω output impedance of the function generator. The factor of 20 leads to more comfortable element values. You should find that you can build the low-pass final stage of the circuit using the same components you used to build the RC circuit earlier, $R_2 = 22$ kΩ and $C_2 = 0.008$ μF. If you find you need a capacitor value which we don't have, consider the parallel or series combinations of components that we do have.
2 Build a bandpass filter based on your design.
3 Using a 5 V amplitude sine wave from the signal generator, measure the frequency response, both the amplitude and the phase, and compare to the expected response.

What this exercise does not show you, at least if you do the design correctly, is how things go wrong when you do not have the correct progression of input and output impedances. If you have time, you might want to try using $R_1 = R_2$ and $C_1 = C_2$ and see what happens to the response. What gain do you get now at $\omega = \omega_{RC}$?

5.5 Additional Problems

After completing this lab, you should be able to answer the following questions.

1. You measure the data shown in Figure 11 and plot it on a Bode plot as shown.
(a) At approximately what frequency is the 3dB point?
(b) What is the slope of the fall-off in dB/decade?
(c) In the fall-off region, how does the gain, $|G|$, depend on the frequency f?

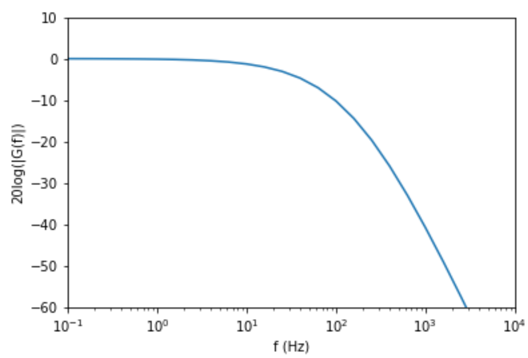

Figure 11: The Bode plot for problem 1.

2. You build the filter circuit shown in Figure 12 in lab where you have chosen the resistor, R, to have a value of 4.70 kΩ and the capacitor, C, to be 339 pF.
(a) For such a circuit, we talk about high-frequency and low-frequency behavior. For this circuit, is f = 20 kHz considered high-frequency or low-frequency?
(b) What is the output impedance of our filter at its characteristic frequency? Give both the complex Z_{out} value and the magnitude, $|Z_{out}|$.
(c) Is this a high-pass or a low-pass filter? Justify your answer with some physics and mathematical arguments.

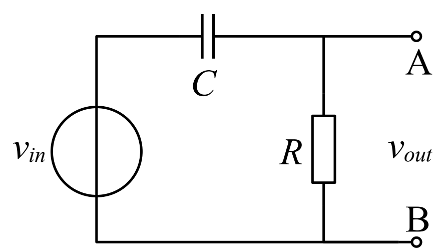

Figure 12: The circuit for problem 2.

3. You are given a black-box circuit with two input terminals, and two output terminals. In the lab, you make the following measurements using your signal generator and your oscilloscope.

(a) For an input voltage given as

$$v_{in}(t) = (3.00\,V)\cos(628\,s^{-1}t)$$

you measure the output voltage to be

$$v_{out}(t) = (2.00\,V)\cos(628\,s^{-1}t)\,.$$

At what frequency, *f*, did you perform this measurement? What is the (complex) gain at this frequency (express as magnitude and phase)?

(b) For an input voltage given as

$$v_{in}(t) = (3.00\,V)\cos(62800\,s^{-1}t)$$

you measure the output voltage to be

$$v_{out}(t) = (2.00\,V)\sin(62800\,s^{-1}t)\,.$$

What is the (complex) gain at this frequency (express as magnitude and phase)?

(c) Estimate the slope of the Bode plot from your measurements.

(d) You now connect a 4.7kΩ resistor across the output of your black box. For the same input voltage as in part (a),

$$v_{in}(t) = (3.00\,V)\cos(628\,s^{-1}t)$$

you measure the voltage across the resistor to be

$$v_{out}(t) = (1.00\,V)\cos(628\,s^{-1}t)\,.$$

What is the magnitude of the output impedance of the black box at this frequency?