Ex 207

5) $\varphi = \neg(\exists x. Q(x)) \leftrightarrow (\forall x. \neg Q(x))$ validă

ddacă $\neg\varphi$ nesatisf.

$\neg\varphi \xrightarrow{FNP} \varphi_1$ - FNP $\xrightarrow{\text{închiderea exist.}} \varphi_2$ - FNP închis $\xrightarrow{\text{skolemizare}} \varphi_3$ - FNS $\xrightarrow{\varphi' - FNC} \varphi_4$ - FNSC

$\varphi_1 \equiv \neg\varphi$ $\qquad$ φ_2 echisat φ_1 $\qquad$ φ_2 echisat φ_3 $\qquad$ $\varphi_3 \equiv \varphi_4$

$$\neg\varphi = \neg\Big(\underbrace{\neg(\exists x. Q(x))} \leftrightarrow \underbrace{(\forall x. \neg Q(x))}\Big)$$

$$\equiv \neg\Big(\big(\neg(\exists x. Q(x)) \rightarrow (\forall x. \neg Q(x))\big) \wedge \big((\forall x. \neg Q(x)) \rightarrow \neg(\exists x. Q(x))\big)\Big)$$

$$\equiv \neg\Big(\big(\neg\neg(\exists x. Q(x)) \vee (\forall x. \neg Q(x))\big) \wedge \big(\neg(\forall x. \neg Q(x)) \vee \neg(\exists x. Q(x))\big)\Big)$$

$$\equiv \neg\Big(\underbrace{\big((\exists x. Q(x)) \vee (\forall x. \neg Q(x))\big)} \wedge \underbrace{\big(\neg(\forall x. \neg Q(x)) \vee \neg(\exists x. Q(x))\big)}\Big)$$

$$\equiv \neg\big((\exists x. Q(x)) \vee (\forall x. \neg Q(x))\big) \vee \neg\big(\neg(\forall x. \neg Q(x)) \vee \neg(\exists x. Q(x))\big)$$

$$\equiv \big(\neg(\exists x. Q(x)) \wedge \neg(\forall x. \neg Q(x))\big) \vee \big(\neg\neg(\forall x. \neg Q(x)) \wedge \neg\neg(\exists x. Q(x))\big)$$

$$\equiv \big(\underbrace{\neg(\exists x. Q(x))} \wedge \underbrace{\neg(\forall x. \neg Q(x))}\big) \vee \big((\forall x. \underbrace{\neg Q(x)}_{\varphi_1}) \wedge \underbrace{(\exists x. Q(x))}_{\varphi_2}\big)$$

$\neg(\exists x. \varphi) \equiv \forall x. \neg\varphi$ $\qquad$ $(\forall x. \varphi_1) \wedge \varphi_2 \equiv \forall x. (\varphi_1 \wedge \varphi_2)$ $\quad x \notin free(\varphi_2)$

$$\equiv \big((\forall x. \underbrace{\neg Q(x)}) \wedge \underbrace{(\exists x. \neg\neg Q(x))}\big) \vee \big(\forall x. (\neg Q(x) \wedge (\exists x. Q(x)))\big)$$

$$\equiv \underbrace{\big(\forall x. (\neg Q(x) \wedge (\exists x. Q(x)))\big)} \vee \underbrace{\big(\forall x. (\neg Q(x) \wedge (\exists x. Q(x)))\big)}$$

$\varphi \vee \varphi \equiv \varphi$

$$\equiv \left(\forall x. \left(\underbrace{\neg Q(x)} \wedge \underbrace{\left(\exists x. Q(x)\right)} \right) \right)$$

$$\equiv \forall x. \left(\left(\exists x. \underbrace{Q(x)}_{\varphi_1}\right) \wedge \underbrace{\neg Q(x)}_{\varphi_2} \right)$$

$(\exists x. \varphi_1) \wedge \varphi_2 \equiv \exists x. (\varphi_1 \wedge \varphi_2)$ dacă $x \notin free(\varphi_2)$

$free(\neg Q(x)) = \{x\}$

$$\overset{LR}{\equiv} \forall x. \left(\left(\exists y. \underbrace{Q(y)}\right) \wedge \underbrace{\neg Q(x)} \right)$$

$$\equiv \forall x. \exists y. \left(Q(y) \wedge \neg Q(x) \right) = \varphi_1 \quad FNP$$

- φ_1 este închisă $\Rightarrow$ nu este necesară închiderea existențială $(\varphi_2 = \varphi_1)$

- skolemizarea

Fie g simbol funcțional nou de aritate 1 $\qquad \Sigma = (\mathcal{P}, \mathcal{F})$

$\varphi_3 = \forall x. \underbrace{\left(Q(g(x)) \wedge \neg Q(x) \right)}_{\varphi'}$ peste $\Sigma_1 = (\mathcal{P}, \mathcal{F} \cup \{g\})$

$\hookrightarrow$ FNS

φ_3 echisat φ_1

- φ_3 - FNSC (φ' în FNC). $\qquad (\varphi_4 = \varphi_3)$

1. $Q(g(x))$ (ip)
2. $\neg Q(x)$ (ip)
3. $\square$ $\qquad$ RB, 1, 2 $\qquad Q(g(x)) \qquad \neg Q(x_1)$

$P = \{ g(x) \doteq x_1 \} \xrightarrow{ORIENT} \{ x_1 \doteq g(x) \}$ formă rez

$\sigma = mgu(P) = \{ x_1 \mapsto g(x) \}$

$\Rightarrow \varphi_3$ nesat

φ_3 echisat φ_1 $\Bigg| \Rightarrow \varphi_1$ nesat, $\neg\varphi \equiv \varphi_1 \Bigg| \Rightarrow \neg\varphi$ nesat $\Rightarrow \varphi$ validă

$\mathbb{P}$ - formă rez.

$\mathbb{P} = \bot \longrightarrow unif(\mathbb{P}) = \emptyset$

sau

$\mathbb{P} = \{ x_1 \doteq t_1, x_2 \doteq t_2, \ldots x_n \doteq t_n \}$

$x_i \neq x_j$

$x_i \notin vars(t_j)$ pt orice i,j

$\sigma = mgu(\mathbb{P}) = \{ x_1 \mapsto t_1, x \mapsto t_2, \ldots x_n \mapsto t_n \}$

Ex 205

4) $\mathbb{P}_4 = \{ f(g(x), y) \doteq f(y, z), z \doteq h(a) \} \xrightarrow{DESC}$

$\{ g(x) \doteq y, y \doteq z, z \doteq h(a) \} \xrightarrow{ORIENT}$

$\{ y \doteq g(x), y \doteq z, z \doteq h(a) \} \xrightarrow{ELIM}$

$\{ y \doteq g(x), g(x) \doteq z, z \doteq h(a) \} \xrightarrow{ELIM}$

$\{ y \doteq g(x), g(x) \doteq h(a), z \doteq h(a) \} \xrightarrow{CONF} \bot$

Deci $unif(\mathbb{P}_4) = \emptyset$

$\varphi = \forall x. \forall y. \left(\overline{\neg P(x)} \wedge \overline{P(i(y))} \wedge \overline{\neg P(e)} \right)$

$P(i(y)) \qquad \neg P(e)$

$$\mathbb{P}_1 = \{ i(y) \doteq e \} \xRightarrow{CONF} \bot$$

$\Rightarrow$ nu putem aplica RB

simb fct ar 1   simb fct ar 0

1. $P(i(y))$
2. $\neg P(x)$
3. $\square$ RB 1,2

$P(i(y)) \qquad \neg P(x)$

$$\mathbb{P}_2 = \{ i(y) \doteq x \} \xRightarrow{ORIENT} \{ x \doteq i(y) \}$$

$$\sigma_2 = mgu(\mathbb{P}_2) = \{ x \mapsto i(y) \}$$

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1. $P(e)$
2. $\neg P(e)$
3. $\square$, R.B 1,2

$$\mathbb{P} = \{ e \doteq e \} \xRightarrow{STERG} \emptyset$$

$$\sigma = mgu(\mathbb{P}) = \{ \}$$

$\sigma : \mathcal{X} \to \mathcal{T} \quad \sigma(x) = x$ pt orice $x \in \mathcal{X}$

$$\varphi_2 = \cancel{\forall y} \forall x . \left(\left(P(x) \vee P(e) \right) \wedge \neg P(y) \right)$$

1. $P(x) \vee P(e)$
2. $\neg P(y)$
3. $P(e)$ FP 1

$$\mathbb{P}_1 = \{ x \doteq e \} \text{ formă rez.}$$

$$\sigma_1 = mgu(\mathbb{P}_1) = \{ x \mapsto e \}$$

4. $\square$ RB 3,2

$$\mathbb{P}_2 = \{ e \doteq y \} \xRightarrow{ORIENT} \{ y \doteq e \} \text{ formă rez.}$$

$$\sigma_2 = \{ y \mapsto e \} = mgu(\mathbb{P}_2)$$

$\Rightarrow \varphi_2$ nesat.

205 5)

$$\mathbb{P}_5 = \{ x_1 \doteq f(x_2, x_2),\ x_2 \doteq f(x_3, x_3),\ \underline{x_3 \doteq f(x_4, x_4)} \} \xrightarrow{ELIM}$$

$$\{ x_1 \doteq f(x_2, x_2),\ \underline{x_2 \doteq f(f(x_4,x_4), f(x_4,x_4))},\ x_3 \doteq f(x_4,x_4) \} \xrightarrow{ELIM}$$

$$\{ x_1 \doteq f(f(f(x_4,x_4), f(x_4,x_4)), f(f(x_4,x_4), f(x_4,x_4))),$$
$$x_2 \doteq f(f(x_4,x_4), f(x_4,x_4)),\ x_3 \doteq f(x_4,x_4) \}$$

formă rez.

$$mgu(\mathbb{P}_5) = \{ x_1 \mapsto f(f(f(x_4,x_4), f(x_4,x_4)), f(f(x_4,x_4), f(x_4,x_4))),$$
$$x_2 \mapsto f(f(x_4,x_4), f(x_4,x_4)),\ x_3 \mapsto f(x_4,x_4) \}$$

Ex 206

1) $\varphi_1 = \forall x. \forall y. \forall z. \left((\neg P(x,z) \vee R(x,x,z)) \wedge \neg R(e,x,e) \wedge P(e,y) \right)$ rezolut $\xrightarrow{}$ FN/SC.

1. $\neg P(x,z) \vee R(x,x,z)$ (ip)
2. $\neg R(e,x,e)$ (ip)
3. $\neg P(e,e)$ R.B 1, 2

$\neg P(x,z) \vee R(x,x,z)$ $\qquad$ $\neg R(e, x_1, e)$

$$\mathbb{P}_1 = \{ \underline{x \doteq e},\ x \doteq x_1,\ z \doteq e \} \xrightarrow{ELIM}$$

$$\Rightarrow \{ x \doteq e,\ \underline{e \doteq x_1},\ z \doteq e \} \xrightarrow{ORIENT}$$

$$\Rightarrow \{ x \doteq e,\ x_1 \doteq e,\ z \doteq e \}$$ formă rez.

$$\sigma_1 = mgu(\mathbb{P}_1) = \{ x \mapsto e,\ x_1 \mapsto e,\ z \mapsto e \}$$

4. $P(e, y)$ (ip)

5. $\square$ RB 4, 3

$$\mathcal{P}_2 = \{ \underline{e \doteq e}, y \doteq e \} \xrightarrow{\text{STERG}} \{ y \doteq e \} \text{ formă rez.}$$

$$\sigma_2 = mgu(\mathcal{P}_2) = \{ y \mapsto e \}$$

$\Rightarrow$ φ_1 nesatisfiabilă

Ex 143 g) $\underbrace{\forall x.(P(x, x) \vee Q(x))}_{\varphi_1} \not\equiv \underbrace{(\forall x. P(x, x)) \vee (\forall x. Q(x))}_{\varphi_2}$

$\varphi_1 \equiv \varphi_2$ ddacă pt orice structură S și pt orice S-atr α avem $S, \alpha \models \varphi_1$ ddacă $S, \alpha \models \varphi_2$

$\varphi_1 \not\equiv \varphi_2$ ddacă există o structură S și există o S-atr α a.î. $(S, \alpha \not\models \varphi_1$ și $S, \alpha \models \varphi_2)$ sau $(S, \alpha \models \varphi_1$ și $S, \alpha \not\models \varphi_2)$ (*)

Căutăm o structură S și o S-atr α a.î. (*)

$S, \alpha \models \varphi_1$ ddacă $S, \alpha \models \forall x.(P(x, x) \vee Q(x))$

ddacă pt orice $u \in D$ avem $\overbrace{S, \alpha[x \mapsto u]}^{\alpha'} \models (P(x, x) \vee Q(x))$

ddacă pt orice $u \in D$ avem $\begin{cases} S, \alpha' \models P(x, x) \\ \text{sau} \\ S, \alpha' \models Q(x) \end{cases}$

ddacă pt orice $u \in D$ avem $\left\{ \begin{array}{l} P^S\left(\overline{\alpha'}(x), \overline{\alpha'}(x)\right) \\ \text{sau} \\ Q^S\left(\overline{\alpha'}(x)\right) \end{array} \right.$

$\alpha : X \to D$ | $\alpha' = \alpha[x \mapsto u]$

$\alpha(x) = \dots$ | $\alpha'(x) = u$

$\alpha(y) = \dots$ | $\alpha'(y) = \alpha(y)$

ddacă pt orice $u \in D$ avem $\left\{ \begin{array}{l} P^S\left(\alpha'(x), \alpha'(x)\right) \\ \text{sau} \\ Q^S\left(\alpha'(x)\right) \end{array} \right.$

ddacă pt orice $u \in D$ avem $\left\{ \begin{array}{l} P^S(u, u) \\ \text{sau} \\ Q^S(u) \end{array} \right.$ (1) A

$S, \alpha \models \varphi_2$ ddacă $S, \alpha \models (\forall x. P(x, x)) \vee (\forall x. Q(x))$

ddacă $\left\{ \begin{array}{l} S, \alpha \models \forall x. P(x, x) \\ \text{sau} \\ S, \alpha \models \forall x. Q(x) \end{array} \right.$

ddacă $\left\{ \begin{array}{l} \text{pt orice } u \in D \text{ avem } S, \overbrace{\alpha[x \mapsto u]}^{\alpha_1} \models P(x, x) \\ \text{sau} \\ \text{pt orice } v \in D \text{ avem } S, \underbrace{\alpha[x \mapsto v]}_{\alpha_2} \models Q(x) \end{array} \right.$

ddacă $\begin{cases} \text{pt orice } u \in D \text{ avem } P^S(\overline{\alpha_1}(x), \overline{\alpha_1}(x)) \\ \text{sau} \\ \text{pt orice } v \in D \text{ avem } Q^S(\overline{\alpha_2}(x)) \end{cases}$

ddacă $\begin{cases} \text{pt orice } u \in D \text{ avem } P^S(u, u) \\ \text{sau} \\ \text{pt orice } v \in D \text{ avem } Q^S(v) \end{cases}$ (2) F

$S = (D, \{P^S, Q^S\}, \{f^S, i^S, e^S\})$

$D = \{\underset{P^S}{\_\_}, \underset{Q^S}{\_\_}, \_\_, \_\_, \_\_, \_\_, \_\_\}$

$S = (\mathbb{N}, \{NOgl, Pal\}, \{+, s, 0\})$ $\alpha : X \to \mathbb{N}$

$\alpha(x) = 1$

$Q^S = $ Palindrom

$Pal(u)$ = u este palindrom.

$Q^S(u) =$ par.

$NOgl(v, w)$ = v nu oglindește w

$P^S(u,u) = u$ imp.

$P^S(u,v) = $ "$\lfloor u/2 \rfloor * 2 \neq v$"

$NOgl(u, u)$ nu este palindrom

u nu oglindește pe u

Pt S, α alese (1) pt orice $u \in \mathbb{N}$, avem $\begin{cases} NOgl(u,u) \\ \text{sau} \\ Pal(u) \end{cases}$ "A"

(2) $\begin{cases} \text{pt orice } u \in \mathbb{N} \text{ avem } NOgl(u,u) \text{ "F"} \\ \text{sau} \\ \text{pt orice } u \in \mathbb{N} \text{ avem } Pal(u) \text{ "F"} \end{cases}$ "F"

$\Rightarrow \varphi_1 \not\equiv \varphi_2$