Sem6-A5

Monday, 9 November 2020   09:55

φ - FNC $\varphi = (p \vee q) \wedge (\neg p \vee r \vee \neg q)$

$\{\{\underbrace{p}_{a}, \underbrace{q}_{C}\}, \{\underbrace{\neg p}_{\neg a}, \underbrace{r, \neg q}_{D}\}\}$

Rez. bin $\dfrac{C \cup \{a\} \qquad D \cup \{\neg a\}}{C \cup D}$

$\tau(p) = 1 \Rightarrow \hat{\tau}(\overbrace{r \vee \neg q}^{D}) = 1$

sau

$\tau(p) = 0 \Rightarrow \hat{\tau}(\underbrace{q}_{C}) = 1$

$\dfrac{q \qquad \neg q}{\square}$

φ nesatisfiabil dacă există derivare pt. $\square$.

φ validă ddacă $\neg\varphi$ nesatisfiabilă

$\varphi_1 \ldots \varphi_n \vDash \varphi$ ddacă $(\varphi_1 \wedge \varphi_2 \wedge \ldots \varphi_n) \to \varphi$ validă

Ex 152

① $\varphi = ((p \wedge q) \to (p \vee q))$ validă ddacă $\neg\varphi$ nesatisf.

$\neg\varphi = \neg((p \wedge q) \to (p \vee q)) \equiv \neg(\neg(p \wedge q) \vee (p \vee q))$

$\equiv \neg\neg(p \wedge q) \wedge \neg(p \vee q)$

$\equiv p \wedge q \wedge \neg p \wedge \neg q$ FNC

$\{\{p\}, \{q\}, \{\neg p\}, \{\neg q\}\}$

1. $\{p\}$ (prem)
2. $\{\neg p\}$ (prem)
3. $\square$ (RB, 1, 2, $a = p$) $\Rightarrow \neg\varphi$ nesatisf $\Rightarrow$ $\Rightarrow \varphi$ validă

② $\varphi = (p \to (q \to p))$ validă ddacă $\neg\varphi$ nesat.

$\neg\varphi = \neg(p \to (q \to p)) \equiv \neg(\neg p \vee (q \to p))$

$\equiv \neg(\neg p \vee (\neg q \vee p))$

$\equiv \neg\neg p \wedge \neg(\neg q \vee p)$

$\equiv p \wedge q \wedge \neg p$ FNC

$\{\{p\}, \{q\}, \{\neg p\}\}$

1. $\{p\}$ (prem)
2. $\{\neg p\}$ (prem)
3. $\square$ (R.B. 1, 2, $a = p$) $\Rightarrow \neg\varphi$ nesatisf $\Rightarrow$ $\Rightarrow \varphi$ validă

③

$\neg\varphi \equiv (\neg p \vee q) \wedge \neg q \wedge p$

$\{\{\neg p, q\}, \{\neg q\}, \{p\}\}$ $\{\neg p\} \cup \emptyset = \{\neg p\}$

1. $\{\neg p, q\}$ (prem)
2. $\{\neg q\}$ (prem)
3. $\{\neg p\}$ RB. 1, 2, $a = q$
4. $\{p\}$ (prem)
5. $\square$ RB, 4, 3, $a = p$ $\Rightarrow \neg\varphi$ nesatisf. $\Rightarrow \varphi$ validă

Ex 153

① $p \vDash (p \vee q)$ ddacă $\varphi = p \to (p \vee q)$ validă

ddacă $\neg\varphi = \neg(p \to (p \vee q))$ nesatisf.

$\neg\varphi = \neg(p \to (p \vee q)) \equiv \neg(\neg p \vee (p \vee q))$

$\equiv p \wedge \neg(p \vee q)$

$\equiv p \wedge \neg p \wedge \neg q$ FNC

$\{\{p\}, \{\neg p\}, \{\neg q\}\}$

1. $\{p\}$ (prem)
2. $\{\neg p\}$ (prem)
3. $\square$ (RB, 1, 2, $a = p$) $\Rightarrow \neg\varphi$ nesatisf $\Rightarrow$ $\Rightarrow \varphi$ validă $\Rightarrow$ $\Rightarrow p \vDash (p \vee q)$

④ $\{(p \to p'), (q \to q')\} \vDash ((p \wedge q) \to (p' \vee q'))$ ddacă

ddacă $\varphi = ((p \to p') \wedge (q \to q')) \to ((p \wedge q) \to (p' \vee q'))$ validă

ddacă $\neg\varphi$ nesatisf.

$\neg\varphi = \neg(((p \to p') \wedge (q \to q')) \to ((p \wedge q) \to (p' \vee q')))$

$\equiv \neg(((\neg p \vee p') \wedge (\neg q \vee q')) \to (\neg(p \wedge q) \vee (p' \vee q')))$

$\equiv \neg(\neg((\neg p \vee p') \wedge (\neg q \vee q')) \vee (\neg p \vee \neg q \vee p' \vee q'))$

$\equiv \neg\neg((\neg p \vee p') \wedge (\neg q \vee q')) \wedge \neg((\neg p \vee \neg q) \vee (p' \vee q'))$

$\neg(\varphi_1 \vee \varphi_2) \equiv \neg\varphi_1 \wedge \neg\varphi_2$

$\equiv (\neg p \vee p') \wedge (\neg q \vee q') \wedge \neg(\neg p \vee \neg q) \wedge \neg(p' \vee q')$

$\equiv (\neg p \vee p') \wedge (\neg q \vee q') \wedge p \wedge q \wedge \neg p' \wedge \neg q'$ FNC

$\{\{\neg p, p'\}, \{\neg q, q'\}, \{p\}, \{q\}, \{\neg p'\}, \{\neg q'\}\}$

1. $\{\neg p, p'\}$ (prem)
2. $\{p\}$ (prem)
3. $\{p'\}$ (RB. 2, 1, $a = p$)
4. $\{\neg p'\}$ (prem)
5. $\square$ (RB, 3, 4, $a = p'$) $\Rightarrow$

$\Rightarrow \neg\varphi$ nesatisf $\Rightarrow \varphi$ validă $\Rightarrow$

$\Rightarrow (p \to p'), (q \to q') \vDash ((p \wedge q) \to (p' \vee q'))$

Ex 5 b)

$\{\neg p \vee \neg q \vee r, p, q\} \vDash \underset{\varphi}{r}$

- ddacă $\varphi = ((\neg p \vee \neg q \vee r) \wedge p \wedge q) \to r$ validă

  ddacă $\neg\varphi$ nesat.

- Th 144   Dacă există o dem. a clauzei φ din $\varphi_1 \ldots \varphi_n$ prin rez. atunci $\varphi_1, \ldots \varphi_n \vDash \varphi$

$\varphi_1 \ldots \varphi_n$ - în FNC.

$\varphi_1 : \{\{\neg p, \neg q, r\}\}$

$\varphi_2 : \{\{p\}\}$

$\varphi_3 : \{\{q\}\}$

prem.

1. $\{\neg p, \neg q, r\}$ (prem)
2. $\{p\}$ (prem)
3. $\{q\}$ (prem)
4. $\{\neg q, r\}$ (RB, 2, 1, $a = p$)
5. $\{r\}$ (RB, 3, 4, $a = q$)

$\xRightarrow{\text{Th 144}} \neg p \vee \neg q \vee r, p, q \vDash r$

$p \vDash (p \vee q)$ ddacă

$\varphi = p \to (p \vee q)$ validă ddacă $\neg\varphi$ nesat.

$\neg\varphi =$