Sem7 AS
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ψ Tautology
ψ open
ψ not above ψ satisfy

$\psi = (p \wedge (q \vee r))$

$p_1 \leftrightarrow (p \wedge q) \equiv$
$\equiv (p \wedge q) \wedge (p \wedge q)$
$\equiv (p \wedge q)$

$\hat{v}(p \wedge q) = 1$ above ....
$\hat{v}(p) = \hat{v}(q)$

$p_2 \leftrightarrow (p \wedge r) \equiv (p \wedge r) \wedge (p \wedge r)$

$p_3 \leftrightarrow (p \wedge (q \vee r)) \equiv$
$\equiv (p \wedge (q \vee r)) \wedge (p \wedge (q \vee r))$
$\equiv (p \wedge (q \vee r)) \wedge ((p \wedge q) \vee (p \wedge r))$
$\equiv (p \wedge (q \vee r)) \wedge (p \wedge q) \wedge (p \wedge r)$ FNC.

$\psi' = p_1 \wedge (\neg p_1 \vee p) \wedge (\neg p_2 \vee p) \wedge (\neg p_3 \vee \neg p_1 \vee p_2)$
$\wedge (\neg p_1 \vee \neg p_2) \wedge (p \vee p)$
$\wedge (\neg p_1 \vee \neg p_3 \vee \neg p) \wedge (\neg p \vee p_3)$

$\llcorner$ equivalent on ψ

ψ satisf. above satisfa $\tau : A \to B$ a.i. $\hat{\tau}(\psi) = 1$

ψ' satisf. above satisfa $\tau' : A \to B$ a.i. $\hat{\tau}'(\psi') = 1$

$\tau : A \to B$
$\hat{\tau}(\psi) = 1$
$\hat{\tau}(p) = 1$
$\hat{\tau}(q) = 0$
$\hat{\tau}(r) = 0$
$\hat{\tau}(p_1) = 0$ $\Leftarrow$ $\hat{\tau}(p \wedge q) = 0$ $\Leftarrow$ $\hat{\tau}(p) = 0$ or $\hat{\tau}(q) = 0$ $\Leftarrow$ $\hat{\tau}(p) = 0$ or $\hat{\tau}(r) = 0$

$\hat{\tau}(\psi) = \hat{\tau}(p) \bullet \hat{\tau}(q \vee r) = 1 \cdot \overset{\max}{1} = 1 \cdot 1 = 1$

ψ satisf. $\Rightarrow$ ψ' satisf.
$\hat{\tau}'(p) = 1$
$\hat{\tau}'(q) = 0$
$\hat{\tau}'(r) = 0$
$\hat{\tau}'(p_1) = 1$
$\hat{\tau}'(p_2) = 1$
$\hat{\tau}'(p_3) = 0$
$\hat{\tau}'(q_1 \vee r) = 0$
$\hat{\tau}'(B) = 0$ $\Leftarrow$ since $b \in \{\psi_1, \psi, p, p_1, p_2, p_3\}$

ψ satisf. $\Leftarrow$ ψ' satisf.

Ex 138

2. $\psi = ((p \to q) \to (p \to r))$ satisfa above ψ unsatisf.

Construire ψ' equivalent cu ψ

$\psi \equiv \neg(\underbrace{(p \to q) \to (p \to r)}_{\psi}) \equiv \underbrace{\neg(\neg(p \to q) \vee (p \to r))}_{\psi'}$

$sub(\psi) =$

- $p_1 \leftrightarrow \neg p_2 \equiv (p_1 \to \neg p_2) \wedge (\neg p_2 \to p_1)$
$\equiv (p_1 \vee \neg p_2) \wedge (p_2 \vee p_1)$

- $p_2 \leftrightarrow (p_3 \vee p_4) \equiv (p_2 \to (p_3 \vee p_4)) \wedge ((p_3 \vee p_4) \to p_2)$
$\equiv (\neg p_2 \vee p_3 \vee p_4) \wedge (\neg(p_3 \vee p_4) \vee p_2)$
$\equiv (\neg p_2 \vee p_3 \vee p_4) \wedge (\neg p_3 \wedge \neg p_4) \wedge (\neg p_2 \vee p_2)$

$p_3 \leftrightarrow \neg p_5 \equiv (\neg p_3 \vee \neg p_5) \wedge (p_5 \vee p_3)$

$p_4 \leftrightarrow (p_5 \to q) \equiv (\neg p_5 \vee q) \wedge (\neg q \vee p_5)$

$p_5 \leftrightarrow (p_6 \to p_7) \equiv (p_5 \to (p_6 \to p_7)) \wedge (p_7 \to (p_6 \to p_5))$
$\equiv (\neg p_5 \vee \neg p_6 \vee p_7) \wedge (\neg p_7 \vee \neg p_6 \vee p_5)$

$p_6 \leftrightarrow \neg p_8$ $p_8 = (p_9 \to r)$
$p_7 = (p_9 \to r)$
$p_9 \to r = (\neg p_9 \vee r)$
$\neg(p_9 \to r) = (p_9 \wedge \neg r)$

$p_8 \leftrightarrow (p_9 \to r) \equiv (p_8 \to (p_9 \to r)) \wedge ((p_9 \to r) \to p_8)$
$\equiv (\neg p_8 \vee \neg p_9 \vee r) \wedge (\neg(p_9 \to r) \vee p_8)$
$\equiv (\neg p_8 \vee \neg p_9 \vee r) \wedge (p_9 \wedge \neg r) \wedge (\neg p_8 \vee p_8)$

$p_9 \leftrightarrow \neg p_{10} \equiv (\neg p_9 \vee \neg p_{10}) \wedge (p_{10} \vee p_9)$

$p_{10} \leftrightarrow (p_{11} \to p_{12}) \equiv (p_{10} \to (p_{11} \to p_{12})) \wedge ((p_{11} \to p_{12}) \to p_{10})$
$\equiv (\neg p_{10} \vee \neg p_{11} \vee p_{12}) \wedge (\neg(p_{11} \to p_{12}) \vee p_{10})$
$\equiv (\neg p_{10} \vee \neg p_{11} \vee p_{12}) \wedge (p_{11} \wedge \neg p_{12}) \wedge (\neg p_{10} \vee p_{10})$

$p_{11} \leftrightarrow \neg p_{13} \equiv (\neg p_{11} \vee \neg p_{13}) \wedge (p_{13} \vee p_{11})$

$p_{12} \leftrightarrow (p_{14} \to 2) \equiv (\neg p_{12} \vee p_{14}) \wedge (\neg p_{14} \vee p_{12})$

$p_{13} \leftrightarrow (p_{15} \to p_{16}) \equiv (\neg p_{13} \vee (p_{15} \to p_{16})) \wedge ((p_{15} \to p_{16}) \to p_{13})$
$\equiv (\neg p_{13} \vee \neg p_{15} \vee p_{16}) \wedge (\neg(p_{15} \to p_{16}) \vee p_{13})$
$\equiv (\neg p_{13} \vee \neg p_{15} \vee p_{16}) \wedge (p_{15} \wedge \neg p_{16}) \wedge (\neg p_{13} \vee p_{13})$

$p_{14} = (p_{15} \to p_{16})$
$p_{15} \to p_{16} = (\neg p_{15} \vee p_{16})$
$\neg(p_{15} \to p_{16}) = (p_{15} \wedge \neg p_{16})$

$\psi' = p_1 \wedge (\neg p_1 \vee \neg p_2) \wedge (p_3 \vee p_1)$
$\wedge (\neg p_2 \vee p_3 \vee p_4) \wedge (\neg p_3 \vee \neg p_4) \wedge (p_2 \vee p_3)$
$\wedge (\neg p_2 \vee \neg p_4) \wedge (\neg p_4 \vee p_5) \wedge (\neg p_5 \vee p_6)$
$\wedge (\neg p_5 \vee \neg p_7) \wedge (p_8 \vee p_6) \wedge (\neg p_8 \vee p_7)$
$\wedge (\neg p_9 \vee \neg p_8) \wedge (p_9 \vee p_8) \wedge (\neg p_{10} \vee p_9)$
$\wedge (\neg p_{10} \vee \neg p_{11} \vee p_{12}) \wedge (p_{11} \vee p_{13})$
$\wedge (\neg p_{12} \vee p_{14}) \wedge (\neg p_{14} \vee p_{12}) \wedge (\neg p_{13} \vee \neg p_{15} \vee p_{16})$
$\wedge (p_{15} \vee p_{13}) \wedge (\neg p_{16} \vee p_{13})$ FNC

ψ' subform on $\psi \equiv \neg\psi$

ψ' subform on ψ $\equiv$ $\neg\psi$ unsatisf. above ψ unsatisf.

$\neg p_1$
$\neg p_2$
$\neg p_3$
$\neg p_4$
$\neg p_5$
$\neg p_6$
$\neg p_7$
$\neg p_8$
$\neg p_9$
$\neg p_{10}$
$\neg p_{11}$
$\neg p_{12}$
$\neg p_{13}$
$\neg p_{14}$
$\neg p_{15}$
$\neg p_{16}$

1. $\{p_1\}$ ψ
2. $\{\neg p_1, \neg p_2\}$ ψ
3. $\{p_3, p_1\}$ ψ
4. $\{\neg p_2 \vee p_3 \vee p_4\}$ ψ
5. $\{\neg p_3, \neg p_4\}$ ψ
6. $\{\neg p_2, \neg p_4\}$ ψ
7. $\{\neg p_4 \vee p_5\}$ ψ
8. $\{\neg p_5 \vee p_6\}$ ψ
9. $\{\neg p_5 \vee \neg p_7\}$ ψ
10. $\{p_8 \vee p_6\}$ ψ
11. $\{\neg p_8 \vee p_7\}$ ψ
12. $\{\neg p_9 \vee \neg p_8\}$ ψ
13. $\{p_9 \vee p_8\}$ ψ
14. $\{\neg p_{10} \vee p_9\}$ ψ
15. $\{p_{11}, \neg p_{12}\}$ $(\text{RD}, 2, 3, a = p_3)$ X
16. $\{\neg p_{12}\}$ $(\text{RD}, 1, 2, a = p_1)$
17. $\{\neg p_{12}\}$ $(\text{RD}, 5, 16, a = p_{12})$
18. $\{\neg p_{13}\}$ $(\text{RD}, 6, 16, a = p_{12})$ X
19. $\{\neg p\}$ $(\text{RD}, 10, 18, a = p_{13})$ X
20. $\{\neg p_{13}\}$ $(\text{RD}, 12, 19, a = p_{13})$
21. $\{\neg p, p_{13}\}$ $(\text{RD}, 14, 20, a = p_{13})$ X
22. $\{p_{13}\}$ $(\text{RD}, 8, 20, a = p_{13})$
23. $\square$ $(\text{RD}, 22, 19, a = p_{13})$

$\Rightarrow \psi'$ unsatisf. $\Rightarrow \psi$ unsatisf. $\Rightarrow \neg\psi$ valid
ψ' subform on ψ $\Rightarrow \neg\psi$ valid
$\psi \equiv \neg\psi$

Ex 139

1. $p \models (p \to q)$ above $\psi = p \to (p \to q)$ satisfa above $\neg\psi$ unsatisf.

$\neg\psi \equiv \neg(\underbrace{\neg p \vee (p \to q)}_{\psi})$

$sub(\psi) =$

$p_1 \leftrightarrow \neg p_2 \equiv (\neg p_1 \vee \neg p_2) \wedge (p_2 \vee p_1)$

$p_2 \leftrightarrow (p_3 \to p_4) \equiv (\neg p_2 \vee (p_3 \to p_4)) \wedge ((p_3 \to p_4) \vee p_2)$
$\equiv (\neg p_2 \vee \neg p_3 \vee p_4) \wedge (\neg(p_3 \to p_4) \vee p_2)$
$\equiv (\neg p_2 \vee \neg p_3 \vee p_4) \wedge (p_3 \wedge \neg p_4) \wedge (\neg p_2 \vee p_2)$

$p_3 \leftrightarrow \neg p_5 \equiv (\neg p_3 \vee \neg p_5) \wedge (p_5 \vee p_3)$

$p_4 \leftrightarrow (p_5 \to q) \equiv (\neg p_4 \vee (p_5 \to q)) \wedge ((p_5 \to q) \to p_4)$
$\equiv (\neg p_4 \vee \neg p_5 \vee q) \wedge (\neg(p_5 \to q) \vee p_4)$
$\equiv (\neg p_4 \vee \neg p_5 \vee q) \wedge (p_5 \wedge \neg q) \wedge (\neg p_4 \vee p_4)$

$\psi'' = p_1 \wedge (\neg p_1 \vee \neg p_2) \wedge (\neg p_2 \vee p_1)$
$\wedge (\neg p_2 \vee \neg p_3 \vee p_4) \wedge (\neg p_3 \vee p_2) \wedge (\neg p_4 \vee p_3)$
$\wedge (\neg p_4 \vee \neg p_5 \vee q) \wedge (\neg p_5 \vee p_4) \wedge (\neg q \vee p_4)$

ψ' subform ψ

1. $\{p_1\}$
2. $\{\neg p_1, \neg p_2\}$
3. $\{p_1, p_2\}$
4. $\{\neg p_2, \neg p_3, p_4\}$
5. $\{\neg p_3, p_2\}$
6. $\{\neg p_4, \neg p_3\}$
7. $\{\neg p_5, \neg p\}$
8. $\{p_5, p\}$
9. $\{\neg p_4, p_5\}$
10. $\{\neg p, p_5\}$
11. $\{\neg q, p_5\}$
12. $\{p_5, q\}$ $(\text{RD}, 1, 3, a = p_1)$
13. $\{\neg p_5\}$ $(\text{RD}, 5, 12, a = p_5)$
14. $\{p\}$ $(\text{RD}, 8, 13, a = p_5)$
15. $\{p_4\}$ $(\text{RD}, 6, 13, a = p_5)$
16. $\{\neg p\}$ $(\text{RD}, 10, 15, a = p_4)$
17. $\square$ $(\text{RD}, 8, 16, a = p)$

$\Rightarrow \psi''$ unsatisf. $\Rightarrow \psi$ unsatisf. $\Rightarrow \neg\psi$ valid
ψ' subform ψ $\psi \equiv \neg\psi$ $\Rightarrow p \models (p \to q)$

Tema Ex 138 2,3
Ex 139 2,3,9] Florian Isaciu