Rez. $\to$ φ - FNSC $\to$ $\square$ ddacă φ nesatisf.

Ex 207

6) $\varphi = (\exists y.\forall x.P(x,y)) \to (\forall x.\exists y.P(x,y))$ validă

ddacă $\neg\varphi$ nesatisf.

$$\neg\varphi = \neg\left(\underbrace{(\exists y.\forall x.P(x,y))} \to \underbrace{(\forall x.\exists y.P(x,y))}\right)$$

$\neg\varphi \xrightarrow{FNP} \varphi_1$ - FNP $\xrightarrow[\exists]{\text{închidere}} \varphi_2$ - FNP închisă $\xrightarrow{\text{Skolem}} \varphi_3$ - FNS $\xrightarrow{\varphi' - FNC} \varphi_4$ - FNSC

$\varphi_1 \equiv \neg\varphi$ $\quad$ φ_1 echisat φ_2 $\quad$ φ_2 echisat φ_3 $\quad$ φ_4 echisat $\neg\varphi$

1. FNP.

$$\neg\varphi \equiv \neg\left(\underbrace{\neg(\exists y.\forall x.P(x,y))} \lor \underbrace{(\forall x.\exists y.P(x,y))}\right)$$

$$\equiv \underbrace{\neg\neg(\exists y.\forall x.P(x,y))}_{\neg\neg\varphi \equiv \varphi} \land \underbrace{\neg(\forall x.\exists y.P(x,y))}_{\neg\forall x.\varphi \equiv \exists x.\neg\varphi}$$

$$\equiv (\exists y.\forall x.P(x,y)) \land \left(\exists x.\neg\underbrace{(\exists y.P(x,y))}_{\neg\exists x.\varphi \equiv \forall x.\neg\varphi}\right)$$

$$\equiv (\exists y.\underbrace{\forall x.P(x,y)}_{\varphi_1}) \land \underbrace{(\exists x.\forall y.\neg P(x,y))}_{\varphi_2} \qquad free(\varphi_2) = \emptyset$$

$(\exists x.\varphi_1) \land \varphi_2 \equiv \exists x.(\varphi_1 \land \varphi_2)$ dacă $x \notin free(\varphi_2)$

$$\equiv \exists y.\left(\forall x.\underbrace{P(x,y)}_{\varphi_1} \land \underbrace{(\exists x.\forall y.\neg P(x,y))}_{\varphi_2}\right)$$

$$\equiv \exists y.\forall x.\left(\underbrace{P(x,y)} \land \underbrace{(\exists x.\forall y.\neg P(x,y))}\right)$$

$\varphi_1 \land \varphi_2 \equiv \varphi_2 \land \varphi_1$

$$\equiv \exists y.\forall x.\left(\left(\exists x \underbrace{\forall y. \neg P(x,y)}_{\varphi_1}\right) \wedge \underbrace{P(x,y)}_{\varphi_2}\right) \qquad free(\varphi_2)=\{x,y\}$$

$$(\exists x.\varphi_1)\wedge\varphi_2 \equiv \exists x(\varphi_1\wedge\varphi_2) \text{ dacă } x\notin free(\varphi_2)$$

$$LR: \quad \exists x.\varphi_1 \equiv \exists z.\ \sigma^b(\varphi_1) \qquad \sigma=\{x\mapsto z\} \qquad z\notin free(\varphi_1)$$

$$\exists x.\underbrace{(Q(x)\wedge\forall x.Q(x))}_{\varphi_1} \equiv \exists z.(Q(z)\wedge\forall x.Q(x))$$

$$\exists x.P(x,y) \not\equiv \exists y.P(y,y)$$

$$\equiv \exists y.\forall x.\left(\left(\exists x_1.\underbrace{\forall y.\neg P(x_1,y)}_{\varphi_1}\right)\wedge\underbrace{P(x,y)}_{\varphi_2}\right)$$

$$\equiv \exists y.\forall x.\exists x_1.\left((\forall y.\neg P(x_1,y))\wedge P(x,y)\right)$$

$$\overset{LR}{\equiv} \exists y.\forall x.\exists x_1.\left((\forall y_1.\neg P(x_1,y_1))\wedge P(x,y)\right)$$

$$\equiv \exists y.\forall x.\exists x_1.\forall y_1.\left(\neg P(x_1,y_1)\wedge P(x,y)\right) = \varphi_1 \text{ - FNP}$$

- închiderea existențială nu este necesară (φ_1 este închis) $(\varphi_2=\varphi_1)$
- skolemizare

Fie a un simb. fct nou de aritate 0 $\qquad \Sigma=(\mathcal{P},\mathcal{F})$ pt φ_1

$\qquad \Sigma'=(\mathcal{P},\mathcal{F}\cup\{a\})$ pt φ_3'

$$\varphi_3' = \forall x.\exists x_1.\forall y_1.\left(\neg P(x_1,y_1)\wedge P(x,a)\right) \text{ — echisat } \varphi_1$$

Fie h un simb fct nou de aritate 1.

$\varphi_3 = \forall x . \forall y_1 . \left(\underbrace{\underbrace{\neg P(h(x), y_1)}_{C_1} \wedge \underbrace{P(x, a)}_{C_2}}_{\varphi' - FNC} \right)$ – FNS – echisat φ_3'

FNSC

1. $\neg P(h(x), y_1)$
2. $P(x, a)$
3. $\square$ RB 2, 1 $P(x, a)$ $\neg P(h(x'), y_1)$

$\mathbb{P} = \{ x \doteq h(x'), \underline{a \doteq y_1} \} \xrightarrow{ORIENT}$

$\{ x \doteq h(x'), y_1 \doteq a \}$ formă rez.

$\sigma = mgu(\mathbb{P}) = \{ x \mapsto h(x), y_1 \mapsto a \}$

$\Rightarrow \varphi_3$ nesatisf. $\Big| \Rightarrow \varphi_3'$ nesat $\Big| \Rightarrow \varphi_1$ nesat $\Big| \Rightarrow \neg\varphi$ nesat $\Rightarrow$

φ_3 echisat φ_3' φ_3' echisat φ_1 $\varphi_1 \equiv \neg\varphi$

$\Rightarrow \varphi$ valid

$$FP \quad \frac{P(t_1 \ldots t_n) \vee P(t_1' \ldots t_n') \vee C}{\sigma^b (P(t_1 \ldots t_n) \vee C)}$$

$\mathbb{P} = \{ t_1 \doteq t_1', \ldots t_n = t_n' \}$

$\sigma = mgu(\mathbb{P})$

$$RB \quad \frac{P(t_1 \ldots t_n) \vee C_1 \quad \neg P(t_1' \ldots t_n') \vee C_2}{\sigma^b (C_1 \vee C_2)}$$

$V_1 \cap V_2 = \phi$

$\mathbb{P} = \{ t_1 \doteq t_1', \ldots t_n \doteq t_n' \}$

$\sigma = mgu(\mathbb{P})$

$P(e) \vee P(i(x)) \vee C$

$\mathbb{P} = \{ e \doteq i(x) \} \xrightarrow{conf} \perp$

$unif(\mathbb{P}) = \phi$

nu pot aplica FP

$P(x) \vee C_1$ $\neg P(i(x')) \vee C_2$

Orice mașină este roșie. "pt orice el, dacă este mașină, atunci este roșie"

$\forall x$

$$\left(\forall x.\left(\text{Mașină}(x) \to \text{Roșu}(x)\right)\right)$$

$\text{Mașină}(u)$ = u este mașină

$\text{Roșu}(u)$ = u este roșu

$$\Sigma = (\{\text{Mașină}, \text{Roșu}\}, \emptyset)$$

$ar(\text{Mașină}) = 1$

$ar(\text{Roșu}) = 1$.

Ex 129

7)

~~k. $\neg(P(a) \wedge Q(a)), P(a) \vdash \bot$~~

~~j. $\neg(P(a) \wedge Q(a)) \vdash \neg P(a)$ $(\neg i, k)$~~

~~h. $\neg(P(a) \wedge Q(a)) \vdash \neg P(a) \vee \neg Q(a)$ $(\vee i_1, j)$~~

1. $\Gamma \vdash \neg(P(a) \wedge Q(a))$ (ip)

2. $\Gamma, \neg P(a) \vdash \neg(\neg P(a) \vee \neg Q(a))$ (ip)

3. $\Gamma, \neg P(a) \vdash \neg P(a)$ (ip)

4. $\Gamma, \neg P(a) \vdash \neg P(a) \vee \neg Q(a)$ $(\vee i_1, 3)$

b. $\Gamma, \neg P(a) \vdash \bot$ $(\neg e, 4, 2)$

a_1. $\Gamma \vdash P(a)$ (PBC, b)

analog cu a_1 (pp. pe $\neg Q(a)$)

$a_2.\quad \Gamma \vdash Q(a)$

$m.\quad \Gamma \vdash P(a) \land Q(a) \quad (\land i,\ a_1, a_2)$

$j\quad \overbrace{\neg(P(a) \land Q(a)),\ \neg(\neg P(a) \lor \neg Q(a))}^{\Gamma} \vdash \bot \quad (\neg e, 1, m)$

$k.\quad \neg(P(a) \land Q(a)) \vdash \neg\neg(\neg P(a) \lor \neg Q(a)) \quad (\neg i, j)$

$n.\quad \neg(P(a) \land Q(a)) \vdash \neg P(a) \lor \neg Q(a) \quad (\neg\neg e, k)\ (PBC, j)$

Ex 33) h)

$\Sigma = (\{ \overset{1}{Nat}, \overset{1}{Int}, \overset{1}{Prim}, \overset{1}{Par}, \overset{2}{>} \}, \{ \overset{2}{+}, \overset{0}{0}, \overset{0}{1}, \overset{0}{2} \})$ aritǎți

$ar(Nat) = 1$

Orice nr prim poate fi scris ca sumă a 4 nr prime.

$\forall$

$$\forall x. \Big(Prim(x) \longrightarrow \big(\exists y_1. \exists y_2. \exists y_3. \exists y_4. (Prim(y_1) \land Prim(y_2) \land Prim(y_3) \land Prim(y_4) \land \neg > (x, +(+(y_1, y_2), +(y_3, y_4))) \land \neg > (+(+(y_1, y_2), +(y_3, y_4)), x)) \big) \Big)$$

$a = b$ ddacă $a \not> b$ și $b \not> a$

Ex 80 $\quad \varphi = \forall x. \exists y. P(x, y)$ nu este validă

φ validă ddacă pt orice S și pt orice S-atr α avem $S, \alpha \vDash \varphi$

φ nu este validă ddacă există S și există o S-atr α a.î. $S, \alpha \nvDash \varphi$

Căutăm S, α a.î. $S, \alpha \nvDash \varphi$

$S, \alpha \not\models \varphi$ ddacă $S, \alpha \not\models \forall x. \exists y. P(x,y)$

ddacă NU e adevarat că $S, \alpha \models \forall x. \exists y. P(x,y)$

ddacă NU e adevarat că pt orice $u \in D$ avem $S, \alpha[x \mapsto u] \models \exists y. P(x,y)$

ddacă NU e adevarat că pt orice $u \in D$, există $v \in D$ a.î.

$$S, \underbrace{\alpha[x \mapsto u][y \mapsto v]}_{\alpha'} \models P(x, y)$$

ddacă NU e adevarat că pt orice $u \in D$, există $v \in D$ a.î. $P^S(\overline{\alpha'}(x), \overline{\alpha'}(y))$

$\alpha : \mathcal{X} \to D$ $\qquad$ $\overline{\alpha} : \mathcal{T} \to D$

$\alpha(x)$

$$\overline{\alpha}(t) = \begin{cases} \alpha(x), & t \in \mathcal{X} \\ c^S, & c \in \mathcal{F}_0 \\ f^S(\overline{\alpha}(t_1) \ldots \overline{\alpha}(t_n)), & t = f(t_1, \ldots t_n),\ n \geq 1 \end{cases}$$

ddacă NU e adevarat că pt orice $u \in D$, există $v \in D$ a.î. $P^S(\alpha'(x), \alpha'(y))$

$\alpha(x) = \ldots$	$\alpha[x \mapsto u](x) = u$	$\alpha'(x) = u$
$\alpha(y) = \ldots$	$(y) = \alpha(y)$	$\alpha'(y) = v$
$\alpha(z) = \ldots$	$(z) = \alpha(z)$	$\alpha'(z) = \alpha(z)$

ddacă NU e adevarat că pt orice $u \in D$, există $v \in D$ a.î. $P^S(u, v)$ $\quad$ "A"

$\Sigma = (\{\overset{2}{P}\}, \{\overset{2}{f}, \overset{1}{i}, \overset{0}{e}\})$

Fie $S = (\mathbb{N}, \{>\}, \{+, succ, 0\})$

$\alpha : \mathcal{X} \to$

pt orice $u \in \mathbb{N}$, există $v \in \mathbb{N}$ a.î. $u > v$ $\quad$ "F"

$u = 0$, nu există v a.î. $0 > v$

$\Rightarrow S, \alpha \not\models \varphi \Rightarrow \varphi$ nu este validă.

Viners 13^{00}.