$\models_S$

Ex 73

$\forall x.\forall y.\forall z.\left(P(x,y)\wedge P(y,z)\to P(x,z)\right),\ P(x_1,x_2),\ P(x_2,x_3)\models P(x_1,x_3)$

ddacă pt orice structură S și orice S-atr α a.î.

oricе "sub"

$S,\alpha\models\forall x.\forall y.\forall z.\left(P(x,y)\wedge P(y,z)\to P(x,z)\right)$ (1)

și

$S,\alpha\models P(x_1,x_2)$ (2)

și

$S,\alpha\models P(x_2,x_3)$ (3)

este "roșu"

~~avem și~~ atunci $S,\alpha\models P(x_1,x_3)$ (4)

Fie S și α o structură și o S-atr oarecare cu

propr că (1), (2) și (3) sunt "A"

Arătăm că (4) este "A".

(1) ddacă pt orice $u\in D$ avem

$S,\underbrace{\alpha[x\mapsto u]}_{\alpha_1}\models\forall y.\forall z.\left(P(x,y)\wedge P(y,z)\to P(x,z)\right)$

ddacă pt orice $u\in D$ avem pt orice $v\in D$ avem

$S,\alpha_1[y\mapsto v]\models\forall z.\left(P(x,y)\wedge P(y,z)\to P(x,z)\right)$

$\alpha_2=\alpha[x\mapsto u][y\mapsto v]$

ddacă pt orice $u\in D$ avem pt orice $v\in D$ avem pt orice $w\in D$ avem

$S,\underbrace{\alpha_2[z\mapsto w]}_{\alpha_3}\models\left(\left(P(x,y)\wedge P(y,z)\right)\to P(x,z)\right)$

ddacă pt orice $u\in D$ avem pt orice $v\in D$ avem pt orice $w\in D$ avem

$S,\alpha_3\not\models P(x,y)\wedge P(y,z)$

sau

$S,\alpha_3\models P(x,z)$

ddacă pt orice $u \in D$ avem pt orice $v \in D$ avem pt orice $w \in D$ avem

$$\text{Nu}\left(\underbrace{S, \alpha_3 \models P(x, y)}_{A} \text{ și } \underbrace{S, \alpha_3 \models P(y, z)}_{B}\right)$$

sau

$$S, \alpha_3 \models P(x, z)$$

ddacă pt orice $u \in D$ avem pt orice $v \in D$ avem pt orice $w \in D$ avem

$$\left(\text{Nu } S, \alpha_3 \models P(x, y)\right) \text{ sau } \left(\text{Nu } S, \alpha_3 \models P(y, z)\right)$$

sau

$$S, \alpha_3 \models P(x, z)$$

ddacă pt orice $u \in D$, pt orice $v \in D$, pt orice $w \in D$ avem

$P^S(\overline{\alpha_3}(x), \overline{\alpha_3}(y))$ nu are loc (este fals)

sau

$P^S(\overline{\alpha_3}(y), \overline{\alpha_3}(z))$ nu are loc

sau

$P^S(\overline{\alpha_3}(x), \overline{\alpha_3}(z))$ are loc.

$\alpha : X \to D$	$\alpha_1 = \alpha[x \mapsto u]$	$\alpha_2 = \alpha_1[y \mapsto v]$	$\alpha_3 = \alpha_2[z \mapsto w]$
$\alpha(x) = \_$	$\alpha_1(x) = u$	$\alpha_2(x) = u$	$\alpha_3(x) = u$
$\alpha(y) = \ominus$	$\alpha_1(y) = \alpha(y)$	$\alpha_2(y) = v$	$\alpha_3(y) = v$
$\alpha(z) = \ominus$	$\alpha_1(z) = \alpha(z)$	$\alpha_2(z) = \alpha(z)$	$\alpha_3(z) = w$
$\alpha(x_1) = \ominus$ pt $x_1 \in X \setminus \{x, y, z\}$	$\alpha_1(x_1) = \alpha(x_1)$	$\alpha_2(x_1) = \alpha(x_1)$	$\alpha_3(x_1) = \alpha(x_1)$ pt orice $x_1 \in X \setminus \{x, y, z\}$

$$\alpha_3 = \alpha[x \mapsto u][y \mapsto v][z \mapsto w]$$

$$\alpha[x \mapsto u][y \mapsto v][x \mapsto w]$$

$\alpha(x) = w$

$\alpha(y) = v$

$\alpha : X \to D \longrightarrow \overline{\alpha} : T \to D$

$$\overline{\alpha}(t) = \begin{cases} \alpha(x) & , t = x \in X \\ c^S & , t = c \in F_0 \\ f^S(\overline{\alpha}(t_1) \dots \overline{\alpha}(t_n)) & , t = f(t_1, \dots t_n) \quad n \geq 1 \end{cases}$$

ddacă pt orice $u \in D$, pt orice $v \in D$, pt orice $w \in D$ avem

$P^S(\alpha_3(x), \alpha_3(y))$ nu are loc (este fals)

sau

$P^S(\alpha_3(y), \alpha_3(z))$ nu are loc

sau

$P^S(\alpha_3(x), \alpha_3(z))$ are loc.

ddacă pt orice $u \in D$, pt orice $v \in D$, pt orice $w \in D$ avem

$P^S(u, v)$ nu are loc (este fals)

sau

$P^S(v, w)$ nu are loc

sau

$P^S(u, w)$ are loc. 1

2 ddacă $S, \alpha \models P(x_1, x_2)$ ddacă $P^S(\overline{\alpha}(x_1), \overline{\alpha}(x_2))$ are loc.

ddacă $P^S(\alpha(x_1), \alpha(x_2))$ are loc 2

3 ddacă $S, \alpha \models P(x_2, x_3)$ ddacă $P^S(\overline{\alpha}(x_2), \overline{\alpha}(x_3))$ are loc.

ddacă $P^S(\alpha(x_2), \alpha(x_3))$ are loc 3

Tb. să dem (4) ddacă $S, \alpha \models P(x_1, x_3)$ ddacă

$P^S(\alpha(x_1), \alpha(x_3))$ are loc

Din (1) și $u = \alpha(x_1)$, $v = \alpha(x_2)$, $w = \alpha(x_3)$ $\Rightarrow$

$$\text{"A"} \begin{cases} P^S(\alpha(x_1), \alpha(x_2)) \text{ nu are loc} & \text{"F" (din (2))} \\ \text{sau} & \\ P^S(\alpha(x_2), \alpha(x_3)) \text{ nu are loc} & \text{"F" (din (3))} \\ \text{sau} & \\ P^S(\alpha(x_1), \alpha(x_3)) \text{ are loc} & \end{cases}$$

$\Rightarrow P^S(\alpha(x_1), \alpha(x_3))$ are loc $\Rightarrow$

$\Rightarrow S, \alpha \models P(x_1, x_3)$

Dar S, α erau <u>oarecare</u> a.î. (1), (2), (3) $\Big| \Rightarrow$

$\Rightarrow \forall x. \forall y. \forall z. (P(x,y) \wedge P(y,z) \rightarrow P(x,z)), P(x_1, x_2), P(x_2, x_3) \models P(x_1, x_3)$

Ex. $Q(e), \neg Q(e) \models P(x)$

pt orice S, α a.î. $S, \alpha \models Q(e)$ și $S, \alpha \models \neg Q(e)$

avem și $S, \alpha \models P(x)$

pt orice S, α

Dacă $S, \alpha \models Q(e)$ și $S, \alpha \models \neg Q(e)$

Atunci $S, \alpha \models P(x)$

pt orice S, α

Dacă $Q^S(\bar{\alpha}(e))$ are loc și $Q^S(\bar{\alpha}(e))$ nu are loc

Atunci $S, \alpha \models P(x)$

mulțime inconsistentă

pt orice S, α

Dacă $Q^S(e^S)$ are loc și $Q^S(e^S)$ nu are loc

fals

"1"

Atunci $S, \alpha \models P(x)$

$\Rightarrow Q(e), \neg Q(e) \models P(x)$

Ex 84 $\varphi = \forall x. \neg P(x,x) \wedge \exists x. P(x,x)$ nu este satisf.

Rationament Semantic $\to$ def.

Rationament Sintactic $\to$ Rezoluția

φ nu este satisf ddacă

Nu există o struct. S, există o S-atr α a.î. $S,\alpha \models \varphi$

ddacă pt orice S și pt orice S-atr α avem $S,\alpha \not\models \varphi$ (*)

Fie S, α alese arbitrar

Arătăm $S,\alpha \not\models \varphi$ ddacă Nu are loc $S,\alpha \models \varphi$

$S,\alpha \models \forall x. (\neg P(x,x) \wedge (\exists x. P(x,x)))$

ddacă pt orice $u \in D$ avem $S, \alpha[x \mapsto u] \models \neg P(x,x) \wedge \exists x. P(x,x)$

ddacă pt orice $u \in D$ avem $\begin{cases} S, \alpha[x\mapsto u] \models \neg P(x,x) \\ \text{și} \\ S, \alpha[x \mapsto u] \models \exists x. P(x,x) \end{cases}$

ddacă pt orice $u \in D$ avem $\begin{cases} S, \underbrace{\alpha[x\mapsto u]}_{\alpha_1} \not\models P(x,x) \\ \text{și} \\ \text{există } v \in D \text{ a.î. } S, \underbrace{\alpha[x\mapsto u][x\mapsto v]}_{\alpha_2 = \alpha_1[x\mapsto v]} \models P(x,x) \end{cases}$

ddacă pt orice $u \in D$ avem $\begin{cases} P^S(\overline{\alpha_1}(x), \overline{\alpha_1}(x)) \text{ nu are loc} \\ \text{și} \\ \text{există } v \in D \text{ a.î. } P^S(\overline{\alpha_2}(x), \overline{\alpha_2}(x)) \text{ are loc.} \end{cases}$

ddacă pt orice $u \in D$ avem $\begin{cases} P^S(u,u) \text{ nu are loc.} \\ \text{și} \\ \text{există } v \in D \text{ a.î. } P^S(v,v) \text{ are loc} \end{cases}$

ddacă $\begin{cases} \text{pt orice } u \in D \text{ avem } (P^S(u,u) \text{ nu are loc}) \\ \text{și} \\ \text{există } v \in D \text{ a.î. } P^S(v,v) \text{ are loc.} \end{cases}$

$u \cancel{P^S} u$ "F"

$v P^S v$

$\Rightarrow S, \alpha \models \varphi$ este fals $\Rightarrow S, \alpha \not\models \varphi$

S, α erau arbitrare | $\Rightarrow \varphi$ nu este satisf. (din ⊛)

$\varphi = P(x, x)$ satisf

ddacă există S și există o S-atr α a.î. $S, \alpha \models \varphi$

Căutam S, α a.î. $S, \alpha \models \varphi$

$S, \alpha \models \varphi$ ddacă $P^S(\alpha(x), \alpha(x))$ are loc ①

$\Sigma = (\{P\}, \{f, i, e\})$ $ar(P)=2$ $ar(f)=2$ $ar(i)=1$ $ar(e)=0$

Fie $S = (\underset{D}{\mathbb{N}}, \underset{P^S}{\{=\}}, \{\underset{f^S}{+}, \underset{i^S}{succ}, \underset{e^S}{0}\})$

$\alpha : X \to \mathbb{N}$ a.î. $\alpha(y) = 1$ pt orice $y \in X$

verificare

① ddacă $\alpha(x) = \alpha(x)$

ddacă $1 = 1$ "A" $\Rightarrow$ am găsit o struct S și o S-atr α a.î. $S, \alpha \models \varphi$

$\Rightarrow \varphi$ satisf

Ex 130

1. $\Gamma \vdash \exists x. \overbrace{\exists y. P(x, y)}^{\varphi}$ (ip)

2. $\underbrace{\Gamma, \exists y. P(x_0, y)}_{\Gamma'} \vdash \exists y. \underbrace{P(x_0, y)}_{\varphi'}$ (ip)

t_x, t_y nu conțin y_0

$y_0 \in vars(\Gamma', \varphi', \psi')$

$t_x = ?$ $\quad t_y = ?$

m. $\Gamma, \exists y. P(x_0, y), P(x_0, y_0) \vdash P(t_x, t_y)$

t_x t_y nu conțin x_0

i. $\Gamma, \exists y. P(x_0, y) \vdash \overbrace{P(t_x, t_y)}^{\psi'}$ ($\exists e$, 2, m) $\quad x_0 \notin vars(\Gamma, \psi, \varphi)$

j. $\Gamma \vdash \overbrace{P(t_x, t_y)}^{\psi}$ ($\exists e$, 1, i)

k. $\Gamma \vdash \exists x. P(x, t_y)$ ($\exists i$, j)

n. $\underbrace{\exists x. \exists y. P(x, y)}_{\Gamma} \vdash \exists y. \underbrace{\exists x. P(x, y)}$ ($\exists i$, k)

$$\exists i \; \frac{\Gamma \vdash \varphi[y \mapsto t]}{\Gamma \vdash \exists y. \varphi}$$

$$\exists e \; \frac{\Gamma \vdash \exists x. \varphi \qquad \Gamma, \varphi[x \mapsto x_0] \vdash \psi}{\Gamma \vdash \psi} \quad x_0 \notin vars(\Gamma, \varphi, \psi)$$

1. $\Gamma \vdash \exists x. \underbrace{\exists y. P(x, y)}_{\varphi}$ (ip)

2. $\underbrace{\Gamma, \exists y. P(x_0, y)}_{\Gamma_1} \vdash \exists y. \underbrace{P(x_0, y)}_{\varphi_1}$ (ip)

3. $\Gamma, \exists y. P(x_0, y), P(x_0, y_0) \vdash P(\underset{t_x}{x_0}, y_0)$ (ip)

4. $\Gamma, \exists y. P(x_0, y), P(x_0, y_0) \vdash \exists x. P(x, \underset{t_y}{y_0})$ $(\exists_i, 3)$

k. $\Gamma, \exists y. P(x_0, y), P(x_0, y_0) \vdash \exists y. \exists x. P(x, y)$ $(\exists_i, 4)$

$y_0 \notin vars(\Gamma, \varphi, \psi)$ ✓ ✓ ✓

j. $\Gamma, \exists y. P(x_0, y) \vdash \exists y. \exists x. P(x, y)$ $(\exists e, 2, k)$

n. $\underbrace{\exists x. \exists y. P(x, y)}_{\Gamma} \vdash \underbrace{\exists y. \exists x. P(x, y)}_{\psi}$ $(\exists e, 1, j)$

<u>Ex 143</u> 9) $\overbrace{\forall x. P(x, x) \vee Q(x)}^{\varphi_1} \not\models \overbrace{(\forall x. P(x, x)) \vee (\forall x. Q(x))}^{\varphi_2}$

ddacă NU pt orice S și pt orice S-atr α avem

$S, \alpha \models \varphi_1$ ddacă $S, \alpha \models \varphi_2$

ddacă există S și există o S-atr α a.î.

$\left(\underset{①\text{"A"}}{S, \alpha \models \varphi_1} \text{ și } \underset{②\text{"F"}}{S, \alpha \not\models \varphi_2} \right)$

sau

$\left(\underset{①\text{"F"}}{S, \alpha \not\models \varphi_1} \text{ și } \underset{②\text{"A"}}{S, \alpha \models \varphi_2} \right)$ $(*)$

Căutăm S, α a.î. $(*)$ "A".

$S, \alpha \models \forall x.(P(x, x) \vee Q(x))$

ddacă pt orice $u \in D$ avem $S, \alpha[x \mapsto u] \models P(x, x) \vee Q(x)$

ddacă pt orice $u \in D$ avem $\begin{cases} S, \alpha[x \mapsto u] \models P(x, x) \\ \text{sau} \\ S, \alpha[x \mapsto u] \models Q(x) \end{cases}$

I

ddacă pt orice $u \in D$ avem $\begin{cases} P^S\left(\overline{\alpha[x \mapsto u]}(x), \overline{\alpha[x \mapsto u]}(x)\right) \\ \text{sau} \\ Q^S\left(\overline{\alpha[x \mapsto u]}(x)\right) \end{cases}$

ddacă pt orice $u \in D$ avem $\begin{cases} P^S(u, u) \\ \text{sau} \\ Q^S(u) \end{cases}$

1 $S, \alpha \models \varphi_1$

A

$S, \alpha \models (\forall x. P(x, x)) \vee (\forall x. Q(x))$

ddacă $\begin{cases} S, \alpha \models \forall x. P(x, x) \\ \text{sau} \\ S, \alpha \models \forall x. Q(x) \end{cases}$

......

ddacă $\begin{cases} \text{pt orice } u \in D, \ P^S(u, u) \\ \text{sau} \\ \text{pt orice } v \in D, \ Q^S(v) \end{cases}$

2 $S, \alpha \models \varphi_2$

F

$$\begin{array}{ccccccccc} & Q^S & & Q^S & & Q^S & \ldots & \ldots & \\ & P^S & & P^S Q^S & & P^S & \ldots & \ldots & \\ D = \{ & u_1 & , & u_2 & , & u_3 & , & \_\_ & , \ \_\_ \ \} \\ & P^S & & Q^S & & P^S, Q^S & & & \end{array}$$

~~$P^S(x, y)$: x predecesor lui y~~ ~~$P^S(u, u)$~~

$\Sigma = (\{P, Q\}, \{f, i, e\})$

Fie $S = (\mathbb{N}, \{BothNull, NotNull\}, \{+, succ, 0\})$

$P^S = BothNull(u, v)$ "A" ddacă $u = 0$ și $v = 0$

$BN(u,v) = \begin{cases} 1 & u = v = 0 \\ 0 & u \neq 0 \text{ sau } v \neq 0 \end{cases}$

$Q^S = NotNull(u)$ "A" ddacă $u \neq 0$

Fie $\alpha : X \to \mathbb{N}$, $\alpha(x) = 1$ pt orice $x \in X$

În S, α alese ① "A" ($P^S(u, u)$ ddacă $u = 0$)

$Q^S(u)$ ddacă $u \neq 0$

$\Rightarrow$

② "F"

$\Rightarrow \varphi_1 \not\equiv \varphi_2$

207. 2)

Ex 33 h)

$S = (\mathbb{R}, \{Nat, Int, Prim, Par, >\}, \{+, 0, 1, 2\})$

$\Sigma = (\{Nat, Int, Prim, Par, >\}, \{+, 0, 1, 2\})$
$\quad\quad f, a, b, c$

Orice nr Par poate fi scris ca suma a două nr pare.

$\forall x. (Par(x) \to \exists y. \exists z. (\neg(x > y+z) \wedge Par(y) \wedge Par(z)))$
$\wedge \neg(y+z > x)$

$a = b$ ddacă $a \not> b$ și $b \not> a$

Ex 130 7)

1. $\overbrace{\forall x. \neg P(x), \exists x. P(x)}^{\Gamma_1} \vdash \exists x. \overbrace{P(x)}^{\varphi}$ (ip)

2. $\forall x. \neg P(x), \exists x. P(x), P(x_0) \vdash P(x_0)$ (ip)

3. $\forall x. \neg P(x), \exists x. P(x), P(x_0) \vdash \forall x. \neg P(x)$ (ip)

4. $\forall x. \neg P(x), \exists x. P(x), P(x_0) \vdash \neg P(x_0)$ $(\forall e, 3, x_0)$

k. $\forall x. \neg P(x), \exists x. P(x), P(x_0) \vdash \bot$ $(\neg e, 2, 4)$

j. $\forall x. \neg P(x), \exists x. P(x) \vdash \overset{\psi}{\bot}$ $(\exists e, 1, k)$

n. $\forall x. \neg P(x) \vdash \neg \exists x. P(x)$ $(\neg i, j)$

$$\neg i \ \frac{\Gamma_1, \varphi \vdash \bot}{\Gamma \vdash \neg \varphi} \qquad \exists e \ \frac{\Gamma_1 \vdash \exists x. \varphi \quad \Gamma_1, \varphi[x \mapsto x_0] \vdash \psi}{\Gamma_1 \vdash \psi} \ x_0 \notin vars(\Gamma, \varphi, \psi)$$

Ex 207 2)

$$\varphi = \Big(\big(\forall x. \forall y. \forall z. (P(x,y) \wedge P(y,z) \to P(x,z)) \big) \wedge P(x,y) \wedge P(y,x) \Big) \to P(x,x)$$

φ validă ddacă $\neg\varphi$ nesat.

$$\neg\varphi = \neg \Big(\underbrace{\big(\forall x. \forall y. \forall z. \overbrace{(P(x,y) \wedge P(y,z) \to P(x,z))}^{\varphi_2} \big) \wedge P(x,y) \wedge P(y,x)}_{\varphi_1} \to P(x,x) \Big)$$

1. FNP

$$\neg\varphi \equiv \neg \big(\neg \varphi_1 \vee P(x,x) \big) \equiv \varphi_1 \wedge \neg P(x,x)$$

$$\varphi_1 = (\underbrace{\forall x. \forall y. \forall z.\, \varphi_2}_{\psi_1}) \wedge \underbrace{P(x,y) \wedge P(y,x)}_{\psi_2}$$

$$(\forall x.\, \psi_1) \wedge \psi_2 \equiv \forall x. (\psi_1 \wedge \psi_2) \text{ dacă } x \notin free(\psi_2)$$

$$\overset{LR}{\equiv} \left(\forall x_1. \underbrace{\forall y. \forall z. (P(x_1, y) \wedge P(y,z) \to P(x_1, z))}\right) \wedge P(x,y) \wedge P(y,x)$$

$$\equiv \forall x_1. \left(\left(\forall y. \underbrace{\forall z. (P(x_1,y) \wedge P(y,z) \to P(x_1,z))}\right)\right) \wedge P(x,y) \wedge P(y,x)$$

$$\overset{LR}{\equiv} \forall x_1. \left(\left(\forall y_1. \forall z. (P(x_1,y_1) \wedge P(y_1,z) \to P(x_1,z))\right)\right) \wedge P(x,y) \wedge P(y,x)$$

$$\equiv \forall x_1. \forall y_1. \forall z. \left(\left(P(x_1,y_1) \wedge P(y_1,z) \to P(x_1,z)\right) \wedge P(x,y) \wedge P(y,x)\right)$$

$$\neg\varphi \equiv \left(\forall x_1. \forall y_1. \forall z. \left(\left(P(x_1,y_1) \wedge P(y_1,z) \to P(x_1,z)\right) \wedge P(x,y) \wedge P(y,x)\right)\right) \wedge \neg P(x,x)$$

$$\equiv \ldots$$

$$\equiv \forall x_1.\forall y_1.\forall z.\left(\left(P(x_1,y_1)\wedge P(y_1,z)\to P(x_1,z)\right)\wedge P(x,y)\wedge P(y,x)\wedge \neg P(x,x)\right)$$

FNP

2. închidere existențială

$$\varphi_2' = \exists x.\exists y.\forall x_1.\forall y_1.\forall z.\left(\left(P(x_1,y_1)\wedge P(y_1,z)\to P(x_1,z)\right)\wedge P(x,y)\wedge P(y,x)\wedge \neg P(x,x)\right)$$

echisat $\neg\varphi$

3. fie a simb fct nou de aritate 0

$$\varphi_3 = \exists y.\forall x_1.\forall y_1.\forall z.\left(\left(P(x_1,y_1)\wedge P(y_1,z)\to P(x_1,z)\right)\wedge P(a,y)\wedge P(y,a)\wedge \neg P(a,a)\right)$$

fie b simb fct nou de aritate 0

$$\varphi_4 = \forall x_1.\forall y_1.\forall z.\left(\left(P(x_1,y_1)\wedge P(y_1,z)\to P(x_1,z)\right)\wedge P(a,b)\wedge P(b,a)\wedge \neg P(a,a)\right)$$

FNS

$$\equiv \forall x_1.\forall y_1.\forall z.\left(\left(\neg\left(P(x_1,y_1)\wedge P(y_1,z)\right)\vee P(x_1,z)\right)\wedge P(a,b)\wedge P(b,a)\wedge \neg P(a,a)\right)$$

$$\equiv \forall x_1.\forall y_1.\forall z.\left(\left(\neg P(x_1,y_1)\vee\neg P(y_1,z)\vee P(x_1,z)\right)\wedge P(a,b)\wedge P(b,a)\wedge \neg P(a,a)\right) = \varphi_5$$

FNSC.

1. $\neg P(x_1, y_1) \vee \neg P(y_1, z) \vee P(x_1, z)$ (ip)

2. $P(a, b)$ (ip)

3. $P(b, a)$ (ip)

4. $\neg P(a, a)$ (ip)

3, 4

$\mathbb{P} = \{ b \doteq a,\ a \doteq a \} \xrightarrow{\text{conflict}} \bot$

5. $\neg P(a, z) \vee P(b, z)$ RB 3, 1

$\mathbb{P}_1 = \{ b \doteq x_1,\ a \doteq y_1 \} \xrightarrow{\text{ORIENT}} \{ x_1 \doteq b,\ a \doteq y_1 \}$ -

$\xrightarrow{\text{ORIENT}} \{ x_1 \doteq b,\ y_1 \doteq a \}$ formă rez.

$\sigma_1 = mgu(\mathbb{P}_1) = \{ x_1 \mapsto b,\ y_1 \mapsto a \}$

6. $P(b, b)$ RB 2, 5

$\mathbb{P}_2 = \{ a \doteq a,\ b \doteq z \} \xrightarrow{\text{STERG}} \{ b \doteq z \}$

$\xrightarrow{\text{ORIENT}} \{ z \doteq b \}$

$\sigma_2 = \{ z \mapsto b \}$

7. $\neg P(a, y_1) \vee \neg P(y_1, a)$ RB 1, 4

$\mathbb{P}_3 = \{ x_1 \doteq a,\ z \doteq a \}$

$\sigma_3 = \{ x_1 \mapsto a,\ z \mapsto a \}$

8. $\neg P(b, a)$ RB 2, 7 $\quad \mathbb{P}_4 = \{ a \doteq a,\ b \doteq y_1 \} \Rightarrow \ldots$

$\sigma_4 = \{ y_1 \mapsto b \}$

9. $\square$ RB 3, 8 $\quad \mathbb{P}_5 = \{ b \doteq b,\ a \doteq a \} \xrightarrow{\text{STERG}} \ldots \Rightarrow \{ \}$

$\sigma_5 = \{ \}$ $\quad (\sigma_5 : X \to T,\ \sigma_5(x) = x)$

$\Rightarrow \varphi_5$ nesatisf
$\varphi_5 \equiv \varphi_4$

$\Rightarrow \varphi_4$ nesat
φ_4 echisat φ_3

$\Rightarrow \varphi_3$ nesat
φ_3 echisat φ_2'

$\Rightarrow$

$\Rightarrow \varphi_2'$ nesat
φ_2' echisat $\neg\varphi$

$\Rightarrow \neg\varphi$ nesat $\Rightarrow \varphi$ valid