(1)

1. $\underbrace{(q \vee (r_3 \wedge r_4)), \neg q}_{\Gamma'} \vdash (\overset{\varphi_3}{q} \vee \overset{\varphi_4}{(r_3 \wedge r_4)})$ (ip)

2. $(q \vee (r_3 \wedge r_4)), \neg q, q \vdash q$ (ip)

3. $(q \vee (r_3 \wedge r_4)), \neg q, q \vdash \neg q$ (ip)

4. $(q \vee (r_3 \wedge r_4)), \neg q, q \vdash \bot$ $(\neg e, 2, 3)$

$$\bot_e \frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}$$

x_1. $(q \vee (r_3 \wedge r_4)), \neg q, q \vdash (r_3 \wedge r_4)$ $(\bot e, 4)$

x_2. $(q \vee (r_3 \wedge r_4)), \neg q, (r_3 \wedge r_4) \vdash (r_3 \wedge r_4)$ (ip)

k. $(q \vee (r_3 \wedge r_4)), \neg q \vdash \overset{\varphi}{(r_3 \wedge r_4)}$ $(\vee e, 1, x_1, x_2)$

n. $(q \vee (r_3 \wedge r_4)) \vdash (\underbrace{\neg q}_{\varphi_1} \to \underbrace{(r_3 \wedge r_4)}_{\varphi_2})$ $(\to_i, k)$

$$\frac{\Gamma, \varphi_1 \vdash \varphi_2}{\Gamma \vdash \varphi_1 \to \varphi_2}$$

$$\vee_e \frac{\Gamma' \vdash \varphi_3 \vee \varphi_4 \qquad \Gamma', \varphi_3 \vdash \varphi \qquad \Gamma', \varphi_4 \vdash \varphi}{\Gamma' \vdash \varphi}$$

$$\forall_e \frac{\Gamma \vdash \forall x . \varphi}{\Gamma \vdash \varphi[x \mapsto t]}$$

$$\forall_i \frac{\Gamma \vdash \varphi[x \mapsto x_0]}{\Gamma \vdash \forall x . \varphi} \quad x_0 \notin vars(\Gamma, \varphi)$$

1. $\Gamma \vdash \forall x . \overset{\varphi}{\boxed{P(x, y)}}$

2. $\Gamma \vdash P(e, y)$ $(\forall e, 1)$

$i(z)$

y

3. $\Gamma \vdash \forall x . \underbrace{\boxed{\exists x . P(x, y)}}_{\varphi}$

legat -> nu poate fi înlocuit.

4. $\Gamma \vdash \exists x . P(x, y)$ $(\forall e, 3)$

variabilă nouă

1. $\Gamma \vdash P(x, y)$

2. $\Gamma \vdash \forall x . \underbrace{P(x, y)}_{\varphi}$ $(\forall_i, 1)$

$$\exists_i \ \frac{\Gamma \vdash \varphi[x \mapsto t]}{\Gamma \vdash \exists x.\varphi}$$

$$\exists e \ \frac{\Gamma \vdash \exists x.\varphi \qquad \Gamma, \varphi[x \mapsto x_0] \vdash \psi}{\Gamma \vdash \psi} \quad x_0 \notin vars(\Gamma, \varphi, \psi)$$

1. $\Gamma \vdash P(\underbrace{i(e)}_{t}, \underbrace{y}_{t'})$
2. $\Gamma \vdash \exists x. P(x, y)$ $(\exists_i, 1)$
3. $\Gamma \vdash \exists z. P(i(e), z)$ $(\exists_i, 1)$

1. $\Gamma \vdash \exists x. \overbrace{P(x)}^{\varphi}$
2. ~~$\Gamma, P(y_0) \vdash P(y_0)$ (ip)~~ !!
3. $\Gamma \vdash \underbrace{P(y_0)}_{\psi}$

orice variabilă, dar NU y_0, x sau var din Γ

$$\vee_e \ \frac{\Gamma \vdash \varphi_1 \vee \varphi_2 \qquad \Gamma, \varphi_1 \vdash \psi \qquad \Gamma, \varphi_2 \vdash \psi}{\Gamma \vdash \psi}$$

(2)

1. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash \forall y_3. (R(y_3) \to Q(y_3))$ (ip)
2. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash R(t_{x_2}) \to Q(t_{x_2})$ $(\forall e, 1)$
3. $\underbrace{\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3))}_{\Gamma} \vdash \exists x_2. \underbrace{R(x_2)}_{\varphi}$ (ip)

m. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)), R(x_0) \vdash R(t_{x_2})$

$x_0 \notin var(\Gamma, \varphi, \psi)$
$x_0 \notin vars(\Gamma)$ ✓
$x_0 \notin vars(\varphi)$ ✓
$x_0 \notin vars(\psi)$
⇓
t_{x_2} nu conține x_0

j. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash \underbrace{R(t_{x_2})}_{\psi}$ $(\exists e, 3, m)$
i. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash Q(t_{x_2})$ $(\to_i, 2, j)$
k. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash \exists x_2. Q(x_2)$ $(\exists_i, i)$
n. $\exists x_2. R(x_2), \ \forall y_3. (R(y_3) \to Q(y_3)) \vdash \neg\neg \exists x_2. Q(x_2)$ $(\neg\neg i, k)$

1. $\overbrace{\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3))}^{\Gamma} \vdash \exists x_2.\ \overbrace{R(x_2)}^{\varphi}$ (ip)

2. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)),\ R(x_0) \vdash \forall y_3.\ (R(y_3)\to Q(y_3))$ (ip)

3. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)),\ R(x_0) \vdash R(x_0)\to Q(x_0)$ ($\forall e$, 2)

4. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)),\ R(x_0) \vdash R(x_0)$ (ip)

j. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)),\ R(x_0) \vdash Q(x_0)$ ($\to e$, 3, 4)

$x_0 \notin vars(\Gamma)$ ✓
$x_0 \notin vars(\varphi)$ ✓
$x_0 \notin vars(\psi)$ ✓

m. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)),\ R(x_0) \vdash \exists x_2.\ Q(x_2)$ ($\exists i$, j)

k. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)) \vdash \underbrace{\exists x_2.\ Q(x_2)}_{\psi}$ ($\exists e$, 1, m)

n. $\exists x_2.\ R(x_2),\ \forall y_3.\ (R(y_3)\to Q(y_3)) \vdash \neg\neg\exists x_2.\ Q(x_2)$ ($\neg\neg i$, k)

(3) $\varphi = (\exists y.\ \neg(P(a) \vee \neg P(x)))$

$\varphi_a = \neg(\forall y.\ (\underbrace{P(b)}_{\varphi_3} \vee \underbrace{Q(a)}_{\varphi_4}))$

$\neg\forall x\ \varphi_1 \equiv \exists x \neg\varphi_1$

$\varphi_1 = \varphi_3 \vee \varphi_4 \qquad \neg\varphi = \neg\varphi_3 \wedge \neg\varphi_4$

$\varphi \equiv \varphi_a$ ddacă pt orice S și orice S-atr α

$S, \alpha \vDash \varphi$ ddacă $S, \alpha \vDash \varphi_a$

$S, \alpha \vDash \varphi$ ddacă există $u \in D$ a.î. $S, \alpha[y \mapsto u] \vDash \neg(P(a) \vee \neg P(x))$

ddacă există $u \in D$ a.î. Nu avem $\left[\begin{array}{l} S, \alpha[y \mapsto u] \vDash P(a) \\ \text{sau} \\ S, \alpha[y \mapsto u] \vDash \neg P(x) \end{array}\right.$

ddacă există $u \in D$ a.î. Nu avem $\left[\begin{array}{l} P^S(\overline{\alpha[y \mapsto u]}(a)) \\ \text{sau} \\ P^S(\overline{\alpha[y \mapsto u]}(x)) \text{ nu are loc} \end{array}\right.$

$S, \alpha \models \varphi$ ddacă ~~există $u \in D$ a.î.~~ Nu avem

$P^S(a^S)$ are loc (Par, 6)

sau

$P^S(\alpha(x))$ nu are loc (3)

A / ¬A / F

$S, \alpha \models \varphi_a$ ddacă Nu ~~pt orice $u \in D$~~ avem

$P^S(b^S)$ are loc ¬F (5)

sau

$Q^S(a^S)$ are loc F (Prim, 6)

¬F / A

$$\varphi_c = \neg(\exists x . P(a)) \wedge \neg(\exists y . \neg P(x))$$

~~Nu există $u \in D$~~ a.î. $P^S(a^S)$ și Nu (nu are loc $P^S(\alpha(x))$)

④ (Trec la logică doar dacă (învăț la logică) sau nu este adevărat că Terra este plată.)

p : Trec la logică

q : învăț la logică

r : este adevărat că Terra este plată

- $((p \to q) \vee \neg r)$
- $(p \to (q \vee \neg r))$

p dacă q $q \to p$

p doar dacă q $p \to q$

p dacă și numai dacă q

$p \leftrightarrow q$

⑤ $\left(\underbrace{(q_3 \wedge q_4)}_{\varphi} \to p\right) \wedge (r \to p) \vdash \left(\underbrace{(q_3 \wedge q_4)}_{\varphi} \vee r\right) \to p$

1. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash (\varphi \vee r)$ (ip)
2. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), \varphi \vdash (\varphi \to p) \wedge (r \to p)$ (ip)
3. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), \varphi \vdash (\varphi \to p)$ ($\wedge e_1$, 2)
4. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), \varphi \vdash \varphi$ (ip)

k_1. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), \varphi \vdash p$ ($\to e$, 3, 4)

x_1 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), r \vdash (\varphi \to p) \wedge (r \to p)$ (ip)

x_2. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), r \vdash (r \to p)$ ($\wedge e_2$, x_1)

x_3 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), r \vdash r$ (ip)

k_2 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), r \vdash p$ ($\to e$, x_2, x_3)

k. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash p$ ($\vee e$, 1, k_1, k_2)

n. $(\varphi \to p) \wedge (r \to p) \vdash \underbrace{(\varphi \vee r)}_{\varphi_1} \to \underbrace{p}_{\varphi_2}$ ($\to i$, k)

$$\to_i \; \frac{\Gamma, \varphi_1 \vdash \varphi_2}{\Gamma \vdash \varphi_1 \to \varphi_2} \qquad \to_e \; \frac{\Gamma \vdash \varphi_3 \to \varphi_4 \qquad \Gamma \vdash \varphi_3}{\Gamma \vdash \varphi_4}$$

$(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash (r \to p)$

x_1 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash (\varphi \vee r)$ (ip)

x_2 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), \varphi \vdash r$ (????)

x_3 $(\varphi \to p) \wedge (r \to p), (\varphi \vee r), r \vdash r$ (ip)

m. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash r$ ($\vee e$, x_1, x_2, x_3)

k. $(\varphi \to p) \wedge (r \to p), (\varphi \vee r) \vdash p$ ($\vee e$, 1, k_1, k_2)

Rez. φ - FNSC

RB
$$\frac{P(t_1,\dots,t_n)\vee C_1 \qquad \neg P(t'_1,\dots,t'_n)\vee C_2}{\sigma^b(C_1 \vee C_2)} \qquad V_1 \cap V_2 = \emptyset$$

$$\sigma = mgu(\{t_1 \doteq t''_1, \dots, t_n \doteq t_n\})$$

$$\varphi = \forall x.\forall y.\forall z.\big((Frate(x,y) \vee Angajat(coleg(x))) \wedge (\neg Frate(z, i)) \wedge \neg Angajat(z)\big)$$

$\forall z \; \forall z_1$

z_1

$Frate(x,y) \vee Angajat(coleg(x)) \qquad \neg Frate(z,i) \qquad \neg Angajat(z)$

$\neg Angajat(z_1)$

$x \mapsto z$
$y \mapsto i$

$Angajat(coleg(z))$

$\mathbb{P}_1 = \{coleg(z) \doteq z_1\} \xrightarrow{ORIENT}$

$\{z_1 \doteq coleg(z)\}$ formă rez.

$\sigma_1 = \{z_1 \mapsto coleg(z)\}$

$\square$

φ nesatisfiabilă

$P(e) \qquad \neg P(i(x))$

$\mathbb{P}_2 = \{e \doteq i(x)\} \xrightarrow{CONFLICT} \bot \Rightarrow$ NU aplicăm RB

FP
$$\frac{P(t_1 \dots t_n) \vee P(t'_1 \dots t'_n) \vee C}{\sigma^b(P(t_1 \dots t_n) \vee C)} \qquad \sigma = mgu(\{t_1 \doteq t'_1, \dots t_n \doteq t'_n\})$$

$$\forall x.(\varphi_1 \wedge \varphi_2) \equiv (\forall x.\varphi_1) \wedge \underbrace{(\forall x.\varphi_2)}_{LR} \equiv (\forall x.\varphi_1) \wedge (\forall x_1.\varphi_2[x \mapsto x_1])$$
$$\equiv \forall x.\forall x_1.(\varphi_1 \wedge \varphi_2[x \mapsto x_1])$$