$\sum |a_n|$ C $\Rightarrow \sum a_n$ AC

$\sum |a_n|$ D

$\sum a_n$ C $\wedge \sum |a_n|$ D mai spunem că seria este SC

Seminar 4

***Exerciţii recomandate:** 4.1(a-f), 4.2(a), 4.3(a-f)

***Rezerve:** 4.1(g,j,k,l), 4.3(g,i,k)

S4.1 Folosind diverse criterii de convergenţă, să se stabilească natura seriilor:

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot n}{2^{n-1}}$;

Studiem seria modulelor

$x_n = \frac{(-1)^{n-1} \cdot n}{2^{n-1}}, n \geq 1 \qquad |x_n| = \frac{n}{2^{n-1}}$

Aplicăm criteriul raportului

$$\frac{|x_{n+1}|}{|x_n|} = \frac{\frac{n+1}{2^n}}{\frac{n}{2^{n-1}}} = \frac{n+1}{2^n} \cdot \frac{2^{n-1}}{n} = \frac{n+1}{2n} \to \frac{1}{2} \xRightarrow{\text{Crit rap}}$$

$\sum x_n$ A.C.

b) $\sum_{n=0}^{\infty} (-1)^n \frac{\ln 2 + 3^n}{\ln 3 + 2^n}$;

$x_n = (-1)^n \frac{\ln 2 + 3^n}{\ln 3 + 2^n}$

$$|x_n| = \frac{\ln 2 + 3^n}{\ln 3 + 2^n} = \frac{\frac{\ln 2}{3^n} + 1}{\frac{\ln 3}{3^n} + \left(\frac{2}{3}\right)^n} \to \infty$$

$\Rightarrow x_n \not\to 0 \Rightarrow \sum x_n$ D

$$\sum \frac{1}{n^\alpha} \begin{cases} \alpha \le 1 & D \\ \alpha > 1 & C \end{cases}$$

c) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\sqrt{n}}{n+1}$;

$$x_n = (-1)^{n+1}\frac{\sqrt{n}}{n+1}$$

$$|x_n| = \frac{\sqrt{n}}{n+1} \qquad y_n = \frac{1}{n^{1/2}} = \frac{1}{\sqrt{n}}$$

$$\frac{|x_n|}{y_n} = \frac{\frac{\sqrt{n}}{n+1}}{\frac{1}{\sqrt{n}}} = \frac{\sqrt{n}}{n+1}\cdot\frac{\sqrt{n}}{1} = \frac{n}{n+1} \longrightarrow 1 \in (0,\infty)$$

$$\xRightarrow{CC\,\overline{III}} \sum |x_n| \sim \sum y_n \quad D$$

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n-1}}{n} \qquad z_n = \frac{\sqrt{n-1}}{n} > 0 \text{ pt } n > 1$$

$$z_n = \sqrt{\frac{1}{n} - \frac{1}{n^2}} \longrightarrow 0 \quad (\bullet)$$

$$\frac{z_{n+1}}{z_n} = \frac{\frac{\sqrt{n}}{n+1}}{\frac{\sqrt{n-1}}{n}} = \frac{n\sqrt{n}}{(n+1)\sqrt{n-1}} =$$

$$\frac{\sqrt{n^3}}{\sqrt{(n+1)^2(n-1)}} \qquad \frac{(n+1)^2 =}{n^2+2n+1}$$

$$z_n \searrow \quad (\bullet\bullet)$$

$$\sqrt{\frac{n^3}{n^3+\cancel{2}n^2+n-\cancel{n^2}-2n-1}} =$$

$$(\bullet),(\bullet\bullet) \xRightarrow{\text{Leibniz}} \sum(-1)^n z^n$$

convergentă

$$\sqrt{\frac{n^3}{n^3+n^2-n-1}} < 1$$

$$\sin a \cdot \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n \cdot \cos n^2}{\sqrt{n}}, n \in \mathbb{N}^*;$

$x_n = \sin n \cdot \cos n^2 \qquad y_n = \frac{1}{\sqrt{n}} \searrow_{\text{descrescător}} 0 \quad (\cdot)$

$$S_n = \sum_{k=1}^{n} \sin k \cos k^2 =$$

$$\frac{1}{2}\sum_{k=1}^{n}[\sin(k+k^2) + \sin(k-k^2)] =$$

$$\frac{1}{2}\sum_{k=1}^{n}[\sin(k+k^2) - \sin(k^2-k)] =$$

$$\frac{1}{2}\sum_{k=1}^{n}\{\sin[k(k+1)] - \sin[k(k-1)]\} =$$

$$\frac{1}{2}\left\{\sum_{k=1}^{n}\sin[k(k+1)] - \sum_{l=0}^{n-1}\sin[(l+1)l]\right\}$$

$$= \frac{1}{2}[\underbrace{-\sin 0}_{0} + \sin n(n+1)] = \frac{1}{2}\sin n(n+1)$$

$-\frac{1}{2} \le |S_n| \le \frac{1}{2} \Rightarrow$ şirul sumelor parţiale pt x_n este mărginit $(\cdot\cdot)$

$(\cdot), (\cdot\cdot) \xrightarrow[\text{Dirichlet}]{\text{crit}} \sum x_n y_n$ convergentă

$$2^n \cdot n! = \overbrace{(2\cdot 1)}^{2}\cdot\overbrace{(2\cdot 2)}^{4}\cdot\overbrace{(2\cdot 3)}^{6}\cdot\ldots\cdot\overbrace{(2\cdot n)}^{2n} = (2n)!!$$

e) $\displaystyle\sum_{n=1}^{\infty} \underbrace{(-1)^n \frac{(2n+1)!!}{2^n\cdot n!}}_{x_n}$;

$$|x_n| = \frac{(2n+1)!!}{(2n)!!} = \frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\cdot\ldots\cdot\frac{2n+1}{2n} > 1 \Rightarrow x_n \not\to 0$$

$\Rightarrow$ seria este divergentă

f) $\displaystyle\sum_{n=1}^{\infty} \underbrace{(-1)^{n-1}\ln\left(\frac{n^2+2}{n^2+1}\right)}_{x_n}$;

$$|x_n| = \ln\left(\frac{n^2+2}{n^2+1}\right) = \ln\left(1+\frac{1}{n^2+1}\right) = \ln\left\{\left[\left(1+\frac{1}{n^2+1}\right)^{n^2+1}\right]^{\frac{1}{n^2+1}}\right\}$$

$$\left(1+\frac{1}{u(n)}\right)^{u(n)}$$

$u(n) \longrightarrow \infty$

$\ln a^b = b\ln a$

$$\frac{1}{n^2+1}\underbrace{\ln\left[\left(1+\frac{1}{n^2+1}\right)^{n^2+1}\right]}_{\downarrow\ 1} < \frac{2}{n^2+1}$$

(deci < 2 eventual de la un rang încolo)

$$\sum \frac{2}{n^2+1} = 2\sum\frac{1}{n^2+1} \overset{CC\ III}{\sim} 2\sum\frac{1}{n^2} \quad C$$

$$\Rightarrow \sum |x_n|\ C \Rightarrow \sum x_n\ AC$$

g) (R) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^{n+1}}{n^{n+2}}$;

h) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$;

i) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln(n)}$;

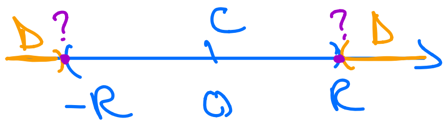
j) (R) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n+1]{\ln(n+1)}}$;

k) (R) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n} \cdot \frac{\sin \frac{n\pi}{6}}{\sqrt{n^3+1}}$;

l) (R) $\displaystyle\sum_{n=0}^{\infty} \frac{a^n + \operatorname{sh} n}{3^n} \cdot b^n, a, b \in \mathbb{R}$;

m) $\displaystyle\sum_{n=1}^{\infty} \operatorname{tg}^n \left(a + \frac{b}{n}\right), a, b \in \left(0, \frac{\pi}{2}\right)$;

n) $\sum_{n=1}^{\infty}(-1)^{n-1}n^{\alpha}\left(\ln\left(\frac{n+2}{n}\right)\right)^{\beta}$, $\alpha,\beta\in\mathbb{R}$.



S4.2 Să studieze convergenţa produsului Cauchy al următoarelor serii:

a) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ şi $\sum_{n=1}^{\infty}\frac{n}{2^n}$;

Serii de puteri

1. Eventual renotez argumentul puterii
2. Determinăm raza de convergență R
- fie cu criteriul raportului
- fie cu criteriul rădăcinii

$\sum a_n x^n$ AC pt $x \in (-R, R)$

D pt $x \notin [-R, R]$

3. Studiem capetele $x=-R$, $x=R$
4. Formulăm concluzii

b) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n+1}}$ şi $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n+1}}$.

S4.3 Să se studieze natura următoarelor serii de puteri:

$x_n = a_n x^n$

a) $\sum_{n=0}^{\infty}\underbrace{[2+(-1)^n]}_{a_n}x^n$, $x\in\mathbb{R}$;

$$a_n = \begin{cases} 3, & n \text{ par} \\ 1 & n \text{ impar} \end{cases}$$

Aplicăm criteriul rădăcinii cu limită superioară

$$\limsup_{n\to\infty}\sqrt[n]{|x_n|} = \limsup_{n\to\infty}\sqrt[n]{3|x|^n} =$$

$$\lim_{n\to\infty}\underbrace{\sqrt[n]{3}}_{\to 1}\cdot|x| = |x|$$

Seria converge pt $|x| < 1$

Dacă $|x| \geq 1 \Rightarrow |x_n| = \begin{cases} 3|x|^n & \text{pt } n \text{ par} \\ |x|^n & \text{pt } n \text{ impar} \end{cases}$

$|x_n| \geq 1 \Rightarrow \sum x_n$ D

$$z = \frac{x+1}{2x+3}$$

Studiem $\sum_{n=0}^{\infty} a_n z^n$

b) $\displaystyle\sum_{n=0}^{\infty} \underbrace{\frac{n+1}{\sqrt{n^4+n^3+1}}}_{a_n} \left(\frac{x+1}{2x+3}\right)^n$, $x \in \mathbb{R} \setminus \left\{\frac{3}{2}\right\}$;

$a_n > 0$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{n+2}{\sqrt{(n+1)^4+(n+1)^3+1}}}{\frac{n+1}{\sqrt{n^4+n^3+1}}} = \underbrace{\frac{n+2}{n+1}}_{\downarrow \, 1} \cdot \underbrace{\frac{\sqrt{n^4+n^3+1}}{\sqrt{(n+1)^4+(n+1)^3+1}}}_{\downarrow \, 1} \to 1$$

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$$

$$R = \frac{1}{l} = 1$$

$\sum a_n z^n$ $\begin{cases} AC \text{ pt } z \in (-1,1) \\ D \text{ pt } z \in (-\infty,-1) \cup (1,\infty) \\ D \text{ pt } z = 1 \\ SC \text{ pt } z = -1 \end{cases}$

$z = 1$

$$\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^4+n^3+1}}$$

$\frac{P(n)}{Q(n)}$ P, Q puteri de lui n

$\deg(P) = 1$

$\deg(Q) = \frac{4}{2} = 2$

↓

notație informată

$\frac{1}{n^\alpha}$ α = puterea maximă a lui n din Q - puterea maximă a lui n din P

$$b_n = \frac{1}{n^{2-1}} = \frac{1}{n}$$

$$\frac{a_n}{b_n} = \frac{\frac{n+1}{\sqrt{n^4+n^3+1}}}{\frac{1}{n}} =$$

$$\frac{n(n+1)}{\sqrt{n^4+n^3+1}} \longrightarrow 1 \in (0,\infty) \xRightarrow{CC\ III}$$

$\sum a_n \sim \sum b_n$ D $\Rightarrow \sum a_n$ D

$z = -1$

$$\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^4+n^3+1}} (-1)^n \qquad a_n > 0$$

$$\frac{a_{n+1}}{a_n} = \sqrt{\frac{(n+2)^2(n^4+n^3+1)}{(n+1)^2\left[(n+1)^4+(n+1)^3+1\right]}} =$$

$$\sqrt{\frac{(n^2+4n+4)(n^4+n^3+1)}{(n^2+2n+1)\left(n^4+4n^3+6n^2+4n+1 + n^3+3n^2+3n+1+1\right)}}$$

$$= \sqrt{\frac{n^6+4n^5+4n^4+n^5+4n^4+4n^3+n^2+4n+4}{(n^2+2n+1)(n^4+5n^3+9n^2+7n+3)}}$$

$$\sqrt{\frac{n^6+5n^5+8n^4+4n^3+n^2+4n+4}{n^6+2n^5+n^4+5n^5+10n^4+5n^3+9n^4+18n^3+9n^2+7n^3+14n^2+7n+3n^2+6n+3}}$$

$$\sqrt{\frac{n^6+5n^5+8n^4+4n^3+n^2+4n+4}{n^6+7n^5+20n^4+30n^3+26n^2+13n+3}} < 1$$

$\Rightarrow a_n$ descrescător

$$a_n = \frac{n+1}{\sqrt{n^4+n^3+1}} \quad (n^2 \quad = \frac{\frac{1}{n}+\frac{1}{n^2}}{\sqrt{1+\frac{1}{n}+\frac{1}{n^4}}} \to 0$$

$\xrightarrow[\text{Leibniz}]{\text{crit}} \sum (-1)^n a_n \quad C$

$\sum a_n z^n$ $\begin{cases} AC \text{ pt } z \in (-1,1) \\ D \text{ pt } z \in (-\infty,-1) \cup (1,\infty) \\ D \text{ pt } z = 1 \\ SC \text{ pt } z = -1 \end{cases}$

$\boxed{z = \frac{x+1}{2x+3}}$

La seminar am scris din greșeală $z = \frac{x+1}{2x+1}$

$z = 1$

$\frac{x+1}{2x+3} = 1 \Leftrightarrow$

$x+1 = 2x+3 \Leftrightarrow x = -2$

$z = -1$

$\frac{x+1}{2x+3} = -1 \Leftrightarrow$

$x+1 = -2x-3 \Leftrightarrow 3x = -2 \Rightarrow$

$x = -\frac{4}{3}$

$$\sum a_n x^n \quad \begin{cases} AC \text{ pt } x \in (-\infty, -2) \cup (-\frac{4}{3}, \infty) \\ D \text{ pt } x - [-2, -\frac{4}{3}) \\ SC \text{ pt } x = -\frac{4}{3} \end{cases}$$

$-1 < z < 1 \iff$

$$\begin{cases} -1 < \dfrac{x+1}{2x+3} \iff \dfrac{x+1}{2x+3} + 1 > 0 \iff \\ \qquad \dfrac{x+1+2x+3}{2x+3} > 0 \iff \\ \qquad \dfrac{3x+4}{2x+3} > 0 \iff \\ \dfrac{x+1}{2x+3} < 1 \iff 1 - \dfrac{x+1}{2x+3} < 0 \iff \\ \qquad \dfrac{x+2}{2x+3} > 0 \end{cases}$$

		-2		$-\frac{3}{2}$		$-\frac{4}{3}$	
$x+2$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2x+3$	$-$	$-$	$-$	0	$+$	$+$	$+$
$3x+4$	$-$	$-$	$-$	$-$	$-$	0	$+$
$\frac{x+2}{2x+3}$	$+$	0	$-$	$\vert$	$+$		$+$
$\frac{3x+4}{2x+3}$	$+$	$+$	$+$	$\vert$	$-$	0	$+$

$$0 < \frac{1}{n} < \frac{\pi}{2}$$

$$\cos\frac{1}{n} > 0$$

c) $\sum\limits_{n=1}^{\infty} \underbrace{\left(\cos\frac{1}{n}\right)^{\frac{n^2+2}{n+2}}}_{a_n} \cdot x^n, x \in \mathbb{R};$

$$\sqrt[n]{a_n} = \left(\cos\frac{1}{n}\right)^{\frac{n^2+2}{(n+2)\cdot n}} \longrightarrow 1$$

$$\Rightarrow R = 1 \quad \begin{cases} AC \text{ pt } x \in (-1, 1) \\ D \text{ pt } x \in (-\infty, -1) \cup (1, \infty) \\ D \text{ pt } \underbrace{x = 1, -1}. \end{cases}$$

$$\Big(\underbrace{\cos\frac{1}{n}}_{\downarrow 1}\Big)^{\overbrace{\frac{n^2+2}{n+2}}^{\nearrow \infty}} \overset{1^\infty}{=} \left[\left(1 + \underbrace{\cos\frac{1}{n} - 1}_{\to 0}\right)^{\frac{1}{\cos\frac{1}{n}-1}}\right]^{\frac{n^2+2}{n+2}\left(\cos\frac{1}{n}-1\right)}$$

$$= \left[\left(1 + \cos\frac{1}{n} - 1\right)^{\frac{1}{\cos\frac{1}{n}-1}}\right]^{-\frac{n^2+2}{n+2} \cdot 2\underbrace{\sin^2\frac{1}{2n}}} =$$

$$\cos\frac{1}{n} - 1 = \cos\frac{1}{n} - \cos 0 = -2\sin^2\frac{1}{2n}$$

$$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$$

Limită fundamentală

$$\frac{\sin x}{x} \xrightarrow{x \to 0} 1$$

$$\sin^2\frac{1}{2n} = \frac{\sin\frac{1}{2n}}{\frac{1}{2n}} \cdot \frac{\sin\frac{1}{2n}}{\frac{1}{2n}} \cdot \left(\frac{1}{2n}\right)^2$$

$$\left[\left(1 + \cos\frac{1}{n} - 1\right)^{\frac{1}{\cos\frac{1}{n}-1}}\right]^{-\frac{n^2+2}{n+2} \cdot 2 \cdot \frac{\sin^2\frac{1}{2n}}{\left(\frac{1}{2n}\right)^2} \cdot \left(\frac{1}{2n}\right)^2}$$

$\sin^2\frac{1}{2n}$ gata

$$= \underbrace{\left[\left(1 + \cos\frac{1}{n} - 1\right)^{\frac{1}{\cos\frac{1}{n}-1}}\right]}_{\to e}^{-\underbrace{\frac{(n^2+2) \cdot 2}{(n+2) \cdot 4n^2}}_{\downarrow 0} \cdot \underbrace{\frac{\sin^2\frac{1}{2n}}{\left(\frac{1}{2n}\right)^2}}_{\downarrow 1}} \to 1$$

$$\Rightarrow \sum \left(\cos \frac{1}{n}\right)^{\frac{n^2+2}{n+2}} \quad D$$

$$\underbrace{-\frac{(n^2+2)\cdot 2}{(n+2)\,(2n)^2}}_{0}$$

$$e^{0\cdot 1} \to 1$$

$$1^{\infty}$$

$$\left(1+x_n\right)^{\frac{1}{x_n}} \xrightarrow[x_n \to 0]{} e$$

d) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-4)^n}{n\cdot 3^n}, x\in\mathbb{R};$

$$= \sum_{n=1}^{\infty} \frac{(-1)}{n}\cdot(-1)^n\cdot\frac{(x-4)^n}{3^n}$$

$$z = \frac{(-1)\cdot(x-4)}{3}$$

$$\sum_{n=1}^{\infty}\frac{(-1)\cdot z^n}{n} \qquad a_n = \frac{-1}{n}$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{\left|-\frac{1}{n+1}\right|}{\left|-\frac{1}{n}\right|} = \frac{n}{n+1} \longrightarrow 1$$

$$R = 1 \Rightarrow \begin{cases} AC \text{ pt } z\in(-1,1) \\ D \text{ pt } z\in(-\infty,-1)\cup(1,\infty) \\ D \text{ pt } \underline{z=1} \\ C \text{ pt } \underline{z=-1} \end{cases}$$

$$\text{Pt } z=1 \quad \sum_{n=1}^{\infty}\frac{-1}{n} = -\sum_{n=1}\frac{1}{n} \ D$$

$$\text{Pt } z=-1 \quad \sum_{n=1}^{\infty} -\frac{(-1)^n}{n} = -\sum_{n=1}^{\infty}\frac{(-1)^n}{n} \ C$$

$$b_n = \frac{1}{n} \searrow 0 \text{ descrescător} \xRightarrow{\text{Leibniz}}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n} \ C$$

$$z=1 \Rightarrow \frac{(-1)(x-4)}{3} = 1 \Rightarrow$$

$$4-x=3 \Rightarrow \underline{x=1}$$

$$z=-1 \Rightarrow \frac{x-4}{3} = 1 \Rightarrow x=7$$

$$\begin{cases} AC \text{ pt } x\in(1,7) \\ D \text{ pt } x\in(-\infty,1]\cup(7,\infty) \\ SC \text{ pt } x=7 \end{cases}$$

e) $\sum_{n=1}^{\infty} \frac{x^n}{n^p},\ p \in \mathbb{R};$ $= \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^p}}_{a_n} x^n$ $a_n > 0$

$\frac{a_{n+1}}{a_n} = \frac{(n+1)^p}{n^p} \to 1 \Rightarrow R = 1$

$\begin{cases} AC \text{ pt } x \in (-1,1) \\ D \text{ pt } |x| > 1 \\ ? \text{ pt } x = \pm 1 \end{cases}$

AC pt $x = \pm 1$ $p > 1$

D pt $x = 1$ $p \leq 1$

C pt $x = -1$ $p \in (0,1]$

D pt $x = -1$ $p \leq 0$

Pt $|x| = 1$

<u>seria modulelor</u> $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\begin{cases} p \leq 1 & D \\ p > 1 & C \end{cases}$

$x = -1,\ p \leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^p} \cdot (-1)^n$

$\frac{1}{n^p} \searrow 0$ pentru $p > 0$ $\xRightarrow[\text{seria converge}]{\text{Leibniz}}$

Pt $x = -1$, $p \leq 0$ $q = -p > 0$

$\sum_{n=1}^{\infty} n^q \cdot (-1)^n$ D

$|n^q \cdot (-1)^n| \longrightarrow \infty$

e) $\sum_{n=1}^{\infty} \frac{x^n}{n^p},\ p \in \mathbb{R};$ $= \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^p}}_{a_n} x^n$ $a_n > 0$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^p}{n^p} \to 1 \Rightarrow R = 1$$

f) $\sum_{n=2}^{\infty} \frac{x^n}{3^n \cdot n \cdot \ln n}\ x \in \mathbb{R};$ $\frac{x}{3} = z$

$$\sum_{n=2}^{\infty} \underbrace{\frac{1}{n \cdot \ln n}}_{a_n} \cdot z^n \qquad a_n > 0$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)\ln(n+1)}{n \ln n} =$$

$$\underbrace{\frac{n+1}{n}}_{\downarrow\, 1} \cdot \underbrace{\frac{\ln(n+1)}{\ln n}}_{\downarrow\, 1}$$

$\ln n \to \infty$

Considerăm fct. asociată

$\frac{\ln(n+1)}{\ln n}$

$f : \mathbb{R}_+ \to \mathbb{R}$

$$f(x) = \frac{\ln(x+1)}{\ln x}$$

$$\lim_{x\to\infty} f(x) \xrightarrow{\text{Dacă } \exists} \lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x}} = 1$$

$\Rightarrow$ Raza de convergență $= 1$

$$\begin{cases} AC \text{ pt } z \in (-1, 1) \\ D \text{ pt } z \in (-\infty, -1) \cup (1, \infty) \\ ?_{\text{ }} \text{ pt } z = \pm 1 \end{cases}$$

$z = 1$

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} \overset{\text{condensare}}{\sim} \sum_{n=2}^{\infty} \cancel{2^n} \frac{1}{\cancel{2^n} \ln 2^n} = \sum_{n=2}^{\infty} \frac{1}{n \ln 2} = \frac{1}{\ln 2} \sum_{n=2}^{\infty} \frac{1}{n} \; D$$

$a_n \searrow$

Încercau crit log $\frac{\ln(n \ln n)}{\ln n} = \frac{\ln n + \ln(\ln n)}{\ln n} =$

$$1 + \frac{\ln(\ln n)}{\ln n} \longrightarrow 1$$

$z = -1$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

$$\frac{1}{n \ln n} \longrightarrow 0 \quad (\cdot)$$

$$\frac{a_{n+1}}{a_n} = \frac{n \ln n}{(n+1)\ln(n+1)} = \frac{n}{n+1} \frac{\ln(n)}{\ln(n+1)} < 1$$

$\Rightarrow a_n \searrow \quad (\cdot\cdot) \xrightarrow{\text{Leibniz}} \sum a_n (-1)^n \; C$

$$\begin{cases} AC \text{ pt } z \in (-1,1) \\ D \text{ pt } z \in (-\infty, -1] \cup (1, \infty) \\ SC \text{ pt } z = -1 \\ D \text{ pt } z = 1 \end{cases}$$

$$z = \frac{x}{3}$$

$$\begin{cases} AC \text{ pt } x \in (-3, 3) \\ D \text{ pt } x \in (-\infty, -3) \cup [3, \infty) \\ SC \text{ pt } x = -3 \end{cases}$$

g) (R) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x+1)^{2n}}{(4n+1)^2},\ x \in \mathbb{R};$

h) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n}-1\right)^n \cdot x^n, x \in \mathbb{R};$

i) (R) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n \left(\frac{1-x}{1-2x}\right)^n, x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\};$

j) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\ln n}\left(\frac{1-x^2}{1+x^2}\right)^n, x \in \mathbb{R};$

k) (R) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{3^{\frac{n}{2}}\sqrt{1+n^2}}\operatorname{tg}^n x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right);$

l) $\sum_{n=1}^{\infty}\frac{n!}{(a+1)(a+2)\cdot \ldots \cdot (a+n)}x^n, a > 0, x \in \mathbb{R}.$

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