Sem12-A5

Monday, 21 December 2020 10:03

Ex 130

(i)

1. $\Gamma \vdash \exists x. \underbrace{\exists y. P(x,y)}_{\varphi''}$ (ip)

t_x nu poate fi x_0

i. $\Gamma, \exists y. P(x_0, y) \vdash P(t_x, t_y)$

k. $\Gamma \vdash \underbrace{P(t_x, t_y)}_{\varphi}$

j. $\Gamma \vdash \exists x. \underbrace{P(x, t_y)}_{\varphi'}$ $(\exists_i, k)$

n. $\underbrace{\exists x. \exists y. P(x,y)}_{\Gamma} \vdash \exists y. \underbrace{\exists x. P(x,y)}_{\varphi}$ $(\exists_i, j)$

$\exists_i$ ✗

$\exists_e$ $\varphi = \exists y. \varphi$

$$\exists_e \frac{\Gamma \vdash \exists x. \varphi'' \qquad \Gamma, \varphi''[x \mapsto x_0] \vdash \psi}{\Gamma \vdash \psi} \; x_0 \notin vars(\Gamma, \varphi'', \psi)$$

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1. $\Gamma \vdash \exists x. \underbrace{\exists y. P(x,y)}_{\varphi''}$ (ip)

2. $\Gamma, \exists y. P(x_0, y) \vdash \exists y. \underbrace{P(x_0, y)}_{\varphi''}$ (ip)

k_2. $\Gamma'' \vdash P(x_0, y_0)$ (ip)

k_1 $\Gamma'' \vdash \exists x. P(x, y_0)$ $(\exists_i, k_2)$

k. $\overbrace{\Gamma, \exists y. P(x_0, y), P(x_0, y_0)}^{\Gamma''} \vdash \exists y. \underbrace{\exists x. P(x,y)}_{\varphi_1}$ $(\exists_i, k_1)$

j. $\overbrace{\Gamma, \exists y. P(x_0, y)}^{\Gamma'} \vdash \underbrace{\exists y. \exists x. P(x,y)}_{\psi}$ $(\exists_e, 2, k)$

$x \mapsto x_0$ orice în afară de x, y

n. $\underbrace{\exists x. \exists y. P(x,y)}_{\Gamma} \vdash \underbrace{\exists y. \exists x. P(x,y)}_{\psi}$ $(\exists_e, 1, j)$

$$\exists_i \frac{\Gamma \vdash \varphi[x \mapsto t]}{\Gamma \vdash \exists x. \varphi}$$

$y \mapsto t$

$\exists y. \varphi$

Ex 130 (7) → similar: $\exists_e$ înainte de $\exists_i$

Ex 143

$\Sigma = (\{P, Q\}, \{f, i, e\})$

(P: 2, Q: 1; f: 2, i: 1, e: 0)

1) $S = (\mathbb{N}, \{<, Par\}, \{+, s, 0\})$

$P(e, x) \overset{S}{\equiv} P(e, f(x,x))$ ddacă

ddacă pt orice S-atribuire α avem

$S, \alpha \models P(e, x)$ ddacă $S, \alpha \models P(e, f(x,x))$ (*)

Fie α o S-atribuire arbitrară

$S, \alpha \models P(e, x)$

ddacă $P^S(\bar{\alpha}(e), \bar{\alpha}(x))$

ddacă $0 < \alpha(x)$

ddacă

ddacă $0 < \alpha(x) + \alpha(x)$

ddacă $0 < f^S(\bar{\alpha}(x), \bar{\alpha}(x))$

ddacă $P^S(\bar{\alpha}(e), \bar{\alpha}(f(x,x)))$

ddacă $S, \alpha \models P(e, f(x,x))$

$\Rightarrow S, \alpha \models P(e, x)$ ddacă $S, \alpha \models P(e, f(x,x))$ pt. orice α arbitrar. (*)

$\Rightarrow P(e, x) \overset{S}{\equiv} P(e, f(x,x))$

(2) $\neg \forall x. \varphi \equiv \exists x. \neg \varphi$

ddacă pt orice structură S_1 și pt orice S-atribuire α_1

avem $S_1, \alpha_1 \models \neg \forall x. \varphi$ ddacă $S_1, \alpha_1 \models \exists x. \neg \varphi$ (*)

Fie S_1, α_1 arbitrare.

- $S_1, \alpha_1 \models \neg \forall x. \varphi$

ddacă $S_1, \alpha_1 \not\models \forall x. \varphi$ $\qquad$ $S_1, \alpha_1 \models \forall x. \varphi$

ddacă NU pt orice $u \in D_1$ avem $S_1, \alpha_1[x \mapsto u] \models \varphi$

ddacă

ddacă există $u \in D_1$ a.î. $S_1, \alpha_1[x \mapsto u] \not\models \varphi$

ddacă există $u \in D_1$ a.î. $S_1, \alpha_1[x \mapsto u] \models \neg \varphi$

- ddacă $S_1, \alpha_1 \models \exists x. \neg \varphi$

$\Rightarrow S_1, \alpha_1 \models \neg \forall x. \varphi$ ddacă $S_1, \alpha \models \exists x. \neg \varphi$ pt orice S_1 și α_1 arbitrare (*)

$\Rightarrow \neg \forall x. \varphi \equiv \exists x. \neg \varphi$

(4) $(\forall x. \varphi_1) \wedge \varphi_2 \equiv \forall x. (\varphi_1 \wedge \varphi_2)$ $\qquad x \notin free(\varphi_2)$

ddacă pt orice structură S și pt orice S-atribuire α

avem $S, \alpha \models (\forall x. \varphi_1) \wedge \varphi_2$ ddacă $S, \alpha \models \forall x. (\varphi_1 \wedge \varphi_2)$

Fie S, α arbitrare

$S, \alpha \models (\forall x. \varphi_1) \wedge \varphi_2$

ddacă $\begin{cases} S, \alpha \models \forall x. \varphi_1 \\ \text{și} \\ S, \alpha \models \varphi_2 \end{cases}$

a) ddacă $\begin{cases} \text{pt orice } u \in D \text{ avem } S, \alpha[x \mapsto u] \models \varphi_1 \\ \text{și} \\ S, \alpha \models \varphi_2 \end{cases}$

ddacă

b) ddacă $\underline{\text{pt orice } u \in D \text{ avem}}$ $\begin{cases} S, \alpha[x \mapsto u] \models \varphi_1 \\ \text{și} \\ S, \alpha[x \mapsto u] \models \varphi_2 \end{cases}$ $\overset{x \notin free(\varphi_2)}{\Longleftrightarrow}$ $S, \alpha \models \varphi_2$

ddacă pt orice $u \in D$ avem $S, \alpha[x \mapsto u] \models \varphi_1 \wedge \varphi_2$

ddacă $S, \alpha \models \forall x. (\varphi_1 \wedge \varphi_2)$

Ex 144

(1) $\sigma = \{x \mapsto i(y), y \mapsto f(x, z)\}$ $\qquad \sigma : X \to T$

$\varphi = (\forall x. P(x, y)) \to P(i(y), x)$ $\qquad \sigma^\#: T \to T$

legat $\qquad \sigma^b : LP1 \to LP1$

$\sigma^b(\varphi) = \sigma^b(\forall x. P(x,y)) \to \sigma^b(P(\overbrace{i(y)}^{t_1}, \overbrace{x}^{t_2}))$

$= (\forall x. \rho^b(P(x,y))) \to P(\sigma^\#(i(y)), \sigma^\#(x))$

$\rho = \sigma|_{dom(\sigma) \setminus \{x\}} = \sigma|_{\{y\}} = \{y \mapsto f(x, z)\}$

$= (\forall x. P(\rho^\#(x), \rho^\#(y))) \to P(i(\sigma^\#(y)), \sigma(x))$

$= (\forall x. P(\rho(x), \rho(y))) \to P(i(\sigma(y)), \sigma(x))$

$= (\forall x. P(x, f(x,z))) \to P(i(f(x,z)), i(y))$

$\sigma^b(\varphi) = (\forall x. P(x, f(x,z))) \to P(i(f(x,z)), i(y))$

Ex 145

$\varphi' = \underset{\exists}{Q_1} x_1. \underset{\forall}{Q_2} x_2. \underset{\forall}{Q_3} x_3 \ldots (\varphi'')$ fără alți quantificatori

Th. redenumire $\qquad \underset{\exists}{\forall} x. \varphi \equiv \underset{\exists}{\forall} y. \underbrace{\boxed{\varphi[x \mapsto y]}}_{\sigma^b(\varphi)}$ $\qquad y \notin free(\varphi)$

$\sigma = \{x \mapsto y\}$

(3) $\varphi = (\exists z. \underbrace{P(x, z)}_{\varphi_1}) \wedge (\underbrace{\forall x. P(x, z)}_{\varphi_2})$

liber

LR

→ $(\exists z. \varphi_1) \wedge \varphi_2 \equiv \exists z. (\varphi_1 \wedge \varphi_2)$ dacă $z \notin free(\varphi_2)$

$free(\varphi_2) = \{z\} \ni z \Rightarrow$ nu putem aplica

z înlocuit cu orice variabila $\notin free(\varphi_1) \cup free(\varphi_2)$

$\overset{LR}{\underset{(z \mapsto y)}{\equiv}} (\exists y. \underbrace{P(x, y)}_{\varphi_1}) \wedge (\underbrace{\forall x. P(x, z)}_{\varphi_2})$

$y \notin free(\varphi_2)$

$\equiv \exists y. (\underbrace{P(x, y)}_{\varphi_1} \wedge \forall x. \underbrace{P(x, z)}_{\varphi_2})$ ← LR $x \mapsto x_0$

orice fără x, y, z

→ $\varphi_1 \wedge (\forall x. \varphi_2) \equiv \forall x. (\varphi_1 \wedge \varphi_2)$ dacă $x \notin free(\varphi_1)$

$free(\varphi_1) = \{x, y\} \ni x$

$\overset{LR}{\equiv} \exists y. (\underbrace{P(x, y)}_{\varphi_1} \wedge \forall x_0. \underbrace{P(x_0, z)}_{\varphi_2})$

$x_0 \notin free(\varphi_1)$

$\equiv \exists y. \forall x_0. (P(x, y) \wedge P(x_0, z))$ $\quad$ – FNP