$$\text{MT}\ \frac{\Gamma \vdash \varphi_1 \to \varphi_2 \qquad \Gamma \vdash \neg\varphi_2}{\Gamma \vdash \neg\varphi_1}$$

$$1.\quad \underline{(r_3 \wedge r_4) \to q},\ q \to (r_3 \wedge r_4) \vdash \underbrace{((r_3 \wedge r_4) \wedge q) \vee (\neg(r_3 \wedge r_4) \wedge \neg q)}_{(\varphi_1 \vee \varphi_2)}$$

$$\vee_{i_1}\ \frac{\Gamma \vdash \varphi_1}{\Gamma \vdash \varphi_1 \vee \varphi_2}$$

$$\text{PBC}\ \frac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \underbrace{(\varphi_1 \vee \varphi_2)}_{\varphi}}$$

1. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash ((r_3\wedge r_4)\to q)$ (ip)
2. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash \neg q$ (ip)
3. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash \neg(r_3\wedge r_4)$ (MT, 1, 2)
4. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash (\neg(r_3\wedge r_4)\wedge\neg q)$ ($\wedge_i$, 3, 2)
5. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash (((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))$ ($\vee_{i_2}$, 4)
6. $\Gamma, \underline{\neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))}, \neg q \vdash \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))$ (ip)

~~7~~ k_1. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)), \neg q \vdash \bot$ ($\neg e$, 5, 6)

$$\neg_i\ \frac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \neg\neg\varphi} \qquad \neg\neg e\ \frac{\Gamma \vdash \neg\neg\varphi}{\Gamma \vdash \varphi}$$

k_2. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash \neg\neg q$ ($\neg i$, k_1)

k_3. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash q$ ($\neg\neg e$, k_2)

k_4. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash (q \to (r_3\wedge r_4))$ (ip)

k_5. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash (r_3\wedge r_4)$ ($\to e$, k_4, k_3)

k_6. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash ((r_3\wedge r_4)\wedge q)$ ($\wedge_i$, k_5, k_3)

$\neg(\varphi_1 \vee \varphi_2)$

$(\underline{\varphi_1} \vee \varphi_2)$

k_7. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash (((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))$ ($\vee_{i_1}$, k_6)

k_8. $\Gamma, \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q)) \vdash \neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))$ (ip)

j. $\Gamma, \underline{\neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))} \vdash \bot$ ($\neg e$, k_7, k_8)

j_1 $\Gamma \vdash \neg\neg(((r_3\wedge r_4)\wedge q) \vee (\neg(r_3\wedge r_4)\wedge\neg q))$ ($\neg i$, j)

$$n.\quad \underbrace{((r_3\wedge r_4)\to q),\ (q \to (r_3\wedge r_4))}_{\Gamma} \vdash \underline{((r_3\wedge r_4)\wedge q)} \vee (\neg(r_3\wedge r_4)\wedge\neg q) \quad (\neg\neg e, j_1)$$

Ex 32 Nu toți oamenii sunt studenți.

Nu → $\neg$, toți → $\forall$

$$\neg(\forall x.(Om(x) \to Student(x)))$$

$\cancel{\wedge}$

simb. pred.

- $Student(u)$: u este student; $ar(Student) = 1$
- $Om(t)$: t este om; $ar(Om) = 1$

$D = \{\underline{a}, \underline{b}, \underline{c}, d, \ldots\}$

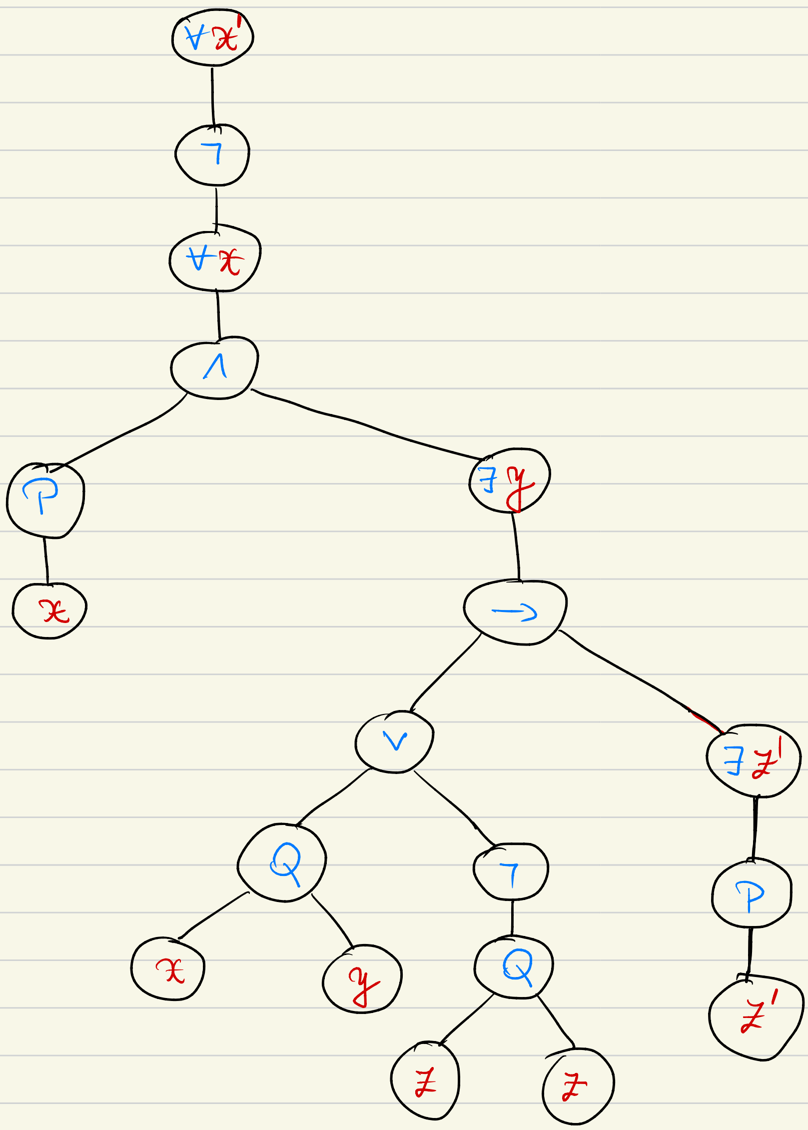
✓ ✓

Ordinea operatorilor : $\perp \; \neg \; \wedge \; \vee \; \rightarrow \; \leftrightarrow \; \forall \; \exists$

$\{\neg \varphi\}$

Ex 36 5)

$$\forall x'. \neg \forall x. P(x) \wedge \exists y. \big((Q(x,y) \vee \neg Q(z,z)) \rightarrow (\exists z'. P(z')) \big)$$



Relatie $Nat \subseteq \mathbb{R}$, $u \in Nat$ "A"/"F" $Nat : \mathbb{R} \to \{0,1\}$

$+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

Ex 33 $S = (\mathbb{R}, \{Nat, Int, Prim, Par, >\}, \{+, 0, 1, 2\})$

1. $\Sigma = (\{Nat, Int, Prim, Par, >\}, \{+, 0, 1, 2\})$

nume + ar

$ar(Nat) = 1 = ar(Int) = ar(Prim) = ar(Par)$

$Nat^S = Nat$ $ar(>) = 2$

simb pred   Pred

$ar(+) = 2$ $ar(0) = ar(1) = ar(2) = 0$

2. c) Oricum am alege un nr natural, există un nr prim care este mai mare decât nr respectiv.

$\forall x$ dacă   Nat   atunci $\exists y$ Prim $\wedge$

$y > x$

$$\left(\forall x . \left(Nat(x) \to \left(\exists y . \left(Prim(y) \wedge (y > x)\right)\right)\right)\right)$$

$>(y, x)$

f) Suma a două numere pare este nr par.

oricăror $\forall$ Suma

$$\left(\forall x . \left(\forall y . \left(\left(Par(x) \wedge Par(y)\right) \to Par((x+y))\right)\right)\right)$$

h') Orice nr par poate fi scris ca sumă a două numere impare.

$\forall x$ $\exists y . \exists z$

$$\left(\forall x . \left(Par(x) \to \left(\exists y . \exists z . \left(\neg Par(y) \wedge \neg Par(z) \wedge \neg((y+z) > x) \wedge \neg(x > (y+z))\right)\right)\right)\right)$$

$y + z = x$

$a = b$ dacă $a \not> b$ și $b \not> a$

$((((y_1 + y_2) + y_3) + y_4) = x$

Ex 39 $\varphi_2 = \Big(\forall x.\big(P\big(\underbrace{f(x,x),\ i(x)}_{\text{legate}}\big) \wedge \big(\exists y.\big(P(x,y) \wedge P(x,z)\big)\big)\big)\Big)$

ap liberă

$var(\varphi_2) = \{x, y, z\}$

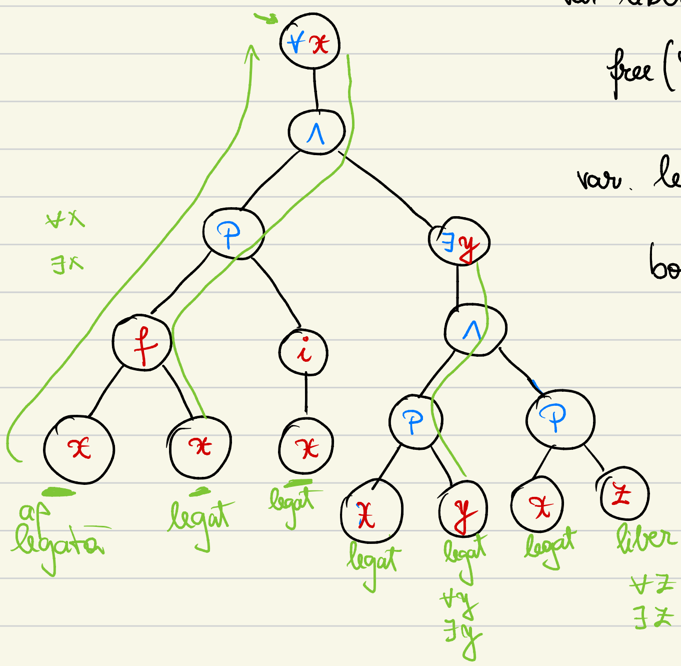
var libere (au măcar o ap. liberă)

$free(\varphi_2) = \{z\}$

var. legate (sunt cuantificate)

$bound(\varphi_2) = \{x, y\}$

∃x
∀x
noduri



$\varphi = \forall x.\,P(y, z)$

ap. libere

$free(\varphi) = \{y, z\}$

$bound(\varphi) = \{x\}$