# Logic for Computer Science

# Propositional Logic

## Examination - November 19th, 2021

Examination subject for: VARIANTA SUPLIMENTARA 39

Rules:

- Read the questions carefully.
- You should work individually.
- Do not exceed the space allocated for each question.
- Please first solve the questions on a scrap piece of paper and then write down the final solutions.
- Please have an extra copy of the subject, in case you make any mistakes.
- It is not allowed to use extra sheets of paper. You may use any number of scrap pages for drafts. You should not upload the drafts. It is not allowed to share the drafts with anyone.
- You may consult the bibliography.
- It is not allowed to communicate with other persons to solve the tasks.
- Scan the 5 A4 pages into a single PDF document at most 10MB in size.
- Before uploading, check carefully the quality of the scan.
- If the quality of the scan is poor, your final result will reflect only what can be seen without significant effort in the scan.
- Upload your solution in the Google Forms document at the URL:

  `https://forms.gle/nvovtDzq2b132vuG8`.

- Solutions sent through any other channel (e.g., email, Discord) are not accepted.
- The Forms document does not allow uploading solutions having more than 10MB in size.

1. Translate the following proposition into propositional logic: I pass all exams only if I go to school and it is not true that $2 + 2 = 4$.

2. Show, by using a semantic argument, that the following formula is satisfiable:

$$((\mathtt{r}_1 \rightarrow \mathtt{r}_2) \wedge (\mathtt{q} \wedge \mathtt{r}_2)).$$

3. Show, by using a semantic argument, that the following formula is valid:

$$((\mathtt{r} \wedge \mathtt{p}) \rightarrow (\mathtt{p} \vee \mathtt{q})).$$

4. Show, by using a semantic argument, that: for all $\varphi_2 \in \mathbb{PL}$,

$$(\mathtt{p} \wedge \neg\varphi_2) \equiv (\neg\mathtt{p} \leftrightarrow (\varphi_2 \wedge \mathtt{p})).$$

5. Show, by using a semantic argument, that:

$$\{(\neg q \wedge q), ((p \wedge q) \vee r)\} \models (q \wedge (p \wedge p)).$$

6. Compute a CNF of the following formula:

$$(q \leftrightarrow ((r \vee q) \wedge p)).$$

7. Find a refutation for the following set of clauses:

$$\{(r \lor \neg r), (\neg q \lor r_1 \lor \neg r_2), q, \neg r, (r \lor \neg r_1), r_2\}.$$

8. Find a formal proof for the following sequent:

$$\{((q_1 \lor q_2) \land (q \lor p))\} \vdash (((q_1 \lor q_2) \land q) \lor ((q_1 \lor q_2) \land p)).$$