

Let $\varphi_1, \varphi_2 \in \mathcal{L}_P$ arbitrary

$$\neg(\varphi_1 \vee \varphi_2) \equiv (\neg\varphi_1 \wedge \neg\varphi_2) \text{ ddac}$$

Def. pt orice $\tau: A \rightarrow B$ avem $\hat{\tau}(\neg(\varphi_1 \vee \varphi_2)) = \hat{\tau}(\neg\varphi_1 \wedge \neg\varphi_2)$

$$\text{ddac} \text{ pt orice } \tau: A \rightarrow B, \overline{\hat{\tau}(\varphi_1 \vee \varphi_2)} = \hat{\tau}(\neg\varphi_1) * \hat{\tau}(\neg\varphi_2)$$

$$\text{ddac} \text{ pt orice } \tau: A \rightarrow B, \underline{\hat{\tau}(\varphi_1) + \hat{\tau}(\varphi_2) = \hat{\tau}(\varphi_1) * \hat{\tau}(\varphi_2)} \quad (*)$$

Case 1: Let $\tau: A \rightarrow B$ arbitrary a.i. $\hat{\tau}(\varphi_1) = 0$ & $\hat{\tau}(\varphi_2) = 0$

$$\Rightarrow \underline{0+0 = 0 * 0} \Leftrightarrow 0 = 1 * 1 \Leftrightarrow 1 = 1 \quad "A" \Rightarrow (*) \text{ are loc in case 1.}$$

Case 2: Let $\tau: A \rightarrow B$ arbitrary a.i. $\hat{\tau}(\varphi_1) = 0$ & $\hat{\tau}(\varphi_2) = 1$

$$\Rightarrow \underline{0+1 = 0 * 1} \Leftrightarrow 1 = 1 * 0 \Leftrightarrow 0 = 0 \quad "A"$$

Case 3: Let $\tau: A \rightarrow B$ arbitrary a.i. $\hat{\tau}(\varphi_1) = 1$ & $\hat{\tau}(\varphi_2) = 0$

$$\Rightarrow \underline{1+0 = 1 * 0} \Leftrightarrow 1 = 0 * 1 \Leftrightarrow 0 = 0 \quad "A"$$

Case 4: Let $\tau: A \rightarrow B$ arbitrary a.i. $\hat{\tau}(\varphi_1) = 1$ & $\hat{\tau}(\varphi_2) = 1$

$$\Rightarrow \underline{1+1 = 1 * 1} \Leftrightarrow 1 = 0 * 0 \Leftrightarrow 0 = 0 \quad "A"$$

Dim cele 4 cazuri \Rightarrow relatia (*) are loc pt orice $\tau: A \rightarrow B \Rightarrow$

$$\Rightarrow \neg(\varphi_1 \vee \varphi_2) \equiv (\neg\varphi_1 \wedge \neg\varphi_2)$$

$$p \equiv \neg\neg p \text{ ddac} \text{ pt orice } \tau: A \rightarrow B, \hat{\tau}(p) = \hat{\tau}(\neg\neg p)$$

Let $\tau: A \rightarrow B$ arbitrary.

$$\hat{\tau}(\neg\neg p) = \overline{\hat{\tau}(\neg p)} = \overline{\overline{\hat{\tau}(p)}} = \hat{\tau}(p)$$