Sud 53

$\mathbb{C}_{2\times 2}^{\mathbb{R}}$ $\mathbb{R}^{\mathbb{C}}$ $A_{\mathbb{R}}$ $A_{\mathbb{C}}$ $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \in M_2^{\mathbb{R}}$

a) $T(x_1, x_2, x_3, x_4) = ?$

$T\left(\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}\right) \Rightarrow A\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}\right) = \begin{pmatrix} \frac{1}{2}x_1 + 0x_2 + 0x_3 + 0x_4 \\ 0x_1 + \frac{1}{2}x_2 + 0x_3 + 0x_4 \\ 0x_1 + 0x_2 + \frac{1}{2}x_3 + 0x_4 \\ 0x_1 + 0x_2 + 0x_3 + \frac{1}{2}x_4 \end{pmatrix}$

$T(x_1, x_2, x_3, x_4) = \left(\frac{1}{2}x_1, \frac{1}{2}x_2, \frac{1}{2}x_3, \frac{1}{2}x_4\right)$

NEB

$T(e_1) = (1,0,0,0)$ $T(e_2) = (0,1,0,0)$ $T(e_3) = (0,0,1,0)$ $T(e_4) = (0,0,0,1)$

$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} = x_1 T(e_1) + x_2 T(e_2) + x_3 T(e_3) + x_4 T(e_4)$

$= x_1(\frac{1}{2},0,0,0) + x_2(0,\frac{1}{2},0,0) + x_3(0,0,\frac{1}{2},0) + x_4(0,0,0,\frac{1}{2})$

$= (\frac{1}{2}x_1, \frac{1}{2}x_2, \frac{1}{2}x_3, \frac{1}{2}x_4)$

$\Rightarrow T(x_1, x_2, x_3, x_4) = (\frac{1}{2}x_1, \frac{1}{2}x_2, \frac{1}{2}x_3, \frac{1}{2}x_4)$

b) $\ker T$, $\operatorname{Im}(\ker T)$, $\operatorname{rang} T$

$\ker T = \{ x \in \mathbb{R}^4 \mid T x = 0_{\mathbb{R}^4} \}$

$\Leftrightarrow T(x_1, x_2, x_3, x_4) = (0,0,0,0) \Leftrightarrow (\frac{1}{2}x_1, \frac{1}{2}x_2, \frac{1}{2}x_3, \frac{1}{2}x_4) = (0,0,0,0)$

$\begin{cases} \frac{1}{2}x_1 = 0 \\ \frac{1}{2}x_2 = 0 \\ \frac{1}{2}x_3 = 0 \\ \frac{1}{2}x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{cases}$ $\Rightarrow \ker T = \{0_{\mathbb{R}^4}\}$ $\dim(\ker T) = 0$

$\operatorname{Im}(\ker T) = \{0_{\mathbb{R}^4}\}$ $\operatorname{dim}(\operatorname{Im}(\ker T)) = 0$

$\operatorname{rang} T = \operatorname{dim}(\operatorname{Im} T)$

$\operatorname{Im} T = \{ y \in \mathbb{R}^4 \mid \exists x \in \mathbb{R}^4 : T x = y \}$

$= \{ y \in \mathbb{R}^4 \mid \exists (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : (\frac{1}{2}x_1, \frac{1}{2}x_2, \frac{1}{2}x_3, \frac{1}{2}x_4) = (y_1, y_2, y_3, y_4) \}$

$= \{ (y_1, y_2, y_3, y_4) \in \mathbb{R}^4 \mid \exists (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : (x_1, x_2, x_3, x_4) = (2y_1, 2y_2, 2y_3, 2y_4) \}$

$= \{ (y_1, y_2, y_3, y_4) \in \mathbb{R}^4 \mid \exists (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \begin{cases} x_1 = 2y_1 \\ x_2 = 2y_2 \\ x_3 = 2y_3 \\ x_4 = 2y_4 \end{cases} \}$

$\operatorname{dim}(\operatorname{Im} T) = \operatorname{dim}(\mathbb{R}^4) = 4$

$\operatorname{rang} T = 4$

c) $\operatorname{rang} T = 4 \Rightarrow T$ bijectif

$\operatorname{dim}(\ker T) = 0$

$\operatorname{dim}(\operatorname{Im} T) = \operatorname{rang} T = \operatorname{dim}(\mathbb{R}^4)$

$\Rightarrow \operatorname{Im} T = \mathbb{R}^4$

$\Rightarrow T$ surjectif

$\Rightarrow T$ bijectif

$T(e_1) = T(1,0,0,0) = (\frac{1}{2}, 0, 0, 0)$

$T(e_2) = T(0,1,0,0) = (0, \frac{1}{2}, 0, 0)$

$T(e_3) = T(0,0,1,0) = (0, 0, \frac{1}{2}, 0)$

$T(e_4) = T(0,0,0,1) = (0, 0, 0, \frac{1}{2})$

$\det \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16} \neq 0$

$\det(T) = 0$

d) $\operatorname{Im}(\operatorname{Im}(\ker T)) = \operatorname{Im}(\{0_{\mathbb{R}^4}\}) = \{0_{\mathbb{R}^4}\}$

$\operatorname{dim}(\operatorname{Im}(\operatorname{Im}(\ker T))) = 0$

$\operatorname{dim}(\operatorname{Im}(\ker T)) = 0$

$\operatorname{rang} T = \operatorname{dim}(\operatorname{Im} T) = \operatorname{dim}(\mathbb{R}^4) = 4$

$\operatorname{dim}(\operatorname{Im}(\operatorname{Im}(\ker T))) \neq \operatorname{rang} T$

Sud 54

Règle A B = AB The $\mathbb{R}^{\mathbb{C}}$ $\mathbb{C}^{\mathbb{R}}$ $\mathbb{R}^{\mathbb{R}}$ $\mathbb{R}^{\mathbb{R}}$

$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

$\begin{cases} (1-i)\cdot(1-i) + (-1)\cdot(1+i) = 0 + 0 \cdot i \\ (1-i)\cdot(-1) + (-1)\cdot(1-i) = -2 + 0 \cdot i \\ (1+i)\cdot(1-i) + 1\cdot(1+i) = 2 + 0 \cdot i \\ (1+i)\cdot(-1) + 1\cdot(1-i) = 0 + 0 \cdot i \end{cases}$

$\begin{pmatrix} -1+i & -2+i \\ 2+i & 1+i \end{pmatrix}$ $\operatorname{rang}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 1 \end{pmatrix} = 2$ $\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Sud 55 Exo 1

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2