Solving Day #25 ICQ Recurrence

T(1) = 1
T(N) = 2T(N/2) + N

Write recurrence using *x* as the variable: T(*x*) = *x* + 2T(*x*/2)

- *x* is a variable
- N is a constant equal to the size of the problem

T(N)	= N + 2T(N/2)	plug N in for x
= N + 2(N/2 + 2T(N/2/2))	= N + N + 4T(N/4)	plug N/2 in for x
= N + N + 4(N/4 + 2T(N/4/2))	= N + N + N + 8T(N/8)	plug N/4 in for x
= N + N + N + 8(N/8 + 2T(N/8/2))	= N + N + N + N + 16T(N/16)	plug N/8 in for x
= …	= …	

= N + N + N + N + … + N*T(N/N)
= N + N + N + N + … + N*T($N/2^k$)
= N + N + N + N + … + N*T(1)
= N + N + N + N + … + N

$\log_2(N)$

because N/2 in recurrence

N + N*$\log_2(N)$ which is $\theta(Nlog_2(N))$

Example: N = 2^5 = 32 (k = 5)

T(32)	= 32 + 2T(16)	begin by plugging N = 32 in for x
= 32 + 2(16 + 2T(8))	= 32 + 32 + 4T(8)	
= 32 + 32 + 4(8 + 2T(4))	= 32 + 32 + 32 + 8T(4)	
= 32 + 32 + 32 + 8(4 + 2T(2))	= 32 + 32 + 32 + 32 + 16T(2)	
= 32 + 32 + 32 + 32 + 16(2 + 2T(1))	= 32 + 32 + 32 + 32 + 32 + 32T(1)	

= 32 + 32 + 32 + 32 + 32 + 32

There are five 32's here, and k = 5

= 32 + 5(32)

= N + k(N) Note: 5 = $\log_2(2^5)$ = $\log_2(N)$

= N + $\log_2(N)$*N or N + N*$\log_2(N)$