Reasoning Table for goToTheRear

Use: T.Init(x) as a predicate to state that variable x has initial value for its type T

```
void goToTheRear(QueueOfT& q);
   //! updates q
   //! requires   |q| > 0
   //! ensures    q = #q[1,|#q|) * #q[0,1)
```

State	Code	Assume	Confirm
0		S0F: |q0| > 0	true This *true* is requires clause from Type T’s constructor
	T y;		
1		S1F: T.Init(y1) ^ q1 = q0 ^ S0F	q1 /= <>
	q.dequeue(y);		
2		S2F: <y2> is prefix of q1 ^ q2 = q1[1,|q1|) ^ S1F	true This *true* is requires clause from enqueue
	q.enqueue(y);		
3		S3F: q3 = q2 * <y2> ^ T.Init(y3) ^ S2F	q3 = q0[1,|q0|) * q0[0,1)

VCs written using SxFs

VC0: S0F → true
VC1: S1F → q1 /= <>
VC2: S2F → true
VC3: S3F → q3 = q0[1,|q0|) * q0[0,1)

Prove:

VC3: S3F → q3 = q0[1,|q0|) * q0[0,1)

Direct Proof

1. Assume facts on left hand side of implication are true
2. Must show right hand side of implication cannot be false – i.e., show row 2 of truth table cannot happen

TABLE 5 **The Truth Table for the Conditional Statement** $p \rightarrow q$**.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Recall our Facts – the highlighted facts (in this list) are used in the proof steps below**:**

S0F: |q0| > 0

S1F: T.Init(y1) ^ q1 = q0 ^ S0F

S2F: <y2> is prefix of q1 ^ q2 = q1[1,|q1|) ^ S1F

S3F: q3 = q2 * <y2> ^ T.Init(y3) ^ S2F

Proof Steps – Using a *backwards sweep* approach

Which means we start with q3 on the lhs of the equals sign and try to make it look like the rhs of the equals sign

Step 1.	q3 = q0[1,|q0|) * q0[0,1)	Begin with right hand side of VC3
Step 2.	q2 * <y2> = q0[1,|q0|) * q0[0,1)	By substitution for q3 in Step 1 using facts S3F
Step 3.	q1[1,|q1|) * <y2> = q0[1,|q0|) * q0[0,1)	By substitution for q2 in Step 2 using facts S2F
Step 4.	q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1)	By substitution for q1 in Step 3 using facts S1F

Note: from Step 4 the following highlighted parts are equal:
q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1)

At this point if we can show <y2> = q0[0,1), we will have successfully completed the proof

Step 5.	<y2> = q0[0,1)	Continue with this portion of the equation
Step 6.	<y2> is prefix of q1	Fact from S2F
Step 7.	<y2> is prefix of q0	By substitution for q1 in Step 6 using facts S1F
Step 8.	<y2> = q0[0,1)	Lemma: proof is based on definition of prefix

That successfully completes the proof, since the lhs and rhs of equals sign are equal (from Step 1)