A Detailed Look at *append*'s Reasoning Table

append's Reasoning Table

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
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append's RT - Code

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append's RT - States

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append's RT – State 0 Assume

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void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
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//! ensures: r = #r * #g
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append's RT – State 0 Confirm

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append's RT – State 1 Assume

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append's RT – State 1 Confirm

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append's RT – State 2 Assume

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//! requires: true
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//! clears g
//! requires: true
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	while(g.length() > 0) { //! updates g, r //! maintains //! r * g = #r * #g //! decreases |g|			r2 = r1 g2 = g1
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	T y;			
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append's RT – State 2 Confirm

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append's RT – State 3 Assume

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

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append's RT – State 3 Confirm

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r4 = r3 * <y3></td><td>Unchanged
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</table>

append’s RT – State 4 Assume

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

<table>
<tr><th>S</th><th>Code</th><th colspan="2">Assume</th><th>Confirm</th></tr>
<tr><td>0</td><td></td><td colspan="2">true</td><td>r0 * g0 = r0 * g0</td></tr>
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<tr><td></td><td></td><td colspan="2"></td><td></td></tr>
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</table>

append's RT – State 4 Confirm

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

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</table>

append's RT – State 4 Confirm

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

S	Code	Assume		Confirm
0		true		r0 * g0 = r0 * g0
	while(g.length() > 0) { //! updates g, r //! maintains //! r * g = #r * #g //! decreases |g|			
1		|g1| > 0 ^ r1 * g1 = r0 * g0		
	T y;			
2		T.Init(y2)	Unchanged r, g	g2 /= <>
	g.dequeue(y);			
3		g3 = g2[1,|g2|) ^ <y3> = prefix of g2	Unchanged r	
	r.enqueue(y);			
4		T.Init(y4) ^ r4 = r3 * <y3>	Unchanged g	|g4| < |g1| ^ r4 * g4 = r0 * g0
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append's RT – State 5 Assume

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

S	Code	Assume		Confirm
0		true		r0 * g0 = r0 * g0
	while (g.length() > 0) { //! updates g, r //! maintains //! r * g = #r * #g //! decreases |g|			
1		|g1| > 0 ^ r1 * g1 = r0 * g0		
	T y;			
2		T.Init(y2)	Unchanged r, g	g2 /= <>
	g.dequeue(y);			
3		g3 = g2[1,|g2|) ^ <y3> = prefix of g2	Unchanged r	
	r.enqueue(y);			
4		T.Init(y4) ^ r4 = r3 * <y3>	Unchanged g	|g4| < |g1| ^ r4 * g4 = r0 * g0
	}			
5		~(|g5| > 0) ^ r5 * g5 = r0 * g0		r5 = r0 * g0 ^ g5 = <>

append's RT – State 5 Assume

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void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

S	Code	Assume		Confirm
0		true		r0 * g0 = r0 * g0
	while (g.length() > 0) { //! updates g, r //! maintains //! r * g = #r * #g //! decreases |g|			
1		|g1| > 0 ^ r1 * g1 = r0 * g0		
	T y;			
2		T.Init(y2)	Unchanged r, g	g2 /= <>
	g.dequeue(y);			
3		g3 = g2[1,|g2|) ^ <y3> = prefix of g2	Unchanged r	
	r.enqueue(y);			
4		T.Init(y4) ^ r4 = r3 * <y3>	Unchanged g	|g4| < |g1| ^ r4 * g4 = r0 * g0
	}			
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append's RT – State 5 Assume

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append’s RT – State 5 Confirm

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void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
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append's RT – State 5 Confirm

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
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S	Code	Assume		Confirm
0		true		r0 * g0 = r0 * g0
	while(g.length() > 0) { //! updates g, r //! maintains //! r * g = #r * #g //! decreases |g|			
1		|g1| > 0 ^ r1 * g1 = r0 * g0		
	T y;			
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	}			
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Loop Invariant Requires Confirming

In locations **1** and **2** is where we must prove the loop invariant holds

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<tr><td>2</td><td></td><td>T.Init(y2)</td><td>Unchanged
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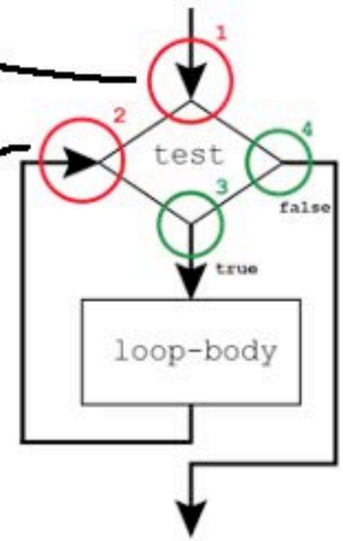

Loop Invariant Assumed True

In locations **3** and **4** is where we get to assume the loop invariant holds

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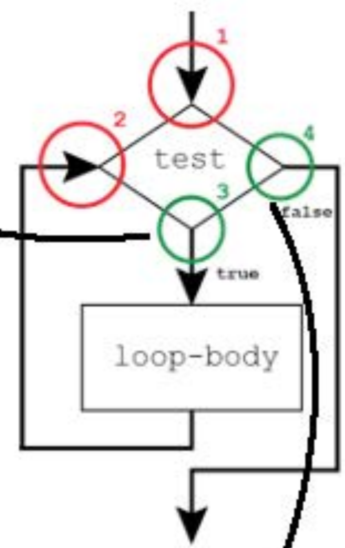

append's RT – Major Assertions

```
void appendV1 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! requires: true
//! ensures: r = #r * #g
```

s	Code	Assume		Confirm
0		true		r0 * g0 = r0 * g0 ①

	`while(g.length() > 0) {`
	`//! updates g, r`
	`//! maintains`
	`//! r * g = #r * #g`
	`//! decreases |g|`			

1		|g1| > 0 ^ r1 * g1 = r0 * g0 ③		
	`T y;`			
2		T.Init(y2)	Unchanged r, g	g2 /= <>
	`g.dequeue(y);`			
3		g3 = g2[1,|g2|) ^ <y3> = prefix of g2	Unchanged r	
	`r.enqueue(y);`			
4		T.Init(y4) ^ r4 = r3 * <y3>	Unchanged g	|g4| < |g1| ^ r4 * g4 = r0 * g0 ②
	`}`			
5		~(|g5| > 0) ^ r5 * g5 = r0 * g0 ④		r5 = r0 * g0 ^ g5 = <>
7				

Up Next – *append*'s VCs

1. Using the assertions in this RT to generate *append*'s VCs
2. Proving the VCs to prove *append* is correct, i.e., meets its spec