To Do:

1. Complete loop invariant at * by identifying the 2nd and 3rd conjuncts
2. Use completed loop invariant – Confirm at 1 and 2
3. After Step #2 – Assume it holds at 3 and 4
4. Prove the Confirm at State 8

Name 1:

Name 2:

One CM:

```
void appendV2 (QueueOfT& r, QueueOfT& g) // Using r for receiver, g for giver
//! updates r
//! clears g
//! ensures r = #r * #g
```

<table>
<tr><th>S</th><th>Code</th><th colspan="2">Assume</th><th>Confirm</th></tr>
<tr><td>0</td><td></td><td colspan="2"></td><td></td></tr>
<tr><td></td><td>Integer k, z;</td><td colspan="2"></td><td></td></tr>
<tr><td>1</td><td></td><td>k1 = 0 ^ z1 = 0</td><td>Unchanged
r, g</td><td></td></tr>
<tr><td></td><td>z = g.length();</td><td colspan="2"></td><td></td></tr>
<tr><td>2</td><td></td><td>z2 = |g1|</td><td>Unchanged
k, r, g</td><td>1</td></tr>
<tr><td></td><td>while(k < z) {
//! updates k, g, r
//! maintains
//! r * g = #r * #g ^
//! * 2nd conjunct ^
//! 3rd conjunct
//! decreases (z - k)</td><td colspan="2">Note: conjunct – is an operand of ^

Come up with 2nd conjunct for loop invariant that is invariant and involves k and either r or g

Come up with a 3rd conjunct involving k that places an upper limit on k</td><td></td></tr>
<tr><td>3</td><td></td><td>3 ^ k3 < z3</td><td></td><td></td></tr>
<tr><td></td><td>T y;</td><td colspan="2"></td><td></td></tr>
<tr><td>4</td><td></td><td>T.Init(y4)</td><td>Unchanged
k, z, r, g</td><td>g4 /= <></td></tr>
<tr><td></td><td>g.dequeue(y);</td><td colspan="2"></td><td></td></tr>
<tr><td>5</td><td></td><td>g5 = g4[1,|g4|) ^
<y5> = prefix of g4</td><td>Unchanged
k, z, r</td><td></td></tr>
<tr><td></td><td>r.enqueue(y);</td><td colspan="2"></td><td></td></tr>
<tr><td>6</td><td></td><td>T.Init(y6) ^
r6 = r5 * <y5></td><td>Unchanged
k, z, g</td><td>k6 < maxInt</td></tr>
<tr><td></td><td>k++;</td><td colspan="2"></td><td></td></tr>
<tr><td>7</td><td></td><td>k7 = k6 + 1</td><td>Unchanged
z, y, r, g</td><td>(z7 - k7) < (z3 - k3) ^
2</td></tr>
<tr><td></td><td>}</td><td colspan="2"></td><td></td></tr>
<tr><td>8</td><td></td><td>~(k8 < z8) ^ 4</td><td>Unchanged(2)
z</td><td>r8 = r0 * g0 ^
g8 = <></td></tr>
<tr><td></td><td></td><td colspan="2"></td><td></td></tr>
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