Prove *goToRear's VC3*:

```
VC3: (A0 ^ A1 ^ A2 ^ A3) → q3 = q0[1,|q0|) * q0[0,1)
```

Direct Proof

1. Assume facts on left hand side of implication are true (1st 2 rows of truth table where p = true)
2. Must show right hand side (rhs) of implication cannot be false (where q = false), i.e., show row 2 of truth table cannot happen
3. For the rhs to not be false we must show that the equality in the rhs holds:
   `q3 = q0[1,|q0|) * q0[0,1)`

TABLE 5 **The Truth Table for the Conditional Statement** $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Recall our Facts – the highlighted facts (in this list) are used in the proof steps below:

```
A0: |q0| > 0
A1: T.Init(y1) ^ q1 = q0
A2: <y2> is prefix of q1 ^ q2 = q1[1,|q1|)
A3: q3 = q2 * <y2> ^ T.Init(y3)
```

Proof Steps – Using a *backwards sweep* approach

Which means we start with q3 on the lhs of the equals sign and use substitution to transform the lhs into something that is similar to the rhs

```
Step 1.  q3 = q0[1,|q0|) * q0[0,1)                  Start with consequent side of VC3
Step 2.  q2 * <y2> = q0[1,|q0|) * q0[0,1)           By substitution for q3 from Step 1 using facts A3
Step 3.  q1[1,|q1|) * <y2> = q0[1,|q0|) * q0[0,1)   By substitution for q2 from Step 2 using facts A2
Step 4.  q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1)   By substitution for q1 from Step 3 using facts A1


         Note: from Step 4 the following highlighted parts are equal:
         q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1)

         At this point if we can show <y2> = q0[0,1), we will have successfully completed the proof

Step 5.  <y2> = q0[0,1)                             Continue with this portion of the equation
Step 6.  <y2> is prefix of q1                       Fact from A2
Step 7.  <y2> is prefix of q0                       By substitution for q1 from Step 6 using facts A1
Step 8.  <y2> = q0[0,1)                             Lemma: proof is based on definition of prefix
That successfully completes the proof, since the lhs and rhs of equals sign are equal (from Step 1)
```