

```

void flipStack2 (StackOfT& s)
///! updates s
///! requires |s| = 2
///! ensures s = #s[1,2) * #s[0,1)

```

Name: \_\_\_\_\_

Name: \_\_\_\_\_

One CM: \_\_\_\_\_

Reasoning Table for *flipStack2*

S	Code	Assume	Confirm
0		$ s0  = 2$	true
	T y; StackOfT t;	/ / / / / / / / / / / /	/ / / / / / / /
1		$T.Init(y1) \wedge$ $t1 = \langle \rangle \wedge$ $s1 = s0$	$s1 \neq \langle \rangle$
	s.pop(y)	/ / / / / / / / / / / /	/ / / / / / / /
2		$\langle y2 \rangle$ is prefix of $s1 \wedge$ $s2 = s1[1,  s1 ) \wedge$ $t2 = t1$	true
	t.push(y);	/ / / / / / / / / / / /	/ / / / / / / /
3		$t3 = \langle y2 \rangle * t2 \wedge$ $s3 = s2 \wedge$ $T.Init(y3)$	$s3 \neq \langle \rangle$
	s.pop(y)	/ / / / / / / / / / / /	/ / / / / / / /
4		$\langle y4 \rangle$ is prefix of $s3 \wedge$ $s4 = s3[1,  s3 ) \wedge$ $t4 = t3$	true
	t.push(y)	/ / / / / / / / / / / /	/ / / / / / / /
5		$t5 = \langle y4 \rangle * t4 \wedge$ $s5 = s4 \wedge$ $T.Init(y5)$	true
	s.transferFrom(t);	/ / / / / / / / / / / /	/ / / / / / / /
6		$s6 = t5 \wedge$ $t6 = \langle \rangle \wedge$ $y6 = y5$	$s6 = s0[1,2) * s0[0,1)$

VC1:  $|s0| = 2 \wedge T.Init(y1) \wedge t1 = \langle \rangle \wedge s1 = s0 \rightarrow s1 \neq \langle \rangle$

VC3:  $|s0| = 2 \wedge T.Init(y1) \wedge t1 = \langle \rangle \wedge s1 = s0 \wedge$   
 $\langle y2 \rangle$  is prefix of  $s1 \wedge s2 = s1[1, |s1|) \wedge t2 = t1 \wedge$   
 $t3 = \langle y2 \rangle * t2 \wedge s3 = s2 \wedge T.Init(y3) \rightarrow s3 \neq \langle \rangle$

VC6:  $|s0| = 2 \wedge T.Init(y1) \wedge t1 = \langle \rangle \wedge s1 = s0 \wedge$   
 $\langle y2 \rangle$  is prefix of  $s1 \wedge s2 = s1[1, |s1|) \wedge t2 = t1 \wedge$   
 $t3 = \langle y2 \rangle * t2 \wedge s3 = s2 \wedge T.Init(y3) \wedge$   
 $\langle y4 \rangle$  is prefix of  $s3 \wedge s4 = s3[1, |s3|) \wedge t4 = t3 \wedge$   
 $t5 = \langle y4 \rangle * t4 \wedge s5 = s4 \wedge T.Init(y5) \wedge$   
 $s6 = t5 \wedge t6 = \langle \rangle \wedge y6 = y5 \rightarrow s6 = s0[1,2) * s0[0,1)$

Steps:

1. Assume all the *premises* on the lhs of  $\rightarrow$  are true
2. Use the premises from the lhs to show that the *conclusion* on the rhs cannot be false

- The proof goes through if the conclusion is shown to be true
- The proof fails if the conclusion cannot be shown to be true

To do: Prove VC6

## Direct Proof of VC6

$$\begin{aligned}
 & |s_0| = 2 \wedge T.Init(y_1) \wedge \boxed{t_1 = \langle \rangle} \wedge \boxed{s_1 = s_0} \wedge \\
 & \langle y_2 \rangle \text{ is prefix of } s_1 \wedge \boxed{s_2 = s_1[1, |s_1|)} \wedge \boxed{t_2 = t_1} \wedge \\
 & \boxed{t_3 = \langle y_2 \rangle * t_2} \wedge \boxed{s_3 = s_2} \wedge T.Init(y_3) \wedge \\
 & \langle y_4 \rangle \text{ is prefix of } s_3 \wedge s_4 = s_3[1, |s_3|) \wedge \boxed{t_4 = t_3} \wedge \\
 & \boxed{t_5 = \langle y_4 \rangle * t_4} \wedge s_5 = s_4 \wedge T.Init(y_5) \wedge \\
 & \boxed{s_6 = t_5} \wedge t_6 = \langle \rangle \wedge y_6 = y_5 \rightarrow \boxed{s_6 = s_0[1, 2) * s_0[0, 1)}
 \end{aligned}$$

Steps:

- 1) Assume all of VC6's premises are true
- 2) Show that the equality in VC6's conclusion holds utilizing its premises

Start with the VC6's conclusion (i.e., the rhs of VC6)

Do a backwards sweep starting with  $s_6$  using VC6's premises to make transformations to  $s_6$

1. $s_6 = s_0[1, 2) * s_0[0, 1)$	Conclusion from VC6
2. $t_5 = s_0[1, 2) * s_0[0, 1)$	Substitution using #1
3. $\langle y_4 \rangle * t_4 = s_0[1, 2) * s_0[0, 1)$	Substitution using #2
4. $\langle y_4 \rangle * t_3 = s_0[1, 2) * s_0[0, 1)$	Substitution using #3
5. $\langle y_4 \rangle * \langle y_2 \rangle * t_2 = s_0[1, 2) * s_0[0, 1)$	Substitution using #4
6. $\langle y_4 \rangle * \langle y_2 \rangle * t_1 = s_0[1, 2) * s_0[0, 1)$	Substitution using #5
7. $\langle y_4 \rangle * \langle y_2 \rangle * \langle \rangle = s_0[1, 2) * s_0[0, 1)$	Substitution using #6
8. $\langle y_4 \rangle * \langle y_2 \rangle = s_0[1, 2) * s_0[0, 1)$	Concatenation identity
9. $s_0[1, 2) * s_0[0, 1) = s_0[1, 2) * s_0[0, 1)$	Lemma #1 and Lemma #2 (below)

Lemma 1:

1. $\langle y_2 \rangle$ is prefix of $s_1 = \langle y_2 \rangle$ is prefix of $s_0$	Substitution using #7
2. $\langle y_2 \rangle = s_0[0, 1)$	Definition of prefix

Lemma 2:

1. $\langle y_4 \rangle$ is prefix of $s_3 = \langle y_4 \rangle$ is prefix of $s_2$	Substitution using #8
2. $\langle y_4 \rangle = s_2[0, 1)$	Definition of prefix
3. $\langle y_4 \rangle = s_1[1,  s_1 )[0, 1)$	Substitution using #9
4. $\langle y_4 \rangle = s_0[1,  s_0 )[0, 1)$	Substitution using #7
5. $\langle y_4 \rangle = s_0[1, 2)$	Application of Concise Notation