## Prove goToRear's VC3:

VC3: 
$$(A0 ^ A1 ^ A2 ^ A3) \rightarrow q3 = q0[1, |q0|) * q0[0, 1)$$

## **Direct Proof**

- 1. **Assume** facts on *antecedent* (lhs) of implication are true (i.e., where p = true)
- 2. Must **show** *consequent* (rhs) of implication cannot be false, i.e., show row 2 of truth table cannot happen (i.e., where q = false)
- 3. For the rhs to not be false we must show that the equality in the rhs holds: q3 = q0[1, |q0|) \* q0[0, 1)

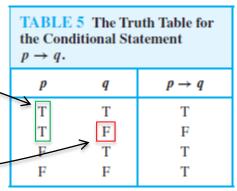
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Recall our Facts – the highlighted facts (in this list) are used in the proof steps below:
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A0: |q0| > 0

A1: T.Init(y1)  $^{q1} = q0$ 

A2:  $\langle y2 \rangle$  is prefix of q1 ^ q2 = q1[1, |q1|)

A3:  $q3 = q2 * < y2 > ^ T.Init(y3)$ 



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## **Proof Steps** – Using a *backwards sweep* approach

Which means we start with q3 on the lhs of the equals sign and use substitution to transform the lhs into something that is similar to the rhs

Step 1. q3 = q0[1, |q0|) \* q0[0,1)

Start with consequent side of VC3

Step 2.  $q2 * \langle y2 \rangle = q0[1, |q0|) * q0[0, 1)$ 

By substitution for q3 from Step 1 using facts A3

Step 3. q1[1,|q1|) \* < y2> = q0[1,|q0|) \* q0[0,1)

By substitution for q2 from Step 2 using facts A2

Step 4. q0[1,|q0|) \* <y2> = q0[1,|q0|) \* q0[0,1)

By substitution for q1 from Step 3 using facts A1

Note: from Step 4 the following highlighted parts are equal:

$$q0[1,|q0|)$$
 \*  =  $q0[1,|q0|)$  \*  $q0[0,1)$ 

At this point if we can show  $\langle y2 \rangle = q0[0,1)$ , we will have successfully completed the proof

Step 5.  $\langle y2 \rangle = q0[0,1)$ 

Continue with this portion of the equation

Step 6. <y2> is prefix of q1

Fact from A2

Step 7.  $\langle y2 \rangle$  is prefix of q0

By substitution for q1 from Step 6 using facts A1

Step 8.  $\langle y2 \rangle = q0[0,1)$ 

Lemma: proof is based on definition of prefix

That successfully completes the proof, since the lhs and rhs of equals sign are equal (from Step 1)