Prove *goToRear's VC3*:

VC3: (A0 ^ A1 ^ A2 ^ A3) → q3 = q0[1,|q0|) * q0[0,1)

Direct Proof

1. **Assume** facts on *antecedent* (lhs) of implication are true (i.e., where *p* = true)
2. Must **show** *consequent* (rhs) of implication cannot be false,
   i.e., show row 2 of truth table cannot happen (i.e., where *q* = false)
3. For the rhs to not be false we must show that the equality in the rhs holds:
   q3 = q0[1,|q0|) * q0[0,1)

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Recall our Facts – the highlighted facts (in this list) are used in the proof steps below:

A0: |q0| > 0
A1: T.Init(y1) ^ q1 = q0
A2: <y2> is prefix of q1 ^ q2 = q1[1,|q1|)
A3: q3 = q2 * <y2> ^ T.Init(y3)

Proof Steps – Using a *backwards sweep* approach
Which means we start with q3 on the lhs of the equals sign and use substitution to transform the lhs into something that is similar to the rhs

Step 1. q3 = q0[1,|q0|) * q0[0,1) — Start with consequent side of VC3
Step 2. q2 * <y2> = q0[1,|q0|) * q0[0,1) — By substitution for q3 from Step 1 using facts A3
Step 3. q1[1,|q1|) * <y2> = q0[1,|q0|) * q0[0,1) — By substitution for q2 from Step 2 using facts A2
Step 4. q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1) — By substitution for q1 from Step 3 using facts A1

Note: from Step 4 the following highlighted parts are equal:
q0[1,|q0|) * <y2> = q0[1,|q0|) * q0[0,1)

At this point if we can show <y2> = q0[0,1), we will have successfully completed the proof

Step 5. <y2> = q0[0,1) — Continue with this portion of the equation
Step 6. <y2> is prefix of q1 — Fact from A2
Step 7. <y2> is prefix of q0 — By substitution for q1 from Step 6 using facts A1
Step 8. <y2> = q0[0,1) — Lemma: proof is based on definition of prefix

That successfully completes the proof, since the lhs and rhs of equals sign are equal (from Step 1)