

## UNIVERSIDADE FEDERAL DO CEARÁ

CAMPUS DE RUSSAS

# Algoritmos em Grafos

Aula 12: Caminho Mínimo (Dijkstra)

**Professor Pablo Soares** 

2022.1

#### Sumário

- Busca em Largura(últimas aula);
  - a. Distância mínima em número de arestas;
  - b. Árvore Primeiro na Extensão.
- 2. Caminho Mínimo.
  - a. Motivação;
    - i. Modelagem;
  - b. Algoritmo de Dijkstra.
    - i. Estruturas utilizadas;
    - ii. Tempo de Execução.

### Caminho Mínimo(Motivação)

### Situação Comum







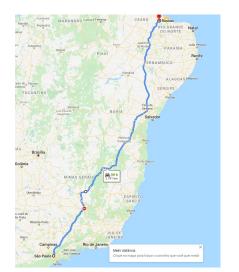
## Caminho Mínimo(Motivação)

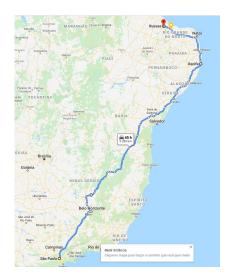
#### Situação Comum

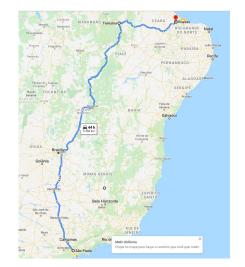












### Caminho Mínimo(Modelagem)

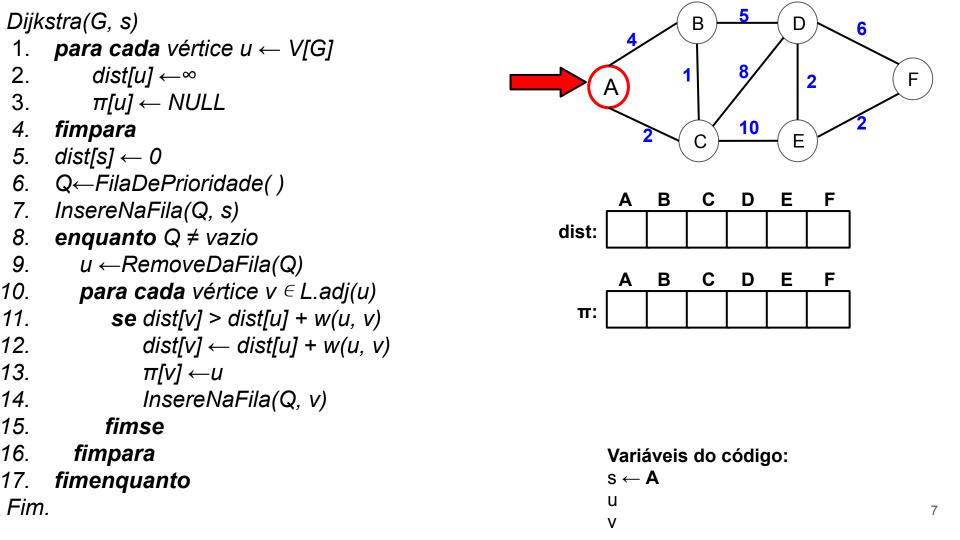
- 1. Mapa rodoviário;
  - a.  $G = (V, E) \rightarrow \text{grafo ponderado};$ 
    - i.  $V \rightarrow Conjunto de cidades;$
    - ii.  $E \rightarrow Segmentos de estradas entre cidades.$ 
      - $w(u, v) \rightarrow \text{Distância entre a cidade } u \text{ e } v(\text{peso da aresta}).$
- 2. Seja "C" um caminho em G e P() uma função de peso;
  - a.  $P(\mathbf{C}) = \sum w(u, v), \ \forall (u, v) \in \mathbf{C}$

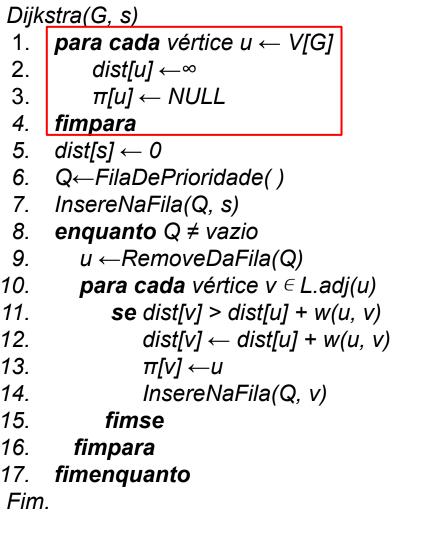
$$\delta(u, v) = \begin{cases} min\{P(c): u \sim v\}, \text{ se existir caminho de } u \text{ até } v \\ \infty, \text{ caso contrário} \end{cases}$$

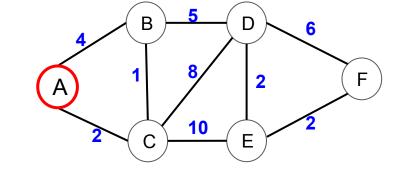
#### Algoritmo de Dijkstra

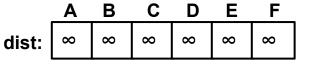
Pseudocódigo

```
Dijkstra(G, s)
 1. para cada vértice u ← V[G]
         dist[u] ←∞
         \pi[u] \leftarrow NULL
     fimpara
     dist[s] \leftarrow 0
 6. Q←FilaDePrioridade()
     InsereNaFila(Q, s)
     enquanto Q ≠ vazio
        u ←RemoveDaFila(Q)
10.
        para cada vértice v ∈ L.adj(u)
11.
           se dist[v] > dist[u] + w(u, v)
12.
               dist[v] \leftarrow dist[u] + w(u, v)
13.
               \pi[v] \leftarrow u
14.
               InsereNaFila(Q, v)
15.
           fimse
16.
       fimpara
17.
     fimenquanto
Fim.
```

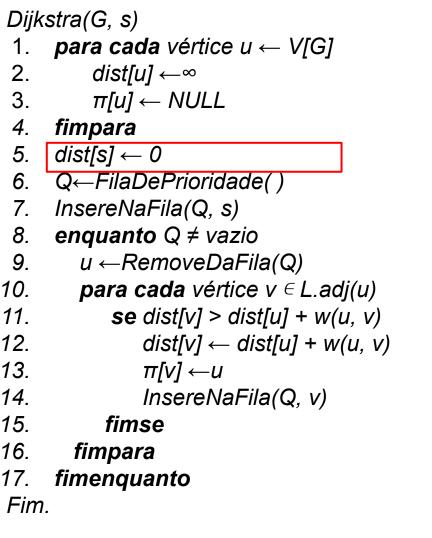


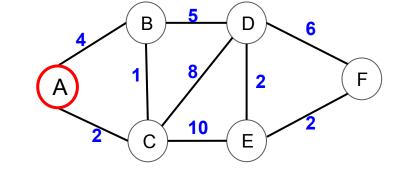


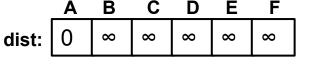




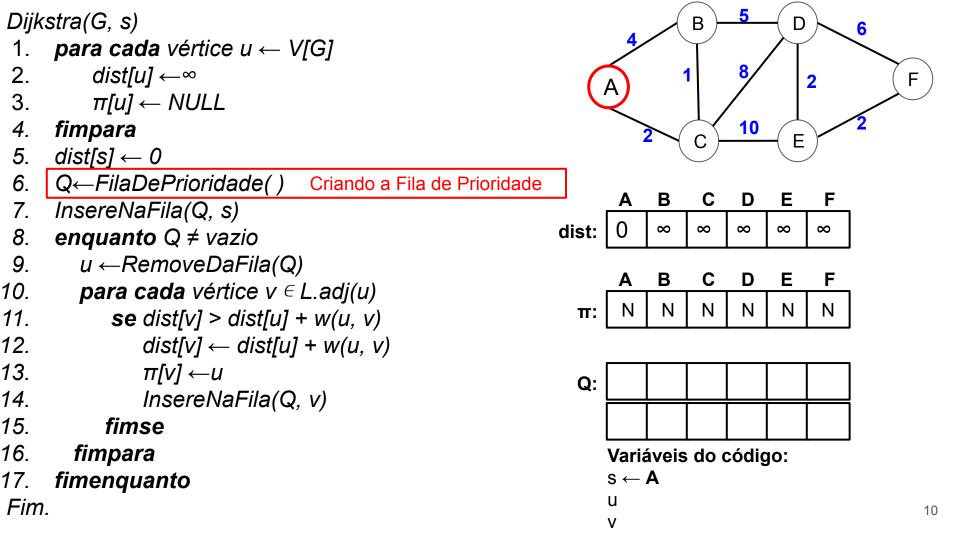
u

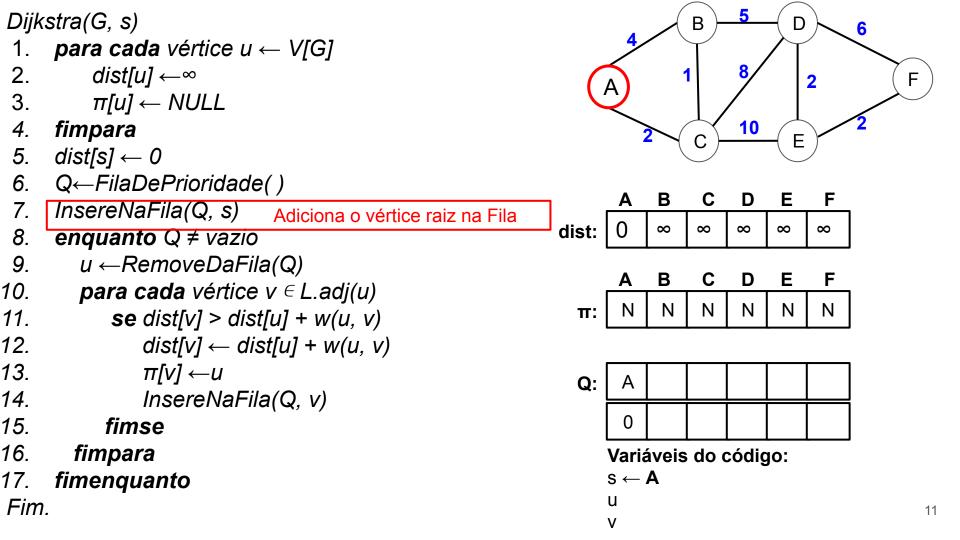


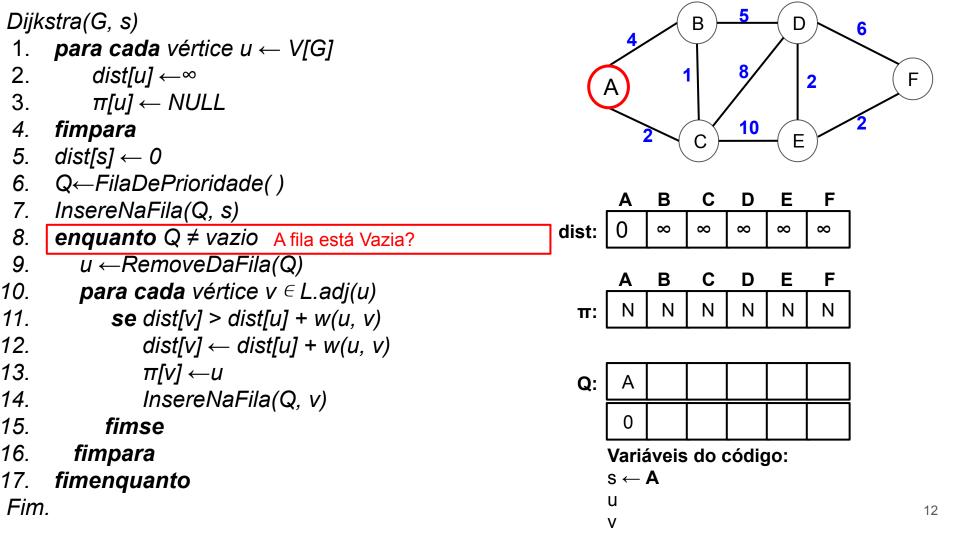


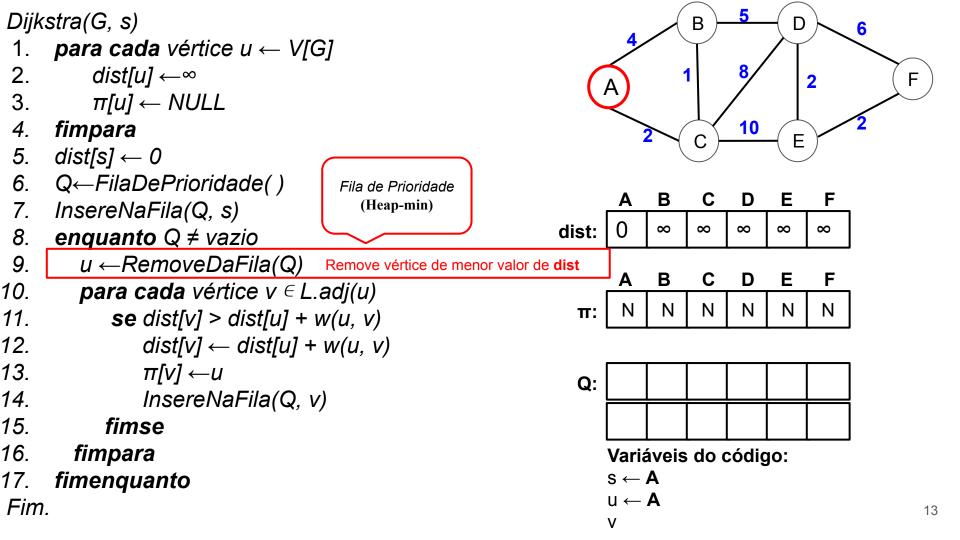


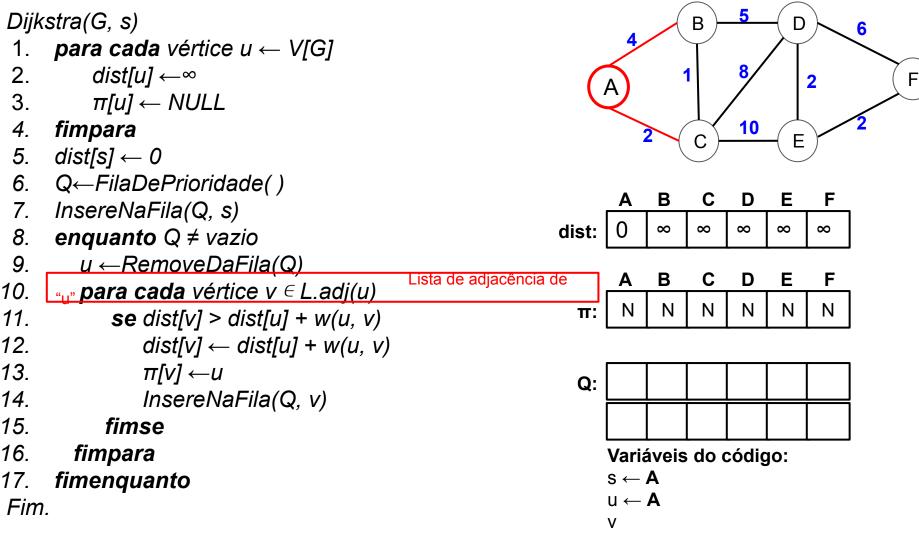
$$s \leftarrow \mathbf{A}$$
 U

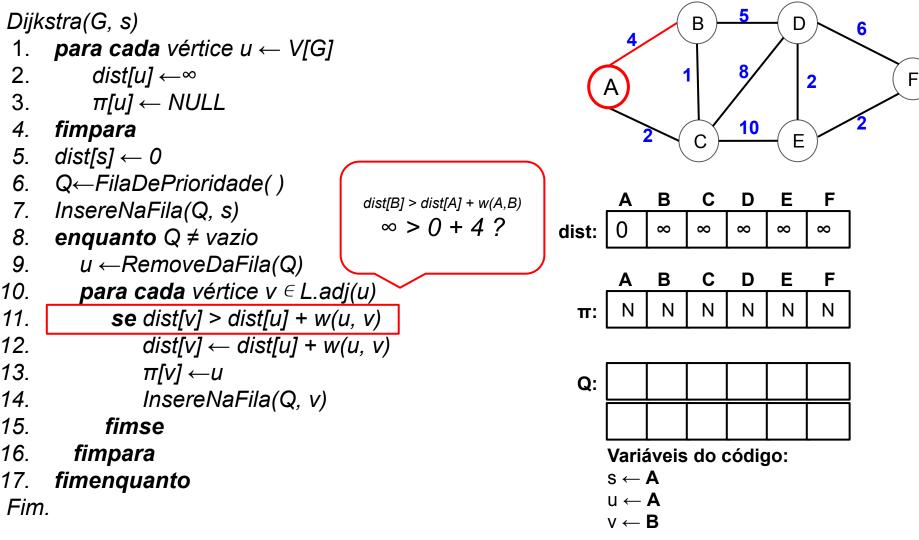


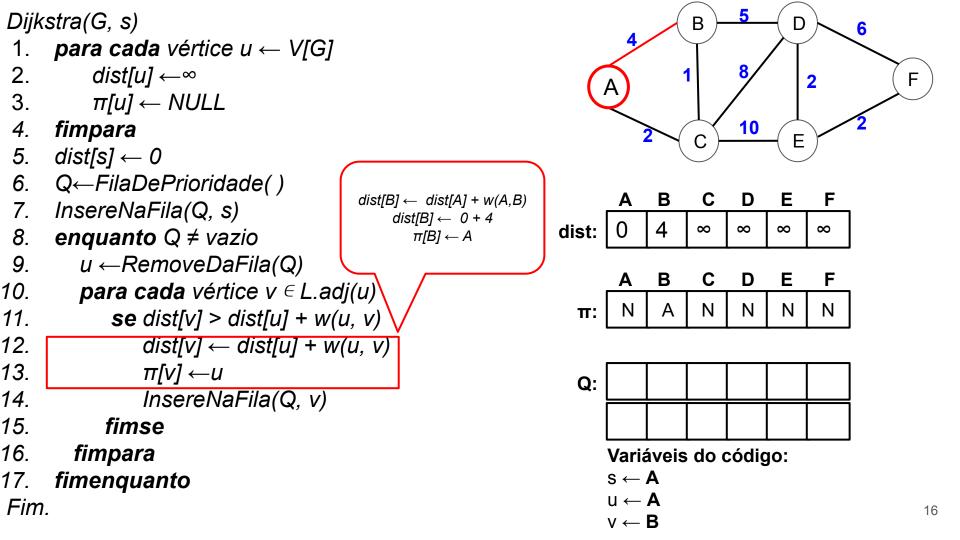


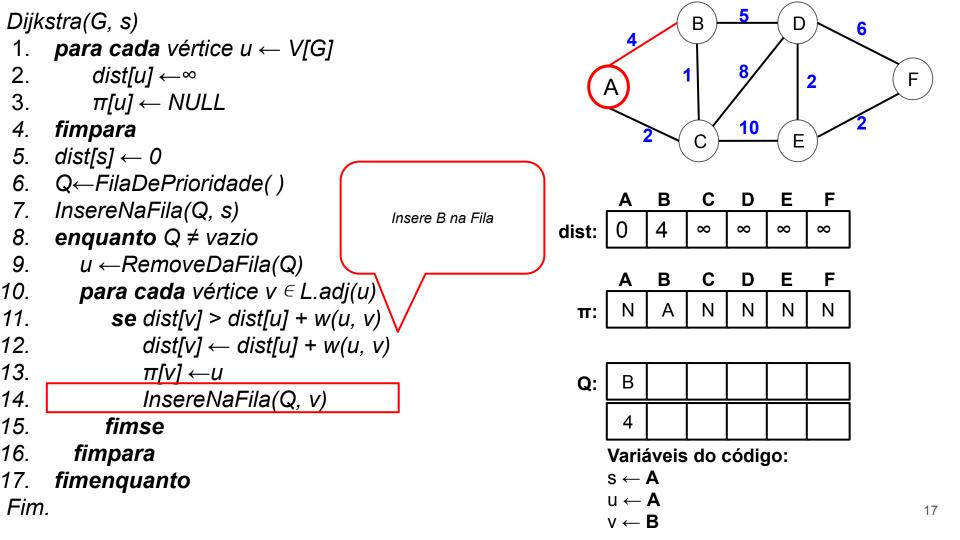


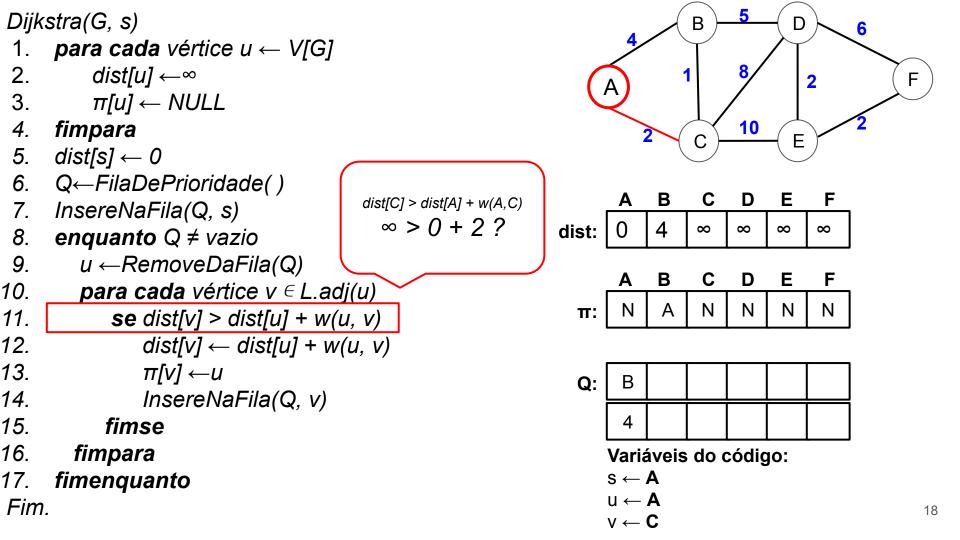


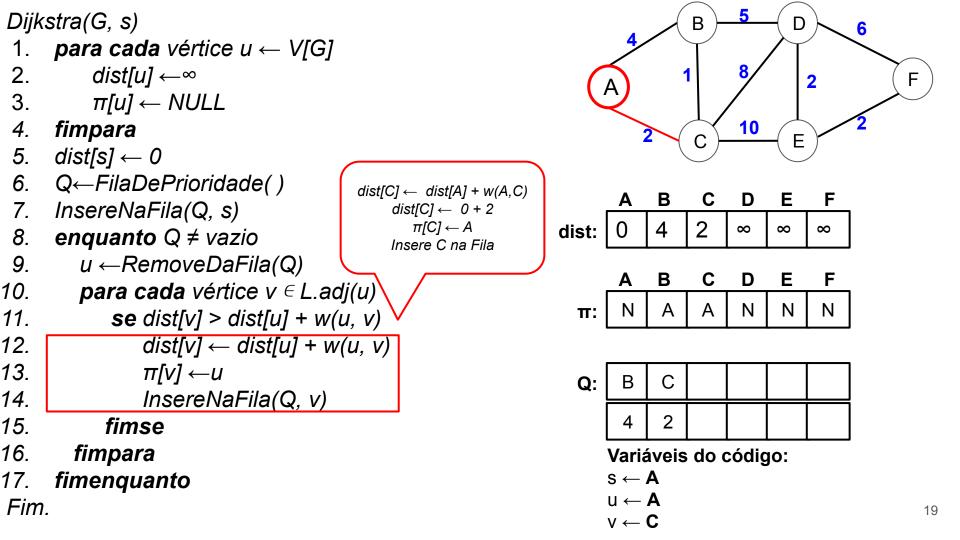


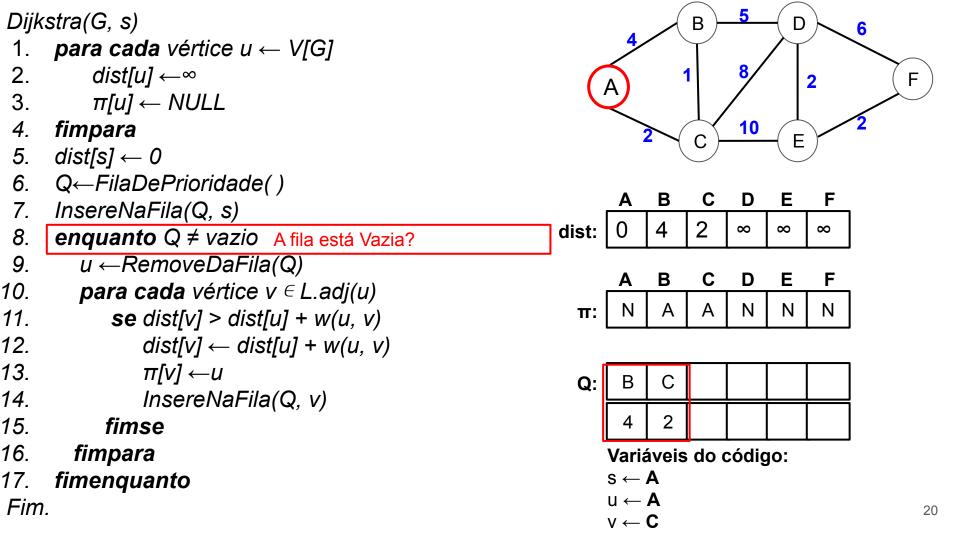


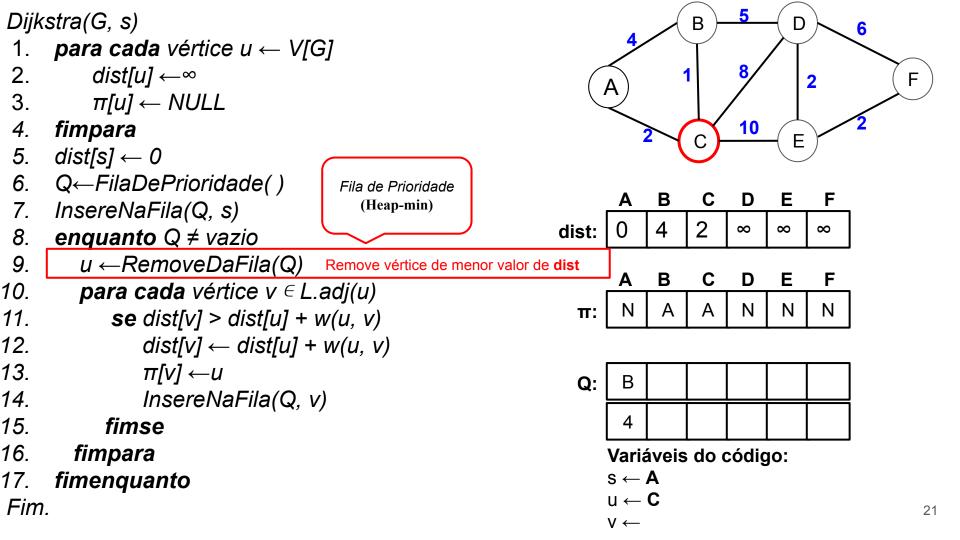


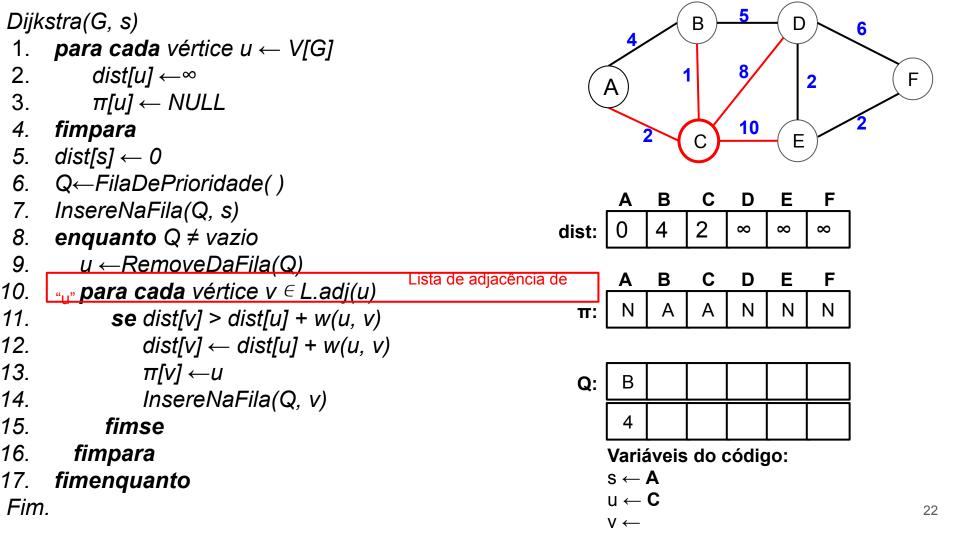


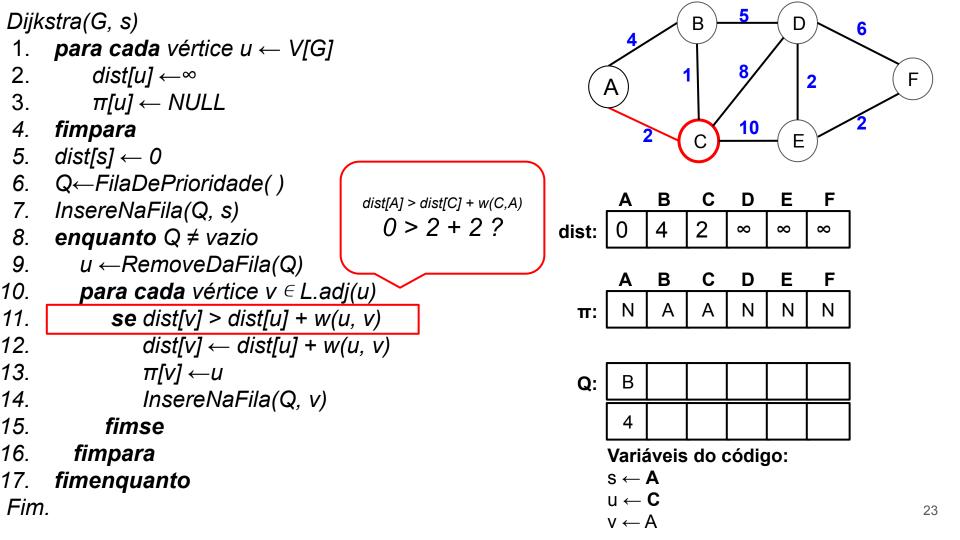


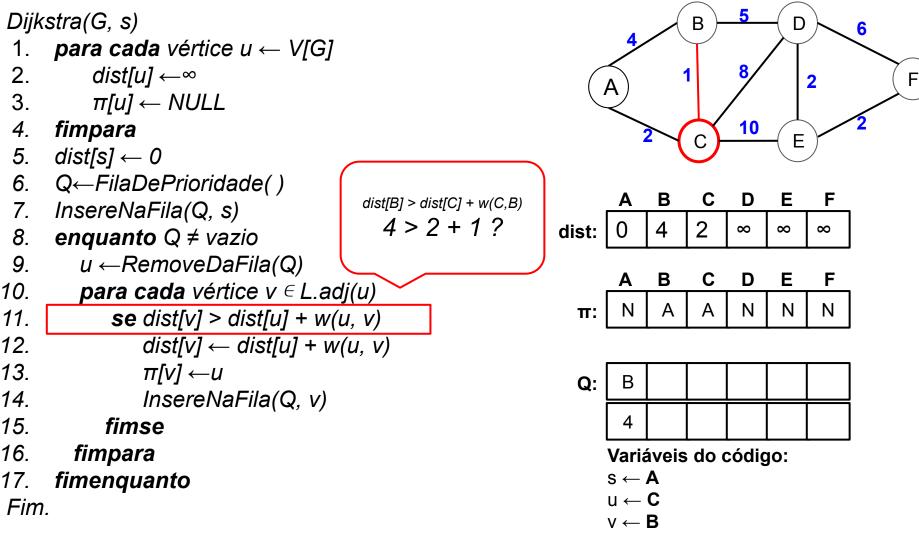


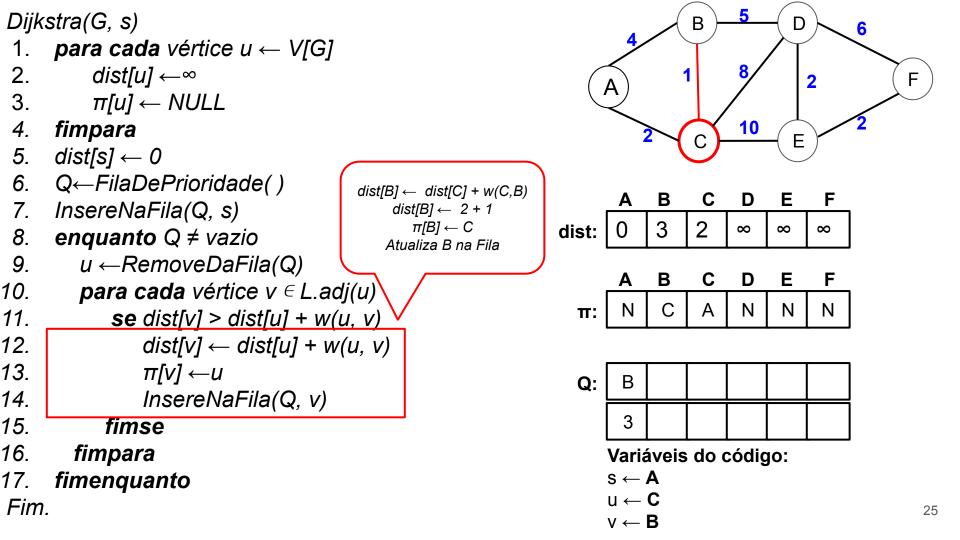


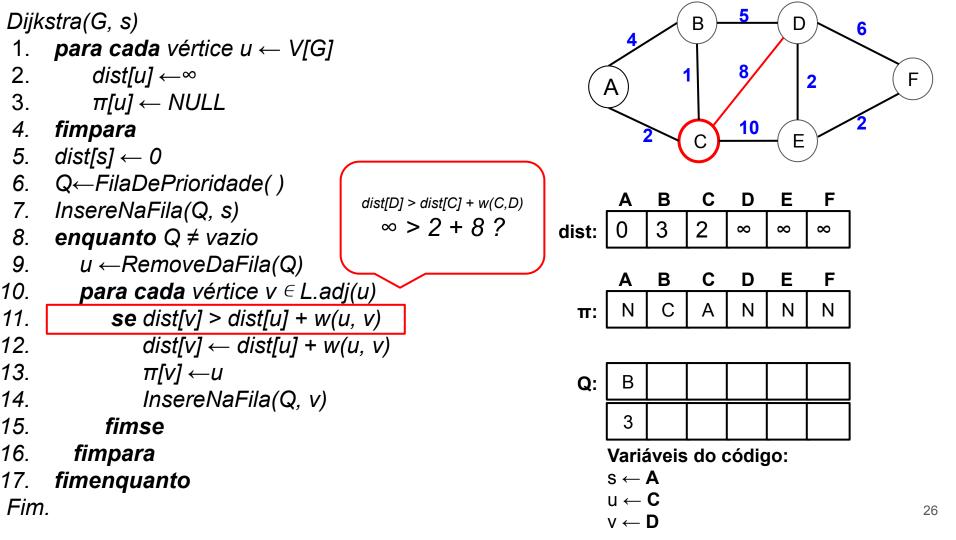


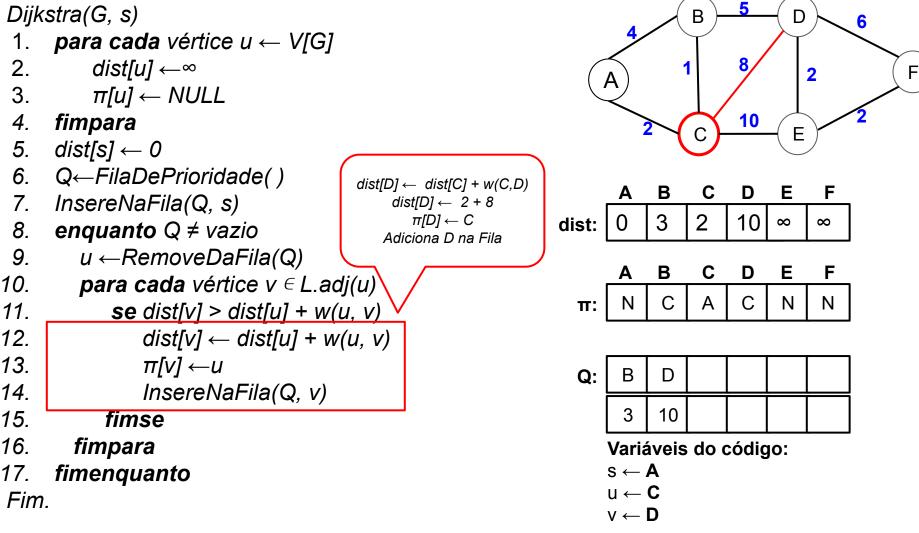


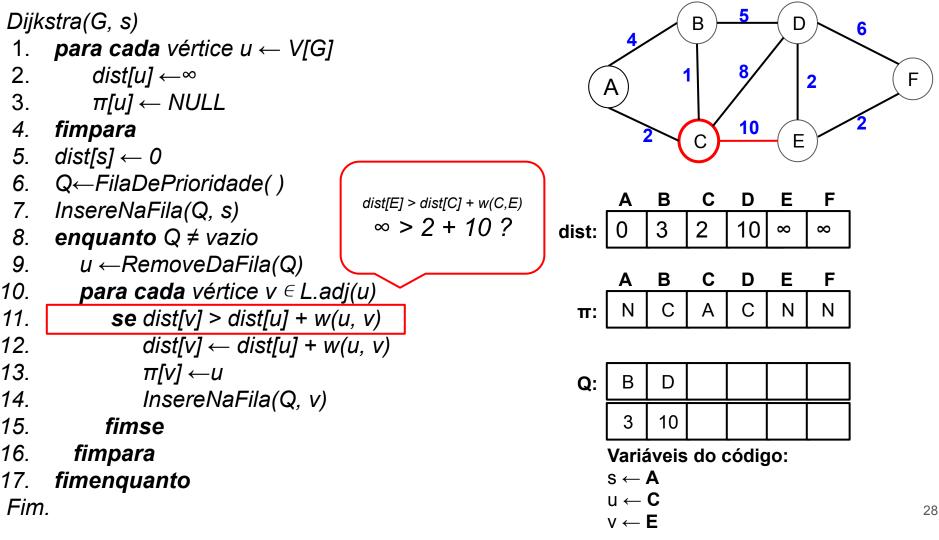


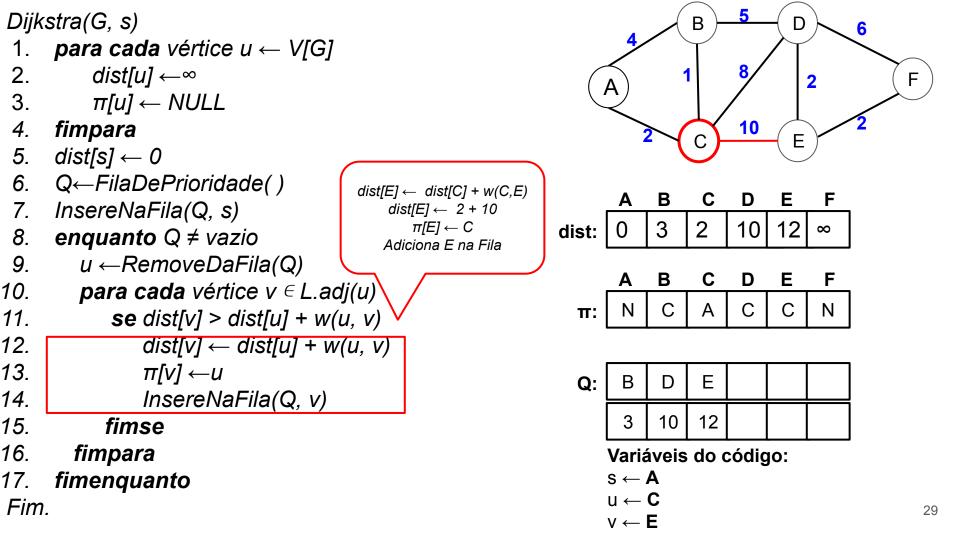


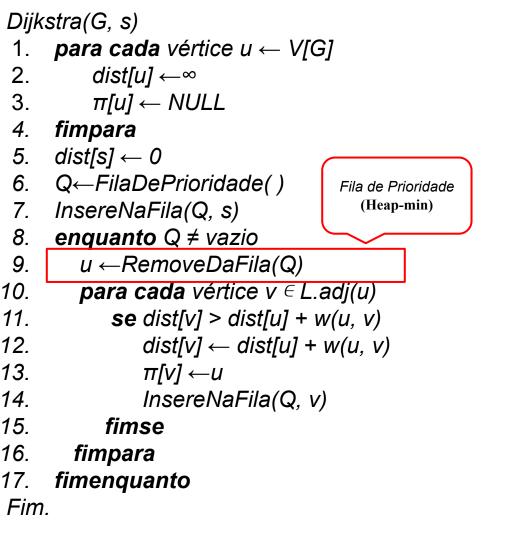


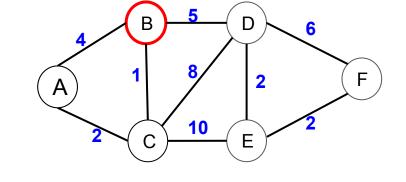


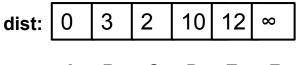






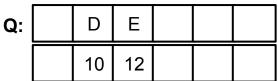






В

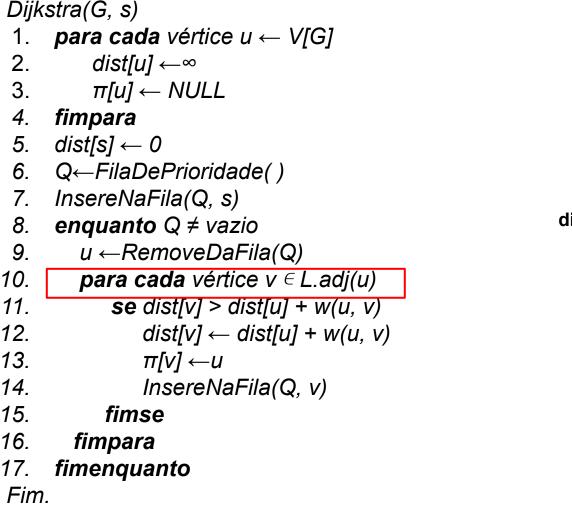


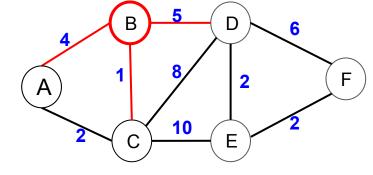


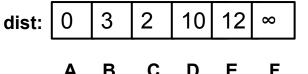
Variáveis do código:

u **← B ∨** ← **E** 

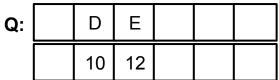
 $s \leftarrow A$ 



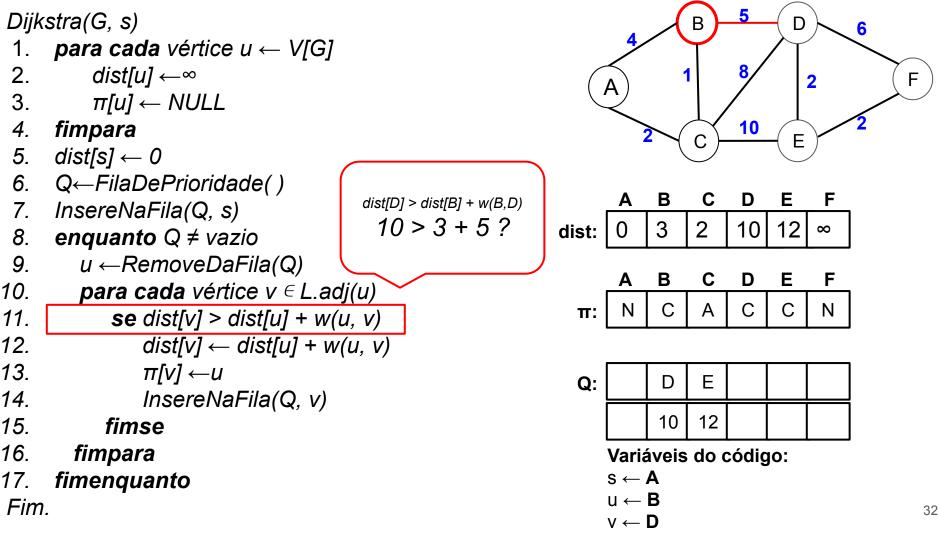


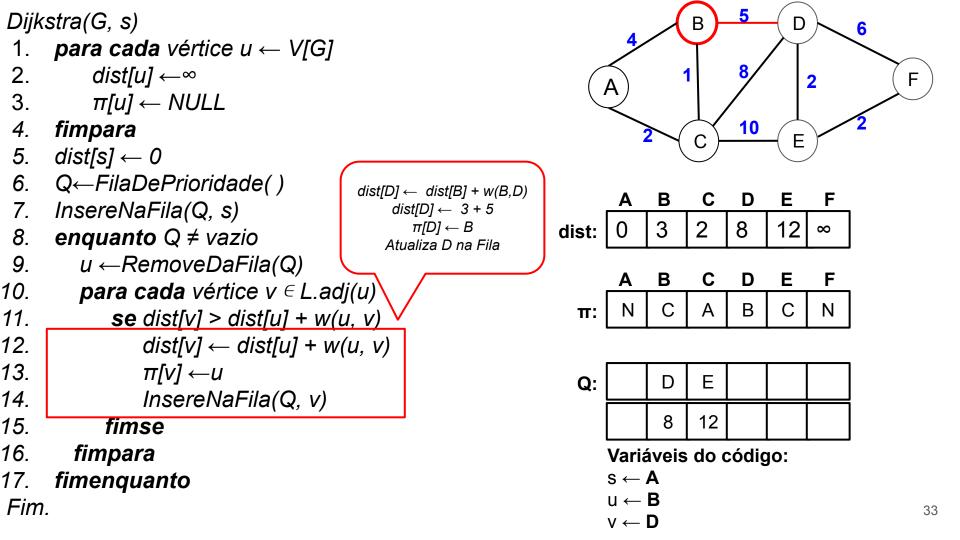


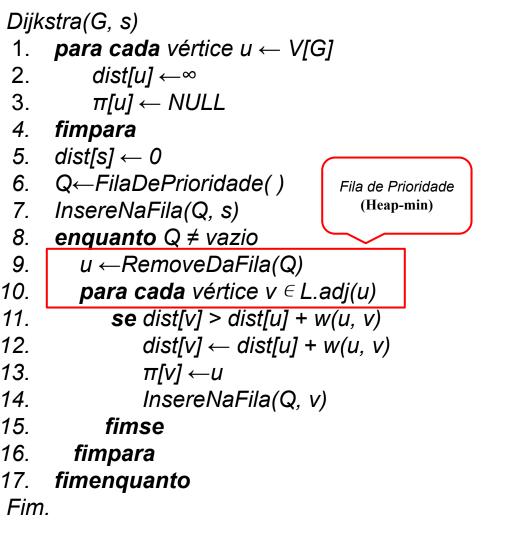
π:	Ν	С	Α	С	С	Ν

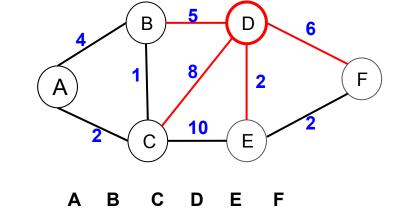


$$s \leftarrow \mathbf{A}$$
  
 $u \leftarrow \mathbf{B}$   
 $v \leftarrow \mathbf{E}$ 





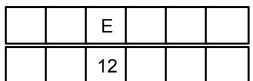






8

 $\infty$ 



Variáveis do código:

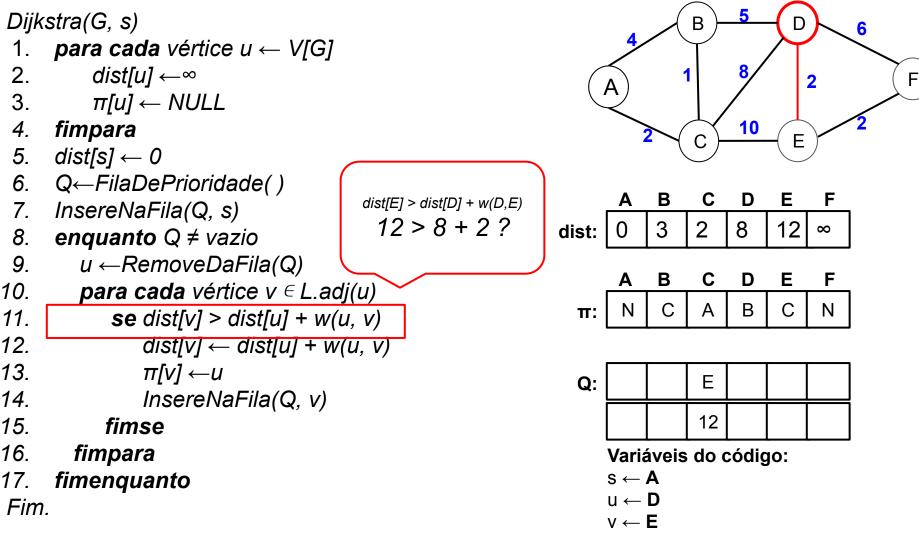
$$s \leftarrow \mathbf{A}$$
  
 $u \leftarrow \mathbf{D}$ 

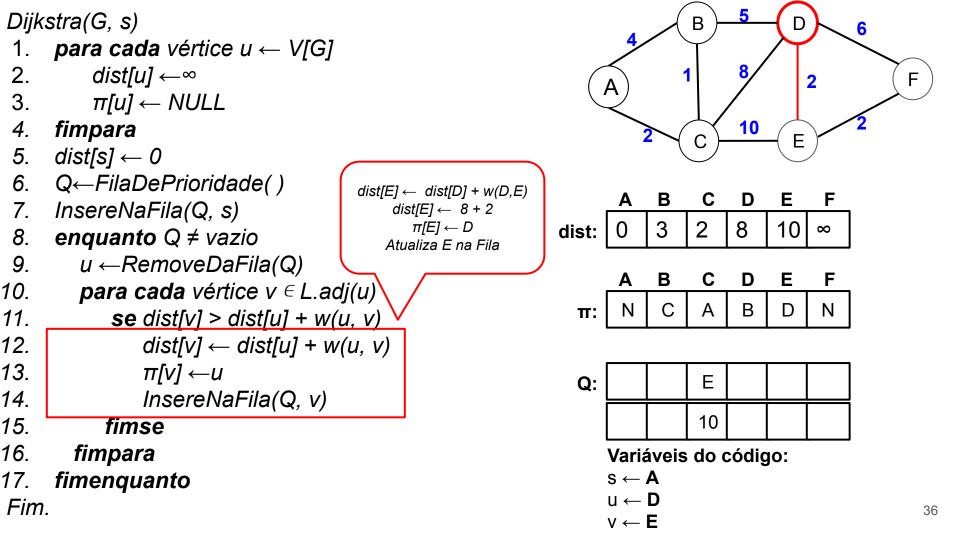
**∨** ← **D** 

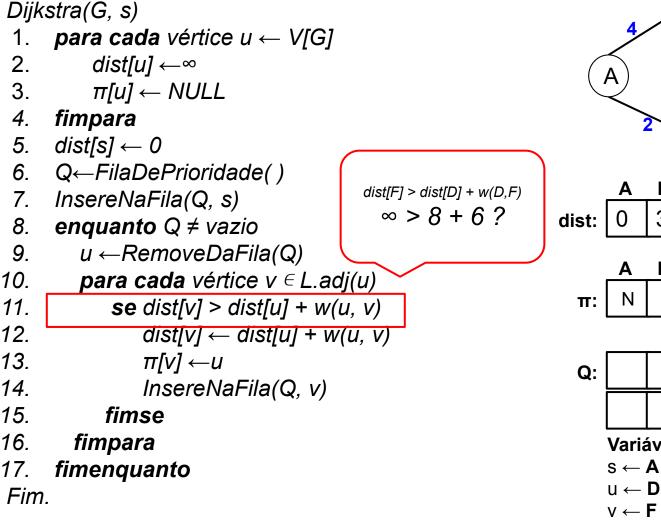
3

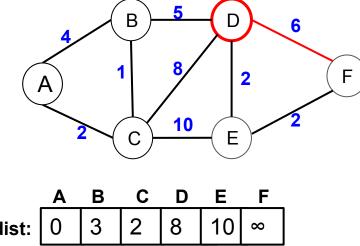
dist:

Q:







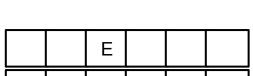


D

В

F

Ν



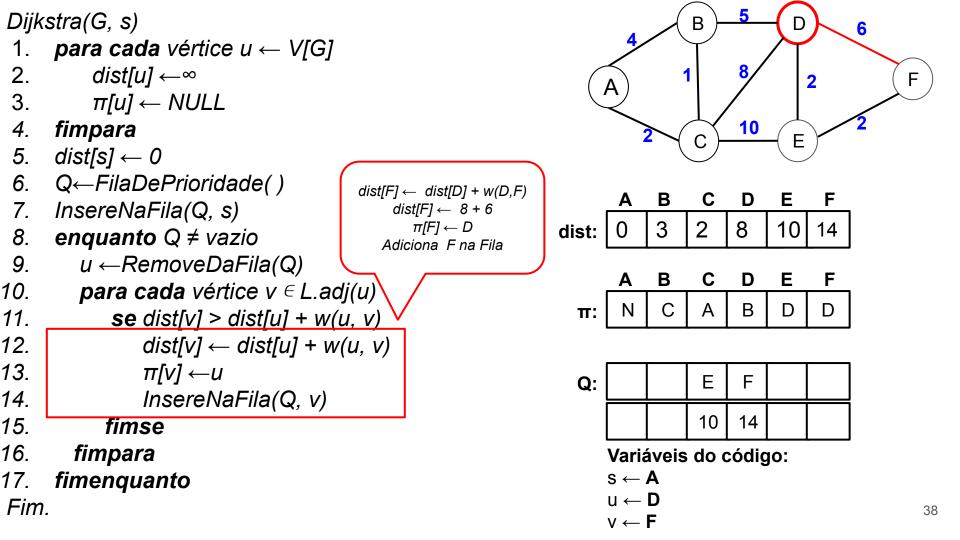
10

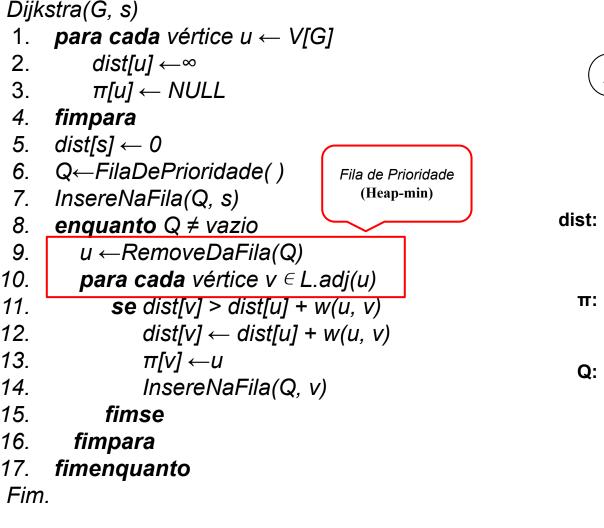
Variáveis do código:

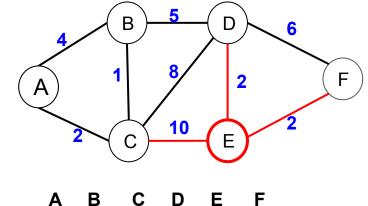
u ← **D** 

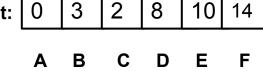
В

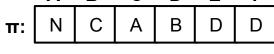
C





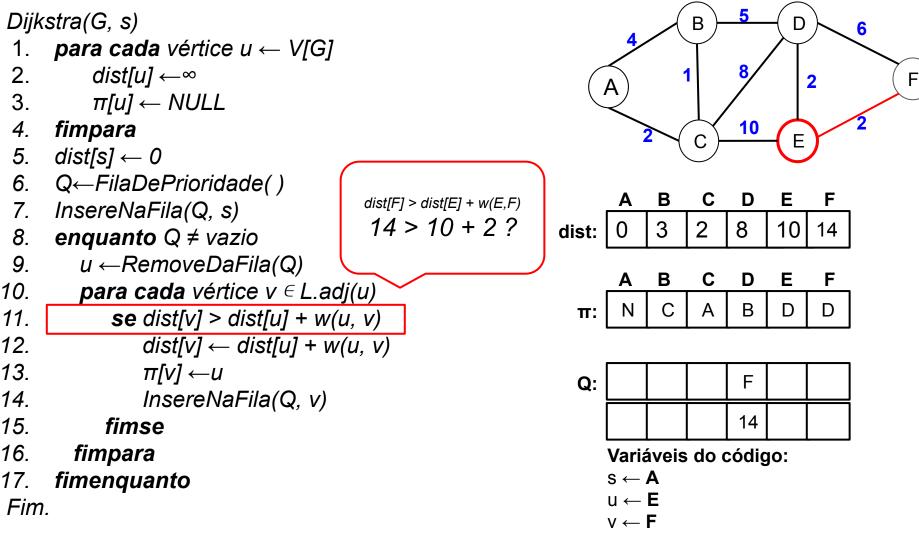


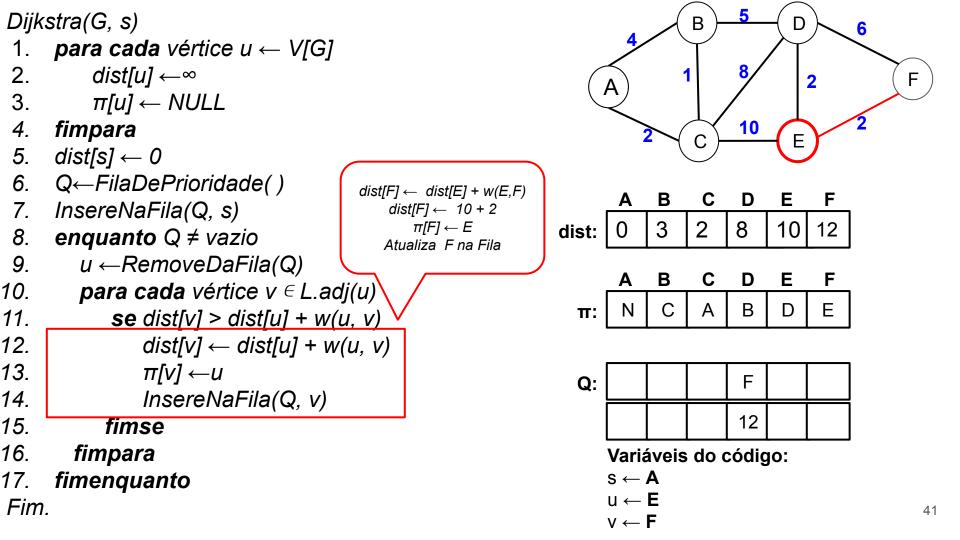


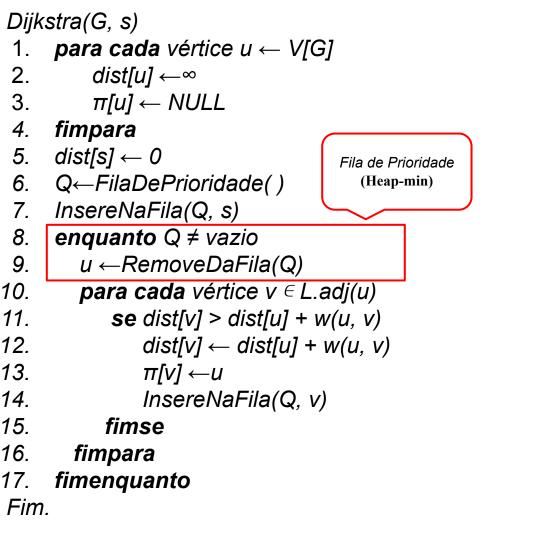


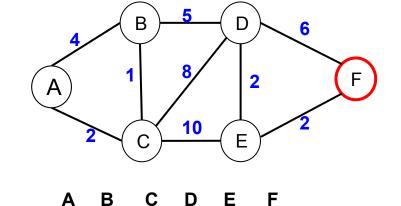


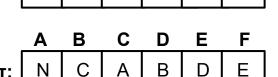
$$s \leftarrow A$$
  
 $u \leftarrow E$   
 $v \leftarrow F$ 











8

10 I

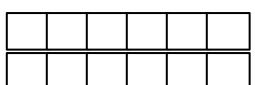
12

3

dist:

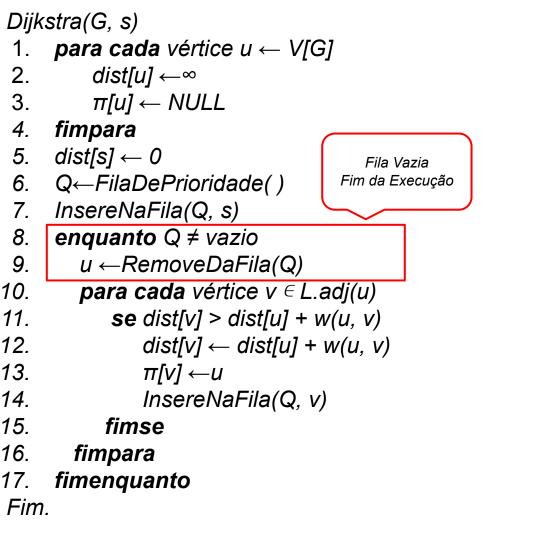
π:

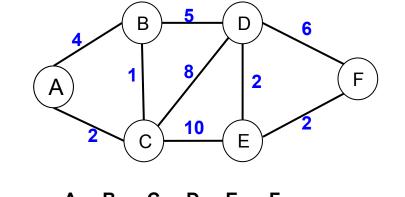
Q:

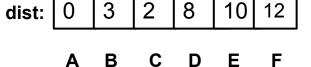


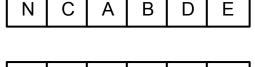
Variáveis do código:

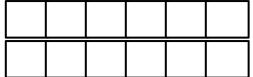
v ← **F** 











 $u \leftarrow \mathbf{F}$   $v \leftarrow \mathbf{F}$ 

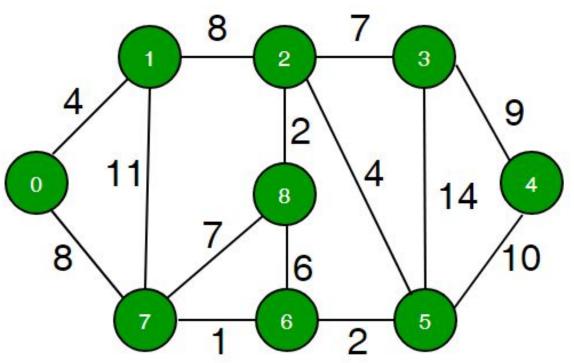
π:

Q:

### Exercício de Fixação

Encontre a menor distância do vértice 8 para todos os outros vértices do

grafo.





## UNIVERSIDADE FEDERAL DO CEARÁ

CAMPUS DE RUSSAS

# Algoritmos em Grafos

Aula 12: Caminho Mínimo (Dijkstra)

**Professor Pablo Soares** 

2022.1