UNIVERSIDADE FEDERAL DO CEARÁ

CAMPUS DE RUSSAS

Algoritmos em Grafos

Aula 17: Fluxo Máximo em Redes(Push-Relabel)

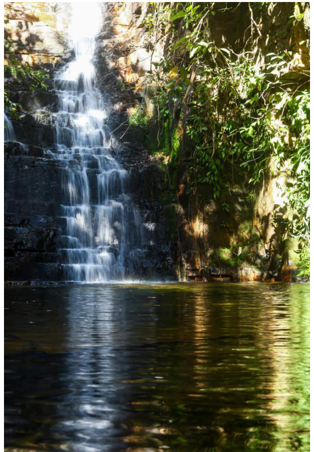
Professor Pablo Soares

2022.1

Sumário

1. Fluxo Máximo em Redes(**última aula**);
    a. Rede Residual;
    b. Caminho aumentante.
2. Intuição;
3. Pré-Fluxo;
4. Operações.
    a. Push;
    b. Relabel;
    c. Algoritmo Genérico Push-Relabel.
        i. Exemplo.

Intuição



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Intuição

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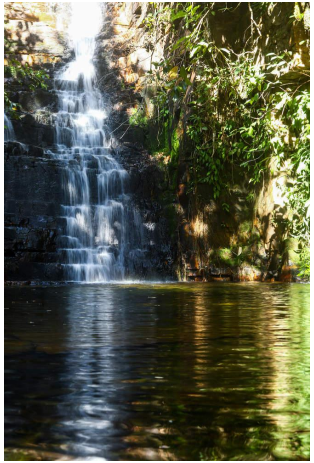

Push-Relabel(Pré-fluxo)

- Propriedade de conservação de fluxo não é mantida;

- Um Pré-fluxo é uma uma função f: $V \times V \rightarrow R$;
  a. $f(V, u) \geq 0$ para todo $u \in V$ - {s}
     - $e(u) = f(V, u)$ **(Excesso)**
  b. se $u \in V - \{s, t\}$ & $e(u) > 0$
     - u está transbordando

- Também usa-se o conceito de **rede residual**.

Push-Relabel(Operações Básicas)

- <u>Relabel</u>(u)
  a. Seja $G = (V, E)$ uma rede com origem $\mathbf{s}$ e destino $\mathbf{t}$, e seja f um pré-fluxo em G. Uma função $h : V \rightarrow N$ é uma função de altura se
     - $h(\mathbf{s}) = |V|$, $h(\mathbf{t}) = 0$ e $h(u) \leq h(v) + 1 \; \forall \; (u, v) \in E_f$
  b. Aplica-se quando:
     - $e(u) > 0$
     - $\forall \; v \in V$, *tal que* $(u, v) \in E_f$, $h(u) \leq h(v)$

Relabel(u)
1. $h[u] \leftarrow 1 + \min\{ h[v] : (u, v) \in E_f \}$

Fim.

Push-Relabel(Operações Básicas)

- <u>Exemplo Relabel</u>(u)
  a. Aplica-se quando:
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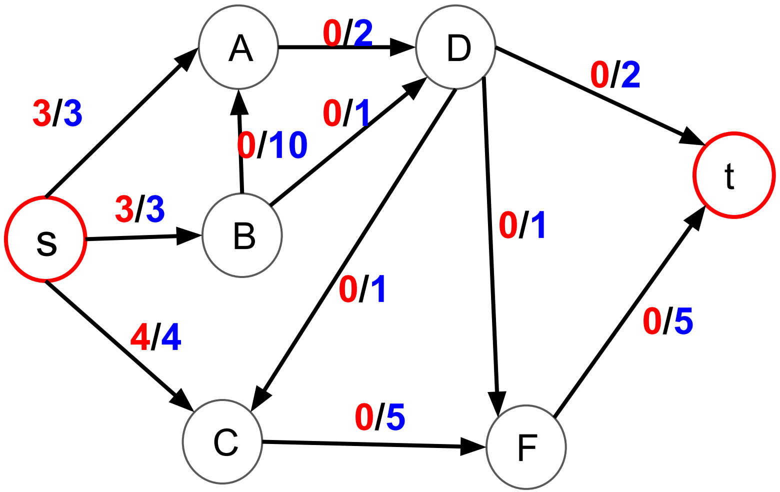
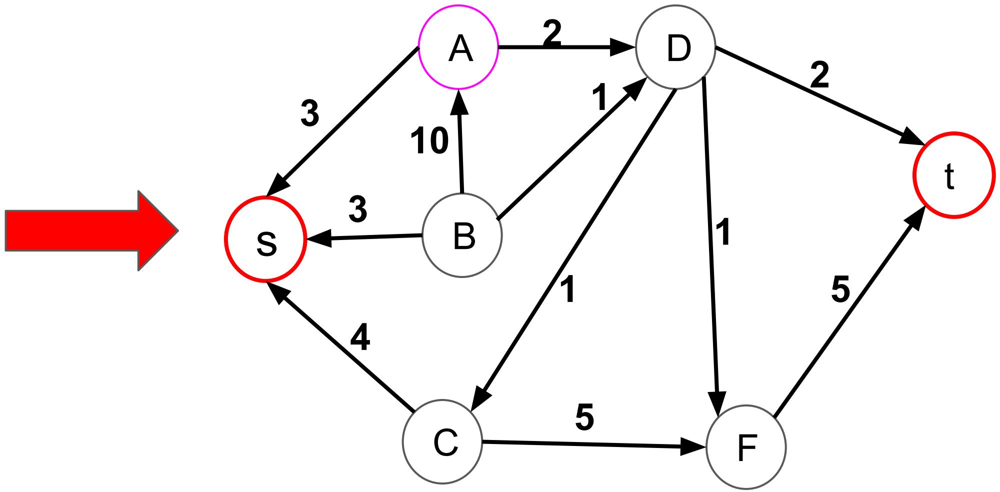
Fim.

Vértice	*e*	*h*
s	∞	7
A	3	0
B	3	0
C	4	0
D	0	0
F	0	0
t	0	0



$G = (V, E)$



$G_f = (V, E_f)$

Push-Relabel(Operações Básicas)

- Exemplo Relabel(u)
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Fim.

Vértice	e	h
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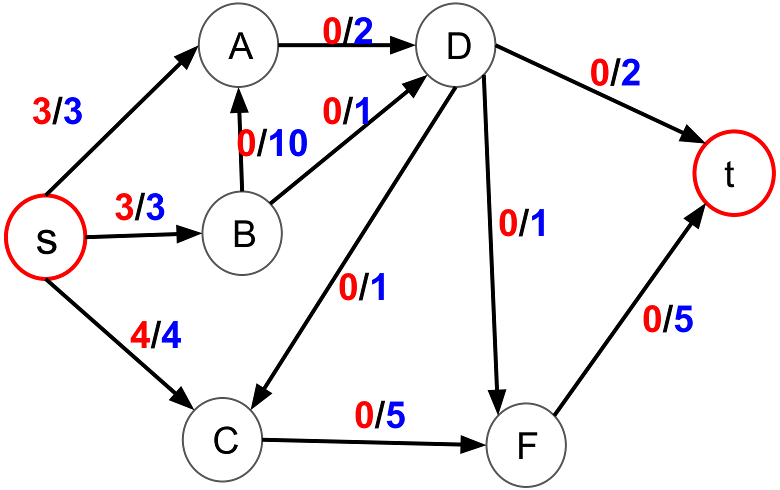
B	3	0
C	4	0
D	0	0
F	0	0
t	0	0



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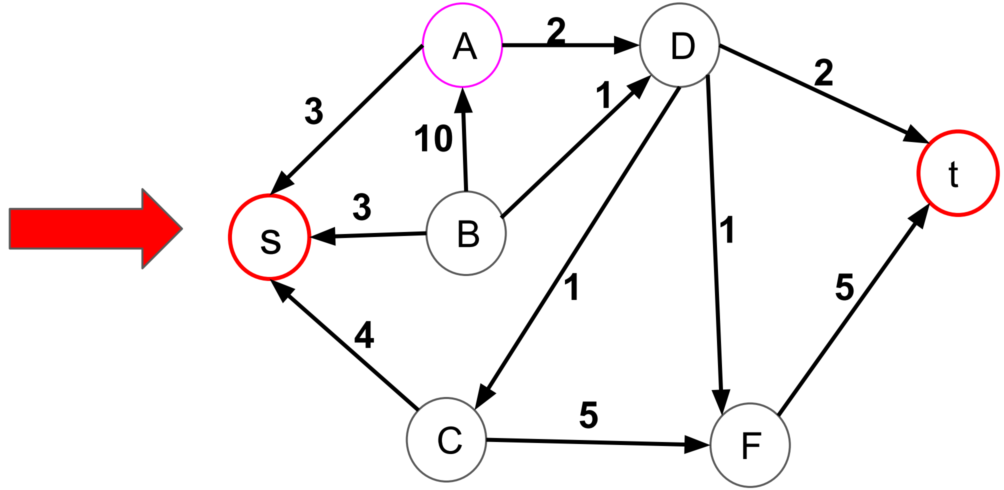

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Vértice	*e*	*h*
s	∞	7
A	3	1
B	3	0
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Relabel(u)
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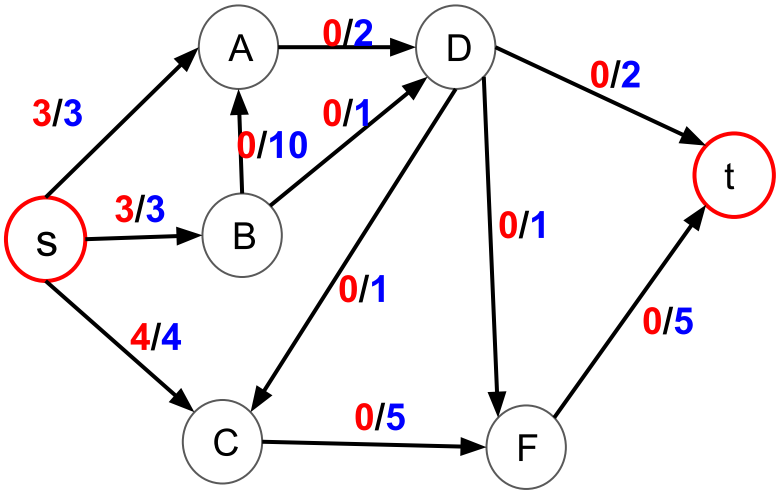

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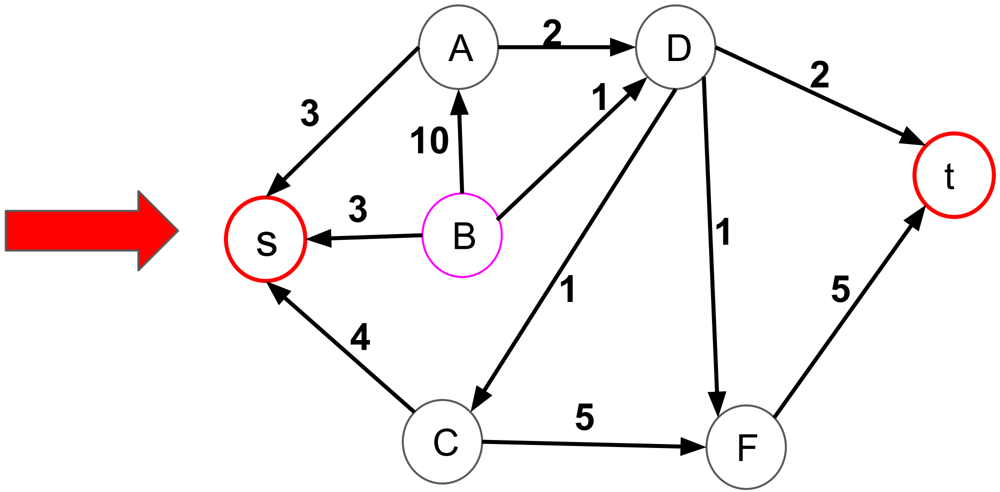

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Push-Relabel(Operações Básicas)

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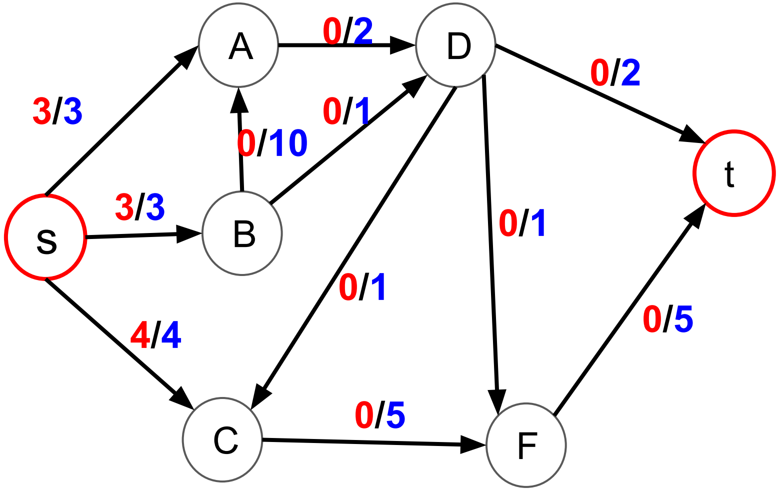

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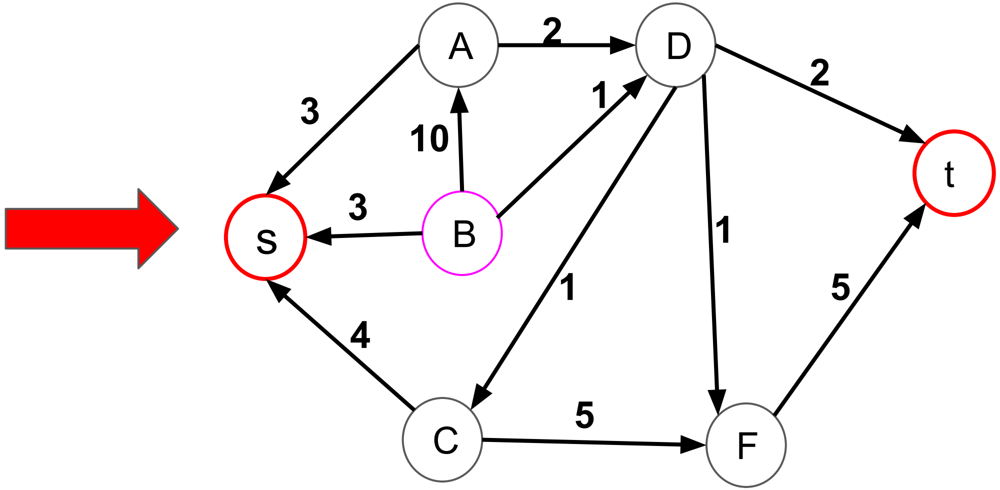

$G_f = (V, E_f)$

Push-Relabel(Operações Básicas)

- Push(u, v)
  a. Seja $G = (V, E)$ uma rede com origem ***s*** e destino ***t***, e seja f um pré-fluxo em G.
  b. Aplica-se quando:
     - $e(u) > 0$
     - $c_f(u, v) > 0$
     - $h(u) = h(v) + 1$

Push(u, v)

1. $d_f[u, v] = min\ (e[u], c_f[u, v])$
2. $f[u, v] \leftarrow f[u, v] + d_f[u, v]$
3. $f[v, u] \leftarrow - f[u, v]$
4. $e[u] \leftarrow e[u] - d_f[u, v]$
5. $e[v] \leftarrow e[v] + d_f[u, v]$

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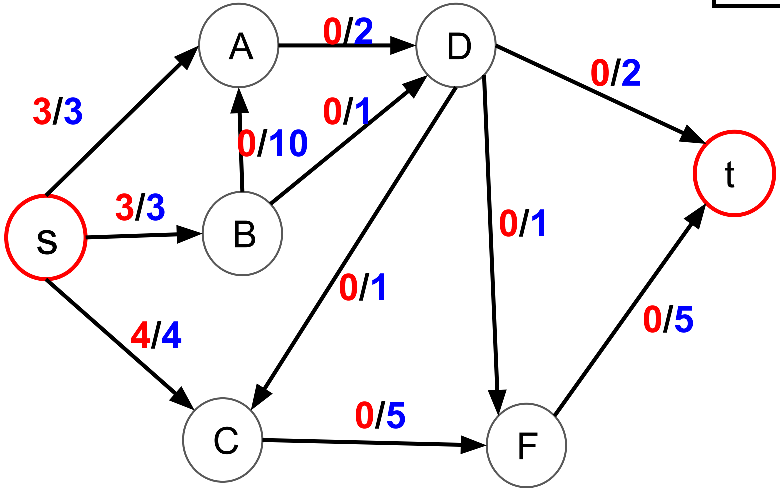

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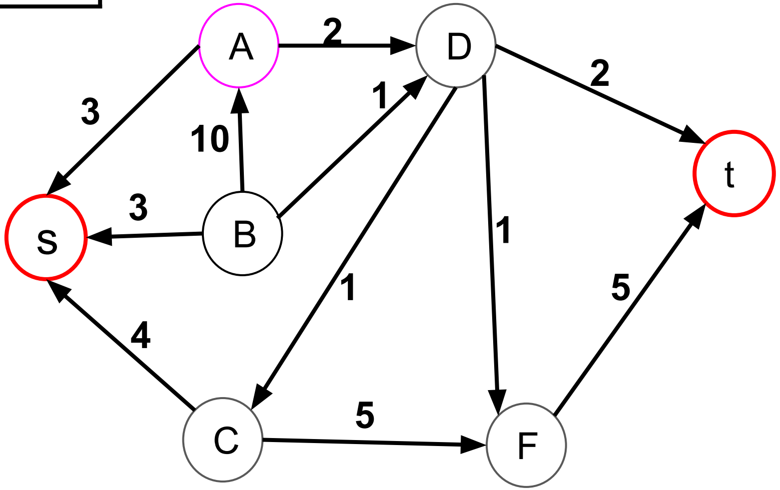

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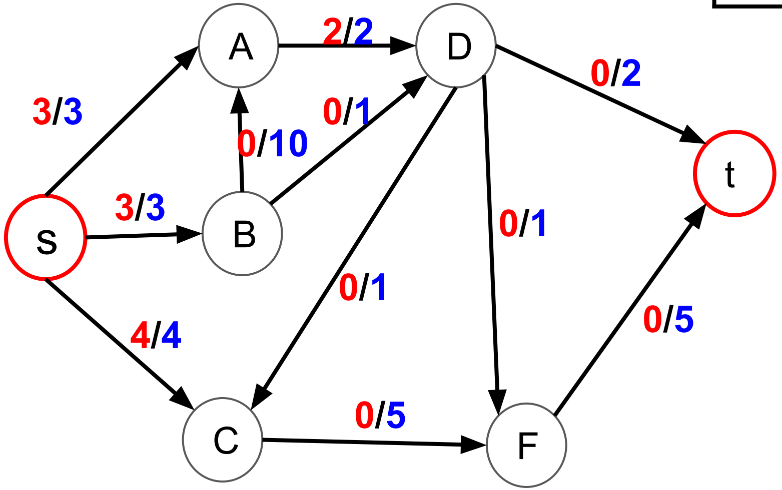

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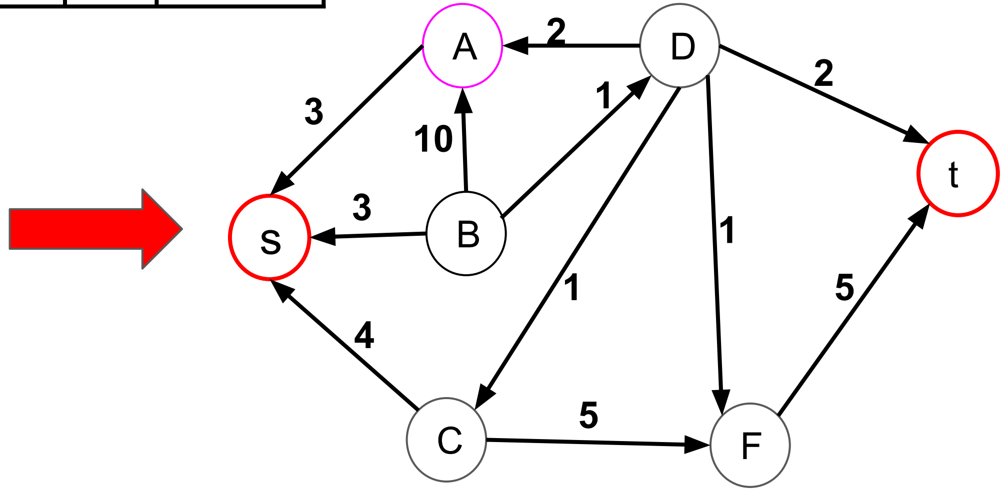

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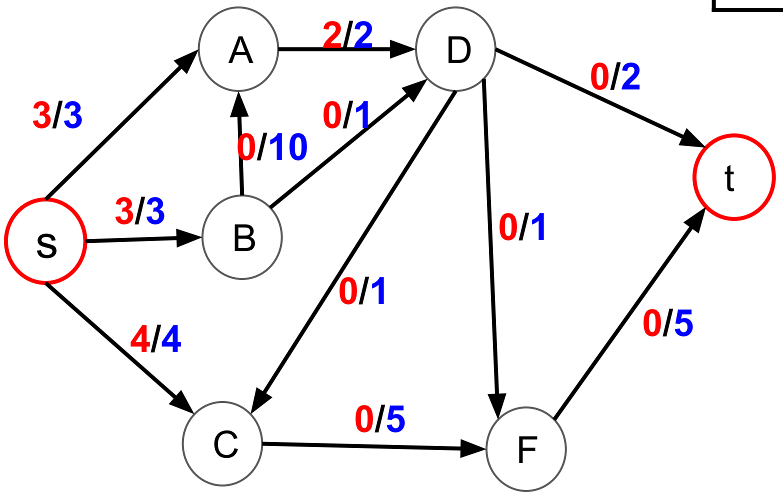

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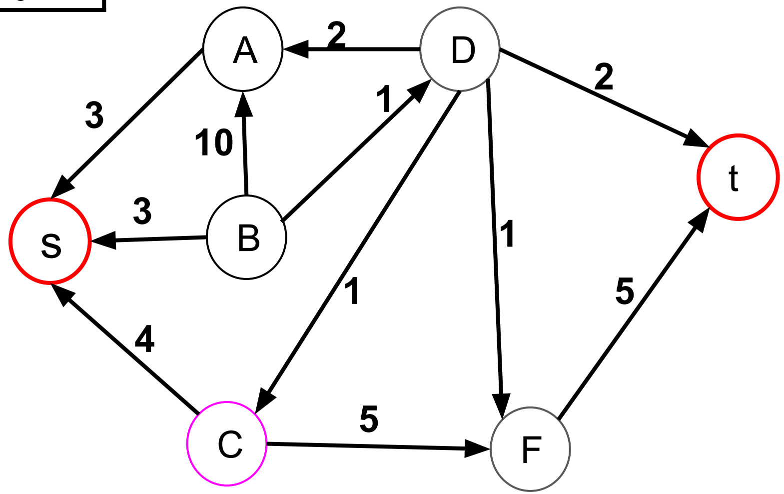

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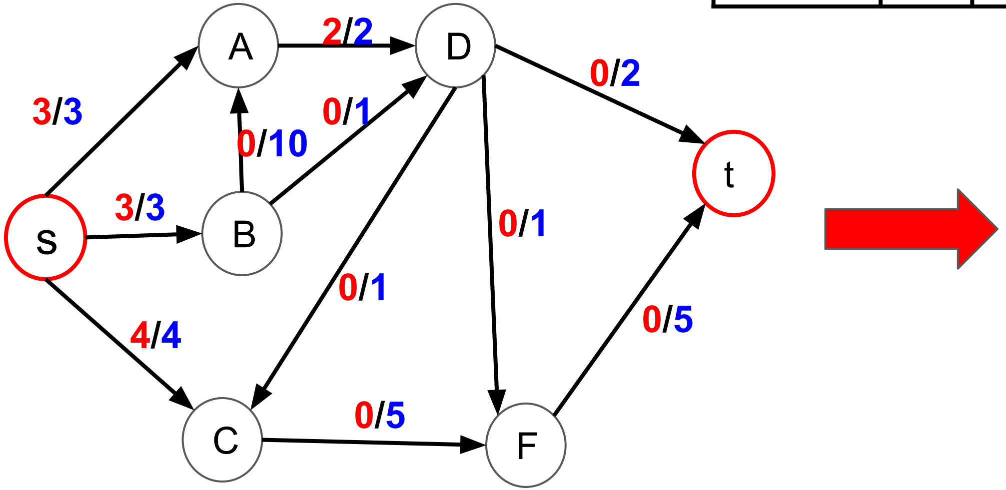

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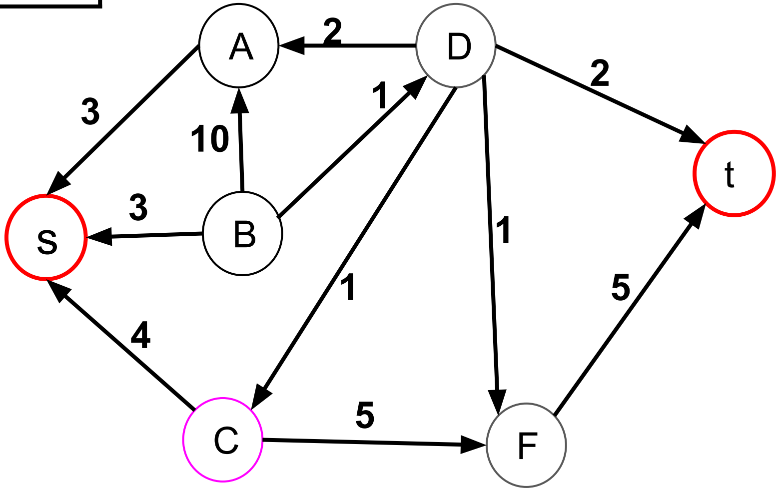

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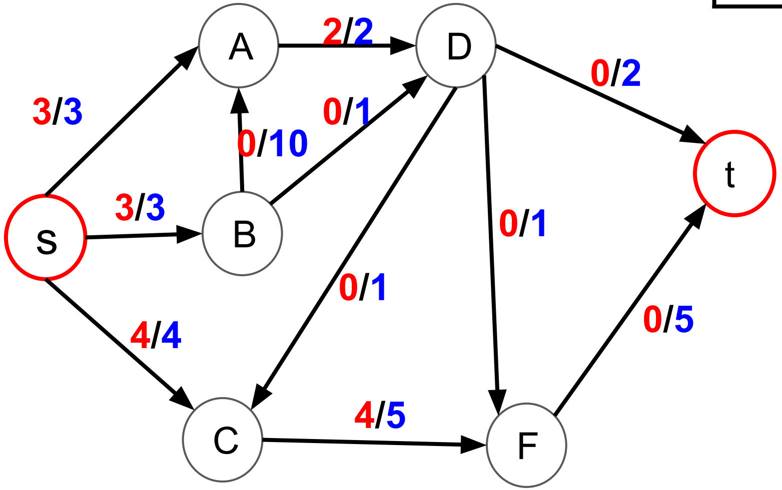

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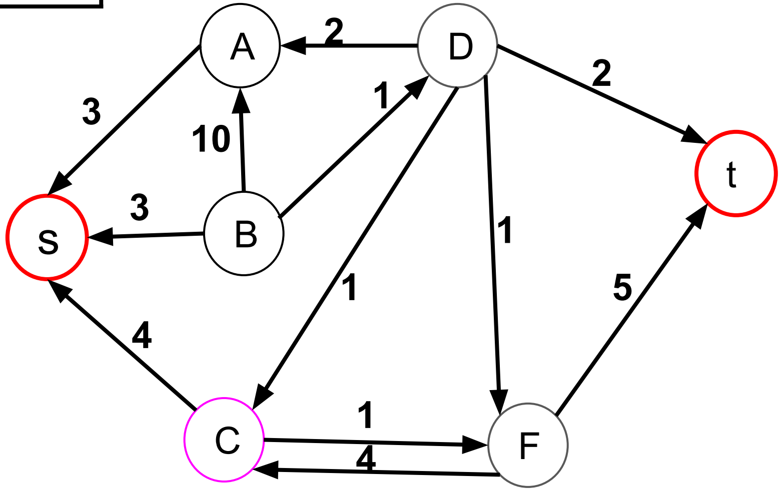


$G_f = (V, E_f)$

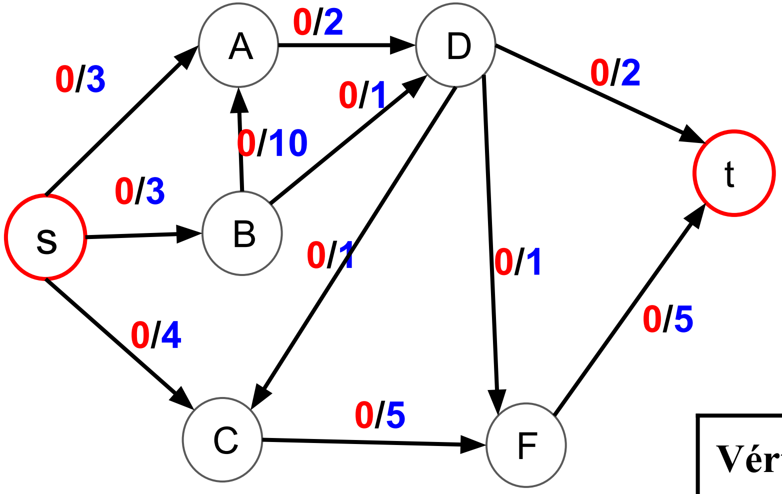
Pseudocódigo Push-Relabel

```
Push-Relabel-Generico(G, s)
1.   para cada vértice u ← V[G]
2.        h[u]← e[u]←0
3.   fimpara
4.   para cada aresta (u,v) ← E[G]
5.      f[u, v]←f[v, u] ← 0
6.   fimpara
7.   h[s] ← |V|
8.   para cada u ∈ L.adj[s]
9.          f[s, u]←c(s, u)
10.         f[u, s]←- c(s, u)
11.         e[u]←c(s, u)
12.         e[s]←e[s]- c(s, u)
13.  fimpara
14.  enquanto existir uma operação de Push ou Relabel faça
15.       selecione uma operação e execute
16.  fimenquanto
Fim.
```

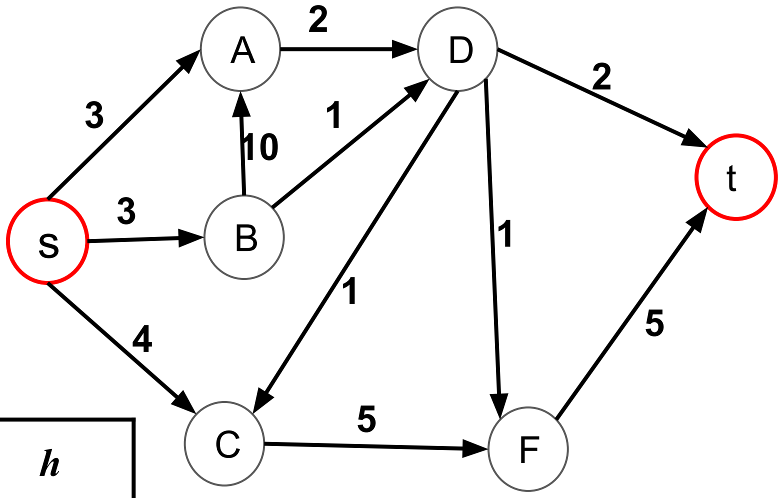
```
Push(u, v)
1.   d_f[u, v] = min (e[u], c_f[u, v])
2.   f[u, v]←f[u, v] + d_f[u, v]
3.   f[v, u]←- f[u, v]
4.   e[u]←e[u] - d_f[u, v]
5.   e[v]←e[v] + d_f[u, v]
Fim.
```

```
Relabel(u)
1.    h[u]←1+ min{ h[v]: (u , v) ∈ E_f}
Fim.
```

Push-Relabel (Exemplo)



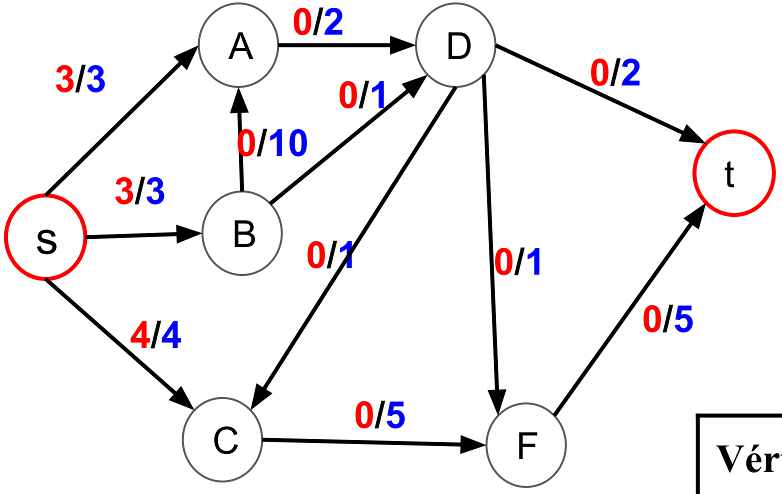
Grafo G



Rede Residual

Vértice	*e*	*h*
s	∞	0
A	0	0
B	0	0
C	0	0
D	0	0
F	0	0
t	0	0

Push-Relabel (Exemplo)



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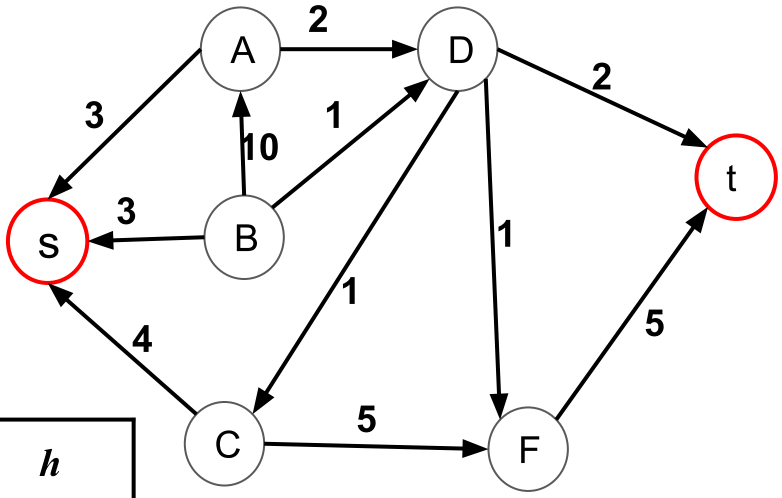
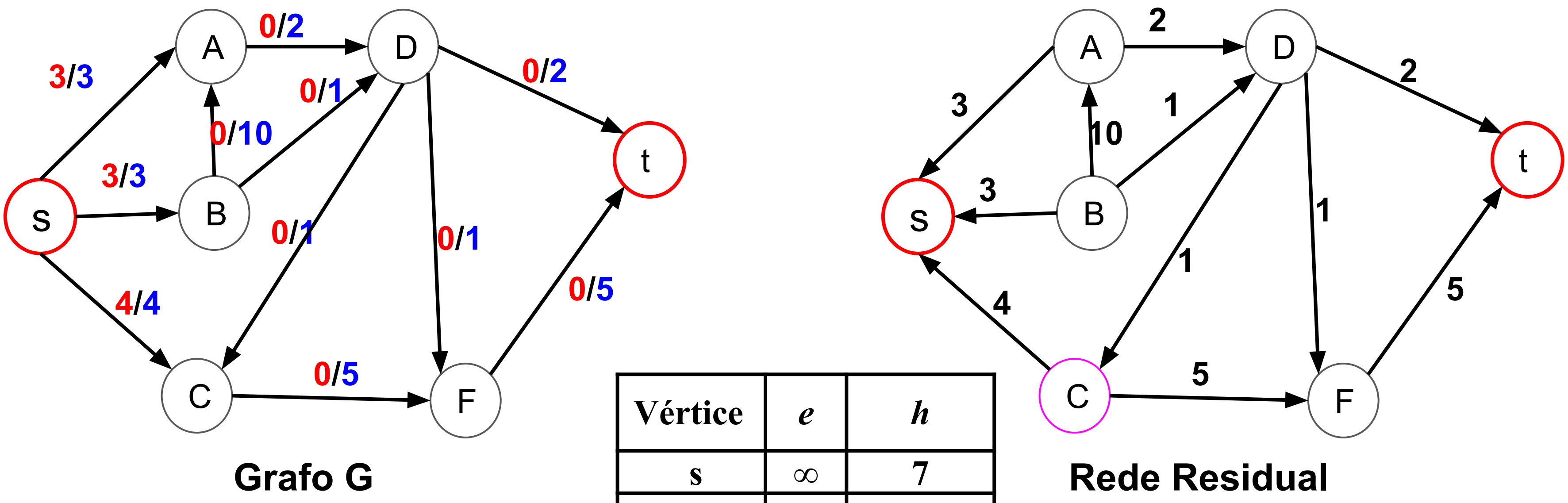


Rede Residual

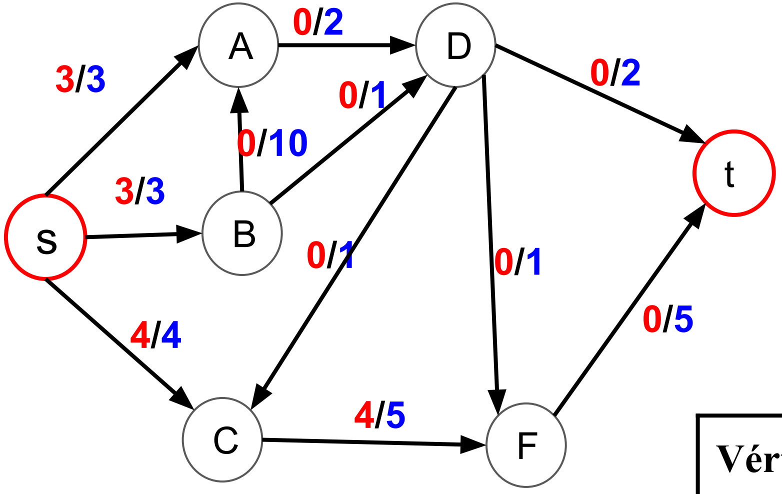
Vértice	*e*	*h*
s	∞	7
A	3	0
B	3	0
C	4	0
D	0	0
F	0	0
t	0	0

Push-Relabel (Exemplo)



Vértice	*e*	*h*
s	∞	7
A	3	0
B	3	0
C	4	1
D	0	0
F	0	0
t	0	0

Push-Relabel (Exemplo)



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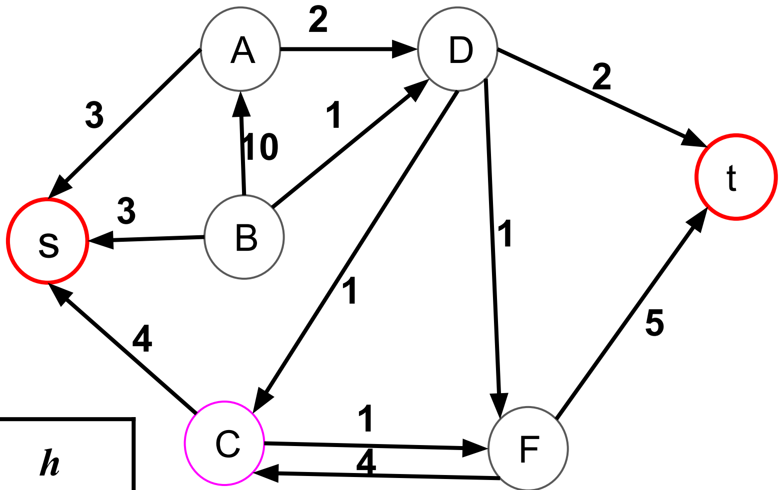

Rede Residual

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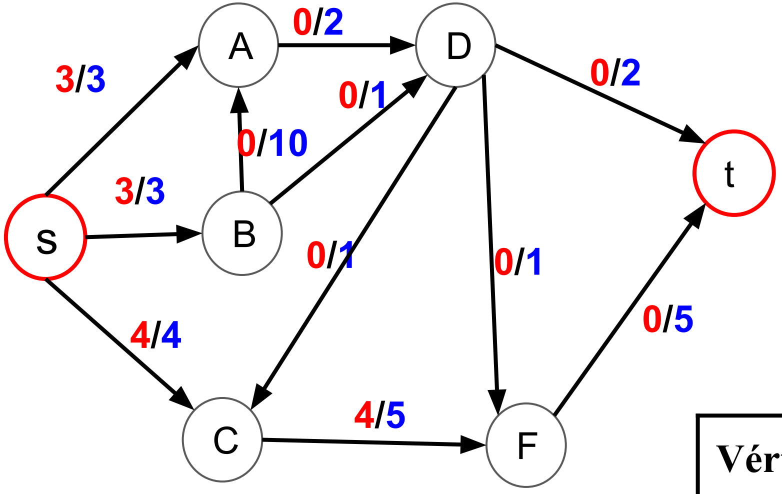
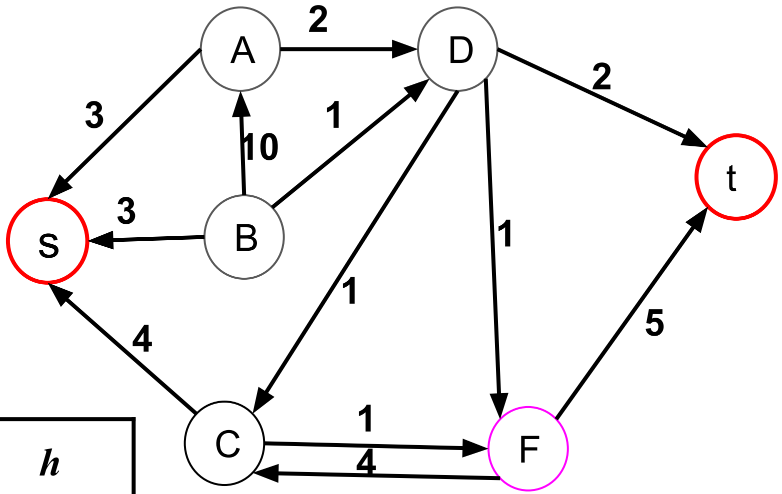
Push-Relabel (Exemplo)



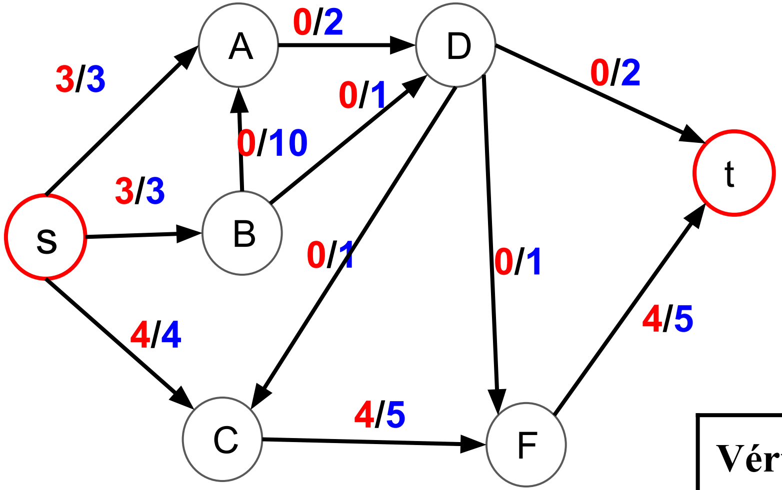
Grafo G

Vértice	*e*	*h*
s	∞	7
A	3	0
B	3	0
C	0	1
D	0	0
F	4	1
t	0	0



Rede Residual

Push-Relabel (Exemplo)



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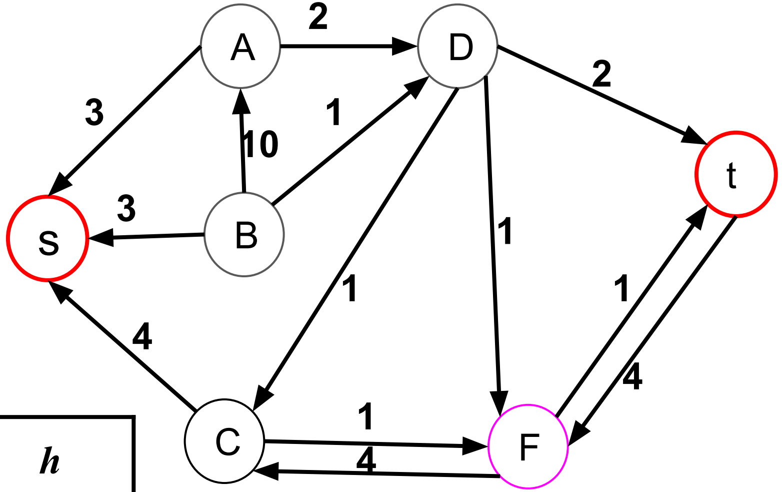

Rede Residual

Vértice	*e*	*h*
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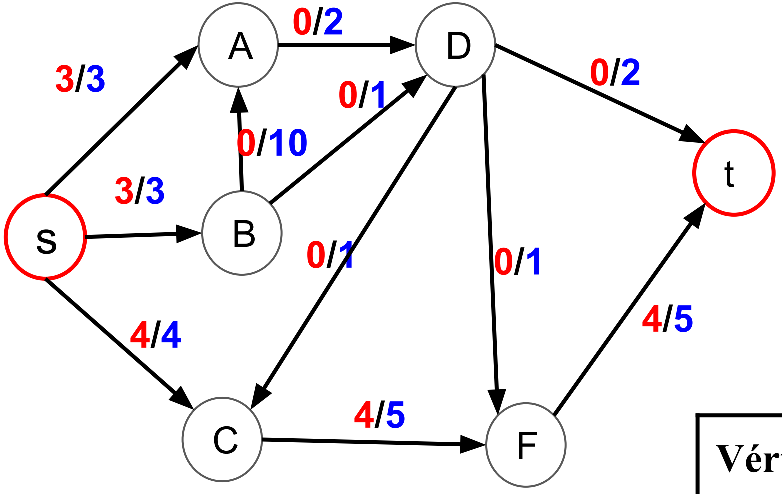
A	3	0
B	3	0
C	0	1
D	0	0
F	0	1
t	4	0

Push-Relabel (Exemplo)



Grafo G

Vértice	*e*	*h*
s	∞	7
A	3	1
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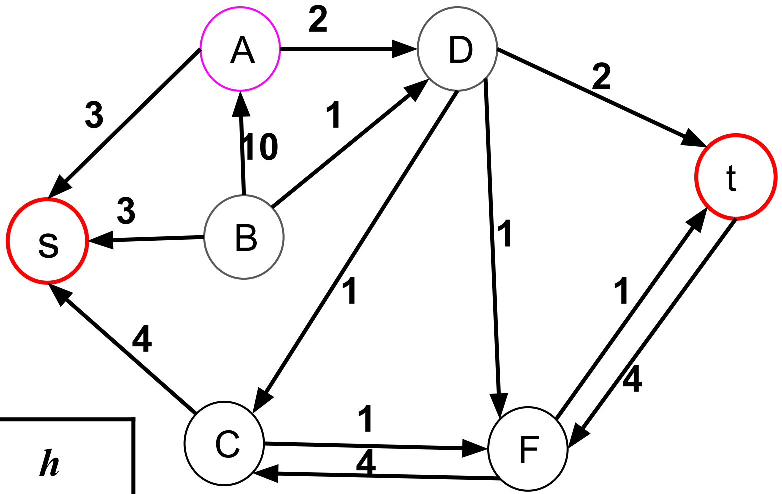
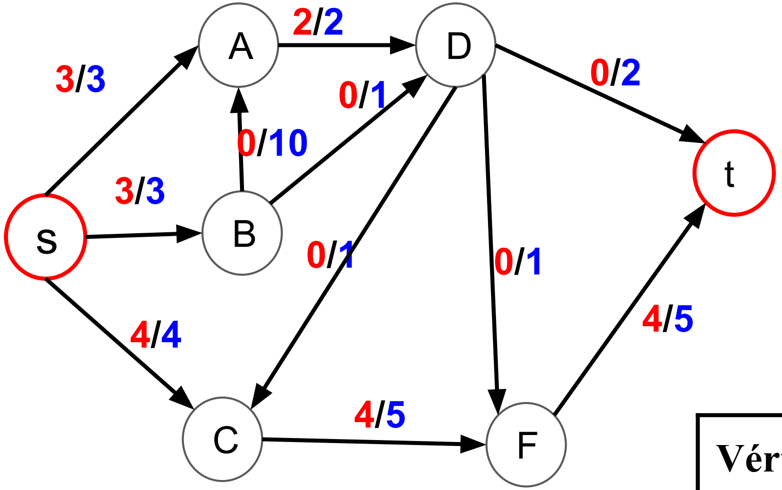
D	0	0
F	0	1
t	4	0



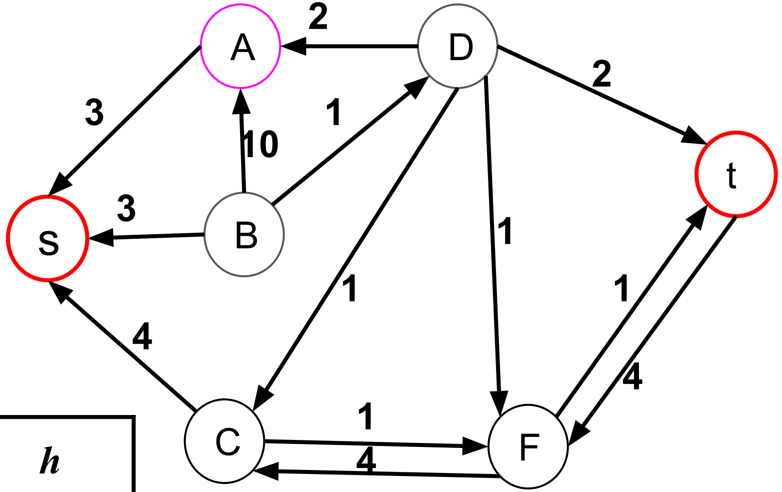
Rede Residual

Push-Relabel (Exemplo)



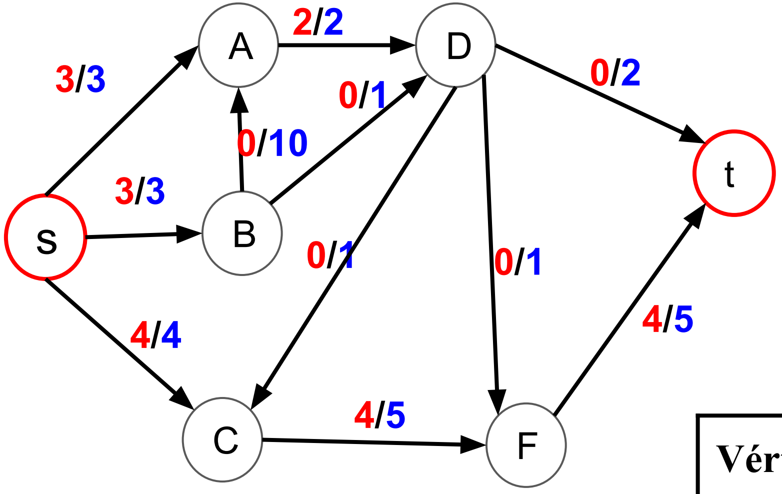
Grafo G

Vértice	*e*	*h*
s	∞	7
A	1	1
B	3	0
C	0	1
D	2	0
F	0	1
t	4	0



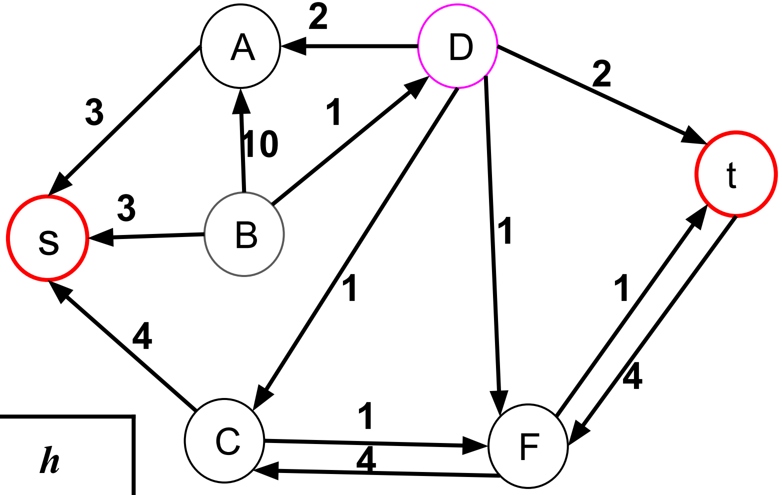
Rede Residual

Push-Relabel (Exemplo)



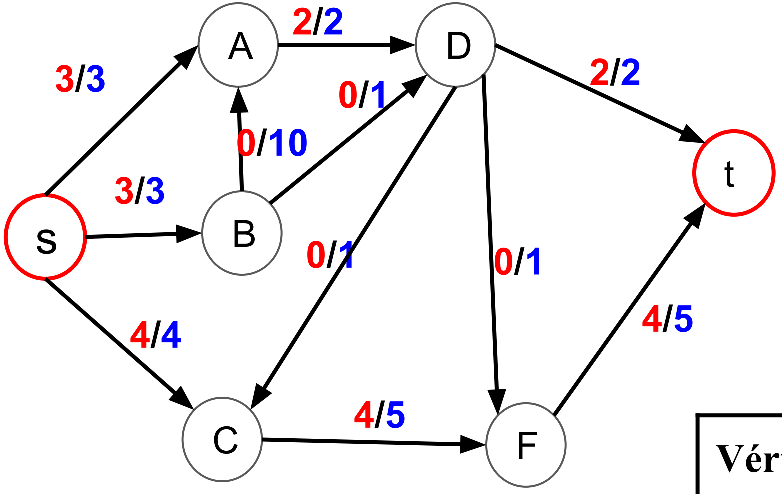
Grafo G

Vértice	*e*	*h*
s	∞	7
A	1	1
B	3	0
C	0	1
D	2	1
F	0	1
t	4	0

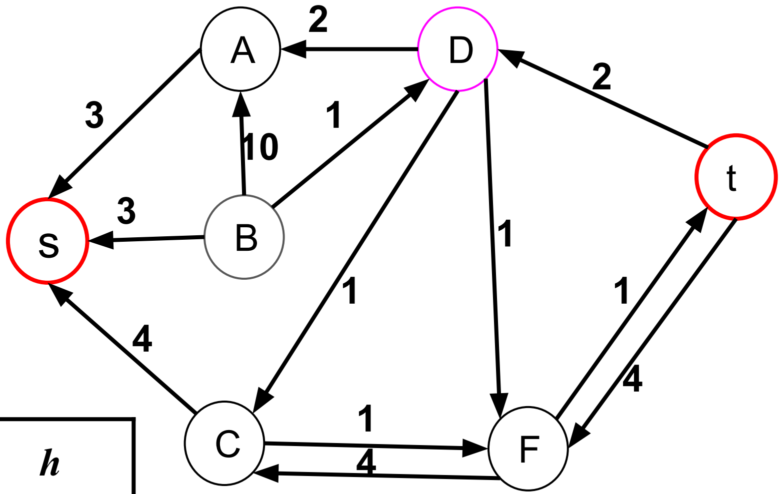


Rede Residual

Push-Relabel (Exemplo)



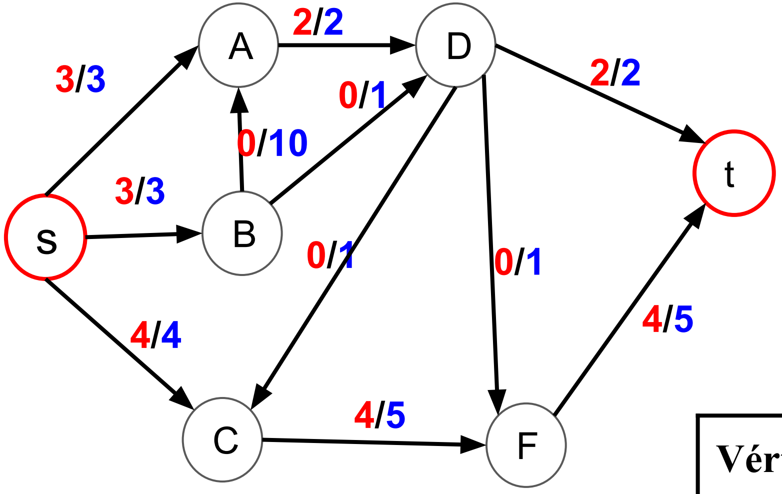
Grafo G



Rede Residual

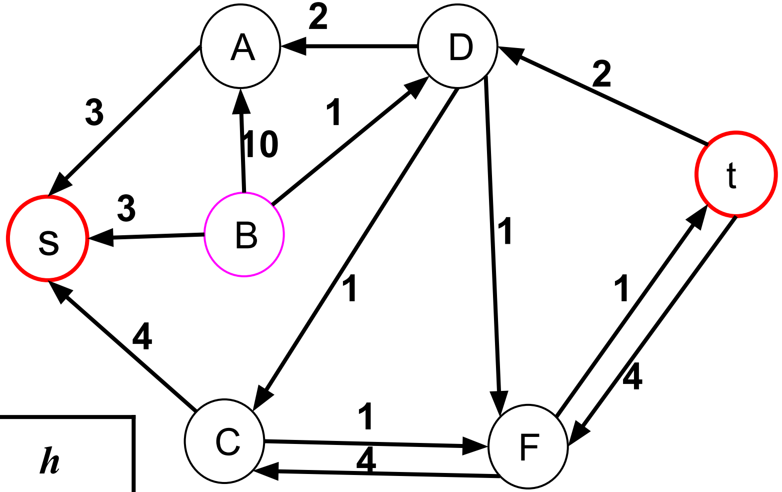
Vértice	*e*	*h*
s	∞	7
A	1	1
B	3	0
C	0	1
D	0	1
F	0	1
t	6	0

Push-Relabel (Exemplo)



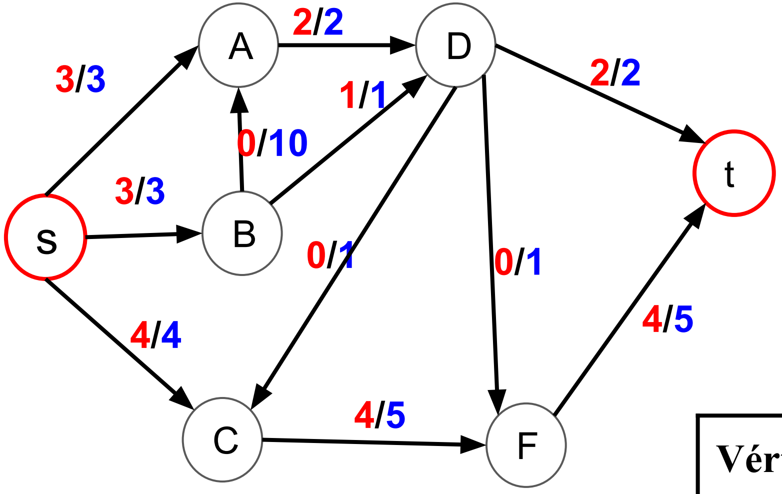
Grafo G

Vértice	*e*	*h*
s	∞	7
A	1	1
B	3	2
C	0	1
D	0	1
F	0	1
t	6	0



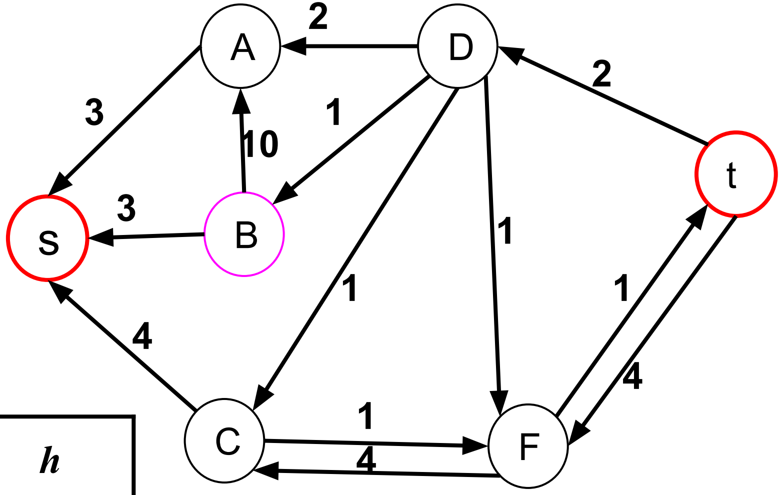
Rede Residual

Push-Relabel (Exemplo)



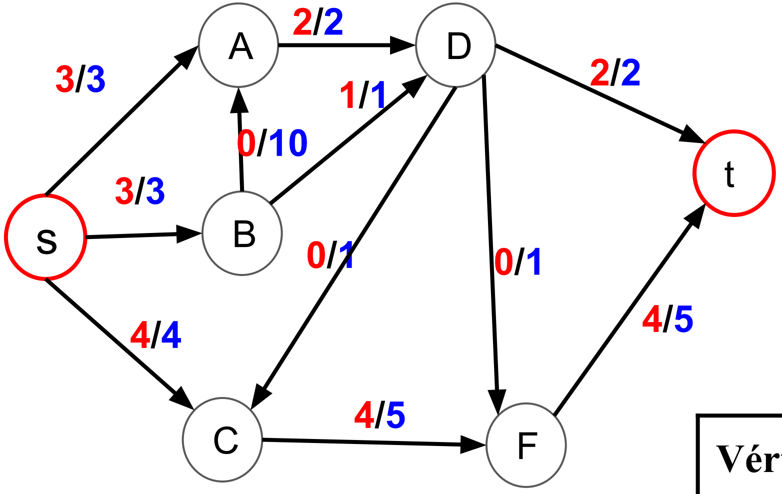
Grafo G

Vértice	*e*	*h*
s	∞	7
A	1	1
B	2	2
C	0	1
D	1	1
F	0	1
t	6	0



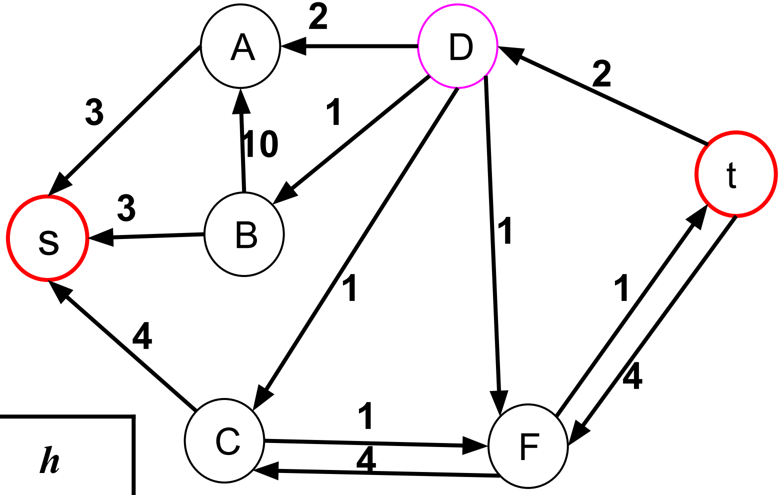
Rede Residual

Push-Relabel (Exemplo)



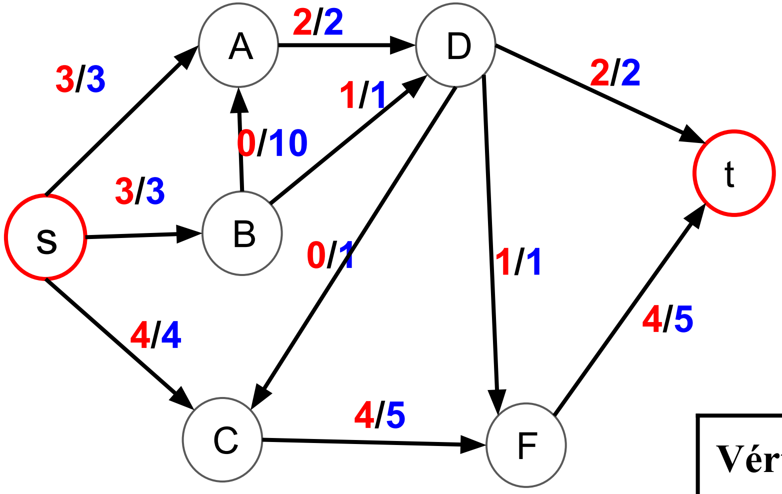
Grafo G

Vértice	*e*	*h*
s	∞	7
A	1	1
B	2	2
C	0	1
D	1	2
F	0	1
t	6	0

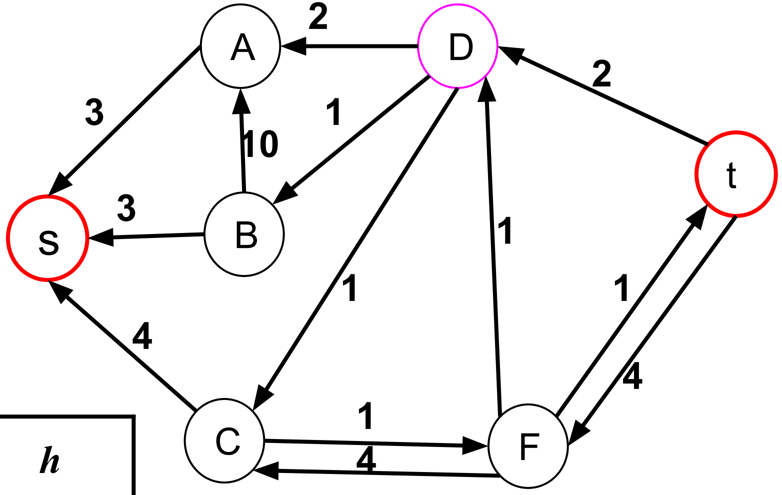


Rede Residual

Push-Relabel (Exemplo)



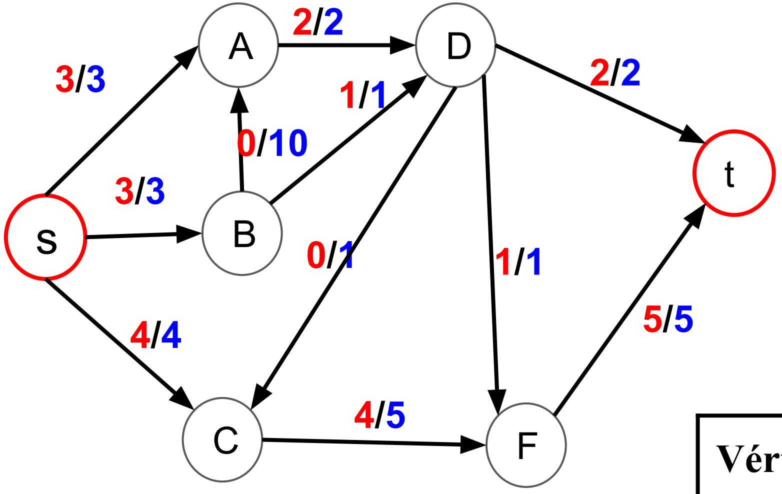
Grafo G



Rede Residual

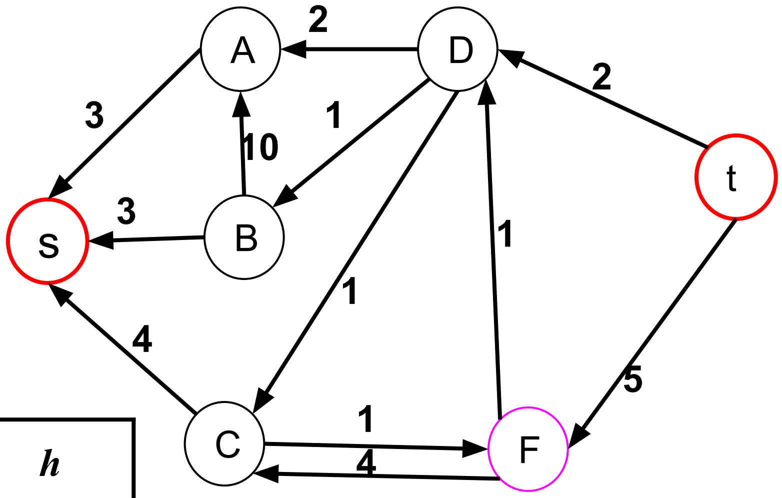
Vértice	*e*	*h*
s	∞	7
A	1	1
B	2	2
C	0	1
D	0	2
F	1	1
t	6	0

Push-Relabel (Exemplo)



Grafo G

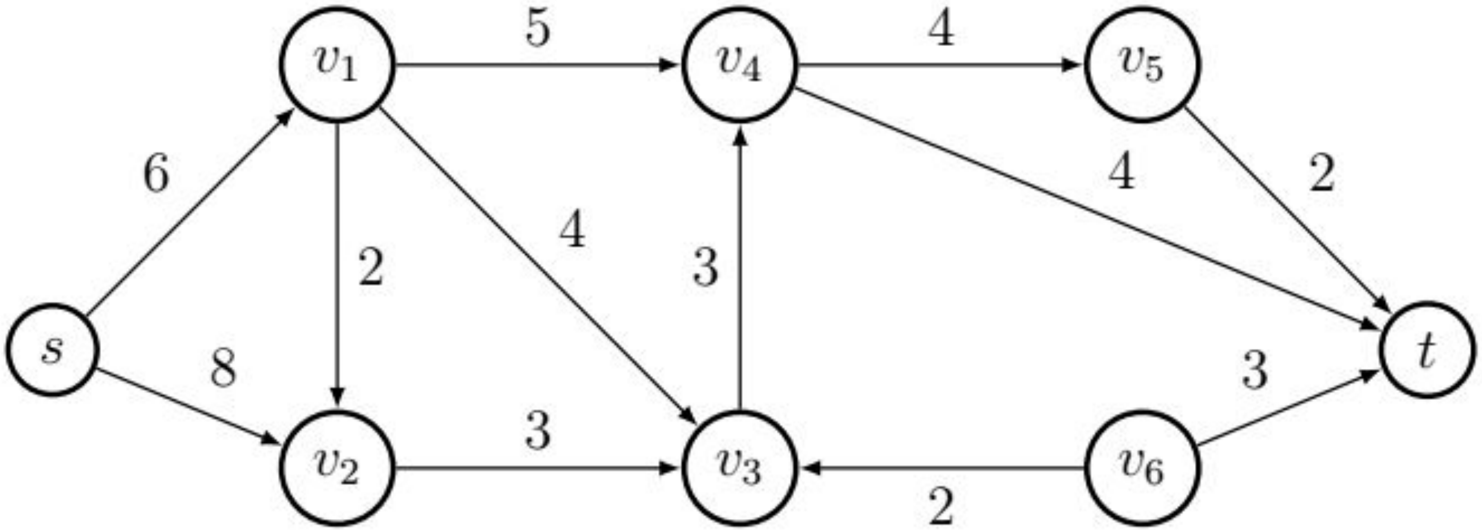
Vértice	*e*	*h*
s	∞	7
A	1	1
B	2	2
C	0	1
D	0	2
F	0	1
t	7	0



Rede Residual

Exercício de Fixação

Determine o fluxo máximo na rede abaixo.



UNIVERSIDADE FEDERAL DO CEARÁ

CAMPUS DE RUSSAS

Algoritmos em Grafos

Aula 17: Fluxo Máximo em Redes(Push-Relabel)

Professor Pablo Soares

2022.1