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COM231

ASSIGNMENT 2

Linear Regression Activity: Predicting Exam Scores

We want to predict a student's **Exam Score** (y) based on the number of **Hours Studied** (x).

Data

Student	Hours Studied (x)	Exam Score (y)
1	1	52
2	2	57
3	3	61
4	4	65
5	5	70

Task

We want to fit a **linear regression line** of the form:

$$y = mx + b$$

A new student studied **6 hours**. We want to predict the **Exam Score** using the regression equation.

1. Fill in the table (14 points)

- Compute x^2 for each student.
- Compute xy for each student.
- Find the totals: Σx , Σy , Σx^2 , and Σxy .

Student	Hours Studied (x)	Exam Score (y)	xy	x^2
1	1	52	?	?
2	2	57	?	?
3	3	61	?	?
4	4	65	?	?
5	5	70	?	?
	$\Sigma x = ?$	$\Sigma y = ?$	$\Sigma xy = ?$	$\Sigma x^2 = ?$

Student	Hours studied (x)	Exam score (y)	xy	x^2
1	1	52	52	1
2	2	57	114	4
3	3	61	183	9
4	4	65	260	16
5	5	70	350	25
	$\Sigma x = 15$	$\Sigma y = 305$	$\Sigma xy = 959$	$\Sigma x^2 = 55$

$\Sigma x = 1 + 2 + 3 + 4 + 5 = 15$
 $\Sigma y = 52 + 57 + 61 + 65 + 70 = 305$
 $\Sigma xy = 52 + 114 + 183 + 260 + 350 = 959$
 $\Sigma x^2 = 1 + 4 + 9 + 16 + 25 = 55$

2. Compute the Slope m (5 points)

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = ?$$

3. Compute the Intercept b (5 points)

$$b = \frac{\sum y - m \sum x}{n}$$

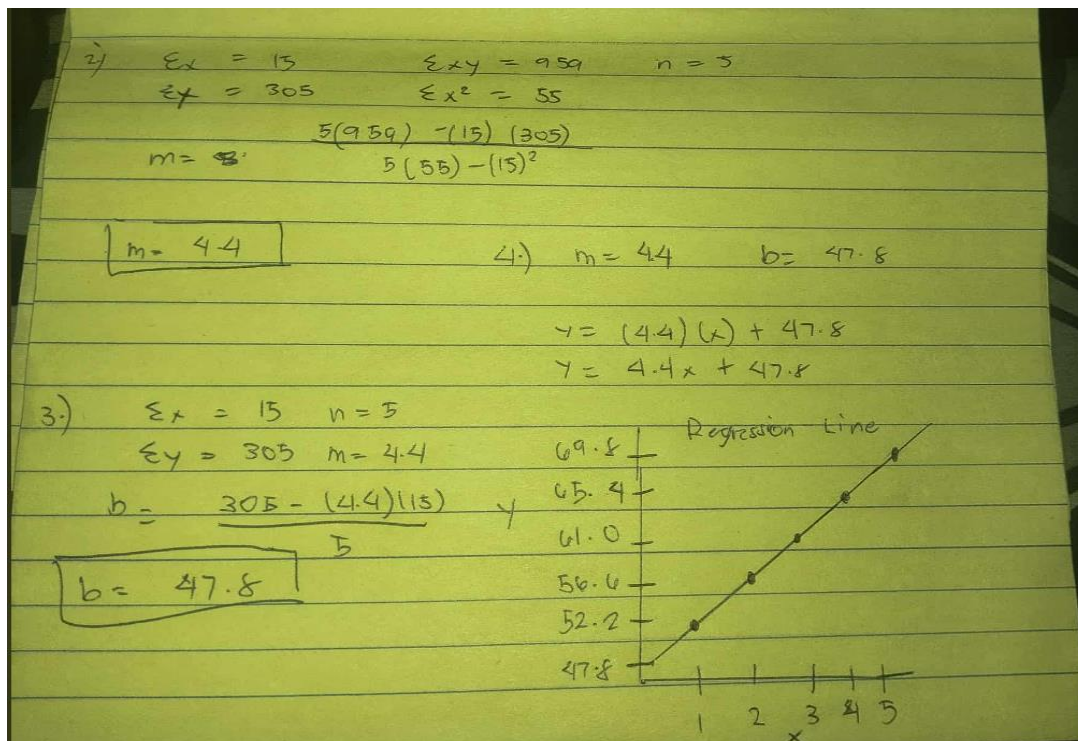
$$b = ?$$

4. Regression Equation (5 points)

Write the regression line:

$$y = mx + b$$

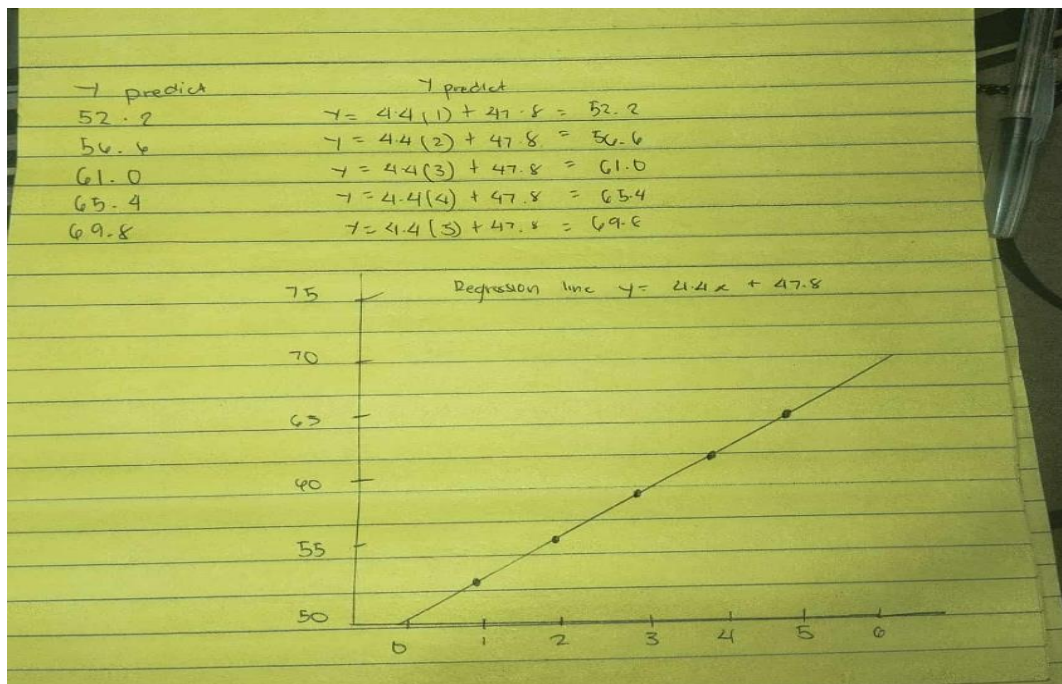
$$y = ?$$



5. Draw the regression line using a scatter plot (10 points)

- Calculate $y_{predict}$ for each data points
- Draw a regression line using $y_{predict}$
- Use a circle ● for all data points
- Use a red line for the regression line

Student	Hours Studied (x)	Exam Score (y)	Predicted Exam Score ($y_{predict}$)
1	1	52	?
2	2	57	?
3	3	61	?
4	4	65	?
5	5	70	?



6. Calculate the Sum of Squared Errors (20 points)

$$SSE = \sum (y_i - y_{\text{predict}})^2$$

Student	Hours Studied (x)	Exam Score (y)	Predicted Exam Score (y_{predict})	$y_i - y_{\text{predict}}$	$(y_i - y_{\text{predict}})^2$
1	1	52	?	?	?
2	2	57	?	?	?
3	3	61	?	?	?
4	4	65	?	?	?
5	5	70	?	?	?
					$SSE = ?$

7. Calculate the Sum of Squared Total (20 points)

- Get \bar{y} using this formula :

$$\bar{y} = \frac{\sum y_i}{n}$$

- Get SST using this formula :

$$SST = \sum (y_i - \bar{y})^2$$

Student	Hours Studied (x)	Exam Score (y)	Mean (\bar{y})	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
1	1	52	?	?	?
2	2	57	?	?	?
3	3	61	?	?	?
4	4	65	?	?	?
5	5	70	?	?	?
					$SST = ?$

$$y_i - y_{\text{predict}}$$

$$(y_i - y_{\text{predict}})^2$$

$$52 - 52.2 = -0.2$$

$$0.2^2 = 0.2 \times 0.2 = 0.04$$

$$57 - 56.6 = 0.4$$

$$0.4^2 = 0.4 \times 0.4 = 0.16$$

$$61 - 61 = 0$$

$$0^2 = 0 \times 0 = 0$$

$$65 - 65.4 = -0.4$$

$$0.4^2 = 0.4 \times 0.4 = 0.16$$

$$70 - 69.8 = 0.2$$

$$0.2^2 = 0.2 \times 0.2 = 0.04$$

7. mean (\bar{y})

$$y_i - \bar{y}$$

$$(y_i - \bar{y})^2$$

$$61$$

$$9$$

$$81$$

$$SST =$$

$$61$$

$$4$$

$$16$$

$$81 + 16 + 0 + 16 + 81 = 194$$

$$61$$

$$0$$

$$0$$

$$61$$

$$4$$

$$16$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{305}{5} \quad \sum y = 305$$

$$61$$

$$9$$

$$81$$

$$= 61$$

$$SST = 194$$

$$y_i - \bar{y}$$

$$(y_i - \bar{y})^2$$

$$52 - 61 = -9 = 9$$

$$9^2 = 9 \times 9 = 81$$

$$57 - 61 = -4 = 4$$

$$4^2 = 4 \times 4 = 16$$

$$61 - 61 = 0$$

$$0^2 = 0 \times 0 = 0$$

$$65 - 61 = 4$$

$$4^2 = 4 \times 4 = 16$$

$$70 - 61 = 9$$

$$9^2 = 9 \times 9 = 81$$

8. Compute R^2 (20 points)

- Get R^2 using this formula :

$$R^2 = 1 - \frac{SSE}{SST}$$

$$R^2 = ?$$

9. Prediction (1 point)

Use your equation to predict the exam score for a student who studied 6 hours.

$$y = m(6) + b$$

$$y = ?$$

8.) $\frac{SSE}{SST} = \frac{0.4}{194}$ $R^2 = 1 - \frac{0.4}{194} = \underline{0.99}$

9.) $m = 4.4, \quad b = 47.8$
 $y = m(x) + b$
 $= 4.4(4) + 47.8$
 $= \underline{74.2}$