Stochastic Unit Commitment Problem

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1 Introduction

This document describes the formulation of a stochastic unit commitment problem under uncertainty in demand. The demand follows a multivariate normal distribution with varying correlation levels across time periods. The problem is mathematically formulated as deterministic with binary approximation.

2 Problem Formulation

The goal is to minimize total operating costs while meeting demand with a high reliability level p. Demand scenarios are generated from a multivariate normal distribution $D \sim \mathcal{N}_T(\mu, \Sigma)$, with mean vector μ and covariance matrix Σ.

Objective Function

$$\min \sum_{t=1}^{T} \sum_{g=1}^{G} \left(z_{g,t}^{\text{on}} \cdot C_{\text{startup},g} + z_{g,t}^{\text{off}} \cdot C_{\text{shutdown},g} + b_g p_{g,t} + c_g u_{g,t} \right) \tag{1}$$

Constraints

$$u_{g,t} - u_{g,t-1} = z_{g,t}^{\text{on}} - z_{g,t}^{\text{off}} \qquad \forall g, t \tag{2}$$

$$z_{g,t}^{\text{off}} + z_{g,t}^{\text{on}} \leq 1 \qquad \forall g, t \tag{3}$$

$$P_g^{\min} \cdot u_{g,t} \leq p_{g,t} \leq P_g^{\max} \cdot u_{g,t} \qquad \forall g, t \tag{4}$$

$$p_{g,t} - p_{g,t-1} \leq R_{\text{up},g} u_{g,t-1} + R_{\text{up},g}^{SU} z_{g,t}^{\text{on}} \qquad \forall g, t \tag{5}$$

$$p_{g,t-1} - p_{g,t} \leq R_{\text{down},g} u_{g,t} + R_{\text{down},g}^{SD} z_{g,t}^{\text{off}} \qquad \forall g, t \tag{6}$$

$$\sum_{g=1}^{G} p_{g,t} \geq y_i D_t^i \qquad \forall i, t \tag{7}$$

$$\sum_{i=1}^{N} y_i \geq pN \tag{8}$$

3 Model Parameters

Generator Characteristics

Generating unit #	**1**	**2**	**3**
$P_i^{\min}$	50	80	40
$P_i^{\max}$	350	200	140
$R_{\text{down},i}$	300	150	100
$R_{\text{down},i}^{SD}$	300	150	100
$R_{\text{up},i}$	200	100	100
$R_{\text{up},i}^{SU}$	200	100	100
C_i^{startup}	20	18	5
C_i^{shutdown}	0.5	0.3	1.0
b_i	0.1	0.125	0.150
c_i	5	7	6

Table 1: Parameters for each generator.

4 Demand Modeling

Demand is modeled as a multivariate normal random vector:

$$D \sim \mathcal{N}_T(\mu, \Sigma) \tag{9}$$

Time	**1**	**2**	**3**
μ_t	225	630	400
σ_t	25	40	28

Table 2: Multivariate normal distribution parameters.

Correlation Structure

The covariance matrix Σ is computed as:

$$\Sigma_{i,j} = \rho_{ij} \cdot \sigma_i \cdot \sigma_j$$

We consider three different correlation cases, as shown in Table 3:

Case	ρ_{01}	ρ_{02}	ρ_{12}
Non-correlated	0.0	0.0	0.0
Moderately correlated	0.3	0.4	0.5
Highly correlated	0.6	0.7	0.8

Table 3: Correlation coefficients for demand across time periods.