Deterministic Unit Commitment Problem

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1 Introduction

This dcument describes the formulation of a deterministic unit commitment problem. The problem is mathematically formualted as a Quadratic Unconstrained Binary Optimization (QUBO) problem, suited for D-Wave's simulated and quantum annealers. As a QUBO problem, all variables are binary, and constraints are introduced as penalty terms in the objective function

2 Mixed-Integer Linear Program formulation

To introduce the optimization problem, we will first introduce it as a constrained Mixed-Integer Linear Program (MILP), which we will afterwards convert into QUBO. Thus, the MILP formualtion reads as follows.

Objective function

$$\min \sum_{t=1}^{T} \sum_{g=1}^{G} \left(z_{g,t}^{\text{on}} \cdot C_{\text{startup},g} + z_{g,t}^{\text{off}} \cdot C_{\text{shutdown},g} + b_g p_{g,t} + C_g u_{g,t} \right) \tag{1}$$

Constraints

$$u_{g,t} - u_{g,t-1} = z_{g,t}^{\text{on}} - z_{g,t}^{\text{off}} \qquad \forall g,t \tag{2}$$

$$z_{g,t}^{\text{off}} + z_{g,t}^{\text{on}} \leq 1 \qquad \forall g,t \tag{3}$$

$$P_g^{\min} \cdot u_{g,t} \leq p_{g,t} \leq P_g^{\max} \cdot u_{g,t} \qquad \forall g,t \tag{4}$$

$$-R_g^{\text{down}} \leq p_{g,t} - p_{g,t-1} \leq R_g^{\text{up}} \qquad \forall g,t \tag{5}$$

$$\sum_{g=1}^{G} p_{g,t} \geq D_t \qquad \forall t \tag{6}$$

3 QUBO Reformulation

Binary Encoding

To remove the continuous variables $p_{g,t}$, we use the following binary encoding

$$p_{g,t} = P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k}, \tag{7}$$

which introduces a new set of binary variables $p_{g,t,k}$. The value of n is chosen to be at least the the number of bits required to span the admissible generation range, following the rule $n = \lceil \log_2\bigl(\max_g\{P_g^{\max} - P_g^{\min}\} + 1\bigr)\rceil$.

Constraint Penalization

Here we will introduce all the constraints by adding penalty terms to the objective function.

First, using a binary encoding requires the addition of an extra constraint that couples the status variables $u_{g,t}$ with the power generation variables $p_{g,t,k}$. This is, imposing that when a generator is in operation, at least one variable $p_{g,t,k}$ has to be 1. Thus, we add the penalization term

$$\sum_{g=1}^{G}\sum_{t=1}^{T}\left[(1 - u_{g,t})\sum_{k} p_{g,t,k}\right]. \tag{8}$$

The equality constraint (2) is enforced by adding squared deviations,

$$\left(u_{g,t} - u_{g,t-1} - z_{g,t}^{\text{on}} + z_{g,t}^{\text{off}}\right)^2. \tag{9}$$

Constraint (3) is enforced by adding the penalty term

$$z_{g,t}^{\text{off}} z_{g,t}^{\text{on}}, \tag{10}$$

which imposes that a generator cannot be switched on and off simultaneously.

The capacity constraint (4) can be slightly modified because the binary encoding and the coupling constraint already impose a lower bound for the generation. Thus, the resulting constraint becomes $p_{g,t} \leq P_g^{\max}$, which can straightforwardly be imposed using slack variables. The final penalty term with binary encoded variables becomes

$$\left(P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k} + \sum_{q} 2^q s_{g,t,q} - P_g^{\max}\right)^2. \tag{11}$$

The inequality constraint (5) requires the use of slack variables as well. The resulting equality becomes

$$p_{g,t} - p_{g,t-1} = s_{g,t}, \quad s_{g,t} \in [-R_g^{\text{down}}, R_g^{\text{up}}], \tag{12}$$

and the binary encoded penalty term

$$\left(P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k} - P_g^{\min} u_{g,t-1} - \sum_{k=0}^{n-1} 2^k p_{g,t-1,k} + R_g^{\text{down}} - \sum_{q} 2^q s_{g,t,q}\right)^2. \tag{13}$$

The demand constraint (6) is enforced analogously to the capacity constraint, thus the penalty term becomes

$$\left(P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k} - D_t - \sum_{q} 2^q s_{g,t,q}\right)^2. \tag{14}$$

Thus, the final optimization problem is

$$
\begin{aligned}
\min \Bigg\{ & \underbrace{\sum_{t=1}^{T}\sum_{g=1}^{G}\left(z_{g,t}^{\text{on}}\, C_{\text{startup},g} + z_{g,t}^{\text{off}}\, C_{\text{shutdown},g} + b_g\, p_{g,t} + c_g\, u_{g,t}\right)}_{\text{Original cost function}} \\
& + \lambda_1 \underbrace{\sum_{g=1}^{G}\sum_{t=1}^{T}\left[(1-u_{g,t})\sum_{k} p_{g,t,k}\right]}_{\text{Coupling penalty (generator on/off with binary power bits)}} \\
& + \lambda_2 \underbrace{\sum_{g=1}^{G}\sum_{t=1}^{T}\left(u_{g,t} - u_{g,t-1} - z_{g,t}^{\text{on}} + z_{g,t}^{\text{off}}\right)^2}_{\text{Logical constraint (startup/shutdown consistency)}} \\
& + \lambda_3 \underbrace{\sum_{g=1}^{G}\sum_{t=1}^{T} z_{g,t}^{\text{off}} z_{g,t}^{\text{on}}}_{\text{Mutual exclusivity of on/off events}} \\
& + \lambda_4 \underbrace{\sum_{g=1}^{G}\sum_{t=1}^{T}\left(P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k} + \sum_{q} 2^q s_{g,t,q} - P_g^{\max}\right)^2}_{\text{Capacity constraint penalty}} \\
& + \lambda_5 \underbrace{\sum_{g=1}^{G}\sum_{t=1}^{T}\left(P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k} - P_g^{\min} u_{g,t-1} - \sum_{k=0}^{n-1} 2^k p_{g,t-1,k} + R_g^{\text{down}} - \sum_{q} 2^q s_{g,t,q}\right)^2}_{\text{Ramp constraint penalty}} \\
& + \lambda_6 \underbrace{\sum_{t=1}^{T}\left(\sum_{g=1}^{G}\left[P_g^{\min} u_{g,t} + \sum_{k=0}^{n-1} 2^k p_{g,t,k}\right] - D_t - \sum_{q} 2^q s_{g,t,q}\right)^2}_{\text{Demand satisfaction penalty}} \Bigg\}
\end{aligned} \tag{15}
$$

4 Model Parameters

Generator Characteristics

Unit #	**1**	**2**	**3**
$P_g^{\min}$ [MW]	50	80	40
$P_g^{\max}$ [MW]	350	200	140
R_g^{down} [MW/h]	300	150	100
R_g^{up} [MW/h]	200	100	100
C_g^{StartUp} [$]	20	18	5
C_g^{ShutDown} [$]	0.5	0.3	1.0
b_g [$/MWh]	0.10	0.125	0.150
C_g [$]	5	7	6

Table 1: Generator characteristics.

Unit #	**1**	**2**	**3**
$u_{g,0}$	0	0	1
$p_{g,0}$ [MW]	0	0	100

Table 2: Initial on/off and output states of the generating units.

Time period	**1**	**2**	**3**
D_t [MW]	160	500	400

Table 3: Deterministic demand profile.