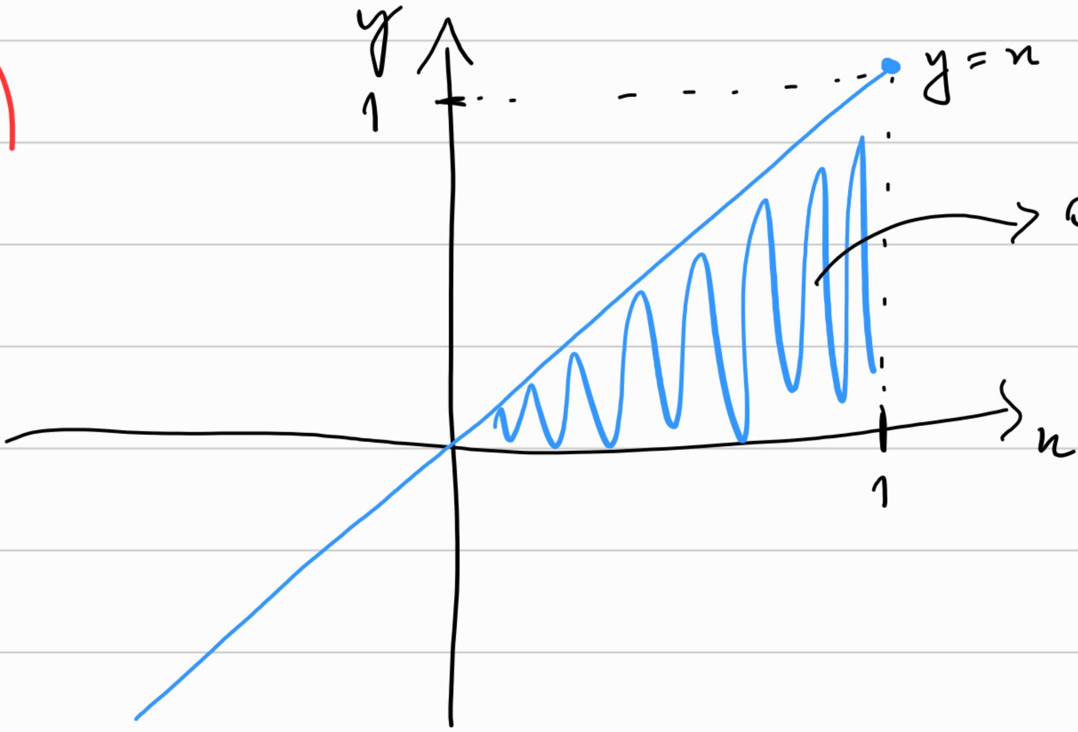
(18) $(X,Y) \sim f_{X,Y}(x,y) = \begin{cases} k(x-y), & 0 \le y \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$

a)



b)

$$1 = \int_A k(x-y)\, dx\, dy$$

$$= k \int_0^x \int_0^1 (x-y)\, dx\, dy$$

$$= k \int_0^1 \int_0^x (x-y)\, dy\, dx$$

$$= k \int_0^1 \left. xy - \frac{y^2}{2} \right|_0^x dx$$

$$= k \int_0^1 x^2 - \frac{x^2}{2}\, dx = \frac{k}{2} \int_0^1 x^2\, dx$$

$$1 = \frac{k}{2} \frac{x^3}{3}\Big|_0^1 \Rightarrow 1 = \frac{k}{6} \Rightarrow k = 6$$

c) $$f_X(x) = \int_0^x f_{X,Y}(x,y)\,dy$$

$$= \int_0^x 6(x-y)\,dy = 6\left(xy - \frac{y^2}{2}\right)\Big|_0^x$$

$$= 6\left(x^2 - \frac{x^2}{2}\right) = 6\frac{x^2}{2} = 3x^2$$

$$f_Y(y) = \int_0^1 f_{X,Y}(x,y)\,dx$$

$$= \int_0^1 6(x-y)\,dx$$

$$= 6\left(\frac{x^2}{2} - xy\right)\Big|_0^1 = 6\left(\frac{1}{2} - y\right)$$

$$= 3 - 6y$$

d) $$f_{Y/X}(y;x) = f_{X,Y}(x,y) / f_X(x)$$

$$= \frac{6(x-y)}{3x^2} = 2\,\frac{x-y}{x^2} = \frac{2}{x} - \frac{2y}{x^2}$$

$$f_{X/Y}(x;y) = f_{X,Y}(x,y) \,/\, f_Y(y)$$

$$= \frac{6(x-y)}{3-6y}$$

(24) $P \sim Unif[0,1] \to$ contínua = PRIOR DENSITY

$X|P=p \sim Bern(p) \to$ discreta

$P|X = ?$

$$f_{X|P}(x;p) = \mathbb{P}(X=x|P=p)$$

$$= p^x(1-p)^{1-x}, \quad x \in \{0,1\}$$

$$f_{X,P}(x,p) = f_{X|P}(x;p) \cdot \cancel{f_P(p)} \to 1$$

$$= p^x(1-p)^{1-x} \cdot 1$$

$\to$ joint probability of (X,P)

$$f_{P|X}(p;x) = \frac{f_{X,P}(x,p)}{f_X(x)}$$

$\to$ falta a marginal de X

$$=$$

$$f_X(x) = \int_0^1 p^x(1-p)^{1-x}\,dp = \frac{\Gamma(x+1)\Gamma(2-x)}{\Gamma(3)}$$

$b-1 = 1-x$ $\quad x = a-1$

$b = 2-x$ $\quad a = x+1$

Beta Function

$$\int_0^1 p^{a-1}(1-p)^{b-1}\,dp = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

when $k \in \mathbb{N}$, $\Gamma(n) = (n-1)!$

$$f_X(x) = \frac{\Gamma(x+1)\,\Gamma(2-x)}{\Gamma(3)} = \frac{x!\,(2-x-1)!}{3!}$$

$$= \frac{x!\,(1-x)!}{3!}$$

$\therefore$

$$f_{P|X}(p;x) = \frac{f_{X,P}(x,p)}{}$$