Physics 2002, Fall 2017
Problem Set 4
6 problems; 126 points; estimated time: 2 hr.

1.(5) A seagull grabs a crab that you left on a dock, flies up to a considerable height, and then drops it on a rocky outcropping, breaking the carapace. Describe the energy and momentum conversions that take place.

1. (Answer, 5 points)
To start with, (1) the seagull does work as it climbs to that considerable height, (2) putting gravitational potential energy into the system. When the bird drops the crab, (3) the gravitational potential energy is converted to kinetic energy as the crab's speed increases and its height decreases. Finally, when the crab hits the rock, that (4) velocity generates an impulse that (5) creates a force sufficient to break the crab's shell.

2. (2) Which of these balls has a greater potential energy, and why: (a) a 12-pound bowling ball held 10 feet above the ground, or (b) a 12-pound bowling ball held 10 feet above the rim of a 50-foot-deep well?

2. (Answer, 2 points) The ball held above the well has greater potential energy, because gravitational potential energy uses a relative zero-point that we can define ourselves: namely, the height above the point beyond which the bowling ball will no longer fall—the ground at your feet, in ball (a)'s case, and the bottom of the well, in ball (b)'s case.

3. (31) A force of fifty newtons drags a crate four meters across a floor; the force is applied at an angle above the horizontal reference of 36.87 degrees. (a) What is the work done by the horizontal component of the force? (b) What is the work done by the vertical component of the force? (c) What is the total work done by the fifty-newton force? (d) If it takes ten seconds to drag the crate the four meters, how much power is generated? If the both the crate and the floor are wooden, and if the crate is one kilogram in mass, (e) what is the work done by friction, and (f) what is the net work done by the pulling force?

3. (Answer, 31 points)
(a)

$$F_x = F\cos\theta$$
$$F_x = (50\ N)\cos(36.87°)$$
$$F_x = 40\ N$$
$$W = Fd$$
$$W_x = F_x d_x$$
$$W_x = (40\ N)(4\ m)$$
$$W_x = 160\ J$$

(b)

$$F_y = F\sin\theta$$
$$F_y = (50\ N)\sin(36.87°)$$
$$F_y = 30\ N$$
$$W = Fd$$
$$W_y = F_y d_y$$
$$W_y = (30\ N)(0\ m)$$
$$W_y = 0\ J$$

(c)

$$W_{total} = 160\ J$$

(d)

$$P = \frac{W}{t}$$
$$P = \frac{160\ J}{10\ s}$$
$$P = 16\ W$$

(e)

$$\sum F_y = 0\ N$$
$$0\ N = F_g + F_N$$
$$F_N = -F_g$$
$$F_N = -mg$$
$$F_N = -(1\ kg)(-9.81\ m/s^2)$$
$$F_N = 9.81\ N$$
$$F_f = \mu_k F_N$$
$$F_f = (0.2)(9.81\ N)$$
$$F_f = 1.96\ N$$
$$W_f = -F_f d$$
$$W_f = -(1.96\ N)(4\ m)$$
$$W_f = -7.84\ J$$

(f)

$$W_{total} = W_x + W_f = 160\ J - 7.84\ J = 152.16\ J$$

4. (25) A twenty-kilogram wooden block is attached to a horizontal spring, which is then compressed by ten centimeters such that the potential energy of the system is one hundred twenty joules. (a) What is the spring constant? (b) Ignoring friction, what is the maximum speed of the block? (c) If friction were a factor and if the floor beneath the block were also wood, what would the maximum speed of the block be?

4. (Answer, 25 points)
(a)

$$PE_s = \frac{1}{2}kx^2$$
$$120\,J = \frac{1}{2}k(0.1\,m)^2$$
$$k = 24{,}000\,N/m$$

(b)
At maximum speed, all the potential energy has been converted into kinetic energy:

$$E_{total} = KE_{max} = \frac{1}{2}mv_{max}{}^2$$
$$120\,J = \frac{1}{2}(20\,kg)v_{max}{}^2$$
$$v_{max}{}^2 = 12\,\,m^2/s^2$$
$$v_{max} = 3.46\,m/s$$

(c)

$$\sum F_y = 0\,N$$
$$0\,N = F_g + F_N$$
$$F_N = -F_g$$
$$F_N = -mg$$
$$F_N = -(20\,kg)(-9.81\,m/s^2)$$
$$F_N = 196.2\,N$$
$$F_f = \mu_k F_N$$
$$F_f = (0.2)(196.2\,N)$$
$$F_f = 39.24\,N$$

The distance when the speed is at the maximum comes from the compression in the spring—the compression in the spring is zero meters when the potential energy of the spring is fully converted to kinetic energy.

$$W_f = -F_f d$$
$$W_f = -(39.24\,N)(0.1\,m)$$
$$W_f = -3.924\,J$$
$$E_{total} = KE_{max} = \frac{1}{2}mv_{max}{}^2$$
$$PE_s + W_f = \frac{1}{2}(20\,kg)v_{max}{}^2$$
$$120\,J - 3.924\,J = \frac{1}{2}(20\,kg)v_{max}{}^2$$
$$116\,J = \frac{1}{2}(20\,kg)v_{max}{}^2$$
$$v_{max}{}^2 = 11.6\,\,m^2/s^2$$
$$v_{max} = 3.41 m/s$$

5. (42) You release a ten-gram superball from a height of one meter. The ball is in contact with the ground for four milliseconds and reaches a height of ninety centimeters. (a) Is energy conserved? Why? (b) Find the work that the ground does on the ball to take energy out of the system. Use (b.1.) kinematics, and (b.2) energy, taking the compression of the ball to do negative work, at one centimeter with the effective spring constant of the ball as one four newtons-per-meter.

5. (Answer, 42 points)
(a – 2 points)
Energy is not conserved, because the superball will be deformed, generating heat and sound, transferring a small portion of its energy into the ground in these forms.
(b.1 – 22 points)
First, find the speed with which the ball strikes the ground.

$$v_f{}^2 = v_i{}^2 + 2ad$$
$$v_f{}^2 = (0\,m/s)^2 + 2(9.81\,\,m/s^2)(1\,m)$$
$$v_f{}^2 = 19.62\,\,m^2/s^2$$
$$v_f = -4.43\,m/s$$

Next, find the force that the ball applies: the only one is gravity, but we must check and make sure that it's the overall force applied to the ground. We need to find the time it takes the ball to fall.

$$v_f = v_i + at$$
$$4.43\,m/s = 0\,m/s + (-9.81\,\,m/s^2)t$$
$$t = 0.45\,s$$

Now, we can look at the force from the acceleration:

$$F_{net} = ma_{net}$$
$$F_{net} = m\left(\frac{v_f - v_i}{t}\right)$$
$$F_{net} = (0.01\,kg)\left[\frac{(-4.43\,m/s) - (0\,m/s)}{0.45\,s}\right]$$

$$F_{net} = 0.098\,N$$

This value is the gravitational force of a ten-gram object. Now, we can calculate the impulse and the change of velocity of the ball during its bounce.

$$J = F\Delta t = m\Delta v = \Delta p$$
$$J = F\Delta t$$
$$J = (0.098\,N)(0.004\,s)$$
$$J = 0.000392\,N \cdot m = \Delta p$$
$$\Delta p = m\Delta v$$
$$0.000392\,N \cdot m = (0.01\,kg)\Delta v$$
$$\Delta v = 0.0392\,m/s$$

So, our upward velocity will be our downward velocity minus this value.

$$v_i = 4.43\,m/s - 0.04\,m/s = 4.39\,m/s$$

Use this to find the height after the first bounce.

$$v_f^2 = v_i^2 + 2ad$$
$$0\,m^2/s^2 = (4.39\,m/s)^2 + 2(-9.81\ m/s^2)d$$
$$d = 0.98\,m$$

(b.2 – 18 points)

For energy, start with the total energy at the top, which is entirely made of up gravitational potential energy.

$$E_T = PE_g + KE$$
$$E_T = PE_g{}^{max} + 0\,J$$
$$E_T = PE_g{}^{max} = mgh_{initial}$$
$$E_T = PE_g{}^{max} = (0.01\,kg)(9.81\ m/s^2)(1\,m)$$
$$E_T = PE_g{}^{max} = 0.0981\,J$$
$$E_T = PE_g{}^{max} = KE_{bottom} = \frac{1}{2}mv_{bottom}{}^2$$
$$0.0981\,J = \frac{1}{2}(0.01\,kg)v_{bottom}{}^2$$
$$v_{bottom}{}^2 = 19.62\ m^2/s^2$$
$$v_{bottom} = 4.43\,m/s$$

Now, calculate the potential energy that the force of the ball does on the ground, dissipating energy into the ground.

$$W_s^{out} = -PE_s = \frac{1}{2}kx^2$$
$$W_s^{out} = \frac{1}{2}(4\ N/m)(0.01\,m)^2$$
$$W_s^{out} = 0.002\,J$$

Subtract this from the total energy, since energy is not conserved.

$$E_T{}^{After\ bounce} = E_T - W_s^{out}$$
$$E_T{}^{After\ bounce} = 0.0981\,J - 0.002\,J$$
$$E_T{}^{After\ bounce} = 0.0961\,J$$

Now, use this result to calculate the new maximum height.

$$PE_g^{max\,After\ Bounce} = mgh_{after\ bounce}$$
$$0.0961\,J = (0.01\,kg)(9.81\ m/s^2)h_{after\ bounce}$$
$$h_{after\ bounce} = 0.98\,m$$

6. (21) A two-hundred-gram wooden block lies on a wooden table attached to a horizontal spring with a spring constant of four hundred newtons-per-meter; the spring is stretched at a distance of forty centimeters. (a) What is the initial potential energy of the system? (b) Absent friction, what is the speed of the block as it travels through the spring's equilibrium point? (c) If friction is present, show whether the block-and-spring system stop at the equilibrium point immediately after you release it.

6. (Answer, 21 points)

(a)

$$PE_s = \frac{1}{2}kx^2$$
$$PE_s = \frac{1}{2}(400\,N/m)(0.4\,m)^2$$
$$PE_s = 32\,J$$

(b)

$$E_T = PE_s + KE$$
$$E_T = PE_s{}^{max} + 0\,J$$
$$E_T = KE_{equilibrium\ point} = \frac{1}{2}mv_{equilibrium\ point}{}^2$$
$$32\,J = \frac{1}{2}(0.2\,kg)v_{equilibrium\ point}{}^2$$
$$v_{equilibrium\ point} = 17.9\,m/s$$

(c)

$$\sum F_y = 0\,N$$
$$0\,N = F_g + F_N$$
$$F_N = -F_g$$
$$F_N = -mg$$
$$F_N = -(0.2\,kg)(-9.81\,m/s^2)$$
$$F_N = 1.962\,N$$
$$F_f = \mu_k F_N$$
$$F_f = (0.2)(1.962\,N)$$
$$F_f = 0.39\,N$$

We're looking for the work that friction removes from the system from the 40-cm extension to the equilibrium point.

$$W_f = -F_f d$$
$$W_f = -(0.39\,N)(0.4\,m)$$

$$W_f = -0.1\,J$$

So, the block will not stop at the equilibrium point (it will, in fact, go through 203.5 oscillations).