

Physics 2002L, Spring 2018

Problem Set 7

3 problems; 74 points; estimated time: 1.5 hr.

1.(12) A Roman catapult aimed at Vercingetorix's stronghold is capable of launching a fifty-pound boulder at a velocity of 50 Roman miles per hour at an angle of 45 degrees (1 Roman mile is 5000 pedes; 1 pes is 296 millimeters). (a) In the absence of atmospheric drag, how far may Julius Caesar station his catapult from the stronghold in order to hit the base of the wall (the corner the wall makes with the ground)? (b) Using the approximation techniques

1. (Answer; 12 points) First, convert initial velocity into meters per second.

$$50 \frac{\text{Roman mile}}{\text{hr}} \times \frac{5000 \text{ pedes}}{1 \text{ Roman mile}} \times \frac{296 \text{ mm}}{1 \text{ pes}} \times \frac{1 \text{ m}}{1000 \text{ mm}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 20.6 \text{ m/s}$$

Split into two pictures, looking at the vertical first.

$$v \rightarrow \begin{cases} v_{ix} = v \cos \theta = (20.6 \text{ m/s}) \cos(45^\circ) = 14.6 \text{ m/s} \\ v_{iy} = v \sin \theta = (20.6 \text{ m/s}) \sin(45^\circ) = 14.6 \text{ m/s} \end{cases}$$

Vertically, we have the acceleration because of gravity (use 10 meters per second squared), and we know the velocity at the maximum height is zero:

$$v_f = v_i + at$$

$$v_{fy} = v_{iy} + a_y t_{\text{max height}}$$

$$0 \text{ m/s} = 14.6 \text{ m/s} + (-10 \text{ m/s}^2) t_{\text{max height}}$$

$$t_{\text{max height}} = 1.46 \text{ s}$$

This value is half of the boulder's travel time, so we need to double it.

$$t_{\text{total}} = 2.92 \text{ s}$$

Now, turn to the horizontal picture, since we know the time the boulder will fly.

$$d = v_i t + \frac{1}{2} at^2$$

$$d_x = v_{ix} t + \frac{1}{2} a_x t^2$$

$$d_x = (14.6 \text{ m/s})(2.92 \text{ s}) + \frac{1}{2} (0 \text{ m/s}^2)(2.92 \text{ s})^2$$

$$d_x = 42.6 \text{ m}$$

1. (42) The International Space Station orbits the Earth at a height 254 miles above the surface, and its mass is 419,455 kilograms. The Earth's radius is 6,371 kilometers, and its mass is 5.972 septillion kilograms. (a) Calculate the linear velocity of the ISS

around the Earth. (Hint: consider the forces.) (b) How much time does one orbit take? (Hint: use geometry.) (c) What is the kinetic energy of the station? (d) What is the gravitational potential energy? (e) What is the total energy in this stable orbit? (f) If the ISS needs to double its orbit height to avoid a cataclysm, what would the station's new velocity be? (Hint: use the conservation of energy.) (f) How long will it take a frictional force of 0.1 newtons to return the station to its original orbit height? (Hint: use momentum and impulse.)

1. (Answer; 12 points) (a) First, convert height to meters.

$$254 \text{ mi.} \times \frac{5,280 \text{ ft.}}{1 \text{ mi.}} \times \frac{12 \text{ in.}}{1 \text{ ft.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 408,7773.376 \text{ m}$$

Now, equate gravitational force and centripetal force to find velocity.

$$\begin{aligned} F_G &= F_C \\ \frac{Gm_1m_2}{r^2} &= \frac{m_2v^2}{r} \\ \frac{Gm_{\text{Earth}}m_{\text{ISS}}}{r^2} &= \frac{m_{\text{ISS}}v^2}{r} \\ \frac{Gm_{\text{Earth}}}{r} &= v^2 \end{aligned}$$

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg}^2 \cdot \text{s}^2}\right) (5.972 \times 10^{24} \text{ kg})}{(6,371 \times 10^3 \text{ m} + 408,7773.376 \text{ m})}}$$

$$v = 7,665 \text{ m/s}$$

(b) Using general geometry, we can calculate the period of one orbit. We found the velocity in part (a), so now we need the distance the ISS will travel.

$$C = \pi d$$

$$C = 2\pi r$$

$$C = 2\pi(6,371 \times 10^3 \text{ m} + 408,7773.376 \text{ m})$$

$$C = 42598572.46 \text{ m}$$

Using this as the distance traveled, we can calculate the period.

$$d = vt$$

$$t = \frac{d}{v}$$

$$t = \frac{C}{v_{\text{ISS}}}$$

$$t = \frac{42598572.46 \text{ m}}{7,665 \text{ m/s}}$$

$$t = 5557.54 \text{ s} \approx 90 \text{ min}$$

(c) Using the result from part (a), we can calculate the kinetic energy.

$$KE = \frac{1}{2}mv^2$$

$$KE_{ISS} = \frac{1}{2} m_{ISS} v_{ISS}^2$$

$$KE_{ISS} = \frac{1}{2} (419,455 \text{ kg})(7,665 \text{ m/s})^2$$

$$KE_{ISS} = 1.232 \times 10^{13} \text{ J}$$

(d) Gravitational potential energy is:

$$PE_G = -\frac{Gm_1m_2}{r}$$

$$PE_{G_{ISS}} = -\frac{Gm_{Earth}m_{ISS}}{r}$$

$$PE_{G_{ISS}} = -\frac{\left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg}^2 \cdot \text{s}}\right)(5.972 \times 10^{24} \text{ kg})(419,455 \text{ kg})}{(6,371 \times 10^3 \text{ m} + 408,773.376 \text{ m})}$$

$$PE_{G_{ISS}} = -2.464 \times 10^{13} \text{ J}$$

(e) The total energy is the sum of kinetic energy and potential energy.

$$E_T = KE + PE$$

$$E_{T_{ISS}} = KE_{ISS} + PE_{G_{ISS}}$$

$$E_{T_{ISS}} = 1.232 \times 10^{13} \text{ J} - 2.464 \times 10^{13} \text{ J}$$

$$E_{T_{ISS}} = -1.232 \times 10^{13} \text{ J}$$

(f) The total energy is constant, so the kinetic and potential energy constituents of the total energy must change. Since we're changing the radius, let's look at gravitational potential energy first.

$$PE_G^{new} = \frac{Gm_1m_2}{r_{new}}$$

$$PE_{G_{ISS}}^{new} = \dots$$

This result means that the kinetic energy must be the difference between the total energy and this new result.

$$E_T = KE + PE$$

$$E_{T_{ISS}} = KE_{ISS}^{new} + PE_{G_{ISS}}^{new}$$

$$E_{T_{ISS}} - PE_{G_{ISS}}^{new} = KE_{ISS}^{new}$$

$$\dots = KE_{ISS}^{new}$$

$$KE_{ISS}^{new} = \dots$$

So, the new velocity is:

$$KE = \frac{1}{2} mv^2$$

$$KE_{ISS}^{new} = \frac{1}{2} m_{ISS} v_{ISS_{new}}^2$$

$$v_{ISS_{new}} = \sqrt{\frac{2KE_{ISS}^{new}}{m_{ISS}}}$$

$$v_{ISS_{new}} = \dots$$

$$v_{ISS_{new}} = \dots \text{ m/s}$$

(f) The momenta of the station at its two orbit heights are

$$p_{old} = mv_{old}$$

$$p_{old} = (419,455 \text{ kg})(7,665 \text{ m/s})$$

$$p_{old} = 3,215,122,575 \text{ kg} \cdot \text{m/s}$$

$$p_{new} = mv_{new}$$

$$p_{new} = (419,455 \text{ kg})(\dots \text{ m/s})$$

$$p_{new} = \dots \text{ kg} \cdot \text{m/s}$$

So the change in momentum is:

$$\Delta p = \dots \text{ kg} \cdot \text{m/s}$$

This value is also the impulse.

$$J = \dots \text{ kg} \cdot \text{m/s}$$

Now, we can find the time for friction to slow the station down to its original orbit.

$$J = Ft$$

$$t = \frac{J}{F}$$

$$t = \frac{\dots \text{ kg} \cdot \text{m/s}}{0.1 \text{ N}}$$

$$t = \dots \text{ s} \approx \dots \text{ years}$$

3. (20) You decide to jump on your bed. The two springs in the mattress you're standing over have a spring constant of 5,000 newtons-per-meter. If you weigh the same as the global average, namely 62 kilograms, how high will you jump up if the bed's springs compress 10 centimeters, and they push up on your feet for half a second? (Please note that the compressed spring is compressed by -10 centimeters.)

Roadmap: (a) find the force of gravity; (b) find the force of the springs; (c) find the net upward force; (d) find the change in momentum delivered over one second (hint: use impulse); (e) find the velocity when you begin the jump; and (f) find the maximum height.

3. (Answer, 20 points)

To solve this problem, we need to use the impulse equation. We'll need to find the net upward force, calculate the change in momentum, and then find the change in velocity. After that, we can use the square-velocity equation to find the distance.

(a) For the gravitational force:

$$F_g = mg$$

$$F_g = (62 \text{ kg})(-9.81 \text{ m/s}^2)$$

$$F_g = -608.22 \text{ N}$$

(b) For the spring force in each spring:

$$F_s = -kx$$

$$F_s = -(5,000 \text{ N/m})(-10 \times 10^{-2} \text{ m})$$

$$F_s = 500 \text{ N}$$

For two springs:

$$F_{s, \text{ total}} = 2 \times 500 \text{ N}$$

$$F_{s, \text{ total}} = 1,000 \text{ N}$$

(c) Now, we plug into our net-force equation:

$$\sum F = F_{net}$$

$$F_g + F_s = F_{net}$$

$$(-608.22 \text{ N}) + (1,000 \text{ N}) = F_{net}$$

$$F_{net} = 391.78 \text{ N}$$

(d) Use the impulse equation to find the change in momentum and the velocity:

$$J = \Delta p = m\Delta v = F\Delta t$$

$$m\Delta v = F\Delta t$$

$$(62 \text{ kg})\Delta v = (391.78 \text{ N})(0.5 \text{ s})$$

$$\Delta v = 3.16 \text{ m/s}$$

(e) Use the velocity-squared equation to find the max height, mindful that at the top of the jump, your velocity is zero:

$$v_f^2 = v_i^2 + 2ad$$

$$(0 \text{ m/s})^2 = (3.16 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)d$$

$$d = \frac{-9.99 \text{ m}^2/\text{s}^2}{-19.62 \text{ m/s}^2}$$

$$\mathbf{d = 0.51 \text{ m}}$$