Physics 2002, Fall 2017
Problem Set 3
8 problems; 106 points; estimated time: 1.5hr.

1.(50) A car with a mass of 1,000 kilograms travels around a curve with a constant speed of 27 meters-per-second, about 60 miles-per-hour; the radius of curvature is 40 meters: (a) what is the centripetal acceleration of the car, (b) what is the centripetal force, (c) what is the ideal speed around the curve if it is banked at 30 degrees, and (d) what is the angle at which the curve above is banked in the absence of friction. If the coefficient of kinetic friction between rubber and dry concrete is 0.7, (e) if the friction force points towards the center of circle that the bend lies on, at what angle is the curve banked, and (g) using algebra and assuming that the only forces in the vertical picture are gravity and the normal force, what is the general relation for the angle at which any curve—with any radius, for any mass, going around at any speed, with any coefficient of friction, with the force towards the center of the circle, on any planet—must be banked, and (g) what is the formula if friction points outwards?

1. (Answer, 7 points)

(a)

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{(27\ m/s)^2}{(40\ m)}$$
$$a_c = 18.225\ m/s^2$$

(b)

$$F_c = ma_c$$
$$F_c = (1000\ kg)(18.225\ m/s^2)$$
$$F_c = 18{,}225\ N$$

(c)

First, we have to get a formula for the ideal velocity. A good place to start is Newton's Second Law. The ideal velocity occurs when friction does not act on the car, so we can leave it out for now—in the ideal velocity, the horizontal component of the normal force provides all of the centripetal force.

$$\sum F_x = ma_x \qquad \sum F_y = 0\ N$$
$$F_{N_x} = ma_x = ma_c = F_c \qquad F_{N_y} - F_g = 0\ N$$
$$F_N \sin\theta = F_c \qquad F_N \cos\theta = F_g$$
$$F_N = \frac{mg}{\cos\theta}$$

Now, we can substitute our result from the vertical picture into the horizontal picture.

$$F_N \sin\theta = F_c$$
$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = F_c$$
$$F_c = mg\tan\theta$$
$$\frac{mv_{ideal}{}^2}{r} = mg\tan\theta$$
$$v_{ideal} = \sqrt{gr\tan\theta}$$

For a bank set at 30 degrees,

$$v_{ideal} = \sqrt{(9.81\,m/s^2)(40\ m)\tan(30°)}$$
$$v_{ideal} = 15\,m/s \approx 33\ mph$$

(d)

In the absence of friction, we're calculating the angle based on the ideal speed, which is 27 m/s.

$$v_{ideal} = \sqrt{gr\tan\theta}$$
$$27\,m/s = \sqrt{(9.81\,m/s^2)(40\ m)\tan(\theta)}$$
$$\tan\theta = 1.8578$$
$$\theta = \arctan(1.8578)$$
$$\theta = 61.7°$$

(e)

As in (c), use vector components—and be sure to use the inclined-plane formulas—and look at Newton's Second Law, for which we have a net acceleration towards the center of the circle and no net force vertically:

$$\sum F_x = ma_x \qquad \sum F_y = 0\ N$$
$$F_{f_x} + F_{N_x} = ma_x = ma_c = F_c \qquad F_{N_y} - F_g = 0\ N$$
$$F_f \sin\theta + F_N \sin\theta = F_c \qquad F_N\cos\theta = F_g$$
$$F_c = \left(F_f + F_N\right)\sin\theta \qquad F_N = \frac{mg}{\cos\theta}$$

The horizontal picture tells us that the friction force and the horizontal component of the normal force create our centripetal acceleration and force. The vertical picture tells us that the force of gravity and the vertical component of the normal force are equal. We use both facts to solve for the angle:

$$F_c = 18{,}225\ N = \left(F_f + F_N\right)\sin\theta$$
$$18{,}225\ N = (\mu_k F_N + F_N)\sin\theta$$
$$18{,}225\ N = \left(\mu_k\frac{mg}{\cos\theta} + \frac{mg}{\cos\theta}\right)\sin\theta$$
$$18{,}225\ N = (\mu_k + 1)\ mg\tan\theta$$
$$18{,}225\ N = (0.7 + 1)\ (1000\ kg)(9.81\,m/s^2)\tan\theta$$
$$\tan\theta = 1.0928$$
$$\theta = 47.5°$$

(f)

Start from our consideration of Newton's Second Law, as in part (e).

$$F_c = \left(F_N + F_f\right)\sin\theta$$
$$F_c = (F_N + \mu_k F_N)\sin\theta$$
$$\frac{mv^2}{r} = \left(\frac{mg}{\cos\theta} + \mu_k\frac{mg}{\cos\theta}\right)\sin\theta$$
$$\frac{mv^2}{r} = (1+\mu_k)\ mg\tan\theta$$
$$\tan\theta = \frac{v^2}{(1+\mu_k)gr}$$
$$\theta = \arctan\left(\frac{v^2}{(1+\mu_k)gr}\right)$$
$$\theta_{friction\ towards\ center}(r, v, \mu_k, g) = \arctan\left(\frac{v^2}{(1+\mu_k)gr}\right)$$

(g)

The force of friction is pointing in the opposite direction, so we can change its sign in our equation from part (d), and do the same algebra as in part (f):

$$F_c = (F_N - F_f)\sin\theta$$
$$\tan\theta = \frac{v^2}{(1-\mu_k)gr}$$
$$\theta = \arctan\left(\frac{v^2}{(1-\mu_k)gr}\right)$$
$$\theta_{friction\ away\ from\ center}(r, v, \mu_k, g) = \arctan\left(\frac{v^2}{(1-\mu_k)gr}\right)$$

2. (14) Calculate the acceleration caused by gravity at the surface of (a) Earth, (b) the moon, (c) Venus, and (d) Uranus. Use your mass, or the mass of any object you like, at your latitude and your elevation above sea level; use the mean radii of the other celestial objects; compare your results with the Earth's gravitational acceleration. Suggested approach: derive a formula to plug numbers into.

2. (Answer, 14 points)

(a)

Start as we did in class by equating the kinematic and universal gravitation formulas:

$$F_g = F_G$$
$$m_{object}g = -\frac{Gm_{planet}m_{object}}{r^2}$$
$$g = -\frac{Gm_{planet}}{r^2}$$

The mass of Earth is 5.972×10^{24} kilograms, and its mean radius is 6,371 kilometers.

$$g_{Earth} = -\frac{\left(6.67\times10^{-11}\frac{m^3}{kg\cdot s^2}\right)(5.972\times10^{24}kg)}{(6.371\times10^6 m)^2}$$
$$g_{Earth} = 9.81\,m/s^2$$

(b)

The mass of the moon is 7.342×10^{22} kilograms, and its mean radius is 1,737.1 kilometers.

$$g_{Moon} = -\frac{\left(6.67\times10^{-11}\frac{m^3}{kg\cdot s^2}\right)(7.342\times10^{22}kg)}{(1.7371\times10^6 m)^2}$$
$$g_{Moon} = 1.61\,m/s^2$$
$$g_{Moon} \approx 0.16g_{Earth}$$

(c)

The mass of Venus is 4.867×10^{24} kilograms, and its mean radius is 6,052 kilometers.

$$g_{Venus} = -\frac{\left(6.67\times10^{-11}\frac{m^3}{kg\cdot s^2}\right)(4.867\times10^{24}kg)}{(6.052\times10^6 m)^2}$$
$$g_{Venus} = 8.86\,m/s^2$$
$$g_{Venus} \approx 0.9g_{Earth}$$

(d)

The mass of Jupiter is 1.898×10^{27} kilograms, and its mean radius is 69,911 kilometers.

$$g_{Jupiter} = -\frac{\left(6.67\times10^{-11}\frac{m^3}{kg\cdot s^2}\right)(1.898\times10^{27}kg)}{(69.911\times10^6 m)^2}$$
$$g_{Jupiter} = 26\,m/s^2$$
$$g_{Jupiter} \approx 2.7g_{Earth}$$

3. (35) A box (10 kilograms) sits on top of another box (100 kilograms), which is attached to a truck. The wood-wood coefficients of friction are 0.4 (static) and 0.2 (kinetic); the wood-concrete coefficients are 0.62 (static) and 0.5 (kinetic). (a) At what velocity will the top box slide off of the box attached to the truck? (b) At what acceleration will the system of boxes begin to slide along with the truck? (c) At what acceleration will the top box begin to slide off of the lower box? (d) Ultimately, is this a secure towing configuration, and why?

3. (Answer, 35 points)

(a)

Trick question! The quantity that matters is acceleration—if you accelerate nicely and slowly, then the top box will not slide. Good luck actually getting it to work, though, considering all the variables at stake.

(b)

Start with the boxes as a system, and calculate the friction felt by the lower box—this will tell you how strong the pull of the truck has to be to move anything. First, we need to calculate gravity.

$$m_{system} = 110kg$$
$$F_g = mg$$
$$F_g = (110\ kg)(-9.81\ m/s^2)$$
$$F_g = -1{,}079.1\ N$$

Now consider the forces in the vertical direction—where there is no net acceleration.

$$\sum F_y = 0\ N$$
$$0\ N = F_N + F_G$$
$$F_N = -F_G = 1{,}079.1\ N$$

Now, use this result to calculate the static friction on the system of boxes.

$$F_{f_s} = \mu_s F_N$$
$$F_{f_s} = (0.62)(1079.1\ N)$$
$$F_{f_s} = 669.0\ N$$

However, this force is a threshold: we need a pull greater than the static friction for the boxes to move:

$$F_{pull} > 669.0\ N$$

Now, let's calculate the acceleration required to get over the threshold.

$$F_{net} = ma_{net}$$
$$F_{f_s} = m_{system}a_{net}$$
$$a_{net} = \frac{669.0\ N}{110\ kg}$$
$$a_{net} = 6.08\ m/s^2$$

However, this acceleration, like the force, is a threshold, so we need an acceleration greater than this to move our boxes.

$$a_{net} > 6.08\ m/s^2$$

(c)

We have friction as a threshold, so let's calculate the static friction that the top box will feel.

$$m_{top} = 10kg$$
$$F_g = mg$$
$$F_g = (10\ kg)(-9.81\ m/s^2)$$
$$F_g = -98.1\ N$$

Now consider the forces in the vertical direction—where there is no net acceleration.

$$\sum F_y = 0\ N$$
$$0\ N = F_N + F_G$$
$$F_N = -F_G = 98.1\ N$$

Now, use this result to calculate the static friction on the system of boxes.

$$F_{f_s} = \mu_s F_N$$
$$F_{f_s} = (0.4)(98.1\ N)$$
$$F_{f_s} = 39.24\ N$$

However, static friction is a threshold: we the lower box to slide under the upper box with a force greater than the static friction for the boxes to move:

$$F_{lower\ box\ sliding} > 39.24\ N$$

Now, let's calculate the acceleration required to get over the threshold.

$$F_{net} = ma_{net}$$
$$F_{f_s} = m_{top\ box}a_{net}$$
$$a_{net} = \frac{39.24\ N}{10\ kg}$$
$$a_{net} = 3.924\ m/s^2$$

However, this acceleration, like the force, is a threshold, so we need an acceleration greater than this to move our boxes.

$$a_{net} > 3.924\ m/s^2$$

(d)

This configuration is not secure—the top box will, certainly, slide off as the truck accelerates, because the threshold acceleration to get both boxes moving is higher than the threshold acceleration for the upper box to remain in place.

5. (7) How fast must a 10,000-kilogram satellite orbit the Earth to be in a stable, circular orbit at about 3,629 kilometers above the Earth's surface?

5. (Answer, 7 points)

(a)

For a circular orbit, the centripetal force is entirely provided by gravitational pull, so we equate Newton's Universal Law of Gravitation with the centripetal force:

$$|F_C| = |F_G|$$
$$\frac{mv^2}{r} = \frac{Gm_1m_2}{r^2}$$
$$\frac{m_{satellite}v^2}{r} = \frac{Gm_{Earth}m_{satellite}}{r^2}$$
$$v^2 = \frac{Gm_{Earth}}{r}$$

Note the similarities to the formula you derived when you found the acceleration because of gravity.

$$v = \sqrt{\frac{Gm_{Earth}}{r}}$$

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{m^3}{kg \cdot s^2}\right)(5.972 \times 10^{24}\ kg)}{(6.371 \times 10^6\ m + 3.629 \times 10^6\ m\ )}}$$
$$v = 199{,}582.7\ m/s \approx\ 446{,}453.8\ mph$$