Physics 2002L, Fall 2017
Problem Set 7
1 problems; 94 points; estimated time: 1 hr.

1.(12) A Roman catapult aimed at Vercingetorix's stronghold is capable of launching a fifty-pound boulder at a velocity of 50 Roman miles per hour at an angle of 45 degrees (1 Roman mile is 5000 pedes; 1 pes is 296 millimeters). (a) In the absence of atmospheric drag, how far may Julius Caesar station his catapult from the stronghold in order to hit the base of the wall (the corner the wall makes with the ground)? (b) Using the approximation techniques

1. (Answer; 12 points) First, convert initial velocity into meters per second.

$$50\,\frac{Roman\ mile}{hr}\times\frac{5000\ pedes}{1\ Roman\ mile}\times\frac{296\ mm}{1\ pes}\times\frac{1\ m}{1000\ mm}$$
$$\times\frac{1\ hr}{60\ min}\times\frac{1\ min}{60\ s}=20.6\ m/s$$

Split into two pictures, looking at the vertical first.

$$v\rightarrow\begin{cases}v_{i_x}=v\cos\theta=(20.6\ m/s)\cos(45^\circ)=14.6\ m/s\\ v_{i_y}=v\sin\theta=(20.6\ m/s)\sin(45^\circ)=14.6\ m/s\end{cases}$$

Vertically, we have the acceleration because of gravity (use 10 meters per second squared), and we know the velocity at the maximum height is zero:

$$v_f=v_i+at$$
$$v_{f_y}=v_{i_y}+a_y t_{\max height}$$
$$0\ m/s=14.6\ m/s+(-10\ m/s^2)t_{\max height}$$
$$t_{\max height}=1.46s$$

This value is half of the boulder's travel time, so we need to double it.

$$t_{total}=2.92\ s$$

Now, turn to the horizontal picture, since we know the time the boulder will fly.

$$d=v_i t+\frac{1}{2}at^2$$
$$d_x=v_{i_x}t+\frac{1}{2}a_x t^2$$
$$d_x=(14.6\ m/s)(2.92\ s)+\frac{1}{2}(0\ m/s^2)(2.92\ s)^2$$
$$d_x=42.6\ m$$

1. (42) The International Space Station orbits the Earth at a height 254 miles above the surface, and its mass is 419,455 kilograms. The Earth's radius is 6,371 kilometers, and its mass is 5.972 septillion kilograms. (a) Calculate the linear velocity of the ISS around the Earth. (*Hint*: *consider the forces*.) (b) How much time does one orbit take? (*Hint*: *use geometry*.) (c) What is the kinetic energy of the station? (d) What is the gravitational potential energy? (e) What is the total energy in this stable orbit? (f) If the ISS needs to double its orbit height to avoid a cataclysm, what would the station's new velocity be? (*Hint*: *use the conservation of energy*.) (f) How long will it take a frictional force of 0.1 newtons to return the station to its original orbit height? (Hint: use momentum and impulse.)

1. (Answer; 12 points) (a) First, convert height to meters.

$$254\ mi.\times\frac{5{,}280\ ft.}{1\ mi.}\times\frac{12\ in.}{1\ ft.}\times\frac{2.54\ cm}{1\ in.}\times\frac{1\ m}{100\ cm}$$
$$=408{,}7773.376\ m$$

Now, equate gravitational force and centripetal force to find velocity.

$$F_G=F_C$$
$$\frac{Gm_1m_2}{r^2}=\frac{m_2v^2}{r}$$
$$\frac{Gm_{Earth}m_{ISS}}{r^2}=\frac{m_{ISS}v^2}{r}$$
$$\frac{Gm_{Earth}}{r}=v^2$$
$$v=\sqrt{\frac{\left(6.67\times10^{-11}\frac{m^3}{kg^2\cdot s}\right)(5.972\times10^{24}kg)}{(6{,}371\times10^3m+\ 408{,}7773.376\ m)}}$$
$$\boldsymbol{v=7{,}665\ m/s}$$

(b) Using general geometry, we can calculate the period of one orbit. We found the velocity in part (a), so now we need the distance the ISS will travel.

$$C=\pi d$$
$$C=2\pi r$$
$$C=2\pi\left(6{,}371\times10^3m+\ 408{,}7773.376\ m\right)$$
$$C=42598572.46\ m$$

Using this as the distance traveled, we can calculate the period.

$$d=vt$$
$$t=\frac{d}{v}$$
$$t=\frac{C}{v_{ISS}}$$
$$t=\frac{42598572.46\ m}{7{,}665\ m/s}$$
$$\boldsymbol{t=5557.54\ s\ \approx 90\ min}$$

(c) Using the result from part (a), we can calculate the kinetic energy.

$$KE=\frac{1}{2}mv^2$$

$$KE_{ISS} = \frac{1}{2} m_{ISS} v_{ISS}^2$$
$$KE_{ISS} = \frac{1}{2}(419{,}455\ kg)(7{,}665\ m/s)^2$$
$$\boldsymbol{KE_{ISS} = 1.232\times10^{13} J}$$

(d) Gravitational potential energy is:

$$PE_G = -\frac{Gm_1m_2}{r}$$
$$PE_{G_{ISS}} = -\frac{Gm_{Earth}m_{ISS}}{r}$$
$$PE_{G_{ISS}} = -\frac{\left(6.67\times10^{-11}\frac{m^3}{kg^2\cdot s}\right)(5.972\times10^{24} kg)(419{,}455\ kg)}{(6{,}371\times10^3 m + 408{,}7773.376\ m)}$$
$$\boldsymbol{PE_{G_{ISS}} = -2.464\times10^{13} J}$$

(e) The total energy is the sum of kinetic energy and potential energy.

$$E_T = KE + PE$$
$$E_{T_{ISS}} = KE_{ISS} + PE_{G_{ISS}}$$
$$E_{T_{ISS}} = 1.232\times10^{13} J - 2.464\times10^{13} J$$
$$\boldsymbol{E_{T_{ISS}} = -1.232\times10^{13} J}$$

(f) The total energy is constant, so the kinetic and potential energy constituents of the total energy must change. Since we're changing the radius, let's look at gravitational potential energy first.

$$PE_G{}^{new} = \frac{Gm_1m_2}{r_{new}}$$
$$PE_{G_{ISS}}{}^{new} = \cdots$$

This result means that the kinetic energy must be the difference between the total energy and this new result.

$$E_T = KE + PE$$
$$E_{T_{ISS}} = KE_{ISS}{}^{new} + PE_{G_{ISS}}{}^{new}$$
$$E_{T_{ISS}} - PE_{G_{ISS}}{}^{new} = KE_{ISS}{}^{new}$$
$$\ldots = KE_{ISS}{}^{new}$$
$$KE_{ISS}{}^{new} = \cdots$$

So, the new velocity is:

$$KE = \frac{1}{2} mv^2$$
$$KE_{ISS}{}^{new} = \frac{1}{2} m_{ISS} v_{ISS_{new}}{}^2$$
$$v_{ISS_{new}} = \sqrt{\frac{2KE_{ISS}{}^{new}}{m_{ISS}}}$$
$$v_{ISS_{new}} = \cdots$$
$$\boldsymbol{v_{ISS_{new}} = \cdots\ m/s}$$

(f) The momenta of the station at its two orbit heights are

$$p_{old} = mv_{old}$$
$$p_{old} = (419{,}455\ kg)(7{,}665\ m/s)$$
$$p_{old} = 3{,}215{,}122{,}575\ kg\cdot m/s$$
$$p_{new} = mv_{new}$$
$$p_{new} = (419{,}455\ kg)(\ldots\ m/s)$$
$$p_{new} = \ \ldots kg\cdot m/s$$

So the change in momentum is:

$$\Delta p = \cdots\ kg\cdot m/s$$

This value is also the impulse.

$$J = \cdots\ kg\cdot m/s$$

Now, we can find the time for friction to slow the station down to its original orbit.

$$J = Ft$$
$$t = \frac{J}{F}$$
$$t = \frac{\ldots\ kg\cdot m/s}{0.1\ N}$$
$$\boldsymbol{t = \cdots\ s \approx \cdots\ years}$$