Physics 2002L, Fall 2017
Problem Set 8
3 problems; 55 points; estimated time: 1 hr.

1. (20) You decide to jump on your bed. The two springs in the mattress you're standing over have a spring constant of 5,000 newtons-per-meter. If you weigh the same as the global average, namely 62 kilograms, how high will you jump up if the bed's springs compress 10 centimeters, and they push up on your feet for half a second? (Please note that the compressed spring is compressed by – 10 centimeters.)

Roadmap: (a) find the force of gravity; (b) find the force of the springs; (c) find the net upward force; (d) find the change in momentum delivered over one second (hint: use impulse); (d) find the velocity when you begin the jump; and (e) find the maximum height.

1. (Answer, 20 points)

To solve this problem, we need to use the impulse equation. We'll need to find the net upward force, calculate the change in momentum, and then find the change in velocity. After that, we can use the square-velocity equation to find the distance.

(a) For the gravitational force:

$$F_g = mg$$
$$F_g = (62\ kg)(-9.81\ m/s^2)$$
$$F_g = -608.22\ N$$

(b) For the spring force *in each spring*:

$$F_s = -kx$$
$$F_s = -(5{,}000\ N/m)(-10\times10^{-2}\ m)$$
$$F_s = 500\ N$$

For two springs:

$$F_{s,\ total} = 2 \times 500\ N$$
$$F_{s,\ total} = 1{,}000\ N$$

(c) Now, we plug into our net-force equation:

$$\sum F = F_{net}$$
$$F_g + F_s = F_{net}$$
$$(-608.22\ N) + (1{,}000\ N) = F_{net}$$
$$F_{net} = 391.78\ N$$

(d) Use the impulse equation to find the change in momentum and the velocity:

$$\mathrm{J} = \Delta p = m\Delta v = F\Delta t$$
$$m\Delta v = F\Delta t$$
$$(62\ kg)\Delta v = (391.78\ N)(0.5\ \mathrm{s})$$
$$\Delta v = 3.16\ m/s$$

(e) Use the velocity-squared equation to find the max height, mindful that at the top of the jump, your velocity is zero:

$$v_f^{\,2} = v_i^{\,2} + 2ad$$
$$(0\ m/s)^2 = (3.16\ m/s)^2 + 2(-9.81\ m/s^2)d$$
$$d = \frac{-9.99\ m^2/s^2}{-19.62\ m/s^2}$$
$$\boldsymbol{d = 0.51\ m}$$

2. (18) You're driving down a highway, and you decide to accelerate. Flaunting the speed limit, you accelerate way up there. At one point, you're going (a) 100 km/hr. Later, you're going (b) 0.25 times the speed of light. For each case, how much is your car's length contracted, and how slow does time go for you? Your car is 15 feet long at rest (4.5 meters), and take t_0 to be 1 second.

2. (Answer, 18 points)

(a) For v=100 km/hr:

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1 - \frac{v^2}{c^2}}$$
$$l = (4.5\ m)\sqrt{1 - \frac{(100\times10^3\ m/s)^2}{(3\times10^8\ m/s)^2}}$$
$$l = (4.5\ m)\sqrt{1 - \frac{10000\times10^6\ m^2/s^2}{9\times10^{16}\ m^2/s^2}}$$
$$l = (4.5\ m)(0.9999999444)$$
$$\boldsymbol{l = 4.49999975}$$

(a/k/a: length of the 15-ft. car contracts by about 0.0001%)

$$t = t_0\gamma = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{(1\,s)}{\sqrt{1 - \frac{(100\times10^3\ m/s)^2}{(3\times10^8\ m/s)^2}}}$$

$$t = \frac{(1\,s)}{(0.9999999444)}$$

$$\boldsymbol{t = 1.000000056\,s}$$

(a/k/a, for every one second that passes for people standing on the street, just a split-hair over 1 second passes for you.)

(b) For v=0.25c:

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$l = (4.5\,m)\sqrt{1 - \frac{\left(\frac{c}{4}\right)^2}{(c)^2}}$$

$$l = (4.5\,m)\sqrt{1 - \frac{1}{16}}$$

$$l = (4.5\,m)(0.9682458366)$$

$$\boldsymbol{l = 4.357\,m}$$

(a/k/a: length of the 15-ft. car contracts by 15 cm)

$$t = t_0\gamma = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{(1\,s)}{\sqrt{1 - \frac{1}{16}}}$$

$$t = \frac{(1\,s)}{(0.9682458366)}$$

$$\boldsymbol{t = 1.0328\,s}$$

(As one second passes for bystanders, 1.0328 seconds pass for you.)

3. (17) How much heat must be added to 120 grams of water at an initial temperature of 60 degrees Celsius to (a) heat the water to its boiling point, and (b) completely convert the boiling water to steam? (c) Convert the total energy you've added to joules. The specific heat capacity of water is 1 calorie per gram per degree Celsius; the latent heat of vaporization of water is 540 calories per gram. (d) For comparison, how fast would a 1-kilogram mass have to go to achieve this energy? (e) For further comparison, how far would you have to push a 25-kilogram armchair with a force of 100 newtons to have done the same amount of work?

3. (Answer, 17 points)

(a) Use the heat equation, leave everything in grams:

$$Q = mc\Delta T$$

$$Q = mc\left(T_f - T_i\right)$$

$$Q = (120\ g)\left(1\,\frac{cal}{g\cdot{}^\circ\mathrm{C}}\right)[100^\circ\mathrm{C} - 60^\circ\mathrm{C}]$$

$$\boldsymbol{Q = 4{,}800\ calories} = 4.8\ Calories$$

(b) Use the latent-heat equation:

$$Q = mL_v$$

$$Q = (120g)(540\ cal/g)$$

$$\boldsymbol{Q = 64{,}800\ calories} = 64.8\ Calories$$

(c) The total heat you must add is:

$$Q = Q_{change}^{temperature} + Q_{change}^{phase}$$

$$Q = 4{,}800\ calories + 64{,}800\ calories$$

$$Q = 69{,}600\ calories$$

$$Q = 69{,}600\ calories \times \left(\frac{4{,}184\,J}{1\,Cal.}\right) \times \left(\frac{1\,Cal.}{1{,}000\ cal}\right)$$

$$\boldsymbol{= 291{,}206.4\,J}$$

(d) Use the kinetic energy formula:

$$KE = \frac{1}{2}mv^2$$

$$291{,}206.4\,J = \frac{1}{2}(1\,kg)v^2$$

$$\boldsymbol{v = 763.16\ m/s} = 1{,}679\ mph$$

(e) Use the work formula:

$$W = Fd\cos\theta$$

$$291{,}206.4\,J = (100\ N)d(1)$$

$$\boldsymbol{d = 2{,}913.06\,m} = 2.913\ km \approx 1.8\ mi.$$