Physics 2002
Test 1
Thursday, October 5, 2017
50 points
9:05 a.m. — 10:25 a.m.
No additional time will be allotted — use 10 m/s^2 for *g*

An old pond,
a frog jumps in —
Splash!
Bashō (1644-1694)

1. (12) A Roman catapult aimed at King Vercingetorix's stronghold can launch a fifty-pound boulder at a velocity of 50 milia passus per hour at an angle of 45 degrees (1 mille passus is 5000 pedes; 1 pes is 296 millimeters). How far may General Julius Caesar station his catapult from the stronghold to hit the base of the wall (the corner the wall makes with the ground)?
2. (5) Colonel Murgatroyd has a weight of 720 N (about 162 lbs.) when he is standing on the surface of the Earth. What would his weight (the gravitational force exerted by the Earth) be if he doubled his distance from the center of the Earth by flying up in a spacecraft? If you use Newton's Universal Law of Gravitation, take Earth's radius to be 6,371 km.
3. (32) A box (10 kilograms) sits on top of another box (100 kilograms), which is attached to a truck. The wood-wood coefficients of friction are 0.4 (static) and 0.2 (kinetic); the wood-concrete coefficients are 0.62 (static) and 0.5 (kinetic). (a) At what acceleration will the system of boxes begin to slide along with the truck? (b) At what acceleration will the top box begin to slide off the lower box?

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1. (12) A Roman catapult aimed at Vercingetorix's stronghold is capable of launching a fifty-pound boulder at a velocity of 50 Roman miles per hour at an angle of 45 degrees (1 Roman mile is 5000 pedes; 1 pes is 296 millimeters). How far may Julius Caesar station his catapult from the stronghold in order to hit the base of the wall (the corner the wall makes with the ground)?

1. (Answer; 12 points) First, convert initial velocity into meters per second.

$$50\,\frac{Roman\ mile}{hr}\times\frac{5000\ pedes}{1\ Roman\ mile}\times\frac{296\ mm}{1\ pes}\times\frac{1\ m}{1000\ mm}$$
$$\times\frac{1\ hr}{60\ min}\times\frac{1\ min}{60\ s}=20.6\ m/s$$

Split into two pictures, looking at the vertical first.

$$v\rightarrow\begin{cases}v_{i_x}=v\cos\theta=(20.6\ m/s)\cos(45^\circ)=14.6\ m/s\\ v_{i_y}=v\sin\theta=(20.6\ m/s)\sin(45^\circ)=14.6\ m/s\end{cases}$$

Vertically, we have the acceleration because of gravity (use 10 meters per second squared), and we know the velocity at the maximum height is zero:

$$v_f=v_i+at$$
$$v_{f_y}=v_{i_y}+a_yt_{\max height}$$
$$0\ m/s=14.6\ m/s+(-10\ m/s^2)t_{\max height}$$
$$t_{\max height}=1.46s$$

This value is half of the boulder's travel time, so we need to double it.

$$t_{total}=2.92\ s$$

Now, turn to the horizontal picture, since we know the time the boulder will fly.

$$d=v_it+\frac{1}{2}at^2$$
$$d_x=v_{i_x}t+\frac{1}{2}a_xt^2$$
$$d_x=(14.6\ m/s)(2.92\ s)+\frac{1}{2}(0\ m/s^2)(2.92\ s)^2$$
$$d_x=42.6\ m$$

2. (5) Colonel Murgatroyd has a weight of 720 N (about 162 lbs.) when he is standing on the surface of the Earth. What would his weight (the gravitational force exerted by the Earth) be if he doubled his distance from the center of the Earth by flying a spacecraft? If you use Newton's Universal Law of Gravitation, take Earth's radius to be 6,371 km.

2. (Answer; 5 points)
Step 1: Find Col. Murgatroyd's mass.

$$F_g=m\times g$$
$$-720\ N=m\times(-10\ m/s^2)$$
$$m_{Col.Murgatroyd}=72\ kg$$

Step 2: Find the force at the doubled distance — we saw, following from Newton's Universal Law of Gravitation, that doubling the distance quarters the force:

$$F_G|_{doubled\ distance}=\frac{1}{4}F_G|_{single\ distance}$$
$$F_G=-180\ N$$

3. (35) A box (10 kilograms) sits on top of another box (100 kilograms), which is attached to a truck. The wood-wood coefficients of friction are 0.4 (static) and 0.2 (kinetic); the wood-concrete coefficients are 0.62 (static) and 0.5 (kinetic). (a) At what velocity will the top box slide off of the box attached to the truck? (b) At what acceleration will the system of boxes begin to slide along with the truck? (c) At what acceleration will the top box begin to slide off of the lower box? (d) Ultimately, is this a secure towing configuration, and why?

3. (Answer, 35 points)
(a)
Start with the boxes as a system, and calculate the friction felt by the lower box—this will tell you how strong the pull of the truck has to be to move anything. First, we need to calculate gravity.

$$m_{system}=110kg$$
$$F_g=mg$$
$$F_g=(110\ kg)\left(-10\ m/s^2\right)$$
$$F_g=-1{,}100\ N$$

Now consider the forces in the vertical direction—where there is no net acceleration.

$$\sum F_y = 0\ N$$
$$0\ N = F_N + F_G$$
$$F_N = -F_G = 1{,}100\ N$$

Now, use this result to calculate the static friction on the system of boxes.

$$F_{f_s} = \mu_s F_N$$
$$F_{f_s} = (0.62)(1{,}100\ N)$$
$$F_{f_s} = 682\ N$$

However, this force is a threshold: we need a pull greater than the static friction for the boxes to move:

$$F_{pull} > 682\ N$$

Now, let's calculate the acceleration required to get over the threshold.

$$F_{net} = ma_{net}$$
$$F_{f_s} = m_{system}a_{net}$$
$$a_{net} = \frac{682\ N}{110\ kg}$$
$$a_{net} = 6.2\ m/s^2$$

However, this acceleration, like the force, is a threshold, so we need an acceleration greater than this to move our boxes.

$$a_{net} > 6.2\ m/s^2$$

(c)

We have friction as a threshold, so let's calculate the static friction that the top box will feel.

$$m_{top} = 10kg$$
$$F_g = mg$$
$$F_g = (10\ kg)(-10\ m/s^2)$$
$$F_g = -100\ N$$

Now consider the forces in the vertical direction—where there is no net acceleration.

$$\sum F_y = 0\ N$$
$$0\ N = F_N + F_G$$
$$F_N = -F_G = 100\ N$$

Now, use this result to calculate the static friction on the system of boxes.

$$F_{f_s} = \mu_s F_N$$
$$F_{f_s} = (0.4)(100\ N)$$
$$F_{f_s} = 40\ N$$

However, static friction is a threshold: we the lower box to slide under the upper box with a force greater than the static friction for the boxes to move:

$$F_{lower\ box\ sliding} > 40\ N$$

Now, let's calculate the acceleration required to get over the threshold.

$$F_{net} = ma_{net}$$
$$F_{f_s} = m_{top\ box}a_{net}$$
$$a_{net} = \frac{40\ N}{10\ kg}$$
$$a_{net} = 4\ m/s^2$$

However, this acceleration, like the force, is a threshold, so we need an acceleration greater than this to move our boxes.

$$a_{net} > 4\ m/s^2$$