Physics 2002, Spring 2018
Problem Set 5
6 problems; 126 points; estimated time: 2 hr.

1.(3) A coin rolls down a ramp, gaining speed as it rolls, never slipping. Does the coin have a rotational acceleration, and why?

1. (Answer, 3 points)
The coin does have rotational acceleration, because it never slips as it rolls down. In order for its linear speed to increase, the speed of the rotations must increase as well.

2. (3) Is it possible for the net force acting on an object to be zero, but the net torque to be greater than zero? Explain. (Hint: forces contributing to the net force may not lie along the same line.)

2. (Answer, 3 points) Of course! Imagine pushing oppositely sideways on the two ends of a pencil lying on a table. Since forces will act on the center of mass of the pencil, the net force is zero. However, since the two forces are displaced towards the ends of the pencil, the net torque is not zero, causing a rotation.

3.(3) Why do you feel resistance when you try to flip over a spinning fidget spinner? (Try it, if you don't believe me.)

3. (Answer, 3 points)
Conservation of angular momentum requires that the angular momentum of the spinner to be constant, unless an external torque is applied. The fight you feel comes from the work you have to do to flip the angular-momentum vector (that is to say, you change the direction of rotation when you flip the spinner over).

4. (15) A four-meter plank weighing eighty newtons is placed along the gunwale of a ship, with one meter of its length extended over the water. The plank is of uniform density, such that its center of gravity is in the center of the plank. A stowaway child, having fallen from the favor of a pirate captain, walks out, slowly. He weighs one hundred fifty newtons. (a) What are the three torques on acting on the plank, and what is the relevant pivot point? (b) Calculate them. The two from the plank will result in a numerical answer; the one from the boy will result in a radius-dependent relation. (c) How far may the child walk before the plank starts to rotate?

4. (Answer, 15 points)
(a) The three torques are (1) the torque caused by the part of the plank lying along the ship; (2) the torque caused by the part of the plank jutting out over the water; and (3) the torque caused by the child. The relevant pivot point is the edge of the gunwale.
(b)
Taking counterclockwise to be positive:

$$\tau = r\,F\sin\theta$$
$$\tau_{plank} = (1.5\,m)\,(60N)\sin(90°)$$
$$\tau_{plank} = 90\,N\cdot m$$
$$\tau_{extension} = (0.5\,m)\,(20N)\sin(90°)$$
$$\tau_{extension} = 10\,N\cdot m$$
$$\tau_{child}(r) = r\,(150N)\sin(90°)$$
$$\tau_{child}(r) = (150\,N)\times r$$

(c)

$$\sum \tau = 0\,N\cdot m$$
$$\tau_{plank} - (\tau_{extension} + \tau_{child}) = 0\,N\cdot m$$
$$90\,N\cdot m - (10\,N\cdot m + (150\,N)\times r) = 0\,N\cdot m$$
$$r = 0.53\,m$$

4. (25) A force of fifty newtons is applied to the end of a wrench handle stuck deep in the engine of a car by a string, making an angle of thirty degrees with the perpendicular reference axis of the wrench handle. The wrench handle is twenty-four centimeters long. If the wrench is, happily, secured on a nut, (a) what is the force of the torque being applied? (b) If the string slips down to a point halfway along the handle of the wrench, what is the new torque?

5. (Answer, 6 points)
(a)

$$\tau = r\,F\sin\theta$$
$$\tau = (0.24\,m)\,(50\,N)\sin(30°)$$
$$\tau = 6\,N\cdot m$$

(b)

$$\tau = r\,F\sin\theta$$
$$\tau = (0.12\,m)\,(50\,N)\sin(30°)$$
$$\tau = 3\,N\cdot m$$

5. (6) A torque of sixty newton-meters produces a counterclockwise rotation of a wheel about its axle. A frictional torque of ten newton-meters acts against this rotation. (a) What is the net torque? (b) If the wheel accelerates at 2 radians-per-second-squared, what is the moment of inertia of the wheel?

5. (Answer, 7 points)

(a)

First of all, there's a net angular acceleration, which means that the system is rotating. Thus, there is a net torque on the wheel.

$$\sum \tau = \tau_{net}$$
$$\tau_{net} = \tau_{CCW} - t_f$$
$$\tau_{net} = 60\,N\cdot m - 10\,N\cdot m = 50\,N\cdot m$$

(b)

$$\tau_{net} = I\,\alpha_{net}$$
$$50\,N\cdot m = I\,(2\,rad/s^2)$$
$$I = 25\,kg\cdot m^2$$

6. (6) A mass of eight hundred grams is located at the end of a very light—almost massless—and rigid rod fifty centimeters long. The rod rotates about an axis at its opposite end with a rotational velocity of 3 radians-per-second. (a) What is the rotational inertia of the system? (The formula for the moment of inertia for a mass rotating on the end of a massless, rigid rod is $I_{point} = mr^2$) (b) What is the angular momentum of the system?

6. (Answer, 6 points)

(a)

$$I_{point\ mass} = mr^2$$
$$I_{point\ mass} = (0.8\,kg)(0.5\,m)^2$$
$$I_{point\ mass} = 0.2\,kg\cdot m^2$$

(b)

$$L = I\omega$$
$$L = (0.2\,kg\cdot m^2)(3\,rad/s)$$
$$L = 0.6\,\frac{kg\cdot m^2}{s}$$

6. (43) A fifty-four-kilogram student, sitting on a stool rotation at a rate of twenty rotations per minute holds 2.2-pound weights in her hands. When her arms are extended—a full two meters from hand to hand—the total rotational inertia of the system is 4.5 kilogram-meters-squared. She pulls her arms in close to her body—holding her arms twenty centimeters form the center of her sternum—reducing the total rotational inertia to 1.5 kilogram-meters-squared. (a) Using the parallel-axis theorem, determine the moment of inertia of the system (treat the student and weights as a dumbbell: a massive, rigid rod, rotating about its center, with two masses on the ends of the rod; the moment of inertia for a rod rotating about its center is $I_{rod,center} = \frac{1}{12}ml^2$.) (b) What is the angular momentum of the system before the student pulls her hands in? (c) If there are no external torques, what is the new rotational velocity of the system (please convert your answer into rotations per minute)? (d) As we saw in class, this model is not that accurate. How would you make the model more accurate? (e) Show that the change in angular momentum of the system is zero. (f) How much work does the student do when she pulls her hands in? (g) What is the force that she must overcome to do this work?

6. (Answer, 43 points)

(a)

First, let's calculate the moments of inertia of two point masses in the student's hands, before she pulls the weights close.

$$I_{point\ mass}^{before} = mr^2$$
$$I_{point\ mass}^{before} = (1\,kg)(1\,m)^2$$
$$I_{point\ mass}^{before} = 1\,kg\cdot m^2$$

This values is the same for both dumbbells. Then, we need to know the moment of inertia for her rotating arms.

$$I_{rod,center}^{before} = \frac{1}{12}ml^2$$
$$I_{rod,center}^{before} = \frac{1}{12}(54\,kg)(2\,m)^2$$
$$I_{rod,center}^{before} = 18\,kg\cdot m^2$$

Now, we use the parallel axis theorem and add these three moments together, since they're all rotating around a common axis.

$$I_{total}^{before} = 2I_{point\ mass}^{before} + I_{rod,center}^{before}$$
$$I_{total}^{before} = 2(1\ kg \cdot m^2) + (18\ kg \cdot m^2)$$
$$I_{total}^{before} = 20\ kg \cdot m^2$$

(b)

$$L_{before} = I_{before}\omega_{before}$$

We must first convert the angular speed in rotations per minute to radians per second, first.

$$\omega_{before} = 20\frac{rotations}{minute} \times \frac{6.28\ radians}{1\ rotation} \times \frac{1\ minute}{60\ seconds} = 2\ rad/s$$

Now, fill in the formula:

$$L_{before} = (20\ kg \cdot m^2)(2\ rad/s)$$
$$L_{before} = 40\ \frac{kg \cdot m^2}{s}$$

(c)

When there are no net torques, conservation of angular momentum makes angular momentum a constant.

$$L_{before} = L_{after}$$
$$I_{before}\omega_{before} = I_{after}\omega_{after}$$

We calculated the value of the angular momentum when the student had her arms extended in part (b), so we can fill in our equation a little bit.

$$40\ \frac{kg \cdot m^2}{s} = I_{after}\omega_{after}$$

So, this relation makes it clear that we need to first calculate the moment of inertia after the student pulls her hands in, then solve for our new angular speed. First, let's calculate the moments of inertia of two point masses in the student's hands, after she pulls the weights close.

$$I_{point\ mass}^{after} = mr^2$$
$$I_{point\ mass}^{after} = (1\ kg)(0.2\ m)^2$$
$$I_{point\ mass}^{after} = 0.04\ kg \cdot m^2$$

This values is the same for both dumbbells. Then, we need to know the moment of inertia for her rotating arms.

$$I_{rod,center}^{after} = \frac{1}{12}ml^2$$
$$I_{rod,center}^{after} = \frac{1}{12}(54\ kg)(0.4\ m)^2$$
$$I_{rod,center}^{after} = 0.72\ kg \cdot m^2$$

Now, we use the parallel axis theorem and add these three moments together, since they're all rotating around a common axis.

$$I_{total}^{after} = 2I_{point\ mass}^{after} + I_{rod,center}^{after}$$
$$I_{total}^{after} = 2(0.04\ kg \cdot m^2) + (0.72\ kg \cdot m^2)$$
$$I_{total}^{after} = 0.8\ kg \cdot m^2$$

Plug this result into our conservation of angular momentum equation.

$$40\ \frac{kg \cdot m^2}{s} = I_{after}\omega_{after}$$
$$40\ \frac{kg \cdot m^2}{s} = (0.8\ kg \cdot m^2)\omega_{after}$$

Now, solve for the angular speed in radians per second.

$$\omega_{after} = 50\ rad/s$$

Convert back to rotations per minute.

$$\omega_{after} = 50\frac{rad}{s} \times \frac{1\ rotation}{6.28\ radians} \times \frac{60\ seconds}{1\ minute} = 477.7\ rotations\ per\ minute$$

(d)

One way to improve this model is to change our moment of inertia. Instead of a dumbbell, we can use a dumbbell with a central cylinder (which closer mimics a person with outstretched arms), estimating the weight of the arms, putting the mass of the student's trunk in the cylinder. For this, our new moment of inertia before pulling the arms in would be:

$$I_{total}^{before} = I_{point} + I_{arm} + I_{trunk} + I_{arm} + I_{point}$$

whereas the moment of inertia afterwards could be:

$$I_{total}^{after} = I_{point} + I_{trunk} + I_{point}$$

For the central cylinder, the formula for moment of inertia is

$$I_{cylindar}^{central\ axis} = \frac{1}{2}mr^2$$

(e)

$$L_{after} = I_{after}\omega_{after}$$
$$L_{after} = (0.8\ kg \cdot m^2)(50\ rad/s)$$
$$L_{after} = 40\ \frac{kg \cdot m^2}{s}$$

$$L_{before} - L_{after} = 0\ \frac{kg \cdot m^2}{s}$$

(f)

In the reference frame of the room—that is to say, for an observer looking on the spinning student—let's calculate the rotational kinetic energy before and after the student pulls her hands in.

$$KE_{before} = \frac{1}{2} I_{before} \omega_{before}{}^2$$

$$KE_{before} = \frac{1}{2}(20\ kg \cdot m^2)(2\ rad/s)^2$$

$$KE_{before} = 40\ J$$

$$KE_{after} = \frac{1}{2} I_{after} \omega_{after}{}^2$$

$$KE_{after} = \frac{1}{2}(0.8\ kg \cdot m^2)(50\ rad/s)^2$$

$$KE_{after} = 1{,}000\ J$$

$$W = \Delta KE = 960\ J$$

(g)

She must overcome the centrifugal force that keeps her hands apart.