Physics 2002
Test 2
Thursday, October 16, 2017
53 points
9:05 a.m. — 10:20 a.m.

In the sea-surf edge,
mingling with bright, small shells,
bush-clover petals.
Bashō (1644 – 1694)

1.(15) Blood courses through your aorta at about 0.33 meters per second. The aorta is roughly 18 millimeters in diameter, and the blood flows into capillaries 8 micrometers in diameter. The density of whole blood is about 1,060 kilograms per cubic meter.
(a) What's the flux in the aorta?
(b) What is the speed of blood through a capillary bed comprising about 5,000,000,000 capillaries?

2. (15) A uniformly dense, four-meter-long plank weighing 80 newtons is placed along the edge of a ship, with one meter of its length extended over the water. A one-hundred-fifty-newton child walks out, slowly.
(a) Calculate the three torques acting on the plank. (*Hint*: *the two from the plank will result in a numerical answer; the one from the boy will result in a radius-dependent relation*.)
(b) How far may the child walk before the plank starts to rotate? (*Hint*: *use Newton's Second Law for torque*.)

3. (23) The International Space Station orbits the Earth at a height 409 kilometers above the surface, and its mass is 419,455 kilograms. Its linear velocity is 7,665 meters per second. The Earth's radius is 6,371 kilometers, and its mass is 5.972 septillion kilograms.
(a) What is the kinetic energy of the station?
(b) What is the gravitational potential energy?
(c) What is the total energy in this stable orbit?
(d) If the orbit height doubles, what is the station's new velocity? (*Hint*: *use conservation of energy*.)

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58 points
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Bashō

1.(15) Blood courses through your aorta at about 0.33 meters per second. The aorta is roughly 18 millimeters in diameter, and the blood flows into capillaries 8 micrometers in diameter. The density of whole blood is about 1,060 kilograms per cubic meter.

(a) What's the flux in the aorta?

(b) What is the speed of blood through a capillary bed comprising about 5,000,000,000 capillaries?

1. (Answer, 15 points)

(a) We'll need the cross-sectional area of the interior of the aorta. Then we can use the mass-flux equation.

$$A_{aorta} = \pi {r_{aorta}}^2 = \pi \left(\frac{d_{aorta}}{2}\right)^2$$

$$A_{aorta} = \pi \left(\frac{18 \times 10^{-3}\ m}{2}\right)^2$$

$$A_{aorta} = 0.000254469\ m^2$$

$$J_{aorta} = \rho_{blood} A_{aorta} v_{aorta}$$

$$J_{aorta} = (1{,}060\ \ kg/m^3)(0.000254469\ m^2)(0.33\ \ m/s)$$

$$\boldsymbol{J_{aorta} = 0.089\ \ m^3/s}$$

(b) Start with the area of a capillary and work up to a capillary bed.

$$A_{capillary} = \pi {r_{capillary}}^2 = \pi \left(\frac{d_{capillary}}{2}\right)^2$$

$$A_{capillary} = \pi \left(\frac{8 \times 10^{-6}\ m}{2}\right)^2$$

$$A_{capillary} = 5.027 \times 10^{-11}\ m^2$$

$$A_{bed} = 5{,}000{,}000{,}000 \times A_{capillary}$$

$$A_{bed} = 0.251\ m^2$$

Now, use the continuity equation.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$J_{aorta} = \rho_{blood} A_{bed} v_{bed}$$

$$0.089\ \ m^3/s\ = (1{,}060\ \ kg/m^3)(0.251\ m^2)v_{bed}$$

$$\boldsymbol{v_{bed} = 3 \times 10^{-4}\ m/s = 0.003\ cm/s}$$

2. (15) A uniformly dense, four-meter-long plank weighing 80 newtons is placed along the edge of a ship, with one meter of its length extended over the water. A one-hundred-fifty-newton child walks out, slowly.

(a) Calculate the three torques acting on the plank. (*Hint: the two from the plank will result in a numerical answer; the one from the boy will result in a radius-dependent relation*.)

(b) How far may the child walk before the plank starts to rotate? (*Hint*: *use Newton's Second Law for torque*.)

2. (Answer, 15 points)

(a)

Taking counterclockwise to be positive:

$$\tau = r\ F \sin\theta$$

$$\tau_{plank} = (1.5\ m)\ (60N) \sin(90°)$$

$$\boldsymbol{\tau_{plank} = 90\ N \cdot m}$$

$$\tau_{extension} = (0.5\ m)\ (20N) \sin(90°)$$

$$\boldsymbol{\tau_{extension} = 10\ N \cdot m}$$

$$\tau_{child}(r) = r\ (150N) \sin(90°)$$

$$\boldsymbol{\tau_{child}(r) = (150\ N) \times r}$$

(b) Use Newton's Second Law for torques.

$$\sum \tau = 0\ N \cdot m$$

$$\tau_{plank} - (\tau_{extension} + \tau_{child}) = 0\ N \cdot m$$

$$90\ N \cdot m - (10\ N \cdot m + (150\ N) \times r) = 0\ N \cdot m$$

$$\boldsymbol{r = 0.53\ m}$$

(The solution for question 3 appears on the next page.)

3. (23) The International Space Station orbits the Earth at a height 409 kilometers above the surface, and its mass is 419,455 kilograms. Its linear velocity is 7,665 meters per second. The Earth's radius is 6,371 kilometers, and its mass is 5.972 septillion kilograms.

(a) What is the kinetic energy of the station?

(b) What is the gravitational potential energy?

(c) What is the total energy in this stable orbit?

(d) If the orbit height doubles, what is the station's new velocity? (*Hint*: *use conservation of energy*.)

3. (23 points)

(a)

$$KE = \frac{1}{2}mv^2$$

$$KE_{ISS} = \frac{1}{2}m_{ISS}{v_{ISS}}^2$$

$$KE_{ISS} = \frac{1}{2}(419{,}455\ kg)(7{,}665\ m/s)^2$$

$$\boldsymbol{KE_{ISS} = 1.232 \times 10^{13} J}$$

(b)

$$PE_G = -\frac{Gm_1m_2}{r}$$

$$PE_{G_{ISS}} = -\frac{Gm_{Earth}m_{ISS}}{r}$$

$$PE_{G_{ISS}} = -\frac{\left(6.67 \times 10^{-11}\frac{m^3}{kg^2 \cdot s}\right)(5.972 \times 10^{24} kg)(419{,}455\ kg)}{(6{,}371{,}000\ m + 409{,}000\ m)}$$

$$\boldsymbol{PE_{G_{ISS}} = -2.464 \times 10^{13} J}$$

(c) The total energy is the sum of kinetic energy and potential energy.

$$E_T = KE + PE$$

$$\boldsymbol{E_{T_{ISS}} = -1.232 \times 10^{13} J}$$

(d) The total energy is constant, so the kinetic and potential energy constituents of the total energy must change. Since we're changing the radius, let's look at gravitational potential energy first. Remember that doubling the orbit height does not mean doubling the total radius between the center of the Earth and the station.

$$PE_G{}^{new} = -\frac{Gm_1m_2}{r^{new}}$$

$$PE_G{}^{new} = -\frac{\left(6.67 \times 10^{-11}\frac{m^3}{kg^2 \cdot s}\right)(5.972 \times 10^{24} kg)(419{,}455\ kg)}{(6{,}371{,}000\ m + 2 \times 409{,}000\ m)}$$

$$PE_G{}^{new} = -2.324 \times 10^{13}\ J$$

This result means that the kinetic energy must be the difference between the total energy and this new result.

$$E_T = KE + PE$$

$$E_{T_{ISS}} = KE_{ISS}{}^{new} + PE_{G_{ISS}}{}^{new}$$

$$E_{T_{ISS}} - PE_{G_{ISS}}{}^{new} = KE_{ISS}{}^{new}$$

$$-1.232 \times 10^{13} J - (-2.324 \times 10^{13} J) = KE_{ISS}{}^{new}$$

$$KE_{ISS}{}^{new} = 1.092 \times 10^{13} J$$

So, the new velocity is:

$$KE = \frac{1}{2}mv^2$$

$$KE_{ISS}{}^{new} = \frac{1}{2}m_{ISS}{v_{ISS_{new}}}^2$$

$$v_{ISS_{new}} = \sqrt{\frac{2KE_{ISS}{}^{new}}{m_{ISS}}}$$

$$v_{ISS_{new}} = \sqrt{\frac{2 \times 1.092 \times 10^{13} J}{419{,}455\ kg}}$$

$$\boldsymbol{v_{ISS_{new}} = 7{,}215.8\ m/s}$$