Homework Sheet 9: Supplemental

5. (26) A buoy is attached to the ocean floor with a rusty spring; the buoy is a green can (right cylinder), has a mass of 785 kg, is 2 m high, and has a diameter of 1 m. The depth of the water at low tide is 11 m. At high tide, the depth is 15 m.

(a) (1) What volume of water does the buoy displace?

(2) How deeply will the buoy float?

(3) What is the equilibrium position of the spring (length at low tide)?

(b) If the rusty spring does not break when high-tide comes, and if the buoy displaces 75% of its total volume, what is the spring constant?

(c) A spring tide is an exceptionally high tide, and it makes a depth near the buoy of 17 m. If the rust has made the spring so brittle that it will break if a force greater than 2,000 N is applied to it, and if the buoy displaces its whole volume, will the buoy break free during the next spring tide?

5. (Answer, 26 points)

(a) We can find the buoy's buoyant force using Newton's Second Law, keeping in mind that at low tide, the spring is, on average, at its equilibrium position.

(a.1)

$$\sum F_y = 0\ N$$
$$F_b - F_g - F_s = 0\ N$$
$$\rho g V - mg - ky = 0\ N$$
$$(1{,}000\ kg/m^3)(9.81\ m/s^2)V - (785\ kg)(9.81\ m/s^2) - k(0\ m) = 0\ N$$
$$V = 0.785\ m^3$$

(a.2)

$$V_{cylinder} = \pi r^2 h$$
$$0.785\ m^3 = \pi(0.5\ m)^2 h$$
$$h = 1\ m$$

(a.3)

The spring's equilibrium length is the height of the surface of the water minus the depth to which the buoy sinks.

$$y_{equilib} = y_{surface} - y_{buoy} = 11\ m - 1\ m = 10\ m$$

(b)

Here, again, we have a stationary system, but, now, there is an additional force: the spring force is no longer zero.

$$\sum F_y = 0\ N$$
$$F_b + F_g + F_s = 0\ N$$

Three quarters of the volume of the can buoy is 1.178 m^3. Remember that the volume displaced is negative.

$$\rho g V + mg - ky = 0\ N$$
$$(1{,}000\ kg/m^3)(-9.807\ m/s^2)(-1.178\ m^3) + (785\ kg)(-9.807\ m/s^2) - k|(15\ m - 1.5\ m) - 10\ m| = 0\ N$$
$$11{,}552.646\ N - 7698.495\ N - k(3.5\ m) = 0\ N$$
$$k = \frac{3{,}854.15\ N}{3.5\ m}$$
$$k = 1{,}101.186\ N/m$$

(c) Let's look to the net force acting on the spring during a spring tide. Note that the whole volume is displaced—let's start there:

$$V_{cylinder} = \pi r^2 h$$
$$V_{cylinder} = \pi(0.5\ m)^2(2\ m)$$
$$V_{cylinder} = 1.571\ m^3$$

Now, use Newton's Second Law with a net force.

$$\sum F_y = F_{net}$$
$$F_b + F_g + F_s = F_{net}$$
$$\rho g V + mg - ky = F_{net}$$
$$(1{,}000\ kg/m^3)(-9.807\ m/s^2)(-1.571\ m^3) - (785\ kg)(-9.807\ m/s^2) - (1{,}101.186\ N/m)|(17\ m - 2\ m) - 10\ m| = F_{net}$$
$$15{,}406.797\ N - 7698.495\ N - 5505.93 N = F_{net}$$
$$F_{net} = -2{,}202.372\ N$$

The buoy will break free if the magnitude of the net force is greater than 1,500 N:

$$|F_{net}| = |-2{,}202.372\ N| > 2{,}000\ N$$

So the buoy will break free.