Physics 2002L, Spring 2018
Problem Set 8
2 problems; 35 points; estimated time: 0.5 hrs.

1. (18) You're driving down a highway, and you decide to accelerate. Flaunting the speed limit, you accelerate way up there. At one point, you're going (a) 100 km/hr. Later, you're going (b) 0.25 times the speed of light. For each case, how much is your car's length contracted, and how slow does time go for you? Your car is 15 feet long at rest (4.5 meters), and take $t_0$ to be 1 second.

1. (Answer, 18 points)
(a) For v=100 km/hr:

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$l = (4.5\,m)\sqrt{1-\frac{(100\times10^3\ m/s)^2}{(3\times10^8\ m/s)^2}}$$

$$l = (4.5\,m)\sqrt{1-\frac{10000\times10^6\ m^2/s^2}{9\times10^{16}\ m^2/s^2}}$$

$$l = (4.5\,m)(0.9999999444)$$

$$\boldsymbol{l = 4.49999975}$$

(a/k/a: length of the 15-ft. car contracts by about 0.0001%)

$$t = t_0\gamma = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{(1\,s)}{\sqrt{1-\frac{(100\times10^3\ m/s)^2}{(3\times10^8\ m/s)^2}}}$$

$$t = \frac{(1\,s)}{(0.9999999444)}$$

$$\boldsymbol{t = 1.000000056\,s}$$

(a/k/a, for every one second that passes for people standing on the street, just a split-hair over 1 second passes for you.)
(b) For v=0.25c:

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$l = (4.5\,m)\sqrt{1-\frac{\left(\frac{c}{4}\right)^2}{(c)^2}}$$

$$l = (4.5\,m)\sqrt{1-\frac{1}{16}}$$

$$l = (4.5\,m)(0.9682458366)$$

$$\boldsymbol{l = 4.357\,m}$$

(a/k/a: length of the 15-ft. car contracts by 15 cm)

$$t = t_0\gamma = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{(1\,s)}{\sqrt{1-\frac{1}{16}}}$$

$$t = \frac{(1\,s)}{(0.9682458366)}$$

$$\boldsymbol{t = 1.0328\,s}$$

(As one second passes for bystanders, 1.0328 seconds pass for you.)

2. (17) How much heat must be added to 120 grams of water at an initial temperature of 60 degrees Celsius to (a) heat the water to its boiling point, and (b) completely convert the boiling water to steam? (c) Convert the total energy you've added to joules. The specific heat capacity of water is 1 calorie per gram per degree Celsius; the latent heat of vaporization of water is 540 calories per gram. (d) For comparison, how fast would a 1-kilogram mass have to go to achieve this energy? (e) For further comparison, how far would you have to push a 25-kilogram armchair with a force of 100 newtons to have done the same amount of work?

2. (Answer, 17 points)
(a) Use the heat equation, leave everything in grams:

$$Q = mc\Delta T$$

$$Q = mc\left(T_f - T_i\right)$$

$$Q = (120\ g)\left(1\ \frac{cal}{g \cdot {}^\circ\mathrm{C}}\right)[100^\circ\mathrm{C} - 60^\circ\mathrm{C}]$$

$$\boldsymbol{Q = 4,800\ calories} = 4.8\ Calories$$

(b) Use the latent-heat equation:

$$Q = mL_v$$

$$Q = (120g)(540\ \ cal/g)$$

$$\boldsymbol{Q = 64,800\ calories} = 64.8\ Calories$$

(c) The total heat you must add is:

$$Q = Q_{change}^{temperature} + Q_{change}^{phase}$$

$$Q = 4{,}800\ calories + 64{,}800\ calories$$

$$Q = 69{,}600\ calories$$

$$Q = 69{,}600\ calories \times \left(\frac{4{,}184\ J}{1\ Cal.}\right) \times \left(\frac{1\ Cal.}{1{,}000\ cal}\right)$$

$$= \boldsymbol{291,206.4\ J}$$

(d) Use the kinetic energy formula:

$$KE = \frac{1}{2}mv^2$$

$$291{,}206.4\ J = \frac{1}{2}(1\ kg)v^2$$

$$\boldsymbol{v = 763.16\ \ m/s} = 1{,}679\ mph$$

(e) Use the work formula:

$$W = Fd\cos\theta$$

$$291{,}206.4\ J = (100\ N)d(1)$$

$$\boldsymbol{d = 2,913.06\ m} = 2.913\ km \approx 1.8\ mi.$$