Physics 2002, Spring 2018
Problem Set 1
6 problems; 64 points; estimated time: 1.5 hrs.

1. (1) Convert knots to meters-per-second.

1. (1 point) A knot (*kn*) is one nautical mile (*M*) per hour, and the relevant conversion from nautical miles to meters is that one nautical mile is 1,852 meters.

$$1\,kn = 1\ M/hr. = 1{,}852\ m/hr. = 0.5147\ m/s$$

2. (5) Grass clippings are found to have an average length of 4.8 cm when a lawn mowed 12 days after the previous mowing; what is the average speed of growth of this grass in (a) cm/day, (b) miles-per-hour; (c) furlongs-per-fortnight?

2. (5 points) (a) Get the average speed.

$$v = \frac{d}{t}$$
$$v = \frac{4.8\ cm}{12\ days}$$
$$v = 0.4\ cm/day$$

(b) Convert.

$$0.4\frac{cm}{day}\times\frac{1\,in.}{2.54\ cm}\times\frac{1\,ft.}{12\ in.}\times\frac{1\,mi.}{5{,}280\ ft.}\times\frac{1\,day}{24\ hr.}$$
$$= 0.0000001\,mph$$

(c) Convert.

$$0.4\ \frac{cm}{day}\times\frac{1\ in.}{2.54\ cm}\times\frac{1\,ft.}{12\ in.}\times\frac{1\,yd.}{3\,ft.}\times\frac{1\,fur.}{220\ yd.}$$
$$\times\frac{14\ days}{1\,ftn.}$$
$$= 0.0003\ fur./ftn.$$

3. (4) If a world-class sprinter ran a distance of 100 meters, running at a constant rate of 11 m/s, how long did it take?

3. (4 points)

$$d = vt$$
$$t = \frac{d}{v}$$
$$t = \frac{100\ m}{11\ m/s}$$
$$t = 9.09\ s$$

4. (32) A the velocity of an object can be recorded as a series of ordered pairs to be graphed (pairs consist of *(time, velocity)* values): (0,0), (1,2), (2,2), (3,1), (4,3), and (5,0). Calculate (a) the acceleration between each two points; (b) the average acceleration; and (c) the distance traveled overall (for part (c), please use geometry instead of kinematics formulas).

4. (Answer, 32 points)

(a) Acceleration is the slope of a velocity curve.

Between (0,0) and (1,2):

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{2\ m/s - 0\ m/s}{1\,s - 0s}$$
$$a = 2\ m/s^2$$

Between (1,2) and (2,2):

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{2\ m/s - 2\ m/s}{2\,s - 1s}$$
$$a = 0\ m/s^2$$

Between (2,2) and (3,1):

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{1\ m/s - 2\ m/s}{3\,s - 2s}$$
$$a = -1\ m/s^2$$

Between (3,1) and (4,3):

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{3\ m/s - 1\ m/s}{4\,s - 3s}$$
$$a = 2\ m/s^2$$

Between (4,3) and (5,0):

$$a = \frac{\Delta v}{\Delta t}$$
$$a = \frac{0\ m/s - 3\ m/s}{5\,s - 4s}$$
$$a = -3\ m/s^2$$

(b) Average the four accelerations:

$$\bar{a} = (2 + 0 - 1 + 2 - 3)\ m/s^2 = 0\ m/s^2$$

(c) The distance is the area under the velocity curve, so we use squares, trapezoids, and triangles to calculate it geometrically:

Between (0,0) and (1,2), we have a right triangle:

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(1\,s)(2\ m/s)$$
$$A = 1\,m$$
Between (1,2) and (2,2), we have a rectangle:
$$A = lw$$
$$A = (1\,s)(2\ m/s)$$
$$A = 2\,m$$
Between (2,2) and (3,1), we have a trapezoid (take the time-value as the height, since the velocities are changing):
$$A = \frac{1}{2}(a+b)h$$
$$A = \frac{1}{2}(2\ m/s + 1\ m/s)(1\,s)$$
$$A = 1.5\,m$$
Between (3,1) and (4,3), we have a trapezoid (take the time-value as the height, since the velocities are changing):
$$A = \frac{1}{2}(a+b)h$$
$$A = \frac{1}{2}(1\ m/s + 3\ m/s)(1\,s)$$
$$A = 2\,m$$
Between (4,3) and (5,0), we have a right triangle:
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(1\,s)(3\ m/s)$$
$$A = 1.5\,m$$
The total area is the sum, giving us the total distance:
$$d = A_{total} = 8\,m$$

5. (3) Aristotle concluded from his observations 2,350 years ago that heavier objects fall faster than lighter objects. Was he correct? Why or why not? Is there any way he could be considered correct or incorrect, depending on your choice.

5. (3 points) Although his observations were reasonable, they were not correct when we consider acceleration. Too many variables went into what he observed. We know, now (and have, since Galileo observed the descent of spheres of varying masses down ramps), that all objects fall at the same acceleration: every object on earth falls at an acceleration of – 9.8 m/s².

6. (14) A Roman catapult aimed at Vercingetorix's stronghold can launch a 50-pound boulder at a velocity of 50 Roman miles per hour at an angle of 45 degrees. How far may Julius Caesar station his catapult from the stronghold in order to hit the base of the wall (the corner the wall makes with the ground) (express your answer in both meters and pedes)? Note that 1 Roman mile is 5,000 pedes; 1 pes is 296 millimeters.

6. (14 points) First, convert initial velocity into meters per second.
$$50\,\frac{Roman\ mile}{hr} \times \frac{5000\ pedes}{1\ Roman\ mile} \times \frac{296\ mm}{1\ pes} \times \frac{1\ m}{1000\ mm} = 74{,}000\ m/hr$$
$$74{,}000\ \frac{m}{hr} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ s} = 20.6\ m/s$$
Split into two pictures, looking at the vertical first.
$$v_i \to \begin{cases} v_{i_x} = v_i\cos\theta = (20.6\ m/s)\cos(45°) = 14.6\ m/s \\ v_{i_y} = v_i\sin\theta = (20.6\ m/s)\sin(45°) = 14.6\ m/s \end{cases}$$
Vertically, we have the acceleration because of gravity, and we know the velocity at the maximum height is zero:
$$v_f = v_i + at$$
$$v_{f_y} = v_{i_y} + a_y t_{\max height}$$
$$0\ m/s = 14.6\ m/s + (-9.81\ m/s^2)t_{\max height}$$
$$t_{\max height} = 1.49s$$
This value is half of the boulder's travel time, so we need to double it.
$$t_{total} = 2.98\ s$$
Now, turn to the horizontal picture, since we know the time the boulder will fly.
$$d = v_i t + \frac{1}{2}at^2$$
$$d_x = v_{i_x}t + \frac{1}{2}a_x t^2$$
$$d_x = (14.6\ m/s)(2.98\ s) + \frac{1}{2}(0\ m/s^2)(2.98\ s)^2$$
$$d_x = 43.5\ m$$
Convert back to pedes:
$$43.5\ m \times \frac{1{,}000 mm}{1\ m} \times \frac{1\ pes}{296\ mm} = 147\ pedes$$

7. (7) An old house in Paris, covered in vines, where live twelve children who like to queue up, has windows on the first floor that are 2 meters

high. One morning, someone on the second floor knocks a flowerpot off a balcony (take its initial velocity to be 0 meters per second). One of the children living in the house recorded that the flowerpot took exactly 0.25 seconds to pass across the window. How high above the first-floor window is the second-floor balcony?

7. (7 points) For this problem, we can split our analysis into two parts: (1) using the time it takes the flowerpot to pass the first-floor window, we can determine the velocity the flowerpot at the top of the window. Note that the flowerpot travels downward, so we take the height of the window to be negative.

$$d = v_i t + \frac{1}{2} a t^2$$
$$-2\ m = v_i(0.25\ s) + \frac{1}{2}(-9.81\ m/s^2)(0.25\ s)^2$$
$$-2\ m = v_i(0.25\ s) - 1.226\ m$$
$$v_i = -3.10\ m/s$$

Then, (2) using this velocity as a final velocity, we can determine the distance the flowerpot has already fallen—namely, the distance above the first-floor window.

$$v_f{}^2 = v_i{}^2 + 2ad$$
$$(-3.10\ m/s)^2 = (0\ m/s)^2 + 2(-9.81\ m/s^2)d$$
$$d = 0.16\ m = 16\ cm$$