Mission Plan: Neptune Orbiter

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AEE 4263: Space Flight Mechanics

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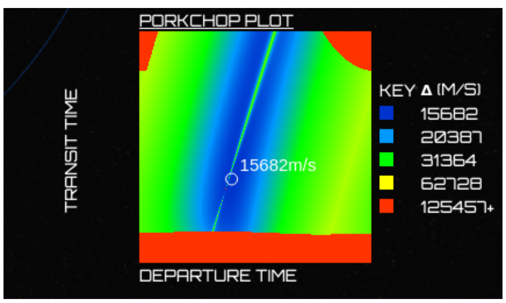
Target Selection and Departure Date

I chose Neptune as a target for some of its intriguing characteristics: it is the outermost planet, invisible to the naked eye, and was only discovered via perturbations to Uranus's orbit. It was only visited once by Voyager 2, which used gravity assists from Jupiter, Saturn, and Uranus before performing a flyby of Neptune twelve years and five days into its mission. My objective for this mission plan is to send a probe directly to Neptune orbit from Earth, pray it survives the 30 year coast phase, and use a scientific payload to study the planet along with its unique atmosphere.

Figure 1. Porkchop plot of Earth→Neptune transfer



To choose a departure date, I did not start with NASA's "Eyes on the Solar System" because it cannot display Earth and Neptune's orbits at the same time. Instead, I used the "Planetary Transfer Calculator" at *transfercalculator.com* to generate a porkchop plot (figure 1) and choose the optimal departure date. A porkchop plot is useful for weighing the Δv cost of a mission versus the phasing time. After obtaining the date (2025-05-14), I checked it against "Eyes on the Solar System" using a vector graphics program (figure 2): $\varphi_0 \approx 127°$ as opposed to the calculated minimum $\varphi_0 = 113.2°$. Because the true phase angle *exceeds* the minimum phase angle by $\approx 14.2°$, I chose to continue calculations departing Earth on May 14 2025. Neptune is nearly still with respect to Earth, resulting in a synodic period of approximately one year ($T_{syn} = 367.48\,\text{day}$).

Figure 2. Relative phase of Earth and Neptune

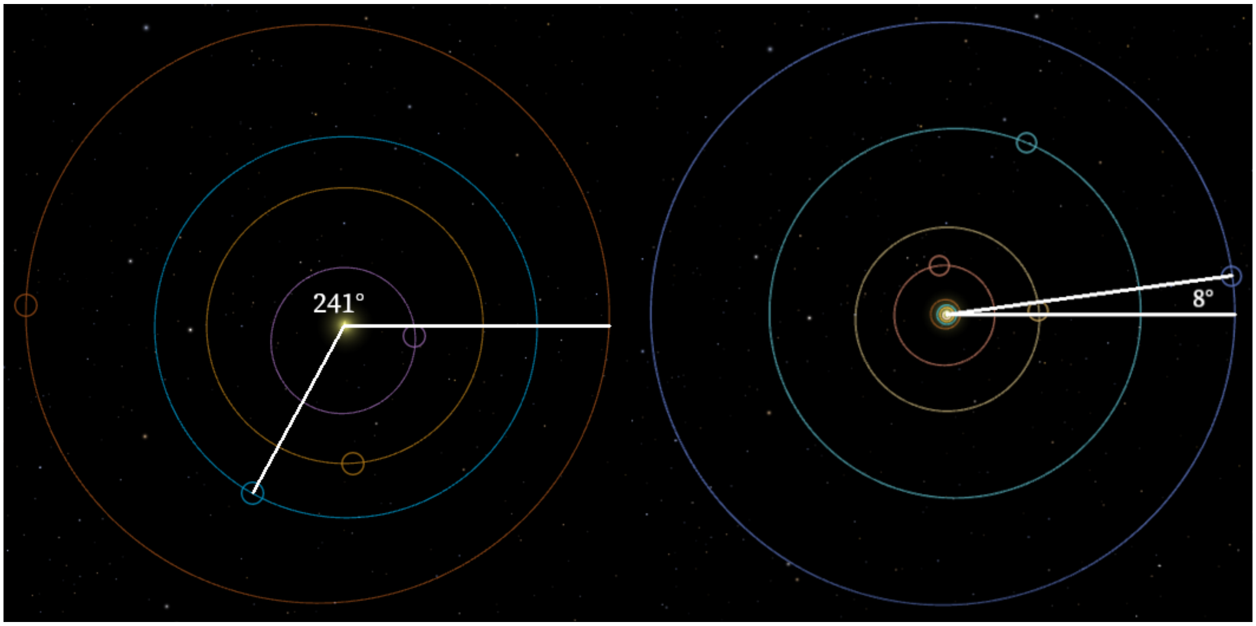


Table 1

Part 1 Deliverables

Variable	Value	Unit
Departure date	2025-05-14	date
Synodic period T_{syn}	367.48	day
φ_0 (Minimum)	113.2°	degree
φ_0 (Actual)	127°	degree
Transfer time t_{12}	30.6	year

Patched Conics

For the method of patched conics, I followed examples 8.4 and 8.5 and checked my departure solution against HW6 problem 8. Specifically, the equations for the circular and hyperbolic velocities, then setting Δv equal to the difference between hyperbolic and parking orbit speeds. The departure makes up the majority of the delta-v cost (7.726 km/s), while the arrival only takes 2.946 km/s. To calculate the elliptical heliocentric orbital elements, I assumed that the perihelion and aphelion are equal to Earth and Neptune's heliocentric semi-major axes respectively.

Table 2

Part 2 Deliverables (Departure)

Variable	Value	Unit
Elliptical semi-major axis a_T	$2.3223 \cdot 10^9$	km
Elliptical eccentricity e_T	0.936	
Elliptical period T_H	61.1	year
Hyperbolic excess speed $v_{\infty,1}$	11.653	km/s
Transfer time t_{12}	30.6	year
Parking orbit speed v_c	7.726	km/s
Hyperbolic delta-v Δv_1	8.248	km/s
Departure perigee angle β_1	72.2°	degree

Table 3

Part 2 Deliverables (Arrival)

Variable	Value	Unit
Hyperbolic excess speed $v_{\infty,2}$	4.055	km/s
Optimal periapsis r_{p2}	447764	km
Optimal apoapsis r_{a2}	831561	km
Parking orbit eccentricity e_2	0.3	
Capture delta-v Δv_2	2.946	km/s
Aiming radius Δ	756859	km
Arrival perigee angle β_2	61.2°	degree

Launch and Constraints

Assuming my Δv calculations are correct, the spacecraft can enter an elliptical orbit around Neptune with a final mass fraction of 0.022, corresponding to 66.8 kg of payload or 1.3 scientific instruments. To launch more mass to Neptune, I added a Delta IV 5 meter DCSS (Delta Cryogenic Second Stage) with the extendable nozzle RL10B-2: among the most efficient chemical engines ($I_{sp} = 462\,\mathrm{s}$) and my personal favorite. With the 15000 kg spacecraft as payload, the DCSS contributes 4.10 km/s of Δv, leaving 7.092 km/s of burns for the spacecraft. The capture burn is identical to the capture burn without a kick stage. This corresponds to a final mass fraction of 0.090, payload of 269 kg, and 5 scientific instruments; a 302% improvement.

Table 4

Part 3 Deliverables

Variable	Value	Value (DCSS)	Unit
$(\Delta m/m)_1$	0.939	0.756	
$(\Delta m/m)_2$	0.633	0.633	
m_f/m_0	0.022	0.090	
Final mass	334	1348	kg
Payload mass	66.9	269	kg
Instruments	1	5	

Code Explanation

I wrote the code in Julia using a Pluto.jl notebook for formatting, making heavy use of Unicode input to shorten variable names and improve readability; however, not all English letters have a Unicode subscript. Unlike the Spacecraft Observation Report, the code is almost entirely comprised of basic expressions.

The program starts by defining some of the bodies' properties: Neptune and Earth's semi-major axes, mean motion, and the Sun's gravitational parameter. Using these variables, the synodic period, transfer time, and minimum phase angle are calculated then printed in a human-readable format.

Next, several patched conic variables are calculated for the departure phase: hyperbolic excess speed, hyperbolic eccentricity, and angular momentum of the hyperbolic orbit. Periapsis speeds for the parking and hyperbolic orbits are calculated, then Δv is set equal to the difference. The departure periapsis angle is calculated from the inverse of hyperbolic eccentricity. Finally, eccentricity, semi-major axis, and period of the elliptical heliocentric orbit are calculated from both planets' semi-major axes.

The arrival phase is similar: the hyperbolic excess speed at Neptune's SOI is calculated, along with the optimal periapsis, apoapsis, and aiming radius for an elliptical orbit with eccentricity $e_2 = 0.3$. The hyperbolic and elliptical periapsis speeds are calculated, then the difference is set equal to the capture burn Δv. The arrival periapsis angle is also calculated with an equation equivalent to the equation for the departure periapsis angle.

To calculate mass ratios and payload, the program starts by defining variables for specific impulse, standard gravity, and launch mass. A propellant mass fraction for each burn is calculated, then both are combined to find the final payload fraction, payload mass, and the number of instruments. For the kick stage, I start by calculating the maximum Δv for a DCSS with 15 tons of payload. That number is subtracted from the first burn, which the spacecraft completes with its lower specific impulse thruster. The capture burn with a kick stage is identical to the capture burn without one.

Verbatim Code (next page)

Mission Plan: Neptune Flyby

AEE 4263: Orbital Mechanics

by Dakota Richline

```
md"""# Mission Plan: Neptune Flyby
AEE 4263: Orbital Mechanics

by Dakota Richline"""
```

```
using Printf
```

1. Set a departure date

Calculate φ_0 and T_{syn} :

```
md"""## 1. Set a departure date

### Calculate $\varphi_0$ and $T_{syn}:$"""
```

```
▸ (1.496e8, 4.495e9, 1.99102e-7, 1.20814e-9, 1.32712e11)
```

```
# Define planetary variables
R₁, R₂, n₁, n₂, μ_sun = 149.6e6, 4.495e9, 2π/(365.25 * 24 * 3600), 2π/(164.8 * 365.25 * 24 * 3600), 132712e6
```

```
t_syn = 3.175025934065934e7
```

```
# Synodic period
t_syn = 2π/abs(n₁ - n₂)
```

```
t₁₂ = 9.650998739274707e8
```

```
# Transfer time (s)
t₁₂ = (π/sqrt(μ_sun)*((R₁+R₂)/2)^(3/2))
```

```
30.58216955432196
```

```
# Transfer time (year)
t₁₂/(365.25*24*3600)
```

```
φ₀ = 1.9756130516153354
```

```
# Initial phase angle
φ₀ = π - n₂*t₁₂
```

```
# Formatted output
@printf "synodic period = %.2f day \nφ₀ = %.4f radian = %.2f°" t_syn/(24 * 3600) φ₀ rad2deg(φ₀)
```

```
synodic period = 367.48 day
φ₀ = 1.9756 radian = 113.19°
```

NOTE: Because Neptune has a very large orbital period (164.8 year), it is nearly still relative to Earth and therefore reaches φ_0 approximately once per year

```
md"""**NOTE:** Because Neptune has a very large orbital period (164.8 year), it is nearly still relative to Earth and therefore reaches φ₀ approximately once per year"""
```

2. Patched Conics

Departure

```
md"""## 2. Patched Conics
### Departure"""
```

```
(6678, 398600)
```

```
# Parking/departure variables
rₚ₁, μ₁ = 6378+300, 398600
```

```
v∞₁ = 11.653207051375738
```

```
# Hyperbolic periapsis speed v∞₁
v∞₁ = sqrt(μ_sun/R₁)*(sqrt(2*R₂/(R₁+R₂))-1)
```

```
e₁ = 3.2750976732066066
```

```
# Hyperbolic Eccentricity e₁
e₁ = 1 + rₚ₁*v∞₁^2/μ₁
```

```
h = 106675.54622078645
```

```
# Transfer orbit angular momentum
h = μ₁*sqrt(e₁^2-1)/v∞₁
```

```
true
```

```
# Double-check h
h == rₚ₁*sqrt(v∞₁^2 + 2*μ₁/rₚ₁)
```

```
vₚ_hyp = 15.974175834199828
```

```
# Hyperbolic periapsis speed
vₚ_hyp = h/rₚ₁
```

```
v_c₁ = 7.725835197559566
```

```julia
# Parking orbit speed
v_c₁ = sqrt(μ₁/rₚ₁)
```

```
Δv₁ = 8.248340636640261
```

```julia
# Departure burn Δv₁
Δv₁ = vₚ_hyp - v_c₁
```

```
β₁ = 72.2217163509277
```

```julia
# Departure periapsis angle β₁
β₁ = acosd(1/e₁)
```

```
true
```

```julia
# Double-check β₁
β₁ == acosd(μ₁/(μ₁ + rₚ₁*v∞₁^2))
```

```
eₜ = 0.935581104939069
```

```julia
# Elliptical eccentricity eₜ
eₜ = (R₂-R₁)/(R₂+R₁)
```

```
aₜ = 2.3223e9
```

```julia
# Elliptical semimajor axis aₜ
aₜ = (R₁+R₂)/2
```

```
Tₕ = 61.16433910864393
```

```julia
# Elliptical orbit period Tₕ
Tₕ = 2π*sqrt(aₜ^3/μ_sun)/(24*3600*365.25)
```

Arrival

```julia
md"""### Arrival"""
```

```
(6835100, 24760, 0.3)
```

```julia
# Define μ, R_neptune, and e=0.3 for capture orbit
μ₂, Rₙ, e₂ = 6835100, 24760, 0.3
```

```
v∞₂ = 4.054528763084226
```

```julia
# Hyperbolic excess speed v∞₂
v∞₂ = sqrt(μ_sun/R₂) * (1-sqrt(2*R₁/(R₁+R₂)))
```

```
rₚ₂ = 447763.5992066942
```

```julia
# Optimal periapsis radius
rₚ₂ = 2μ₂*(1-.3)/(v∞₂^2*(1+0.3))
```

```
rₐ₂ = 831560.9699552894
```

```julia
# Optimal apoapsis radius
rₐ₂ = rₚ₂*(1+e₂)/(1-e₂)
```

```
vₚ_hyp₂ = 6.8534044700795675
```

```julia
# Hyperbolic periapsis speed
vₚ_hyp₂ = sqrt(v∞₂^2 + 2μ₂/rₚ₂)
```

```
vₚ_cap₂ = 3.9070416775246515
```

```julia
# Capture periapsis speed
vₚ_cap₂ = sqrt(μ₂/rₚ₂)
```

```
Δv₂ = 2.946362792554916
```

```julia
# Capture burn Δv₂
Δv₂ = vₚ_hyp₂ - vₚ_cap₂
```

```
Δ = 756858.6219641838
```

```julia
# Aiming radius Δ
Δ = rₚ₂*sqrt(2/(1-e₂))
```

```
β₂ = 61.21779531941862
```

```julia
# Capture periapsis angle β₂
β₂ = acosd(μ₂/(μ₂ + rₚ₂*v∞₂^2))
```

3. The Launch and Constraints

Stock spacecraft

```julia
md"""## 3. The Launch and Constraints
### Stock spacecraft"""
```

```
▸ (300, 9.81, 15000)
```

```julia
# Define Iₛₚ, g₀, and launch mass m₀
Iₛₚ, g₀, m₀ = 300, 9.81, 15000
```

```
Δm₁ = 0.9393537905075505
```

```julia
# 1st burn prop. mass fraction (Δm/m)₁
Δm₁ = 1-exp(-Δv₁*1e3/(Iₛₚ * g₀))
```

```
Δm₂ = 0.6325406729070855
```

```julia
# 2nd burn prop. mass fraction (Δm/m)₂
Δm₂ = 1-exp(-Δv₂*1e3/(Iₛₚ * g₀))
```

```
Δmₗ = 0.02228501533083141
```

```julia
# Final mass fraction m_final/m₀
Δmₗ = (1-Δm₁) * (1-Δm₂)
```

```
mₗ = 66.85504599249423
```

```julia
# Payload mass
mₗ = 0.2 * m₀ * Δmₗ
```

```
instruments = 1.3371009198498844
```

```julia
# Scientific payload
instruments = mₗ/50
```

Add a 5 m DCSS kick stage

(RL10B-2 with the nozzle extension, my favorite engine)

```julia
md"""### Add a 5 m DCSS kick stage
(RL10B-2 with the nozzle extension, my favorite engine)"""
```

```
5.390169586574405
```

```julia
let mₗ=15000, mₛ=3490, m₀=30710+mₗ, Iₛₚ₁ = 462, Iₛₚ₂=300, Δv₁=8.248, Δv₂=2.946
    Δv_DCSS = Iₛₚ₁ * g₀ * log(m₀/(mₗ+mₛ))/1e3
    #  DCSS provides 4.10 km/s of Δv

    Δv₁ -= Δv_DCSS
    # 4.146 km/s of Δv₁ remaining for SV

    Δm₁ = 1-exp(-Δv₁*1e3/(Iₛₚ₂ * g₀))
    # 0.756 prop. mass fraction

    Δm₂ = 1-exp(-Δv₂*1e3/(Iₛₚ₂ * g₀))
    # 0.632 prop. mass fraction

    Δmₗ = (1-Δm₁) * (1-Δm₂)
    # 0.090 mass fraction m_final/m₀

    payload = 0.2*mₗ*Δmₗ
    # 269 kg of science!

    instruments = payload/50
    # 5 instruments
end
```