

# Labo Signaalverwerking

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## Opdracht 2A: Analyse v.e. actieve filtertrap

### Specificatie

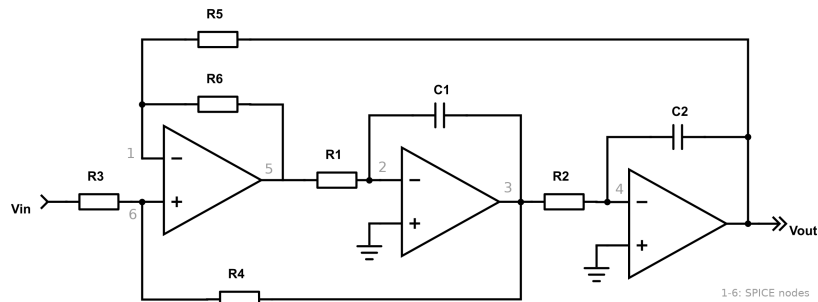


Figure 1: Het schema.

- **Low Pass KHN - Non Inverting** (schema nr 5)
- Filter is een *LDL*
  - $|H(0)| = 6dB$
  - $|H(10kHz)| = -34dB$
  - $Q_p = 4$

### Analyse

#### 1. Bepaal de DC- en HF-weergave

##### DC

Bij DC zijn condensatoren open kring, dus wordt de versterking bepaald door de feedback weerstanden  $R_4$ ,  $R_5$ , en  $R_6$ . Dit is dus een vaste versterking.  $|H(DC)| = A$ .

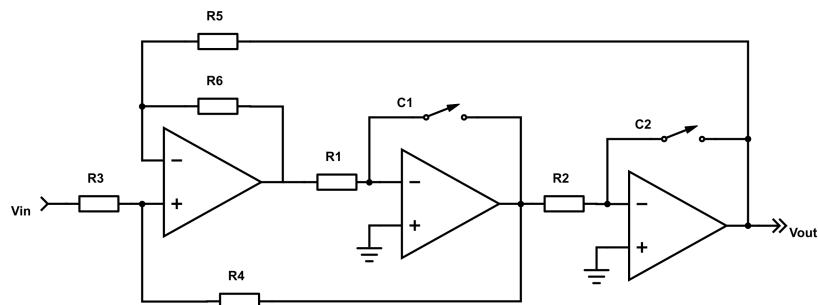


Figure 2: Schema met alle condensatoren open kring.

##### HF

The diagram shows two inverting operational amplifier stages. The first stage has an input voltage  $V_4$  connected to a resistor  $R_1$ , which is connected to the inverting input (marked with a minus sign). The non-inverting input (marked with a plus sign) is grounded. A feedback capacitor  $C_1$  connects the output of the first stage to its inverting input. The output of the first stage is labeled  $V_5$ . The second stage has an input voltage  $V_5$  connected to a resistor  $R_2$ , which is connected to the inverting input (marked with a minus sign). The non-inverting input (marked with a plus sign) is grounded. A feedback capacitor  $C_2$  connects the output of the second stage to its inverting input. The final output of the system is labeled  $V_{out}$ .

2

**Geval 2:**  $v_5, v_{out} = v_{in} = 0$

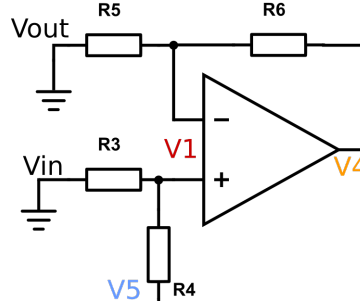


Figure 6: Superpositie schema geval 2

De opamp is nu een niet inverterende versterker.

$$v_4 = v_1 \cdot \left(1 + \frac{R_6}{R_5}\right) \quad v_1 = v_5 \cdot \frac{R_3}{R_3 + R_4} \Rightarrow v_4 = v_5 \cdot \frac{R_3}{R_3 + R_4} \cdot \left(1 + \frac{R_6}{R_5}\right) = v_5 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5}$$

**Geval 3:**  $v_{out}, v_5 = v_{in} = 0$

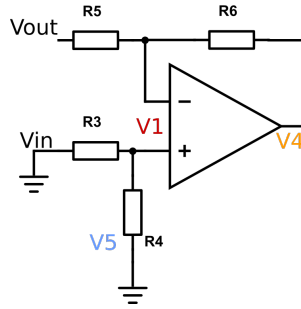


Figure 7: Superpositie schema geval 3

De opamp is nu een inverterende versterker.

$$v_4 = \frac{-R_6}{R_5} \cdot v_{out}$$

**Totaal**

$$v_4 = \sum v_4 = v_{in} \cdot \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + v_5 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + \frac{-R_6}{R_5} \cdot v_{out}$$

$$v_{in} \cdot \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} = -v_5 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + \frac{R_6}{R_5} \cdot v_{out} + v_4$$

Vervang in deze formule  $v_5$  en  $v_4$  door de formules van de twee integrators:

$$v_{in} \cdot \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} = v_{out} \cdot \left(s R_2 C_2 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + \frac{R_6}{R_5} + s^2 R_1 R_2 C_1 C_2 v\right)$$

$$\frac{v_{in}}{v_{out}} \cdot \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} = s^2 R_1 R_2 C_1 C_2 + s R_2 C_2 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + \frac{R_6}{R_5}$$

$$\frac{v_{out}}{v_{in}} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} \cdot \frac{1}{s^2 R_1 R_2 C_1 C_2 + s R_2 C_2 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} + \frac{R_6}{R_5}}$$

$$\frac{v_{out}}{v_{in}} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_5} \cdot \frac{1}{\frac{R_6}{R_5} \cdot \left(s^2 \cdot \frac{R_1 R_2 C_1 C_2 R_5}{R_6} + s R_2 C_2 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_6} + 1\right)}$$

$$\text{Het resultaat: } H(s) = \frac{v_{out}}{v_{in}} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_6} \cdot \frac{1}{s^2 \cdot \frac{R_1 R_2 C_1 C_2 R_5}{R_6} + s R_2 C_2 \cdot \frac{R_3}{R_3 + R_4} \cdot \frac{R_6 + R_5}{R_6} + 1}$$

### 3. Vergelijk transfer functie met de algemene

Algemene vorm LDL filter:  $H(s) = K \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{1}{Q} \left(\frac{s}{\omega_n}\right) + 1}$

- $K = \frac{R_4}{R_3 + R_4} \cdot \frac{R_5 + R_6}{R_6}$

- $\frac{1}{\omega_n^2} = \frac{C_1 C_2 R_1 R_2 R_5}{R_6}$
- $\frac{1}{Q\omega_n} = C_2 R_2 \cdot \frac{R_3}{R_4 + R_3} \cdot \frac{R_5 + R_6}{R_6}$

#### 4. Pole-zero plot

- Geen zeros
- Wel polen, namelijk:

$$\frac{s^2}{\omega_n^2} + \frac{s}{Q\omega_n} + 1 = 0$$

$$\frac{s^2}{(2000\pi)^2} + \frac{s}{4 \cdot 2000\pi} + 1 = 0$$

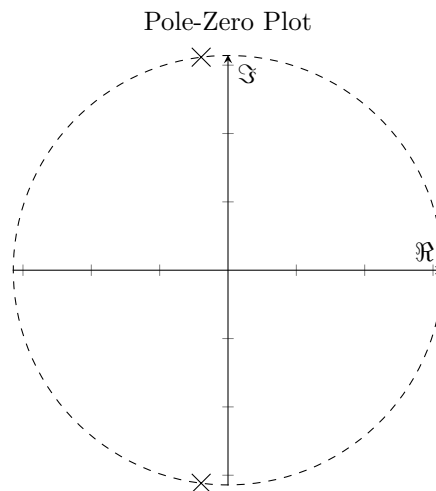
$$\frac{s^2}{(2000\pi)^2} + \frac{s}{8000\pi} + 1 = 0$$

2 complexe polen:

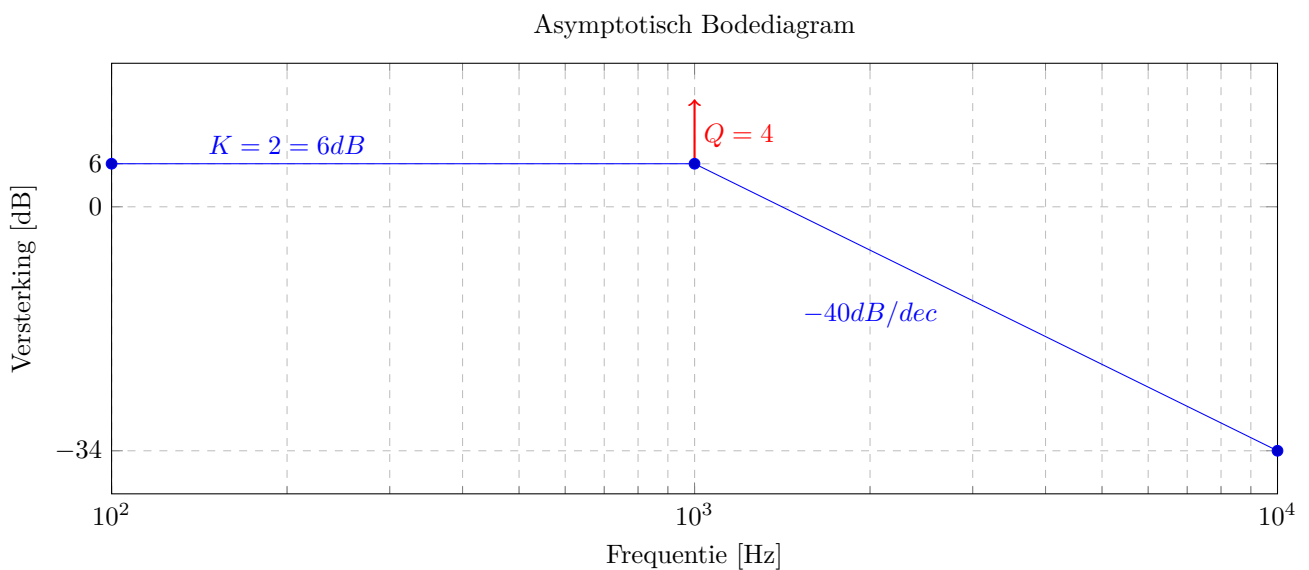
$$250\pi(-1 + 3\sqrt{7}i) \text{ en } 250\pi(-1 - 3\sqrt{7}i)$$

of ongeveer

$$-785 + 6234i = 6283\angle 97^\circ \text{ en } -785 - 6234i = 6283\angle -97^\circ$$



#### 5. Frequentiegedrag



De lijn van  $-40\text{dB/dec}$ , het beginpunt bij  $10\text{kHz}$ ,  $-34\text{dB}$ , en het filtertype (LDF) laat toe  $f_n$  te berekenen. We moeten  $40\text{dB}$  zakken van  $6\text{dB}$  to  $-34\text{dB}$ , dit is dus 1 decade, ofwel  $f_n = 1\text{kHz}$ . Door de dubbele pool is er maar 1 knik in de grafiek, daar gaat de helling van 0 naar  $-40\text{dB/dec}$ .

## 5. Tijdsgedrag

ToDo: Dit heel deel

ToDo: Grafiek

ToDo: Bespreek ligging polen

## Synthese

### Ontwerpvergelijkingen

Kies:  $+ C_2 = c^{te} = 1$  Kies  $C_2$  omdat van  $C_1$  makkelijker een ontwerpvergelijking te vinden is.  $+ R = R_1 = R_2 = R_3 = R_4 = R_6$   $R_5$  variabel omdat die enkel in tellers zit. Dit maakt ontwerpvergelijkingen makkelijker.

De transfer functie wordt dan:

$$H(s) = \frac{R+R_5}{2R} \cdot \frac{1}{s^2 RC_1 C_2 R_5 + s \cdot (R+R_5) \cdot \frac{C_2}{2} + 1}$$

Met de vergelijkingen van uit de transfer functie:  $+ K = \frac{R_4}{R_3+R_4} \cdot \frac{R_5+R_6}{R_6} + \frac{1}{\omega_n^2} = \frac{C_1 C_2 R_1 R_2 R_5}{R_6} + \frac{1}{Q \omega_n} = C_2 R_2 \cdot \frac{R_3}{R_4+R_3} \cdot \frac{R_5+R_6}{R_6}$

Geeft:

- $K = \frac{R}{2R} \cdot \frac{R_5+R}{R} = \frac{R_5+R}{2R} \Rightarrow R_5 + R = 2KR \Rightarrow R_5 = R(2K - 1)$
- $\frac{1}{\omega_n^2} = \frac{C_1 C_2 R^2 R_5}{R} = C_1 C_2 R R_5 \Rightarrow C_1 = \frac{1}{\omega_n^2 C_2 R_5 R}$
- $\frac{1}{Q \omega_n} = C_2 R_2 \cdot \frac{R_3}{R_4+R_3} \cdot \frac{R_5+R_6}{R_6} = C_2 R \cdot \frac{R}{2R} \cdot \frac{R_5+R}{R} = \frac{C_2(R_5+R)}{2} \Rightarrow Q = \frac{2}{\omega_n C_2 2KR} \Rightarrow R = \frac{1}{Q \omega_n C_2 K}$

De ontwerpvergelijkingen:

- $R = \frac{1}{Q \omega_n C_2 K}$
- $R_5 = R(2K - 1)$
- $C_1 = \frac{1}{\omega_n^2 C_2 R_5 R}$

### Impedantieschaling

Waarden zonder impedantieschaling:

- $R = 0.0000198943... \Omega$
- $R_5 = 0.0000596831... \Omega$
- $C_1 = 21.33... F$
- $C_2 = 1 F$

Met schalingsfactor  $10^9$ :

- $R = R * ISF = 19894.36... = 19.89 k\Omega$
- $R_5 = R_5 * ISF = 59683.10... = 59.68 k\Omega$
- $C_1 = \frac{C_1}{ISF} = 0.000000021333... = 21.33 nF$
- $C_2 = \frac{C_2}{ISF} = 0.000000001 = 1 nF$

## Simulatie op basis van de transferfunctie (Matlab)

### Matlab code

```
% Gegevens
fn = 1000 % 1kHz
K = 2 % 6dB
Q = 4

wn = 2*pi*fn

H_N = K * [0 0 1]
H_D = [1/wn^2 1/(Q*wn) 1]
```

```

H = tf(H_N, H_D) % H_N / H_D

% Figuren uit gegevens
figure(1);
hold on;
pzmap(H);
figure(2);
hold on;
bode(H);
figure(3);
hold on;
step(H);

% Ontwerpvergelijkingen
C2 = 1
R=1/(C2*K*Q*wn)
R5=R*(2*K-1)
C1=1/(wn^2*C2*R5*R)

% Impedantieschaling
ISF= 10^9
C1 = C1/ISF
C2 = C2/ISF
R = R*ISF
R5 = R5*ISF

% K, wn, fn, en Q uit componenten
Kc = (R+R5)/(2*R)
wnc = 1/sqrt(C1*C2*R*R5)
fnc = wnc/(2*pi)
Qc = 2/(C2*wn*(R5+R))

% H uit componenten
H_Nc = ((R5+R)/(2*R)) * [0 0 1]
H_Dc = [C1*C2*R*R5 C2*(R5+R)/2 1]
Hc = tf(H_Nc, H_Dc)

% Figuren uit componentwaarden
figure(1);
pzmap(Hc);
figure(2);
bode(Hc);
figure(3);
step(Hc);

Output:

fn = 1000
K = 2
Q = 4
wn = 6.2832e+03
H_N = 0 0 2
H_D = 0.0000 0.0000 1.0000
H =
      2
-----
2.533e-08 s^2 + 3.979e-05 s + 1

Continuous-time transfer function.

C2 = 1
R = 1.9894e-05

```

```
R5 = 5.9683e-05
C1 = 21.3333

ISF = 1.0000e+09
C1 = 2.1333e-08
C2 = 1.0000e-09
R = 1.9894e+04
R5 = 5.9683e+04

Kc = 2
wnc = 6.2832e+03
fnc = 1.0000e+03
Qc = 4

H_Nc = 0      0      2
H_Dc = 0.0000 0.0000 1.0000
Hc =
      2
-----
2.533e-08 s^2 + 3.979e-05 s + 1
```

Continuous-time transfer function.

Pole Zero plot

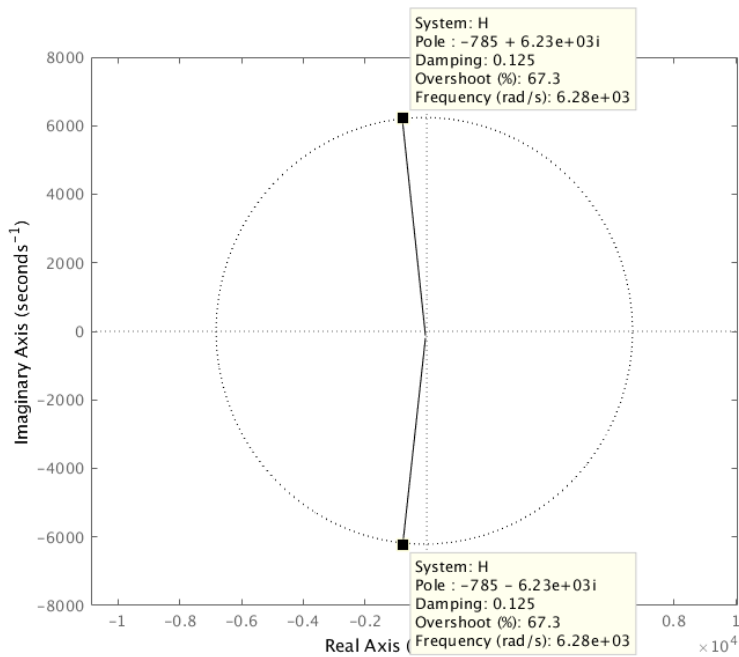


Figure 8: Pole zero plot

## Bode plot

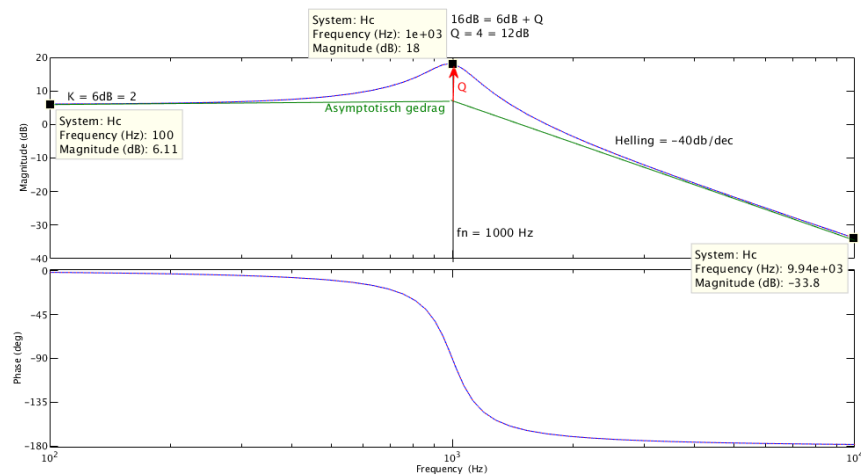


Figure 9: Bode Plot

Door de dubbele pool is er maar 1 knik in de (anymptotische) grafiek, daar gaat de helling van 0 naar  $-40\text{dB/dec}$ .

## Stapresponsie

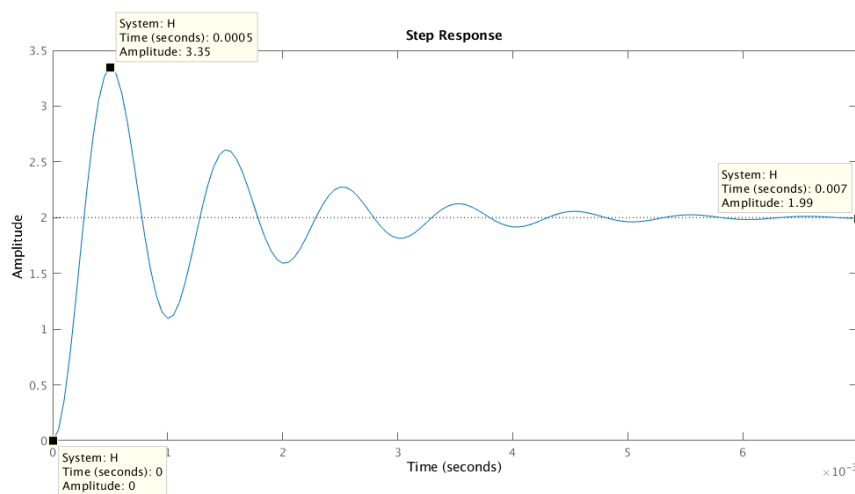


Figure 10: Stapresponsie

## Simulatie op basis van de componenten (SPICE)

**Note:** Ik gebruik LTspice, dus de numering van de nodes is niet systematish. Ze zijn aangeduid op het schema uit de opgave in lichtgrijs.

### Ideaal

```
* H Ideaal
.inc opampIdeal.cir
R3 N006 vin 19894
R1 N002 N005 19894
R2 N004 N003 19894
C2 Vout N004 1n
```



```

C1 N003 N002 21.33n
R6 N005 N001 19894
R4 N003 N006 19894
R5 Vout N001 59683
V1 vin 0 AC 1
XU4 N001 N006 N005 opampIdeal
XU5 N002 0 N003 opampIdeal
XU6 N004 0 Vout opampIdeal
.ac dec 100 100 1MEG
.probe
.end

```

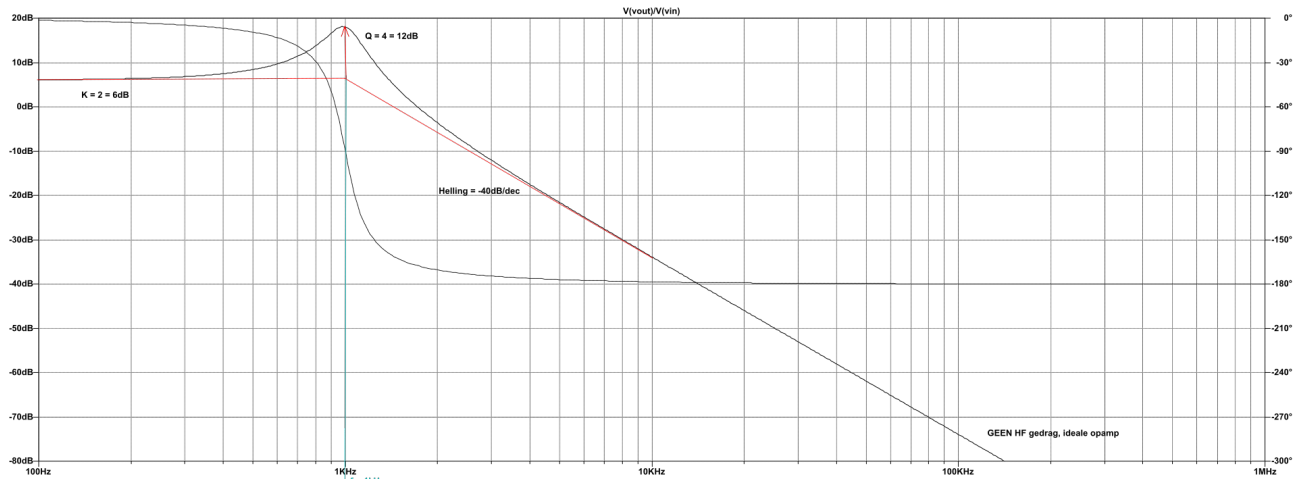


Figure 11: Ideaal Bode Plot

## VCVS

```

* H VCVS
.inc opamp84.cir
R3 N006 vin 19894
R1 N002 N005 19894
R2 N004 N003 19894
C2 Vout N004 1n
C1 N003 N002 21.33n
R6 N005 N001 19894
R4 N003 N006 19894
R5 Vout N001 59683
V1 vin 0 AC 1
XU1 N001 N006 N005 opamp84
XU2 N002 0 N003 opamp84
XU3 N004 0 Vout opamp84
.ac dec 100 100 1MEG
.probe
.end

```

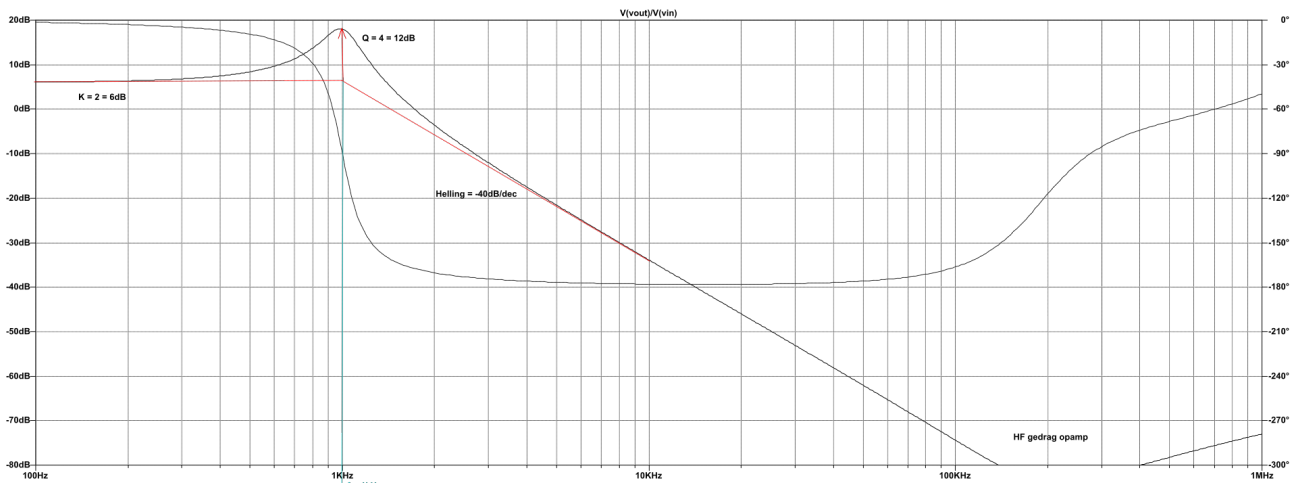


Figure 12: VCVS Bode Plot

tl084

```
* H TL084
.inc TL084.cir
R3 N006 vin 19894
R1 N002 N005 19894
R2 N004 N003 19894
C2 Vout N004 1n
C1 N003 N002 21.33n
R6 N005 N001 19894
R4 N003 N006 19894
R5 Vout N001 59683
V1 vin 0 AC 1
XU1 N006 N001 vp vn N005 TL084
XU2 0 N002 vp vn N003 TL084
XU3 0 N004 vp vn Vout TL084
V2 vp 0 15
V3 0 vn 15
.ac dec 100 100 1000000
.probe
.end
```

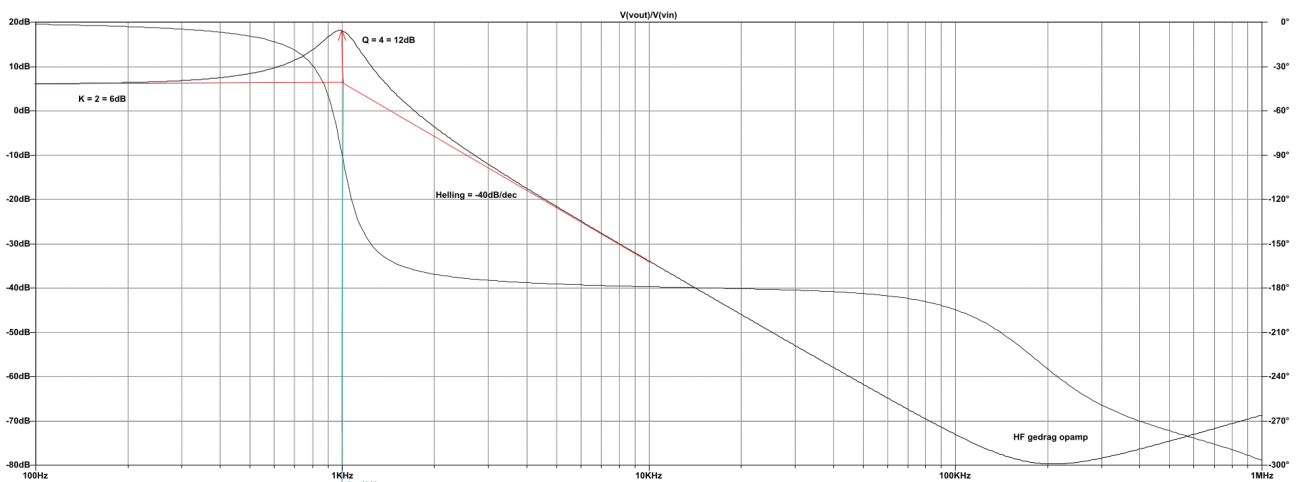


Figure 13: TL084 Bode Plot

### Monte Carlo analyse R5% - C20%

```
* H TL084, MC 5-20
.inc tl084.cir
.model rmod res(r = 1 DEV/GAUSS 5%)
.model cmod cap(c = 1 DEV/GAUSS 20%)
R3 6 vin rmod 19894
R1 2 5 rmod 19894
R2 4 3 rmod 19894
C2 Vout 4 cmod 1n
C1 3 2 cmod 21.33n
R6 5 1 rmod 19894
R4 3 6 rmod 19894
R5 Vout 1 rmod 59683
V1 vin 0 AC 1
V2 vp 0 15
V3 0 vn 15
XU4 1 6 vp vn 5 tl084
XU5 2 0 vp vn 3 tl084
XU6 4 0 vp vn Vout tl084
.ac dec 100 100 1MEG
.mc 10 ac V(V1) ymax list output all
.probe
.end
```

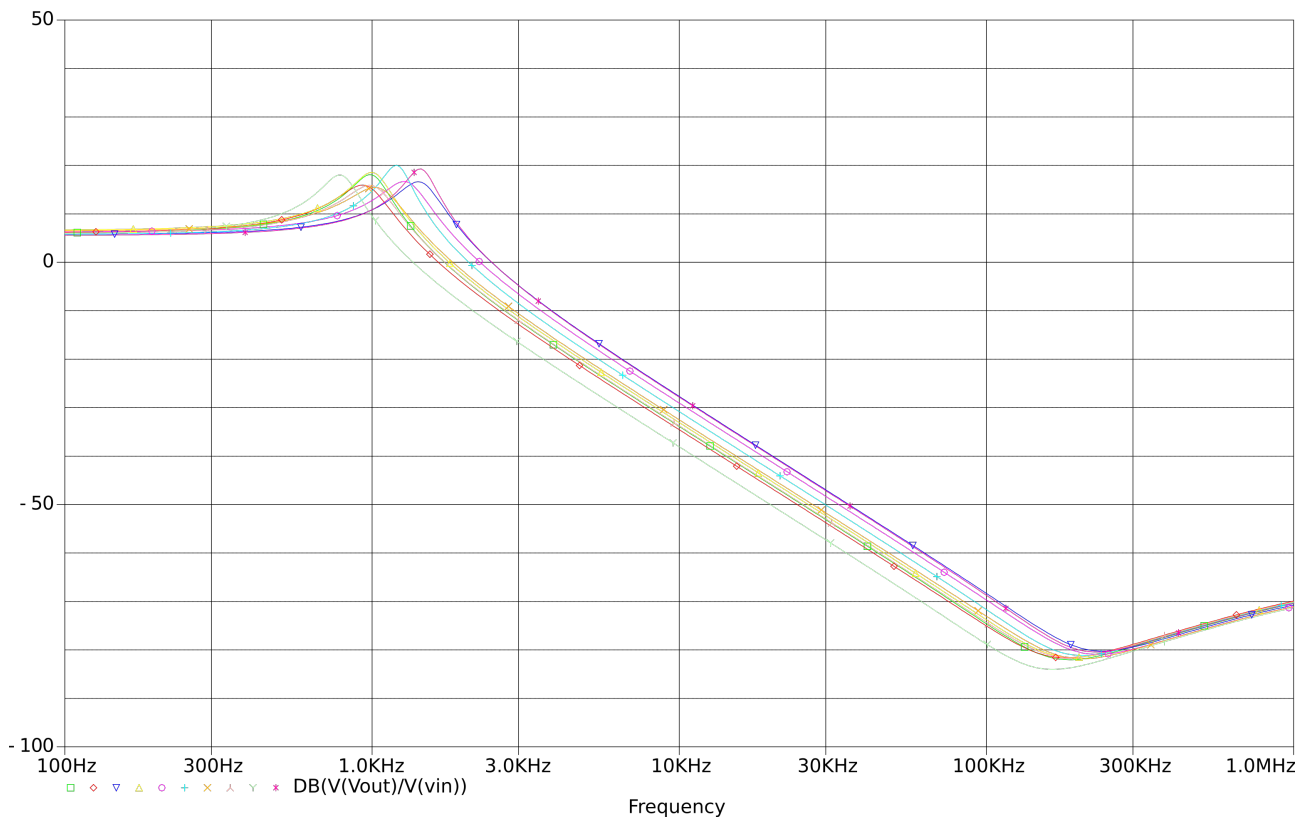


Figure 14: Monte Carlo analyse 5%

### Monte Carlo analyse 1%

```
* H TL084, MC 1
.inc tl084.cir
.model rmod res(r = 1 DEV/GAUSS 1%)
.model cmod cap(c = 1 DEV/GAUSS 1%)
R3 6 vin rmod 19894
```

```

R1 2 5 rmod 19894
R2 4 3 rmod 19894
C2 Vout 4 cmod 1n
C1 3 2 cmod 21.33n
R6 5 1 rmod 19894
R4 3 6 rmod 19894
R5 Vout 1 rmod 59683
V1 vin 0 AC 1
V2 vp 0 15
V3 0 vn 15
XU4 1 6 vp vn 5 tl084
XU5 2 0 vp vn 3 tl084
XU6 4 0 vp vn Vout tl084
.ac dec 100 100 1MEG
.mc 10 ac V(V1) ymax list output all
.probe
.end

```

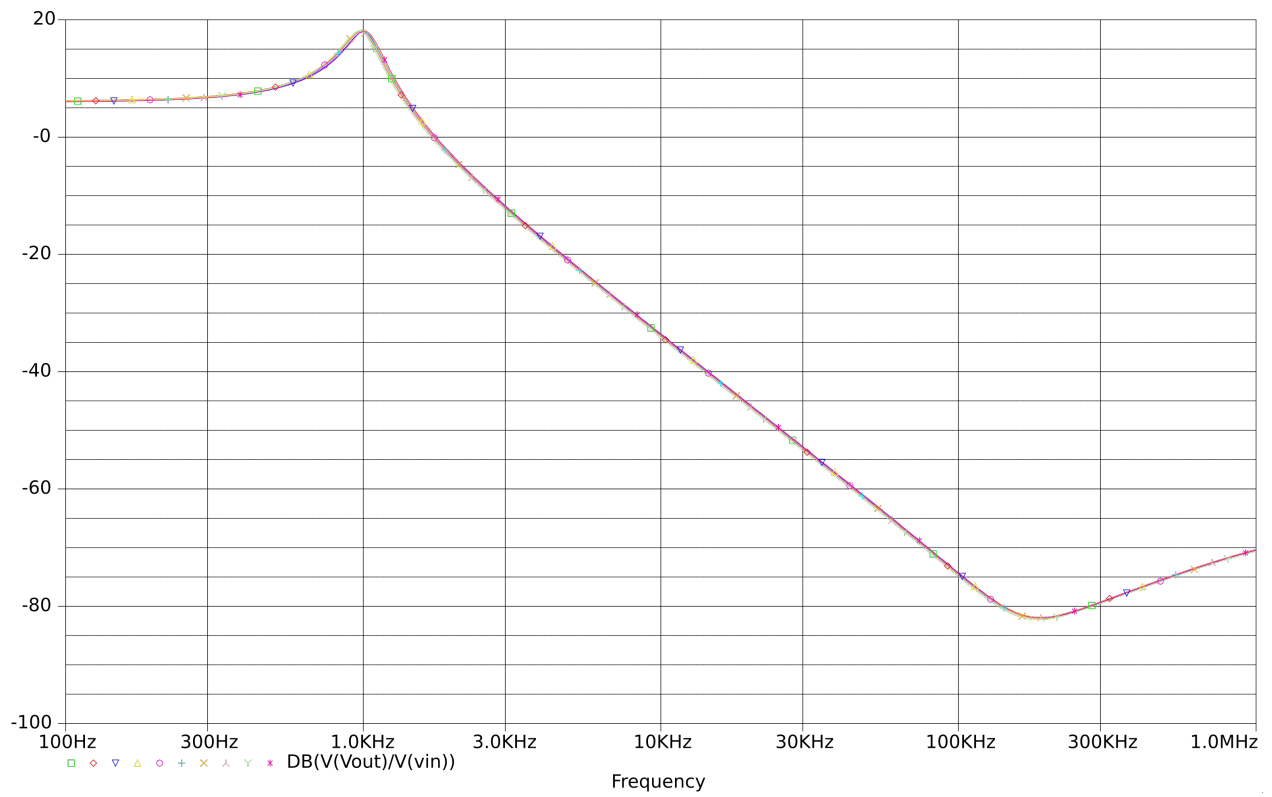


Figure 15: Monte Carlo analyse 1%

## Ingangsimpedantie

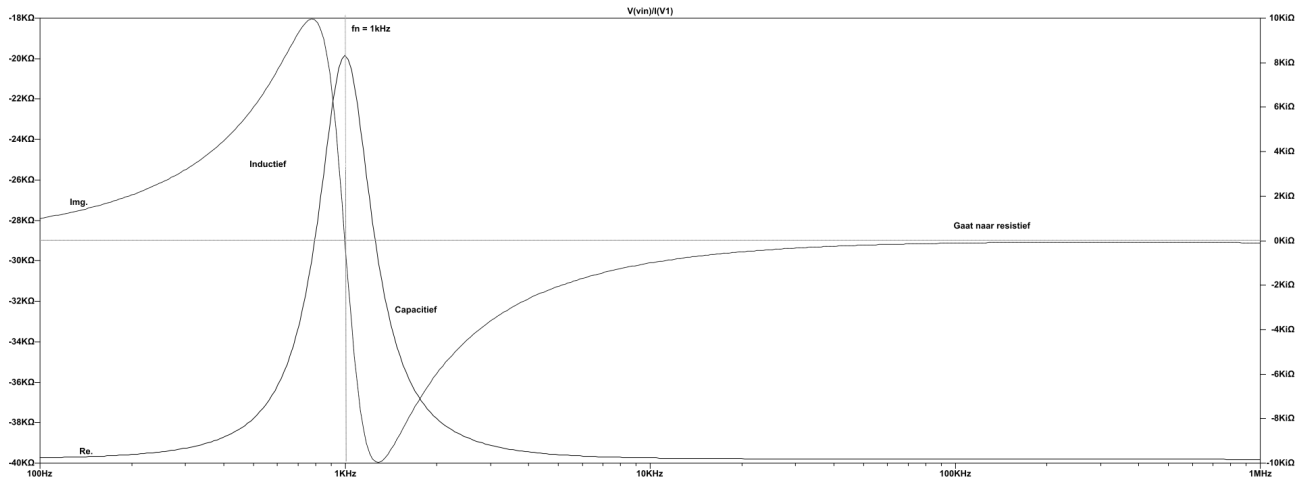


Figure 16: Cartesiaanse Ingangsimpedantie

Omdat er  $180^\circ$  fasedraaiing zit op de ingangsstroom is de reële as (links) negatief en lijkt deze ondersteboven te staan. De reële impedantiecomponent *daalt* rond de kantelfrequentie. De maximale ingangsimpedantie is  $40k\Omega$ , de minimale  $20k\Omega$ .

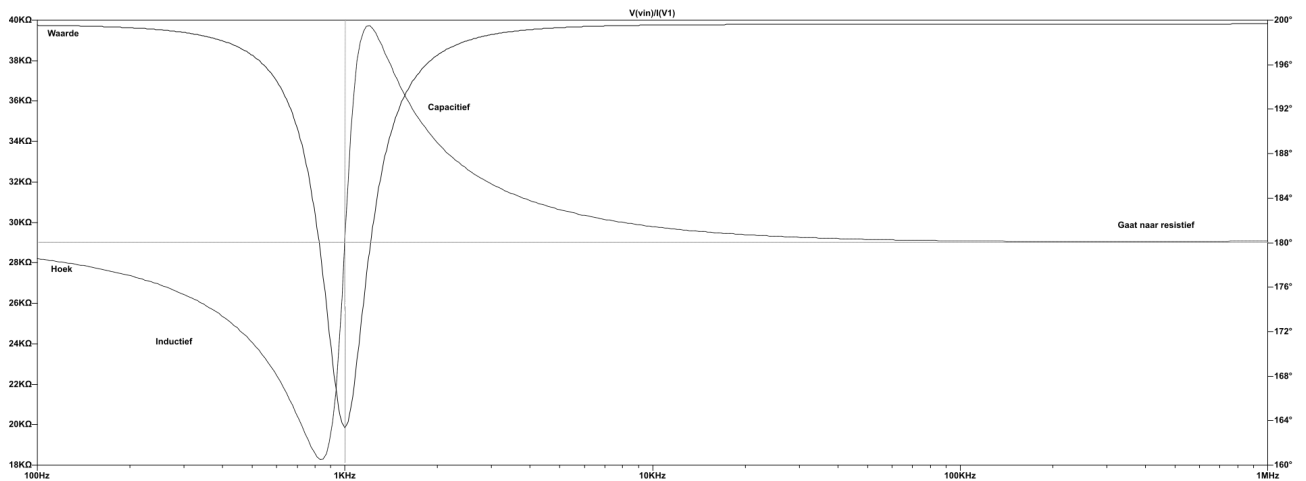


Figure 17: Polaire Ingangsimpedantie

## Stapresponsie

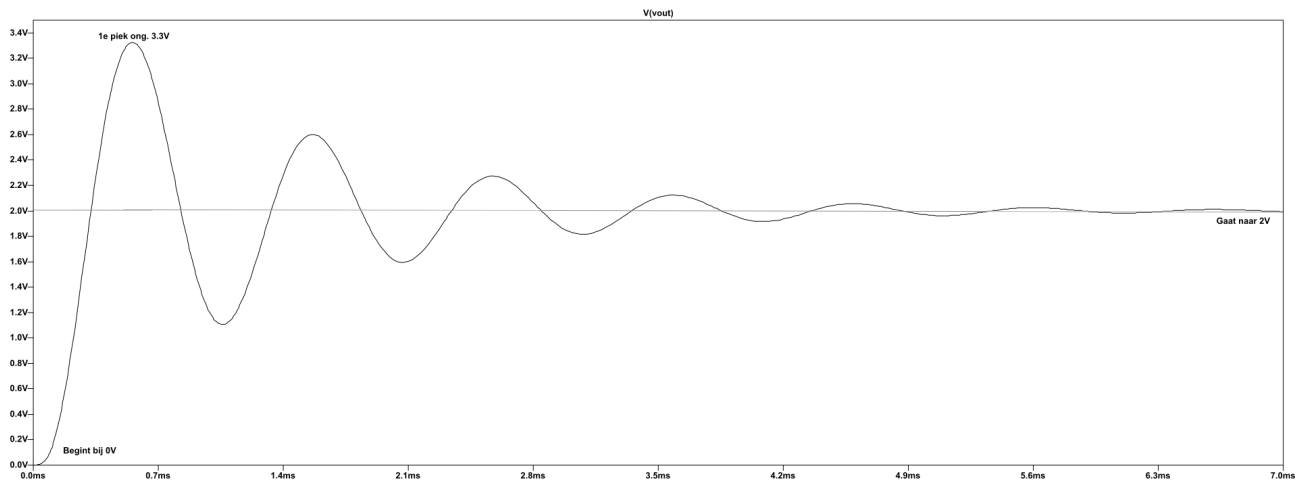


Figure 18: Stapresponsie

De stapresponsie berekend via SPICE is vrijwel identiek aan die berekend via Matlab.