CG Assignment 1

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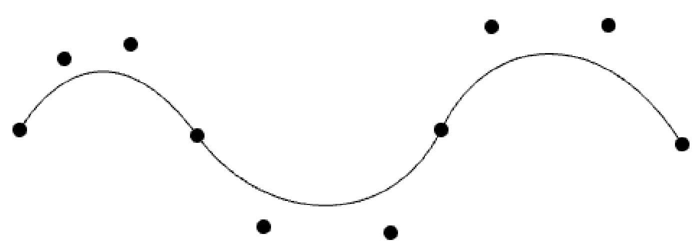
September 14, 2020

Abstract

Piecewise Bezier curves provide good local control and C1 continuity at joins. A sequence of mouse clicks on the application canvas draw a piecewise linear curve connecting the points. We also allow control points to be modified with mouse pick and drag.

1 Introduction

A Bezier Curve is an approximation curve passing through the first and last point but not necessarily through the intermediate points. In this case we need to implement a piecewise cubic bezier curve, i.e. every 4 points should form a cubic bezier curve with c1 continuity throughout the curve.
The control points are provided by the user which decides what the curve will look like. However, a control point is also added by the program to ensure C1 continuity at the joints. Following is what a piecewise cubic bezier curve with c1 continuity looks like -



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2 Strategy

2.1 Forming a Single Bezier Curve

We were already given the code for finding the linear bezier curve between 2 points which works as follows -
For any 2 points P_0 *and* P_1 we find n intermediate points between them lying on the line joining P_0 *and* P_1 and then plot a line from P_0 to P_1 by joining each adjacent intermediate point.

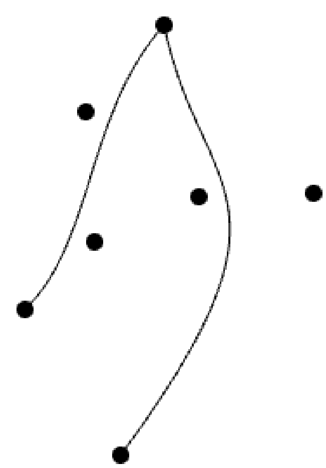
We can create a quadratic bezier curve using 2 linear bezier curves as follows -
For 3 points $P_0,\ P_1,\ and\ P_2$ we first construct a linear bezier curve L_0 between points points $P_0,\ and\ P_1$ and another Linear bezier curve L_1 between $P_1,\ and\ P_2$.
Now, if there are n intermediate points on L_0 and L_1, then for each i^{th} point on each curve

we make a new linear bezier curve between them and take the i^{th} intermediate point on that curve. We do this process for all points from i = 1 to n. If we join the collection of points hence obtained we get the required quadratic bezier curve.

Now, to form a cubic bezier curve with 4 points, say $P_0,\ P_1,\ ,P_2\ and\ P_3$, we can do the following -
We form three linear bezier curves between P_0 and P_1; P_1 and P_2; P_2 and P_3; call it L_0, L_1 and L_2 respectively.
Now we form a Linear Bezier curve between each intermediate point on L_0 and L_1; L_1 and L_2, call them L'_0 and L'_1 respectively. We now follow the same procedure as we did for quadratic bezier curve to get a cubic bezier curve.
Hence we have a cubic bezier curve given 4 points. For each additional 3 points, we will form a new bezier curve using the last point of the previous bezier curve and the 3 additional points.

2.2 Maintaining C1 continuity

Even though we have a piecewise cubic bezier curve at this point, it is not necessarily C1 continuous if choice of all control points for the next bezier curve is left up to the user. Hence we can get a graph looking like this -



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This clearly isn't C1 continuous. To maintain C1 continuity we need second last point of the first curve, last point of the first curve(also the first point of the second curve) and the second point of the second curve to be co-linear and equidistant. To do so whenever the user chooses the last point of a cubic bezier curve, the program automatically adds the second point of the next curve. In this way we are able to maintain C1 continuity.

3 Mathematical proof of C1 continuity

To show that a piecewise cubic bezier curve is C1 continuous we need to show that the segments have equal 1^{st} derivative at the interpolation point.
Consider 2 cubic bezier curves C_0 and C_1 containing control points P_0, P_1, P_2, P_3 and P_3, P_4, P_5, P_6 respectively.
The parametric equation of a bezier curve is -
C(t) = $(1-t)_3$*P_0 + 3*$(1-t)^2$*t*P_1 + 3*(1-t)*t^2*P_2 + t^3*P_3

On differentiating we get,

$\frac{dC(t)}{dt}$ = -3*$(1-t)^2$*P_0 + 3*[-2*(1-t)*t + $(1-t)^2$]*P_1 + 3*[-t^2 + (1-t)*2*t]*P_2 + 3*t^2*P_3

Now for curve C_0 at t = 1,

$\frac{dC_0(t)}{dt}$ = -3*P_2 + 3*P_3

and curve C_1 at t = 0,

$\frac{dC_1(t)}{dt}$ = -3*P_3 + 3*P_4

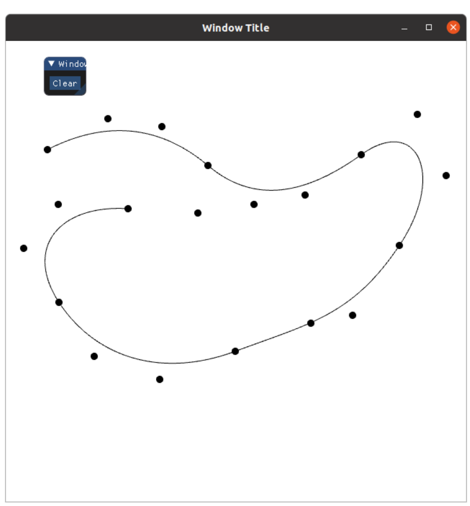
Now we know have set up points P_2, P_3 and P_4 such that they all lie on the same line and P_2 and P_4 are equidistant from P_3.

Therefore, P_3 - P_2 = P_4 - P_3

Hence, we have $\frac{dC_0(t)}{dt}$ = $\frac{dC_1(t)}{dt}$ at t = 0 and t = 1 respectively

So we can say that curve is C1 continuous

4 Output



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