

Machine Learning for Control

by

Rodrigo Queiro (DOW)

Fourth-year undergraduate project in
Group F, 2010/2011

I hereby declare that, except where specifically indicated, the work submitted herein is my own original work.

Signed: _____ Date: _____

Technical Abstract

technical abstract...

Contents

1	Introduction	3
1.1	Division of Labour	4
2	Reinforced Model Learnt Control	4
2.1	Practical Concerns	6
2.1.1	GPR Implementation	6
2.1.2	Choice of State Vector	6
2.1.3	Controller Form	7
2.1.4	Gathering Initial Data	8
2.1.5	Loss Function	9
2.1.6	Timestep	10
3	Hardware and Software	10
3.1	Sensing	11
3.1.1	Angle Sensing	11
3.1.2	Position Sensing	14
3.1.3	Wheel Speed Sensing	15
3.2	Fall Protection	16
3.3	Inability to Spin	17
3.3.1	Spin Analysis	18
3.3.2	Solving the Spin Problem	20
3.4	Known Issues	21
3.4.1	Toothed Belt	21
3.4.2	Motor Control	21
3.4.3	Flywheel Encoder	22
3.4.4	I ² C Communications	23
4	Results of Learning	24
4.1	Learning the 2D System	24
4.2	Learning the 3D Unicycle	27
5	Conclusions	29
6	Further Work	29
A	Evaluation of Risk Assessment	31

1 Introduction

Balancing a unicycle is a very challenging task for a human rider. Many attempts have been made to achieve this task, using a variety of models for the action of the rider. Some represent the rider as a flywheel or pendulum in the coronal plane, allowing direct compensation of falling to the side [1,2], as in Figure 1(a). Other use a more realistic (and challenging) model of a flywheel in the horizontal plane [3,4], as in Figures 1(b), but none of these have reliably balanced a real unicycle.

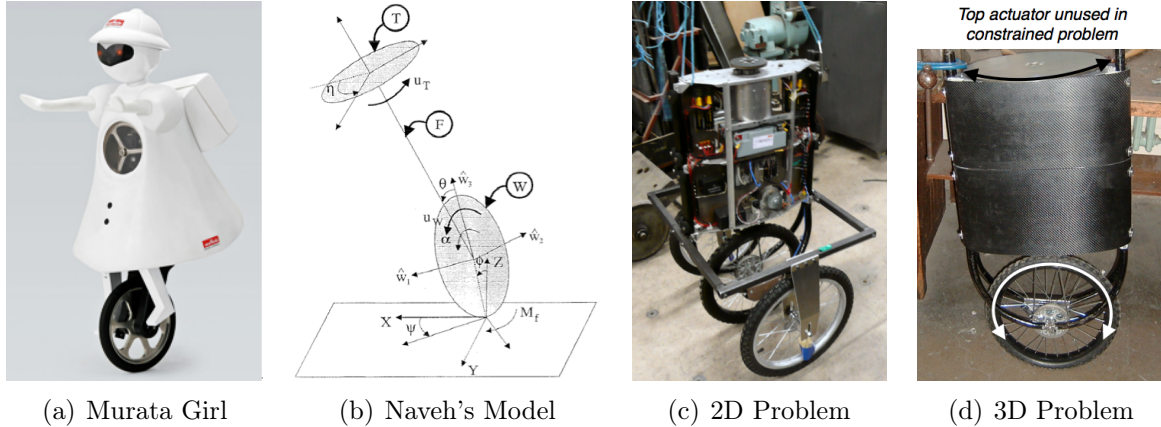


Figure 1: Different balance problems

In 2004/2005 Mellors and Lamb [5,6] built a robotic unicycle, shown in Figure 1(d), intending to design a controller to balance it. However, they were only able to complete the construction of the unicycle. In 2007/2008, D’Souza-Mathew resumed work, replacing a wheel sensor and attempting to design a controller. He simplified the problem by removing the ability to fall to the side, reducing it to 2D dynamic control: the inverted pendulum (from now on referred to as the **2D system**). This is shown in Figure 1(c). He was unable to balance the unicycle due to hardware problems.

Next, in 2008/2009 Forster analysed the dynamics of both the 2D problem and the unrestricted 3D unicycle [7]. Again, hardware problems prevented him from balancing the 2D system, and although he designed a controller for the 3D unicycle, it was not even tested in simulation. Given the simplicity of his approach compared to those of Vos and Naveh [3,4], it appears unlikely to work.

One thing all these approaches have in common is that their first step is a series of simplifying assumptions about the dynamic system. They ignore the non-linearities like motor dead-zones and wheel friction that are present in any real-world system, and attempt to design a controller to stabilise the idealised system. In many cases, this approach is very successful. However, D’Souza-Mathew and Forster found that their model was invalid since the unicycle’s motor drive didn’t react faster enough. Vos and Naveh had to use complex, approximate techniques to model the unicycle.

An alternative “intelligent” approach to control involves learning the dynamics of the system directly, instead of relying on assumptions and mechanical analysis. Various methods for this have been used, but many require prohibitively large amounts of data from the system. One method due to Rasmussen and Deisenroth, known here as Reinforced Model Learnt Control (RMLC), achieves unprecedented data efficiency, and has been successfully used to stabilise a computer simulation of a 3D unicycle [8, 9].

In 2009/2010, McHutchon successfully applied RMLC to the 2D system [10]. However, since he had to make significant changes to the unicycle hardware and software to achieve this, he did not have time to attempt to balance the 3D unicycle.

The principle objective of this project is to apply RMLC to the unrestricted 3D unicycle. This includes the solution of problems identified by McHutchon, as well as other problems identified during the project.

1.1 Division of Labour

This project was conducted as a joint project between the author and Alan Douglass. The vast majority of the work was done together, and as such it is difficult to attribute ideas or components of the system to one or the other. When only one of us was responsible for a particular component, such as a particular circuit or module of the code, it is focussed upon in their report, and the other report will simply refer to it.

Although this may raise difficulties for marking the projects, it was essential for the actual work. We would never have been able to make such progress if we had not been able to build on each other’s ideas, to spot each others mistakes and to rely upon another point of view when stuck.

2 Reinforced Model Learnt Control

The main technique in this project is Reinforced Model Learnt Control, diagrammed in Figure 2. At its core, it assumes that the system (in this case, the unicycle) can be modelled in discrete time as:

$$\mathbf{q}[n + 1] = \mathbf{f}(\mathbf{q}[n], \mathbf{u}[n])$$

In this equation, $\mathbf{q}[n]$ is the state of the system at time n , and $\mathbf{u}[n]$ is control input at time n . In the case of the unicycle, \mathbf{q} consists of angles and angular velocities of the components of the unicycle, and the position of the unicycle. \mathbf{u} consists of the commands sent to the wheel and flywheel motors.

This function, \mathbf{f} , is modelled as a Gaussian Process (GP). By using Gaussian Process Regression (GPR), we can estimate any continuous function from sampled inputs and outputs. For a description of the mechanics of GPR, refer to [11]. When the unicycle

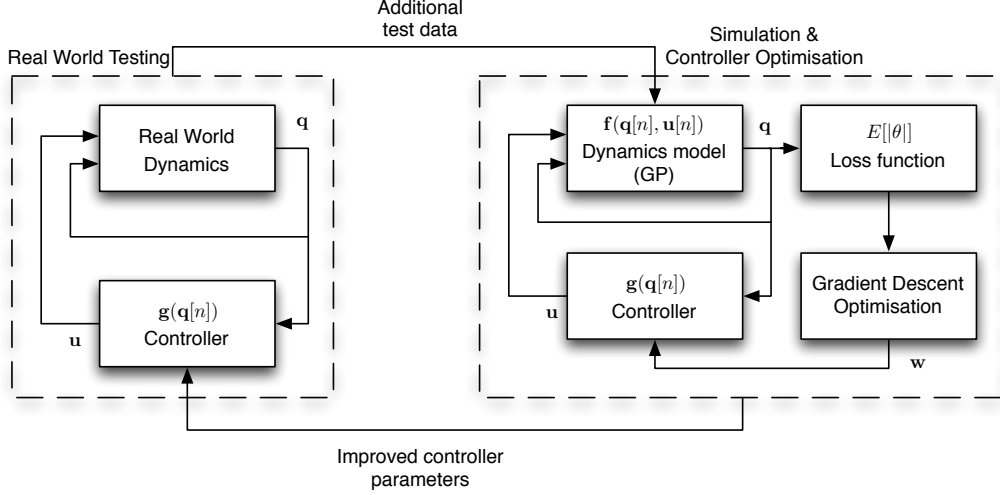


Figure 2: Reinforced Model Learnt Control

runs, we get a series of states and control inputs that can be converted to samples of \mathbf{f} , and this allows us to use GPR to estimate \mathbf{f} at any point. This estimated \mathbf{f} is referred to as the **dynamics model**.

By successively applying \mathbf{f} to an initial distribution of possible starting states, we can estimate, with confidence bounds, a distribution of states over some finite horizon. This is referred to as **simulation** of the system. Then, a **loss function** is applied to the state distributions—this might find, for example, the expected distance between the top of the unicycle and the upright position. Summing these losses over the horizon gives a numerical score that rates how well the dynamics model believes a given controller will balance the unicycle. This loss score penalises uncertainty as well as falling.

The gradient of the loss with respect to the controller parameters can be calculated, and this allows standard gradient descent optimisation methods to be used to find a locally optimal controller (for the estimated dynamics model). This process is shown as the right-hand box, “Simulation & Controller Optimisation”, in Figure 2, and is referred to as **training** a controller.

Once a optimal controller has been trained on the simulated system, a **trial** is performed on the real system (“Real World Testing” in Figure 2). This generates a log of states and control inputs, which can be converted into more samples of \mathbf{f} , improving the quality of the dynamics model and allowing a better controller to be trained. This process is repeated iteratively until the dynamics model is sufficiently accurate that the trained controllers perform well on the real system. This process of gathering initial data (see Section 2.1.4) and then iterating between training a new controller and gathering data with it, continuing until steady-state is reached, is referred to as a training **sequence**.

2.1 Practical Concerns

There are many different decisions to make when implementing the RMLC strategy, which are detailed in this section. Previously, Rasmussen used Forster’s analytical model of the unicycle to simulate it, and used RMLC to train a controller for this ideal unicycle. Thanks to this, and to McHutchon’s work on the real 2D system, we have a lot of information on which choices can work well in these situations, and which are most important.

Some of these decisions were made on the results of various training sequences, which are presented in Section 4.

2.1.1 GPR Implementation

The GPR system has many configurable parameters, but fortunately the form used previously had proven very robust. This project uses a zero-mean GP with a squared exponential covariance function, with automatic relevance detection. This has the form:

$$k(\mathbf{x}, \mathbf{x}') = \alpha^2 \exp \left(\sum_{d=1}^D -\frac{(x_d - x'_d)^2}{2l_d^2} \right)$$

This expression contains hyperparameters for the signal variance α^2 and the length scales l_d . An additional hyperparameter involved in the regression is the noise variance, σ_ϵ^2 . These parameters are chosen to best fit the data without overfitting with the maximum likelihood (ML) method [11]. The optimal values of these hyperparameters are very useful for interpreting how accurate the dynamics model is: a high SNR $\frac{\alpha}{\sigma_\epsilon}$ suggests the model can predict very well. Furthermore, automatic relevance detection is provided by the length scales - if a variable is not useful for predicting, the ML length scale will tend to inf.

2.1.2 Choice of State Vector

The state vector $\mathbf{q}[n]$ should be chosen to ensure that the future states are a function only of the current state, and current and future control inputs. In other words, the states should form a Markov Chain:

$$P(\mathbf{q}[n+1]|\mathbf{q}[i] \text{ for } i = 1, \dots, n) = P(\mathbf{q}[n+1]|\mathbf{q}[n])$$

Forster’s analysis suggested the following state to be suitable, which was found by Rasmussen to be sufficient to model the ideal unicycle.

$$\mathbf{q}[n] = \begin{bmatrix} \dot{\theta} & \text{roll angular velocity} \\ \dot{\phi} & \text{yaw angular velocity} \\ \dot{\psi}_w & \text{wheel angular velocity} \\ \dot{\psi}_f & \text{pitch angular velocity} \\ \dot{\psi}_t & \text{flywheel (turntable) angular velocity} \\ x_c & \text{position of target in unicycle's reference frame} \\ y_c & \\ \theta & \text{roll angle} \\ \psi_f & \text{pitch angle} \end{bmatrix}$$

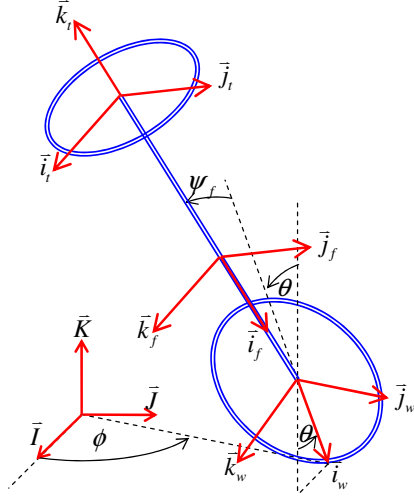


Figure 3: Diagram showing Euler angles for the rotation of the unicycle (from [7])

However, this ignores the presence of unobserved states in the system like delays, backlash in the gears, etc. To help the dynamics model deal with these problems, we tried giving it access to the previous state and control input, in effect modelling it as a 2nd order Markov chain:

$$\mathbf{q}_2[n] = \begin{bmatrix} \mathbf{q}[n-1] \\ \mathbf{u}[n-1] \\ \mathbf{q}[n] \end{bmatrix}$$

This significantly improved the accuracy of the dynamics model, which in turn suggests that unobserved states are significant in the behaviour of the real unicycle.

2.1.3 Controller Form

The most basic form of controller is a linear controller, $\mathbf{g}(\mathbf{q}[n]) = \mathbf{W}\mathbf{q}[n] + \mathbf{p}$, where \mathbf{W} is a matrix of weights and \mathbf{p} is a vector of offsets. This form is capable of stabilising the

ideal inverted pendulum, and indeed proved sufficient to stabilise the 2D system.

However, the linear controller cannot generate the correct turning command as shown in Figure 4—this is equivalent to the XOR problem, and can be solved by using a quadratic controller. This takes the form:

$$g_i(\mathbf{q}[n]) = p_i + \sum_{j=1}^D w_{i,j} q_j[n] + \sum_{j=1}^D \sum_{k=j}^D h_{i,j,k} q_j[n] q_k[n]$$

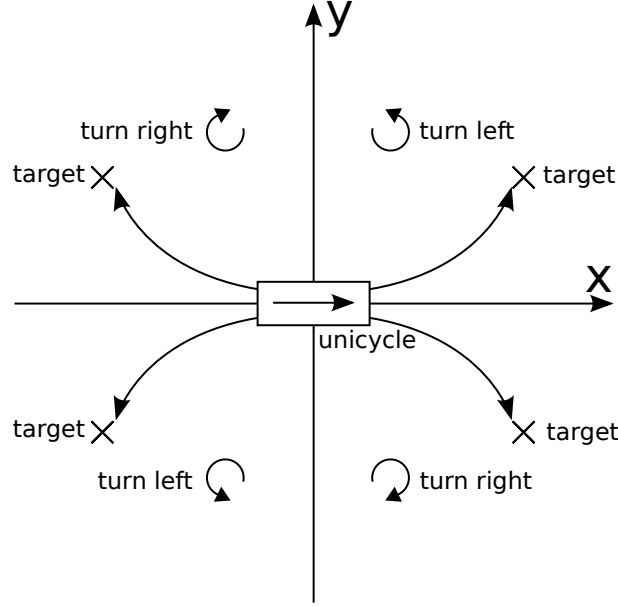


Figure 4: Correct turning command in unicycle-centred coordinates

It was found that the controllers performed significantly better when using $\mathbf{q}_2[n]$ as input, instead of $\mathbf{q}[n]$. To understand this, consider the effect with a linear policy: the controller for the augmented state is equivalent to a combination of a two-tap FIR filter and a first-order IIR filter on the original state:

$$\begin{aligned} \mathbf{u}[n] &= \mathbf{g}(\mathbf{q}_2[n]) = \mathbf{W} \mathbf{q}_2[n] \\ &= \mathbf{W}_1 \mathbf{q}[n-1] + \mathbf{W}_2 \mathbf{u}[n-1] + \mathbf{W}_3 \mathbf{q}[n] \end{aligned}$$

This allows the RMLC system to create basic low or high-pass filters in the controller, and this additional freedom improved the subjective quality of the controllers trained.

2.1.4 Gathering Initial Data

In order to train a controller, some experience of the system is required. The way this has been done before is with **random trials**: the system inputs are chosen randomly, and the response is recorded. Rasmussen chose to use independent and identically distributed

(IID) motor commands when stabilising the ideal unicycle. McHutchon chose to use a controller with randomly chosen weights.

We initially chose to use IID motor commands, chosen uniformly from the full range. However, the dynamics model could not model the resulting data well. We suspect that the same problems that mean that modelling the system as Markov order 2 (described in Section 2.1.2) mean that if the motor command changes radically between cycles, the behaviour is very unpredictable. Since the dynamics model cannot model signal-dependent noise well, these datapoints cause a very uncertain model to be learnt.

We solved this problem by low-pass filtering the random motor commands: this prevents sudden changes in the motor command, while providing a wider survey of the space than a random controller, and avoiding the risk of large positive feedback.

2.1.5 Loss Function

The main concerns when choosing a loss function are accurately represent what is desired of the controller, and to ensure that different desires are appropriately weighted. When stabilising the ideal unicycle, Rasmussen was successful using the following form of loss function:

$$1 - \mathcal{N}(\mathbf{q}[n]; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = 1 - \exp \left(\sum_{d=1}^D -\frac{(q_d[n] - \mu_d)^2}{w_d^2} \right)$$

This is configured using w_d , the characteristic widths for each variable. To ignore a variable, take $w_d^{-2} = 0$. While the RMLC system is not too sensitive to these values, it is important to set them appropriately. During a test run on the 2D system, we accidentally set the characteristic pitch angle to 10° and the characteristic distance (from the origin) to around 10cm. This meant that the preferred policy was to fall over immediately, to prevent travelling away from the origin. Loosening the characteristic distance to 30cm led to a controller that balanced the system.

For the ideal unicycle, Rasmussen chose to penalise the pitch and roll angles, to encourage the system to keep the robot vertical. In addition, he chose to penalise the yaw rate $\dot{\phi}$ and flywheel rate $\dot{\psi}_t$ to prevent the system from using a “spinning-top” like approach to balance, and to penalise the distances from the origin, x_c and y_c , to prevent the system from driving the robot very fast to enhance stability.

We used the same loss function structure, with characteristic widths of 9° on the angles, 1 metre on the distances and 1 rps & 3 rps on the yaw & flywheel rates respectively. Unlike the ideal unicycle, the real unicycle has an upper limit on the flywheel speed, so the “spinning-top” strategy is not viable, so it is possible that the yaw & flywheel rates need not be penalised. This was not tested.

2.1.6 Timestep

The RMLC approach assumes a discrete-time system - to apply it to a continuous time system, it must be discretised. We used a zero-order hold (ZOH) on the control signal: $u(t) = u[\lfloor \frac{t}{T} \rfloor]$

Training for the ideal unicycle, Rasmussen found it desirable to choose the largest timestep T with which the system can be stabilised, to reduce the computational cost of the optimisation, and to avoid problems with noise buildup from many repeated applications of the transition function \mathbf{f} . He chose a timestep of $\frac{1}{7}$ seconds.

However, for the real unicycle, both McHutchon and we found that the predicted trajectories are more confident and reliable, and controllers perform better, when using a timestep of $\frac{1}{20}$ seconds. It was hoped that this was because the sensors were noisier in real life, and so a shorter time in the ZOH led to several successive noisy control settings being averaged by the low-pass effect of the system, reducing the effect of noise. However, upgrading the sensors didn't seem to change this, so there is possibly another cause.

3 Hardware and Software

When the project started, some of the requirements were already clear, as a result of the previous work on the unicycle. As such, extensive changes to the hardware and a complete rewrite of the controller software were required to fix issues that had been turned up by the previous year's work, and to prepare it for 3D balance. A brief summary of the changes is below, followed by greater detail where necessary.

- The cumbersome FPGA-based controller used previously was replaced with an Arduino microcontroller, allowing changes to be made faster and more easily.
- Older gyroscopes and very noisy angle sensors were replaced by modern MEMS gyroscopes and accelerometers.
- Algorithms for keeping track of angle and position in 3D were written.
- A new motor controller was built for the flywheel, as well as a quadrature encoder for speed sensing.
- A sensor for the battery voltage was added, to ensure it could supply sufficient power to the motors.
- We designed and tested a number of methods of protecting the unicycle from falling, while allowing 3D movement.
- Some faults were identified and could not be fixed, so tests and error-checking were added to ensure these did not corrupt the data.

- To improve safety, the motor is disabled when a user presses a switch on the chassis or when the unicycle nears the ground.

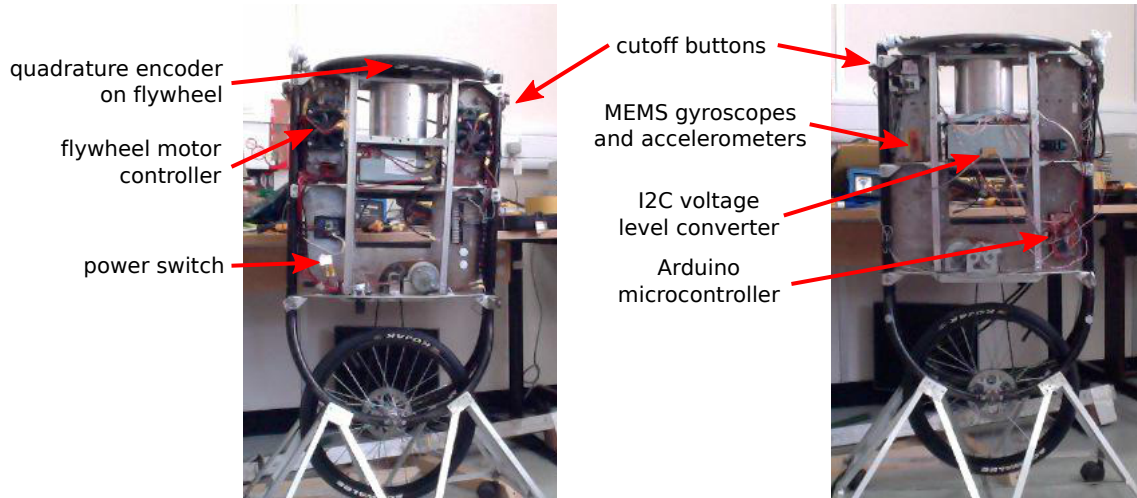


Figure 5: Summary of changes to unicycle hardware

3.1 Sensing

3.1.1 Angle Sensing

The unicycle had previously used a MEMS rate gyroscope and a pair of infra-red distance sensors to keep track of its angle. However, these were reported to be extremely noisy, and were blamed for a large part of the poor performance of the previous system. Our supervisor had already purchased the ITG-3200, a 3-axis MEMS rate gyroscope with in-built temperature compensation and low-pass filtering. We later purchased an ADXL345, a 3-axis MEMS accelerometer, to determine the absolute angle of the unicycle.

The ITG-3200 rate gyro determines the angular velocity of the chip (and thus the unicycle) around each of its 3 axes. These values are offset by some unknown bias - this is determined before the trial by averaging the output when stationary, and is assumed to be approximately constant during a trial. They can then be converted to radians per second with a calibration factor from the datasheet [12].

To keep track of the absolute angle, these angular velocities must be integrated. There are 3 possible ways to keep track of rotations in 3D, of which unit quaternions were chosen as best suited to the project.

Euler angles These are the yaw, roll and pitch angles shown in Figure 3. They are required for the controller, and so other forms must be converted to them. However, they suffer from gimbal lock—in certain positions, it is impossible to represent a

small rotation with a small change in the Euler angles. They also require trigonometric functions in the integration loop: a problem for fast integration on embedded platforms.

Direction Cosine Matrices (DCMs) A DCM is an orthonormal rotation matrix, representing the rotation from the global coordinate system to the body's coordinate system. They require only basic linear algebra to understand. However, when errors in integration accumulate, the matrix will no longer be orthonormal, and there is no clear way to fix this.

Unit quaternions Quaternions extend the complex numbers into 3D. Just as a complex number of unit magnitude can represent a rotation in the 2D plane, a quaternion of unit magnitude can represent a rotation in 3D. Although they may appear confusing, they have the tightest integration loop, do not suffer from gimbal lock and can be normalised by simply dividing by the magnitude.

Quaternion Integration Given the unit quaternion representing a rotation from the global coordinate system to that of the unicycle, $\mathbf{q}[n] = q_w + q_x\mathbf{i} + q_y\mathbf{j} + q_z\mathbf{k}$, and rate gyro outputs ω_x , ω_y and ω_z , we can integrate the angular velocities with $\mathbf{q}[n+1] = \mathbf{q}[n] \left(1 + \frac{\omega_x \Delta t}{2}\mathbf{i} + \frac{\omega_y \Delta t}{2}\mathbf{j} + \frac{\omega_z \Delta t}{2}\mathbf{k} \right)$. For more details on this, see [13].

Euler Angles of a Quaternion Unfortunately, the mechanical analysis of Forster (and thus the simulation of the ideal unicycle) uses a different angle convention to all other sources located. We chose to continue with this convention, and so the expressions for converting a quaternion to Euler angles had to be rederived. Using a convention of yaw around the vertical y -axis, roll around the forward x -axis and pitch around the sideways z -axis (the axes of the mounted gyroscope) we get:

$$\begin{aligned}\phi &= \tan^{-1} \left(\frac{2q_x q_y + 2q_y q_w}{-q_x^2 - q_y^2 + q_z^2 + q_w^2} \right) \\ \theta &= \sin^{-1} (2q_w q_x - 2q_y q_z) \\ \psi_f &= \tan^{-1} \left(\frac{2q_x q_y + 2q_z q_w}{-q_x^2 + q_y^2 - q_z^2 + q_w^2} \right)\end{aligned}$$

Angular Velocities To calculate the angular rates, $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}_f$, we convert the quaternion to a DCM, and then apply expressions for the derivatives of the Euler angles of a DCM. This could cause a division by zero in a gimbal lock situation, but fortunately the

unicycle never reaches such positions.

$$\begin{aligned}
\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} &= \begin{bmatrix} 1 - 2q_y^2 - 2q_z^2 & 2q_xq_y - 2q_zq_w & 2q_xq_z + 2q_yq_w \\ 2q_xq_y + 2q_zq_w & 1 - 2q_x^2 - 2q_z^2 & 2q_yq_z - 2q_xq_w \\ 2q_xq_z - 2q_yq_w & 2q_yq_z + 2q_xq_w & 1 - 2q_x^2 - 2q_y^2 \end{bmatrix} \\
\dot{\phi} &= \frac{(d_{12}d_{32} - d_{12}d_{33})\omega_x + (d_{11}d_{33} - d_{13}d_{31})\omega_y}{d_{13}^2 + d_{33}^2} \\
\dot{\theta} &= \frac{d_{22}\omega_x - d_{21}\omega_y}{\sqrt{1 - d_{23}^2}} \\
\dot{\psi}_f &= -\frac{d_{23}(d_{21}\omega_x + d_{22}\omega_y)}{d_{21}^2 + d_{22}^2} + \omega_z
\end{aligned}$$

Initial Angle To determine the initial angle of the unicycle, we take an average accelerometer reading while the unicycle is initially stable. This is a 3D vector, representing the direction of gravity in the reference frame of the rotated unicycle. By considering the DCM resulting from rotations in pitch and roll, we can determine the pitch and roll angles, and use these to construct an initial rotation quaternion.

$$\begin{aligned}
\mathbf{D} &= \begin{bmatrix} \cos \psi_f & -\sin \psi_f & 0 \\ \cos \theta \sin \psi_f & \cos \theta \cos \psi_f & -\sin \theta \\ \sin \theta \sin \psi_f & \sin \theta \cos \psi_f & \cos \theta \end{bmatrix} \\
\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} &= \begin{bmatrix} \cos \theta \sin \psi_f \\ \cos \theta \cos \psi_f \\ -\sin \theta \end{bmatrix} \\
\theta &= -\sin^{-1}(a_z) \\
\psi_f &= \tan^{-1}\left(\frac{a_x}{a_y}\right) \\
\mathbf{q} &= \left(\cos\left(\frac{\theta}{2}\right) + \mathbf{i} \sin\left(\frac{\theta}{2}\right)\right) \left(\cos\left(\frac{\psi_f}{2}\right) + \mathbf{k} \sin\left(\frac{\psi_f}{2}\right)\right)
\end{aligned}$$

Gyro Noise and Drift The approach described above is vulnerable to drift: accumulating errors in the integration of the gyroscope, especially when the zero-offset of the gyroscope changes. To evaluate the effect of this, the gyro was sampled for 10 seconds when stationary. The rate readings are shown in Figure 6. It is clear that the noise and drift are on the same order of magnitude, over a 10 second trial, the zero-offset changed by about 0.1 deg s^{-1} , leading to a drift of below 1° . This was judged as acceptable. If we wished to conduct longer trials, we could use one of a variety of fusion methods such as state observers, Kalman filters, or one of many highly tuned implementations developed by UAV hobbyists [14].

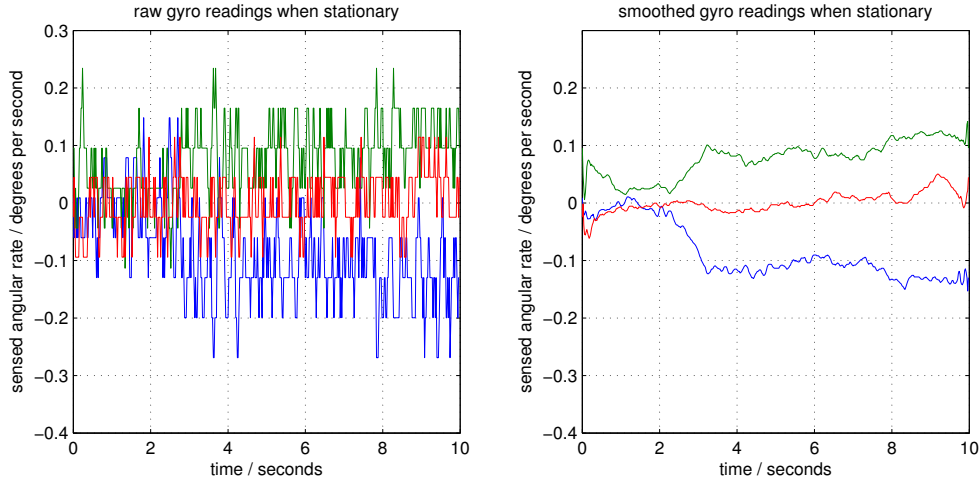


Figure 6: Gyro readings when stationary

Accelerometer Noise Figure 7 shows the accelerometer noise when stationary (the mean reading has been subtracted). (Note that the noise characteristic is very different for one axis compared to the other 2 - this is because this axis is perpendicular to the chip, and is built differently.) This corresponds to noise standard deviations for pitch and roll of about 0.5° , but this can be reduced by averaging many readings. More detail on the use of the accelerometer can be found in Douglass's report.

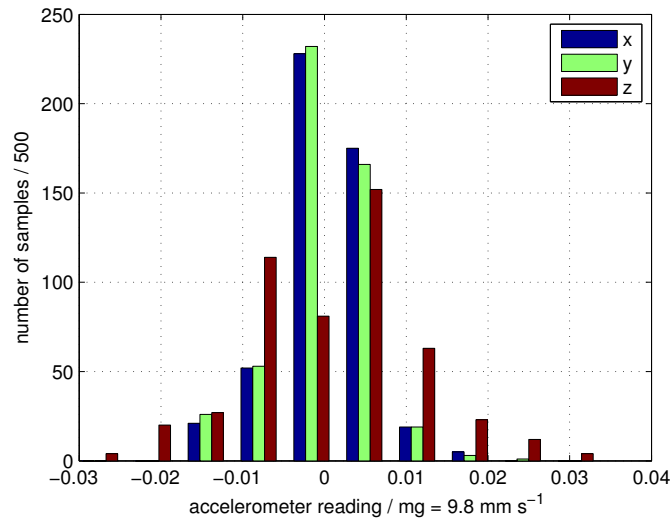


Figure 7: Accelerometer readings when stationary

3.1.2 Position Sensing

In order to reduce the number of state variable necessary, Rasmussen used self-centred coordinates to keep track of the position of the target. In these coordinates, the unicycle

is at $(0, 0)$ and faces along the positive x -axis, as shown in Figure 4. At each timestep, we must use the change in yaw angle $\Delta\phi$ and the change in wheel angle $\Delta\psi_w$ (along with wheel radius r_w) to calculate the new target position. From Figure 8 we can see that x_c and y_c must be modified as follows:

$$\begin{bmatrix} x_c[n+1] \\ y_c[n+1] \end{bmatrix} = \begin{bmatrix} \cos(-\Delta\phi) & -\sin(-\Delta\phi) \\ \sin(-\Delta\phi) & \cos(-\Delta\phi) \end{bmatrix} \begin{bmatrix} x_c[n] \\ y_c[n] \end{bmatrix} + \begin{bmatrix} 0 \\ -r_w\Delta\psi_f \end{bmatrix}$$

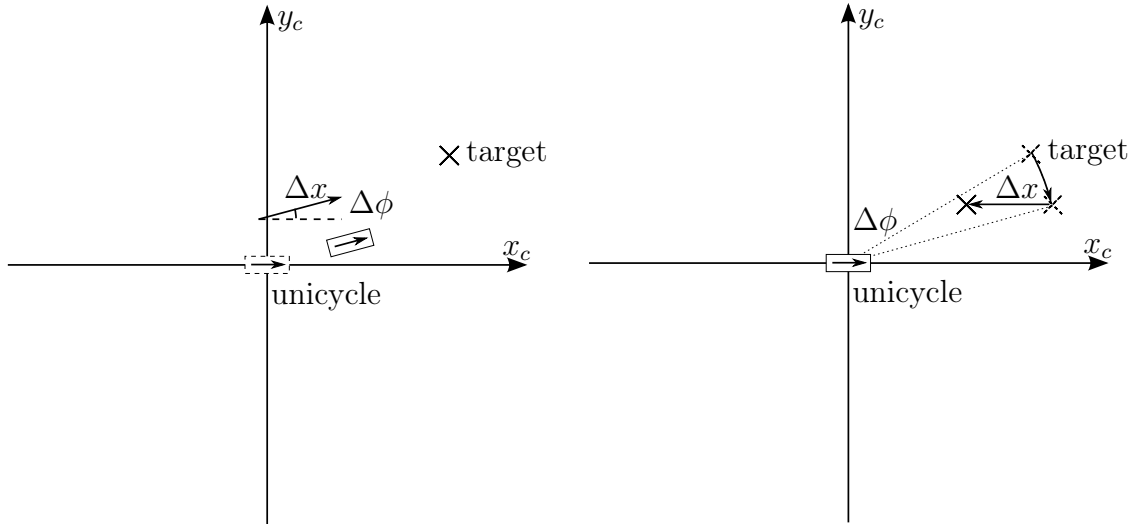


Figure 8: Effect of unicycle movement on target position in self-centred coordinates

The shaft encoder used to detect wheel angle changes has a quantisation error of around 1mm. The buildup in yaw error over the trial should be less than 1° , so when the distance from the target is around 1m, we can expect errors of around $1\text{mm} + 1^\circ \cdot \frac{\pi}{180} \cdot 1\text{m} \approx 2\text{cm}$. This should be more than accurate enough for the application.

3.1.3 Wheel Speed Sensing

When we started the project, a quadrature encoder was being used to measure the position and speed of the wheel. This is a digital sensor, which generates a pulse every time the wheel moves through $\frac{1}{512}$ of a revolution. Merely counting these pulses gives extremely good measurements of position (accurate to around 1mm) but it is harder to measure speed. The approach being used previously was to measure the number of pulses in 40ms, but this leads to a quantisation error of 25 pulses per second, or 2.5cm s^{-1} .

More details on how the quadrature encoders are used to sense position can be found in Douglass's report .

An alternative method is to measure the amount of time between the last two pulses, but at high speeds this time may be very short, and this time may be hard to measure accurately.

This became more important when we added a quadrature encoder for the flywheel: the sensors used are much less accurate than those on the wheel. There are only 72 pulses per revolution, and there is significant non-uniformity around the wheel. This means that the first method would have a quantisation error of $125^\circ/\text{s}$, and the second method would only be accurate to within around 50%. This is unacceptable accuracy.

After reading a comparative analysis of various methods [16], we decided on a method that combines the benefits of the two approaches above. It adapts the size of the 40ms window to contain an integer number of pulses, eliminating quantisation error. At high speeds, it is similar to the first scheme, considering the length of many pulses to reduce the effect on non-uniformity and timing errors. At sufficiently low speeds, it adapts to only using the most recent pulse, avoiding quantisation error. Figure 9 explains how it works—note that some thought is required about what should be returned when no pulses are observed in the window.

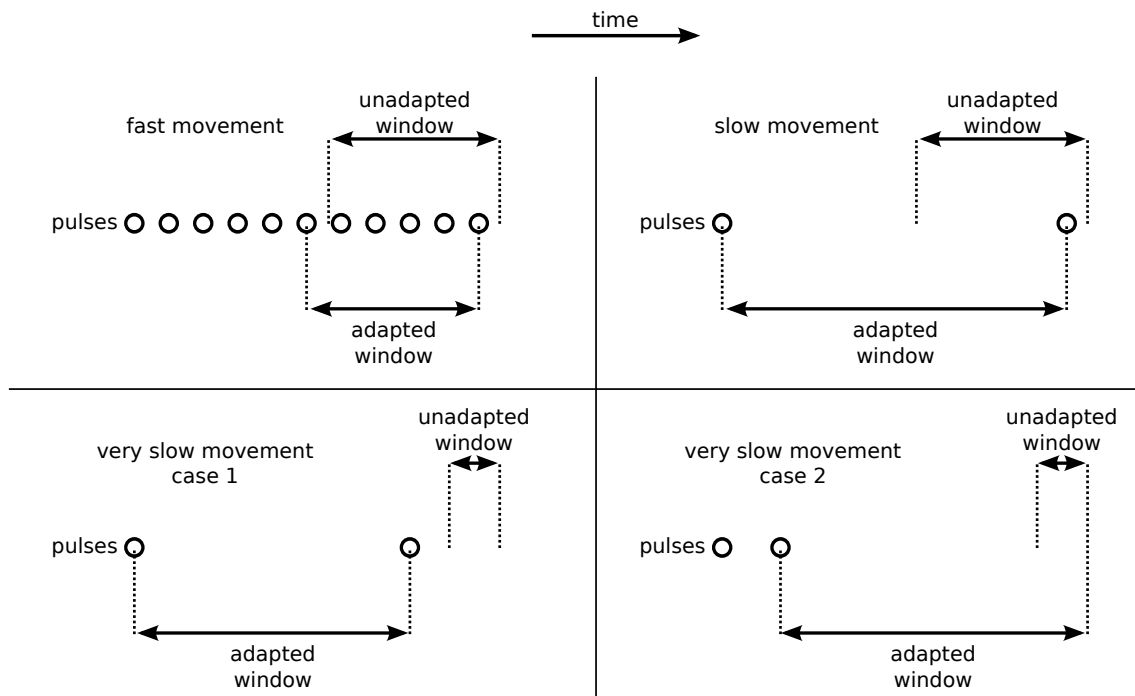


Figure 9: Adaptive-window encoder speed measurement

3.2 Fall Protection

Due to the trial-and-error nature of the RMLC method, the unicycle must be able to survive using a controller that may be worse than none at all. When the dynamics model is inaccurate, it may choose a controller that throws the 30kg unicycle towards the ground with all the force of a 240W motor. Figure 10 shows the effect of repeated trials on the aluminium bar used in the previous year.

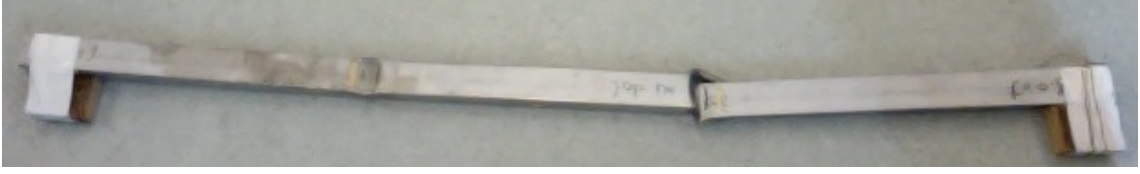


Figure 10: Damaged aluminium box section from previous year's testing

Two approaches had been used previously: suspending the unicycle from a crane in the Structures Lab, and equipping the unicycle with a support structure that hits the ground and stops the fall before it becomes dangerous. Suspending the unicycle limits its range of motion, as if it travels too far the rope will pull it over, as shown in Figure 11(b).

We suspected that the ability to travel freely would be important for the unicycle, since if an RMLC system is unable to experience control policies that travel a long way, the dynamics model will never learn that this can happen and that it should try to avoid it. As such, we chose to build a wooden skirt which would travel with the unicycle and protect it from falling, as shown in Figure 11(a). More details on the design of the skirt, including a CAD drawing of the mounted skirt, can be found in Douglass's report.

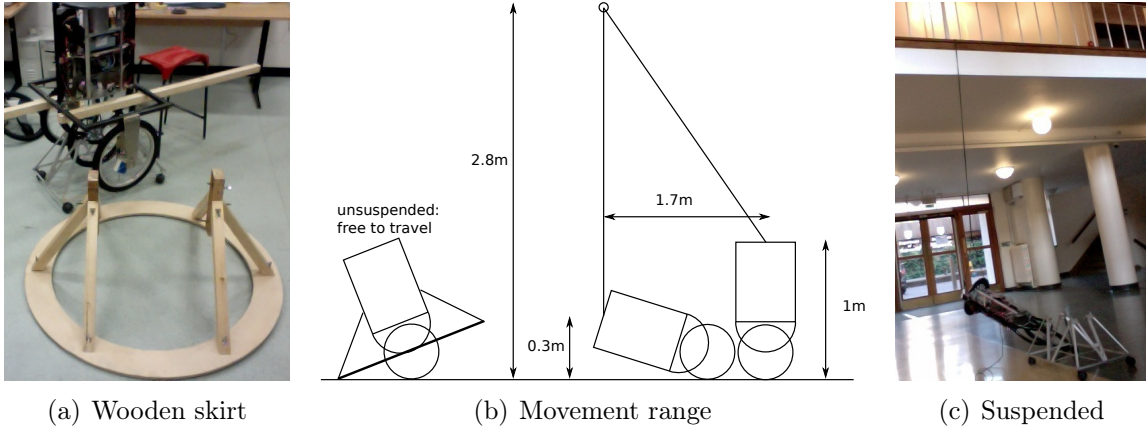


Figure 11: Unicycle fall protection methods

3.3 Inability to Spin

After building the skirt and preparing the necessary hardware and software, we started training the unicycle. However, it made very little progress, and would consistently seem to fall to the side, without turning in that direction and driving forwards to pick itself up. We tried modifying the controller to send the maximum motor command to the flywheel at all times, and found that it could only turn around 30° before falling.

It is very important that the robot be able to turn sufficiently fast to be able to point in the direction in which it is falling, before it has fallen. We considered various possible

causes for the slow turning:

- Adding the skirt had increased the moment of inertia, making it spin slowly.
- Friction with the ground was causing it to turn too slowly.
- The gear ratio for the flywheel was chosen badly.
- The flywheel had insufficient moment of inertia to turn the robot fast enough, even without the above effects.
- The flywheel motor was insufficiently powerful, even without the above effects.

To try to quantise this, we suspended the robot above the ground, and tested the response of the system to a step increase in flywheel command. We then used a mathematical model to quantify how changing various variables would affect the spin speed.

3.3.1 Spin Analysis

The response of a DC motor can be well modelled as [17]:

$$\frac{\Omega_{\text{motor}}(s)}{V(s)} = \frac{K_t}{(R + Ls)(Js + b) + K_t^2}$$

where we define

ω Speed of motor shaft

V Motor input voltage

K_t Motor torque constant

R Motor armature resistance

L Motor armature inductance (usually negligible)

J Moment of inertia of object on motor shaft

b Damping on motor shaft (sometimes negligible)

The effect of the gear train is analogous to that of a transformer: the flywheel MoI, J_f appears to the motor as an MoI of $J = \frac{J_f}{n^2}$. More obviously, it has the effect that $\omega_{\text{flywheel}} = \frac{\omega_{\text{motor}}}{n}$.

So, neglecting L and b , we have:

$$\begin{aligned} \frac{\Omega_{\text{flywheel}}(s)}{V(s)} &= \frac{K_t}{R \frac{J_f}{n^2} s + K_t^2} \\ &= n^{-1} K_t^{-1} \frac{1}{\frac{R J_f}{n^2 K_t^2} s + 1} \end{aligned}$$

The corresponding step response is of the form:

$$\omega_{\text{flywheel}}(t) = K(1 - e^{-\frac{t}{\tau}})$$

Fitting this to the observed step response, as in Figure 12(a), tells us that $K = 33$ and $\tau = 0.55$ seconds.

If we integrate to get the flywheel angle, and then assume that there are no external torques on the robot and that the robot has MoI J_r :

$$\begin{aligned}\omega_{\text{flywheel}}(t) &= K(1 - e^{-\frac{t}{\tau}}) \\ \theta_{\text{flywheel}}(t) &= K(t + \tau(e^{-\frac{t}{\tau}} - 1)) \\ \theta_{\text{robot}}(t) &= \frac{J_f}{J_r} K(t + \tau(e^{-\frac{t}{\tau}} - 1))\end{aligned}$$

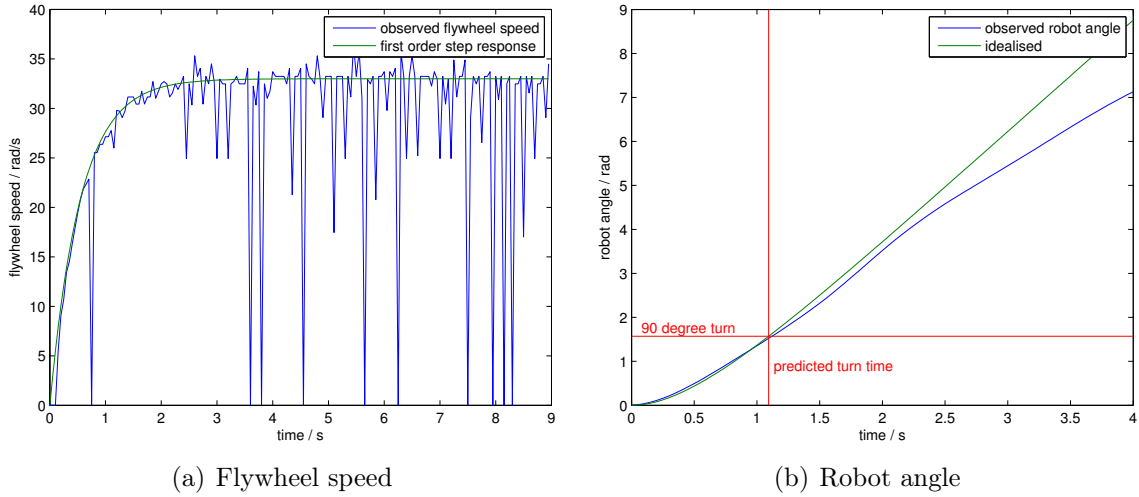


Figure 12: Response to step in flywheel command

By fitting (see Figure 12(b)) we get $\frac{J_r}{J_f} = 13$ (including the skirt). As the angle increased, the rope supporting the unicycle twisted, slowing the robot's spin, but the initial fit is good. We can now solve for the time to turn 90° . This model predicts a turn time of 1.1 seconds for the suspended robot: this is far too slow, and so clearly contact with the ground is not entirely to blame—we must look elsewhere to solve the problem.

We can check the observed value of $\frac{J_r}{J_f}$ by estimating the moments of inertia. Modelling the flywheel as a uniform disc of mass 8kg, the robot as a uniform cuboid of mass 24kg, and the skirt as mass concentrated at 55cm (the ring) and 40cm (the struts) from the

body:

$$\begin{aligned}
J_f &= \frac{1}{2}MR^2 = \frac{1}{2} \cdot 8 \cdot (0.2)^2 \\
&= 0.16 \text{ kg m}^2 \\
J_r &= \frac{1}{12}ML^2 = \frac{1}{12} \cdot 24 \cdot (0.4)^2 \\
&= 0.32 \text{ kg m}^2 \\
\text{mass of skirt} &= 4.4 \text{ kg} \\
\text{mass of struts} &= 3.5 \text{ kg} \\
J_{\text{skirt}} &= 3.5 \cdot 0.4^2 + 4.4 \cdot 0.55^2 \\
&= 1.89 \text{ kg m}^2 \\
\frac{J_r}{J_f} &= 2 \\
\frac{J_r + J_{\text{skirt}}}{J_f} &= 13.8
\end{aligned}$$

This backs up the result of the previous analysis, which suggested $\frac{J_r + J_{\text{skirt}}}{J_f} = 13$. With this model, we can predict the effect on the turn time of various changes to the suspended robot—Table 3.3.1 shows various possibilities, and makes it clear that reducing or removing the skirt is most valuable improvement to make.

gear ratio	flywheel MoI	robot MoI	turn time
1x	1x	1x	1.09s
1x	2x	1x	0.94s
1.6x (optimum)	2x	1x	0.87s
1x	10x	1x	0.85s
4.8x (optimum)	10x	1x	0.51s
1x	1x	$\frac{3.6}{13}$ x (lightweight skirt)	0.50s
1x	1x	$\frac{2}{13}$ x (removed skirt)	0.36s

Table 1: Ideal turn time under various conditions

3.3.2 Solving the Spin Problem

Having realised this, we redesigned the skirt to reduce its MoI. We reduced the width of the rim from 12cm to 2cm, and replaced the wooden struts with aluminium members. This reduced the MoI of the skirt by a factor of 7x, and could still support the weight of the unicycle. However, after a few falls, the wood had started to crack and one of the aluminium struts broke at the joint. Despite reinforcing the wood with glue and aluminium struts, we were unable to reach a design for the skirt that was both sturdy and light.



Figure 13: Reduced-weight skirt

Eventually, we chose to use a rope tied to the mezzanine in the foyer of the Engineering department. This is a very busy area, and so testing can only be done outside of the department’s normal opening hours, but at least the problematic effects of the skirt are completely removed.

3.4 Known Issues

Throughout the design process we found some problems with the unicycle’s hardware and software that we were unable to fix. When this occurred, we did our best to stop them from interfering with the testing process.

3.4.1 Toothed Belt

The robot uses a toothed belt to transfer torque to the wheel, shown in Figure 14. During tests, it would frequently slip when the controller demanded large changes in motor velocity. This leads to a mismatch between the encoder reading at the actual position, which would cause great problems for the dynamic model. As such, it was critical to make sure that we observed carefully for trials in which the belt slipped, and removed the corrupted data from the logs. Towards the end of the testing process, we installed an extra strut, also shown in Figure 14 to maintain tension in the belt.

3.4.2 Motor Control

Unpredictably, one or both of the motors would fail move at all. This proved extremely hard to debug, since sometimes even touching the circuit with a single probe of a voltmeter would cause them to start working again. At one time, we were able to investigate the problem with an oscilloscope—it appeared that the inputs to the motor controller were behaving correctly, and that while the PWM input went through to the output, both



Figure 14: Toothed belt and tensioning strut

terminals of the motor were driven with exactly the same PWM signal, with no potential difference across the motor.

The cause of this is unknown, but likely an electrical problem due to the unpredictable behaviour and sensitivity to contact from the voltmeter probe. Because we were unable to prevent it from happening, we modified the software to test both motors before every trial. This allowed us to power cycle the system to avoid the problem.

3.4.3 Flywheel Encoder

The flywheel encoder is prone to generating spurious pulses—these can manifest as sudden changes in direction, which are perceived as sudden stops by the speed-sensing algorithm described earlier. Figure 15 shows the effect of these spurious transmissions—clearly extremely severe, and very problematic for the dynamic model to predict.

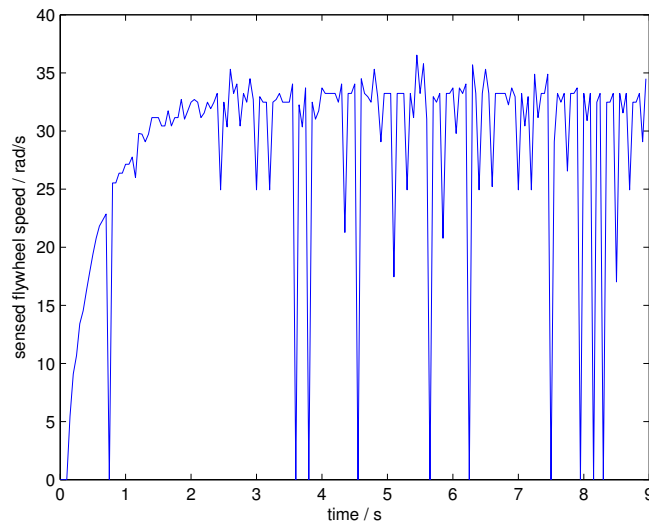


Figure 15: Corrupted flywheel speed measurements

As described in Douglass’s report, the flywheel encoder uses Schmitt triggers to digitise

the analog encoder outputs. Figure 16 shows a trace of the input and output to this signal cleaning circuit (the two traces have been traced in an image editor for clarity). There appears to be a negative spike in the signal, part-way through the high cycle. This is due to interference between the two quadrature sensors (covering one emitter removed the spike). Unfortunately, the low hysteresis voltage on the model of Schmitt trigger used (0.4V min., 0.7V typ.) means that this is enough to reach the threshold and cause problems. The extent of this problem was limited slightly by placing a larger piece of cardboard between the two sensors.

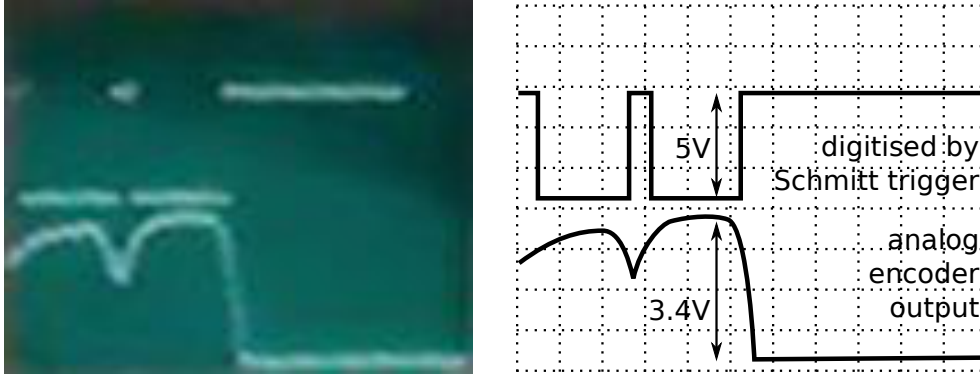


Figure 16: Oscilloscope trace of flywheel encoder outputs

The other problem visible is that the range of the analog signal is not 0–5V, but around 0–3.4V. This is because the photodiode in the sensor is not fully conducting when pointed at a white part of the pattern. To ensure that the Schmitt trigger acted properly, we reduced its supply voltage to 3.3V. While this meant that the input exceeded the supply voltage, and that the high-level output was 3.3V, yet the AVR (the processor on the Arduino) operates on 5V, both the Schmitt trigger [18] and the AVR’s [19] datasheets stated that these were acceptable. This does have the effect of further reducing the hysteresis voltage, but combined with the added occlusion, it helped control the problem.

Despite these changes, the problem is still very sensitive to the orientation of the sensors, and the shocks undergone during trials can cause it to resurface. To avoid this, we adapted the software to try to test for this problem before every trial, and we carefully inspected the logs after the trials, in an attempt to detect signs of corrupted data.

3.4.4 I²C Communications

The ITG-3200 and ADXL345 both reside on the same I²C bus, a digital communications protocol that allows the Arduino to read and write data to its peripherals. We chose to use the I²C driver provided by the Arduino project, which served us well initially. However, we found that occasionally, and unpredictably, the Arduino would stop responding. We narrowed this down to an infinite loop in the I²C driver, waiting for a response from the

ADXL345.

We were unable to identify or remove the cause of this problem, which could be anything from a race-condition in the software to damage to one of the involved ICs (soldering mistakes led to a number of short circuits and over-volting of the ICs during the development process). Fortunately, the problem occurred reasonably rarely, so we chose to implement timeouts in the I²C driver. Whenever one of these timeouts occurs, the error is signalled, and the data can be discarded.

4 Results of Learning

After implementing the necessary subset of the new hardware and software, we recreated the results of McHutchon’s work on the 2D system. This initially resulted in failure, and helped us to find bugs in our implementations and to understand the RMLC system. After iterating through a few sequences of trial and improvement, we had discovered various techniques that would help us stabilise the 3D unicycle. We then removed the training wheels, wrote and built the remaining parts of the unicycle and started training the 3D unicycle.

4.1 Learning the 2D System

McHutchon’s report describes the processes that the RMLC system goes through when successfully learning to balance the 2D system. Our aim in learning the 2D system was to verify that our sensors and controller worked correctly, and that we were using the machine learning system appropriately. We ran 3 training sequences before deciding to move onto the 3D unicycle.

Due to a calculation error, the first sequence used a loss function that very heavily penalised the distance moved (see Section 2.1.5)—a movement of 10cm was considered “as bad as” an angle of 10°. This led to a controller which would intentionally fall as fast as possible to stop the unicycle dead. Figure 17 shows the effect of this on the trajectory of the trials: by the end, the distance moved is consistently extremely small. Although the optimal controller would be one that kept both distance and angle close to zero, the problem is that intentionally falling is a local optimum: initially, it is more attractive to fall than to try to balance, and it never gathers any data suggesting that it can balance.

The second sequence fixed this error, and as shown in Figure 18, this sequence successfully generated controllers that could balance the unicycle for 15 seconds, with no sign of growing instability. (The blue lines show the prediction: the mean is dashed and the 95% confidence bounds are solid. The red line shows the trial result.) However, this is despite the fact that uncertainty the prediction grows extremely rapidly. Clearly, the dynamics model has managed to capture enough of the dynamics of the real system to

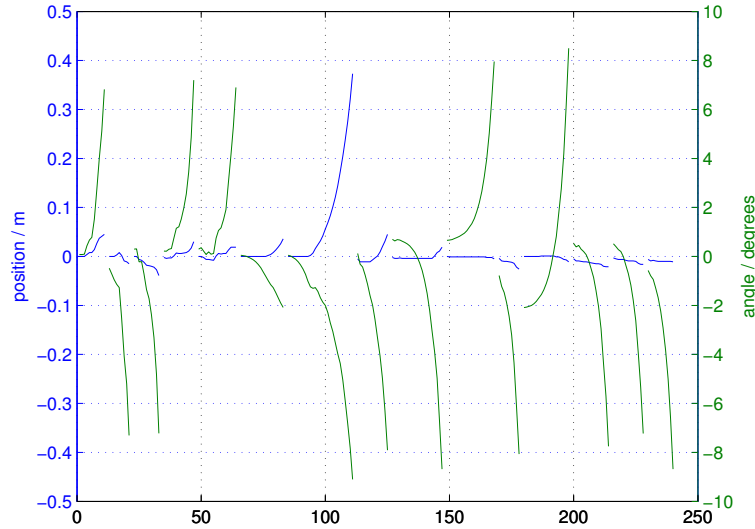


Figure 17: Trials from the first sequence

allow balancing the unicycle, yet is extremely uncertain about its predictions.

We suspected this was due to the high levels of noise in the sensors. In particular, the wheel speed is detected by differentiating the output of a digital position sensor, which dramatically amplifies the quantisation noise. McHutchon reported that decreasing the timestep (Section 2.1.6) help address problems with sensor noise, so we started another sequence with the timestep reduced from 100ms to 50ms. This produced the controller and prediction shown in Figure 19(a)—much improved, but still inadequate.

During trials, we observed the unicycle shaking, and a lot of noise coming from the gear-train. We suspected that problems like gear backlash were contributing to the poor performance of the system, and so we gave the dynamic model access to two consecutive states when predicting (as described in Section 2.1.2). This further improved the confidence of the prediction, shown in Figure 19(b) and the quality of the controller. Unfortunately, the log data for this trial was lost, but a video remains: the unicycle is observed to remain balanced, but to be in a limit cycle oscillation, slowly falling one way, before picking itself up and returning to slowly falling.

This poor performance of the resulting controller is likely due to the large dead-zone in the motor: on a PWM scale from -100% to 100%, there is a dead-zone from -10% to 10%. Figure 20 shows a trial exhibiting stable limit cycle behaviour, along with the motor command and dead-zone. The motor command is within the dead zone for quite a significant duration of the cycle. There are a number of possibilities for remedying this problem, such as manually compensating for the dead-zone, or allowing the RMLC system to use non-linear controllers that could learn to compensate for the dead-zone. Due to time constraints, however, we accepted the performance as satisfactory and moved on to the 3D unicycle.

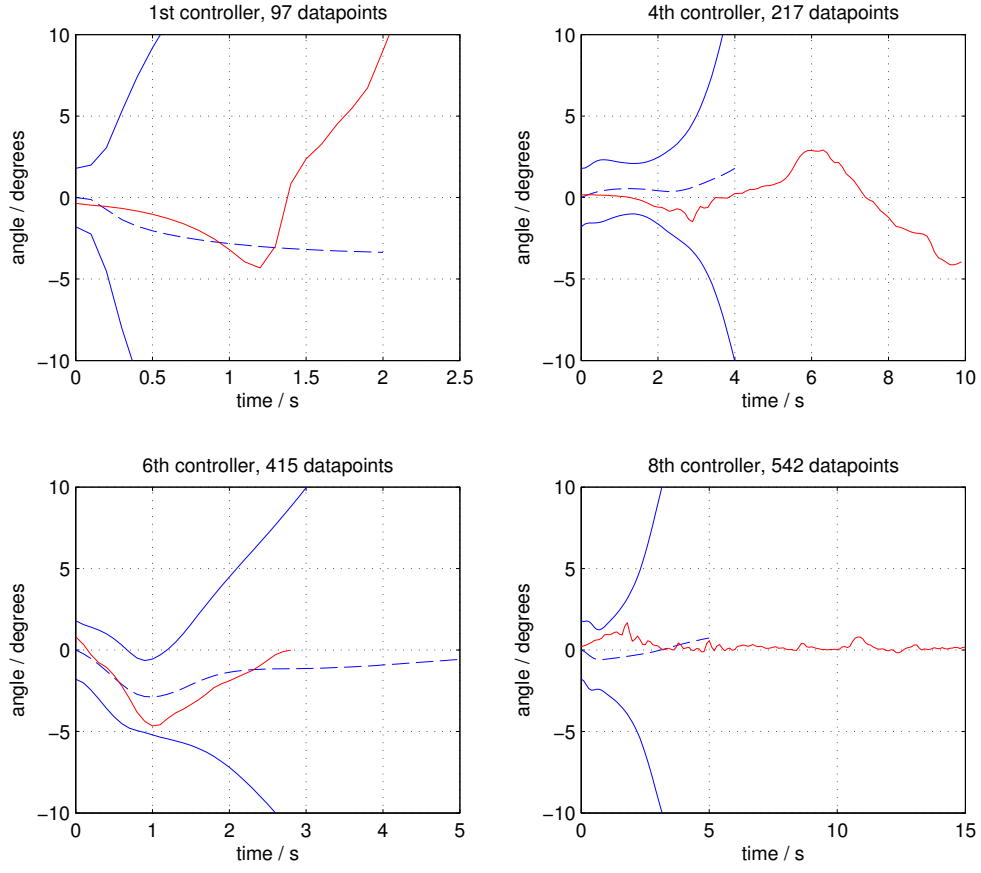


Figure 18: Sampled predictions and trials from 2nd training sequence

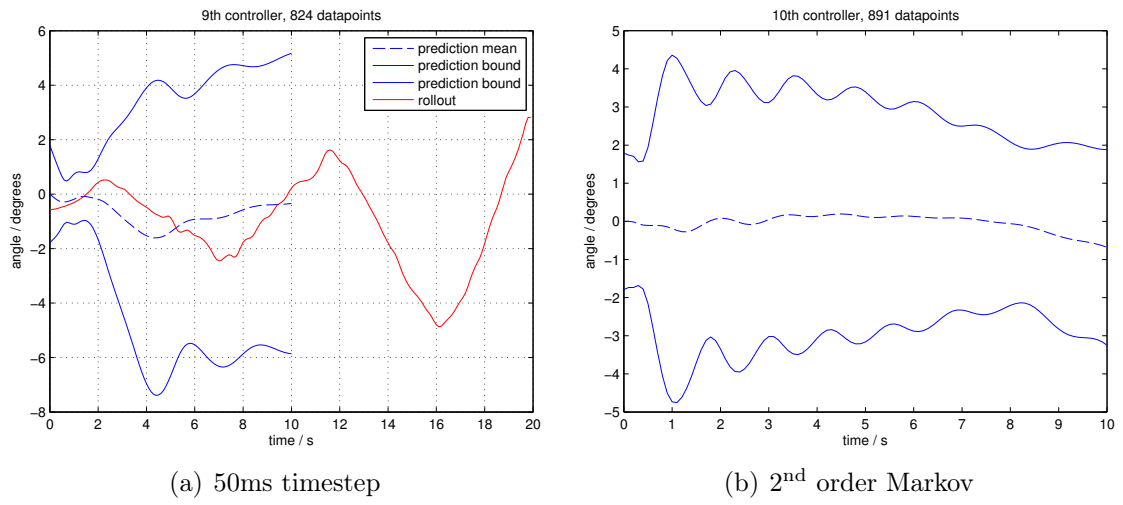


Figure 19: Final predictions of 2D training sequence

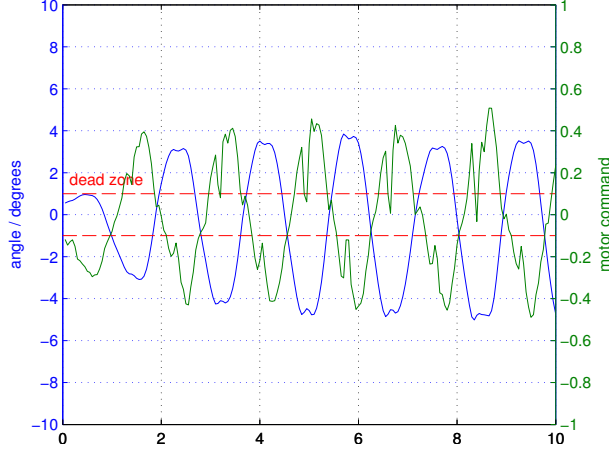


Figure 20: Trial exhibiting limit cycle behaviour

As a sidenote: there is a technique for dealing with motor dead-zones known as “dithering”, which involves adding a high-frequency periodic signal to the motor command, which will push the motor in and out of its dead-zone. The system will filter this rapidly oscillating force out, effectively smoothing the dead-zone out. McHutchon observed much better performing controllers for the 2D system—ironically, it is possible that the high noise levels of his angle sensors had a dithering effect on the motor, improving the performance.

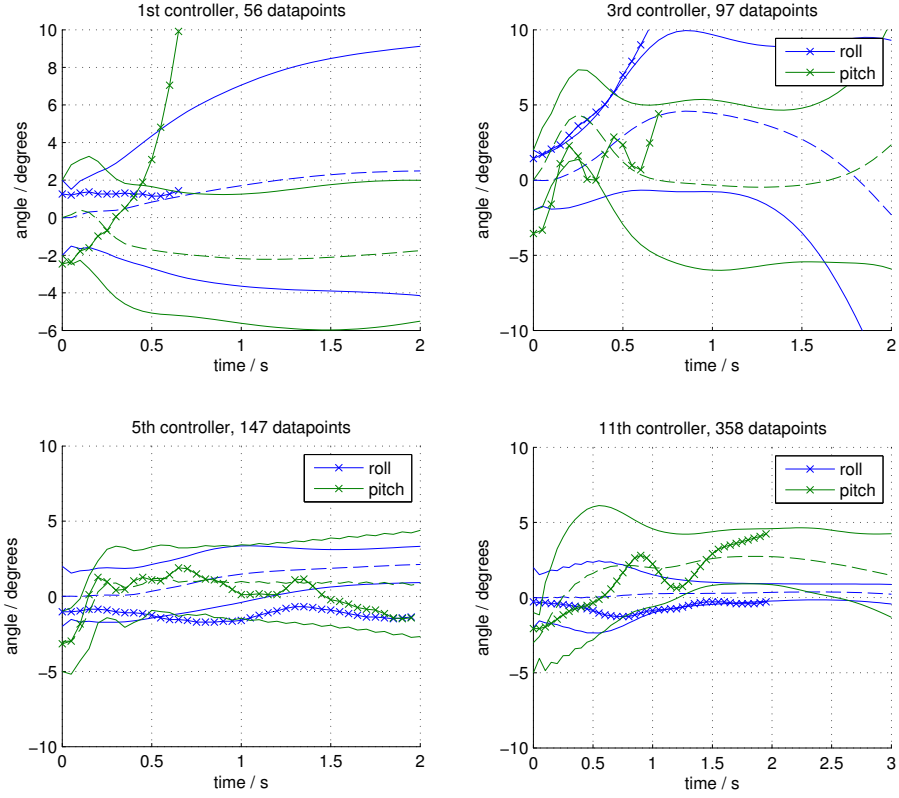
4.2 Learning the 3D Unicycle

Once all necessary work was complete, we initiated a training sequence for the 3D unicycle. As detailed in Section 3.2, we realised that we couldn’t use the skirt for fall protection, and decided to suspend the unicycle to protect it. We then iterated through a training sequence, and observed that the system learned to balance the robot very quickly.

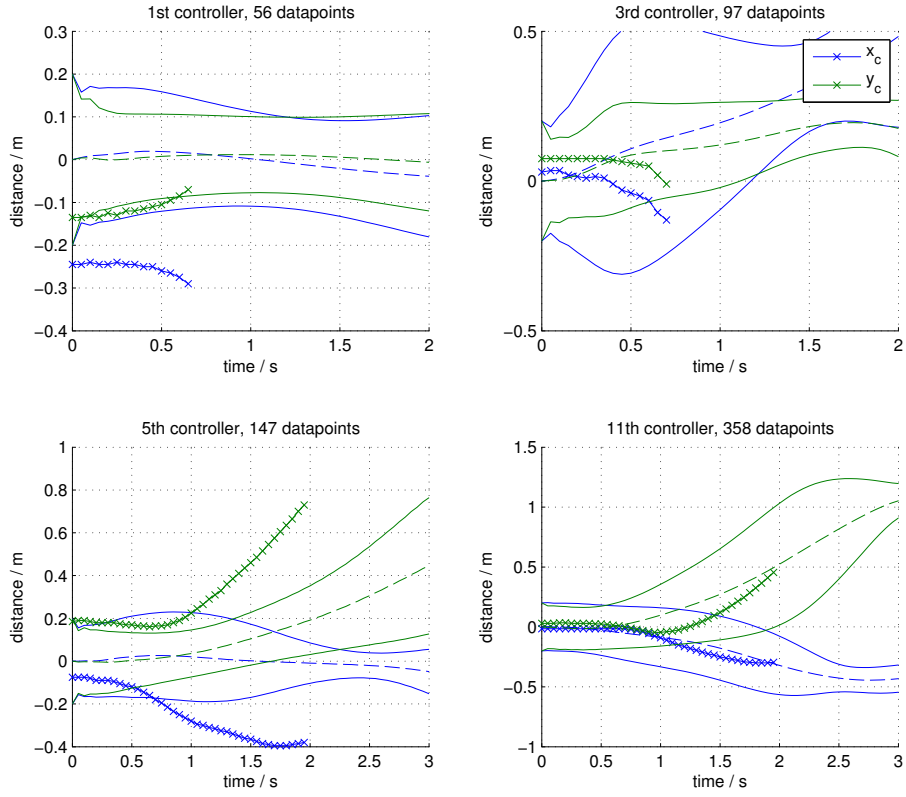
As shown in Figure 21(a), though initial trials disagreed greatly with the prediction, by the 3rd trial the system was learning to balance in pitch, and the trials start to resemble the predictions. By the 5th trial the unicycle could control both pitch and roll, and when the sequence ended after 11 trials, the system could confidently predict both roll and pitch angles.

However, the length of the trials is limited by the rope: once the unicycle travels a certain distance to the side, the rope becomes tight and pull the unicycle over, interfering with the dynamics in an unpredictable way. If the unicycle is to balance for any length of time it must learn to stay within this limited region. Figure 21(b) shows that the unicycle failed to achieve this: by the 11th controller, the dynamics model predicts reliably the position trajectory, but the system is unable to keep itself near the centre.

Upon investigation, we realised that there was an error in the position representation: the self-centred position integration described in Section 3.1.2 was not being performed



(a) Unicycle angles



(b) Unicycle position

Figure 21: 1st sequence of 3D trials

properly, and so there was no way for the quadratic controller to generate the correct commands to return to the origin. We fixed this and proceeded with a second training sequence, but it failed to learn a satisfactory dynamics model—we looked for causes, and found the problem with the flywheel speed sensor described in Section 3.4.3. We resolved this and proceeded with a 3rd sequence.

5 Conclusions

6 Further Work

References

- [1] D. Zenkov et al. The Lyapunov-Malkin theorem and stabilization of the unicycle with rider. *Systems and Control Letters*, 46:293–302, 2002.
- [2] Murata Manufacturing Co. Development of the unicycle-riding robot: Murata girl. Retrieved 18/05/2011 from http://www.murata.com/new/news_release/2008/0923.html, 2008.
- [3] D. Vos. Nonlinear control of an autonomous unicycle robot: Practical issues. MIT PhD Thesis, 1992.
- [4] Y. Naveh et al. Nonlinear modelling and control of a unicycle. *Dynamics and Control*, 9:279–296, 1999.
- [5] M. Mellors. Robotic unicycle: Mechanics & control. CUED Master’s Project, 2005.
- [6] A. Lamb. Robotic unicycle: Mechanics & control. CUED Master’s Project, 2005.
- [7] D. Forster. Robotic unicycle. CUED Master’s Project, 2009.
- [8] Carl Edward Rasmussen and Marc Peter Deisenroth. Recent advances in reinforcement learning. chapter Probabilistic Inference for Fast Learning in Control, pages 229–242. Springer-Verlag, Berlin, Heidelberg, 2008.
- [9] Marc P. Deisenroth and Carl E. Rasmussen. PILCO: A Model-Based and Data-Efficient Approach to Policy Search. In L. Getoor and T. Scheffer, editors, *Proceedings of the 28th International Conference on Machine Learning*, Bellevue, WA, USA, June 2011.
- [10] A. McHutchon. Machine learning for control. CUED Master’s Project, 2010.
- [11] Carl Edward Rasmussen. Gaussian processes for machine learning. MIT Press, 2006.

- [12] ITG-3200 data sheet.
- [13] J. Favre, B. M. Jolles, O. Siegrist, and K. Aminian. Quaternion-based fusion of gyroscopes and accelerometers to improve 3D angle measurement. *Electronics Letters*, 42(11):612–614, 2006.
- [14] Tom Pycke. gluonpilot’s attitude estimation. Retrieved 20/05/2011 from http://www.gluonpilot.com/wiki/Matlab_attitude_estimation, 2010.
- [15] A. Douglass. Machine learning for control. CUED Master’s Project, 2011.
- [16] R. Petrella, M. Tursini, L. Peretti, and M. Zigliotto. Speed measurement algorithms for low-resolution incremental encoder equipped drives: a comparative analysis. In *Electrical Machines and Power Electronics, 2007. ACEMP '07. International Aegean Conference on*, pages 780 –787, sept. 2007.
- [17] K. Lundberg. Notes on feedback systems. 6.302 Lecture Notes, 2008.
- [18] NXP HEF4093B data sheet.
- [19] ATmega1290/V data sheet.

A Evaluation of Risk Assessment

The main concern in our risk assessment was the mechanical risk to the unicycle, the operator and their surroundings. The trial-and-error nature of RMLC means that initial controllers may well have positive feedback, and the unicycle will throw itself toward the ground much faster than it would fall naturally. Additionally, even when this is not the case, the unicycle is composed of more than 30kg of steel in a very unstable configuration.

As described in the body of the report, a very large proportion of the work on this project was spent trying to determine a way to test the unicycle without risking it hitting the ground when it fell. Fortunately, all the methods used did successfully protect the unicycle during trials. Although the technique used to protect the unicycle changed (switching from the skirt to the rope) we were careful to ensure that the rope would not allow the unicycle to hit the ground or any bystanders.

When we were using the rope, the range of the unicycle was limited, but when using the skirt and testing the 2D system, it was free to travel as far as it wanted. This presented the risk of the unicycle building up speed and driving into a wall, or one of the desks in the Control Lab. Previously, the only way to disable the unicycle during a trial was to use a nearby computer—to address this, we added cutoff buttons to the unicycle, allowing us to quickly disable it before it damaged itself or the surroundings. This made testing easy and safe.

During the project, we discovered a risk that we didn’t foresee. When we left the unicycle on its stand in the Control Lab, the cleaner tried to move it. Since, even on the stand, the unicycle is difficult to handle, it was knocked over. Fortunately, the cleaner was not hurt, and damage to the unicycle and floor were inconsequential. To prevent this happening again, we left a “Do Not Touch” sign on the unicycle whenever it was unattended, and made sure that the unicycle was left safely out of the way.

There was also a small electrical risk in the project. Although the unicycle’s power supply is only 12V, it has a very high current capacity, and can produce sparks capable of welding contacts together. Previously, the key to the unicycle’s power switch had been lost, and to turn the power on, the unicycle had to be “hot-wired” by directly connecting the main power supply wires every time. Early in the project, we installed a power switch, removing this risk. We also endeavoured to insulate much of the exposed wiring on the unicycle to prevent short circuits.