States $S \{$ Sunny, Cloudy $\}$
Actions $\{$ Go to school, Stay at home $\}$
Discount $\gamma = 0.9$

	go to school	stay at home
S	+5	-5
C	+3	+1

$P_{school} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$ $R_{school} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ $R_{stay} \begin{bmatrix} -5 \\ 1 \end{bmatrix}$

$P_{stay} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$

Step #1

For Sunny
$$r\pi = 0.5 \times (5) + 0.5 \times (-5) = 2.5 - 2.5 = 0$$

For Cloudy
$$r\pi \quad 0.5 \times (3) + 0.5 \times (1) = 1.5 + 0.5 = 2$$

Step #2

Row 1 (Sunny)
- $P\pi (1,1) = 0.5 \times 0.8 + 0.5 \times 0.9 = 0.4 + 0.45 = 0.85$
- $P\pi (1,2) = 0.5 \times 0.2 + 0.5 \times 0.1 = 0.1 + 0.5 = 0.15$

Row 2 (Cloudy)
- $P\pi (2,1) = 0.5 \times 0.4 + 0.5 \times 0.3 = 0.2 + 0.15 = 0.35$
- $P\pi (2,2) = 0.5 \times 0.6 + 0.5 \times 0.7 = 0.3 + 0.35 = 0.65$

$$P\pi \begin{bmatrix} 0.85 & 0.35 \\ 0.15 & 0.65 \end{bmatrix}$$

Step 3

$V_1 = 0 + 0.9 (0.85 v_1 + 0.15 v_2)$

$V_1 = 0 + 0.765 v_1 + 0.135 v_2$

$V_1 = -0.765 v_1 = v_1 - 0.135 v_2 = 0$

$\boxed{0.235 v_1 - 0.135 v_2 = 0}$

$V_2 = 2 + 0.9 (0.35 v_1 + 0.65 v_2)$

$V_2 = 2 + 0.315 v_1 + 0.585 v_2$

$V_2 = -0.315 v_1 + -0.585 v_2 = 2$

$\boxed{-0.315 v_1 + 0.415 v_2 = 2}$

Step 4

$0.235 v_1 - 0.135 v_2 = 0 \qquad -0.315 v_1 + 0.415 v_2 = 2$

$0.235 v_1 = 0 + 0.135 v_2 \qquad\qquad -0.315 \left(\dfrac{0 + 0.135 v_2}{0.235} \right) + 0.415 v_2 = 2$

$\dfrac{0.235 v_1}{0.235} = \dfrac{0 + 0.135 v_2}{0.235} \qquad \left(-0.315 \times \dfrac{0}{.235} \right) \left(-0.315 \times \dfrac{.135}{.235} v_2 \right) + 0.415 v_2$

$\boxed{V_1 = \dfrac{0 + 0.135 v_2}{0.235}} \qquad \quad (-0.315 \times 0)(-0.315 \times \cancel{0.574} 0.425 v_2) + 0.4$

$\qquad\qquad\qquad\qquad\qquad\qquad 0.574 \qquad = 2$

$\qquad\qquad\qquad\qquad\qquad 0 - 0.180 v_2 + 0.415 v_2 = 2$

$\qquad\qquad\qquad\qquad\qquad 0 + (0.415 - 0.180) v_2 = 2$

$\qquad\qquad\qquad\qquad\qquad 0 + 0.235 v_2 = 2$

$\qquad\qquad\qquad\qquad\qquad 0.235 v_2 = 2 + 0$

$$V_R \text{ (Cloudy)} = \frac{2+0}{0.235} = \frac{2}{0.235} = 8.510$$

A

$$V_1 = \frac{0+0.135\,v_2}{0.235}$$

$$V_1 = \frac{0+0.135\,(8.510)}{0.235} = \frac{0+1.148}{0.235} = \frac{1.148}{0.235}$$

$$\boxed{V_R \text{ (Sunny)} = 4.885}$$

Step 5

5

Jan Sunny

$$V*(\text{sunny}) = \cancel{0.9} + 0.9\,(0.8\,v_1 + 0.2\,v_2)$$

$$V*(\text{Cloudy}) = 3 + 0.9\,(0.4\,v_1 + 0.6\,v_2)$$

Sunny:

$$V_1 = 5 + 0.72\,v_1 + 0.18\,v_2$$

$$V_1 = 0.72\,v_1 - 0.18\,v_2 = 5$$

$$\boxed{0.28\,v_1 - 0.18\,v_2 = 5}$$

Cloudy: $V_2 = 3 + \cancel{0.9} \; 0.36\,v_1 + 0.54$

$$\boxed{= -0.36\,v_1 + 0.46\,v_2 = 3}$$

step 6 (cloudy)

$$0.28 v_1 = 5 + 0.18 v_2$$

$$v_1 = \frac{5 + 0.18 v_2}{0.28}$$

Using cloudy : $-0.36 v_1 + 0.46 v_2 = 3$

$$-0.36 \left(\frac{5 + 0.18 v_2}{0.28} \right) + 0.46 v_2 = 3$$

$$-0.36 \times \left(\frac{5}{0.28} \right) = -\cancel{} 6 - 6.428$$

$$-0.36 \times \left(\frac{0.18}{28} \right) = -0.231 v_2$$

$-6.428 - 0.231 v_2 + 0.46 v_2 = 3$

$-6.428 - (0.46 - 0.231) v_2 = 3$

$-6.428 + 0.229 v_2 = 3$

$$0.\overset{229}{\cancel{299}} v_2 = 3 + 6.428 = 9.428$$

$$\cancel{0.2999} v_2 = 9.428$$

$$0.229$$

$$\boxed{v^* (\text{cloudy}) = \frac{9.428}{0.229} = 41.170}$$

step 6 (Sunny)

$$V_1 = \frac{5 + 0.18\, V_2}{0.28}$$

$$V_1 = \frac{5 + 0.18 \times 41.170}{0.28}$$

$$V_1 = \frac{5 + 7.41}{0.28} = \frac{12.41}{0.28}$$

$V_\# (Sunny) = 44.32$

$V_\# (Cloudy) = 41.170$

$s + (0.9)(0.8)$

$s + 0.9 \cancel{0.8}()$

$s + 0.9\,(0.8(44.32) + 0.2(41.170)$

Step 7

- $q(1, School) = 5 + 0.9\,(0.8\,v_1 + 0.2\,v_2) = 44.32$
- $q(1, Stay) = -5 + 0.9\,(0.9\,v_1 + 0.1\,v_2) = \cancel{44.604} \; 54.604$
- $q(2, School) = 3 + 0.9\,(0.4\,v_1 + 0.6\,v_2) = \cancel{44.170} \; 42.604$
- $q(2, Stay) = 1 + 0.9\,(0.3\,v_1 + 0.7\,v_2) = 38.903$