Lösungen zu Übung 005

1. Aufgabe:

a) $(a \vee \bar{b}) \wedge b = b \wedge (a \vee \bar{b}) = b \wedge a \vee b \wedge \bar{b} = b \wedge a \vee 0 = b \wedge a = a \wedge b$

(Kommutativgesetz; Distributivgesetz; Inverses Element; neutrales Element; Kommutativgesetz)

b) $a \wedge \bar{b} \vee b$ [Beachten Sie die Bindungsregel!]

$= b \vee a \wedge \bar{b} = (b \vee a) \wedge (b \vee \bar{b}) = (b \vee a) \wedge 1 = b \vee a = a \vee b$

c) $(a \vee b) \wedge (a \vee \bar{b}) = a \wedge a \vee a \wedge \bar{b} \vee b \wedge a \vee b \wedge \bar{b}$ [Distributivgesetz]

$= a \vee a \wedge \bar{b} \vee a \wedge b \vee 0$ (Absorption)

$= a \vee 0 = a$

2. Aufgabe:

a) $y = \overline{a \wedge b} \vee \bar{a} \vee \bar{c} \vee \overline{a \wedge \bar{b} \wedge c}$

$= \bar{a} \vee \bar{b} \vee \bar{a} \vee \bar{c} \vee \bar{a} \vee b \vee \bar{c}$ (de Morgan)

$= \bar{a} \vee \underbrace{\bar{b} \vee b}_{=1} \vee \bar{c}$ (Idempotenz, Element $\wedge$ Negation)

$= 1 \vee \bar{a} \vee \bar{c} = 1$ ($1 \vee a = a$)

b) $S = \overline{A \wedge B} \vee \overline{\bar{B} \wedge C} \vee A \wedge B$

$S = (A \wedge B) \wedge (\bar{B} \wedge C) \wedge (\overline{A \wedge B})$

$S = A \wedge \underbrace{B \wedge \bar{B}}_{=0} \wedge C \wedge (\bar{A} \vee \bar{B})$

$S = 0 \wedge A \wedge C \wedge (\bar{A} \vee \bar{B}) = 0$ ($0 \wedge a = 0$ Grundregel)

c) $e = a \wedge \bar{b} \wedge c \vee \underbrace{(\bar{b} \vee \bar{c}) \wedge (\bar{b} \vee \bar{d})}_{\text{ausdistribuieren}} \wedge \overline{(a \vee c \vee d)}$

$e = a \wedge \bar{b} \wedge c \vee \underbrace{\bar{b} \wedge \bar{b}}_{=\bar{b}} \vee \bar{b} \wedge \bar{d} \vee \bar{c} \wedge \bar{b} \vee \bar{c} \wedge \bar{d} \vee \bar{a} \wedge \bar{c} \wedge \bar{d}$ (Absorption)

$e = \bar{b} \vee \bar{c} \wedge \bar{d}$