1

$$\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 9 \\ 6 & 2 & 0 & -2 & 0 \\ -1 & 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow[\mathrm{I} + \mathrm{III} = \mathrm{III}_b]{6 \cdot \mathrm{I} + (-1) \cdot \mathrm{II} = \mathrm{II}_b} \begin{array}{c} \mathrm{I} \\ \mathrm{II}_b \\ \mathrm{III}_b \end{array} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 9 \\ 0 & 16 & 12 & 2 & 54 \\ 0 & 3 & 3 & 1 & 12 \end{array}\right)$$

$$\xrightarrow{(-3) \cdot \mathrm{II}_b + 16 \cdot \mathrm{III}_b = \mathrm{III}_c} \begin{array}{c} \mathrm{I} \\ \mathrm{II}_b \\ \mathrm{III}_c \end{array} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 9 \\ 0 & 16 & 12 & 2 & 54 \\ 0 & 0 & 12 & 10 & 30 \end{array}\right)$$

Sei $\boxed{x_4 = k}$

Aus III_c: $12x_3 + 10k = 30 \quad | -10k \quad :12$

$$\boxed{x_3 = 2{,}5 - \frac{10}{12}k}$$

Aus II_b: $16x_2 + 12 \cdot \left(2{,}5 - \frac{10}{12}k\right) + 2k = 54$

$$16x_2 + 30 - 10k + 2k = 54 \quad | -30 \quad +8k \quad :16$$

$$\boxed{x_2 = 1{,}5 + 0{,}5k}$$

Aus I: $x_1 + 3 \cdot (1{,}5 + 0{,}5k) + 2 \cdot \left(2{,}5 - \frac{10}{12}k\right) = 9$

$$x_1 + 4{,}5 + 1{,}5k + 5 - \frac{5}{3}k = 9 \quad | -9{,}5$$

$$x_1 - \frac{1}{6}k = -0{,}5 \quad | + \frac{1}{6}k$$

$$\boxed{x_1 = -\frac{1}{2} + \frac{1}{6}k}$$

$$\mathbb{L} = \left\{ \begin{pmatrix} -\frac{1}{2} + \frac{1}{6}k \\ \frac{3}{2} + \frac{1}{2}k \\ \frac{5}{2} - \frac{10}{12}k \\ k \end{pmatrix} \middle| \; k \in \mathbb{R} \right\}$$

②

a) keine Lösung.

↳ 3 Gleichungen im Widerspruch

$$2x_1 + 5x_2 = 1$$
$$2x_1 + 5x_2 = 2$$
$$2x_1 + 5x_2 = 3 \quad \Longrightarrow \text{keine Lösung}$$

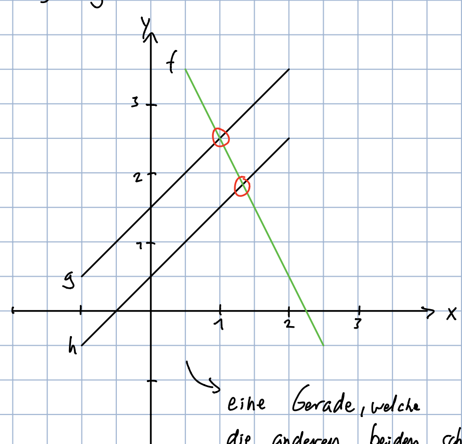
b) genau eine.

↳ x_1 und x_2 bestimmen

$x_1 = 4; \quad x_2 = 2$

$$x_1 + 3x_2 = 10$$
$$x_1 + x_2 = 6$$
$$x_1 + 2x_2 = 8 \quad \Longrightarrow \mathbb{L} = \left\{ \begin{matrix} 4 \\ 2 \end{matrix} \right\}$$

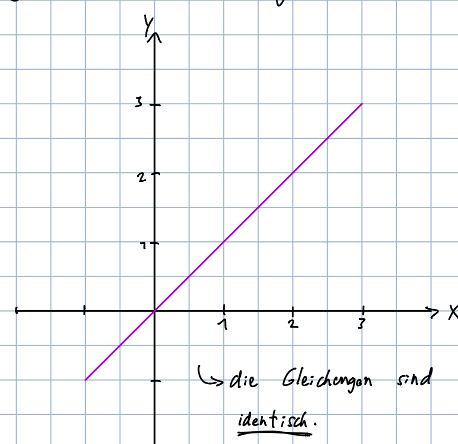
c) genau zwei.



↳ eine Gerade, welche die anderen beiden schneidet.

$g \parallel h$

d) unendlich viele Lösungen



↳ die Gleichungen sind <u>identisch</u>.

$g = h = f$

③

$$\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 2 & -\frac{5}{2} & b \end{array}\right) \begin{array}{c} \mathrm{I}-\mathrm{II}=\mathrm{II}_b \\ \mathrm{I}-\mathrm{III}=\mathrm{III}_b \\ \Longrightarrow \end{array} \begin{array}{c} \mathrm{I} \\ \mathrm{II}_b \\ \mathrm{III}_b \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & -1 & \frac{3}{2} & 1-b \end{array}\right)$$

$$\begin{array}{c} \mathrm{II}b + 2\cdot\mathrm{III}b = \mathrm{III}c \\ \Longrightarrow \end{array} \quad \begin{array}{c} \mathrm{I} \\ \mathrm{II}_b \\ \mathrm{III}_c \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 1-2b \end{array}\right)$$

Aus $\mathrm{III}c$: $1-2b = 0 \quad | -1 \; :(-2)$

$$\boxed{b = 0{,}5}$$

↳ Für $b=0{,}5$ sei x_3 beliebig.

Aus $\mathrm{II}b$: $2x_2 - 3x_3 = -1 \quad | +3x_3 \; :2$

$$\boxed{x_2 = -\frac{1}{2} + \frac{3}{2}x_3}$$

Aus I: $x_1 + \left(-\frac{1}{2} + \frac{3}{2}x_3\right) - x_3 = 1 \quad | +\frac{1}{2} \; -\frac{1}{2}x_3$

$$\boxed{x_1 = 1{,}5 - \frac{1}{2}x_3}$$

$\mathbb{L}$ für $b=0{,}5$: $\left\{\begin{pmatrix} \frac{3}{2} - \frac{1}{2}x_3 \\ -\frac{1}{2} + \frac{3}{2}x_3 \\ x_3 \end{pmatrix}\right\}$

(4)

a = Fett; b = Kamille; c = Zink

I. alles zusammen wiegt 32g

↳ $\boxed{a + b + c = 32}$

II. Fett wiegt 4 mal Kamille

↳ $\boxed{a = 4 \cdot b}$

III. Zink wiegt wie Kamille +2 gramm

↳ $\boxed{c = b + 2}$

! Variablen links, Zahlen rechts

II. $a = 4b \quad | -4b$

$\underline{\underline{a - 4b = 0}}$

III. $c = b + 2 \quad | - b$

$\underline{\underline{-b + c = 2}}$

! LGS

$$\begin{array}{l} \text{I.} \\ \text{II.} \\ \text{III.} \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 1 & -4 & 0 & 0 \\ 0 & -1 & 1 & 2 \end{array}\right) \xRightarrow{\text{I} - \text{II} = \text{IIb}} \begin{array}{l} \\ \text{IIb} \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 0 & 5 & 1 & 32 \\ 0 & -1 & 1 & 2 \end{array}\right) \xRightarrow{\text{IIb} + 5 \cdot \text{III} = \text{IIIb}} \begin{array}{l} \\ \\ \text{IIIb} \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 0 & 5 & 1 & 32 \\ 0 & 0 & 6 & 42 \end{array}\right)$$

Aus IIIb: $6 \cdot c = 42 \quad | :6$

$\boxed{c = 7}$

Aus IIb: $5 \cdot b + 7 = 32 \quad | -7 \; :5$

$\boxed{b = 5}$

Aus I: $a + 5 + 7 = 32 \quad | -12$

$\boxed{a = 20}$

$\mathbb{L} = \{$ Fett = 20 Gramm; Kamille = 5 Gramm; Zink = 7 Gramm $\}$