SUSTech DSP tutorial 11

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Q1

$X(z) = \frac{7}{1+0.3z^{-1}-0.1z^{-2}}$ can be partial fraction into $X(z) = \frac{5}{1+0.5z^{-1}} + \frac{2}{1-0.2z^{-1}}$

so， the inverse z-transforms could be

- $5 * (-0.5)^n \mu[n] + 2 * 0.2^n \mu[n]\ ROC = \{z : |z| > 0.5\}$
- $5 * (-0.5)^n \mu[n] - 2 * 0.2^n \mu[n]\ ROC = \{z : |z| > 0.5\} \cap \{z : |z| < 0.2\}$ don't exist
- $-5 * (-0.5)^n \mu[n] + 2 * 0.2^n \mu[n]\ ROC = \{z : 0.2 < |z| < 0.5\}$
- $-5 * (-0.5)^n \mu[n] - 2 * 0.2^n \mu[n]\ ROC = \{z : |z| < 0.2\}$

Q2

Using long division
$X(z) = \frac{1}{1-z^{-3}} = 1 + z^{-3} + z^{-6} + \cdots + z^{-3k} + \ldots$
so, $x[n] = \sum_{k=0}^{\infty} \delta[n-3k]$

Q3

let $X(z) = 1$, so the output of $H(z)$ is $H(z)$ itself,

$H_1(z) = 2.1 + 3.3z^{-1} + 0.7z^{-2}$
$H_2(z) = 1.4 - 5.2z^{-1} + 0.8z^{-2}$
$H_1(z) = 3.2 + 4.5z^{-1} + 0.9z^{-2}$

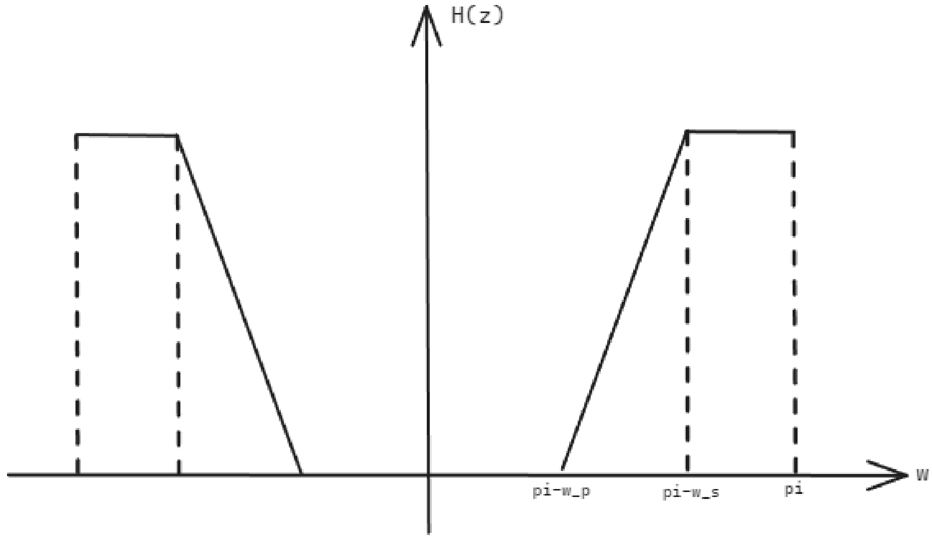
$H(z) = [H_1(z) + 1]H_2(z) + H_1(z)H_3(z) = 11.06 + 8.51z^{-1} + 5.28z^{-2} + 5.12z^{-3} + 1.19z^{-4}$

Q4

> For convenience, we call $H_{LP}(z)$ with $H(z)$ and $G_1(z)$ with $G(z)$

Frequency inversion in the Z-domain corresponds to a reflection of the frequency response about the unit circle. So, it is a right frequncy shift with π of $H(z)$

Sketch:

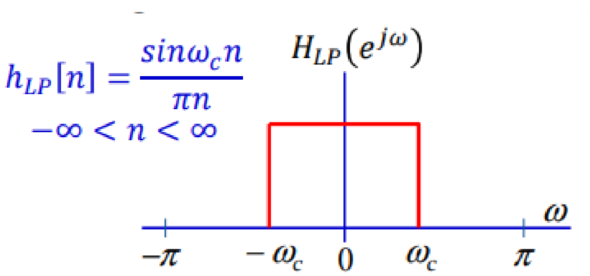


So,$G(z)$is a highpass filter, the bandedge is from $\pi-\omega_s$ to $\pi-\omega_p$, the stopband ripple is δ_p, the passband ripple is δ_s

$$G(z)=H(-z)=\sum_{n=-\infty}^{\infty} h[n](-z)^{-n}=\sum_{n=-\infty}^{\infty} h[n](z)^{-n}(-1)^{-n}=\sum_{n=-\infty}^{\infty} h\left[n_{\text{even}}\right](z)^{-n_{\text{even}}}-\sum_{n=-\infty}^{\infty} h\left[n_{\text{odd}}\right](z)^{-n_{\text{odd}}}$$

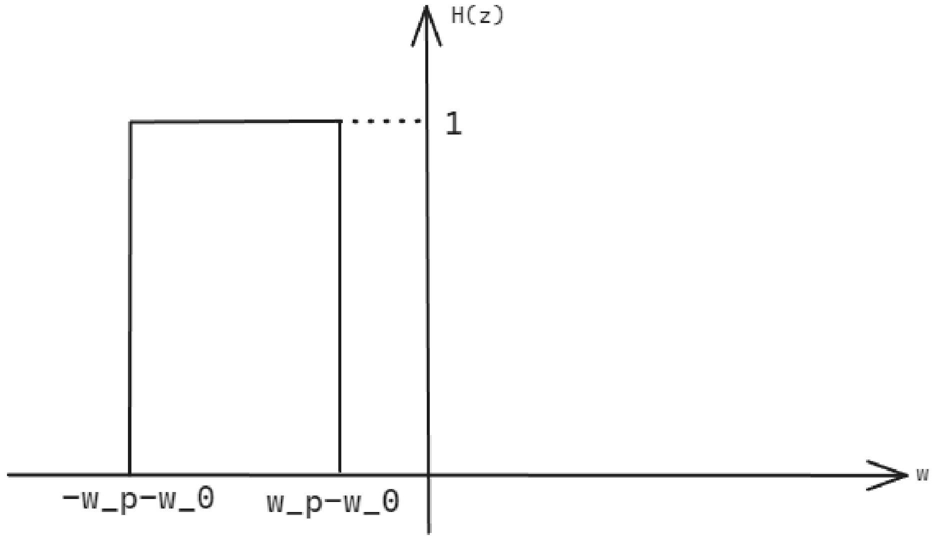
so, $g[n]=(-1)^n h[n]$

Q5

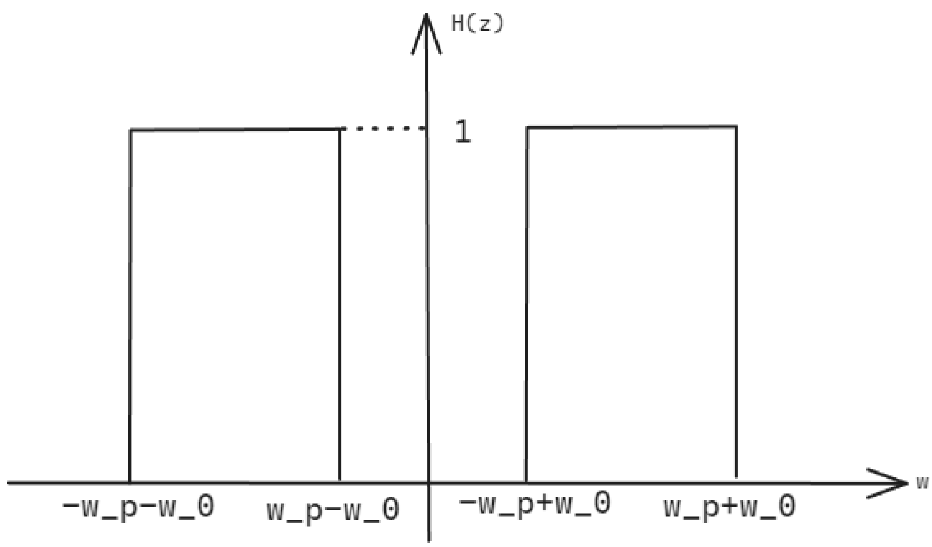


let $z=e^{j\omega}$, so we can analyse the magnitude response.
When $H(e^{j\omega}*e^{j\omega_0})=H(e^{j(\omega+\omega_0)})$, there is a left frequncy shift with ω_0 with $H(z)$

The Sketch of $H(e^{j\omega_0}z)$ is as follows:



$G(z)$ is two $H(z)$ shift two different side and add them together,so $g[n] = 2h[n] * cos(n\omega_0)$, since multiple with cos in time domain is frequency shift to tow sides. So the Sketch of $G(z)$ is as follows:



And the bandwith is $2\omega_p$

Q6

To make the output $y[n]$ a delayed and scaled replica of the input $x[n]$ in a discrete-time system with the transfer function $H_0(z) = 1 + \alpha z^{-1}$, you need to choose a feedback filter $F_0(z)$.

The output $y[n]$ is related to the input $x[n]$ through the following equation:

$$Y(z) = X(z) \cdot (H_0(z) \cdot F_0(z) - H_0(-z) \cdot F_0(-z))$$

In this case, we want $y[n]$ to be a delayed and scaled version of $x[n]$, which means:

$$Y(z) = c \cdot X(z) \cdot z^{-k}$$

where c is a scaling factor, and k is the delay.

Now, substitute $Y(z)$ into the equation:

$$c \cdot X(z) \cdot z^{-k} = X(z) \cdot (H_0(z) \cdot F_0(z) - H_0(-z) \cdot F_0(-z))$$

$$c \cdot z^{-k} = H_0(z) \cdot F_0(z) - H_0(-z) \cdot F_0(-z)$$

when $F_0(z) = H_0(z)$, the $H_0(z) \cdot F_0(z) - H_0(-z) \cdot F_0(-z) = 4\alpha z^{-1}$ satisfy the $c \cdot z^{-k}$.

Q7

take z-transform to Y :

$Y(z) = a_1 z^{-(k+1)} X(z) + a_2 z^{-k} X(z) + a_3 z^{-(k-1)} X(z) + a_2 z^{-(k-2)} X(z) + a_1 z^{-(k-3)} X(z)$

So the transfer function is :

$H(z) = a_1 z^{-(k+1)} + a_2 z^{-k} + a_3 z^{-(k-1)} + a_2 z^{-(k-2)} + a_1 z^{-(k-3)}$

find the frequency response $H(e^{j\omega})$ by replacing z with $e^{j\omega}$:

$H(e^{j\omega}) = a_1 e^{-j(k+1)\omega} + a_2 e^{-jk\omega} + a_3 e^{-j(k-1)\omega} + a_2 e^{-j(k-2)\omega} + a_1 e^{-j(k-3)\omega}$

When $k = 1$, the system is symmetric so all the imaginary part cancel out.