SUSTech DSP tutorial8

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Q1

(a)

DFT:

$$X[k]=\sum_{n=0}^{N-1}x[n]\cdot e^{-j(2\pi/N)kn}$$

let $n'=0,L,2L,,,(N-1)L, 0\leq n'\leq NL-L$, so $n=\frac{n'}{L}, 0\leq n\leq N-1$

$$Y[k]=\sum_{n=0}^{NL-1}y[n]\cdot e^{-j(2\pi/NL)kn}=\sum_{n'=0}^{(N-1)L}x[\frac{n'}{L}]\cdot e^{-j(2\pi/NL)kn'}$$

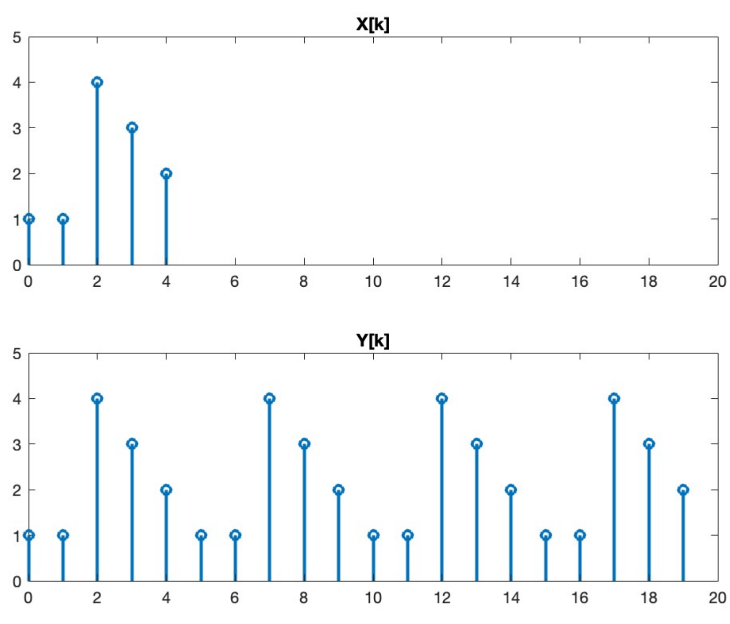
because $n=\frac{n'}{L}$, so

$$Y[k]=\sum_{n'=0}^{(N-1)L}x[\frac{n'}{L}]\cdot e^{-j(2\pi/NL)kn'}=\sum_{n=0}^{N-1}x[n]\cdot e^{-j(2\pi/N)kn}=X[k]$$

this means that when $0\leq k\leq N-1, y[k]=X[k]$

And $e^{-j(2\pi/N)kn}$ is periodic, when $N\leq k\leq LN-1$, the value of $e^{-j(2\pi/N)kn}$ will rotate to N samples before, that is when $N\leq k\leq LN-1$, $y[k]=X[\langle k\rangle_N]$

(b)



Q2

(a)

$y[n]=\{-3,4,-4,0,-7,8,2,-5,6\}, -5\leq n\leq 3$
So $y[-3]=-4$

(b)

$z[n] = \{-3, 4, -4, 0, -7, 8, 2, -5, 6\}, -5 \leq n \leq 3$
So $z[2] = -5$

Q3

$$x[n] = \{-3, 2, -1, 4\}, 0 \leq n \leq 3$$

$$h[\langle n-m \rangle_4] = \{1, 3, 2, -2, 1, 3, 2, -2, 1, 3, 2, -2, 1, 3, 2, -2\}, -8 \leq n \leq 7$$

$y[n] = \sum_{m=0}^{N-1} x[m]h[\langle n-m \rangle_4] =$
$\{(-3*1+2*-2+-1*2+3*4), (-3*3+2*1+-1*-2+4*2), (-3*2+2*3+-1*1+4*-2), (-3*-2+2*2+-1*3$ -
So $y[n] = \{3, 3, -9, 11\}, 0 \leq n \leq 3$

Q4

(a)

Without loss of generality, let length of $x[n]$ is 4, length of $h[n]$ is 3.
$y_L[n] = x[n] * h[n] = \sum_{k=0}^{7} x[k]h[k-n]$

> $y_L[0] = h[0]x[0]$
> $y_L[1] = h[0]x[1] + h[1]x[0]$
> $y_L[2] = h[0]x[2] + h[1]x[1] + h[2]x[0]$
> $y_L[3] = h[0]x[3] + h[1]x[2] + h[2]x[1]$
> $y_L[4] = h[1]x[3] + h[2]x[2]$
> $y_L[5] = h[2]x[3]$

$y_C[n] = \sum_{k=0}^{3} x[k]h[\langle k-n \rangle_4]$

> $y_C[0] = h[0]x[0] + h[1]x[3] + h[2]x[2]$
> $y_C[1] = h[0]x[1] + h[1]x[0] + h[2]x[3]$
> $y_C[2] = h[0]x[2] + h[1]x[1] + h[2]x[0]$
> $y_C[3] = h[0]x[3] + h[1]x[2] + h[2]x[1]$

So, $y_C[n] = y_L[n] + y_L[n+N]$, N is the length of $x[n]$

(b)

$y_L[n] = [-6, 22, -3, -54, 77, 9, -28, 63, -6, 13, 12]$

So, $y_C[n] = [-6-28, 22+63, -3-6, -54+13, 77+12, 9] = [-34, 85, -9, -41, 89, 9]$

Q5

> Correct: "determine $X[k]$ in terms of $G[k]$ and $X[k]$." -> "determine $X[k]$ in terms of $G[k]$ and $H[k]$."

According to Decimation-in-time FFT algorithm

$$G[k] = \frac{1}{2}(X_0[\langle k \rangle_{\frac{N}{2}}] + X_1[\langle k \rangle_{\frac{N}{2}}])$$

$$H[k] = \frac{1}{2}(X_0[\langle k \rangle_{\frac{N}{2}}] - X_1[\langle k \rangle_{\frac{N}{2}}])$$

and

$$X[k] = X_0[\langle k \rangle_{\frac{N}{2}}] + W_N^k X_1[\langle k \rangle_{\frac{N}{2}}]$$

So $X[k]=(1+W_N^k)G\left[\langle k\rangle_{N/2}\right]+(1-W_N^k)H\left[\langle k\rangle_{N/2}\right]$$

Q6

(a)

when N is even so that $N/2$ is an integer.

$$X[N/2] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j(2\pi/N)(N/2)n}$$

$$X[N/2] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\pi n}$$

$e^{-j\pi n}$ takes values of $(-1)^n$. therefore:

$$X[N/2] = \sum_{n=0}^{N-1} x[n] \cdot (-1)^n$$

separate the sum into two part, one is positive and the other is minus:

$$X[N/2] = \sum_{n=0}^{N-1} x[n] \cdot (-1)^n = \sum_{n=0}^{N/2-1} x[n] - \sum_{n=N/2}^{N-1} x[n]$$

since the [x[n]] is symmetric, the sum cancels out:

$$X[N/2] = 0$$

(b)

$x[n]$ is an antisymmetric sequence,so $x[n] = -x[\langle N-1-n \rangle_N]$, so:

$$X[k] = \sum_{n=0}^{N-1} (-x[\langle N-1-n \rangle_N]) \cdot e^{-j(2\pi/N)kn}$$

when $k=0$:

$$X[0] = \sum_{n=0}^{N-1} (-x[\langle N-1-n \rangle_N]) \cdot e^{-j(2\pi/N)\cdot 0 \cdot n}$$

$$X[0] = \sum_{n=0}^{N-1} (-x[\langle N-1-n \rangle_N])$$

Now, the sum of an antisymmetric sequence over a symmetric range is zero because the positive and negative terms cancel each other out, that is:

$$X[0] = 0$$

(c)

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j(2\pi/N)kn}$$

substitute n with $2l$:

$$X[2l] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j(2\pi/N)(2l)n} = \sum_{n=0}^{N-1} x[n] \cdot e^{-j(4\pi/N)ln}$$

because $x[n] = -x[\langle n+M \rangle_N]$:

$$X[2l] = \sum_{n=0}^{N-1} (-x[\langle n+M \rangle_N]) \cdot e^{-j(4\pi/N)ln}$$

since $N = 2M$, we have:

$$X[2l] = \sum_{n=0}^{N-1} (-x[\langle n+M \rangle_{2M}]) \cdot e^{-j(4\pi/(2M))ln}$$

and $\langle n+M \rangle_{2M}$ is equivalent to $n + M - 2M = n - M$ for $0 \leq n \leq 2M - 1$. so:

$$X[2l] = \sum_{n=0}^{N-1} -x[n-M] \cdot e^{-j(4\pi/(2M))ln}$$

for $l = 0, 1, \ldots, M-1$, the term $e^{-j(4\pi/(2M))ln}$ will be periodic with a period of $2M/l$, and since $N = 2M$, the period is N/l. Therefore, the sum over n will include terms that form a geometric series with a common ratio of $e^{-j(4\pi/(2M))l(N/l)} = e^{-j2\pi l} = 1$.

$$X[2l] = \sum_{n=0}^{N-1} -x[n-M] \cdot e^{-j(4\pi/(2M))ln} = \sum_{n=0}^{N-1} x[n-M]$$

Q7

DFT of $x[n] : X[k] = DFT([2, 1, 2, 0]) = [5, -2-i, 1, -2+i]$
DFT of $w[n] : X[k] = DFT([-4, 0-3, 2]) = [-5, -3-2i, 1, -3+2i]$

$Y[k] = \frac{W[k]}{X[k]} = [1, -1-i, 1, -1+i]$
so

$$y[n] = [-2, 1, -2, 1]$$