SUSTech DSP tutorial 11

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Q1

Type I: The impulse response is symmetric and N an odd integer

When $N_1 = 2, N_2 = 0, N_3 = 0, N_4 = 0$; $H(z) = (1 + z^{-1})^2 = 1 + 2z^{-1} + z^{-2}$
When $N_1 = 0, N_2 = 2, N_3 = 0, N_4 = 0$; $H(z) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2}$
When $N_1 = 0, N_2 = 0, N_3 = 1, N_4 = 0$; $H(z) = 1 + \alpha_i z^{-1} + z^{-2}$

Type II: The impulse response is symmetric and N an even integer

When $N_1 = 1, N_2 = 0, N_3 = 0, N_4 = 0$; $H(z) = 1 + z^{-1}$

Type III: The impulse response is anti-symmetric and N an odd integer

When $N_1 = 1, N_2 = 1, N_3 = 0, N_4 = 0$; $H(z) = (1 + z^{-1})(1 - z^{-1}) = 1 + 0z^{-1} - z^{-2}$

Type IV: The impulse response is anti-symmetric and N an even integer

When $N_1 = 0, N_2 = 1, N_3 = 0, N_4 = 0$; $H(z) = 1 - z^{-1}$

Q2

$\alpha = \frac{1-sin(\omega_c)}{cos(\omega_c)} = \frac{1-sin(0.42)}{cos(0.42)} = 0.648611018$

So $H(z) = \frac{1+\alpha}{2}\frac{1—z^{-1}}{1—\alpha z^{-1}} = \frac{0.1757(1+z^{-1})}{1-0.6486z^{-1}}$

Q3

(a)

$|H_{BS}(z)|^2 = (\frac{1+\alpha}{2})^2 * (\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}})^2 = (\frac{1+\alpha}{2})^2 * \frac{1-4\beta z^{-1}+(4\beta^2+2)z^{-2}-4\beta z^{-3}+z^{-4}}{\alpha^2-(2\alpha^2\beta-2\alpha\beta)z^{-1}+(\alpha^2\beta^2+2\alpha\beta^2+2\alpha+\beta^2)z^{-2}-(2\alpha\beta+2\beta)z^{-3}+z^{-4}}$

(b)

$|H_{BS}(e^{j\omega})|^2 = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{1-4\beta e^{-j\omega}+(4\beta^2+2)e^{-2j\omega}-4\beta e^{-3j\omega}+e^{-4j\omega}}{\alpha^2-(2\alpha^2\beta-2\alpha\beta)e^{-j\omega}+(\alpha^2\beta^2+2\alpha\beta^2+2\alpha+\beta^2)e^{-2j\omega}-(2\alpha\beta+2\beta)e^{-3j\omega}+e^{-4j\omega}} = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{2\cos 2\omega-8\beta\cos\omega+2+4\beta^2}{2\alpha\cos 2\omega-2\beta(1+\alpha)^2\cos\omega+1+\alpha^2+\beta^2(1+\alpha)^2}$

when $\omega = cos^{-1}(\beta)$, $|H_{BS}(e^{jcos^{-1}(\beta)})|^2 = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{2\cos 2cos^{-1}(\beta)-8\beta\cos cos^{-1}(\beta)+2+4\beta^2}{2\alpha\cos 2cos^{-1}(\beta)-2\beta(1+\alpha)^2\cos cos^{-1}(\beta)+1+\alpha^2+\beta^2(1+\alpha)^2} = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{0}{\dots} = 0$

(c)

$|H_{BS}(e^{j\omega})|^2 = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{2\cos 2\omega-8\beta\cos\omega+2+4\beta^2}{2\alpha\cos 2\omega-2\beta(1+\alpha)^2\cos\omega+1+\alpha^2+\beta^2(1+\alpha)^2}$

when $\omega = 0$, $|H_{BS}(e^{j\omega})|^2 = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{4}{1+2\alpha+\alpha^2} = 1$

when $\omega = \pi$, $|H_{BS}(e^{j\omega})|^2 = \left(\frac{1+\alpha}{2}\right)^2 \cdot \frac{4}{1+2\alpha+\alpha^2} = 1$

(d)

$|H_{BS}(e^{j\omega})|^2 = \frac{1}{2} \Rightarrow B_\omega = cos^{-1}(\frac{2\alpha}{1+\alpha^2})$

Q4

$cos^{-1}(\beta) = 0.35\pi \Rightarrow \beta = cos(0.35\pi) = 0.4539905$

$B_w = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) = 0.15\pi \Rightarrow \alpha^2 + 1 - \frac{1}{cos(0.15\pi)}\alpha = 0 \Rightarrow \alpha = e^{-0.31\pi}$

So $H_{BS}(z) = \frac{1+e^{-0.31\pi}}{2} * \frac{1-0.9z^{-1}+z^{-2}}{0.55(1+e^{-0.31\pi})z^{-1}+e^{-0.31\pi}z^{-2}}$

Q5

$|A_M(e^{j\omega})|^2 = A_M(e^{j\omega}) \cdot A_M(e^{-j\omega}) = 1$
So, it is a allpass filter

As for causality and stability, similar to real-coefficient allpass transfer function, the root of the denominator is also within the unit circle, the poles and zeros are conjugate pairs.

Q6

$H_1(z) = 2.5(1-1.6z^{-1}+2z^{-2})(1+1.6z^{-1}+z^{-2})(1+z^{-1})(1-0.8z^{-1}+0.5z^{-2})$

(a)

the only zero outside the unit circle is come from $1 - 1.6z^{-1} + 2z^{-2}$, so substitude it with $2 - 1.6z^{-1} + 1z^{-2}$
$H_2(z) = H_1(z)\frac{2-1.6z^{-1}+1z^{-2}}{1-1.6z^{-1}+2z^{-2}} = 2.5(2 - 1.6z^{-1} + 1z^{-2})(1 + 1.6z^{-1} + z^{-2})(1 + z^{-1})(1 - 0.8z^{-1} + 0.5z^{-2})$

(b)

the only zero inside the unit circle is come from $1 - 0.8z^{-1} + 0.5z^{-2}$, so substitude it with $0.5 - 0.8z^{-1} + 1z^{-2}$
$H_3(z) = H_1(z)\frac{0.5-0.8z^{-1}+1z^{-2}}{1-0.8z^{-1}+0.5z^{-2}} = 2.5(1 - 1.6z^{-1} + 2z^{-2})(1 + 1.6z^{-1} + z^{-2})(1 + z^{-1})(0.5 - 0.8z^{-1} + 1z^{-2})$

(c)

there is no more combination of poles and zeros to satisfy the magnitude response.

Q7

To fix the sound issues caused by the channel, we need a special filter at the receiving end. This filter, let's call it $G(z)$, should make the overall sound transfer, $H(z)G(z)$, have an even volume across different frequencies.

The channel's sound behavior is described by $H(z)$, and it's a bit wonky:

$$H(z) = \frac{(2.2 + 5z^{-1})(1 - 3.1z^{-1})}{(1 + 0.81z^{-1})(1 - 0.62z^{-1})}$$

To create $G(z)$, we kind of do the opposite of what the channel does. So, $G(z)$ is like the "flip side" of $H(z)$:

$$G(z) = \frac{(1 + 0.81z^{-1})(1 - 0.62z^{-1})}{(2.2 + 5z^{-1})(1 - 3.1z^{-1})}$$

This $G(z)$ acts as a sort of equalizer, balancing out the sound, making everything sound more natural and clear.

Q8

$0 < K < 0.5$

Q9

$$C(z) = \frac{0.3+0.1167z^{-1}-0.4533z^{-2}-1.0717z^{-3}-0.9338z^{-4}-0.4819z}{z^{-1}+2.35z^{-2}+2.925z^{-3}+1.6875+0.5063z^{-5}}$$