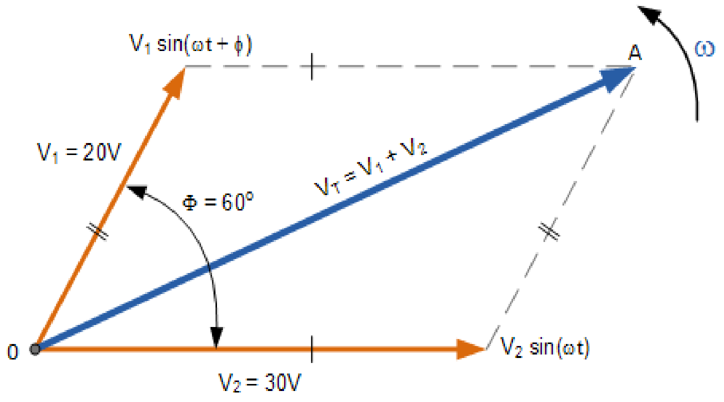
通信原理
习题课

Assignment No. 7

As its name suggests, phasor diagram is used to represent the phase difference between the sinusoidal signals which have the same frequency but different phase and amplitude. Using this phasor diagram, it is easy to analyze different sinusoidal signal signals which have the same frequency. 2017年9月10日

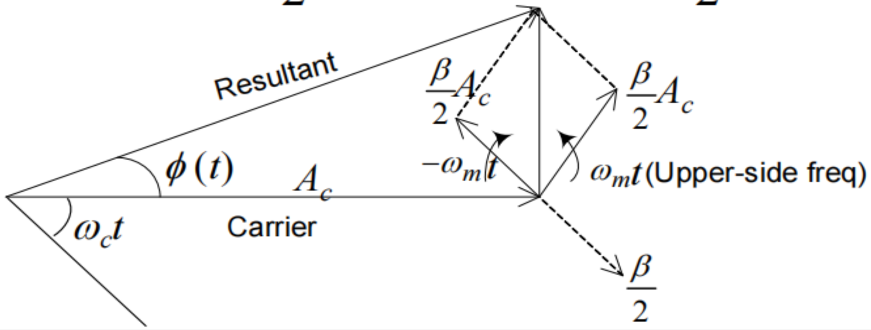


Phasor Representation

$$f_{NBFM}(t) = A_c \cos \omega_c t + \frac{\beta A_c}{2} \cos(\omega_c + \omega_m)t - \frac{\beta A_c}{2} \cos(\omega_c - \omega_m)t$$

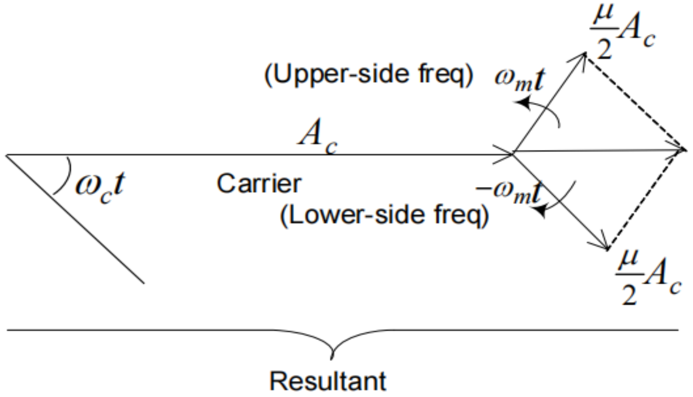
$\omega_m t$ 可以看成是随时间变化的相位

NBFM:



$$f_{AM}(t) = A_c \cos\omega_c t + \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t$$

AM:



https://www.electronics-tutorials.ws/accircuits/phasors.html

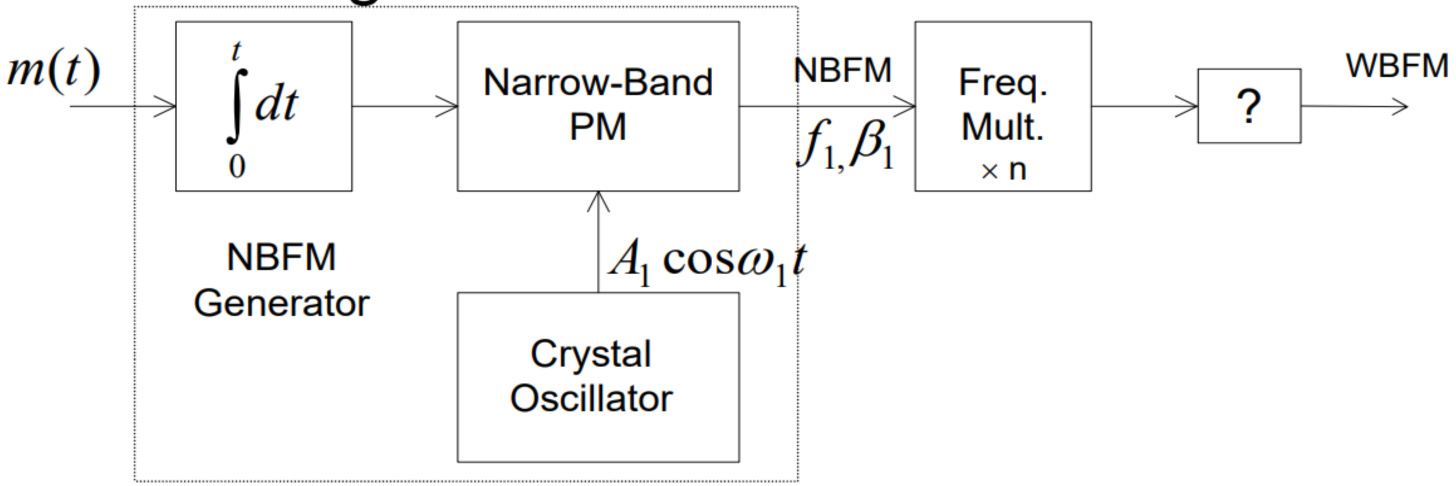
Generation of WBFM signals

- Indirect Method (multi-stage)
  - The message signal is first used to generate an NBFM signal, and frequency multiplication is used next to increase frequency deviation (hence modulation index β) to produce a WBFM signal.
- Direct Method (one stage)
  - The instantaneous freq. of the carrier signal (oscillator's output frequency) is varied directly in accordance with the message signal. That is,

$$f_i(t) = f_c + k_f m(t).$$

Indirect Method (Armstrong Method)

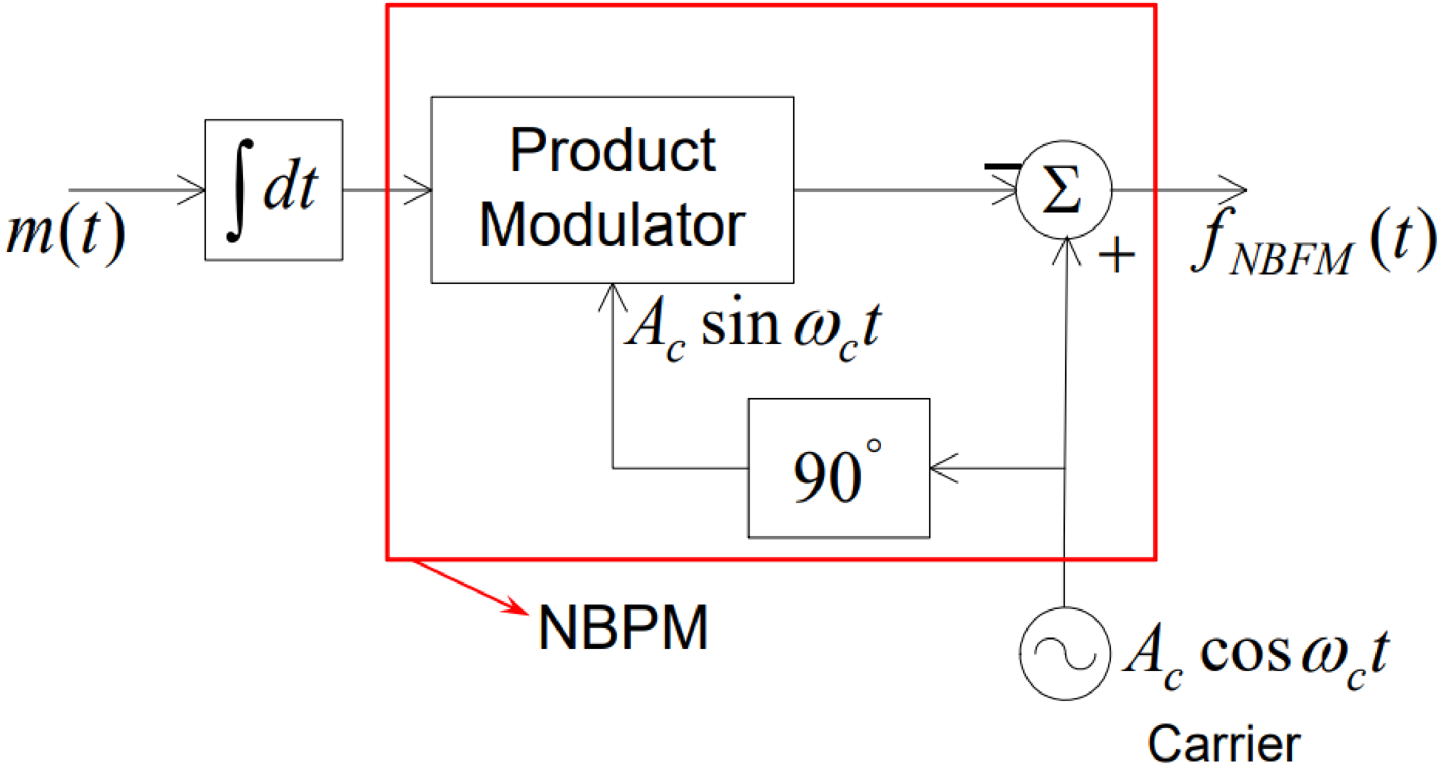
- Message signal $m(t)$ is first integrated and then used to phase-modulate a crystal-controlled oscillator.
- β is kept small to minimize distortion. ($\beta \leq 0.2$)
- Frequency multiplier is used next to produce the WBFM signal.



Block Diagram

Using $f_{NBFM}(t) \approx A_c[\cos\omega_c t - \beta \sin\omega_m t \sin\omega_c t]$

This method can be used to generate NBFM & NBPM signals.



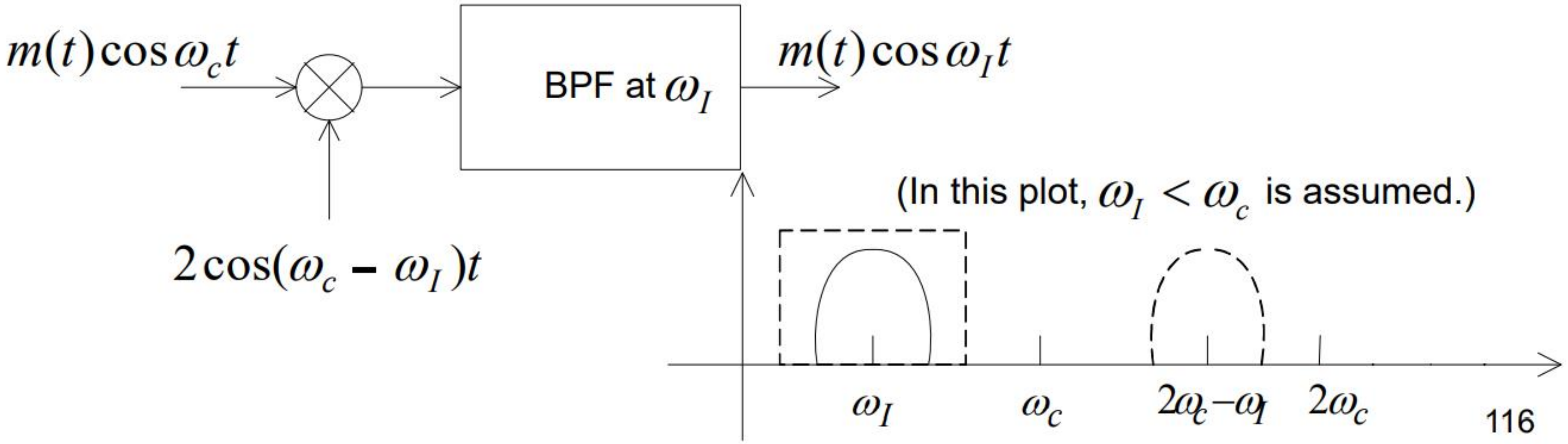
Remarks

1. Frequency multiplier (*n* times):
   - Including a non-linear device with order *n*:

     $s_{out}(t) = k_0 + k_1 s_1(t) + \ldots + k_n s_1^n(t)$, where $s_1(t)$ is an NBFM signal with f_1 and Δf_1.

     - with spectra at $f_1, 2f_1, \ldots, nf_1$
     - peak freq. dev. at $\Delta f_1, 2\Delta f_1, \ldots, n\Delta f_1$
   - And an appropriate filter to obtain nf_1 and $n\Delta f_1$ only.

2. Frequency Converter (Mixer) is then used to translate the spectrum.



10.2.1 倍频器

倍频器由非线性电路（或装置）和BPF组成，如图10.2.1所示：

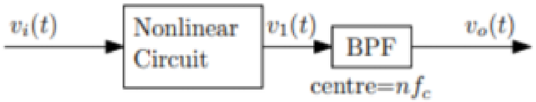


Figure 10.2.1: Frequency multiplier.

假设输入信号是带通信号，频谱集中在fc上：

$$v_i\left(t\right)=A_c\cos\left(2\pi f_c t+\theta\left(t\right)\right) \tag{10.2.1}$$

非线性电路产生谐波。BPF具有中心频率nfc，因此选择N次谐波。

示例：

n=5,v1 (t)包含：

$$v_i^5\left(t\right)=A^5\left(t\right)\cos^5\left(2\pi f_c t+\theta\left(t\right)\right)$$

我们有：

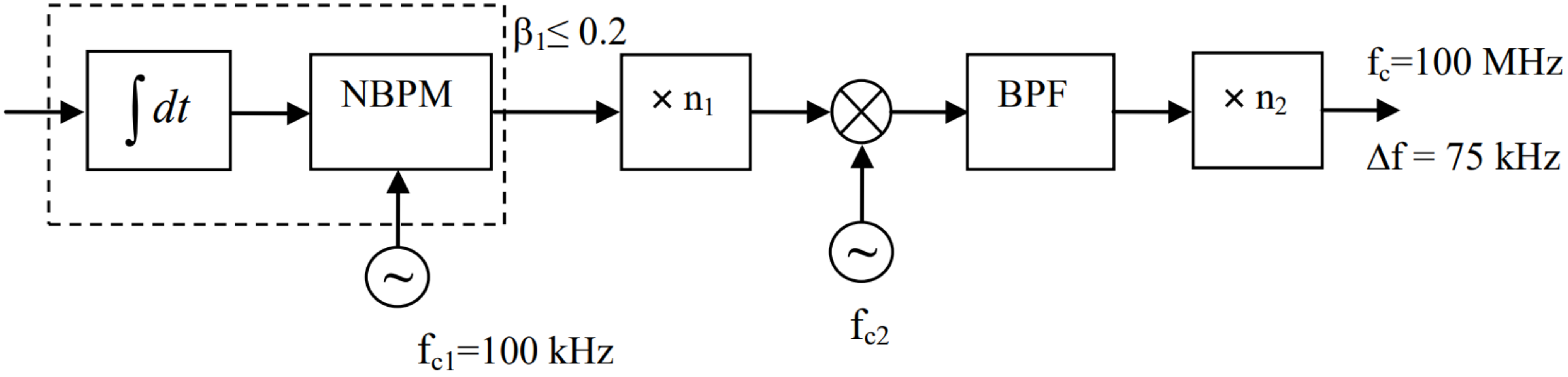
$$\cos^5\left(x\right)=\frac{5}{8}\cos\left(x\right)+\frac{5}{16}\cos\left(3x\right)+\frac{1}{16}\cos\left(5x\right)$$

BPF选择5fc左右的频率分量；因此，输出信号是：

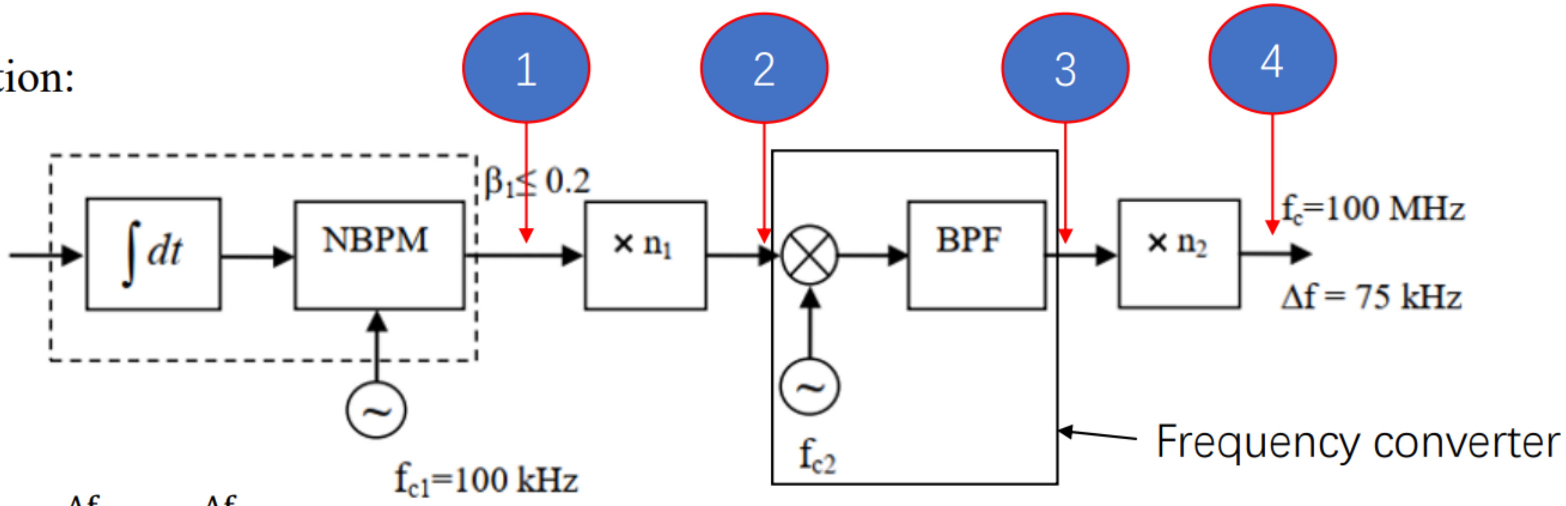
$$v_o\left(t\right)=\frac{A^5\left(t\right)}{16}\cos\left[5\left(2\pi f_c t+\theta\left(t\right)\right)\right]$$

我们看到输出频率相对于输入信号频率增加了n（=5）倍；因此被称为“倍频器”。请注意，相位偏差也增加了因数n（=5）。

1. The indirect FM system shown below is used to transmit a single-tone signal with frequency of 100 Hz. The desired FM signal at the transmitter output has carrier frequency of 100 MHz and maximum frequency deviation of 75 kHz. The modulation index of the narrowband frequency modulator, β_1, is restricted to a maximum value of 0.2 in order to keep distortion level low. Determine the suitable values of n_1, n_2 and f_{c2}.



Solution:



1： $\beta_1 = \frac{\Delta f_1}{f_m} = \frac{\Delta f_1}{100Hz} \leq 0.2 \Rightarrow \Delta f_1 \leq 20Hz$

$fc_1 = 100kHz$

2: $f = n_1 fc_1$

$\Delta f = n_1 \Delta f_1$

3: $fc_3 = f - fc_2$

$\Delta f_3 = \Delta f$

混频后会有 $f + fc_2$ 和$f - fc_2$ 两个频率成分。 保留低频成分即对应（ $f - fc_2$),低频信号相较高频信号容易处理。

4: $f_c = n_2 fc_3 = n_2 n_1 fc_1 - n_2 fc_2 = 100MHz$

$\Delta f = n_2 \Delta f_3 = n_2 n_1 \Delta f_1 = 75kHz$

One example:

suppose $\Delta f_1 = 10$Hz

Then

$n_2 n_1 = 7500$

7500*100kHz- $n_2 fc_2$ = 100MHz => $n_2 fc_2$ = 650MHz

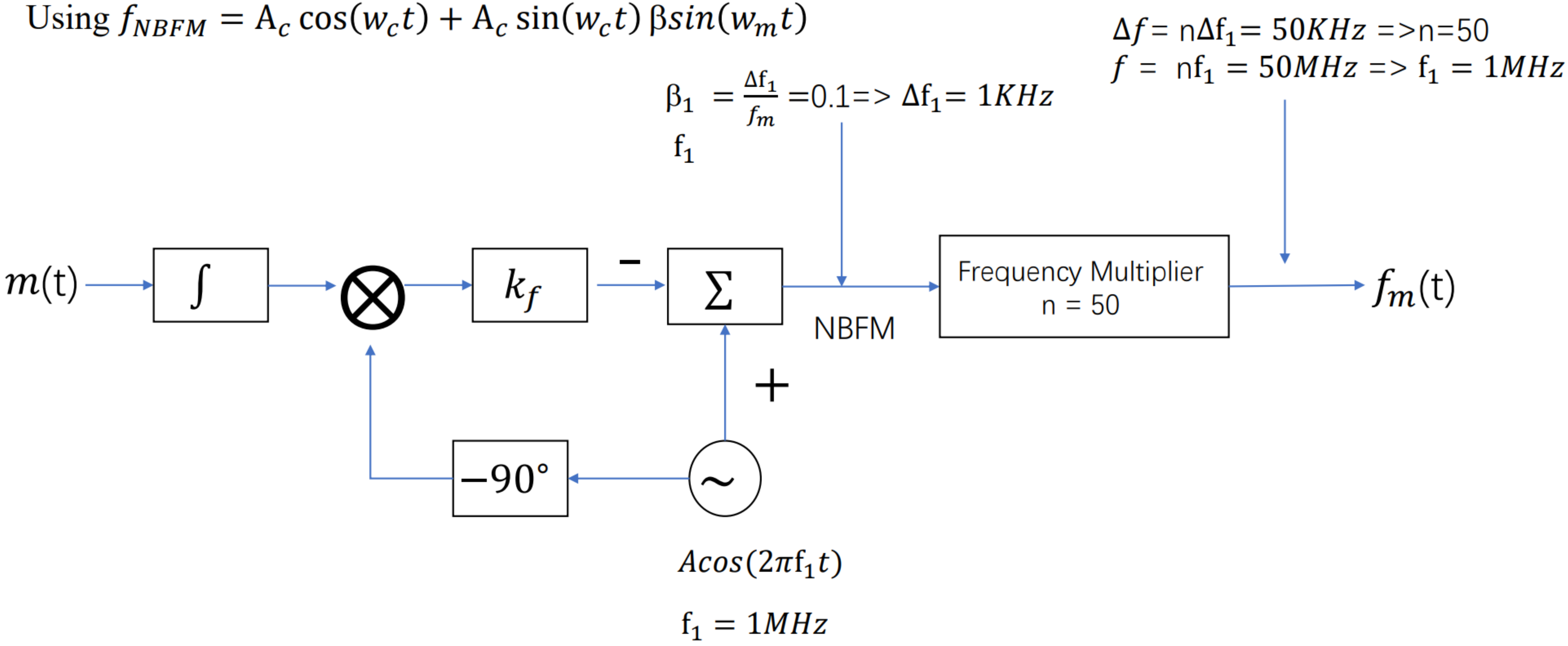
$n_1$100KHz > fc_2

==> we can let n_1=75 n_2=100 fc_2 =6.5MHz

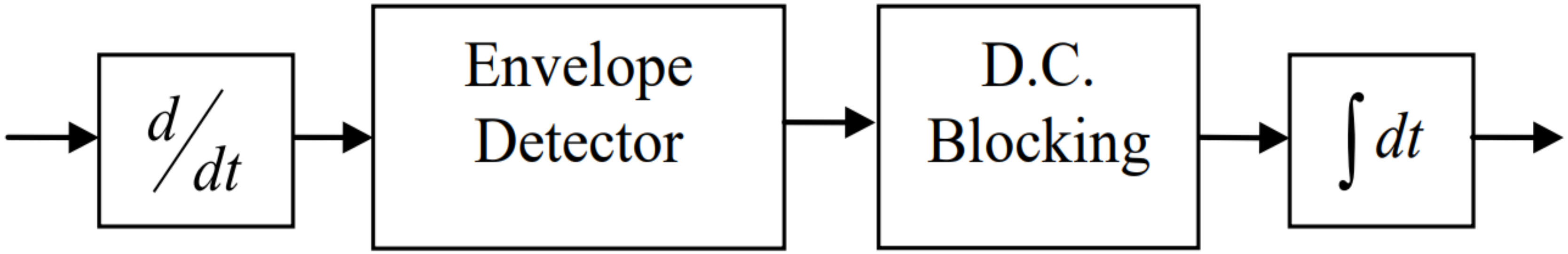
2. Design a wideband FM modulator that uses the indirect method for generating a WBFM signal with the carrier frequency of 50 MHz. The peak frequency deviation of the FM modulator is 50 KHz when modulated by a single tone signal of frequency 10 kHz. Show a complete block diagram of your design, indicating all necessary frequencies and peak frequency deviations of the signals at various points of the modulator. Assume that no frequency converter is used in the system and the modulation index of the involved NBFM modulator is 0.1.

Solution:

Using $f_{NBFM} = A_c \cos(w_c t) + A_c \sin(w_c t)\, \beta sin(w_m t)$



3. A message signal $m(t) = 0.5\cos(2000\pi t)$ phase modulates a carrier signal $f(t)=10\cos(2\pi\, 10^6 t)$ with modulation phase sensitivity $k_p = 5.6$ rad/V. If the PM signal is demodulated using the demodulator shown below, show the demodulating process in which signal $m(t)$ is recovered.



$f_{PM} \rightarrow \boxed{d/dt} \rightarrow \boxed{\text{Envelope Detector}} \rightarrow \boxed{\text{D.C. Blocking}} \rightarrow \boxed{\int dt} \rightarrow$

Solution:

$$\beta_p = k_p A_m = 5.6 \times 0.5 = 2.8$$

$$\omega_c = 2\pi \times 10^6, \quad \omega_m = 2\pi \times 10^3, \quad A_c = 10$$

$$f_{PM}(t) = A_c \cos(\omega_c t + \beta_p \cos \omega_m t)$$

$$\frac{d[f_{PM}(t)]}{dt} = -A_c[\omega_c - \beta_p \omega_m \sin \omega_m t]\sin(\omega_c t + \beta_p \cos \omega_m t)$$

The output of envelope detector: $-A_c[\omega_c - \beta_p \omega_m \sin \omega_m t] \quad = -A_c w_c + A_c \beta_p w_m sin w_m t$

The output of D.C. blocking: $A_c \beta_p \omega_m \sin \omega_m t$

The output of integrator:

$$A_c \beta_p \int \omega_m \sin \omega_m t dt$$
$$= -A_c \beta_p \cos \omega_m t$$
$$= -28 \cos 2\pi \times 10^3 t$$