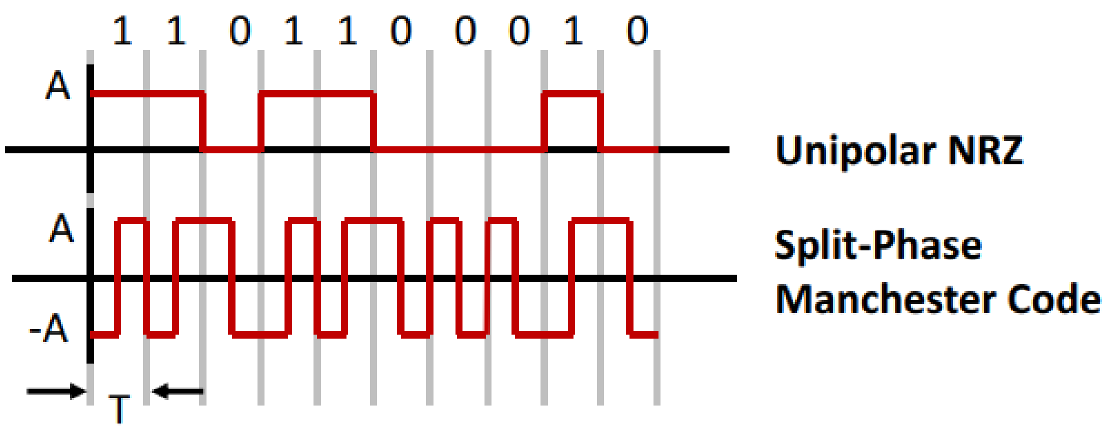
Homework #D5

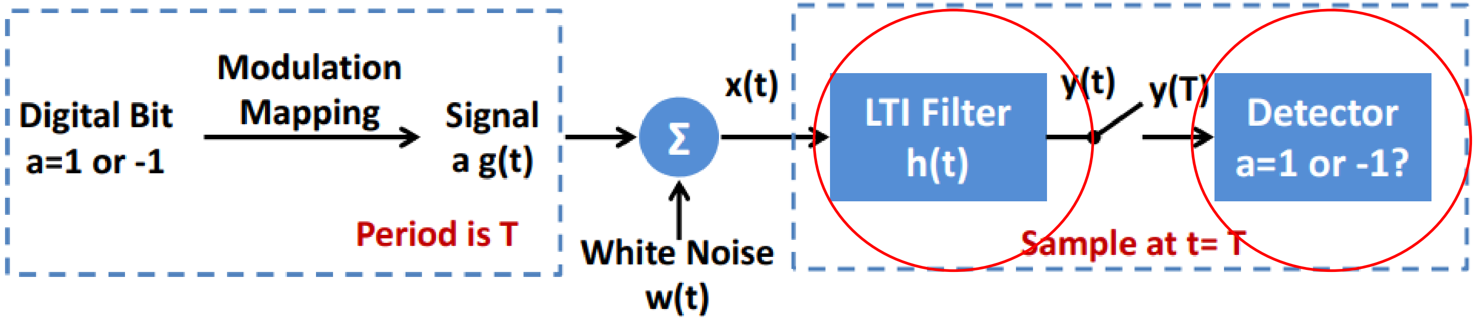
- D5.1

(a) If the Unipolar NRZ code as follows is used at the transmitter, how to design the receiver? What is the BER?

(b) If the Split-Phase Manchester Code as follows is used at the transmitter, how to design the receiver? What is the BER?



Solution:



为了实现最佳接收,接收端需要确定:

1. 滤波器 $h(t)$。
2. 抽样判决,判决界限。

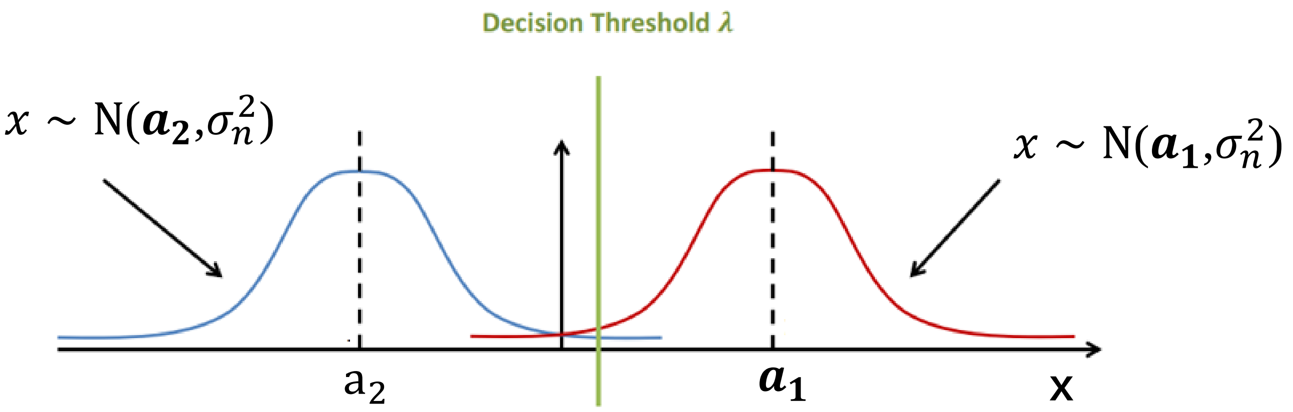
1. 匹配滤波器

$h(t) = kg(T - t)$ 推导过程见Digital-5 page22

2. 抽样判决,判决界限。

$$y(t) = x(t)\otimes h(t) = (ag(t) + w(t))\otimes h(t) = ag(t)\otimes h(t) + w(t)\otimes h(t) = y_s(t) + n(t)$$

抽样处理后信号 $\dfrac{y(T)}{kE} = a + \dfrac{n(T)}{kE}$



$$P_e = P(a=1)P(error|a=1) + P(a=-1)P(error|a=-1)$$

最佳门限 λ，即令$\frac{\partial P_e}{\partial \lambda}$ =0

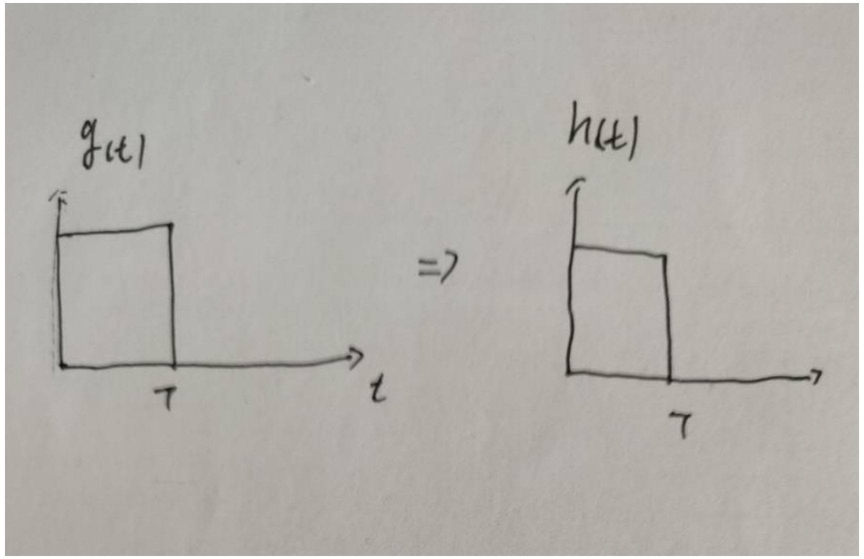
$$P_e = P(a=1)P(error|a=1) + P(a=0)P(error|a=0)$$

$$= P(a=1)\int_{-\infty}^{\lambda}\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{(x-a_1)^2}{2\sigma_n{}^2}}\,dx + P(a=0)\int_{\lambda}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{(x-a_2)^2}{2\sigma_n{}^2}}\,dx$$

$$\lambda = \frac{a_1+a_2}{2} + \frac{\sigma_n^2}{a_1-a_2}\ln\left(\frac{P(a=0)}{P(a=1)}\right)$$

Unipolar NRZ code

$\lambda = 0.5$



$$P_e = P(a = 1)P(error|a = 1) + P(a = -1)P(error|a = -1)$$

抽样值 r = $\frac{y(T)}{kE} = a + \frac{n(T)}{KE} \sim$ N(a, $\frac{N_0}{2E}$)

$P(a = 1) = P(a = -1) = \frac{1}{2}$
$P(error|a = 1) = P(error|a = -1)$

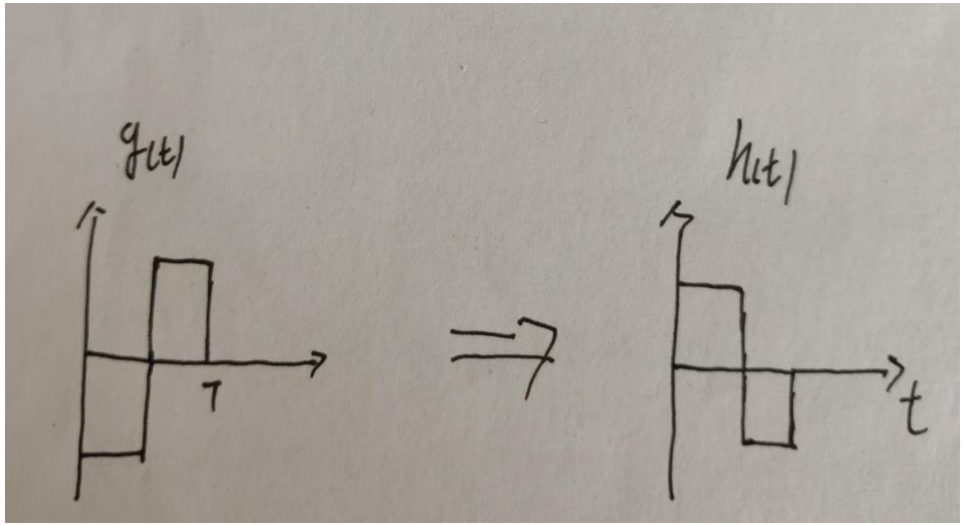
$$P_e = P(error|a = 0) = \int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{\pi N_0/E}} e^{-\frac{x^2}{N_0/E}} dx = \int_{\frac{1}{2}\sqrt{2E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \qquad (z = x\sqrt{\frac{2E}{N_0}})$$

$$= \int_{\sqrt{E/2N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q(\sqrt{\frac{E}{2N_0}}) = Q(\sqrt{\frac{E_b}{N_0}})$$

E is the energy for match filter $E = 2 E_b$ (k is constant can be ignored)

E_b is the transmitter signal energy per bit = $\frac{A^2}{2}T$

Split phase Manchester code



$\lambda = 0$

$$P_e = P(a=1)P(error|a=1) + P(a=-1)P(error|a=-1)$$

抽样值 r = $\frac{y(T)}{kE} = a + \frac{n(T)}{KE} \sim$ N(a, $\frac{N_0}{2E}$)

$P(a=1) = P(a=-1) = \frac{1}{2}$

$P(error|a=1) = P(error|a=-1)$

$$P_e = P(error|a=0) = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0/E}} e^{-\frac{(x+1)^2}{N_0/E}} dx = \int_{0\sqrt{2E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad (z = (x+1)\sqrt{\frac{2E}{N_0}})$$

$$= \int_{\sqrt{\frac{2E}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q(\sqrt{\frac{2E}{N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$$

E_b is the transmitter signal energy per bit = A^2T

E is the energy for match filter, so E = E_b (k is constant can be ignored)