- D7.1

Please design a receiver of the following band-pass modulation for AWGN channel. What is the BER?

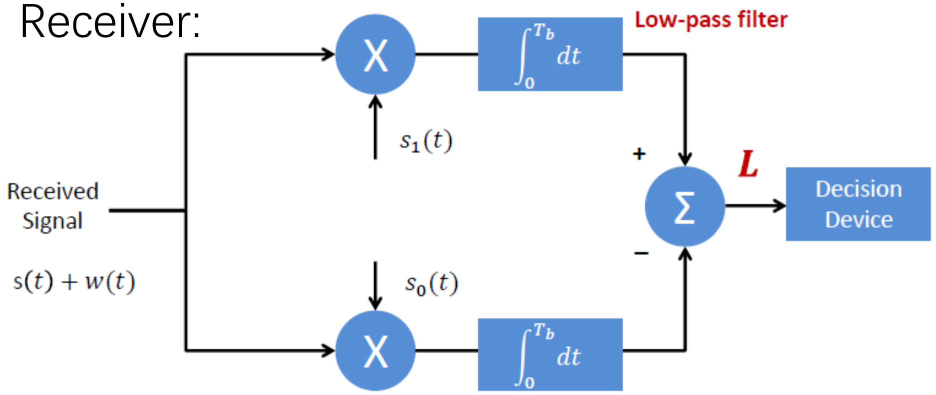
$$Bit\ 1:\ \ s(t) = s_1(t) = A_c \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$
$$Bit\ 0:\ \ s(t) = s_0(t) = A_c \cos(2\pi f_c t + \phi) \quad 0 \le t \le T_b$$

$$Bit\ 1:\ s(t) = s_1(t) = A_c\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$
$$Bit\ 0:\ s(t) = s_0(t) = A_c\cos(2\pi f_c t + \phi) \qquad 0 \le t \le T_b$$

Solution

Assume AWGN channel with white noise of *zero-mean* and *spectral density* $N_o/2$

Receiver:



The receiver output L is given by

$$L = \int_0^{T_b} x(t)[s_1(t) - s_0(t)]dt$$

BER is given by

$$P_e = Q\left(\sqrt{\frac{\boxed{E_b}(1 - \boxed{\rho})}{N_0}}\right)$$

Signal Energy

$$E_b = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_0^2(t)dt = \frac{{A_c}^2 T_b}{2}$$

Correlation coefficient

$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)dt}{E_b} \in [-1,1]$$
$$= cos\phi$$

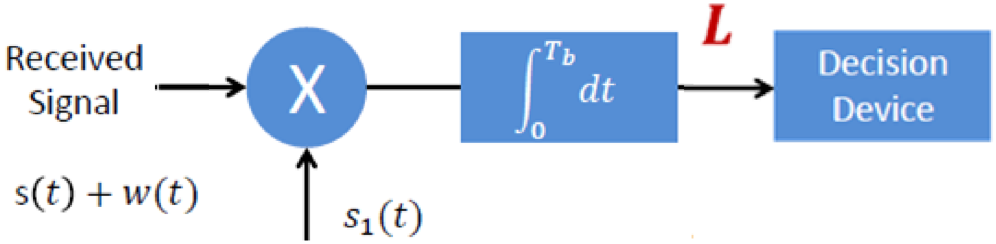
$$\text{Bit 1:}\quad s(t) = s_1(t) = A_c\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$
$$\text{Bit 0:}\quad s(t) = s_0(t) = A_c\cos(2\pi f_c t + \phi) \qquad 0 \le t \le T_b$$

Discuss

Assume $\phi = \pi$, BPSK

Due to s0(t) is the negative of s1(t), the receiver reduces to a single path

Receiver:



The receiver output *L* is given by

$$L = \int_0^{T_b} x(t)\boxed{[s_1(t) - s_0(t)]}dt = \int_0^{T_b} x(t)\boxed{2s_1(t)}dt$$

Signal Energy

$$E_b = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_0^2(t)dt = \frac{{A_c}^2 T_b}{2}$$

Correlation coefficient

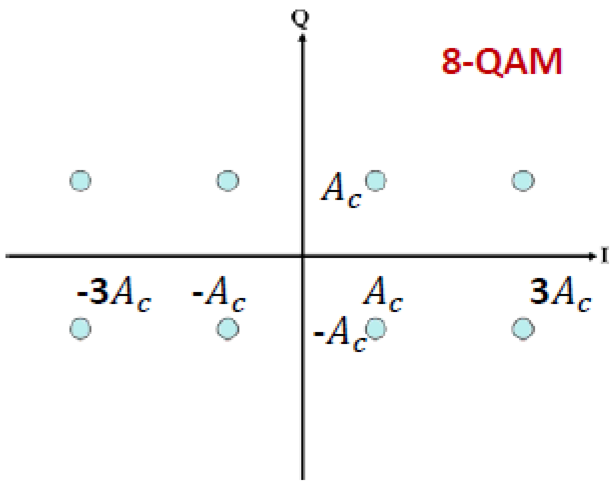
$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)dt}{E_b} \in [-1,1]$$
$$= -1$$

BER is given by

$$P_e = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right) = Q\left(\sqrt{\frac{\boxed{2E_b}}{N_0}}\right)$$

- D7.2

What is the average transmission power of the following 8-QAM modulation scheme? Suppose each symbol is transmitted with equal probability, and the symbol duration is T.



Solution

8 symbols total power

$$P_{total} = 4 \times \left(\frac{1}{2}\left(\sqrt{2}A_c\right)^2\right) + 4 \times \left(\frac{1}{2}\left(\sqrt{10}A_c\right)^2\right) = 24{A_c}^2$$

average transmission power with equal probability

$$P_{symbol} = \frac{E_{total}}{8} = 3{A_c}^2$$