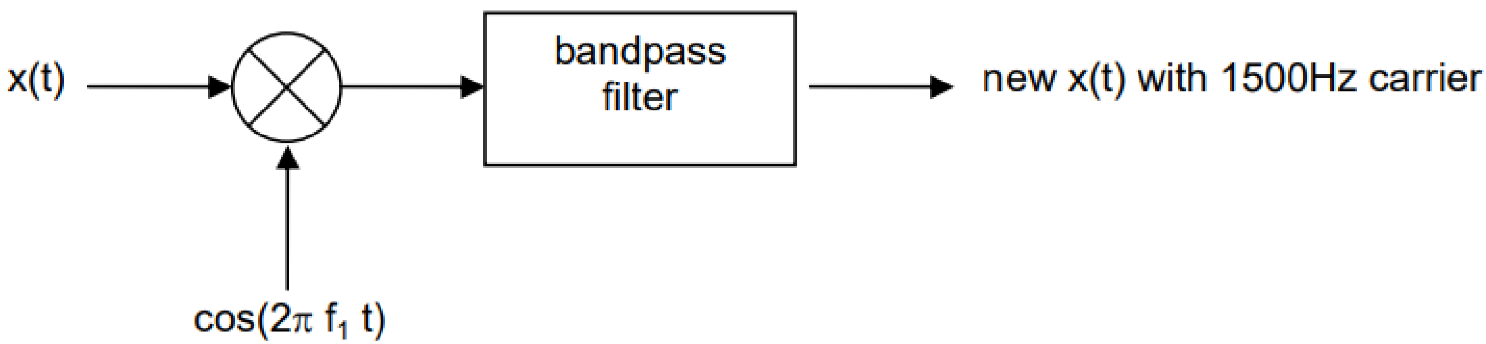
EE206: Communications Principles

Assignment 4

1. The signal $s(t) = \sin(200\pi t + \pi/3) + \cos(200\pi t + \pi/3)$ is modulated using a cosine carrier signal with carrier frequency 500Hz and zero phase to generate a Suppressed-Carrier AM signal $x(t)$.

   (a) Write $s(t)$ as a single cosine term. Then find $x(t)$.

   (b) Use the mixer below to shift the carrier frequency of $x(t)$ to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.



1. The signal s(t) = sin(200πt + π/3) + cos(200πt + π/3) is modulated using a cosine carrier signal with carrier frequency 500Hz and zero phase to generate a Suppressed-Carrier AM signal x(t).

(a) Write s(t) as a single cosine term. Then find x(t).

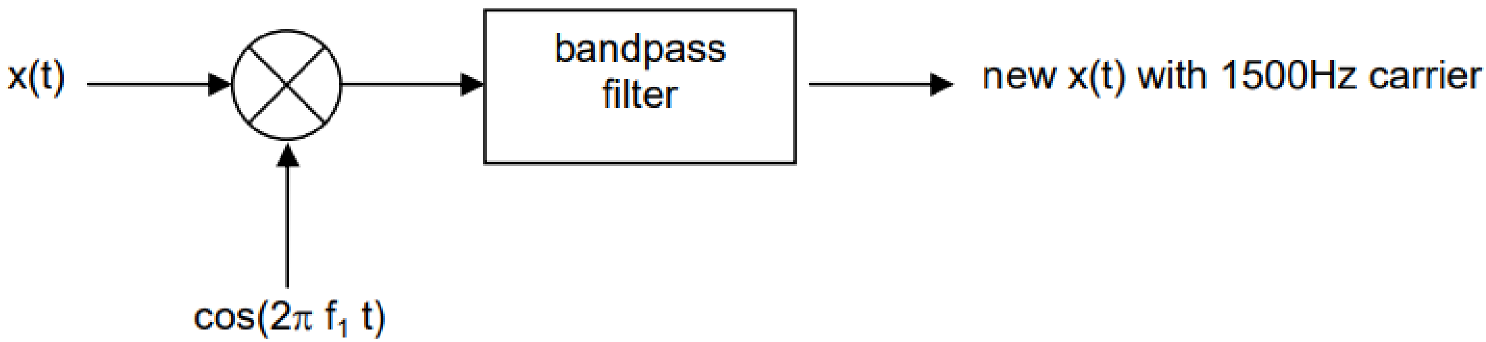
$$\begin{aligned} s(t) &= \sin(2\pi 100t + \frac{\pi}{3}) + \cos(2\pi 100t + \frac{\pi}{3}) \\ &= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{3} - \tan^{-1}\frac{1}{1}) \\ &= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{3} - \frac{\pi}{4}) \\ &= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12}) \\ x(t) &= s(t)\cos(2\pi 500t) \\ &= \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t) \end{aligned}$$

$a\cos x + b\sin x$ can be written as $R\cos(x-\alpha)$

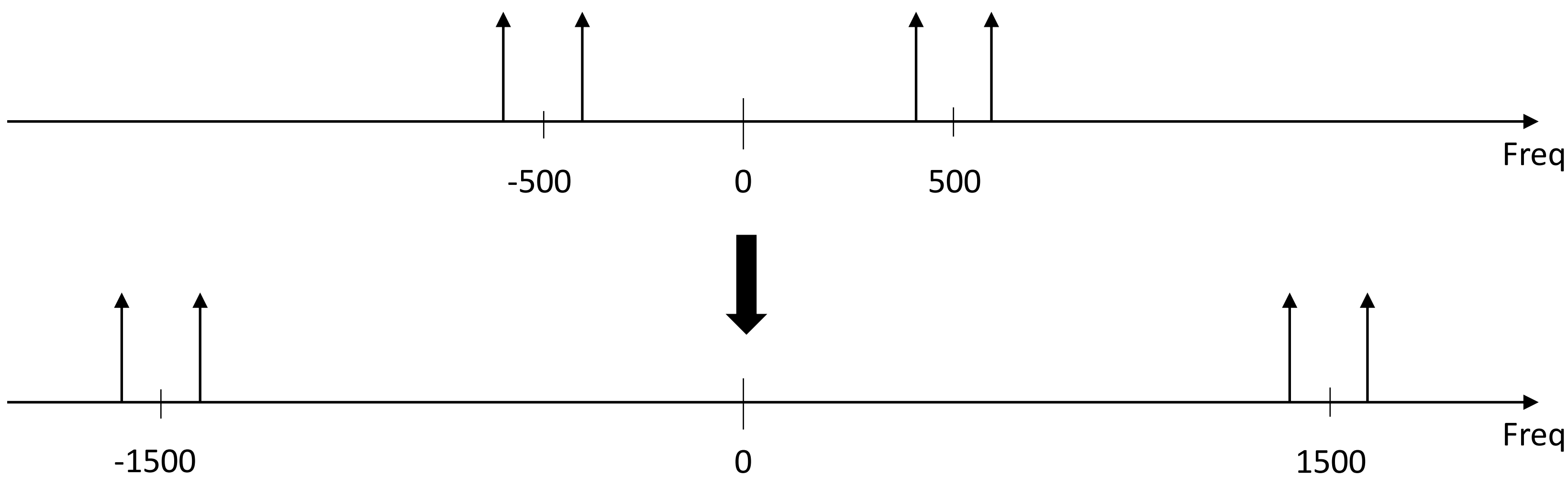
where

$R = \sqrt{a^2+b^2}, \quad \tan\alpha = \frac{b}{a}$

(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.



$$x(t) = \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t)$$



(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.

Frequency Translation (2)

- For a modulated signal $x(t) = m(t) \cos 2\pi f_c t$,

$$v_1(t) = x(t) \cdot \cos 2\pi f_1 t$$
$$= m(t) \cos 2\pi f_c t \cos 2\pi f_1 t$$
$$= \frac{m(t)}{2} [\cos 2\pi(f_c - f_1)t + \cos 2\pi(f_c + f_1)t]$$

- Assuming $f_c > f_1$, if $v_1(t)$ is passed through a bandpass filter with a centre frequency $f_0 = f_c - f_1$, then $x(t)$ will occupy a new frequency band. That is

$$v_2(t) = [v_1(t)]_{BP}$$
$$= \frac{1}{2} m(t) \cos 2\pi(f_c - f_1)t = \frac{1}{2} m(t) \cos 2\pi f_0 t$$

- The device that carries out the frequency translation of a modulated signal is called mixer, and the operation itself is called mixing.

(b) Use the mixer below to shift the carrier frequency of x(t) to 1500Hz. State the 2 applicable values of f_1, the filter center frequency, and the required filter bandwidth.

$$x(t)\cos(2\pi f_1 t) = \sqrt{2}\cos(2\pi 100t + \frac{\pi}{12})\cos(2\pi 500t)\cos(2\pi f_1 t)$$

$$f_1 - 500 = 1500 \text{ or } f_1 + 500 = 1500$$

$$f_1 = 1000Hz \text{ or } 2000Hz$$

filter center frequency: $1500Hz$

bandwidth: $200Hz$

2. a) A received signal a(t) has SNR 13dB and **noise** power 64 μW (μW = 10^{-6} Watt). Another received signal b(t) also has SNR 13dB but **total** (signal+noise) power of 64 μW. Determine the useful signal power in mW in each of these signals.

$$
\begin{aligned}
\mathrm{SNR}_a &= 10\log_{10}\frac{P_{m_a}}{64\times10^{-6}} = 13\mathrm{dB} \\
P_{m_a} &= 0.001277W \\
&= 1.277mW \\
\mathrm{SNR}_b &= 10\log_{10}\frac{P_{m_b}}{64\times10^{-6} - P_{m_b}} = 13\mathrm{dB} \\
P_{m_b} &= 0.000061W \\
&= 0.061mW
\end{aligned}
$$

b) An AM signal x(t) is received with 6mW signal power, 20KHz bandwidth and carrier freq 100MHz. Another AM signal y(t) is received with 100mW signal power, 3MHz bandwidth and carrier freq 500MHz. The channel contains white noise. Which signal has better quality?

$$
\begin{aligned}
\mathrm{SNR_x} &= 10\log_{10}\frac{6\times 10^{-3}}{20\times 10^{3}\times 2\times \frac{\eta}{2}} \\
&= 10\log_{10}(\frac{3}{\eta}\times 10^{-7}) \\
\mathrm{SNR_y} &= 10\log_{10}\frac{100\times 10^{-3}}{3\times 10^{6}\times 2\times \frac{\eta}{2}} \\
&= 10\log_{10}(\frac{1}{3\eta}\times 10^{-7}) \\
\mathrm{SNR_x} &> \mathrm{SNR_Y}
\end{aligned}
$$

signal x has better quality.

3. Two message signals, $s_1(t) = 2$ and $s_2(t) = 10\sin(20\pi t)$, are modulated to form a QAM signal x(t) with carrier frequency 500Hz. $s_1(t)$ is modulated onto the I-phase, $s_2(t)$ onto the Q-phase. During transmission, x(t) is corrupted by white noise with 2-sided PSD of 10^{-5} Watt/Hz. At the receiver, it is demodulated using a coherent demodulator. Determine the SNR of the Q-branch output signal in dB.

$$
\begin{aligned}
x(t) &= s_1(t)\cos(2\pi 500t) + s_2(t)\sin(2\pi 500t) \\
&= 2\cos(2\pi 500t) + 10\sin(2\pi 10t)\sin(2\pi 500t) \\
n(t) &= n_c(t)\cos(2\pi 500t) - n_s(t)\sin(2\pi 500t) \\
x(t)\sin(2\pi 500t) &= 2\cos(2\pi 500t)\sin(2\pi 500t) + 10\sin(2\pi 10t)\sin(2\pi 500t)\sin(2\pi 500t) \\
&= \sin(2\pi 1000t) + \underline{5\sin(2\pi 10t)} - 5\sin(2\pi 10t)\cos(2\pi 1000t) \\
n(t)\sin(2\pi 500t) &= n_c(t)\cos(2\pi 500t)\sin(2\pi 500t) - n_s(t)\sin(2\pi 500t)\sin(2\pi 500t) \\
&= \frac{1}{2}n_c(t)\sin(2\pi 1000t) \underline{- \frac{1}{2}n_s(t)} + \frac{1}{2}n_s(t)\cos(2\pi 1000t)
\end{aligned}
$$

3. Two message signals, $s_1(t) = 2$ and $s_2(t) = 10\sin(20\pi t)$, are modulated to form a QAM signal x(t) with carrier frequency 500Hz. $s_1(t)$ is modulated onto the I-phase, $s_2(t)$ onto the Q-phase. During transmission, x(t) is corrupted by white noise with 2-sided PSD of 10^{-5} Watt/Hz. At the receiver, it is demodulated using a coherent demodulator. Determine the SNR of the Q-branch output signal in dB.

After LPF, the demodulated Q-branch signal is,

$$5\sin(2\pi 10t).$$

The Q-branch output noise is,

$$-\frac{1}{2}n_s(t).$$

Signal power is $\frac{5^2}{2} = 12.5W$, and noise power is $\frac{1}{4}\overline{n_s^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \frac{1}{4} \times 2 \times B \times \mathrm{PSD}_{2-sided} = \frac{1}{4} \times 2 \times 20 \times 10^{-5} = 10^{-4}W$.

$\mathrm{SNR}_o = 10\log_{10}\frac{12.5}{10^{-4}} = 50.97\mathrm{dB}$.