1. Sketch the amplitude spectrum of each of the following signals:

(a) $2\cos^2(2000\pi t)$

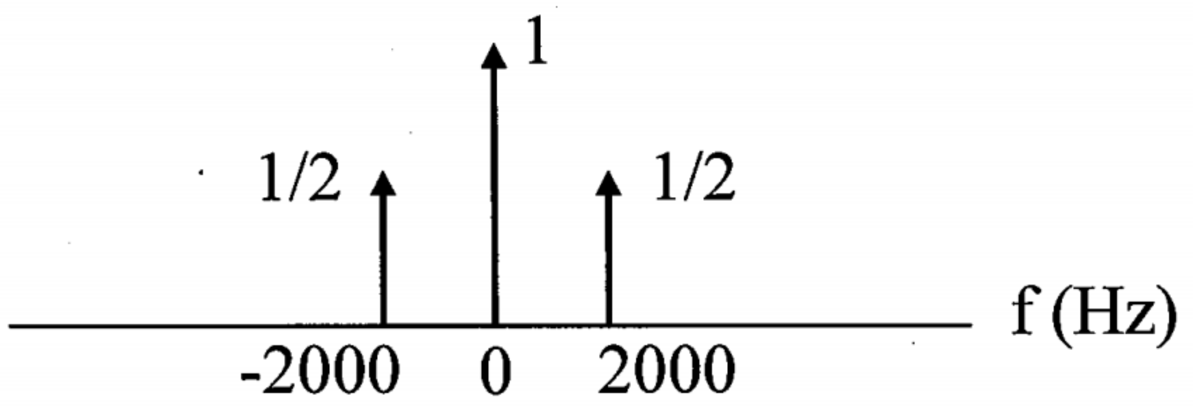
(b) rect(2000t) filtered by an ideal lowpass filter with 4KHz bandwidth

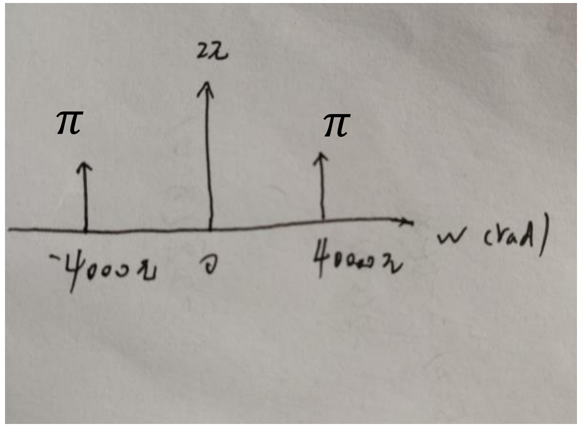
Solution:

(a) $2cos^2(2000\pi t) = 2 * \frac{1+\cos(4000\pi t)}{2} = 1 + \cos(4000\pi t)$

$$1 + \cos(4000\pi t) \xrightarrow{\text{F.T}} \delta(f) + \frac{\delta(f-2000)+\delta(f+2000)}{2}$$

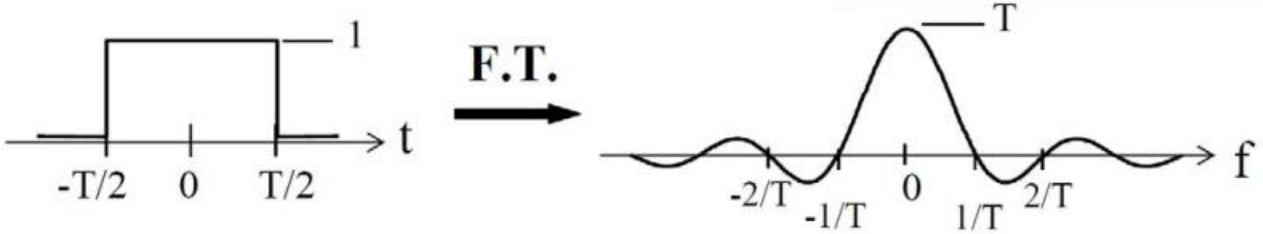
$$2\pi\delta(w) + \pi\delta(w - 4000\pi) + \pi\delta(w + 4000\pi)$$

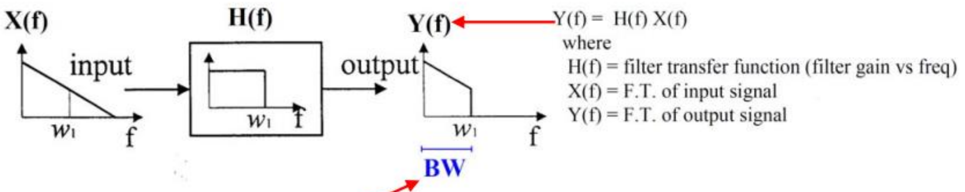




❑ F.T. of $\mathrm{rect}\left(\frac{t}{T}\right) = T\,\mathrm{sinc}(f\,T)$. Note $\boxed{\mathrm{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}}$

$$A\,\mathrm{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\,\mathrm{rect}\left(\frac{f}{2W}\right)$$





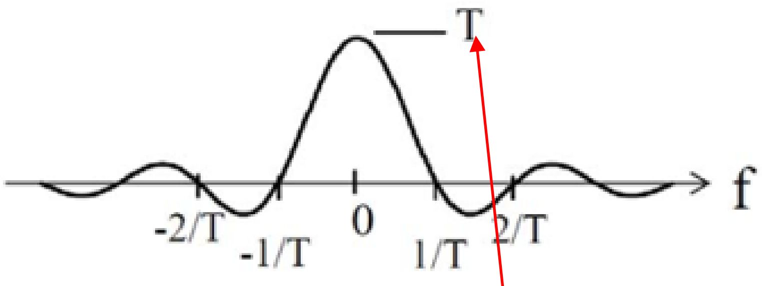
Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

25

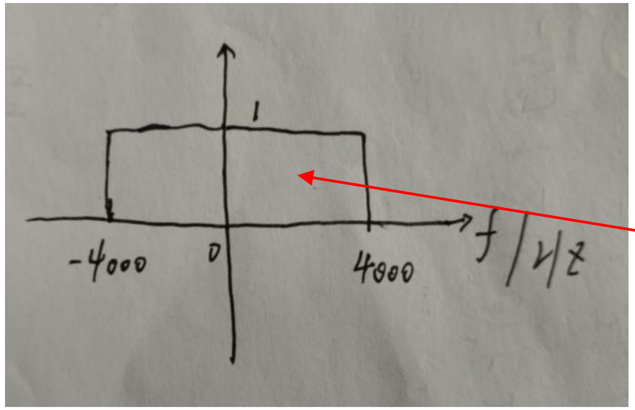
Solution:

(b)

$$rect(2000t) \xrightarrow{\text{F.T}} \frac{1}{2000}\,sinc\left(f * \frac{1}{2000}\right)$$



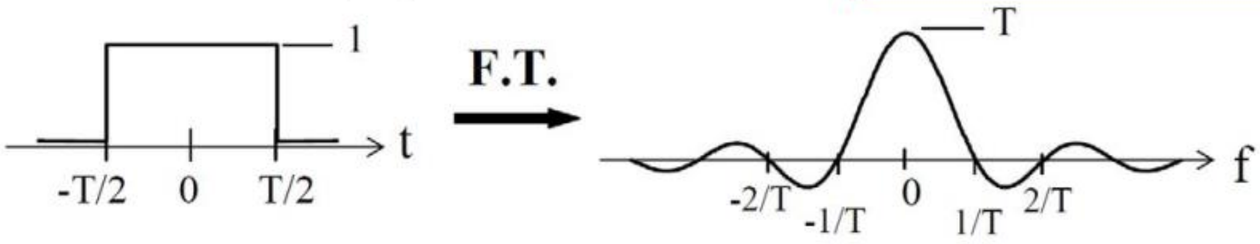
$\frac{1}{2000}$

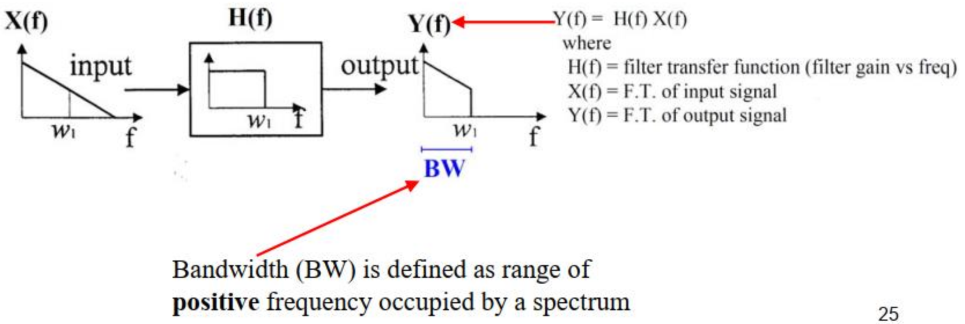


H_{Lp}

❑ F.T. of $\mathrm{rect}\left(\frac{t}{T}\right) = T\,\mathrm{sinc}(f\,T)$. Note $\boxed{\mathrm{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}}$

$$A\,\mathrm{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\,\mathrm{rect}\left(\frac{f}{2W}\right)$$



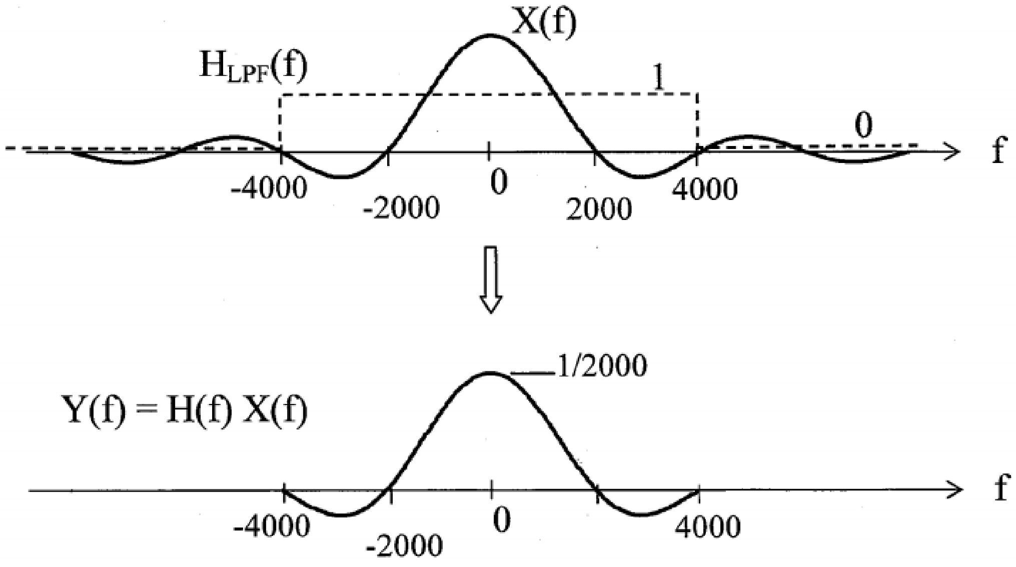


25

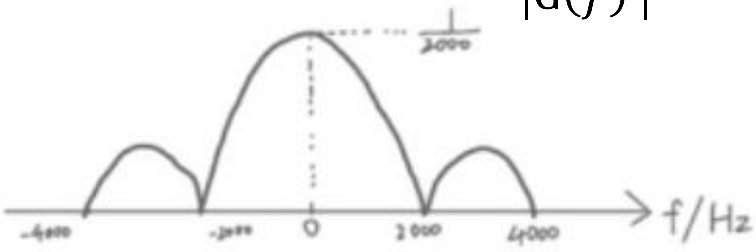
Solution:

(b)

$$rect(2000t) \xrightarrow{\text{F.T}} \frac{1}{2000}\, sinc\left(f * \frac{1}{2000}\right)$$



$|G(f)|$



2. Determine the power of the signal s(t) = 6cos(200πt) + 8 sin(200πt) using the time-domain and frequency-domain formulas. Do not use superposition of power.

<u>Note</u>: PSD of $A\cos(2\pi f_0 t+\theta)$ is $\frac{A^2}{4}\left[\delta\left(f-f_0\right)+\delta\left(f+f_0\right)\right]$, where θ is any phase angle

W/Hz

and $\delta(f)$ is a delta dirac function in f.

✓ Time domain:

$$\overline{v^2(t)}=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}|v(t)|^2\,dt$$

✓ Frequency domain:

$$\overline{v^2(t)}=\int_{-\infty}^{\infty}S_v(f)\,df$$

Solution:

(1) Time domain

$$s(t) = 6\cos(200\pi t) + 8\sin(200\pi t)$$

$$= 10\cos(\boxed{200\pi t - \phi})\quad (\phi = \tan^{-1}(\tfrac{4}{3}))$$

$$\overline{s^2(t)} = \frac{100}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} cos^2(x)\,dt = \frac{100}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\frac{1+\cos(2x)}{2}\,dt = \frac{100}{T_0} * \frac{T_0}{2} = 50$$

(2) Frequency domain

PSD of s(t) $\frac{100}{4}\delta(f-100)+\delta(f+100)$

$$\overline{s^2(t)} = \frac{100}{4} * 2 = 50$$

3. Determine and compare the bandwidths of the following signals:

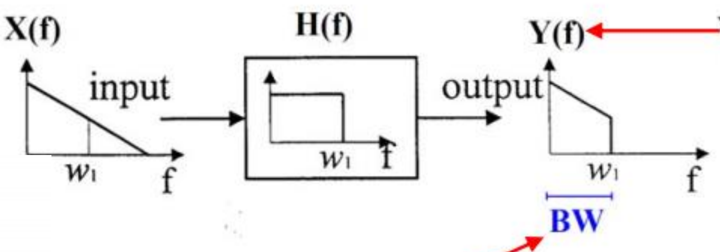
(a) $\operatorname{sinc}\left(\frac{t}{50}\right)$

(b) $\operatorname{sinc}\left(\frac{t-4}{50}\right)$

(c) $\operatorname{sinc}\left(\frac{t}{50}\right)-4$

(d) $\operatorname{sinc}\left(\frac{t}{50}\right)\sin(5000\pi t)$

$$A\,\operatorname{sinc}(2Wt) \rightleftharpoons \frac{A}{2W}\operatorname{rect}\left(\frac{f}{2W}\right)$$



Y(f) = H(f) X(f)
where
H(f) = filter transfer function (filter gain vs freq)
X(f) = F.T. of input signal
Y(f) = F.T. of output signal

Bandwidth (BW) is defined as range of **positive** frequency occupied by a spectrum

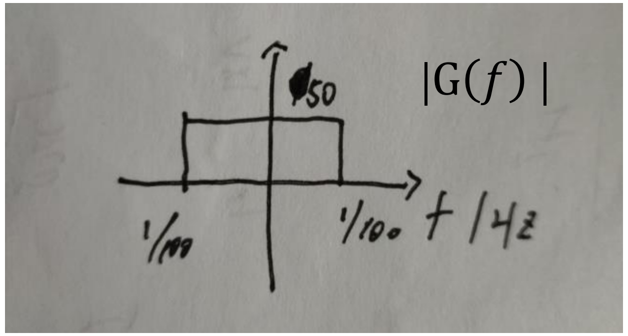
25

Solution:

$$sinc(\frac{t}{50}) \xrightarrow{\text{F.T}} 50rect(\frac{f}{\frac{1}{50}})$$

$$\operatorname{rect}(t) = \begin{cases} 1, & -\frac{1}{2} < t < \frac{1}{2} \\ 0, & |t| \geq \frac{1}{2} \end{cases} \qquad (2.7)$$

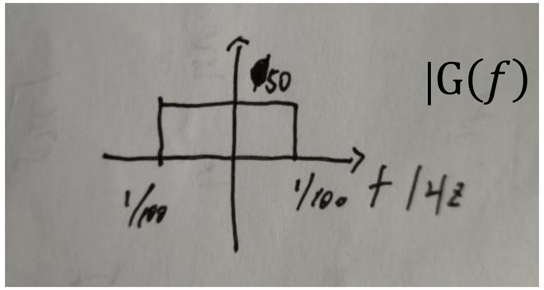
a)



BW = 0.01 HZ.

Solution:

b) $sinc(\frac{t-4}{50}) \xrightarrow{\text{F.T}} 50rect(\frac{f}{\frac{1}{50}}) * e^{-j2\pi f*4}$ 4. Time shifting $g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$



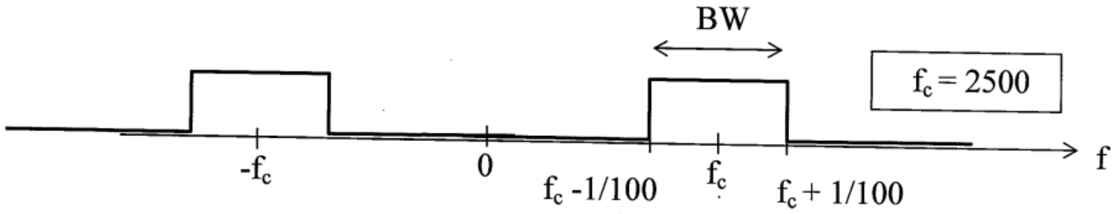
$|G(f)|$

BW = 0.01 HZ.

c)

$sinc(\frac{t}{50})$ -4 $\xrightarrow{\text{F.T}} 50rect(\frac{f}{\frac{1}{50}})$-$4\delta(f)$ BW = 0.01HZ.

d) $sinc(\frac{t}{50}) * \sin(5000\pi t) \xrightarrow{\text{F.T}} X(f) \otimes Y(f)$



BW = 0.02 HZ.