Types of AM

- Conventional/Full AM
  - Analog broadcast radio
- DSB Suppressed-Carrier AM
  - Satellite Communications
- Single Sideband AM
  - Long distance telephone links
- Vestigial Sideband AM
  - Analog broadcast TV

See Section 3.5 on pages 88-91

- Quadrature AM
  - PC modem, wireless LAN, digital TV

Conventional AM

- Consider a sinusoidal carrier wave given by $A_c\cos(2\pi f_c t)$. Let $m(t)$ denote the modulating signal (i.e., message). The full AM is given by

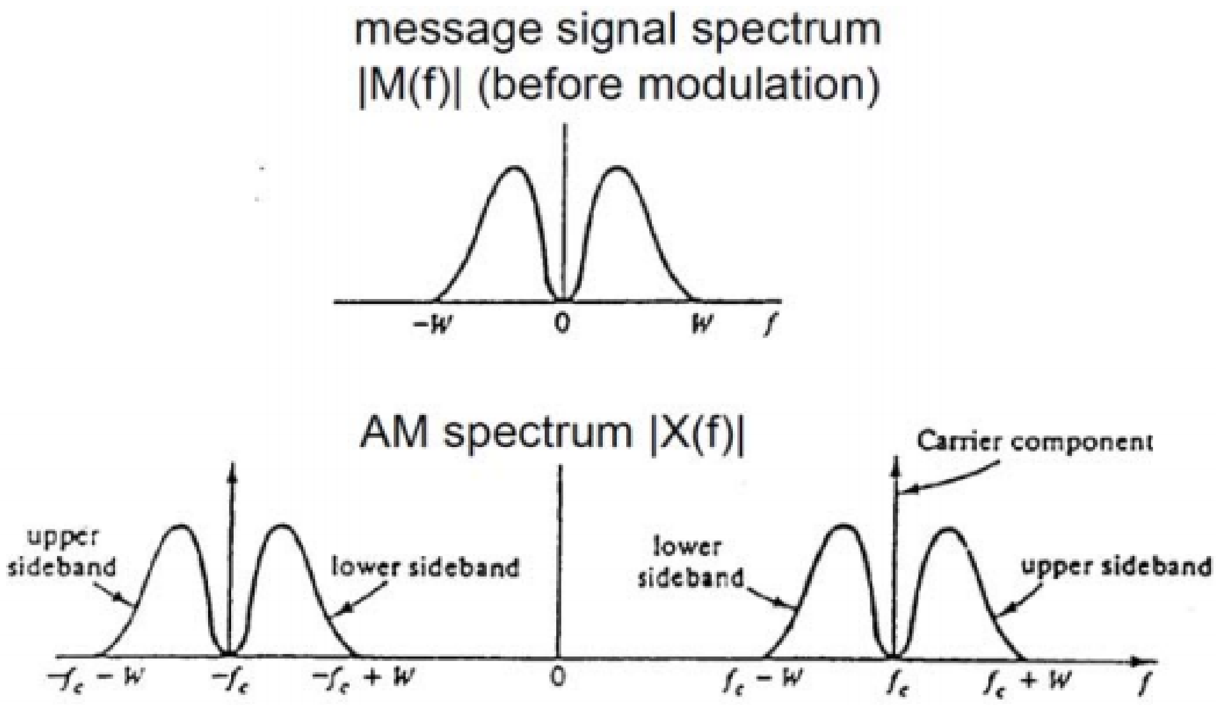
Amplitude sensitivity, a constant (1/volt)

$$x(t) = A_c[1 + \boxed{k_a} m(t)] \cos 2\pi f_c t$$
$$= \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{A_c k_a m(t) \cos 2\pi f_c t}_{\text{sidebands}}$$

- Fourier transform of $x(t)$:

$$X(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

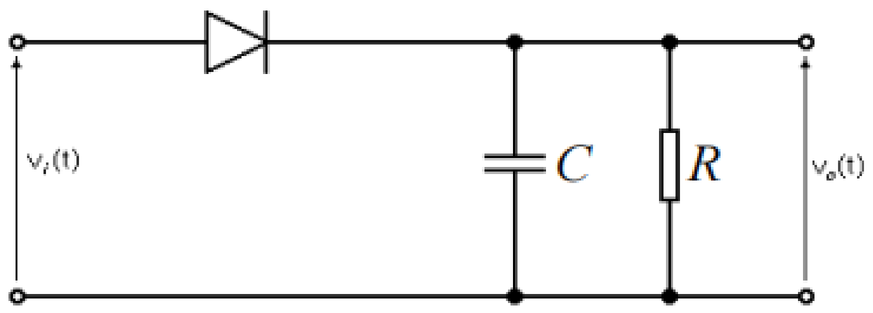
Spectrum of Conventional AM

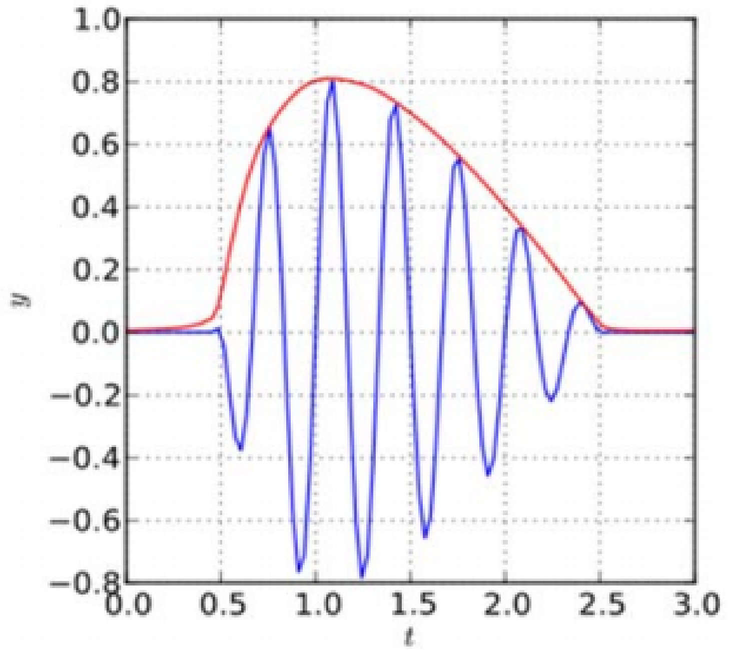


- Upper sideband and lower sideband are symmetrical.
- $f_c > W$ to ensure the sidebands do not overlap.
- Transmission bandwidth of a conventional AM is exactly twice the message bandwidth W, that is, $2W$.

Envelope Detection

- The simplest form of **envelope detector** is the **diode detector:**



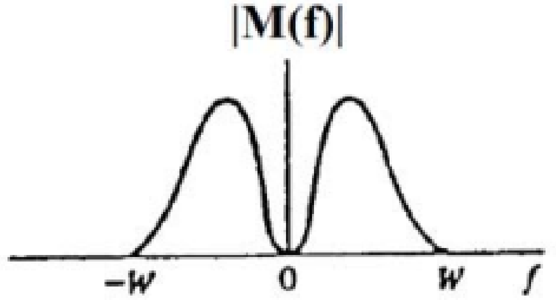


- In order for this diode detector to work, it is required that $\frac{1}{f_c} \ll RC \ll \frac{1}{W}$.

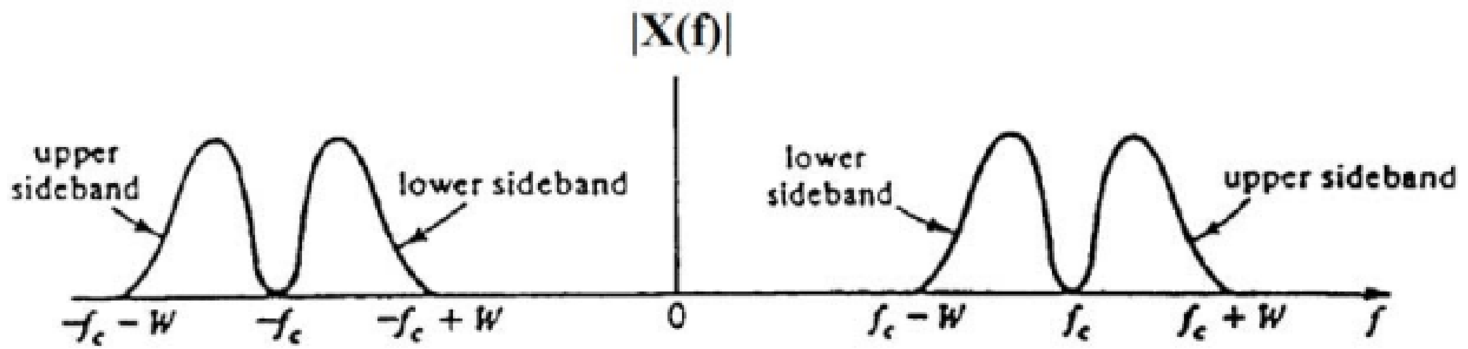
Expression & Spectrum of DSBSC-AM

- The DSBSC AM modulated signal $x(t)$ is
  $x(t) = m(t) \cdot A_c \cos 2\pi f_c t$
- The signal spectrum is obtained as

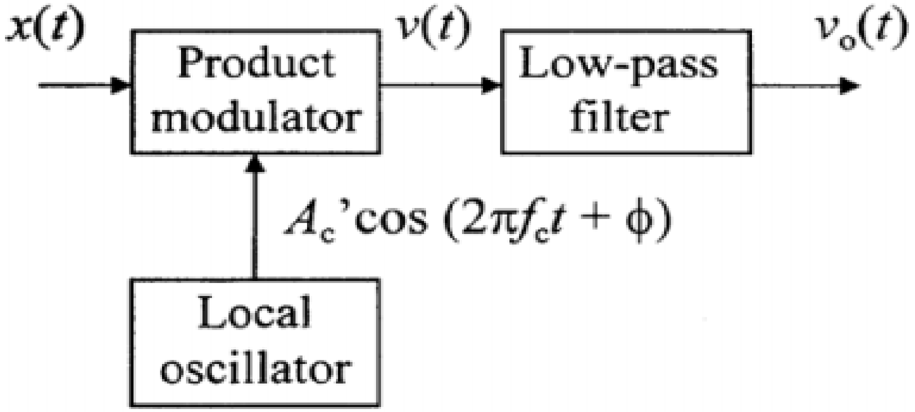
$$X(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Note: DSBSC-AM shifts the center freq. from 0 to $\pm f_c$, and doubles the BW, while maintaining the spectral shape



Coherent/Synchronous Demodulation



The job of demodulation is to recover the message $m(t)$ from the DSBSC-AM signal $x(t)$.

$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$

1. Input (DSBSC): $x(t) = m(t) \cdot A_c \cos 2\pi f_c t$
2. $v(t) = A_c{}' \cos(2\pi f_c t + \phi\,) \cdot m(t) \cdot A_c \cos 2\pi f_c t$

$$= \frac{1}{2} A_c{}' A_c m(t) \cos\phi + \frac{1}{2} A_c{}' A_c m(t) \cos(4\pi f_c t + \phi)$$

3. $v_0(t) = \frac{1}{2} A_c{}' A_c m(t) \cos\phi =$ constant $\times\, m(t)$

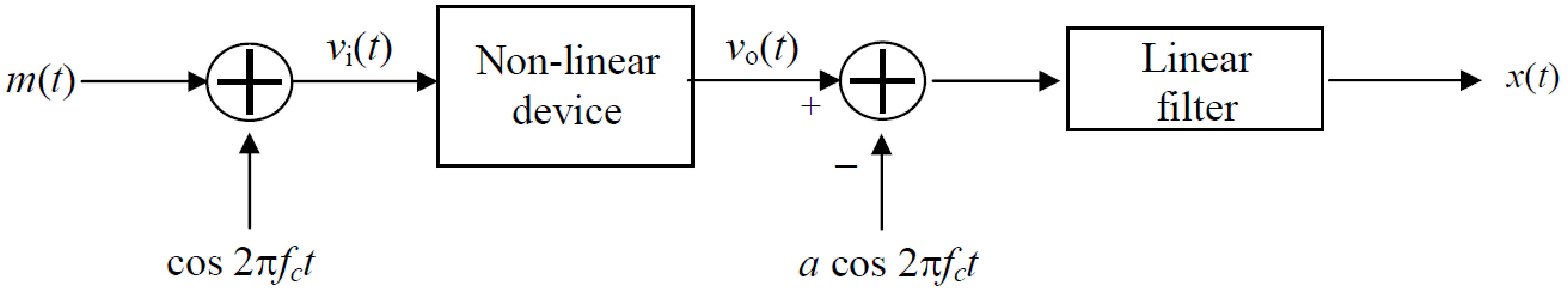
✓ The output contains a phase error, ϕ. Output = 0 if $\phi = \pm\pi/2$. Output is max if $\phi = 0$, i.e., the local oscillator has the same frequency & phase (synchronized, coherent) as the carrier component in the AM signal.

1. The message signal $m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t)$ is input to the system shown below to generate a DSBSC-AM signal $x(t)$. Assume that $v_o(t) = a\, v_i(t) + b\, v_i^2(t)$ where a and b are constants, and the carrier frequency $f_c >> 70$Hz.

   (a) Sketch the amplitude spectrum of the filter input

   (b) Determine the center frequency and bandwidth of the filter in this modulator

   (c) Determine the minimum value of f_c permitted for this modulator



Solution of (a)

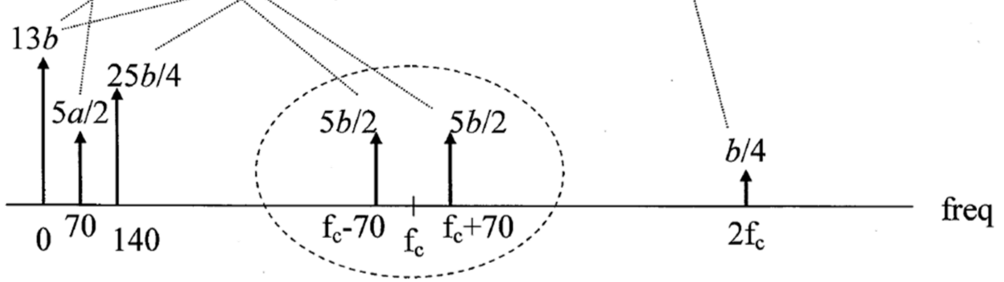
$$v_o(t) = a v_i(t) + b v_i^2(t)$$
$$= a\left[m(t)+\cos(2\pi f_c t)\right] + b\left[m(t)+\cos(2\pi f_c t)\right]^2$$
$$= a\,m(t) + b\,m^2(t) + 2b\,m(t)\cos(2\pi f_c t) + a\cos(2\pi f_c t) + \underbrace{b\cos^2(2\pi f_c t)}_{\frac{b}{2}+\frac{b}{2}\cos(4\pi f_c t)}$$

$$m(t) = 3\cos(2\pi 70t) + 4\sin(2\pi 70t) = 5\cos(2\pi 70t - \theta) \leftarrow \text{from Trigo Table}$$

$$m^2(t) = \frac{25}{2} + \frac{25}{2}\cos(2\pi 140t - 2\theta) \quad \text{where } \theta = \tan^{-1}\frac{4}{3}$$

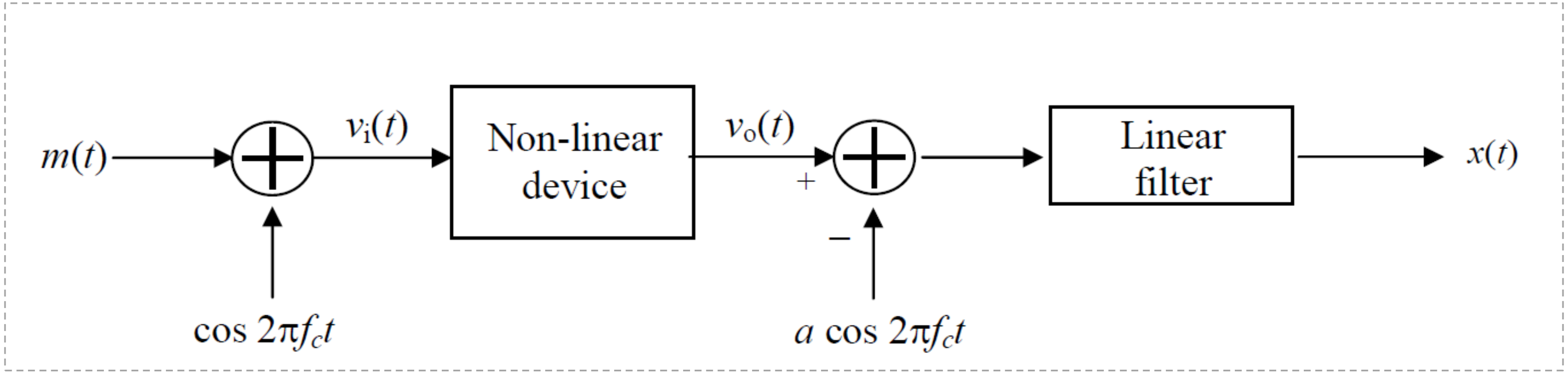
So,

$$\text{filter input} = v_o(t) - a\cos(2\pi f_c t)$$
$$= 5a\cos(2\pi 70t - \theta) + \frac{25b}{2} + \frac{25b}{2}\cos(2\pi 140t - 2\theta)$$
$$+ 10b\cos(2\pi 70t - \theta)\cos(2\pi f_c t) + \frac{b}{2} + \frac{b}{2}\cos(4\pi f_c t)$$



desired DSBSC-AM signal

傅里叶变换是对称的，所以可以只画单边频谱



Solution of (b)

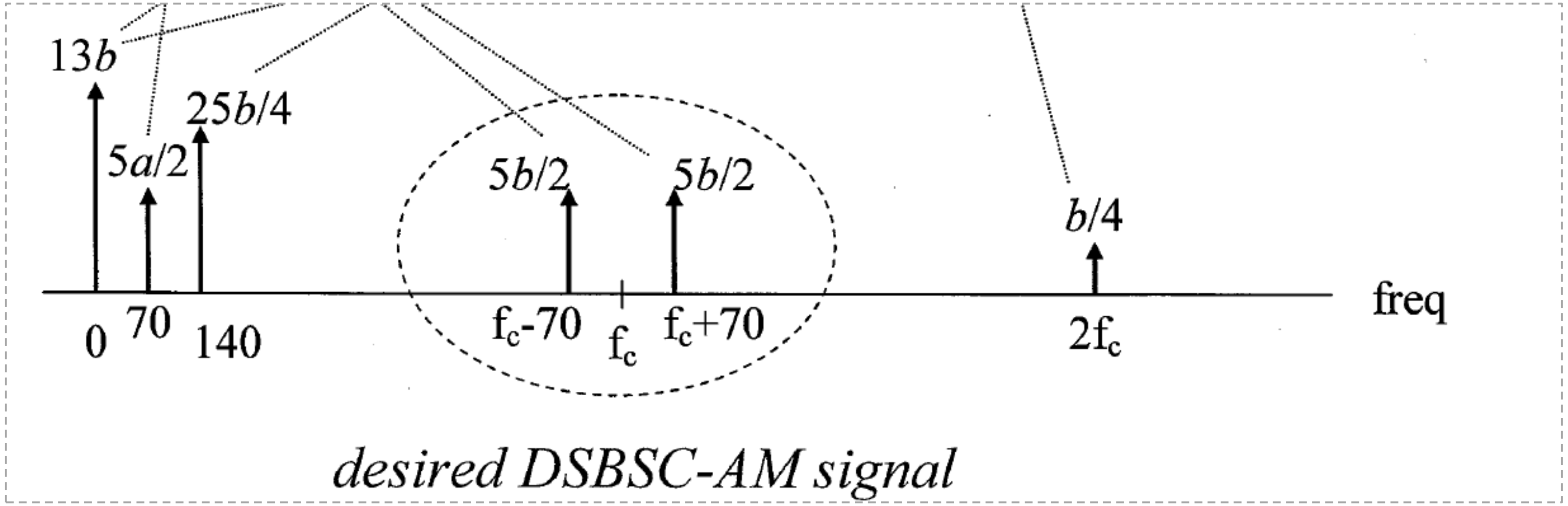
Since the message signal $m(t)$ is a $\cos(2\pi 70t - \theta)$ signal, its DSBSC-AM signal should be the $\cos(2\pi 70t-\theta)\cos(2\pi f_c t)$ term. Hence the linear filter should have center freq $= f_c$ and bandwidth $=$ 140Hz in order to separate it from the other unwanted signal components.

Solution of (c)

From the spectrum of part (a),

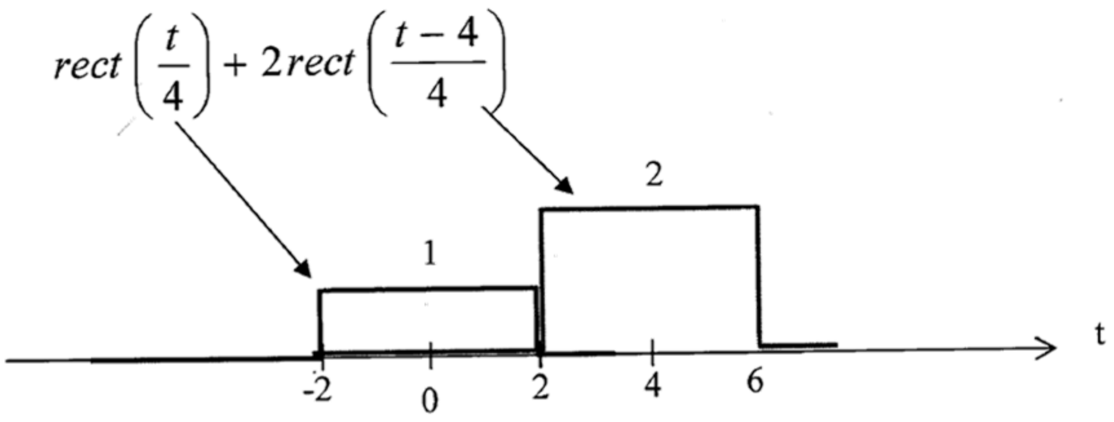
$f_c - 70 > 140$

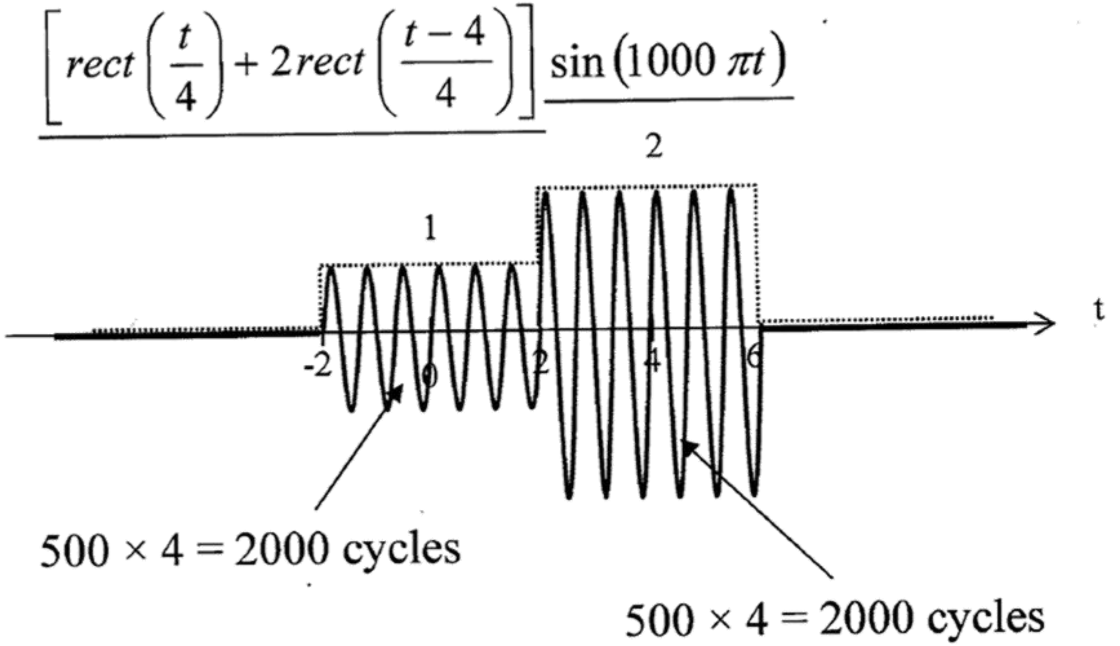
$\Rightarrow f_c > 210, \quad \text{or} \quad f_c > 3 \times \text{message bandwidth}$



2. A suppressed-carrier AM signal $x_1(t)$ is generated by modulating $s_1(t) = rect\left(\frac{t}{4}\right) + 2rect\left(\frac{t-4}{4}\right)$ with $\sin(1000\pi t)$. Sketch the time waveform of $x_1(t)$.

Solution





$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le T/2, \\ 0, & \text{otherwise.} \end{cases}$$

3. A DSBSC-AM signal is

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

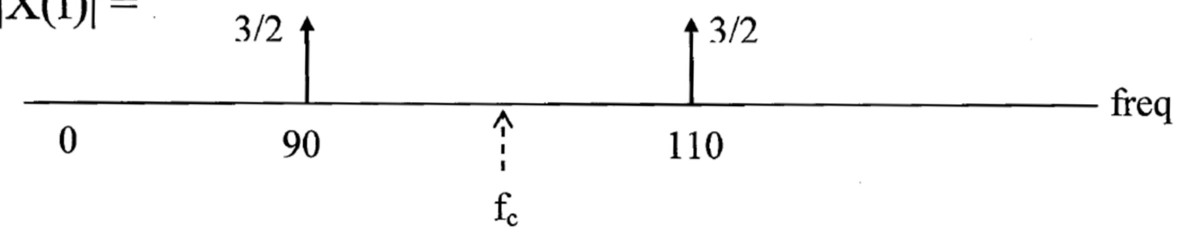
(a) Sketch the amplitude spectrum of x(t) to deduce the carrier frequency in x(t)

(b) Given that x(t) was generated using a sine carrier signal with phase 0, demodulate x(t).

Solution

$$x(t) = 3 \sin 180\pi t + 3 \sin 220\pi t$$

$|X(f)| =$



傅里叶变换是对称的，所以可以只画单边频谱

Carrier freq = centre freq of AM spectrum $\Rightarrow f_c = 100$Hz

To demod x(t), multiply it by $\sin(2\pi\ 100t)$ followed by LPF.

From X(f), AM signal BW = 110-90 = 20Hz.

LPF bandwidth = message signal BW = ½ × 20Hz = 10Hz.

Demodulator output

$= [\ (3 \sin 180\pi t + 3 \sin 220\pi t) \sin 200\pi t\ ]_{LPF}$

$= 3\ [\ \sin 180\pi t \sin 200\pi t + \sin 220\pi t \sin 200\pi t\ ]_{LPF}$

$= 3/2\ [\ \cos(180-200)\pi t - \cos(180+200)\pi t$

$\quad + \cos(220-200)\pi t - \cos(220+200)\pi t]_{LPF}$

$= 3/2\ [\ 2 \cos 20\pi t - \cos 380\pi t - \cos 420\pi t]_{LPF}$

$= 3 \cos 20\pi t$

Since the modulation process would halve the amplitude of message signal, a filter with a gain of 2 should be added into demodulator. Then we would have de demodulator output as

$$6cos20\pi t$$