

EE206: Communications Principles Tutorial

Assignment 5

1. A triangular signal $m(t) = 4\Lambda[(t - 6)/2]$ frequency modulates a carrier signal $f(t) = 100 \cos(2\pi f_c t)$ with $k_f = 30$ Hz/volt.
 - a. Sketch the instantaneous frequency deviation in hertz for the obtained FM signal.
 - b. Sketch the instantaneous phase deviation in radian for this FM signal.

Solution:

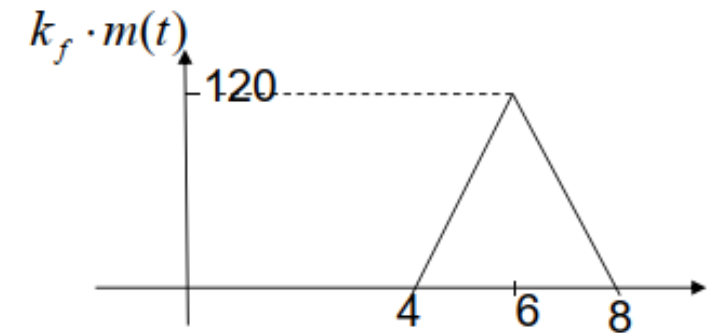
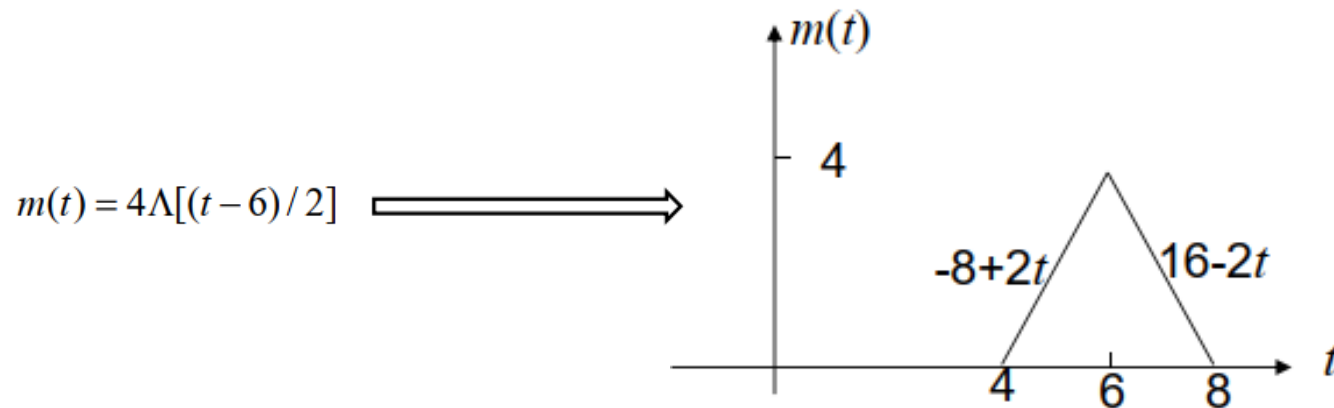
a) For a FM signal

$$f_i(t) = f_c + \boxed{k_f m(t)} \rightarrow \text{Instantaneous carrier frequency deviation, } \Delta f_i(t)$$

f_c : carrier frequency, a constant

k_f : frequency sensitivity, a constant (Hz/volt).

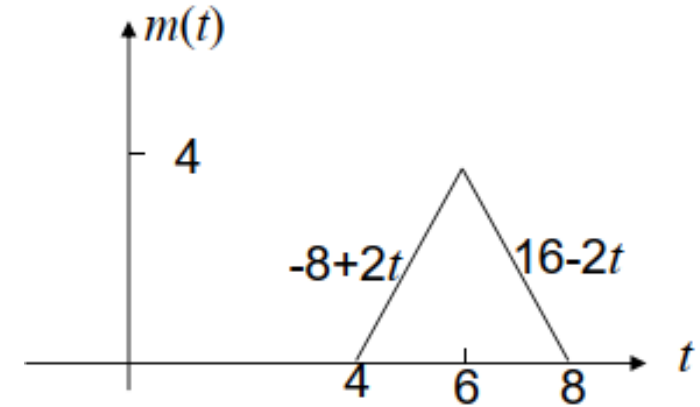
Instantaneous frequency deviation:



Solution:

$$f_i(t) = f_c + k_f m_t$$

$$\theta = \int_0^t 2\pi f_i(\tau) d\tau = 2\pi \int_0^t (f_c + k_f m_\tau) d\tau = 2\pi f_c t + \boxed{2\pi \int_0^t k_f m_\tau d\tau}$$



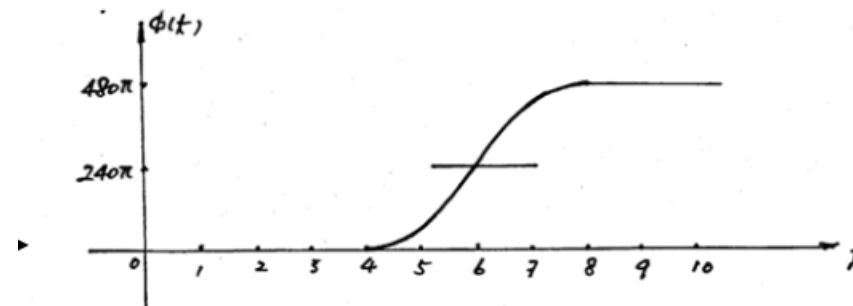
b) The phase deviation in radian is obtained by $\Phi(t) = 2\pi k_f \cdot \int_0^t m(\tau) d\tau$.

$$\Phi(t) = 0 \quad \text{for } t < 4$$

$$\Phi(t) = 2\pi k_f \int_4^t (-8 + 2\tau) d\tau = 60\pi(t^2 - 8t + 16) \quad \text{for } 4 \leq t \leq 6$$

$$\Phi(t) = \Phi(6) + 2\pi k_f \int_6^t (16 - 2\tau) d\tau = 240\pi - 60\pi(t^2 - 16t + 60) \quad \text{for } 6 \leq t \leq 8$$

$$\Phi(t) = 480\pi \quad \text{for } t > 8$$



2. An 18 MHz carrier is frequency modulated by a 400 Hz cosine waveform. If the FM signal has an amplitude of 5 volts and a peak frequency deviation of 30 KHz.
- Write the expression for the obtained FM signal
 - Calculate the peak phase deviation in radian for this FM signal
 - Calculate the peak frequency deviation and the peak phase deviation if the frequency of the modulating signal is tripled.

Solution:**a)**

$$f_i(t) = f_c + k_f A \cos(2\pi f t)$$

$$\theta = \int_0^t 2\pi f_i(\tau) d\tau = 2\pi \int_0^t (f_c + k_f A \cos(2\pi f \tau)) d\tau = 2\pi f_c t + 2\pi \int_0^t k_f A \cos(2\pi f \tau) d\tau = 2\pi f_c t + \frac{A k_f}{f} \sin(2\pi f t)$$

$$f_{FM} = A_{FM} \cos\left(2\pi f_c t + \frac{A k_f}{f} \sin(2\pi f t)\right)$$

$$= 5 \cos(36\pi * 10^6 t + 75 \sin(800\pi t))$$

$$f_c = 18 \text{ MHz}$$

$$f = 400 \text{ Hz}$$

$$\text{Peak frequency deviation: } k_f A = 30 \text{ KHz}$$

$$A_{FM} = 5$$

c)**b)**

$$\theta = 36\pi * 10^6 t + 75 \sin(2\pi f t)$$

The maximum phase deviation is 75 rad

$$f = 1200 \text{ Hz}$$

$$\text{Peak frequency deviation: } k_f A = 30 \text{ KHz}$$

$$\text{The peak phase deviation: } \frac{A k_f}{f} = 25 \text{ rad}$$

3. A message signal $m(t) = 0.5 \cos(2\pi 1000t)$ phase modulates a carrier signal $f(t) = 10 \cos(2\pi 10^6 t)$ with modulation phase sensitivity $k_p = 0.3$ rad/V.

- a. Write the expression of the obtained PM signal.
- b. Construct a phasor diagram for this PM signal.
- c. Re-construct the phasor diagram if $m(t) = 0.5 \sin(2\pi 1000t)$.

Solution:

a)

$$\begin{aligned} f_{PM}(t) &= A_c \cos(2\pi f_c t + k_p m(t)) = 10 \cos(2\pi 10^6 t + 0.15 \cos(2\pi 1000 t)) \\ &= 10 \cos(2\pi 10^6 t) \cos(0.15 \cos(2\pi 1000 t)) - 10 \sin(2\pi 10^6 t) \sin(0.15 \cos(2\pi 1000 t)) \end{aligned}$$

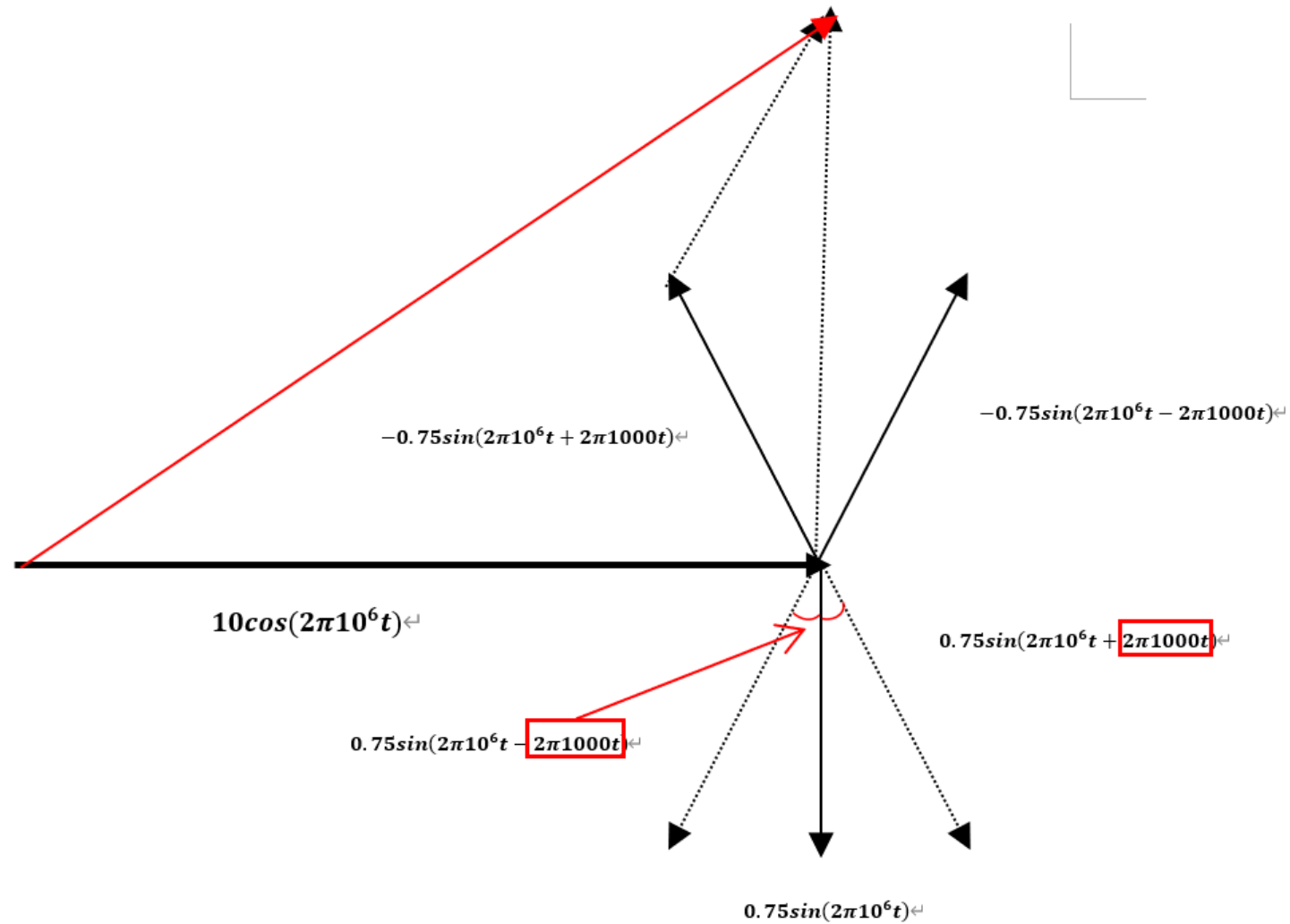
b)

$$\beta_p = k_p A_m = 0.15 < 0.2 \quad \Rightarrow \text{NBPM}$$

$$\begin{aligned} \sin(0.15 \cos(2\pi 1000 t)) &\approx 0.15 \cos(2\pi 1000 t) \\ \cos(0.15 \cos(2\pi 1000 t)) &\approx 1 \end{aligned}$$

$$\begin{aligned} f_{NBPM}(t) &= 10 \cos(2\pi 10^6 t) - 1.5 \sin(2\pi 10^6 t) \cos(2\pi 1000 t) \\ &= 10 \cos(2\pi 10^6 t) - 0.75 \sin(2\pi 10^6 t - 2\pi 1000 t) - 0.75 \sin(2\pi 10^6 + 2\pi 1000 t) \end{aligned}$$

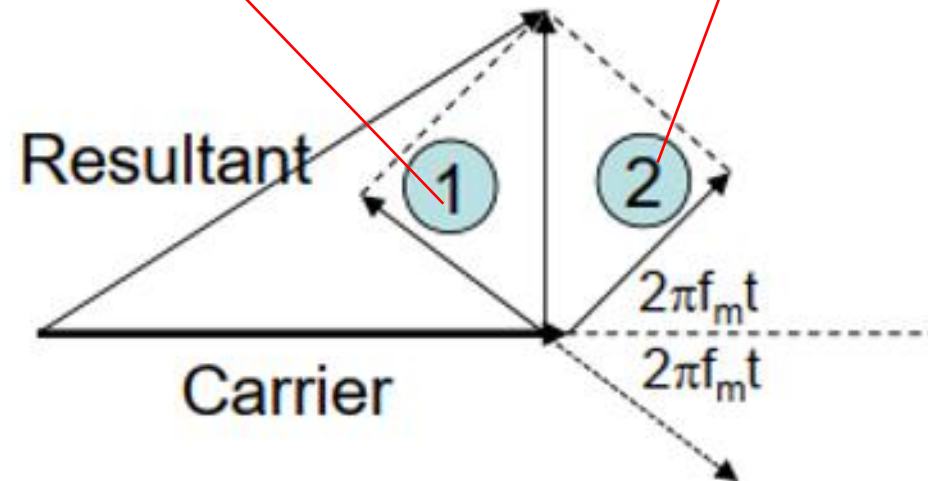
$$f_{NBPM}(t) = 10\cos(2\pi 10^6 t) - 0.75\sin(2\pi 10^6 t - 2\pi 1000 t) - 0.75\sin(2\pi 10^6 + 2\pi 1000 t)$$



$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

c)

$$\begin{aligned} f_{NBPM}(t) &= 10\cos(2\pi 10^6 t) - 1.5\sin(2\pi 10^6 t)\sin(2\pi 1000 t) \\ &= 10\cos(2\pi 10^6 t) - 0.75\cos(2\pi 10^6 t - 2\pi 1000 t) + 0.75\cos(2\pi 10^6 + 2\pi 1000 t) \end{aligned}$$



Phase diagram