An Example of Matlab simulation for an adaptive control system using ode45

Plant:

$\dot{x} = Ax + Bu,\ y = Cx$, where $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix}$.

The constant matrices $A,\ B,\ C$ are unknown.

Reference model:

$y_m(t) = \frac{1}{s+1}[r](t)$ with the reference input $r(t) = \sin t + 2\sin 2t$.

Control input:

$u = \theta^T(t)\omega(t)$, where $\theta(t)$ is a vector of parameter estimates and $\omega(t) = [x(t), r(t)]^T$.

Auxiliary signals:

$\zeta(t) = \frac{1}{s+1}[\omega](t)$;

$\xi(t) = \theta^T(t)\zeta(t) - \frac{1}{s+1}[\theta^T\omega](t)$;

$\varepsilon(t) = e(t) + \rho(t)\xi(t)$, where $e(t) = y(t) - y_m(t)$ and $\rho(t)$ is a parameter estimate.

Adaptive laws:

$\dot{\theta}(t) = -\frac{\Gamma\zeta(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}$, $\Gamma = 5I$;

$\dot{\rho}(t) = -\frac{\gamma\xi(t)\varepsilon(t)}{1+\zeta^T\zeta+\xi^2}$, $\gamma = 1$.

Simulation programs:

simu.m

```
function simu()
y0=[1 1 1 0 0 0 0 0 0 0 0 0 0 0]; % initial condition
[t,y]=ode45('simuequ',[0 200],y0); % ode function

% y: an array of solutions to a set of differential equations
% each row of y corresponds to a time instant returned in vector t

% in this case, y=[state x, reference output y_m, auxiliary signals \zeta
% and \frac{1}{s+1}[\theta^T\omega](t), parameter estimates \theta and \rho]^T

save simudata t y; % save data
```

simuequ.m

```
function dy=simuequ(t,y)
dy=zeros(14,1); % dy: a vector of small increments of y during each time step
% dy also can be considered as \dot{y} in the differential equations.

g11=5;
g12=5;
g13=5;
g14=5; % \Gamma=5I

g2=1; % \gamma=1

r=sin(t)+2*sin(2*t); % reference input

xi=y(10)*y(5)+y(11)*y(6)+y(12)*y(7)+y(13)*y(8)-y(9); % auxiliary signal \xi
eps=4*y(1)-y(4)+y(14)*xi; % auxiliary signal \epsilon

m=1+y(5)*y(5)+y(6)*y(6)+y(7)*y(7)+y(8)*y(8)+xi*xi; % 1+\zeta^T\zeta+\xi^2

u=y(10)*y(1)+y(11)*y(2)+y(12)*y(3)+y(13)*r; % control input

dy(1)=-y(2)+3*y(3)+u;
dy(2)=y(1)-y(3);
dy(3)=y(2)-y(3); % plant dynamics

dy(4)=-y(4)+r; % reference system

dy(5)=-y(5)+y(1);
dy(6)=-y(6)+y(2);
dy(7)=-y(7)+y(3);
dy(8)=-y(8)+r; % auxiliary signal \zeta

dy(9)=-y(9)+u; % \frac{1}{s+1}[\theta^T\omega](t)

dy(10)=-g11*y(5)*eps/m;
dy(11)=-g12*y(6)*eps/m;
dy(12)=-g13*y(7)*eps/m;
dy(13)=-g14*y(8)*eps/m;
dy(14)=-g2*xi*eps/m; % adaptive laws
```

simudraw.m

```
function simudraw()
load simudata; % load data

plot(t,4*y(:,1)-y(:,4));
xlabel ('Tracking error e(t)');
pause; % plot the tracking error

subplot(2,1,1); % plot figures in a tiled way
plot(t,y(:,10)+1);
xlabel ('Parameter error \theta_1(t)-\theta*_1');
subplot(2,1,2);
plot(t,y(:,11)-1);
xlabel ('Parameter error \theta_2(t)-\theta*_2');
pause; % plot the parameter errors

subplot(2,1,1); % plot figures in a tiled way
plot(t,y(:,12)+3);
xlabel ('Parameter error \theta_3(t)-\theta*_3');
subplot(2,1,2);
plot(t,y(:,13)-0.25);
xlabel ('Parameter error \theta_4(t)-\theta*_4');
pause; % plot the parameter errors

subplot(1,1,1); % back to the original setting for plotting
plot(t,y(:,14)-4);
xlabel ('Parameter error \rho(t)-\rho*'); % plot the parameter errors
```

The programs above are in three Matlab **.m** files, i.e., **simu.m**, **simuequ.m**, and **simudraw.m**, respectively. In the Matlab environment, type **simu** at the command window. The function **ode45** in **simu.m** calls the function **simuequ** for each time step. The data generated by the programs is saved in the file **simudata.mat** as the numerical solution to those differential equations to describe the dynamic system. Use **simudraw** to plot the results of simulation.

The simulation results are showed in the following figures.

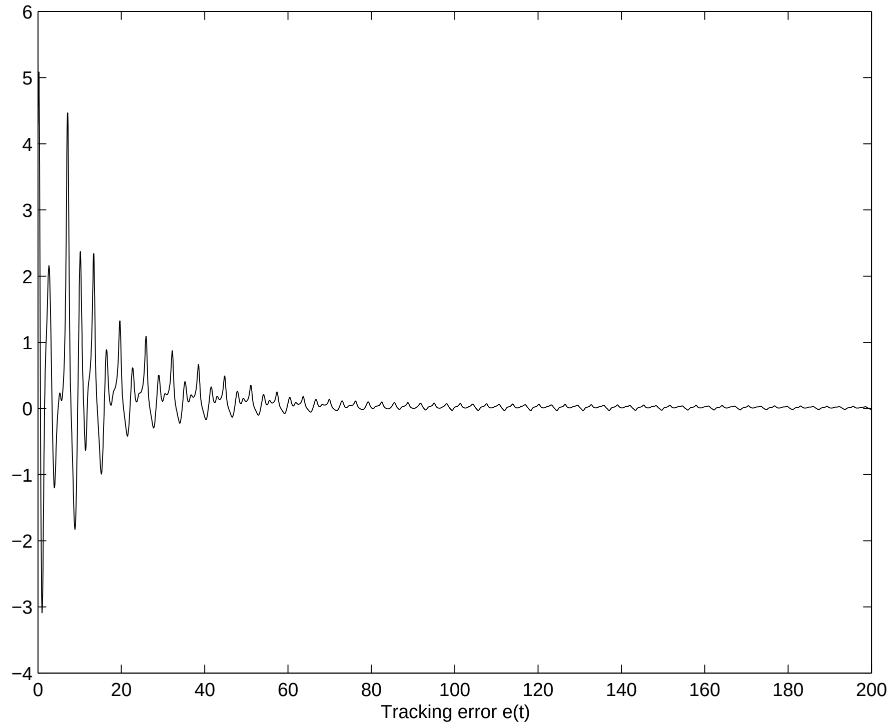

Figure 1: Tracking error $y(t) - y_m(t)$.

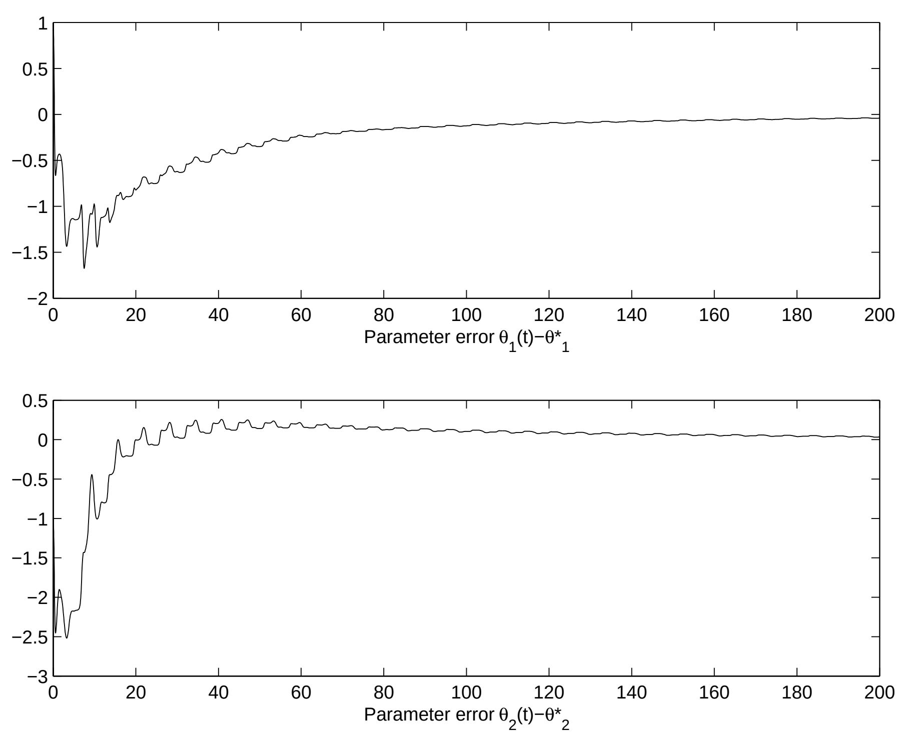

Figure 2: Parameter errors $\theta_1 - \theta_1^*$ ($\theta_1^* = -1$) and $\theta_2 - \theta_2^*$ ($\theta_2^* = 1$).

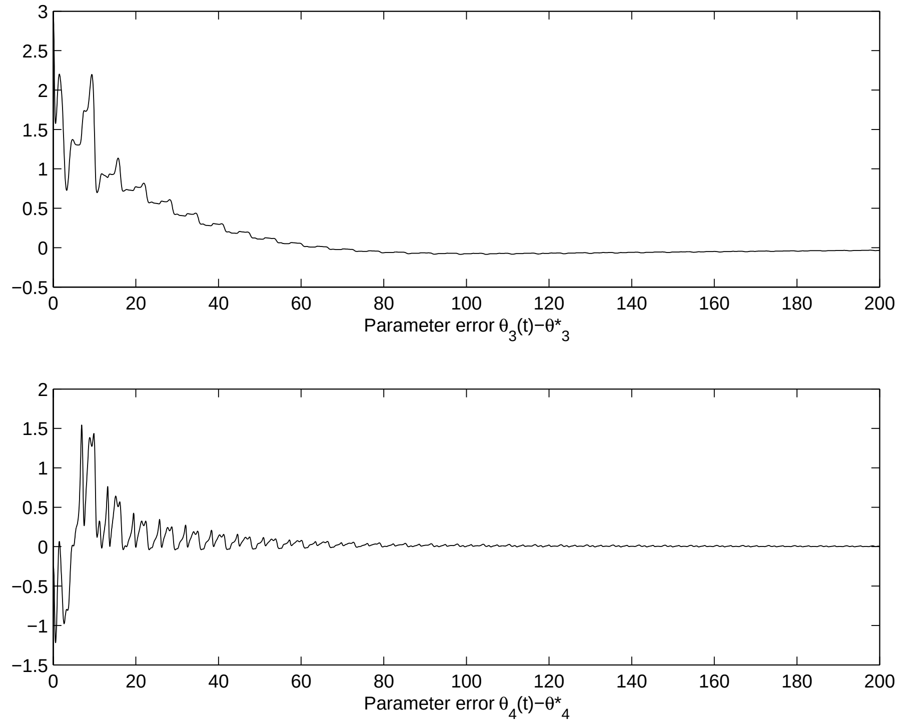

Figure 3: Parameter errors $\theta_3 - \theta_3^*$ $(\theta_3^* = -3)$ and $\theta_4 - \theta_4^*$ $(\theta_4^* = 0.25)$.

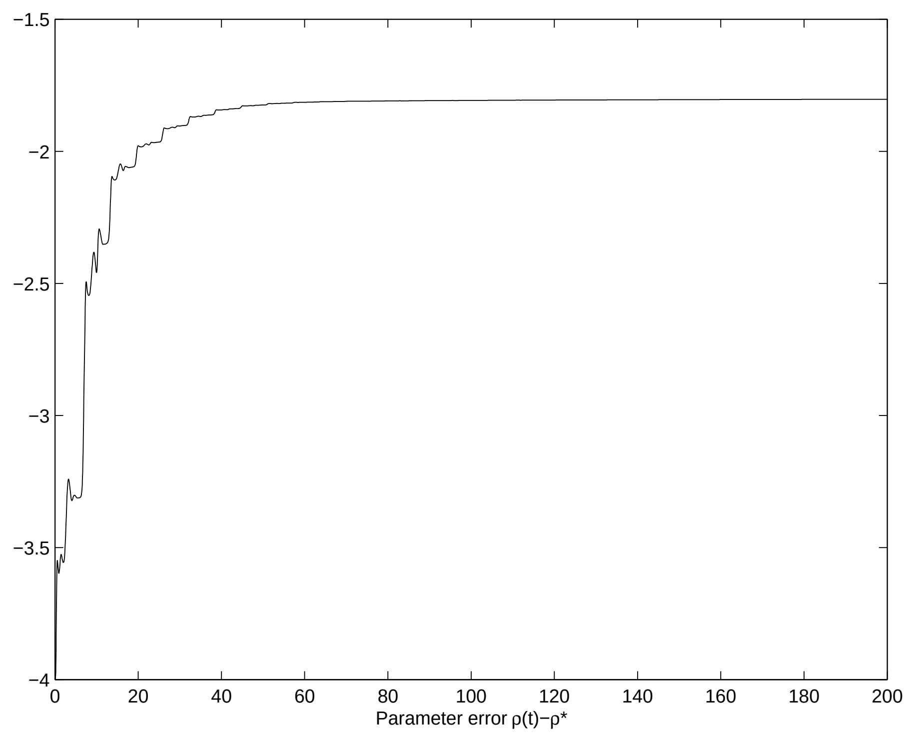

Figure 4: Parameter error $\rho - \rho^*$ $(\rho^* = 4)$.