

TSP & Shortest path



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Agenda

- Problem
- TSP / Shortest path
- Code Shortest path
- Code TSP – Permutation
- Dynamic Programming
- Code TSP – Dynamic Programming
- Q&A



Problem

Problem

출발 \ 도착	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	3	5	48	48	8	8	5	5	3	3	-1	3	5	8	8	5
2	3	0	3	48	48	8	8	5	5	-1	-1	3	-1	3	8	8	5
3	5	3	0	72	72	48	48	24	24	3	3	5	3	-1	48	48	24
4	48	48	74	0	-1	6	6	12	12	48	48	48	48	74	6	6	12
5	48	48	74	-1	0	6	6	12	12	48	48	48	48	74	6	6	12
6	8	8	50	6	6	0	-1	8	8	8	8	8	8	50	-1	-1	8
7	8	8	50	6	6	-1	0	8	8	8	8	8	8	50	-1	-1	8
8	5	5	26	12	12	8	8	0	-1	5	5	5	5	26	8	8	-1
9	5	5	26	12	12	8	8	-1	0	5	5	5	5	26	8	8	-1
10	3	-1	3	48	48	8	8	5	5	0	-1	3	-1	3	8	8	5
11	3	-1	3	48	48	8	8	5	5	-1	0	3	-1	3	8	8	5
12	-1	3	5	48	48	8	8	5	5	3	3	0	3	5	8	8	5
13	3	-1	3	48	48	8	8	5	5	-1	-1	3	0	3	8	8	5
14	5	3	-1	72	72	48	48	24	24	3	3	5	3	0	48	48	24
15	8	8	50	6	6	-1	-1	8	8	8	8	8	8	50	0	-1	8
16	8	8	50	6	6	-1	-1	8	8	8	8	8	8	50	-1	0	8
17	5	5	26	12	12	8	8	-1	-1	5	5	5	5	26	8	8	0

- Path size provided

- Find Shortest Path

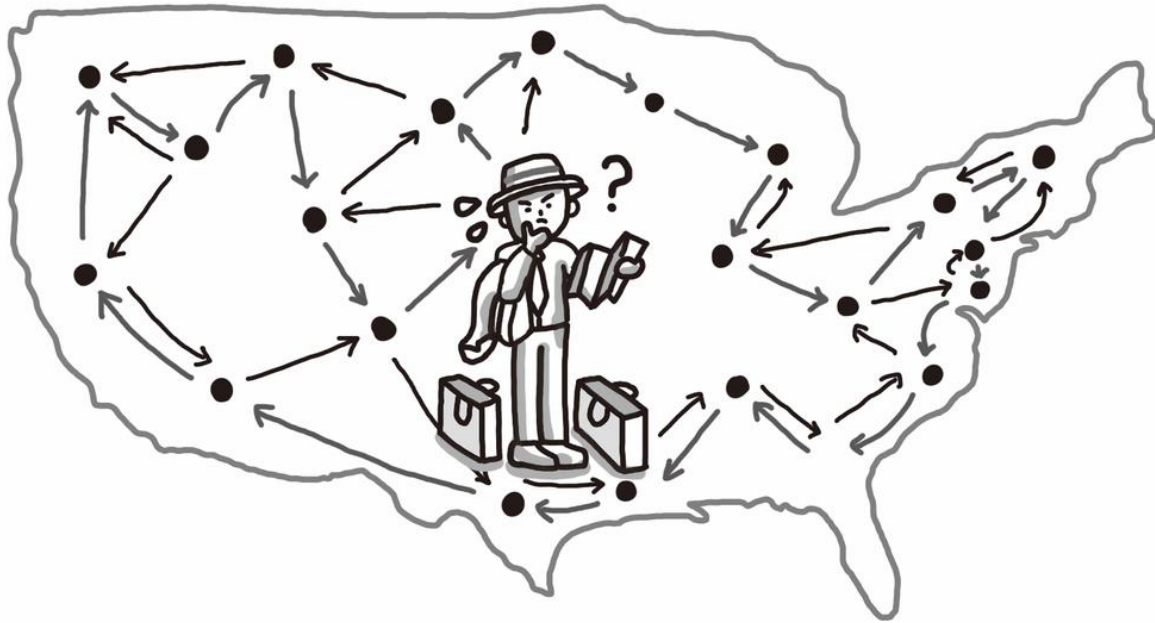
- Choose start & end city
- Find shortest path each city

- TSP

- Travel Salesman Problem
- Uses a two-dimensional array of numbers entered

TSP / Shortest Path(1/3)

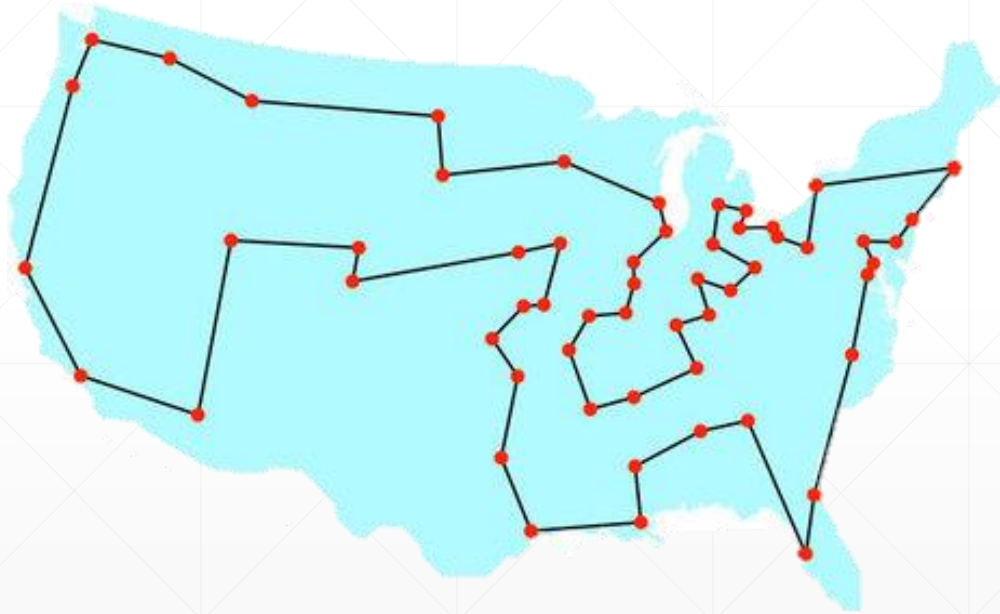
외판원 문제(TSP)



■ TSP

- several cities and the cost
- moving from one city to another
- visit all cities only once
- seek the order of minimum cost travel back to the original starting point

TSP / Shortest Path(2/3)



■ Permutation Algorithm

- List the number of all possible cases
- Calculate the distance of all cases and select the small one

■ DP (Dynamic Programming)

- solve complex problems by dividing them into simple multiple problems



Code

Code : Main

■ Main

- Make city map
- Get user's input and move to right menu

```
if __name__ == '__main__':  
    cmap = cityMap()  
    mn = menu()  
    if mn == 1:  
        start, end = shortestMenu()  
        st = time.time()  
        print(findShortestCase(start, end, cmap))  
        print(time.time()-st)  
  
    elif mn == 2:  
        n = TSPMenu()  
        mp = TSPMap(n)  
        st = time.time()  
        print("%d의 최단경로" % n, TSP(mp))  
        # permute(5)  
        # print(distanceCalc(permute(10), cmap))  
        print(time.time() - st)
```


Code : Menu (1/3)

■ City Map

- Open file and read lines
- Make 2-D list

```
def cityMap():  
    f = open("city.txt", 'r', encoding='UTF8')  
    lines = f.readlines()  
    cityMap = [[int(lines[i].split()[n]) for n in range(17)] for i in range(17)]  
  
    return cityMap
```

Code : Menu (2/3)

```
def menu():  
    while True:  
        try:  
            mn = int(input("Shortest path(1) or Tsp(2)? Please input the m  
            if mn == int(1):  
                return mn  
            break  
            elif mn == int(2):  
                return mn  
            break  
            else:  
                raise ValueError  
  
        except:  
            print("Please input the right value")
```

```
def shortestMenu():  
    while True:  
        try:  
            ans = list(map(int, input("Please input the start & end. ex) 7 11").split()))  
            return ans  
            break  
        except:  
            print("Please input the right value")
```

■ Menu

- Get menu value
- Get start and end city of shortest path

Code : Menu (3/3)

```
def TSPMenu():  
    while True:  
        try:  
            num = int(input("Please input the TSP size(3~17)"))  
            if num < 3 or num > 17:  
                raise ValueError  
            else:  
                return num  
                break  
        except:  
            print("Please input the right value")
```

```
def TSPMap(num):  
    mparr = np.array(cityMap())  
    mp = list(map(list, mparr[:num, :num]))  
    return mp
```

■ TSP Menu

- Get TSP size
- By the size value, determine the map's size

Code : Shortest path (1/3)

■ Find Shortest Case

- Get start, end, city map(distance)
- Return searchCase
- Start searchCase recursion

```
def findShortestCase(start, end, distance):  
    shortest = ["", 999]  
  
    def searchCase(start, end, length, tmplist):  
  
    return searchCase(start, end, 0, "")
```

Code : Shortest path (2/3)

```
def searchCase(start, end, length, tmplist):  
    start -= 1  
    end -= 1  
    # print("시작:", start, "끝:", end, "현재경로:", tmplist, "길이"  
    if tmplist == "":  
        tmplist += "%d" % (start+1)  
    # print("현재경로", tmplist, "길이", length)  
  
    if distance[start][end] == -1 or distance[start][end] == 0:  
        # print("경로가 없거나 자신일때", start, end)  
        return  
  
    if distance[start][end] != -1:  
        cmprLength = length + distance[start][end]  
        cmprList = tmplist + ",%d" % (end+1)  
        # print("도착지까지의 길이:", cmprLength, "도착지까지의 경로:"  
        if cmprLength < shortest[1]:  
            # print("Shortest에 저장", "현재 길이:", cmprLength, "  
            shortest[0] = cmprList  
            shortest[1] = cmprLength
```

■ SearchCase

- Tmplist = path to now
- Start -1, end -1 to index
- When start recursion, add tmplist to start city
- If no path between start & end city or start city is same with end, stop searching
- if there's path, Creates a path that reaches the end and compares it to the currently stored minimum path

Code : Shortest path (3/3)

```
for i in range(17):  
    # print("현재 길이:", length, "현재 경로", tmp_list, "현재 개수", i+1)  
    if start != i and distance[start][i] != -1 and length+distance[start][i] < shortest[1] and distance[start][i] != 0:  
        nlength = length + distance[start][i]  
        nlist = tmp_list + ",%d" % (i+1)  
        searchCase(i+1, end+1, nlength, nlist)  
    else:  
        continue  
return shortest
```

■ Searching different route

- Ex) start 3 end 4, 3,4 -> 3,1,4 ->, 3,1,2,4 ->
- Go to next searchCase when using i case is right
 - The start and the new point are different
 - path is existed
 - now distance is shorter than now shortest route
- Return shortest route

Code : TSP using permutation

```
import itertools
```

```
def permute(num):  
    # n<=10 일 때만 사용가능  
    inplist = [x + 1 for x in range(num)]  
    permutation = list(itertools.permutations(inplist))  
    # print(permutation)  
    # permutation = list(map(lambda x: ''.join(x), perm  
    return permutation
```

■ Make permutation

- Using itertools, put components in inplist
- If num =6 , inplist = [1,2,3,4,5,6]
- Make permutations

Code : TSP using permutation

```
def distanceCalc(list, distance):
    shortest = [" ", 999999]
    for prmt in list:
        flg = True
        tmpdis = 0
        # print("reset", tmpdis)
        for idx, i in enumerate(prmt):
            # print("i", i)
            if i == prmt[-1]:
                if distance[int(i)-1][int(prmt[0])-1] == -1:
                    flg = False
                else:
                    tmpdis += distance[int(i)-1][int(prmt[0])-1]
                    break
            # print("distance", cityDist(int(i), int(prmt[idx+1])),
            if distance[int(i)-1][int(prmt[idx + 1])-1] == -1:
                flg = False
                break
            tmpdis += distance[int(i)-1][int(prmt[idx + 1])-1]
        if tmpdis < shortest[1] and flg:
            shortest[0] = prmt
            shortest[1] = tmpdis
    return shortest
```

■ Calculate distance

- Set shortest arbitrarily
- Calculates the distance for each permutation
- If permutation is end
 - End -> Start path is existed, pass
- path exists and is shorter than the current shortest path
 - renew the shortest path

Limit of Permutation Method

N=10

```
Shortest path(1) or Tsp(2)? Please input the menu number.2  
[(1, 2, 3, 10, 8, 4, 6, 5, 7, 9), 57]  
8.416694402694702
```

N=11

```
Shortest path(1) or Tsp(2)? Please input the menu number.2  
[(1, 2, 3, 10, 8, 4, 6, 5, 7, 9, 11), 60]  
89.18377304077148
```

■ Time Complexity

- $N \leq 10$, permutation method is okay
- $N > 10$, running time is too long

■ To solve this -> Dynamic Programming

Dynamic Programming

Dynamic Programming

- solve complex problems by dividing them into simple multiple problems

- Conditions

- A small problem is repeated.
- The same question has the same answer every time it is asked.

- Memoization

- Save a small calculated problem and use it again

- Top-Down vs Bottom-Up

- Top-down: Solve small problems if big problems are not solved
- Bottom-up: Solving small problems step by step

Bit Mask

1 1 0 1 (2진수) \longrightarrow **13** (10진수)
 $2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$(8 * 1) + (4 * 1) + (2 * 0) + (1 * 1) = 13$$

■ Techniques using binary representations of integers

➤ { 0, 1, 2, 3, 4 } \Rightarrow 11111

➤ { 1, 2, 3, 4 } \Rightarrow 11110

➤ { 1, 2, 4 } \Rightarrow 10110

➤ { 2, 4 } \Rightarrow 10100

➤ { 1 } \Rightarrow 00010

■ Express a collection of cities passed by

Code : TSP using dynamic programming

```
def TSP(distance):
    N = len(distance)
    VISITED_ALL = (1 << N) - 1 #if n이 5일때, 11111(전부 다녀간 경우) 만들어줌
    cache = [[None] * (1 << N) for _ in range(N)]
    INF = float('inf')

    def find_path(last, visited): #last: 중간(현재까지 방문한) 경로의 마지막도시, #visited
        if visited == VISITED_ALL:
            if distance[last][0] == -1:
                return INF, []
            return [last+1], distance[last][0]

        if cache[last][visited] is not None: #cache에 있으면 그거 재사용
            return cache[last][visited]

        tmp = INF
        tmp_list = []

        for city in range(N):
            # print("Come in for statement")
            if visited & (1 << city) == 0 and distance[last][city] != -1: #visited &
                ncity, ntmp = find_path(city, visited | (1 << city))
                ntmp += distance[last][city]
                ntmp_list = [last+1] + ncity
                if ntmp < tmp:
                    tmp_list = ntmp_list
                    tmp = min(tmp, ntmp) #visited | (1 << city): city가 포함된 경로 갱

        cache[last][visited] = tmp_list, tmp
        return tmp_list, tmp

    return find_path(0, 1 << 0)
```

■ Using Bit mask

➤ VISITED_ALL = 11111(all visited)

■ Cache list to store small question's minimum answer

■ Top-down Method

➤ 0001 -> 0011 -> 0111 -> 1111(smallest)

Time Complexity : Dynamic Programing

N=10

Shortest path(1) or Tsp(2)? Please input the menu number.2

[(1, 2, 3, 10, 8, 4, 6, 5, 7, 9), 57]

8.416694402694702

Please input the TSP size(3~17)10

10의 최단경로 ([1, 2, 3, 10, 8, 4, 6, 5, 7, 9], 57)

0.02278304100036621

N=11

Shortest path(1) or Tsp(2)? Please input the menu number.2

[(1, 2, 3, 10, 8, 4, 6, 5, 7, 9, 11), 60]

89.18377304077148

Please input the TSP size(3~17)11

11의 최단경로 ([1, 2, 3, 10, 8, 4, 6, 5, 7, 9, 11], 60)

0.04291272163391113

- Big difference in time

- compared to permutations

- 9nn times difference in n=10

Q&A