Introduction

What is proof?

Info

Proof is method of obtaining / ascertaining a truth.

Example

1. Experimentation
2. Observation
3. Sampling & Counter examples
4. Judge & Juries
5. Religion (Word of God)
6. Word of Boss

Important

In mathematics, a mathematical proof is a verification of a **proposition** by a chain of **logical deductions** from a set of **axioms**.

Proposition

Important

A **proposition** is statement which is **either true or false**.

- For this proposition to be to, the predicate has to come true.

Example

$$\forall n \in N,\ n^2 + n + 41 \ is\ the\ prime\ numbers$$

is called the predicate (depends on the value of variable).

Solution

- for the 1 to 39 -> true
- but, 40 and 41 -> false

Failure

Example

$$a^4 + b^4 + c^4 + d^4 \ has\ no\ positive\ solutions$$

Solution

- But for some 6 digit value, this proposition becomes false, and there exists a solution.
- $$\exists\ a^4 + b^4 + c^4 + d^4 \ has\ no\ positive\ solutions$$
- But for some exists propositions, of which finding the shortest smallest counter example of > 1000 digits.

Failure

- factoring of 1000 digits is useful Crypto-systems works.

Not

Important

p	¬ p
T	F
F	T

And

Important

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Or

Important

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or (xor)

Important

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implies

Important

An Implication $p \Rightarrow q$ is true, when p is false or q is true.

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \iff q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Example

"If pigs fly, I would be king" is true.

if and only if

Important

- either both true or both false

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

$$x^2 - 4 \geq 0 \iff |x| \geq 2$$

Conjecture

Important

Conjecture is an opinion or conclusion formed on the basis of incomplete information.

Axiom

Important

An **axiom** is a **proposition that is assumed to be true.**

Example

- In Euclidean Geometry, Given a line L and a point P not on line L, there is exactly one line through P || L.
- In Euclidean Geometry, Given a line L and a point P not on line L, there is no line through P || L.
- In Hyperbolic Geometry, Given a line L and a point P not on line L, there is infinite lines through P || L.

Attention

- Axioms should be consistent and complete
- A set of Axioms is said to be **consistent if no proposition can be proved to be both true and false**.
- A set of Axioms are said to be **complete if it can be used to prove every proposition is either true or false**.

Notations

- $\forall$ (for all)
- $\exists$ (ther exists)
- $\in$ (belongs to)