Introduction

What is proof?

Proof is method of obtaining / ascertaining a truth.

1. Experimentation
2. Observation
3. Sampling & Counter examples
4. Judge & Juries
5. Religion (Word of God)
6. Word of Boss

> 💧 **Important**
>
> In mathematics, a mathematical proof is a verification of a **proposition** by a chain of **logical deductions** from a set of **axioms**.

Proposition

> 💧 **Important**
>
> A **proposition** is statement which is **either true or false**.
>
> – For this proposition to be to, the predicate has to come true.

> ≔ **Example**
>
> $$\forall n \in N, \ n^2 + n + 41 \ is \ the \ prime \ numbers$$
>
> is called the predicate (depends on the value of variable).

– for the 1 to 39 -> true

– but, 40 and 41 -> false

–
$$a^4 + b^4 + c^4 + d^4 \ has \ no \ positive \ solutions$$

– But for some 6 digit value, this proposition becomes false, and there exists a solution.

–
$$\exists \ a^4 + b^4 + c^4 + d^4 \ has \ no \ positive \ solutions$$

– But for some exists propositions, of which finding the shortest smallest counter example of > 1000 digits.

– factoring of 1000 digits is useful Crypto-systems works.

– Proofs by pictures are very convincing, but wrong.

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$$\forall \ n \in \ X, \ n \ \geq 2 \Rightarrow n^2 \geq 4$$

Implication

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Longleftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

> **Example**
>
> "If pigs fly, I would be king" is true.

Axiom

> **Important**
>
> An **axiom** is a **proposition that is assumed to be true.**

> **Example**
>
> – In Euclidean Geometry, Given a line L and a point P not on line L, there is exactly one line through P || L.
> – In Euclidean Geometry, Given a line L and a point P not on line L, there is no line through P || L.
> – In Hyperbolic Geometry, Given a line L and a point P not on line L, there is infinite lines through P || L.

> **Attention**
>
> – Axioms should be consistent and complete
> – A set of Axioms is said to be **consistent if no proposition can be proved to be both true and false.**
> – A set of Axioms are said to be **complete if it can be used to prove every proposition is either true or false.**