Introduction

What is proof?

Proof is method of obtaining / ascertaining a truth.

1. Experimentation
2. Observation
3. Sampling & Counter examples
4. Judge & Juries
5. Religion (Word of God)
6. Word of Boss

> **Important**
>
> In mathematics, a mathematical proof is a verification of a **proposition** by a chain of **logical deductions** from a set of **axioms**.

Proposition

> **Important**
>
> A **proposition** is statement which is **either true or false**.
>
> - For this proposition to be to, the predicate has to come true.

> **Example**
>
> $$\forall n \in N,\ n^2 + n + 41 \text{ is the prime numbers}$$
>
> is called the predicate (depends on the value of variable).

- for the 1 to 39 -> true
- but, 40 and 41 -> false
- $$a^4 + b^4 + c^4 + d^4 \text{ has no positive solutions}$$
- But for some 6 digit value, this proposition becomes false, and there exists a solution.
- $$\exists\, a^4 + b^4 + c^4 + d^4 \text{ has no positive solutions}$$
- But for some exists propositions, of which finding the shortest smallest counter example of > 1000 digits.
- factoring of 1000 digits is useful Crypto-systems works.
- Proofs by pictures are very convincing, but wrong.
- $$\forall\, n \in\ X,\ n\ \geq 2 \Rightarrow n^2 \geq 4$$

Implication

> **Important**
>
> An Implication $p \Rightarrow q$ is true, when p is false or q is true.

p	q	p ⇒ q	q ⇒ p	p ⟺ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

> **Example**
>
> "If pigs fly, I would be king" is true.

Axiom

> **Important**
>
> An **axiom** is a **proposition that is assumed to be true.**

> **Example**
>
> - In Euclidean Geometry, Given a line L and a point P not on line L, there is exactly one line through P || L.
> - In Euclidean Geometry, Given a line L and a point P not on line L, there is no line through P || L.
> - In Hyperbolic Geometry, Given a line L and a point P not on line L, there is infinite lines through P || L.

> **Attention**
>
> - Axioms should be consistent and complete
> - A set of Axioms is said to be **consistent if no proposition can be proved to be both true and false**.
> - A set of Axioms are said to be **complete if it can be used to prove every proposition is either true or false**.