NATIONAL PUBLIC SCHOOL, INR

GRADE 12 (Mathematics) WORKSHEET 4 (Integration)

Answers.

1. $\int \frac{2x+1}{(x+1)(x-2)}\,dx$ — $\frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$

2. $\int \frac{x^2}{x^2-4x+3}\,dx$ — $x + \frac{9}{2}\log|x-3| - \frac{1}{2}\log|x-1| + c$

3. $\int \frac{(2x+1)\,dx}{4-3x-x^2}$ — $\frac{-7}{5}\log|4+x| - \frac{3}{5}\log|1-x| + c$

4. $\int \frac{(2x-1)\,dx}{(x-1)(x+2)(x-3)}$ — $\frac{-1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x$

5. $\int \frac{(2x-3)\,dx}{(x^2-1)(2x+3)}$ — $\frac{-1}{10}\log|x-1| + \frac{5}{2}\log|x+1| - \frac{12}{5}\log|2x+3|$

6. $\int \frac{(3x-2)\,dx}{(x+1)^2(x+3)}$ — $\frac{11}{4}\log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + c$

7. $\int \frac{x^3\,dx}{x^4+3x^2+2}$ — $\log|x^2+2| - \frac{1}{2}\log|x^2+1| + c$

8. $\int \frac{\sin x\,dx}{(1-\cos x)(2-\cos x)}$ — $\log|1-\cos x| - \log|2-\cos x| + c$

9. $\int \frac{\sec x\,dx}{1+\operatorname{cosec} x}$ — $\frac{1}{4}\log\left|\frac{1+\sin x}{1-\sin x}\right| + \frac{1}{2(1+\sin x)} + c$

10. $\int \frac{dx}{(2-x)(x^2+3)}$ — $\frac{-1}{7}\log|2-x| + \frac{1}{14}\log(x^2+3) + \frac{2}{7}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

11. $\int \frac{2x^2+1}{x^2(x^2+4)}\,dx$ · $\frac{1}{4}\left(\frac{-1}{x}+\frac{7}{2}\tan^{-1}\frac{x}{2}\right)+c$

12. $\int \frac{x^2\,dx}{x^4+x^2-2}$ $\frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}}+\frac{1}{6}\log\left|\frac{x-1}{x+1}\right|+c$

13. $\int \frac{x\,dx}{(x+1)(x+2)}$ $2\log(x+2)-\log(x+1)+c$

14. $\int \frac{1}{x^2-9}\,dx$ $\frac{1}{6}\log\left(\frac{x-3}{x+3}\right)+c$

15. $\int \frac{3x-1\,dx}{(x-1)(x-2)(x-3)}$ $\log|x-1|-5\log|x-2|+4\log|x-3|+c$

16. $\int \frac{2x\,dx}{x^2+3x+2}$ $-2\log(x+1)+4\log(x+2)+c$

17. $\int \frac{x^3\,dx}{(x^2-1)(x-2)}$ $\frac{x^2}{2}+3x-\log(x-1)+8\log(x-2)+c$

18. $\int \frac{\cos\theta\,d\theta}{(2+\sin\theta)(3+4\sin\theta)}$ $\frac{1}{5}\log\left(\frac{3+4t}{2+t}\right)+c$

19. $\int \frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$ $\log(\sec x+\tan x)-2\tan\left(\frac{x}{2}\right)+c$

20. $\int \frac{(x^2+1)\,dx}{(x-1)^2(x+3)}$ $\frac{3}{8}\log(x-1)-\frac{1}{2(x-1)}+\frac{5}{8}\log(x+3)+c$

21. $\int \frac{8\,dx}{(x+2)(x^2+4)}$ $\log\frac{x+2}{(x^2+4)^{1/2}}+\tan^{-1}\left(\frac{x}{2}\right)+c$

22. $\int \frac{\tan\theta+\tan^3\theta}{(1+\tan^3\theta)}\,d\theta$ $\frac{-1}{3}\log(t+1)+\frac{1}{6}\log(t^2-t+1)+\frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{2t}{\sqrt{}}\right.$ $+c$

CLASS 12
Mathematics

NATIONAL PUBLIC SCHOOL, INR

INTEGRATION BY PARTS - WORKSHEET

1. $\int x \cos x \, dx$
2. $\int x \sin 3x \, dx$
3. $\int x \log 2x \, dx$
4. $\int x^2 \cos x \, dx$
5. $\int x \tan^2 x \, dx$
6. $\int x^2 \sin^2 x \, dx$
7. $\int x \cdot \log_2 (x+1) \, dx$
8. $\int 2x^3 e^{x^2} \, dx$
9. $\int \cos \sqrt{x} \, dx$
10. $\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$
11. $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx$
12. $\int x \sin^{-1} x \, dx$
13. $\int x \cos^{-1} x \, dx$
14. $\int \frac{\sin^{-1} x}{x^2} \, dx$
15. $\int e^{3x} \sin 4x \, dx$

Answers

1. $x \sin x + \cos x + c$
2. $\frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} + c$
3. $\frac{1}{2} x^2 \log 2x - \frac{1}{4} x^2 + c$
4. $x^2 \sin x + 2x \cos x - 2 \sin x + c$
5. $x \tan x + \log|\cos x| - \frac{x^2}{2} + c$
6. $\frac{1}{6} x^3 - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$
7. $\frac{1}{2}(x^2-1)\log(x+1) - \frac{1}{4}x^2 + \frac{1}{2}x + c$
8. $e^{x^2}(x^2-1) + c$
9. $2\left[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}\right] + c$
10. $\frac{x}{\log x} + c$
11. $-\sqrt{1-x^2} \sin^{-1} x + x + c$
12. $\frac{1}{2} x^2 \sin^{-1} x + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x + \frac{1}{4} \sin^{-1} x + c$
13. $\frac{1}{2} x^2 \cos^{-1} x - \frac{1}{4} x \sqrt{1-x^2}$
14. $-\frac{\sin^{-1} x}{x} + \log \left| \frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \right| + c$
15. $\frac{e^{3x}}{25}(3 \sin 4x - 4 \cos 4x) + c$

Answers

16. $\int \log(1+x^2)\,dx$ — 16) $x\log(1+x)^2 - 2x + 2\tan^{-1}x + c$

17. $\int \frac{\log x\,dx}{(1+x)^2}$ — 17) $\frac{-\log x}{1+x} + \log\left(\frac{x}{1+x}\right) + c$

18. $\int \tan^{-1}\left(\frac{7x}{1-12x^2}\right)dx$ — 18) $x\tan^{-1}4x - \frac{1}{32}\log(1+16x^2) + x\tan^{-1}3x - \frac{1}{18}\log(1+9x^2) + c$

19) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ — 19) $(x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + c$

20) $\int x\,(\tan^{-1}x)^2\,dx$ — 20) $\frac{1}{2}\theta^2\tan^2\theta - \theta\tan\theta + \log\sec\theta + \frac{\theta}{2}$

21) $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ — 21) $-e^x\cot\frac{x}{2} + c$

22) $\int \frac{\log x}{(1+\log x)^2}\,dx$ — 22) $\frac{x}{1+\log x} + c$

23) $\int e^{2x}\left(\frac{\sin 4x - 2}{1-\cos 4x}\right)dx$ — 23) $\frac{1}{2}e^{2x}\cot 2x + c$

24) $\int \frac{x^2+1}{(x+1)^2}e^x\,dx$ — 24) $e^x\left(\frac{x-1}{x+1}\right) + c$

25) $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$ — 25) $\frac{e^{2x}}{4(1+2x)} + c$

26) $\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\}dx$ (26) $x\left\{\log(\log x) - \frac{1}{\log x}\right\} + c$

27) $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x\,dx$ — (27) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$

Class : 12 — Holiday Homework. — Subject : Maths

Evaluate the following:

1. $\int (ax+b)^3\,dx$

2. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$

3) $\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$

4) $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

5) $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx$

6) $\int \frac{(1+\log x)^2}{x}\,dx$

7) $\int (4x+2)\sqrt{x^2+x+1}\,dx$

8) $\int \frac{1}{1+\cot x}\,dx$

9) $\int \frac{(x+1)e^x}{\cos^2(xe^x)}\,dx$

10) $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}\,dx$

11) $\int \tan^4 x\,dx$

12) $\int \sin 4x \cos 7x\,dx$

13) $\int \frac{\cos x}{1+\cos x}\,dx$

14) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

15) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

16) $\int \frac{dx}{x^2-6x+13}$

17) $\int \frac{5x-2}{1+2x+3x^2}\,dx$

18) $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$

19) $\int \frac{1}{x^3-1}\,dx$

20) $\int \frac{\sin x}{(1+\sin x)(2-\sin x)}\,dx$

21) $\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx$

22) $\int \sqrt{x^2+4x-5}\,dx$

23) $\int \frac{d\theta}{5+4\sin\theta}$

24) $\int \frac{dx}{x(x^5+3)}$

25) $\int (3\cot x-2\tan x)^2\,dx$

26) $\int \frac{dx}{50+2x^2}$

27) $\int (x-1)\sqrt{x^2+x+1}\,dx$

28) $\int \frac{x+2}{(x-2)(x-3)}\,dx$

29) $\int \frac{1}{4\cos\theta-1}\,d\theta$

30) $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$

31) $\int \sec^2(7-4x)\,dx$

32) $\int \left(4x^{10}+\frac{3}{x^{12}}+4\right)dx$

33) $\int 2^x\,dx$

34) $\int \frac{2\cos x}{3\sin^2 x}\,dx$

35) $\int \sqrt{a^2-x^2}\,dx$

36) $\int \frac{1}{\sqrt{(2-x)^2+1}}\,dx$

37) $\int \cos 4x \cos 3x\,dx$

38) $\int \cos\sqrt{x}\,dx$

39) $\int \sin^3 x\cos^2 x\,dx$

40) $\int (2x+4)\sqrt{x^2+4x+3}\,dx$

41) $\int e^x \sec^2(e^x)\,dx$

42) $\int \frac{\log x}{x}\,dx$

43) $\int \cos^4 2x\,dx$

44) $\int \cos^3 x\, e^{\log\sin^3 x}\,dx$

45) $\int \frac{\sin 2x}{a\cos^2 x+b\sin^2 x}\,dx$

46) $\int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$

47) $\int \frac{\sin x}{\sin(x-a)}\,dx$

48) $\int \frac{(\log x)^2}{x}\,dx$

49) $\int \frac{\cos x}{(\cos x/2 + \sin x/2)^2}\,dx$

50) $\int x\sqrt{x^2+1}\,dx$

51) $\int \frac{1}{1+\tan x}\,dx$

52) $\int \frac{\sec x\,dx}{\log(\sec x + \tan x)}$

3) $\int \sin^2 x \cos^3 x\,dx$

4) $\int \frac{\cos x}{\cos(x+\alpha)}\,dx$

5) $\int \frac{1}{1-\tan x}\,dx$

6) $\int \frac{x}{9-4x^2}\,dx$

7) $\int \sin^3 x \cos^5 x\,dx$

8) $\int \frac{x^2}{1+x^3}\,dx$

9) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

60) $\int \frac{\log \sin x}{\tan x}\,dx$

61) $\int \frac{1}{1-\cos 2x}\,dx$

62) $\int \sqrt{\frac{1+x}{1-x}}\,dx$

63) $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

64) $\int \frac{3x+1}{2x^2-2x+3}\,dx$

65) $\int \frac{x}{x^4-x^2+1}\,dx$

66) $\int \frac{x}{2-6x-x^2}\,dx$

67) $\int \frac{1}{x^3+1}\,dx$

68) $\int \frac{\cos x\,dx}{(1+\sin x)(2+\sin^2 x)}$

69) $\int \frac{2x\,dx}{(x^2+1)(x^2+3)}$

70) $\int \frac{2x^2+1}{x^2(x^2+4)}\,dx$

71) $\int \frac{x^2+1}{(x^2+4)(x^2+25)}\,dx$

72) $\int \frac{2x\,dx}{(x^2+1)(x^2+2)}$

73) $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx$

74) $\int \frac{(3\sin\phi - 2)\cos\phi}{5-\cos^2\phi - 4\sin\phi}\,d\phi$

75) $\int \frac{dx}{5+4\cos x}$

76) $\int \sqrt{x^2+4x+6}\,dx$

77) $\int \frac{dx}{x(x^3+8)}$

78) $\int \frac{dx}{5-13\sin x}$

79) $\int \frac{x^2+4}{x^4+16}\,dx$

80) $\int \frac{dx}{(x-1)\sqrt{x^2+4}}$

81) $\int (3\cot x - 2\tan x)^2\,dx$

82) $\int \frac{x+1}{2x^2+4x-7}\,dx$

83) $\int \frac{1}{\sqrt{5x^2-2x}}\,dx$

84) $\int \frac{x}{9-4x^2}\,dx$

85) $\int \sin 7x \sin x\,dx$

86) $\int \cos^4 x\,dx$

87) $\int \frac{1-x^2}{1+x^4}\,dx$

88) $\int \frac{x^2}{x^4+x^2+1}\,dx$

89) $\int \frac{dx}{x(x^3+1)}$

90) $\int \frac{dx}{\sqrt{7-6x-x^2}}$

91) $\int \sqrt{x^2+4x+1}\,dx$

92) $\int \frac{dx}{4-9\cos x}$

93) $\int \frac{\sec^2 x\,dx}{(\tan x+1)(\tan x+2)}$

94) $\int \frac{x}{(x-1)^2(x-3)}\,dx$

95) $\int \frac{1-x^2}{x(1-2x)}\,dx$

96) $\int \frac{x^3+x+1}{x^2-1}\,dx$

97) $\int \frac{3x-1}{(x+2)^2}\,dx$

98) $\int \frac{\cos x + 2\sin x + 3}{4\cos x + 5\sin x + 6}\,dx$

99) $\int \sqrt{\tan x}\,dx$

100) $\int \frac{5x}{(x+1)(x^2-4)}\,dx$

101) $\int \frac{\sin 2x}{\sqrt{\sin^4 x + 4\sin^2 x - 2}}\,dx$

102) $\int \frac{dx}{x\left[(6\log x)^2 + 7\log x + 2\right]}$

103) $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$

104) $\int \frac{2x}{x^2+3x+2}\,dx$

105) $\int \frac{x^3+x}{x^4-9}\,dx$

106) $\int \frac{\cos x}{\cos 3x}\,dx$

107) $\int \frac{2\sin 2\theta - \cos\theta}{6 - \cos^2\theta - 4\sin\theta}\,d\theta$

108) $\int \frac{x^2\,dx}{(x^2+4)(x^2+9)}$

109) $\int \frac{1+x^2}{1+x^4}\,dx$

110) $\int \frac{4x+5}{\sqrt{2x^2+x-3}}\,dx$

111) $\int \sqrt{1-4x-x^2}\,dx$

112) $\int \frac{\sin x}{\sqrt{1+\sin x}}\,dx$

113) $\int \frac{\cos 2x}{(\cos x + \sin x)^2}\,dx$

114) $\int \frac{\tan\alpha - \tan x}{\tan\alpha + \tan x}\,dx$

115) $\int \frac{dx}{x\sqrt{x^2-a^2}}$

116) $\int \frac{1+\cos 2x}{1-\cos 2x}\,dx$

117) $\int \sin x \sin 2x \sin 3x\,dx$

118) $\int \frac{x+3}{x^2-2x-5}\,dx$

119) $\int \frac{1}{x^2-9}\,dx$

120) $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

121) $\int \frac{x^2-1}{x^4+x^2+1}\,dx$

122) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}$

123) $\int \frac{3+2\cos x + 4\sin x}{2\sin x + \cos x + 3}\,dx$

124) $\int \sqrt{\cot x}\,dx$

125) $\int \frac{3x+5}{\sqrt{x^2-8x+7}}\,dx$

126) $\int \frac{1}{4+5\cos x}\,dx$

127) $\int \frac{1}{(x^2+1)(x^2+2)}\,dx$

128) $\int \frac{x}{(x-1)(x-2)(x-3)}\,dx$

129) $\int \frac{dx}{\sin x(3+2\cos x)}$

130) $\int \frac{1-\cos x}{\cos x(1+\cos x)}\,dx$

131) $\int \frac{dx}{3+2\sin x + \cos x}$

132) $\int \frac{2}{(1-x)(1+x^2)}\,dx$

133) $\int \frac{dx}{\sqrt{8+3x-x^2}}$

134) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx$

135) $\int \frac{1}{\sin x \cos^3 x}\,dx$

136) $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\,dx$

137) $\int \frac{\cos^5 x}{\sin x}\,dx$

138) $\int \frac{dx}{3x^2+13x-10}$

139) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

140) $\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 4}}\,dx$

141) $\int \frac{3x+1}{(x-2)^2(x+2)}\,dx$

142) $\int \frac{1}{1-x^3}\,dx$

143) $\int \frac{dx}{1+3\sin^2 x + 8\cos^2 x}$

44) $\int \frac{dx}{2+\cos x}$

45) $\int \sqrt{\frac{2-x}{2+x}}\, dx$

46) $\int \frac{dx}{9x^2+6x+5}$

47) $\int \frac{2x}{x^3-1} dx$

48) $\int \frac{x+2}{\sqrt{x^2+2x+3}}$

49) $\int \frac{1-\cos x}{1+\cos x} dx$

50) $\int \frac{\sin 2x \cos 2x}{\sqrt{9-\cos^4 2x}} dx$

51) $\int \sin 4x \sin 8x\, dx$

52) $\int \frac{x^2}{x^4-x^2+12} dx$

53) $\int \sqrt{\cot x} + \sqrt{\tan x}\, dx$

54) $\int \frac{x^3}{(x-1)(x-2)(x-3)} dx$

55) $\int \frac{3x+5}{x^3-x^2-x+1} dx$

56) $\int \frac{\sin 2x}{(1+\sin x)(2-\sin x)} dx$

57) $\int \frac{dx}{x^4-5x^2+6}$

58) $\int \frac{1}{1-x^3} dx$

59) $\int \frac{x^4}{x^4-16} dx$

160) $\int \frac{x}{x^4+x^2+1} dx$

161) $\int \operatorname{cosec}^4 x\, dx$

162) $\int \operatorname{cosec}^4 x \cot^5 x\, dx$

163) $\int \tan^3 x \sec^2 x\, dx$

164) $\int (2x^2+3)\sqrt{x+2}\, dx$

165) $\int x(1-x)^7 dx$

166) $\int \frac{x^2}{(a+bx)^2} dx$

167) $\int 5^{x+\tan^{-1}x} \cdot \left(\frac{x^2+2}{x^2+1}\right) dx$

168) $\int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$

169) $\int x^3 \sin(3x^4+5) dx$

170) $\int \frac{e^{\sqrt{x}} \cos e^{\sqrt{x}}}{\sqrt{x}} dx$

171) $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+a)}}$

172) $\int \frac{\sin 2x}{a^2\sin^2 x + b^2\cos^2 x} dx$

173) $\int (2x+3)(x^2+3x+1)^7 dx$

174) $\int \frac{\tan x}{a+b\tan^2 x} dx$

175) $\int e^{3\log x} (x^4+5)^{-1} dx$

LEVEL : 1

1. $\int \sec x(\sec x + \tan x)dx$

2. $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

3. $\int \frac{e^{\tan^{-1} x}}{1 + x^2} dx$

4. $\int \frac{1}{\sin x \cos x} dx$

5. $\int \frac{\sec^2(\log x)}{x} dx$

6. $\int \sin^3 x \, dx$

7. $\int \cos x \cos 2x \cos 3x \, dx$

8. $\int \frac{dx}{\sqrt{4x^2 - 9}}$

9. $\int \frac{dx}{x^2 + 2x + 10}$

10. $\int \frac{dx}{9x^2 + 12x + 13}$

11. $\int \frac{2x + 1}{(x + 1)(x - 1)} dx$

12. $\int x \sec^2 x \, dx$

13. $\int e^x(\tan x + \log \sec x)dx$

14. $\int \sqrt{4 + x^2} \, dx$

15. $\int \sqrt{1 - 4x^2} \, dx$

16. $\int (9x + \sin x + \operatorname{cosec} x)dx$

17. $\int_0^2 x\sqrt{x + 2} \, dx$

18. *Evaluate* $\int_0^2 (x + 2)dx$ *as a limit of a sum*

19. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} dx$

20. $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

21. $\int \frac{(2x - 3)}{x^2 - 3x - 18} dx$

22. $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1} dx$

23. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

24. $\int \sin^{-1}(\cos x)dx$

25. $\int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}} dx$, $\int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$

26. $\int \frac{dx}{1 + \tan x}$, $\int \frac{dx}{1 - \cot x}$, $\int \frac{dx}{1 - \sin x}$, $\int \frac{dx}{1 + \cos x}$, $\int \frac{1 + \sin x}{1 - \sin x} dx$

27. $\int \frac{\cos 2x - \cos 2a}{\cos x - \cos a} dx$

28. $\int_0^{\pi} |\cos x| dx$

29. $\int e^x(\tan^{-1} x + \frac{1}{1 + x^2})dx$

30. $\int tan^2 x\, dx$

31 $\int sinxsec^2xdx$

32 $\int \cos^{-1}(sinx)dx$

33 $\int \frac{dx}{e^x + e^{-x}}$

34 $\int \frac{sinx}{1+cosx}dx$

35 $\int \frac{2}{1+cos2x}dx$

36 $\int \frac{1}{16+x^2}dx$

37 $\int sin3xsin2xdx$

38 $\int \log x\, dx$

39 $\int \left(sin^2\frac{x}{2} - cos^2\frac{x}{2}\right)dx$

40 $\int \frac{1}{1+(2-x)^2}dx$

41 $\int_0^{\sqrt{2}} [x^2]\, dx$ Where $[x]$ denotes greatest integer.

42 $\int_{-1}^{1} x|x|\, dx$

43 If $\int_0^a \frac{1}{1+x^2}dx = \frac{\pi}{4}$ then find the value of a.

44 $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx$

45 $\int_1^2 \frac{x^3-1}{x^2}dx$

Evaluate

1. $\int \frac{1}{e^x - 1} dx$

2. $\int \frac{\sin 4x}{\sin x} dx$

3. $\int \frac{\sin x}{\sin(x+a)} dx$

4. $\int \frac{dx}{1+\tan x}$

5. $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

6. $\int \tan^2(2x-3) dx$

7. $\int \frac{1}{x\sqrt{x^6 - 1}} dx$

8. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

9. $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

10. $\int \frac{x+2}{2x^2+6x+5} dx$

11. $\int \frac{x+3}{\sqrt{5-4x+x^2}} dx$

12. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$

13. $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

14. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

15 $\int x \sin^{-1} x \, dx$

16. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

17. $\int \frac{5x^2}{x^2+4x+3} dx$

18. $\int_0^1 xe^x dx$

19. $\int_0^2 \frac{6x+3}{x^2+4}dx$

20. $\int_0^2 (x^2+1)dx$

21. $\int x\sin x dx$

22 $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$

23. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$

24. $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx.$

25. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

26. $\int_1^2 (2x^2+3)dx$ as limit of sum

27. $\int \frac{x^2}{(x+2)(x+1)}dx$

28. $\int \frac{x^2+x+1}{(x+2)(x+1)}dx$

29. $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$

30 $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

3 $\int \frac{2x}{\sqrt{1-x^2-x^4}}dx$

32 $\int \log\left(x+\sqrt{x^2+a^2}\right)dx$

33 $\int \frac{x^{1/2}}{x^{1/2}+x^{1/3}}dx$

34 $\int \frac{x^2}{x^4+x^2-2}dx$

35 $\int \frac{\sqrt{1+x^2}}{x^4}dx$

National Public School

Indiranagar, Bangalore

Indefinite Integrals

Class : 12 **Subject: Maths**

Formulae

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ and n is a natural number
2. $\int \sin x dx = -\cos x + c$
3. $\int \cos x dx = \sin x + c$
4. $\int \tan x dx = -\log|\cos x| + c = \log|\sec x| + c$
5. $\int \cot x dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$
6. $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + c = \log\left|\tan\frac{x}{2}\right| + c$
7. $\int \sec x dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$
8. $\int \sec x \tan x dx = \sec x + c$
9. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$
10. $\int \sec^2 x dx = \tan x + c$
11. $\int \operatorname{cosec}^2 x dx = -\cot x + c$
12. $\int a^x dx = \frac{a^x}{\log a} + c$
13. $\int a^{mx+b} dx = \frac{a^{mx+b}}{m \log a} + c$
14. $\int e^x dx = e^x + c$
15. $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$
16. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$ and n is a natural number
17. $\int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a} + c$
18. $\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + c$
19. $\int \tan(ax+b) dx = -\frac{1}{a}\log|\cos(ax+b)| + c = \frac{1}{a}\log|\sec(ax+b)| + c$
20. $\int \cot(ax+b) dx = \frac{1}{a}\log|\sin(ax+b)| + c = -\frac{1}{a}\log|\operatorname{cosec}(ax+b)| + c$

21. $\int \operatorname{cosec}(ax+b)dx = \frac{1}{a}\log\left|\operatorname{cosec}(ax+b) - \cot(ax+b)\right| + c = \frac{1}{a}\log\left|\tan\left(\frac{ax+b}{2}\right)\right| + c$

22. $\int \sec(ax+b)dx = \frac{1}{a}\log\left|\sec(ax+b) + \tan(ax+b)\right| + c = \frac{1}{a}\log\left|\tan\left(\frac{ax+b}{2} + \frac{\pi}{4}\right)\right| + c$

23. $\int \sec(ax+b)\tan(ax+b)dx = \frac{1}{a}\sec(ax+b) + c$

24. $\int \operatorname{cosec}(ax+b)\cot(ax+b)dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + c$

25. $\int \frac{1}{x}dx = \log|x| + c$

26. $\int \frac{1}{ax+b}dx = \frac{1}{a}\log|ax+b| + c$

27. $\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

28. $\int \frac{-1}{1+x^2}dx = \cot^{-1}x + c$

29. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

30. $\int \frac{-1}{\sqrt{1-x^2}}dx = \cos^{-1}x + c$

31. $\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$

32. $\int \frac{-1}{x\sqrt{x^2-1}}dx = \operatorname{cosec}^{-1}x + c$

33. $\int (f_1(x) \pm f_2(x))dx = \int f_1(x)dx \pm \int f_2(x)dx$

34. $\int kf(x)dx = k\int f(x)dx$

35. $\int f(x)g(x)dx = f(x)\int g(x)dx - \left[\int \frac{d}{dx}f(x)\int g(x)dx\right]dx$

Integration of rational and irrational functions

36. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

37. $\int \frac{-1}{a^2+x^2}dx = \frac{1}{a}\cot^{-1}\frac{x}{a} + c$

38. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

39. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

40. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} + c$

41. $\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\frac{x}{a}+c$

42. $\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$

43. $\int \frac{-1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a}+c$

44. $\int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c$

45. $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c$

46. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

47. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|+c$

48. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

49. $\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$

50. $\int f(x).g(x)dx = f(x).\int g(x)dx - \int\left[\frac{d}{dx}f(x).\int g(x)dx\right]dx$

Methods of integration:

1. **Integration by substitution method**

   a) In this method, we change the integral $\int f(x)dx$, where independent variable is x, to another integral in which independent variable is t different from x such that x and t are related by x = g(t)

   $\int f(x)dx = \int f[g(t)]g'(t)dt$ where $x = g(t)$ and $dx = g'(t)dt$

   b) In this method, if the numerator is the exact derivative of the denominator in the integrand, then the anti-derivative is log |denominator| + c

   $\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c$

   c) If n≠ - 1 then the above integration becomes,

   $\int (f(x))^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}$

   d) If the integrand contains t-ratio of f(x) or logarithmic of f(x) or exponential of f(x) in which the index is f(x), put f(x) = t

   If $\int f(x)dx = \phi(x)+c$ then $\int f(ax+b) = \frac{1}{a}\phi(ax+b)+c$

2. Trigonometric Integration

a) To evaluate $\int \sin^n x dx$ or $\int \cos^n x dx$ $n \leq 4$

Express $\sin^n x$ and $\cos^n x$ in terms of sines and cosines as multiples of x by the foll. Identities

(i) $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $(ii)\ \cos^2 x = \frac{1+\cos 2x}{2}$

$(iii)\ \sin 3x = 3\sin x - 4\sin^3 x$ $(iv)\ \cos 3x = 4\cos^3 x - 3\cos x$

b) To evaluate $\int \sin nx \cos mx dx$; $\int \sin mx \sin nx dx$; $\int \cos mx \cos nx dx$; $\int \cos mx \sin nx dx$

Use the following identities to make the product into sum and then solve

$(i)\ 2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$(ii)\ 2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$(iii)\ 2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$(iv)\ 2\sin A\sin B = \cos(A-B)-\cos(A+B)$

c) Integrals of the form $\int \tan^m x \sec^n x dx$; $\int \tan^n x dx$, $\int \cot^n x dx$ put

$t = \tan x$; $dt = \sec^2 x dx$ (if the power of sec x is more than 2 convert it to tanx and solve.)

d) Integrals of the form $\int \cot^m x \operatorname{cosec}^n x dx$ put $t = \cot x$; $dt = -\operatorname{cosec}^2 x dx$ (if the power of cosec x is more than 2 convert it to cotx and solve.)

e) Integrals of the form $\int \tan^{2m+1} x \sec^{2n+1} x dx$, in this type write it as

$\int \tan^{2m} x \sec^{2n} x \tan x \sec x dx$ put $t = \sec x$; $dt = \tan x \sec x dx$

f) Integrals of the form $\int \sin^m x \cos^n x dx$

1) If m is odd positive put t = cosx
2) If n is odd positive put t = sinx
3) If m and n are both odd positive then consider the smallest one and put either t = cosx or sinx
4) If m and n are both even positive use the formula $\cos 2x = 2\cos^2 x - 1$ or $\cos 2x = 1 - 2\sin^2 x$ or $\sin 2x = 2\sin x \cos x$ to bring the integral into multiple angles.

g) Integrals of the form $\int \sin^{-m} x \cos^{-n} x dx$ or $\int \frac{1}{\sin^m x \cos^n x} dx$

Change it in terms of tanx and $\sec^2 x$ by dividing numerator and denominator by $\cos^k x$ where k – (m + n) and then substitute t = tanx

h) Integrals of the form

$\int \frac{1}{a\sin^2 x + b\cos^2 x} dx$; $\int \frac{1}{a + b\sin^2 x} dx$; $\int \frac{1}{a + b\cos^2 x} dx$; $\int \frac{1}{(a\sin x + b\cos x)^2} dx$;

$\int \frac{1}{a + b\sin^2 x + c\cos^2 x} dx$

1) First try to take $\cos^2 x$ or $\sin^2 x$ common from denominator

2) If we have taken out $\cos^2 x$ as a common factor in the denominator, then make it $\sec^2 x$, if any $\sec^2 x$ is available then replace it by $(1 + \tan^2 x)$

3) Put $t = \tan x$, use $dt = \sec^2 x dx$ and evaluate the integral by usual process

4) If we have taken $\sin^2 x$ as common from the denominator then use $\operatorname{cosec}^2 x$ and if any $\operatorname{cosec}^2 x$ is available replace it by $(1 + \cot^2 x)$

i) Integrals of the form $\int \frac{1}{a\sin x + b\cos x} dx$; $\int \frac{1}{a \pm b\sin x} dx$;

$\int \frac{1}{a \pm b\cos x} dx$; $\int \frac{1}{a\sin x + b\cos x + c} dx$

1. Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$; $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and then solve
2. Replace $1+\tan^2\frac{x}{2}$ by $\sec^2\frac{x}{2}$
3. Put $\tan\frac{x}{2} = t$; $\frac{1}{2}\sec^2\frac{x}{2} dx = dt$

(OR)

Substitute $a = r\sin\theta$ and $b = r\cos\theta$, so that $r = a^2 + b^2$ and $\theta = \tan^{-1}\frac{b}{a}$

j) Integrals of the form $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x} dx$

In this type express numerator as

Numerator = A (denominator) + B d/dx(denominator)

Equating the terms find A and B

k) Integrals of the form $\int \frac{a\sin x + b\cos x + c}{p\sin x + q\cos x + r} dx$

In this type express numerator as

Numerator = A (denominator) + B d/dx(denominator)+ C

Equating the terms find A , B and C

3. Integration by partial fractions

$$\int \frac{p(x)}{(ax+b)^n} dx$$

a) If degree of numerator is less than the degree of denominator then write p(x) in terms of ax+b

(ie) Nr = λ(ax+b) + μ

Find λ and μ and reframe the integral as

$$\int \frac{p(x)}{(ax+b)^n} dx = \int \frac{\lambda(ax+b)+\mu}{(ax+b)^n} dx$$

b) If degree of numerator $\geq$ degree of denominator then do long division and solve

$$\frac{p(x)}{ax+b} = Q + \frac{remainder}{divisor}$$

Table for partial fractions

Form of the rational function	Form of the partial fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{(x^2+bx+c)}$ where x^2+bx+c can't be factorized further

c) To evaluate $\int \sqrt{ax^2+bx+c}dx$ (or) $\int \frac{1}{\sqrt{ax^2+bx+c}}dx$

In this case make the coefficient of x^2 unit and express the integral as $x^2 - a^2$ (or) $a^2 - x^2$ (or) $x^2 + a^2$ by completing the square and apply the formula.

d) Integration of the form $\int linear1\sqrt{linear2}dx$ or $\int \frac{linear1}{\sqrt{linear2}}dx$

In this case write $linear1 = \lambda(linear2) + \mu$ find λ and μ

Replace linear 1 by the new form (ie) λ(linear2) + μ in the given integral and solve

e) Integration of the form $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$ or $\int (px+q)\sqrt{ax^2+bx+c}dx$

In this case write $px+q = A\frac{d}{dx}(ax^2+bx+c)+B$

(ie) Numerator = A (differentiation of denominator) + B. Find A and B by equating the coefficient of like powers and split the integral into two parts and solve.

f) Integrals of the form $\int \frac{p(x)}{ax^2+bx+c}dx$

Check the degree of p(x)

If the degree of p(x) < degree of $ax^2 + bx + c$, then follow the above method

If degree of p(x) ≥ degree of $ax^2 + bx + c$, then do long division

g) To evaluate $\int \frac{ax^2+bx+c}{\sqrt{px^2+qx+r}}dx$

In this case $ax^2+bx+c = l(px^2+qx+r)+m\left[\frac{d}{dx}(px^2+qx+r)\right]+n$

Find l, m, n by equating the coefficient of like powers of x and then split the integral into three parts.

h) To evaluate $\int \frac{dx}{linear\sqrt{linear}}$ put linear = t

i) To evaluate $\int \frac{dx}{Quadratic\sqrt{linear}}$ put linear = t^2

j) To evaluate $\int \frac{dx}{linear\sqrt{quadratic}}$ (or)

$\int \frac{dx}{(linear)^2\sqrt{quadratic}}$ (or) $\int \frac{xdx}{(linear)^2\sqrt{quadratic}}$ put linear = 1/t

k) To evaluate $\int \frac{dx}{purequad\sqrt{pure\ quad}}$ put t = $\sqrt{pure\ quad}$

l) To evaluate $\int \frac{x^2+1}{x^4 \pm x^2+1}dx$; $\int \frac{x^2-1}{x^4 \pm x^2+1}dx$

Step -1 : Divide numerator and denominator by x^2

Step – 2: Express the denominator in the form $\left(x \pm \frac{1}{x}\right)^2 + a^2$

Step – 3: Put $x+\frac{1}{x}$ or $x-\frac{1}{x}$ as t so that the numerator may become dt.

m) If the integrand contains an expression of the form

1) $\sqrt{a^2-x^2}$ or a^2-x^2 put x = a sin0 or x = a cos0
2) $\sqrt{x^2-a^2}$ or x^2-a^2 put x = a sec0 or x = a cosec0
3) $\sqrt{a^2+x^2}$ or a^2+x^2 put x = a tan0 or x = a cot0
4) $\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$ put x = a cos2θ
5) $\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(x-\beta)}$ put $x = \alpha\cos^2\theta + \beta\sin^2\theta$
6) $\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$ put x = a $\sin^2\theta$ or x = a $\cos^2\theta$
7) $\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$ put x = a $\tan^2\theta$ or x = a $\cot^2\theta$

NATIONAL PUBLIC SCHOOL, INDIRANAGAR

CLASS 12- Matrices

Equality

Two matrices are equal if and only if

- The order of the matrices are the same
- The corresponding elements of the matrices are the same

Addition

- Order of the matrices must be the same
- Add corresponding elements together
- Matrix addition is commutative
- Matrix addition is associative

Subtraction

- The order of the matrices must be the same
- Subtract corresponding elements
- Matrix subtraction is not commutative (neither is subtraction of real numbers)
- Matrix subtraction is not associative (neither is subtraction of real numbers)

Scalar Multiplication

A scalar is a number, not a matrix.

- The matrix can be any order
- Multiply all elements in the matrix by the scalar
- Scalar multiplication is commutative
- Scalar multiplication is associative

Zero Matrix

- Matrix of any order
- Consists of all zeros
- Denoted by capital O
- Additive Identity for matrices
- Any matrix plus the zero matrix is the original matrix

Matrix Multiplication

$A_{m\times n} \times B_{n\times p} = C_{m\times p}$

- The number of columns in the first matrix must be equal to the number of rows in the second matrix. That is, the inner dimensions must be the same.
- The order of the product is the number of rows in the first matrix by the number of columns in the second matrix. That is, the dimensions of the product are the outer dimensions.

Matrix multiplication is not commutative

- Multiplication of real numbers is.
- The inner dimensions may not agree if the order of the matrices is changed.

Do not simply multiply corresponding elements together

- Since the order (dimensions) of the matrices don't have to be the same, there may not be corresponding elements to multiply together.
- Multiply the rows of the first by the columns of the second and add.

There is no matrix division

- There is no defined process for dividing a matrix by another matrix.
- A matrix may be divided by a scalar.

Note:

1. If AB is defined then BA need not be defined. In particular if both A and B are square matrices of same order then both AB and BA are defined.

2. Multiplication of diagonal matrices of same order will be commutative

3. We can find zero matrix as a product of two non-zero matrices. If a product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

Transpose : Interchange rows and columns Properties: 1. $((A)')' = A$ 2. $(kA)' = kA'$ 3. $(A+B)' = A' + B'$ 4. $(AB)' = B'A'$ 5. $(ABC)' = C'B'A'$	**Symmetric and skew symmetric:** If $A' = A \Rightarrow symmetric$ $A' = -A \Rightarrow skewsymmetic$ Note: ❖ For any square matrix A with real entries $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix. ❖ Any square matrix can be expresses as a sum of symmetric matrix and skew ❖ A matrix which is both symmetric and skew symmetric is a null matrix ❖ Every element in the principal diagonal of a skew symmetric matrix is zero.

Inverse of a matrix:

For a square matrix, the inverse is written A^{-1}. When A is multiplied by A^{-1} the result is the identity I. Non-square matrices do not have inverses

Note: Not all square matrices have inverses. A rectangular matrix do not have inverse.

If B is the inverse of A then A is the inverse of B

- ❖ Null matrix is a matrix which is both symmetric and skew symmetric
- ❖ Sum of two skew symmetric matrices is always skew symmetric matrix
- ❖ The product of any matrix by a scalar 0 is the null matrix
- ❖ A matrix which is not a square matrix is called rectangular matrix
- ❖ If A is a symmetric matrix then A^3 is a symmetric matrix
- ❖ If A is a symmetric matrix then A^2 is a symmetric matrix
- ❖ If A and B are symmetric matrices of same order, the AB is symmetric if and only if AB = BA

QUESTIONS

1. If a matrix has 24 elemts , what are the possible orders it can have? What, if it has 13 elements?
2. .If the matrix $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & -1 & -5 & 17 \end{bmatrix}$,
   (a) Write the order of the matrix
   (b) The number of elements
   (c) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$
3. Construct a 3×4 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{1}{2}|-3i + j|$
4. Find the values of x, y , z from the following equations $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$
5. Find the value of a,b,c, and d from the equation : $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$
6. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, find each of the following : (i) A + B
   (ii) $3A - C$ (iii) AB
7. Compute

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

8. Compute the product
   (i) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$
9. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
10. Show that $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$
11. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

12. If $A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ and I is the identity matrix of order 2 , show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

13. A trust fund has `30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using Matrix multiplication, determine how to divide `30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of (i) ₹1800 (ii) ₹2000

14. Assume that X,Y,Z,W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ *and* $p \times k$ respectively, What is the restriction on n, k and p so that PW + WY will be defined ? (ii) If n = p, then the order of the matrix 7X- 5Z is ?

15. Find the transpose of (i) $\begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

16. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that (a) $(A + B)' = A' + B'$ (b) $(A - B)' = A' - B'$

17. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

18. Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix

19. For the matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that $(A + A)'$ is a symmetric matrix and $(A - A)'$ is a skew symmetric matrix.

20. Express the following matrix as a sum of symmetric and skew symmetric matrix. (i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

21. Using elementary transformations find the inverse

(i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

22. Let $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, Show that $(aI+bA)^n = a^nI+nd^{n-1}bA$, where I is the identity matrix of order 2, and $n\in N$

23. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ Prove that $A^n=\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix}$ $n\in N$

24. If A and B are symmetric matrices, Prove that AB – BA is a skew symmetric matrix.

25. Show that $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

26. If A and B are square matrices of the same order such that AB = BA, then prove by induction that $(AB)^n = B^nA$. Further prove that $(AB)^n = A^nB^n$ for all $n\in N$

27. Find the values of x, y, z if the matrix $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfy the equation $A'A = I$

Exemplar:

1. Construct a 3×2 matrix whose elements are given by $a_{ij} = e^{ix}\sin jx$

2. If $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, Show that $(A+B)(A-B)\neq A^2-B^2$

3. Find the value of x, if $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$

4. Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$, satisfies the equation $A^2-3A-7I=0$, and hence find A^{-1}

5. Find A, if $\begin{bmatrix}4\\ 1\\ 3\end{bmatrix}A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$

6. If $A=\begin{bmatrix}3 & -4\\ 1 & 1\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}$, the verify $(BA)^2\neq B^2A^2$

7. If $A=\begin{bmatrix}2 & 1\end{bmatrix}$, $B=\begin{bmatrix}5 & 3 & 4\\ 8 & 7 & 6\end{bmatrix}$ and $C=\begin{bmatrix}-1 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$, verify that $A(B+C)=AB+AC$

8. If $A=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$, then verify that $A^2+A=A(A+I)$, where I is a 3×3 unit matrix

9. If $A=\begin{bmatrix}0 & -1 & 2\\ 4 & 3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}4 & 0\\ 1 & 3\\ 2 & 6\end{bmatrix}$, then verify that (i) $(A')' = A$ (ii) $(AB)' = B'A'$ (iii) $(kA)' = (kA')$

10. Show that $A'A$ and AA' are both symmetric matrices for any matrix A

11. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

12. Verify $A^2 = I$, when $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

13. Prove by Mathematical induction that $(A')^n = (A^n)'$, where $n \in N$, for any square matrix A.

14. If $A=\begin{bmatrix}1 & 5\\ 7 & 12\end{bmatrix}$ and $B=\begin{bmatrix}9 & 1\\ 7 & 8\end{bmatrix}$, find a matrix C such that 3A + 5B + 2C is a null matrix.

15. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ and $A^{-1} = A'$, find the value of α

16. If $P(x)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then show that P(x) . P(y) = P(x + y) = P(y) . P(x)

17. If A is a square matrix such that $A^2 = A$, Show that $(I+A)^3 = 7A+I$

18. If A and B are square matrices of same order and B is a skew symmetric matrix, show that $A'BA$ is skew symmetric.

19. Find x, y, z if $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A' = A^{-1}$

20. If possible , using elementary row transformations , find the inverse of the following matrices: (i) $\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$ (ii) $\begin{bmatrix}2 & 3 & -3\\ -1 & -2 & 2\\ 1 & 1 & -1\end{bmatrix}$

21. Express the matrix $\begin{bmatrix}2 & 3 & 1\\ 1 & -1 & 2\\ 4 & 1 & 2\end{bmatrix}$ as a sum of a symmetric and skew symmetric matrix.

22. If A is a square matrix of order such that $A^2 = I$, then find the value of $(A-I)^3+(A+I)^3-7A$

23. If A is a matrix of order $m \times n$ and B is a matrix such that $(AB)'$ and $B'A$ are both defined, then the order of the matrix B is ?

Other sources:

1. If $A=\begin{bmatrix}0 & 0\\ 4 & 0\end{bmatrix}$ find A^{16}

2. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then prove that $A^2-4A-5I=0$

3. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find k such that $A^2=kA-7I_2$

4. If matrix $A=[1 \quad 2 \quad 3]$, write AA'

5. To promote the making of toilets for women, an organisation has to generate awareness through (i) house calls (ii) letters, (iii) announcements. The cost for each mode per attempt is given below:
(i) 50 (ii) 20 (iii) 40

The number of attempts made in three villa ges X, Y, Z are given below

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages, separately, using matrices.

6. If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, then show that the matrix A is the root of the polynomial

$$f(x)=x^3-6x^2+7x+2$$

7. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ and $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP'$ and $X=P'Q^{2005}P$, then X is equal to ?