CH08-320201

Algorithms and Data Structures

ADS

Lecture 3

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Divide & Conquer

- Divide & Conquer (Latin Divide et Impera) is one concept/method/technique that can produce faster algorithms.
- It is based on three steps:
  - Divide the given problem into smaller subproblems.
  - Conquer the subproblems by solving them recursively.
  - Combine the solutions of the subproblems.
- Example: Sort recursively?

Merge Sort Idea

MERGE-SORT $A[1 \,..\, n]$

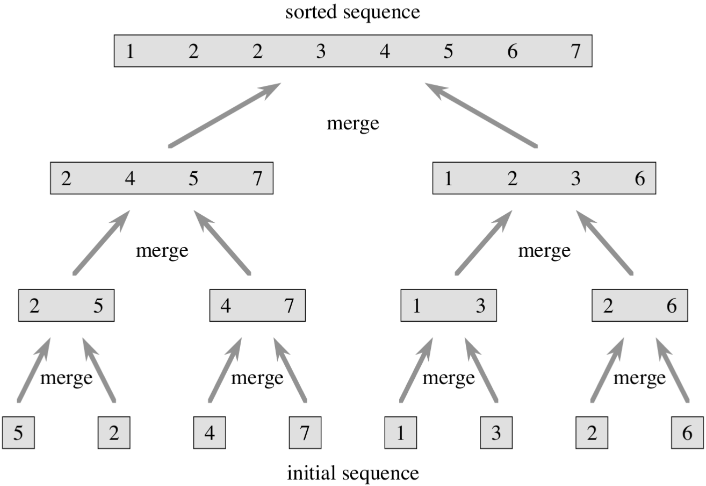
1. If $n = 1$, done.
2. Recursively sort $A[\,1 \,..\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,..\, n\,]$.
3. "***Merge***" the 2 sorted lists.

Merge Sort as Divide & Conquer

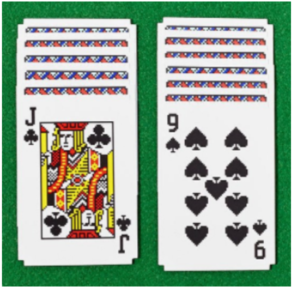
```
MERGE-SORT(A, p, r)
  if p < r                          // check for base case
      q = ⌊(p + r)/2⌋               // divide
      MERGE-SORT(A, p, q)           // conquer
      MERGE-SORT(A, q + 1, r)       // conquer
      MERGE(A, p, q, r)             // combine
```

Initial call: `Merge-Sort(A, 1, A.length)`

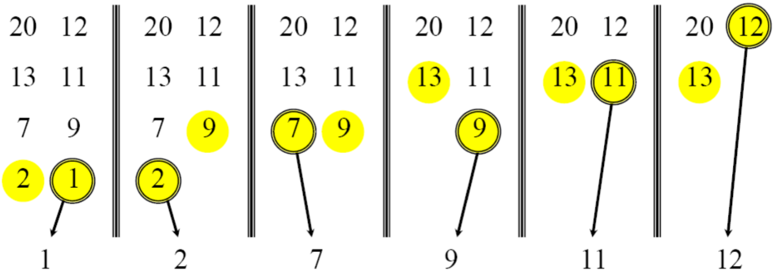
Merge Sort: Example



How Is Merging Done? (1)



How Is Merging Done? (2)



Merge

MERGE(A, p, q, r)

```
n_1 = q - p + 1
n_2 = r - q
let L[1 .. n_1 + 1] and R[1 .. n_2 + 1] be new arrays
for i = 1 to n_1
    L[i] = A[p + i - 1]
for j = 1 to n_2
    R[j] = A[q + j]
L[n_1 + 1] = ∞
R[n_2 + 1] = ∞
i = 1
j = 1
for k = p to r
    if L[i] ≤ R[j]
        A[k] = L[i]
        i = i + 1
    else A[k] = R[j]
        j = j + 1
```

Correctness of Merging

Loop Invariant:

- At the start of each iteration of the for k loop, the subarray $A[p..k-1]$ contains the $k-p$ smallest elements of L and R in sorted order.
- Moreover, $L[i]$ and $R[j]$ are the smallest elements of their arrays which have not been copied back into A.

Asymptotic Analysis (1)

- Merging step?
  Computation time is $\Theta(n)$.
- What is the overall computation time of Merge Sort?

Asymptotic Analysis (2)

$T(n)$	**MERGE-SORT** $A[1 \,..\, n]$
$\Theta(1)$	1. If $n = 1$, done.
$2T(n/2)$	2. Recursively sort $A[\,1 \,..\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,..\, n\,]$.
$\Theta(n)$	3. ***"Merge"*** the 2 sorted lists

Asymptotic Analysis (3)

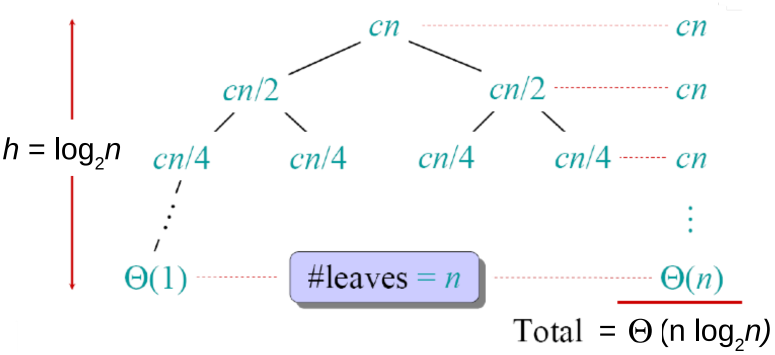
- The overall running time for Merge Sort is given by the recurrence

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1, \\ 2T(n/2) + \Theta(n) & \text{if } n > 1. \end{cases}$$

- The base case may be omitted, if it is obvious that it can be neglected in the asymptotic analysis.

Recursion Tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.



Merge Sort vs. Insertion Sort

- $\Theta(n \log_2 n)$ grows more slowly than $\Theta(n^2)$.
- Merge Sort asymptotically beats Insertion Sort (in the average case and in the worst case).
- In practice, Merge Sort beats Insertion Sort for $n > 30$ or so.

Sorting Algorithms' Complexities

Algorithm	Data structure	Time complexity:Best	Time complexity:Average	Time complexity:Worst	Space complexity:Worst
Quick sort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	O(n)
Merge sort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	O(n)
Heap sort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	O(1)
Smooth sort	Array	$O(n)$	$O(n \log(n))$	$O(n \log(n))$	O(1)
Bubble sort	Array	$O(n)$	$O(n^2)$	$O(n^2)$	O(1)
Insertion sort	Array	$O(n)$	$O(n^2)$	$O(n^2)$	O(1)
Selection sort	Array	$O(n^2)$	$O(n^2)$	$O(n^2)$	O(1)

Here the notation $log(n)$ is mathematically $log_2 n$.