MECH 579: Multidisciplinary Design Optimization
Department of Mechanical Engineering, McGill University

Project #3: Brachistochrone Problem
Due 24th. October, 2013

Write a numerical code to solve the Brachistochrone problem. The Brachistochrone problem, originally introduced by the Bernoulli brothers, is the determination of the path $y(x)$ connecting initial and final points (x_i, y_i) and (x_f, y_f) such that the time taken by a particle traversing this path is subject only to the force of gravity, is a minimum. Additional notes on the derivation of the analytical solution has been posted on MyCourses. The problem statement is as follows

$$\begin{aligned} \text{minimize} \quad & T = \int_{x_i}^{x_f} \frac{ds}{V} \\ \text{with respect to} \quad & x \in \mathbb{R}^n \\ \text{subject to} \quad & \text{gravitational force} \end{aligned}$$

where, T is the total time taken by the particle, V is the velocity of the particle, and $y_o = 0$.

1. Write two codes using the steepest descent, and a quasi-Newton method to solve for the minimum of the function. Employ a backtracking line search that satisfies the 1st. Wolfe condition. You may choose any one algorithm of the quasi-Newton (DFP or BFGS) methods.

2. Provide the following in a written report:

   (a) Convergence of the gradient (y-axis: log of the gradient, x-axis: iteration) and a comparison of the convergence.

   (b) Plot the initial, several solutions during the optimization, and the final solution.

   (c) Plot the log of the norm of the error between your solution and the exact solution.

   (d) Discuss your choice of any parameters that were used for the line search algorithm.

Reports must be handed in a PDF format. All plots must have both x- and y-axis labels, a legend clearly describing the various lines, and a title with a Figure number. Plots generated with MS Excel are not acceptable and assignments will not be graded.