MECH 579: Multidisciplinary Design Optimization
Department of Mechanical Engineering, McGill University

Project #5: MDO via MDF
Due 29th. November, 2013

Write a code to solve the following unconstrained multidisciplinary optimization problem using a direct and adjoint method. The problem exhibits characteristics of a larger MDO problem; however it contains an analytical solution, where the minimum of the function is at $(1, 1)$. The overall design space has two minimums, where $(1, 1)$ is the global minimum.

$$
\begin{aligned}
\text{minimize} \quad & f(x, y) = -20e^{-[(x_1-1)^2+0.25(x_2-1)^2]} + y_1 + \cos(y_2) \\
\text{with respect to} \quad & x_1, x_2 \in \mathbb{R}^n \\
\text{where} \quad & R_1(x, y_1(x, y_2)) : y_1 = -3e^{-[(x_1+1)^2+0.25(x_2+1)^2]} + \sin(y_2) \\
& R_2(x, y_2(x, y_1)) : y_2 = -3e^{-[5(x_1-3)^2+0.25(x_2-3)^2]} + e^{-y_1}
\end{aligned}
$$

1. Provide the following in a written report:
   (a) Compute the derivatives of the objective function with respect to the design variable using three different approaches: finite-difference, direct method, and adjoint method. Compare the derivatives uptop 16 decimal places match. If they do not, then explain why? For the finite difference, use both first- and second-order discretizations and show that as you progressively reduce the value of epsilon (perturbation) that the gradient converges at the expected rates. You may wish to experiment with the complex-derivative as well.
   (b) Use the direct method together with either a Newtons or Quasi-Newton framework to find the optimum solution. Show your derivation of the equations. Use a fixed step length (No line search is required) and select $(-0.1, -1)$ as your initial point.
   (c) Show a contour plot of the function and over plot the optimization path.
   (d) Provide convergence plots of the gradient for both test cases.
   (e) Discuss your choice of any parameters that were used.

Reports must be handed in a PDF format.

For those who have chosen an individual project, repeat the steps above for your own optimization problem defined in the previous proposals. If the proposed optimization algorithm is not suitable for your problem, then propose an alternate by **November 22nd, 2013**.