ACH0021 – Tratamento e Análise de Dados/Informações

Lista de Exercícios 0

Observação : Os exercícios desta lista devem ser resolvidos SEM o uso de ferramentas computacionais

---

1) Calcular:

a) $\sum_{i=1}^{3} i^{-2}$ b) $\sum_{r=0}^{3} \pi$ c) $\sum_{w=3}^{3} \frac{\sqrt{\pi}}{\sqrt{2}-1}$ d) $\sum_{n=1}^{3} n\,(n+1)$ e) $\sum_{k=1}^{3} \left(k^k - 1\right)$ f) $\sum_{k=1}^{3} k^k - 1$

g) $\sum_{i \in \{3,5,7\}} i^{-i}$ h) $\sum_{\substack{j:\text{ ímpares positivos}\\ \text{menores que 8}}} 2$ i) $\sum_{\substack{p:\text{ primos}\\ \text{menores que 20}}} 2^{1/p}$ j) $\sum_{0 \le m \le 3} \pi^\pi$ k) $\sum_{n \in \{0\}} n$ l) $\sum_{n \in \{0\}} 1$

1a) $\sum_{i=1}^{3} i^{-2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} = \frac{49}{36}.$

1b) $\sum_{r=0}^{3} \pi = \overbrace{\pi}^{r=0} + \overbrace{\pi}^{r=1} + \overbrace{\pi}^{r=2} + \overbrace{\pi}^{r=3} = 4\pi.$

1c) $\sum_{w=3}^{3} \frac{\sqrt{\pi}}{\sqrt{2}-1} = \frac{\sqrt{\pi}}{\sqrt{2}-1}.$

1d) $\sum_{n=1}^{3} n\,(n+1) = 1\,(1+1) + 2\,(2+1) + 3\,(3+1) = 20.$

1e) $\sum_{k=1}^{3} \left(k^k - 1\right) = \left(1^1 - 1\right) + \left(2^2 - 1\right) + \left(3^3 - 1\right) = 29.$

1f) $\sum_{k=1}^{3} k^k - 1 = 1^1 + 2^2 + 3^3 - 1 = 31.$

1g) $\sum_{i \in \{3,5,7\}} i^{-i} = \frac{1}{3^3} + \frac{1}{5^5} + \frac{1}{7^7} = \frac{2595891911}{69486440625}.$

1h) $\sum_{\substack{j:\text{ ímpares positivos}\\ \text{menores que 8}}} 2 = \overbrace{2}^{j=1} + \overbrace{2}^{j=3} + \overbrace{2}^{j=5} + \overbrace{2}^{j=7} = 8.$

1i) $\sum_{\substack{p:\text{ primos}\\ \text{menores que 20}}} 2^{1/p} = 2^{1/2} + 2^{1/3} + 2^{1/5} + 2^{1/7} + 2^{1/11} + 2^{1/13} + 2^{1/17} + 2^{1/19}$

1j) $\sum_{0 \le m \le 3} \pi^\pi = \overbrace{\pi^\pi}^{m=0} + \overbrace{\pi^\pi}^{m=1} + \overbrace{\pi^\pi}^{m=2} + \overbrace{\pi^\pi}^{m=3} = 4\pi^\pi.$

1k) $\sum_{n \in \{0\}} n = \overbrace{0}^{n=0} = 0.$

1l) $\displaystyle\sum_{n\in\{0\}} 1 = \overbrace{1}^{n=0} = 1.$

2) Expressar com o símbolo de somatória (notar que existem infinitas possibilidades)

a) $3^{2/3}+4^{2/3}+5^{2/3}$ b) $5+5+5+5+5$ c) $-1-\frac{1}{3}-\frac{1}{5}-\frac{1}{7}$

d) $3\cdot 4^2+3\cdot 5^3+3\cdot 6^4+3\cdot 7^5$ e) $x_4+x_5+\cdots+x_{99}$ f) $x_2+x_4+x_6+x_8$

g) $x_1+2x_2+\cdots+nx_n$ h) $3x_6+6x_{12}+9x_{18}+12x_{24}$ i) $x_{100}+x_{96}+\cdots+x_4+x_0$

2a) $\displaystyle 3^{2/3}+4^{2/3}+5^{2/3}=\sum_{i=3}^{5} i^{2/3}.$

2b) $\displaystyle 5+5+5+5+5=\sum_{j=0}^{4}5=\sum_{j=1}^{5}5.$

2c) $\displaystyle -1-\frac{1}{3}-\frac{1}{5}-\frac{1}{7}=-\frac{1}{1}-\frac{1}{3}-\frac{1}{5}-\frac{1}{7}=-\sum_{k=0}^{3}\frac{1}{2k+1}=-\sum_{k=1}^{4}\frac{1}{2k-1}.$

2d) $\displaystyle 3\cdot 4^2+3\cdot 5^3+3\cdot 6^4+3\cdot 7^5=3\sum_{n=4}^{7}n^{n-2}=3\sum_{n=2}^{5}(n+2)^n.$

2e) $\displaystyle x_4+x_5+\cdots+x_{99}=\sum_{m=4}^{99}x_m.$

2f) $\displaystyle x_2+x_4+x_6+x_8=\sum_{a=1}^{4}x_{2a}.$

2g) $\displaystyle x_1+2x_2+\cdots+nx_n=\sum_{b=1}^{n}bx_b.$

2h) $\displaystyle 3x_6+6x_{12}+9x_{18}+12x_{24}=\sum_{s=1}^{4}3sx_{6s}=3\sum_{s=1}^{4}sx_{6s}.$

2i) $\displaystyle x_{100}+x_{96}+\cdots+x_4+x_0=\sum_{i=0}^{25}x_{100-4i}$

3) Sabendo-se que $x_1=1$, $x_2=6$, $x_3=3$ e $x_j=2^j$ para $j\geq 4$, calcule

a) $\displaystyle\sum_{k=2}^{6}x_k$ b) $\displaystyle\sum_{j=1}^{5}x_j\cdot x_{j+1}$ c) $\displaystyle\sum_{m=3}^{7}m\cdot x_{2m}$ d) $\displaystyle\sum_{t=1}^{5}\frac{x_{2t}}{x_{2t+1}}$ e) $\displaystyle\sum_{y=1}^{4}(2x_y+3)$ f) $\displaystyle\sum_{z=1}^{3}z\cdot x_4$

Inicialmente, tem-se

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14
x_j	1	6	3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}

3a) $\displaystyle\sum_{k=2}^{6} x_k = x_2 + x_3 + x_4 + x_5 + x_6 = 6 + 3 + 2^4 + 2^5 + 2^6 = 121.$

3b) $\displaystyle\sum_{j=1}^{5} x_j \cdot x_{j+1} = x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_5x_6 = 1 \cdot 6 + 6 \cdot 3 + 3 \cdot 2^4 + 2^4 \cdot 2^5 + 2^5 \cdot 2^6 = 2632.$

3c) $\displaystyle\sum_{m=3}^{7} m \cdot x_{2m} = 3x_6 + 4x_8 + 5x_{10} + 6x_{12} + 7x_{14} = 3 \cdot 2^6 + 4 \cdot 2^8 + 5 \cdot 2^{10} + 6 \cdot 2^{12} + 7 \cdot 2^{14} = 145600.$

3d) $\displaystyle\sum_{t=1}^{5} \frac{x_{2t}}{x_{2t+1}} = \frac{x_2}{x_3} + \frac{x_4}{x_5} + \frac{x_6}{x_7} + \frac{x_8}{x_9} + \frac{x_{10}}{x_{11}} = \frac{6}{3} + \frac{2^4}{2^5} + \frac{2^6}{2^7} + \frac{2^8}{2^9} + \frac{2^{10}}{2^{11}} = 4.$

3e) $\displaystyle\sum_{y=1}^{4} (2x_y + 3) = 2\sum_{y=1}^{4} x_y + 3\sum_{y=1}^{4} 1 = 2\,(x_1 + x_2 + x_3 + x_4) + 3\,(1 + 1 + 1 + 1) = 2\left(1 + 6 + 3 + 2^4\right) + 3 \cdot 4$

$= 64.$

3f) $\displaystyle\sum_{z=1}^{3} z \cdot x_4 = 1x_4 + 2x_4 + 3x_4 = 1 \cdot 2^4 + 2 \cdot 2^4 + 3 \cdot 2^4 = 96.$

4) Considere dois conjuntos de n dados cada, $\{y_1, y_2, \cdots, y_n\}$ e $\{z_1, z_2, \cdots, z_n\}$. Assumindo $1 < m < n$ e a e b constantes, mostre que

a) $\displaystyle\sum_{i=1}^{n} (ay_i + bz_i) = a\sum_{i=1}^{n} y_i + b\sum_{i=1}^{n} z_i$

b) $\displaystyle\sum_{i=1}^{m} y_i + \sum_{j=m+1}^{n} y_j = \sum_{r=1}^{n} y_r$

c) $\displaystyle\sum_{i=1}^{n} z_i - \sum_{j=m}^{n} z_j = \sum_{r=1}^{m-1} z_r$

d) $\displaystyle\sum_{i=1}^{n} (y_i - z_i)^2 = \sum_{i=1}^{n} y_i^2 - 2\sum_{i=1}^{n} y_i z_i + \sum_{i=1}^{n} z_i^2$

4a)

$$\begin{aligned}
\sum_{i=1}^{n} (ay_i + bz_i) &= (ay_1 + bz_1) + (ay_2 + bz_2) + \cdots + (ay_n + bz_n) \\
&= (ay_1 + ay_2 + \cdots + ay_n) + (bz_1 + bz_2 + \cdots + bz_n) \\
&= a\,(y_1 + y_2 + \cdots + y_n) + b\,(z_1 + z_2 + \cdots + z_n) \\
&= a\sum_{i=1}^{n} y_i + b\sum_{i=1}^{n} z_i\,.
\end{aligned}$$

4b)

$$\begin{aligned}
\sum_{i=1}^{m} y_i + \sum_{j=m+1}^{n} y_j &= (y_1 + y_2 + \cdots + y_m) + (y_{m+1} + y_{m+2} + \cdots + y_n) \\
&= y_1 + y_2 + \cdots + y_n \\
&= \sum_{r=1}^{n} y_r\,.
\end{aligned}$$

4c)

$$\begin{aligned}\sum_{i=1}^{n} z_i - \sum_{j=m}^{n} z_j &= (z_1 + z_2 + \cdots + z_{m-1} + z_m + z_{m+1} + \cdots + z_n) - (z_m + z_{m+1} + \cdots + z_n) \\ &= z_1 + z_2 + \cdots + z_{m-1} \\ &= \sum_{r=1}^{m-1} z_r \,.\end{aligned}$$

4d)

$$\begin{aligned}\sum_{i=1}^{n} (y_i - z_i)^2 &= \sum_{i=1}^{n} \left(y_i^2 - 2y_i z_i + z_i^2\right) \\ &= \left(y_1^2 - 2y_1 z_1 + z_1^2\right) + \left(y_2^2 - 2y_2 z_2 + z_2^2\right) + \cdots + \left(y_n^2 - 2y_n z_n + z_n^2\right) \\ &= \left(y_1^2 + y_2^2 + \cdots + y_n^2\right) - 2\left(y_1 z_1 + y_2 z_2 + \cdots + y_n z_n\right) + \left(z_1^2 + z_2^2 + \cdots + z_n^2 x\right) \\ &= \sum_{i=1}^{n} y_i^2 - 2\sum_{i=1}^{n} y_i z_i + \sum_{i=1}^{n} z_i^2 \,.\end{aligned}$$

5) Representar através do símbolo de somatória.

a) $\underbrace{x_1 + \cdots + x_1}_{n_1 \text{ vezes}} + \underbrace{x_2 + \cdots + x_2}_{n_2 \text{ vezes}} + \underbrace{x_3 + \cdots + x_3}_{n_3 \text{ vezes}}$

b) $\underbrace{x_1 + \cdots + x_1}_{n_1 \text{ vezes}} + \underbrace{x_2 + \cdots + x_2}_{n_2 \text{ vezes}} + \underbrace{x_3 + \cdots + x_3}_{n_3 \text{ vezes}} + \cdots + \underbrace{x_m + \cdots + x_m}_{n_m \text{ vezes}}$

c) $\underbrace{(x_1 - M)^2 + \cdots + (x_1 - M)^2}_{n_1 \text{ vezes}} + \underbrace{(x_2 - M)^2 + \cdots + (x_2 - M)^2}_{n_2 \text{ vezes}} + \underbrace{(x_3 - M)^2 + \cdots + (x_3 - M)^2}_{n_3 \text{ vezes}}$

d) $\underbrace{(x_1 - M)^2 + \cdots + (x_1 - M)^2}_{n_1 \text{ vezes}} + \underbrace{(x_2 - M)^2 + \cdots + (x_2 - M)^2}_{n_2 \text{ vezes}} + \cdots + \underbrace{(x_m - M)^2 + \cdots + (x_m - M)^2}_{n_m \text{ vezes}}$

5a) $n_1 x_1 + n_2 x_2 + n_3 x_3 = \sum_{i=1}^{3} n_i x_i$.

5b) $n_1 x_1 + n_2 x_2 + n_3 x_3 + \cdots + n_m x_m = \sum_{i=1}^{m} n_i x_i$.

5c) $n_1 (x_1 - M)^2 + n_2 (x_2 - M)^2 + n_3 (x_3 - M)^2 = \sum_{i=1}^{3} n_i (x_i - M)^2$.

5d) $n_1 (x_1 - M)^2 + n_2 (x_2 - M)^2 + \cdots + n_m (x_m - M)^2 = \sum_{i=1}^{m} n_i (x_i - M)^2$.