Equations of Motion for the Automatic Balancing System of 3-DOF Spacecraft Attitude Control Simulator

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***Abstract*—3-DOF spacecraft attitude control simulators have been used for spacecraft attitude determination and control hardware verification and software development. In this paper, a automatic balancing system to reduce the gravitational torques influencing the motion of the simulator was described. The equations of motion and balancing algorithm developed for the system was deduced. The algorithm takes sensor data from the system and gives a vector estimating the location of the center of mass of the platform. It can be shown from the simulated result that the locations of the center of mass are large at first, and begin to diminish rapidly after a 60-second test length is in the automatic balancing algorithm.**

I. Introduction

Many of the missions being assigned to space satellites or space vehicles require that the vehicle be oriented precisely with respect to the earth or to other heavenly bodies. This orientation may be required in order to photograph cloud cover, to point the vehicle for firing course corrections. The torques that may cause disorientation, following initial position, are very small, so that the level of correction torque required is correspondingly small. [1]

In order to develop, prove out, and test attitude-control system, 3-DOF spacecraft attitude control simulator have been used to duplicate a torque-free environment in the space There exist some disturbance torques, which act on the rotor of an air bearing of the spacecraft simulator. They are aerodynamic turbine effect torques from bearing, anisoelasticity torques arising from platform, static and dynamic unbalance torque, the torque arising from vibration, air currents, and so on. In these disturbance torques, the most dominant torque that interferes with the application of the air-bearing spacecraft simulator is unbalance torque. The purpose of the automatic balancing system is to reduce the gravitational torques influencing the motion of the spacecraft simulator. So the study about the system have received considerable attention[2~5]. In this paper, a automatic balancing system to reduce the gravitational torques influencing the motion of the simulator was described. The balancing algorithm developed for the system was deduced.

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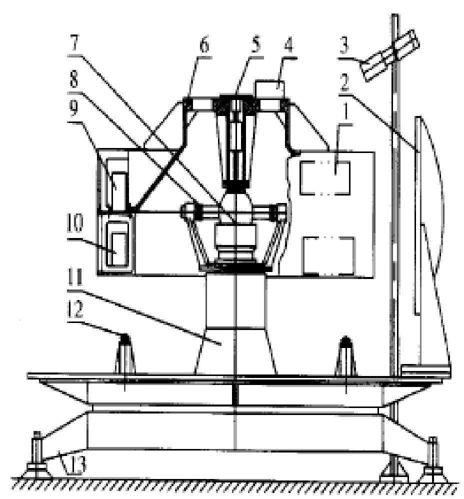
II. Description of the 3-DOF Spacecraft Attitude Control Simulator and Automatic Balancing System

A. The 3-DOF spacecraft attitude control simulator

The 3-DOF spacecraft attitude control simulator consists of a disk-shaped aluminum platform that is supported on a hemispherical air bearing. The platform houses all the various spacecraft components (i.e., sensors, actuators, control computer; see Fig. 1). The air bearing is operated with compressed air from an external source through an air filter. The air filter removes moisture, oil, and other impurities and regulates the air pressure. Pressurized air passes through small holes in the grounded section of the air-bearing and establishes a thin film that supports the weight of the moving section. The air bearing that supports the platform is located on top of a pedestal structure and it allows the platform to move without friction±30°about the two horizontal axes (x and y)and 360° about the vertical (z) axis. Two software applications were developed separately for both the on-board computer and the remote PC computer. The latter is used to send start/stop commands and commands for health monitoring, for postexperiment data analysis and plotting.



1 control system on-board 2.earth simulator
3. sun simulator 4.acceleration sensor
5.auto-balancing system 6. on-board computer
7. air-bearing 8.angle sensor 9. thruster
10. platform 11.support 12.jack 13.foundation

Fig.1 Air-bearing spacecraft simulator

B. Automatic balancing system

The automatic balancing system is composed of acceleration sensor,

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angle sensor, gyros, on-board computer and three actuators. Each actuator consists of a stepper motor moving along a threaded rod with a sliding mass. The set of sliding masses posses the capability to maintain the control of the total mass of the platform, their inertial properties, and the three-axis balancing. These masses can make fine adjustment of the center of mass as well as the principal inertia axis relocation.

III. THE BALANCE ADJUSTING OF THE SPACECRAFT SIMULATOR

The system balancing is crucial to the attitude control of the spacecraft simulator since an unbalanced platform is strongly affected by gravity. An unbalance exist when the center of gravity (CG)and/or the principal inertia axis, has a shift with respect to the rotational axis [6]. Lang has demonstrated that for statically balancing a system (i.e. positioning its center of mass) a single mass is needed[7]. In order to dynamically balance a system (i.e. positioning of the principal inertia axis) two masses are needed, whether, added or subtracted. Static and dynamical unbalance are not alone. There exists another two types of unbalance defined by the International Standards Organization. These are: moment, and quasi-static unbalances; in the first case the principal axis passes through the center of gravity but is not parallel to the rotation axis. In the second case the principal axis intersects the rotation axis at some point other than the CG. By examining the four types of unbalance we see that in balancing a rotating system we must balance not only the forces but also moments. If the system is in dynamic balance it is also in static balance[6].

Manual balancing used to balance the previous spacecraft simulator. This is a time-consuming, iterative process where lead weights of varying mass are placed on the simulator at various locations in an attempt to balance the platform. The process is finalized by carefully adjusting several strategically placed set screws to “zero-in” the center of mass (CM) to the center of rotation (CR). The manual method of balancing has advantages and disadvantages. Manual balancing is relatively simple to conceive and execute, it requires only patience and a little skill in weight placement. On the other hand, the amount of time required to balance the simulator is considerable, often several hours, and the results can be disappointing. After considerable time spent in the manual balancing process, the CM offset was still large enough to create oscillations around the CR with a period of approximately 20 seconds. Using manual balancing, it is extremely difficult to move the CM closer than a half of a millimeter to the CR.

An improvement over manual balancing is an automatic balancing system. Dynamic identification and adjustment of the CM balancing can be performed on it. The system computes the CM by analyzing the dynamical data given by the rate and position sensors, estimates the vector from the CM to the CR, and relocates the CM. Automatic balancing system is very advantageous, especially when compared to manual balancing. Balancing can occur within 10 minutes, move the CM closer than two hundredths of a millimeter to the CR, and requires minimal input from the operator. Creating the automatic balancing system (including component design and creation), and algorithm derivation and coding) is a difficult and time-consuming affair. However, once operation, an automatic balancing system is far more effective than other methods.

IV. DERIVATION OF EQUATION OF MOTION

A right-handed system *X, Y, Z* fixed with respect to the earth (inertial frame). The *Z*-axis is vertically upward and the origin is at the center of rotation of the platform. This system is assumed to be an inertial system. A right-handed system *x, y, z* is rigidly attached to the platform (body-fixed frame). The origin is at the center of rotation of the platform and the axes define the directions of motion of the compensation masses.

We shall derive the coordinate transformation in term of the three Euler angles ψ, θ, ϕ . The transformation is done in three steps by three successive rotations of the platform system about *z, y, x* axis. This transformation is given by [8]

$$\boldsymbol{C} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \tag{1}$$

Where ψ ---yaw, θ ---pitch, ϕ ---roll, so the dynamical equation will be as follows:

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \\ 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

The angular moment vector around the center of rotation is

$$\mathbf{L}_0 = \mathbf{r} \times m\mathbf{v} \tag{3}$$

Where m is the mass of the platform (kg), **r** is the vector that connects the center of rotation with the center of mass, and v is the angular velocity corresponding to the center of mass. The governing equation for the system is that time rate of change of the angular momentum is equal to the applied external moment to the system. this is written as

$$\frac{\mathrm{d}\mathbf{L}_o}{\mathrm{d}t} = \mathbf{M} \tag{4}$$

Where $\mathbf{L}_o$ is the applied external torque to the system. For the case where the CM is not at the CR, $\mathrm{d}\mathbf{L}_o/\mathrm{d}t$ can be written as

$$\mathrm{d}\mathbf{L}_o/\mathrm{d}t = (\mathbf{r} \times m\ddot{\mathbf{r}}) + \dot{\mathbf{L}}_c + [\boldsymbol{\omega} \times (\mathbf{r} \times m\dot{\mathbf{r}})] + (\boldsymbol{\omega} \times \mathbf{L}_c) \tag{5}$$

Where $\mathbf{L}_c$ is the angular momentum around the center of mass, and $\boldsymbol{\omega}$ is the angular velocity vector around the body-fixed axes of the platform. The assumptions for the use of equation (5) are that the translational acceleration of the CR is zero, and the platform is a rigid body. The vector used in the above definitions was defined as

$$\mathbf{L}_c = \begin{pmatrix} I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z \\ I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z \\ I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z \end{pmatrix} \tag{6}$$

Substituting (5),(6) into (4) yields

$$\mathbf{A}\dot{\boldsymbol{\omega}} + \mathbf{B} = \mathbf{M} \tag{7}$$

where $\mathbf{A}$ is a 3×3 matrix, $\mathbf{B}$ is a 3×1 matrix

$$\mathbf{M} = mg\begin{pmatrix} 0 & -\cos\phi\cos\theta & \sin\phi\cos\theta \\ \cos\phi\cos\theta & 0 & \sin\theta \\ -\sin\phi\cos\theta & -\sin\theta & 0 \end{pmatrix}\mathbf{r}$$

Eq.(7)can be also written as

$$\dot{\boldsymbol{\omega}} = (\mathbf{A})^{-1}(\mathbf{M} - \mathbf{B}) \tag{8}$$

This equation, integrated and solved simultaneously with the rotation matrix, describes the dynamics of the simulator. However, this equation of motion can be greatly simplified. If we assume that ω, $\mathbf{r}$, and the cross products of inertia are small compared with the other terms, then we can write as follows:

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dfrac{mg}{I_{xx}}(-r_y\cos\phi\cos\theta + r_z\sin\phi\cos\theta) \\ \dfrac{mg}{I_{yy}}(r_x\cos\phi\cos\theta + r_z\sin\theta) \\ \dfrac{mg}{I_{zz}}(-r_x\sin\phi\cos\theta - r_y\sin\theta) \end{bmatrix} \tag{9}$$

From equations (9) ,(2)and (1) the platform has been modeled like a rigid body whose center of mass is shifted a distance $\mathbf{r}$ with respect to the center of rotation of the air-bearing.

V. CENTER OF MASS LOCATION

Equation (9) can be integrated individually over a short time period to give three equations for each time step. This can be done easily with the assumption that θ and ϕ remain relatively constant over a small time step. After integration, the three equations contain only three unknowns, the CM offset distance vector $\mathbf{r}$. the equation after integration can be written as

$$\omega_{x_{t1}} - \omega_{x_{t2}} = \frac{-mg\Delta t}{2I_{xx}}[\left((\cos\phi\cos\theta)_{t2} + (\cos\phi\cos\theta)_{t1}\right)r_y - \left((\sin\phi\cos\theta)_{t2} + (\sin\phi\cos\theta)_{t1}\right)r_z]$$

$$\omega_{y_{t1}} - \omega_{y_{t2}} = \frac{mg\Delta t}{2I_{yy}}[\left((\cos\phi\cos\theta)_{t2} + (\cos\phi\cos\theta)_{t1}\right)r_x - \left((\sin\theta)_{t2} + (\sin\theta)_{t1}\right)r_z]$$

$$\omega_{z_{t1}} - \omega_{z_{t2}} = \frac{-mg\Delta t}{2I_{zz}}[\left((\sin\phi\cos\theta)_{t2} + (\sin\phi\cos\theta)_{t1}\right)r_x - \left((\sin\theta)_{t2} + (\sin\theta)_{t1}\right)r_y] \tag{10}$$

Placing these equations into matrix form gives

$$\begin{bmatrix} \Delta\omega_x \\ \Delta\omega_y \\ \Delta\omega_z \end{bmatrix} = \begin{bmatrix} 0 & \Phi_{12} & \Phi_{13} \\ \Phi_{21} & 0 & \Phi_{23} \\ \Phi_{31} & \Phi_{32} & 0 \end{bmatrix} \tag{11}$$

These equations can be solved for $\mathbf{r}$ using the method of least squares. In effect, the method of least squares finds a best estimate for a solution using all of the recorded data. With each measurement location, collecting these equations into matrix form, and expanding the matrix for many time steps gives

$$\begin{bmatrix} (\Delta\omega_x)_{t0} \\ (\Delta\omega_y)_{t0} \\ (\Delta\omega_z)_{t0} \\ (\Delta\omega_x)_{t1} \\ (\Delta\omega_y)_{t1} \\ (\Delta\omega_z)_{t1} \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 & (\Phi_{12})_{t0} & (\Phi_{13})_{t0} \\ (\Phi_{21})_{t0} & 0 & (\Phi_{23})_{t0} \\ (\Phi_{31})_{t0} & (\Phi_{32})_{t0} & 0 \\ 0 & (\Phi_{12})_{t1} & (\Phi_{13})_{t1} \\ (\Phi_{21})_{t1} & 0 & (\Phi_{23})_{t1} \\ (\Phi_{31})_{t1} & (\Phi_{32})_{t1} & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{12}$$

The expanded form of (12) can be written as

$$\Delta\mathbf{\Omega}_L = \mathbf{\Phi}_L\ \mathbf{r}$$

The least squares approximation for $\mathbf{r}$ is given by

$$\mathbf{r} = \left[\mathbf{\Phi}_L^T\mathbf{\Phi}_L\right]^{-1}\mathbf{\Phi}_L^T\Delta\mathbf{\Omega}_L \tag{13}$$

Which is the location of the CM of the platform relative to the CR of the bearing.

VI. EXPERIMENTAL RESULT

In this section, we present some experimental results with the algorithm mention above. The weight of the platform is 120.231kg, and its inertia value is given below:

$$I_{xx} = 3.478\ \text{kg}\cdot\text{m}^2$$

$$I_{yy} = 5.784\ \text{kg}\cdot\text{m}^2 \tag{14}$$

Fig.2 The spacecraft attitude control simulator with the automatic balancing system for experiment

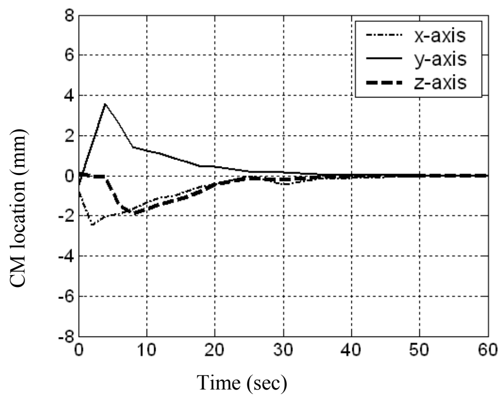

Fig.3 Experimental results of mass center location of the platform with the automatic balancing algorithm.

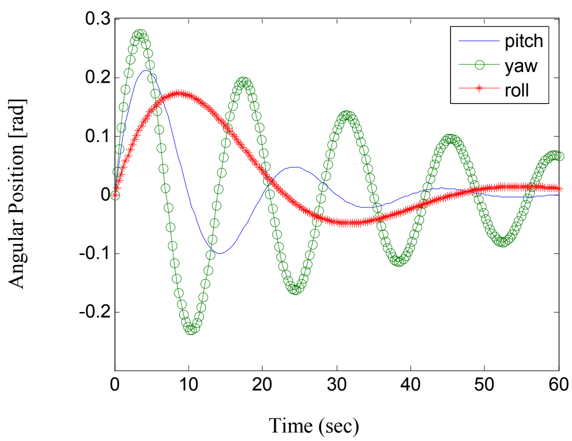

Fig.4 The attitude angles position change in the experiment

$$I_{zz} = 3.821 \text{ kg} \cdot \text{m}^2$$

For convenience, during the experiment, the mass center was moved below the bearing center (+z) to make the platform stable by adjusting a vertical counterweight. The algorithm takes

sensor data from the system and gives a vector estimating the location of the center of mass of the platform.

An important consideration is the amount of data required to accurately locate the CM. To answer this question, a data set of 60 seconds at 25 Hz was taken from the system.

The spacecraft attitude control simulator with the automatic balancing system for the experiment is shown in Fig. 2. the results of the experiments are shown in Fig. 3and Fig.4. The figure shows that a longer test length results in the algorithm setting on a CM value, and the deviation becoming very small. The plots also show that these changes are large at first, and begin to diminish rapidly after a 60-second test length is in the automatic balancing algorithm.

VII. Conclusion

The purpose of the automatic balancing system is to reduce the gravitational torques influencing the motion of the spacecraft simulator. The system calculates the platform center of mass by analyzing the dynamic sensor data along with the integrated equations of motion. The automatic balancing algorithm uses the method of least squares to estimate the vector from the center of rotation to the center of mass. The center of mass of the simulator is then moved near to the center of rotation by means of movable masses that are adjusted to the correct location. The algorithm is able to move the center of mass to a location closer than two hundredths of a millimeter from the center of rotation.

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