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Research on Torque Allocation for Multi-Reaction Wheels System

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***Abstract*—This article deals with torque allocation for multi-reaction wheel in spacecraft. First of all, an energy-based optimal allocation strategy is derived under the constraints of reaction wheel maximum output torque. Secondly, to address the shortened life time problem which results from too large torque output of some specific reaction wheel in energy-based optimal allocation strategy, the concept of output margin is proposed, and a multi-objective optimal model is put forward. Then, based on weighted coefficients theory an energy-output-margin-based optimal allocation strategy is developed, which can flexibly and effectively solved torque allocation in engineering. Finally, eight skewed reaction wheel system numerical simulation results show the effectiveness of the proposed strategy.**

Keywords—Torque allocation; Reaction wheel; Energy optimal; output margin; Multi-objective optimal

I. INTRODUCTION

Due to the long period of spacecraft applications, many spacecraft often employ reaction wheels as its actuator, which only cost electrical energy. In order to improve the reliability of the spacecraft and the torque output capacity in different directions of the spacecraft in the same time, spacecraft often employ redundant reaction wheel configuration. How to allocate the desired torque command to the redundant, saturated torque and angular momentum of each reaction wheels, is one of the key issues when designing spacecraft control allocation algorithm needs to be considered.

YANG Enquan[1] introduced generalized Inverse/Redistributed Pseudo-Inverse, Daisy Chaining and Mathematical Programming. Guo Yanning [2] introduced an energy-based optimal allocation strategy, and then for a four-flywheel and a multi-flywheel system, two torque-based optimal allocation logics are developed. Yan Caixia[3] discussed the capacity of each actuator, and proposed the concept of output margin, and the proposed method is helpful to regulate the balanced margin output of each actuator and thus prevent partial actuators from over loading. YAN Caixia[4] proposed a new method for torque allocation, this approach could better prevent the overloading of partial actuators and greatly ensure the system stability. Using control allocator, CHEN Min [5] proposed a combined control scheme based on the reaction wheels and magnetorquers to be used, two control allocation strategies are proposed to automatically re-distribute the desire control torque among reaction wheels and magnetorquers.

P. A. Servidia[6] stated four possible practical problems and proposed a control allocation algorithm based on a sub gradient optimization. Härkegård[7] indicated that there are two tools for allocating the control torque among a cluster of actuators are optimal control design and control allocation. Fleming[8] Optimized the installation angle of four-skewed reaction wheel system, and proposed Pseudo-Inverse method which minimize the energy cost. Bordingnon[9] pointed out that allocate space of Pseudo-inverse is very limited, its disadvantage is very obvious when it comes to high redundancy system. Hablani[10] dealt with optimization of cant angle(s) of reaction wheel pyramids for minimizing power consumption and required torque and momentum capacity of the wheels to deliver a cylindrical momentum envelope and a rectangular parallelepiped torque envelope. Markley [11] proposed a momentum-based optimal allocation strategy based on maximum torque and momentum envelopes. Tsiotras[12] and Y.H. Jia[13] both investigated the control law of the reaction wheel in an integrated power and attitude control system for a spacecraft.

In this paper, the author proposed a new allocation strategy embodied the combination of output margin and traditional energy-based optimal allocation strategy. Trade-off coefficient theory was used in the new strategy. In the proposed allocation strategy, output torque of the reaction wheel is balanced by adjusting coefficient of each reaction wheel. And it also can achieve the purpose of controlling energy cost at the same time.

II. MATHEMATICAL MODEL OF THE ALLOCATION OF TORQUE

Assume that, n (n>3) reaction wheels are employed on the spacecraft, then the angular momentum vectors of these wheels are $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$. When the magnitude of the angular momentum vectors for each reaction wheels are $\tilde{\boldsymbol{h}}_w = [h_{w1} \quad h_{w2} \quad \cdots \quad h_{wn}]^T$, we can get:

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$$\boldsymbol{h}_w = \boldsymbol{e}_1 h_{w1} + \boldsymbol{e}_2 h_{w2} + \boldsymbol{e}_3 h_{w3} + \cdots + \boldsymbol{e}_n h_{wn} = \boldsymbol{C}_w \tilde{\boldsymbol{h}}_w \tag{1}$$

where $\boldsymbol{C}_w = [\boldsymbol{e}_1 \quad \boldsymbol{e}_2 \quad \boldsymbol{e}_3 \quad \ldots \quad \boldsymbol{e}_n]$.

When choose reaction wheels as actuator, torque command is given by:

$$\boldsymbol{u}_c \equiv \dot{\boldsymbol{h}}_w = \boldsymbol{C}_w \dot{\tilde{\boldsymbol{h}}}_w = \boldsymbol{C}_w \tilde{\boldsymbol{u}}_w \tag{2}$$

where $\tilde{\boldsymbol{u}}_w = [u_{w1} \quad u_{w2} \quad \cdots \quad u_{wn}]^{\mathrm{T}}$, is amplitude vector of allocated torque of each reaction wheel.

III. ENERGY-BASED OPTIMAL ALLOCATION STRATEGY

Energy optimal strategy is a set of feasible solutions that minimize the energy index function under the constraint of equation(2). Since the energy consumption of reaction wheel system is related to the output torque of each reaction wheel closely, so energy index is constructed as follows:

$$J_2 = 0.5 \left\| \tilde{\boldsymbol{u}}_w \right\|_2^2 = 0.5\, \tilde{\boldsymbol{u}}_w^T \tilde{\boldsymbol{u}}_w \tag{3}$$

Now the torque allocation strategy is the solution that minimize J_2 under the constraint of equation(2). Here Lagrange multiplier method is applied to solve this problem, Lagrange function is constructed as follows:

$$H = 0.5 \tilde{\boldsymbol{u}}_w^T \tilde{\boldsymbol{u}}_w + \boldsymbol{\lambda}^{\mathrm{T}} (\boldsymbol{C}_w \tilde{\boldsymbol{u}}_w - \boldsymbol{u}_c) \tag{4}$$

where $\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_n]^{\mathrm{T}}$ is the Lagrange multiplier. Take partial derivative of H with respect to $\boldsymbol{\lambda}$ and $\tilde{\boldsymbol{u}}_w$:

$$\left.\begin{aligned} \frac{\partial H}{\partial \tilde{\boldsymbol{u}}_w} &= \tilde{\boldsymbol{u}}_w + \boldsymbol{C}_w^T \boldsymbol{\lambda} = \boldsymbol{0} \\ \frac{\partial H}{\partial \boldsymbol{\lambda}} &= \boldsymbol{C}_w \tilde{\boldsymbol{u}}_w - \boldsymbol{u}_c = \boldsymbol{0} \end{aligned}\right\} \tag{5}$$

Then we can get:

$$\tilde{\boldsymbol{u}}_w = \boldsymbol{C}_w^T (\boldsymbol{C}_w \boldsymbol{C}_w^T)^{-1} \boldsymbol{u}_c \tag{6}$$

This torque allocation strategy is able to satisfy the requirements of the spacecraft attitude control, but it also has some drawbacks. The most serious thing is the solution of torque allocation strategy is unique. Though the strategy can meet the requirements, but there is no room available for selection. Therefore the solution maybe out of the capability of the reaction wheel, this may lead to reaction wheel torque saturation, and result in maneuver cannot be achieved. So adapt the torque allocation strategy as follows:

$$\tilde{\boldsymbol{u}}_{wf} = \begin{cases} \tilde{\boldsymbol{u}}_w & \left\| \tilde{\boldsymbol{u}}_w \right\|_\infty \le u_{wm} \\ \tilde{\boldsymbol{u}}_w \dfrac{u_{wm}}{\left\| \tilde{\boldsymbol{u}}_w \right\|_\infty} & \left\| \tilde{\boldsymbol{u}}_w \right\|_\infty > u_{wm} \end{cases} \tag{7}$$

Where u_{wm} is the maximum output torque of the reaction wheel, $\left\| \tilde{\boldsymbol{u}}_w \right\|_\infty$ is the infinite norm of $\tilde{\boldsymbol{u}}_w$. After that, it can be ensured that the direction of the output torque of the reaction wheel system unchanged.

IV. OUTPUT MARGIN OF REACTION WHEEL AND MULTI-OBJECTIVE OPTIMIZATION MODEL

For the ith reaction wheel, define its output margin as follows:

$$\Delta u_{wi}(t) = \frac{u_{wm} - \left| u_{wi}(t) \right|}{u_{wm}} \tag{8}$$

where $u_{wi}(t)$ is the output torque of the ith reaction wheel at t moment, u_{wm} is the maximum output torque of the reaction wheel, $\Delta u_{wi}(t)$ is the output margin of the ith reaction wheel at t moment. Output margin intuitively reflects the actual output of the reaction wheel relative to the proportion of its own output capacity, it is a dimensionless parameter.

It can be seen, when $\Delta u_{wi}(t) < 0$, it indicates that the reaction wheel is saturated. Therefore, the situation of output torque of reaction wheel can be knew according to the output margin of reaction wheel in real time, so as to achieve a reasonable allocation of torque.

In order to control the output torque of each single reaction wheel and restrict the energy consumption at the same time. Weighted coefficient method is used to achieve this objective. So, multi-objective optimization model is constructed as follows:

$$\left.\begin{aligned} &J_{mul} = \frac{1}{2} \rho_0 \tilde{\boldsymbol{u}}_w^T \tilde{\boldsymbol{u}}_w + \frac{1}{2} (\Delta \tilde{\boldsymbol{u}}_w - \boldsymbol{E})^T \boldsymbol{\rho} (\Delta \tilde{\boldsymbol{u}}_w - \boldsymbol{E}) \\ &s.t.\ \ \boldsymbol{u}_c = \boldsymbol{C}_w \tilde{\boldsymbol{u}}_w \end{aligned}\right\} \tag{9}$$

where, $\boldsymbol{\rho} = diag(\rho_1 \quad \rho_2 \quad \ldots \quad \rho_n)$ is weighted coefficients of each reaction wheel, They represent individual weight in the objective function, ρ_0 is weighted coefficients of energy consumption, $\Delta \tilde{\boldsymbol{u}}_w$ is output margin of reaction wheel system, and $\boldsymbol{E} = [1 \quad 1 \quad \cdots \quad 1]^T$.

As can be seen from the formula(9), the output torque can be reduced by increasing the weighted coefficients of the reaction wheel. In this way, reaction wheel system can be protected by adjusting the weighted coefficient matrix. And it is an effective way to delay the time of first failure of the reaction wheel system. Here Lagrange multipliers method is applied, Lagrange function is constructed as follows:

$$H = \frac{1}{2} \rho_0 \tilde{\boldsymbol{u}}_w^T \tilde{\boldsymbol{u}}_w + (\Delta \tilde{\boldsymbol{u}}_w - \boldsymbol{E})^T \boldsymbol{\rho} (\Delta \tilde{\boldsymbol{u}}_w - \boldsymbol{E}) + \boldsymbol{\lambda}^T (\boldsymbol{C}_w \tilde{\boldsymbol{u}}_w - \boldsymbol{u}_c) \tag{10}$$

where $\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_n]^{\mathrm{T}}$ is the Lagrange multiplier. Take partial derivative of H with respect to $\boldsymbol{\lambda}$ and $\tilde{\boldsymbol{u}}_w$:

$$\left.\begin{aligned} \frac{\partial H}{\partial \tilde{\boldsymbol{u}}_w} &= \rho_0 \tilde{\boldsymbol{u}}_w + \frac{1}{u_m^2} \boldsymbol{\rho} \tilde{\boldsymbol{u}}_w + \boldsymbol{C}_w^T \boldsymbol{\lambda} = \boldsymbol{0} \\ \frac{\partial H}{\partial \boldsymbol{\lambda}} &= \boldsymbol{u}_c - \boldsymbol{C}_w \tilde{\boldsymbol{u}}_w = \boldsymbol{0} \end{aligned}\right\} \tag{11}$$

Expressions of multi-objective based optimal torque allocation strategy can be derived by solving the above equation:

$$\tilde{\boldsymbol{u}}_w = (\rho_0 + \frac{1}{u_m^2}\rho)^{-1}\boldsymbol{C}_w^T\left[\boldsymbol{C}_w(\rho_0 + \frac{1}{u_m^2}\rho)^{-1}\boldsymbol{C}_w^T\right]^{-1}\boldsymbol{u}_c \qquad (12)$$

So far, the problem for solving multi-objective optimization allocation strategy converted to choice of weighted coefficients.

V. Selection of Weighted Coefficients of Multi-Objective Optimization Strategy

For reaction wheel torque allocation problems, adjusting the output torque only based on the torque of each reaction wheel is not comprehensive. Taking into account the capacity of each reaction wheel and other factors. We can adjust the output torque based on the output margin of each reaction wheel.

According to the concept of the output margin, it can be used as measure parameters to adjust the weighted coefficients. If a large output torque is allocated to one of the reaction wheels, it is reasonable to increase the weighted coefficients appropriately, reducing the output torque to a certain extent, and relieving the output pressure of the reaction wheel. Intuitively, the ratio of ith reaction wheel and the mean of the output margin of the reaction wheel system is applied as the weighted coefficients of ith reaction wheel, that is:

$$\rho_i(t) = ((\sum_{i=1}^{n}\Delta u_{wi}(t-1))/n)/\Delta u_{wi}(t-1) \qquad (13)$$

The above equation can adjust the output margin of each reaction wheel as balanced as possible. The value of weighted coefficients is changing real-time. Because of this strategy adjust the output torque based on the information of last step, it is more effective compared to traditional energy-based optimal strategy.

This strategy reflects the relationship between the weighted coefficients and the output margin of reaction wheel in process of torque allocation. And it also can adjust the output margin of each reaction wheel as balanced as possible. Based on such a weighted coefficients selection method, torque allocation of reaction wheel system can be summarized as follows:

$$\begin{cases} \tilde{\boldsymbol{u}}_w = (\rho_0 + \frac{1}{u_m^2}\rho)^{-1}\boldsymbol{C}_w^T\left[\boldsymbol{C}_w(\rho_0 + \frac{1}{u_m^2}\rho)^{-1}\boldsymbol{C}_w^T\right]^{-1}\boldsymbol{u}_c \\ \rho_i(t) = ((\sum_{i=1}^{n}\Delta u_{wi}(t-1))/n)/\Delta u_{wi}(t-1) \end{cases} \qquad (14)$$

VI. Numerical Simulations

Numerical simulations are performed by using parameters as follows:

The spacecraft employs eight skewed reaction wheel system, configuration of reaction wheel is shown in Figure 1:

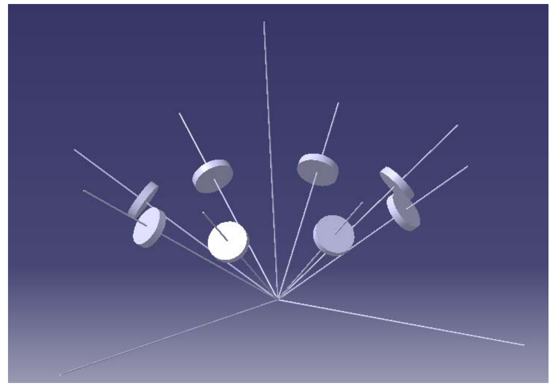

Fig. 1. Schematic of reaction wheel configuration

The configuration matrix is as follows:

$$C_w = \begin{bmatrix} \sin(\alpha)\cos(\beta) & \sin(\alpha)\sin(\beta) & -\sin(\alpha)\sin(\beta) & -\sin(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & -\sin(\alpha)\sin(\beta) & \sin(\alpha)\sin(\beta) & \sin(\alpha)\cos(\beta) \\ \sin(\alpha)\sin(\beta) & \sin(\alpha)\cos(\theta) & \sin(\alpha)\cos(\beta) & \sin(\alpha)\sin(\beta) & -\sin(\alpha)\sin(\beta) & -\sin(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & -\sin(\alpha)\sin(\beta); \\ \cos(\alpha) & \cos(\alpha) & \cos(\alpha) & \cos(\alpha) & \cos(\alpha) & \cos(\alpha) & \cos(\alpha) & \cos(\alpha) \end{bmatrix}$$

where, α=54.74°, β=22.5°. The maximum output torque of reaction wheel is 0.06Nm.

The initial attitude angle of satellite is:

$$\varphi = -3^\circ,\ \theta = 2^\circ,\ \psi = 2^\circ$$

The target attitude angle of satellite is:

$$\varphi = 0^\circ,\ \theta = 0^\circ,\ \psi = 0^\circ$$

The initial angular velocity is:

$$\omega(t_0) = [\,0.01\ ,\ -0.07\ ,\ 0.01]^T\ \ ^\circ/s$$

The following parameters of a satellite dynamics are used:

$$I = \begin{bmatrix} 190.5 & 2.3 & -7.2 \\ 2.3 & 200 & -40.9 \\ -7.2 & -40.9 & 217.6 \end{bmatrix} \mathrm{kg}\cdot m^2$$

$$K_p = [1.9 \quad 2.0 \quad 2.2]^T$$

$$K_d = [26.7 \quad 28.0 \quad 30.5]^T$$

To analyze the effect of each allocation strategy, two evaluations are defined as follows:

$$\Delta u_z = \sum_{t=0}^{T}\sum_{i=1}^{n}\left[\overline{u(t)} - u_i(t)\right]^2 \qquad (15)$$

$$J_z = \int_0^T J_2 dt \qquad (16)$$

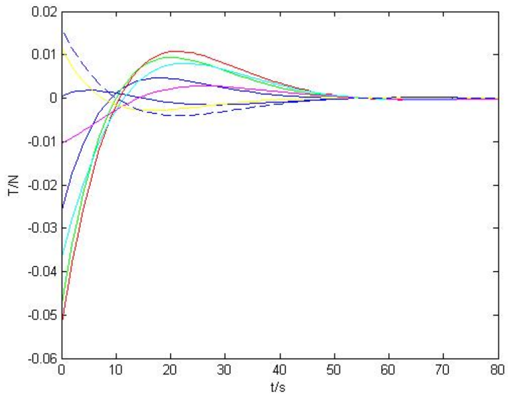

Fig. 2. Torque of reaction wheels under energy-based optimal allocation strategy

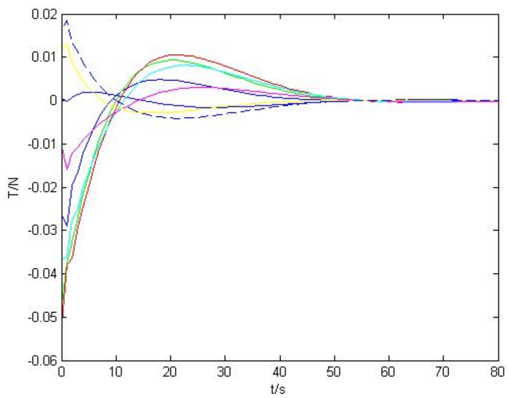

Fig. 3. Torque of reaction wheels under multi-objective optimal strategy ($\rho_0 = 1$)

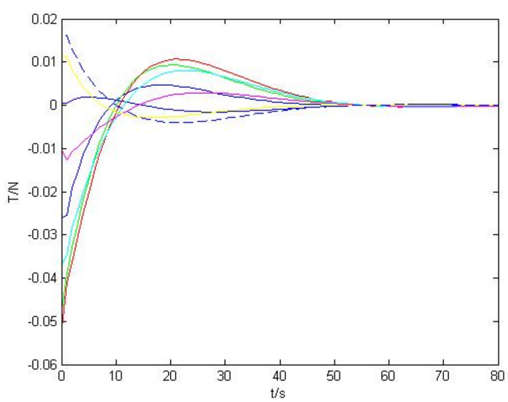

Fig. 4. Torque of reaction wheels under multi-objective optimal strategy ($\rho_0 = 10$)

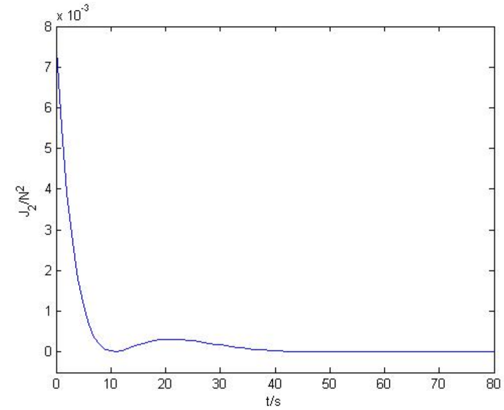

Fig. 5. Energy of reaction wheels under energy-based optimal allocation strategy

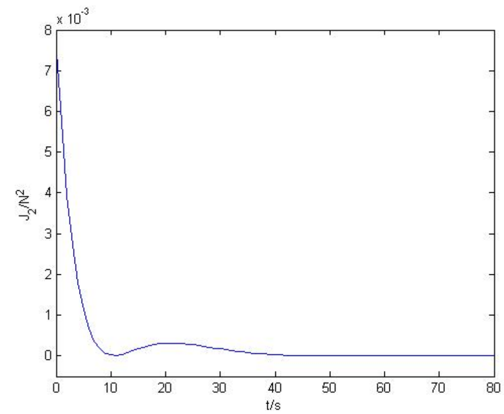

Fig. 6. Energy of reaction wheels under multi-objective optimal strategy ($\rho_0 = 1$)

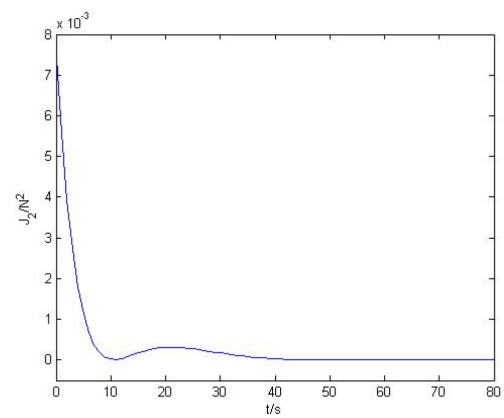

Fig. 7. Energy of reaction wheels under multi-objective optimal strategy ($\rho_0 = 10$)

As can be seen from the expression, Δu_z reflects the extent of the output torque of each reaction wheel offset the mean of the output torque of the reaction wheel system. The smaller Δu_z is, the torque allocated to each of the reaction wheel is more average. J_z represents the energy consumption in process of the attitude stabilization.

For traditional energy-based optimal strategy $\Delta u_{z1} = 0.0092497$, $J_{z1} = 0.029283$.

For multi-objective optimal strategy when $\rho_0 = 1$, $\Delta u_{z2} = 0.008242$, $J_{z2} = 0.029557$, when $\rho_0 = 10$, $\Delta u_{z3} = 0.0083057$, $J_{z3} = 0.02942$.

From the above data can be clearly seen, multi-objective optimal strategy reduce the torque offset index Δu_z by 10%, this shows the effectiveness of multi-objective optimal allocation strategy. And when we increase energy weighted coefficients, it can effectively reduce the evaluation J_z . It proved that the multi-objective optimal allocation strategies achieve the purpose of controlling output margin while balancing energy consumption.

VII. CONCLUSION

In order to solve the torque allocation problem of spacecraft that employed multi-reaction wheel system, taking into account the life of the reaction wheel system and energy consumption, this paper proposed a multi-objective based optimal allocation strategy based on the output margin and energy consumption. Through the simulation under different energy weighted coefficients, and comparing it to traditional energy-based optimal strategy, it proved the advantage of the proposed method. Specially, The proposed method reduced the torque offset index Δu_z by 10%. Output torque was effectively balanced between the reaction wheels, and the life of system was also lengthened effectively.

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