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Pseudospectral Method for Optimal Motion Planning of a Rigid Underactuated Spacecraft

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Abstract—**This paper discusses the problem of optimal motion planning for a rigid underactuated spacecraft. Firstly, we derive the dynamic and kinematic equations of the spacecraft, and for given initial and terminal states, the performance index to be optimized is selected as minimizing the total fuel consumption. Moreover, the control input saturation restriction is transformed into an inequality constraint. Then, the optimal motion planning problem can be converted into a nonlinear programming problem using Legendre pseudospectral method at a set of discretization points. Simulation results demonstrate that the obtained optimal trajectories satisfy the constraint conditions well and the proposed approach can also be used on-line in application practice with its small computational amount and low computational complexity.**

I. Introduction

TYPICALLY, in real space applications, the attitude of a spacecraft is controlled with $n(n \geq 3)$ independent control inputs, which are offered by jets thrusters and/or reaction wheels. However, it is important to consider the attitude control problems in case of actuator failures, that is, when the spacecraft is underactuated.

The problem of rigid spacecraft attitude control using less than three independent control inputs can be traced back to two decades ago. Firstly, controllability of an underactuated spacecraft was studied by Crouch [1]. He proved that an underactuated spacecraft is controllable and locally controllable at any equilibrium only when it is axisymmetric. Later, work by Morin and Samson [2] dealt with the problem of stabilization of an asymmetric rigid underactuated spacecraft using time-varying controllers with exponential convergence; And work by Krishnan *et al*. [3] and Tsiotras *et al*. [4], on the other hand, solved stabilization problem of an axisymmetric underactuated spacecraft using time-invariant feedback controllers when the initial rate of the uncontrolled axis is zero. Especially Tsiotras *et al*. [5]-[7] employed (w,z) attitude parameterization and proposed a series of stabilization and tracking controllers for axisymmetric underactuated spacecraft. Thus the stabilization problem of an axisymmetric underactuated spacecraft can be considered solved, although there is still much work to do for robustness questions. Nevertheless, the states of an underactuated spacecraft usually have unintegral constraints, i.e., for arbitrary given states, it is not sure that there indeed exists a corresponding control command [7]. So from a practical and theoretical point of view, the feasible trajectory generation and motion planning problems are also important in researches of underactuated spacecrafts. Tsiotras *et al*. [7] proposed a method for approximating the origin axisymmetric spacecraft with a differentially flat system, and the feasible trajectories generated can then be used as reference trajectories in tracking control problem; Ge *et al*. [8] discussed the optimal attitude control of a rigid underactuated spacecraft with two momentum wheel actuators.

Pseudospectral method approximates the system dynamics with global orthogonal polynomials, in order to convert the optimal control problem into a mathematical nonlinear programming program. Thus existing and well-developed optimization approaches may be used to solve the transformed problem with a high spectral accuracy. Furthermore, this method can also be used on-line with its small computational amount and low computational complexity. In this paper, the optimal motion planning of a rigid underactuated spacecraft using pseudospectral method is discussed. Firstly, we derive the model of the underactuated spacecraft, and for given initial and terminal attitudes, the performance index to be optimized is selected as minimizing the total fuel consumption. Moreover, since in real applications the control inputs are always bounded, we transform this saturation restriction to an inequality constraint. Then, the optimal motion planning problem can be converted into a nonlinear programming problem using Legendre pseudospectral method. The simulation results illustrate the feasibility and effectiveness of the proposed approach.

II. Model Of Rigid Underactuated Spacecraft

Assuming the rigid body-fixed reference frame along its principle axes of inertia with the origin at the center of mass and without any external disturbances, the dynamics of a rigid underactuated spacecraft can be written as

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}^{\times}]\boldsymbol{I}\boldsymbol{\omega} = \boldsymbol{M} \tag{1}$$

where $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ denotes the angular velocity vector with respect to the principle axes; $\boldsymbol{M} = [M_1, M_2, M_3]^T$ denotes the external independent torques derived by actuators; $\boldsymbol{I} = \mathrm{diag}(I_1, I_2, I_3)$ represents the positive definite symmetric matrix of moment inertia.

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Without loss of generality, assuming there is no external torque along the 3-axis, i.e., $M_3 = 0$, then

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 + M_1 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_1\omega_3 + M_2 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned} \tag{2}$$

For simplicity, defining two new control inputs

$$u_1 = \frac{(I_2 - I_3)\omega_2\omega_3 + M_1}{I_1}$$

$$u_2 = \frac{(I_3 - I_1)\omega_1\omega_3 + M_2}{I_2}$$

and a variable $\alpha = (I_1 - I_2)/I_3$, which gives a measure of the body asymmetry about its underactuated axis. Note that if $\alpha = 0$, then the 3-axis of body is an axis of symmetry. Substituting u_1, u_2 and α into (1), we yield the simplified dynamic equations of the underactuated spacecraft

$$\begin{aligned} \dot{\omega}_1 &= u_1 \\ \dot{\omega}_2 &= u_2 \\ \dot{\omega}_3 &= \alpha\omega_1\omega_2 \end{aligned} \tag{3}$$

Using 3-2-1 Euler angles parameterization, the kinematic equations of the spacecraft can be described as

$$\begin{aligned} \dot{\phi} &= \omega_1 + (\omega_2 \sin\phi + \omega_3 \cos\phi)\tan\theta \\ \dot{\theta} &= \omega_2 \cos\phi - \omega_3 \sin\phi \\ \dot{\psi} &= (\omega_2 \sin\phi + \omega_3 \cos\phi)\sec\theta \end{aligned} \tag{4}$$

where ϕ, θ, ψ corresponds to the roll-pitch-yaw angles of the underactuated spacecraft.

III. Optimal Motion Planning

A. Problem Statement

An optimal motion planning problem can be stated as: determine the state-control pair $\tau \in [0, T] \mapsto (\boldsymbol{X}, \boldsymbol{U}) \in \mathbb{R}^n \times \mathbb{R}^m$, which maximizes or minimizes the cost function

$$J[\boldsymbol{X}, \boldsymbol{U}, T] = H(\boldsymbol{X}(T), T) + \int_0^T G(\boldsymbol{X}(\tau), \boldsymbol{U}(\tau), \tau)\mathrm{d}\tau \tag{5}$$

subject to the dynamic constraints of system

$$\dot{\boldsymbol{X}}(\tau) = \boldsymbol{F}(\boldsymbol{X}(\tau), \boldsymbol{U}(\tau), \tau) \ , \ \boldsymbol{X}(0) = \boldsymbol{X}_0 \tag{6}$$

end points constraints

$$\boldsymbol{E}(\boldsymbol{X}(T), T) = \boldsymbol{0} \tag{7}$$

and inequality constraints of control inputs

$$\boldsymbol{S}[\boldsymbol{X}(\tau), \boldsymbol{U}(\tau), \tau] \le \boldsymbol{0} \tag{8}$$

B. Performance Index and Constraint Conditions

In this paper, for given initial and terminal states, the performance index for optimal motion planning is selected as minimizing the total fuel consumption of the underactuated spacecraft

$$J(\tau) = \frac{1}{2}\int_0^T \langle \boldsymbol{U}, \boldsymbol{U} \rangle \mathrm{d}\tau \tag{9}$$

where, $\boldsymbol{U} = [U_1, U_2]$ is the corresponding control command. Moreover, in order to settle the problem of control input saturation, we design the control torques U_1, U_2 satisfy an explicit inequality constraint

$$|U_i(\tau)| \le U_M \ , \forall 0 \le \tau \le T \ \ i = 1, 2 \tag{10}$$

where U_M is the control input upper limit.

And end points constraints are the initial and terminal conditions of the rigid body

$$\boldsymbol{X}(0) = \boldsymbol{X}_0 \ , \ \boldsymbol{X}(T) = \boldsymbol{X}_T \tag{11}$$

C. Legendre Pseudospectral Method

Pseudospectral method originates from finite element method. In this method, a finite basis of global interpolating polynomials is used to approximate the state and optimal control space at a set of discretization points, and then the optimal control equations can be transformed into nonlinear algebra equations, such that the optimal control problem can be solved by a nonlinear programming method or a matrix analysis method.

Let $t \in [-1, 1]$, $L_N(t)$ denotes the Legendre polynomial of order N, and $\dot{L}_N(t)$ denotes the derivative of $L_N(t)$. The $N-1$ zeros of $\dot{L}_N(t)$, $t_m (m = 1, \ldots, N-1)$ and $t_0 = -1$, $t_N = 1$ are called Legendre-Gauss-Lobatto (LGL) points. We then construct Nth degree Lagrange interpolating basis function

$$\phi_l(t) = \frac{1}{N(N+1)L_N(t_l)} \cdot \frac{(t^2 - 1)\dot{L}_N(t)}{t - t_l} \quad (l = 0, 1, \ldots, N) \tag{12}$$

where

$$\phi_l(t_k) = \delta_{lk} = \begin{cases} 1 & l = k \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Assuming $F(t)$ is a smooth function defined on interval $t \in [-1, 1]$, then it can be approximated by Nth degree interpolating polynomial $F^N(t)$ as

$$F(t) \approx F^N(t) := \sum_{l=0}^{N} F(t_l)\phi_l(t) \tag{14}$$

and by (13), we have

$$F(t_k) = F^N(t_k) \ , \ k = 0, 1, \ldots, N \tag{15}$$

The derivative of $F(t)$ in (14) at a LGL point t_k also can be approximated with $\dot{F}^N(t)$ as follows

$$\dot{F}(t_k) \approx \dot{F}^N(t_k) = \sum_{l=0}^{N} \boldsymbol{D}_{kl} F(t_l) \tag{16}$$

where, $\boldsymbol{D} := (\boldsymbol{D}_{kl})$ is a $(N+1) \times (N+1)$ matrix given by

$$\boldsymbol{D} := (\boldsymbol{D}_{kl}) := \begin{cases} \dfrac{L_N(t_k)}{L_N(t_l)} \cdot \dfrac{1}{t_k - t_l} & k \neq l \\ -\dfrac{N(N+1)}{4} & k = l = 0 \\ \dfrac{N(N+1)}{4} & k = l = N \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

D. Discretization of Optimal Motion Planning Problem

As described above, Legendre pseudospectral method can only be applied on interval $t \in [-1,1]$, so for any smooth functions on interval $\tau \in [0,T]$, we first need a linear transform $2\tau = (\tau_f - \tau_0)t + (\tau_f + \tau_0)$, and then the optimal motion planning problem (5)-(8) can be reformulated as: determine the state-control pair $t \in [-1,1] \mapsto (\boldsymbol{x}, \boldsymbol{u}) \in \mathbb{R}^n \times \mathbb{R}^m$, which minimizes the cost function

$$J[\boldsymbol{x}, \boldsymbol{u}, t] = h(\boldsymbol{x}(1), T) + \int_{-1}^{1} g(\boldsymbol{x}(t), \boldsymbol{u}(t), t, T)\mathrm{d}t \quad (18)$$

subject to

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t, T) \\ &\boldsymbol{x}(-1) = \boldsymbol{x}_0 \ , \ \boldsymbol{e}[\boldsymbol{x}(1), T] = \boldsymbol{0} \\ &\boldsymbol{s}[\boldsymbol{x}(t), \boldsymbol{u}(t), t, T] \leq \boldsymbol{0} \end{aligned} \quad (19)$$

where $h(\cdot), g(\cdot), \boldsymbol{f}(\cdot), \boldsymbol{e}(\cdot)$ and $\boldsymbol{s}(\cdot)$ denote functions obtained from $H(\cdot), G(\cdot), \boldsymbol{F}(\cdot), \boldsymbol{E}(\cdot)$ and $\boldsymbol{S}(\cdot)$ respectively by the linear transform.

The main step of optimal control problem based on pseudospectral method is to discrete the state and control variables of the system at LGL points $t_l (l = 0,1,...,N)$ as (14). The *N*th degree interpolating polynomial approximations to $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$ are stated as follows

$$\boldsymbol{x}(t) \approx \boldsymbol{x}^N(t) := \sum_{l=0}^{N} \boldsymbol{a}_l \phi_l(t) \ , \ \boldsymbol{u}(t) \approx \boldsymbol{u}^N(t) := \sum_{l=0}^{N} \boldsymbol{b}_l \phi_l(t) \quad (20)$$

where $\boldsymbol{a}_l$ and $\boldsymbol{b}_l$ are two sets of unknown coefficients to be determined, and by (15)

$$\boldsymbol{a}_k = \boldsymbol{x}^N(t_k) = \boldsymbol{x}(t_k) \ , \ \boldsymbol{b}_k = \boldsymbol{u}^N(t_k) = \boldsymbol{u}(t_k) \quad (21)$$

Additionally, the integral formula of pseudospectral method can be written as

$$\int_{-1}^{1} F(t)\mathrm{d}t \approx \int_{-1}^{1} F^N(t)\mathrm{d}t = \sum_{l=0}^{N} F(t_l) w_l \quad (22)$$

where the weight function

$$w_l = 2 / \{N(N+1) \cdot [L_N(t_l)]^2\}, k = 0,1,...,N$$

Then, the integral term of cost function in (18) can be approximated by *N*th interpolating polynomials as

$$\begin{aligned} &\int_{-1}^{1} g(\boldsymbol{x}(t), \boldsymbol{u}(t), t, T)\mathrm{d}t \approx \int_{-1}^{1} g(\boldsymbol{x}^N(t), \boldsymbol{u}^N(t), t, T)\mathrm{d}t \\ &= \sum_{k=0}^{N} g(\boldsymbol{x}^N(t_k), \boldsymbol{u}^N(t_k), t_k, T) w_k \end{aligned} \quad (23)$$

Furthermore, the cost function can be discretized

$$\begin{aligned} &J[\boldsymbol{x}, \boldsymbol{u}, t] \approx J[\boldsymbol{x}^N, \boldsymbol{u}^N, t] \\ &= h(\boldsymbol{x}^N(1), T) + \sum_{k=0}^{N} g(\boldsymbol{x}^N(t_k), \boldsymbol{u}^N(t_k), t_k, T) w_k \\ &= h(\boldsymbol{a}_N, T) + \sum_{k=0}^{N} g(\boldsymbol{a}_k, \boldsymbol{b}_k, t_k, T) w_k \end{aligned} \quad (24)$$

subject to corresponding dynamic constraints of system

$$\sum_{l=0}^{N} \boldsymbol{D}_{kl} \boldsymbol{a}_l - \boldsymbol{f}(\boldsymbol{a}_k, \boldsymbol{b}_k, t_k, T) = \boldsymbol{0} \quad (25)$$

end points constraints

$$\boldsymbol{x}_0 = \boldsymbol{a}_0 \ , \ \boldsymbol{e}(\boldsymbol{a}_N, t_N) = \boldsymbol{0} \quad (26)$$

and control inputs saturation constraints

$$\boldsymbol{s}(\boldsymbol{a}_k, \boldsymbol{b}_k, t_k, T) \leq \boldsymbol{0} \ , \ k = 0,1,...,N \quad (27)$$

Thus, the optimal motion planning problem can be stated finally as: determine two sets of coefficients $\boldsymbol{\alpha} := (\boldsymbol{a}_0, \boldsymbol{a}_1, ..., \boldsymbol{a}_N), \boldsymbol{\beta} := (\boldsymbol{b}_0, \boldsymbol{b}_1, ..., \boldsymbol{b}_N)$, which maximize or minimize the discretized cost function

$$J[\boldsymbol{\alpha}, \boldsymbol{\beta}] := h(\boldsymbol{a}_N, T) + \sum_{k=0}^{N} g(\boldsymbol{a}_k, \boldsymbol{b}_k, t_k, T) w_k \quad (28)$$

subject to constraints (25)-(27).

Concreting to the problem in this paper, fuel consumption optimal motion planning of an underactuated spacecraft can be described as: for given initial and final states $\boldsymbol{x}(0), \boldsymbol{x}(T)$, determine two sets of unknown coefficients $\boldsymbol{\alpha} := (\boldsymbol{a}_0, \boldsymbol{a}_1, ..., \boldsymbol{a}_N)$, $\boldsymbol{\beta} := (\boldsymbol{b}_0, \boldsymbol{b}_1, ..., \boldsymbol{b}_N)$, which minimize the discretized cost function

$$J[\boldsymbol{\alpha}, \boldsymbol{\beta}] = \frac{1}{2} \sum_{k=0}^{N} (b^2{}_{1k} + b^2{}_{2k}) w_k \quad (29)$$

subject to the discretized dynamics of the rigid body

$$\begin{aligned} &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{1l}) - b_{1k} = 0 \\ &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{2l}) - b_{2k} = 0 \\ &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{3l}) - \alpha a_{1k} a_{2k} = 0 \\ &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{4l}) - a_{1k} - (a_{2k} \sin a_{4k} + a_{3k} \cos a_{4k}) \tan a_{5k} = 0 \\ &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{5l}) - a_{2k} \cos a_{4k} + a_{3k} \sin a_{4k} = 0 \\ &\frac{2}{T}(\sum_{l=0}^{N} \boldsymbol{D}_{kl} a_{6l}) - (a_{2k} \sin a_{4k} + a_{3k} \cos a_{4k}) \sec a_{5k} = 0 \\ &k = 0,1,...,N \end{aligned} \quad (30)$$

Additionally the control input saturation constraints are transformed into discretized inequality constraints

$$|b_{ik}| \leq U_M \ , \ i = 1,2 \ , \ k = 0,1,...,N \quad (31)$$

where, $U_M > 0$ is the control input upper limit.

Hence, it can be seen that pseudospectral method can convert the optimal fuel consumption motion planning problem into a nonlinear programming problem with equality and inequality constraints by globally approximating the state and optimal control space of the spacecraft, then the existing and well-developed nonlinear programming approaches can be employed.

IV. Numerical Examples

In this section we use numerical examples to illustrate the approach proposed above. Firstly, convergence accuracy of pseudospectral method is introduced as

$$\varepsilon := \| \boldsymbol{x}^N(t) - \boldsymbol{x}^*(t) \|_\infty = \max_{-1 \le t \le 1} | \boldsymbol{x}^N(t) - \boldsymbol{x}^*(t) |$$

where $x^*(t)$ is the so-obtained solution by numerical integration of differential equations of system $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}^N(t), t)$ with the given initial and terminal conditions.

In the following simulation, we choose the number of LGL points as $N = 15$, and other simulation conditions are as follows:

principle moments of inertia

$$\boldsymbol{I} = \mathrm{diag}[55.3\ ,\ 51.5\ ,\ 41.8]\ \mathrm{kg \cdot m^2}$$

initial and terminal constraints conditions

$$x_0 = [0, -\pi/4, 0] \quad , \quad x_T = [0, 0, \pi/6]$$

and the upper limit of control input is $U_M = 0.5\ \mathrm{N \cdot m}$. Simulation results are shown in Fig. 1-3. Fig. 1 shows the optimal fuel consumption trajectories of the underactuated spacecraft from initial point $\boldsymbol{x}_0$ to final point $\boldsymbol{x}_T$. And Fig. 2 shows the corresponding control input history which satisfies the control input constraints well. In Fig. 1 there are actually plotted two separate sets of trajectories, the solid line is planned directly by pseudospectral method and nonlinear programming method, i.e., from (29)-(31); and the other dashed line is generated from the dynamical equations (3) and (4) subject to the control inputs $\boldsymbol{u}^N(t)$ history in Fig. 2, the two sets are almost coincident, so there is no visible discrepancy in Fig. 1. In Fig. 3 the absolute value of the error $| \boldsymbol{x}^N(t) - \boldsymbol{x}^*(t) |$ for each $N+1$ LGL nodes is plotted and the approximation accuracy of Legendre pseudospectral is found to be less than 10^{-5}. The execution time of the whole optimization process is 3.86s, and the resulting value of cost function $J = 6.98522 \times 10^{-3}$.

Euler angles error variables are defined separately as $\phi_e = \varepsilon_\phi = \| \phi^N(t) - \phi^*(t) \|_\infty$, $\theta_e = \varepsilon_\theta = \| \theta^N(t) - \theta^*(t) \|_\infty$, $\psi_e = \varepsilon_\psi = \| \psi^N(t) - \psi^*(t) \|_\infty$ and T_c is the optimization time, Legendre pseudospectral method is solved repeatedly while the number of LGL nodes is increased from 5 to 25. Fig. 4 shows the maximum absolute errors $\varepsilon = \| \boldsymbol{x}^N(t) - \boldsymbol{x}^*(t) \|_\infty$ over all nodes. Additionally, let $N = 5, 10, 15, 20, 25$, the simulation results with Legendre pseudospectral method are reported in Table 1. As seen in Fig.4 and Table 1 that with Legendre pseudospectral method, the increasing number of LGL nodes indeed improves the optimal performance index and convergence rates of the state variables. However, it is also obvious that once the number of LGL points reaches some large enough value, or the approximation errors drop below a tolerance (such as 10^{-5} in Fig. 4), the approximation accuracy will stop improving and oscillate in a very small range, and the performance index will approach a constant without obvious improvement. Further, the program increasing approximation accuracy also comes at a significant computational burden, which makes Legendre pseudospectral method applied on-line in practice impossible.

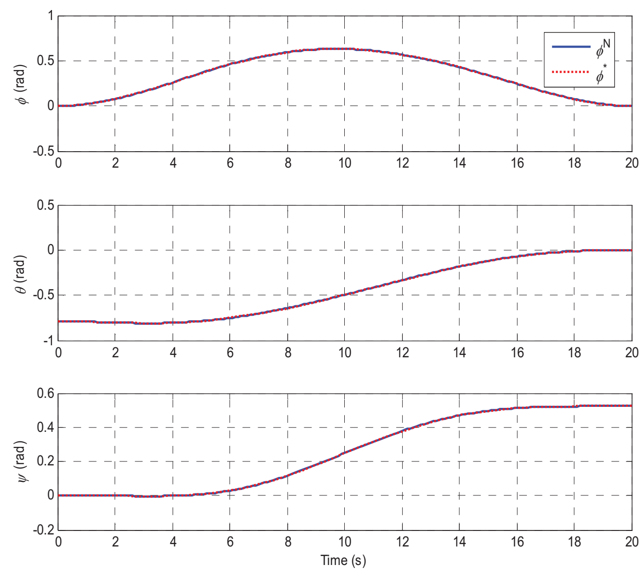

Fig. 1 Optimal Attitude Trajectory

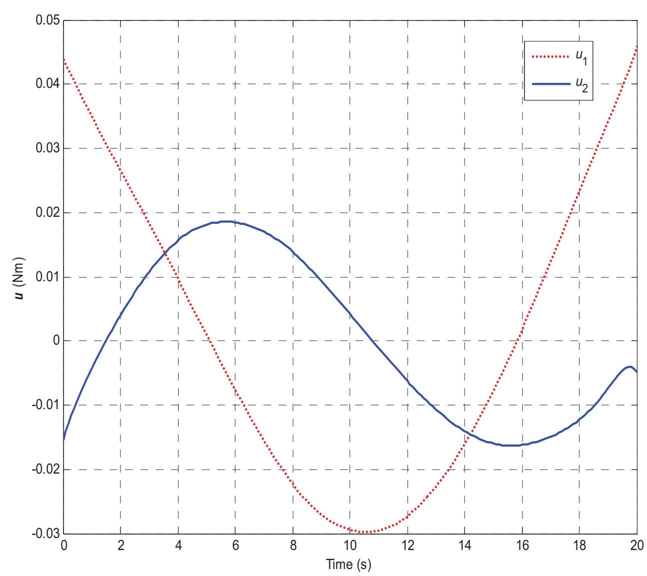

Fig. 2 Control Commands

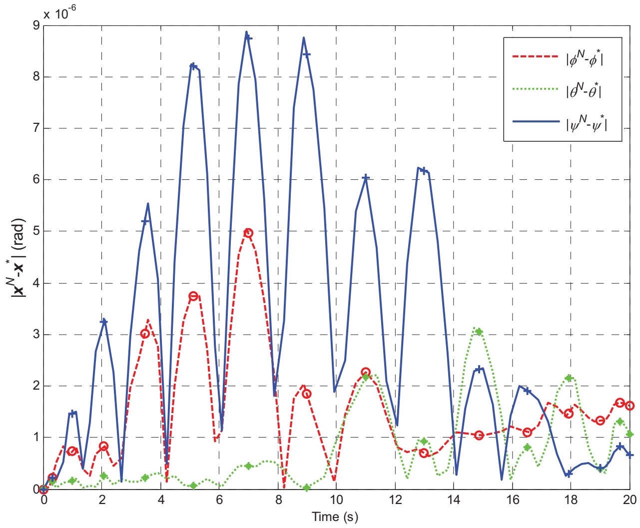

Fig. 3 State Errors

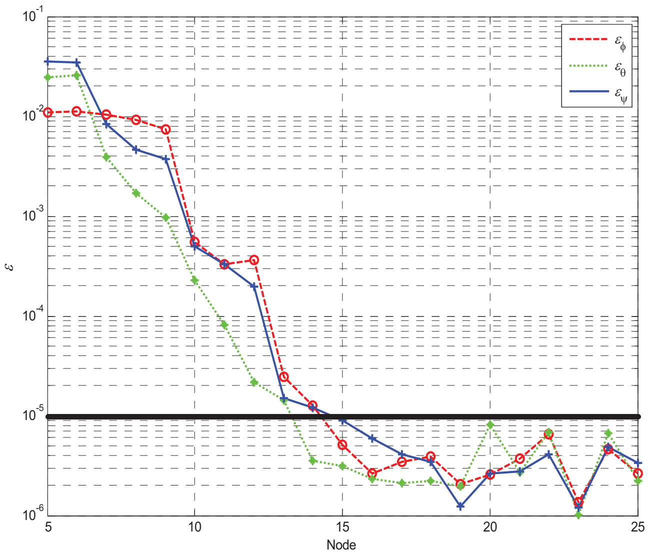

Fig. 4 Convergence of State Errors

TABLE I
OPTIMAL MOTION PLANNING WITH LEGENDRE PSEUDOSPECTRAL METHOD

N	J	ϕ_e (rad)	θ_e (rad)	ψ_e (rad)	T_c (s)
5	1.79602×10^{-2}	1.2×10^{-2}	4.4×10^{-3}	2.1×10^{-2}	3.29
10	6.99174×10^{-3}	3.4×10^{-4}	1.5×10^{-4}	2.9×10^{-4}	3.48
15	6.98522×10^{-3}	5.0×10^{-6}	3.1×10^{-6}	8.9×10^{-6}	3.86
20	6.98463×10^{-3}	2.6×10^{-6}	8.2×10^{-6}	2.7×10^{-6}	8.15
25	6.98420×10^{-3}	2.6×10^{-6}	2.2×10^{-6}	3.4×10^{-6}	14.46

V. CONCLUSIONS

In this paper, we settle the optimal motion planning problem of a rigid underactuated spacecraft using Legendre pseudospectral method. We select the fuel consumption as the cost function for given initial and terminal states, and transform the control input saturation constraints to explicit inequality restrictions. Then the system and constraints are discretized at a set of discretization points using Legendre pseudospectral method, and the optimal motion planning problem can be converted into a nonlinear programming problem, which can be solved by existing nonlinear programming and matrix analysis methods. The simulation results demonstrate that the optimal trajectories obtained by this approach can satisfy the constraints well. Additionally, since this approach approximates the state and control space with global interpolating polynomial basis functions, the approximation accuracy is high. And with its small computational amount and low computational complexity, it also can be used on-line in real space applications.

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