Minimum Energy, Reaction Wheel Based, Satellite Attitude Control: A Comparison of Cost Functions

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Nomenclature

$\mathbf{u}$	Reaction wheel input torque vector
$\boldsymbol{\omega}$	Reaction wheel speed vector
$\boldsymbol{\omega_B}$	Body angular velocity expressed in body frame
$\mathbf{q}$	Unit quaternion representing body frame orientation
m	Mass, kg
Δx	Variable displacement vector
α	Acceleration, m/s^2

Subscript

i	Variable number

I. Introduction

CubeSats operate under severe energy constraints, since their only source of energy is a small covering of solar panels. High performance attitude controllers can increase the capabilities of these small spacecraft, but can use a significant fraction of the energy budget. As such, it is valuable to minimize the energy cost of small satellite attitude maneuvers. We use pseudospectral optimal control methods to compute energy optimal, large angle slew trajectories for a 1U CubeSat using a reaction-wheel-based attitude controller.

A. Hardware Background

Reaction-wheel attitude control is based on the principle of momentum exchange between the wheels and the satellite body. When a torque is exerted on a reaction wheel by a motor (usually a brush-less DC motor), the opposite torque is exerted on the satellite body. Three reaction wheels, arranged orthogonally, allow for control in three degrees. This is a popular attitude control method because it does not require any fuel, only electrical energy which can be generated by solar panels.

II. Problem Description

We are concerned with the computation of a control trajectory for a large angle slew maneuver which minimizes the total electrical energy consumed by the brush-less DC motors that provide actuation torque.

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The torques exerted by the motors on the wheels are the control variables.We consider three sources of energy consumption. The first are frictional losses, which are proportional to wheel speed squared. The second are resistive losses. The torque produced by an electric motor is proportional to the current flowing through the armature, and energy is dissipated as heat through the resistance of the armature. These losses are proportional to the torque exerted by the motor squared. The third source of energy loss arises when the motor driver is not capable of regenerative braking. This means that any electrical energy converted into mechanical energy by the motor cannot be converted back into electrical energy, and must be dissipated as heat when the wheel slows down. We will hereafter refer to this energy as unrecoverable mechanical energy.

III. Methods

We use Legendre Pseudospectral methods to compute an state-control trajectory pair that minimizes a cost function subject to dynamic constraints, as implemented by the software package DIDO(citation). A similar approach is used in (citations), but we consider a wider variety of cost functions than these papers, which only minimize resistive losses.

The optimal control solution has an open loop form, and thus requires a closed closed loop controller to track it, such as in Karpenko.[1]

IV. Optimal Control Problem Formulation

The optimal control problem is formulated as follows:

$$\begin{aligned}\text{Minimize} \quad J = \text{Subject to the constraints :} \quad \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{x}(t_0) &= \mathbf{x_0} \\ \mathbf{x}(t_f) &= \mathbf{x_f} \\ \mathbf{u_L} \leq \mathbf{u} &\geq \mathbf{u_U} \\ \mathbf{x_L} \leq \mathbf{x} &\geq \mathbf{x_U} \\ t_0 \text{ and } t_f & \text{ fixed}\end{aligned}$$

The dynamic constraints, $\mathbf{f}(\mathbf{x}, \mathbf{u})$, can be found in the appendix.

The cost function J takes on one of the following forms, depending on the optimization being performed.

$$\text{To minimize frictional losses:} \qquad J = FL = \int_{t_0}^{t_f} \sum_{i=1}^{3} \omega_i^2 \, dt$$

$$\text{To minimize resistive losses:} \qquad J = RL = \int_{t_0}^{t_f} \sum_{i=1}^{3} u_i^2 \, dt$$

$$\text{To minimize unrecoverable mechanical energy:} \qquad J = UM = \int_{t_0}^{t_f} \sum_{i=1}^{3} F_i \, dt$$

$$F_i = \begin{cases} \omega_i u_i, & \text{if } \omega_i u_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

A. Cost Function Practical Considerations

When performing numerical optimization, it is desirable for the cost function to be continuous and smooth. Cost functions FL and RL satisfy these criteria, while cost function UM is non-smooth at zero. Attempting a numerical optimization using UM as the cost function does not yield a satisfying (i.e. feasible). To improve performance, we define an approximate cost function, $\hat{UM}$which is smooth on the interval $(-\infty, +\infty)$.

$$\hat{UM} = \int_{t_0}^{t_f} \sum_{i=1}^{3} \frac{1}{\alpha} ln(1 + e^{\alpha \omega_i u_i})\, dt, \qquad \alpha > 0$$

As the scaling factor, α, gets large, $\hat{UM}$ becomes arbitrarily close to UM, as illustrated in figure 1.

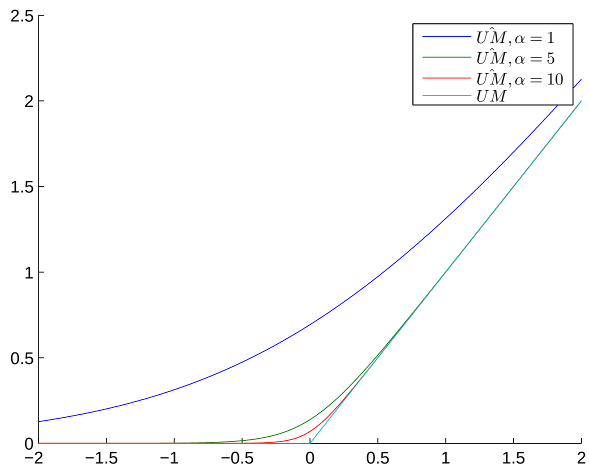

Figure 1. Jhat approaches J as alpha gets larger

V. Results

We present control trajectories for a large angle slew of a 1U CubeSat, whose parameters can be found in the appendix. Table V provides a comparison of the performance of the three numerical solutions with respect to the three cost functions of interest.

	UM	FL	RL
$\hat{UM}$	1.00	1.01	5.10
FL	1.23	1.00	11.30
RL	2.09	1.29	1.00

Table 1. Normalized performance of numerical solutions with respect to different cost functions

A. Discussion

VI. Conclusion

The solutions found w.r.t cost functions FL and $\hat{UM}$ are the nearly the same and have Bang-Off-Bang form. On the other hand, the control which optimizes RL is continuous and smooth, making it apparent why the RL cost function is a popular choice for optimization. It also performs reasonably well with respect to FL. However, it performs poorly with respect to UM, demonstrating that in the absence of regenerative braking, optimizing with respect to RL may not yield a truly energy optimal solution. A better approach would be to optimize a cost function which is a weighted sum of the three, the relative weights being dependent on the specific system at hand. In the full paper, we find a solution of this nature for a commercially available CubeSat attitude controller.

Appendix

Dynamics as derived by Karpenko[1]

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}$$

References

[1] Mark Karpenko, Sagar Bhatt, Nazareth Bedrossian, and I. Michael Ross. Flight Implementation of Shortest-Time Maneuvers for Imaging Satellites. *Journal of Guidance, Control, and Dynamics*, 37(4):1069–1079, 2014.