Minimum Energy, Reaction Wheel Based, Satellite Attitude Control: A Comparison of Cost Functions

Nanosatellites, such as CubeSats, must operate under extreme energy constraints, since their only source of energy is their small covering of solar panels. High performance attitude controllers can significantly increase the capabilities of these small spacecraft, but can use a significant fraction of the energy budget. As such, it is critical to minimize the energy cost of small satellite attitude maneuvers. We use pseudospectral optimal control methods to compute energy optimal, large angle slew trajectories for a 1U CubeSat using a reaction wheel based attitude controller.

Reaction wheel minimal energy studies studies tend to fall into two categories. In the first, a non-optimal, closed loop method is used to compute an overall control torque, and then an energy optimal allocation is found [1], [2]. The other type of energy optimization is concerned with finding an entire control trajectory which minimizes some cost function [3], [4]. This study is of the latter kind. It is common practice to use the square of the control effort (i.e. reaction wheel torque) as a cost function because this function has nice properties for optimization, namely continuity and smoothness. However, finding an optimal trajectory with respect to this cost function may not actually result in minimized energy usage, as shown in this paper.

To define an appropriate cost function, it is critical to examine the hardware at hand. Most CubeSat reaction wheel systems are driven by brushless DC (BLDC) motors *without* regenerative braking capabilities[5]. This means that once energy is invested in spinning up the wheel, there is no way to recover it, and it is dissipated as heat once the wheel is slowed down again. We will call this the work energy loss. In an efficient motor, this is the major source of energy loss. For one wheel, the cost can be written as:

$$Work\ Energy\ Loss = \int_{t_0}^{t_f} F\,dt$$

$$where:$$

$$F = \begin{cases} vT & vT > 0 \\ 0 & otherwise \end{cases}$$

$$v = velocity\ of\ reaction\ wheel\ w.r.t\ satellite\ body$$
$$T = torque\ applied\ by\ motor$$

This cost function is equal to the total work energy input of each wheel. Because F is equal to zero when the product of torque and velocity are negative, the cost function has no penalty for removing energy from the system (i.e. slowing down the reaction wheel). The cost function above is for one wheel: the total cost is obtained by adding the individual wheel costs together.

Because the work energy loss function is non-smooth at zero, it is not well suited for numerical optimization. Instead, we define an alternate, smooth cost function to approximate the desired one:

$$Approximated\ Work\ Energy\ Loss = \int_{t_0}^{t_f} \frac{1}{\alpha} \ln\left(1 + e^{\alpha v T}\right)$$

As the scaling factor, α, gets large, this cost function becomes arbitrarily close to the true work energy loss. By using this cost function, we are able to numerically compute the minimum energy solution. Other sources of losses in the motor include frictional losses, which are proportional to the square of wheel velocity, and resistive losses, which are proportional to the square of the torque. For comparison, we also compute solutions which minimize these losses.

Solution	Work Energy Cost	Frictional Cost	Resistive Cost
Min Approx. Work Energy Loss	1	1.01	5.10
Min Frictional Loss	1.23	1	11.30
Min Resistive Loss	2.09	1.29	1

The table above summarizes the findings. It shows how numerical solutions obtained using the different cost functions perform with respect to the other cost functions. The approximate work energy loss and frictional solutions are quite similar, indicating that the same trajectory is optimal with respect to both cost functions. However, they both incur much higher resistive losses than the resistive loss optimal solution. On the other hand, the resistive loss minimizing solution is significantly more costly in terms of work energy than the other two.

To truly minimize energy consumption, optimization must be made with respect to a cost function which incorporates all three of these costs. The relative weights of each will be dependent on the motor at hand.

References

[1] R. Blenden and H. Schaub, “Regenerative Power-Optimal Reaction Wheel Attitude Control,” *J. Guid. Control. Dyn.*, vol. 35, no. 4, pp. 1208–1217, 2012.

[2] R. Wisniewski and P. Kulczycki, “Slew maneuver control for spacecraft equipped with star camera and reaction wheels,” *Control Eng. Pract.*, vol. 13, no. 3, pp. 349–356, 2005.

[3] U. Lee and M. Mesbahi, “AAS 13-836 QUATERNION BASED OPTIMAL SPACECRAFT REORIENTATION,” pp. 1–16.

[4] G. M. G. Ma, Y. Z. Y. Zhuang, C. L. C. Li, and H. H. H. Huang, “Pseudospectral method for optimal motion planning of a rigid underactuated spacecraft,” *Control Autom. (ICCA), 2010 8th IEEE Int. Conf.*, pp. 684–688, 2010.

[5] Blue Canyon Tech, “Micro Reaction Wheel Datasheet” [Online]. Available: http://bluecanyontech.com/wp-content/uploads/2015/06/Micro-RW-Data-Sheet_2.0.pdf.