Minimum Energy, Reaction Wheel Based, Satellite Attitude Control: A Comparison of Cost Functions

Nanosatellites, such as CubeSats, must operate under extreme energy constraints, since their only source of energy is their small covering of solar panels. High performance attitude controllers can significantly increase the capabilities of these small spacecraft, but can use a significant fraction of the energy budget. As such, it is critical to minimize the energy cost of small satellite attitude maneuvers. We use pseudospectral optimal control methods to compute energy optimal, large angle slew trajectories for a 1U CubeSat using a reaction wheel based attitude controller.

Reaction wheel minimal energy studies studies tend to fall into two categories. In the first, a non-optimal, closed loop method is used to compute an overall control torque, and then an energy optimal allocation is found [1], [2]. The other type of energy optimization is concerned with finding an entire control trajectory which minimizes some cost function [3], [4]. This study is of the latter kind. It is common practice to use the square of the control effort (i.e. reaction wheel torque) as a cost function because this function has nice properties for optimization, namely continuity and smoothness. However, finding an optimal trajectory with respect to this cost function may not actually result in minimized energy usage, as shown in this paper.

To define an appropriate cost function, it is critical to examine the hardware at hand. Most CubeSat reaction wheel systems are driven by brushless DC (BLDC) motors *without* regenerative braking capabilities[5]. This means that once energy is invested in spinning up the wheel, there is no way to recover it, and it is dissipated as heat once the wheel is slowed down again. We will call this the work energy loss. In an efficient motor, this is the major source of energy loss. For one wheel, the cost can be written as:

$$Work\ Energy\ Loss = \int_{t_0}^{t_f} F dt$$

$$where:$$

$$F = \begin{cases} vT & vT > 0 \\ 0 & otherwise \end{cases}$$

$$v = velocity\ of\ reaction\ wheel\ w.r.t\ satellite\ body$$
$$T = torque\ applied\ by\ motor$$

Slowing down the wheel ($vT < 0$) does not use up any of the energy stored in the battery, so it is not penalized the above cost function. This cost function is not well suited for numerical optimization because it is non-smooth at zero. Two other important sources of losses are resistive losses and frictional losses. These two are proportional to $\int T^2$ and $\int v^2$, respectively. Because these cost functions are continuous and smooth, they are well suited for numerical optimization. In this paper, we find solutions which are optimal with respect to each of these cost functions.

The work energy loss function cannot be solved directly due to its non-smoothness at zero. To find a solution, the work energy loss function can be approximated with the smooth function below:

$$Approximated\ Work\ Energy\ Loss = \int_{t_0}^{t_f} \frac{1}{\alpha} \ln\left(1 + e^{\alpha v T}\right)$$

As the scaling factor, α, gets large, this cost function becomes arbitrarily close to the true work energy loss.

The table below summarizes the findings. The leftmost column states the cost function which was used in the numerical optimization. The next three columns show how the solution performs with respect to all three cost functions.

Solution	**Work Energy Cost**	**Frictional Cost**	**Resistive Cost**
Min Approx. Work Energy Loss	1	1.01	5.10
Min Frictional Loss	1.23	1	11.30
Min Resistive Loss	2.09	1.29	1

It is apparent that the same solution minimizes work energy losses and frictional costs (it is obvious upon examination of the trajectories that they are the same). The slight differences arise from numerical imprecision. The resistive cost minimal solution, on the other hand, incurs significantly higher work energy losses. Also, the work energy loss optimal solution performs very poorly with respect to resistive losses.

To truly minimize energy consumption, optimization must be made with respect to a cost function which is a weighted sum of all three costs. The relative weights of each will be dependent on the motor at hand.

References

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