

Point Rotation Formula Proof

Daniel E. Janusch

January 9, 2026

atan2 argument order: $\tan^{-1}(y, x)$

goal: rotate (a, b) θ units around the point (h, k)

Proof:

$$A_1 := (a, b)$$

$$B_1 := (a, k)$$

$$C_1 := (h, k)$$

shift everything by $(-h, -k)$

$$A_2 := (a - h, b - k)$$

$$B_2 := (a - h, 0)$$

$$C_2 := (0, 0)$$

$$m\angle A_1 B_1 C_1 = m\angle A_2 B_2 C_2$$

$t := \tan^{-1}(b - k, a - h)$ (the angle that (a, b) would have if (h, k) was the origin)

$\theta_2 := t + \theta$ (the new angle)

$(\cos \theta_2, \sin \theta_2)$ has the correct angle, with the distance scaled to 1

$$d := \sqrt{(b - k)^2 + (a - h)^2}$$
 (the distance between (a, b) and (h, k))

the point would then be $(d \cos \theta_2, d \sin \theta_2)$

after shifting by (h, k) to counteract the shift from before it becomes $(h + d \cos \theta_2, k + d \sin \theta_2)$

$$= \left(h + \cos(\tan^{-1}(b - k, a - h) + \theta) \sqrt{(b - k)^2 + (a - h)^2}, k + \sin(\tan^{-1}(b - k, a - h) + \theta) \sqrt{(b - k)^2 + (a - h)^2} \right)$$

■