Point Rotation Formula Proof

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atan
2 argument order: $\tan^{-1}(y,x)$ goal: rotate (a,b)
 θ units around the point (h,k)

Proof:

$$A_1 := (a, b)$$

$$B_1 := (a, k)$$

$$C_1 := (h, k)$$

shift everything by (-h, -k)

$$A_2 := (a - h, b - k)$$

$$B_2 := (a - h, 0)$$

$$C_2 := (0,0)$$

 $m \angle A_1 B_1 C_1 = m \angle A_2 B_2 C_2$

 $t := \tan^{-1}(b-k, a-h)$ (the angle that (a,b) would have if (h,k) was the origin)

 $\theta_2 := t + \theta$ (the new angle)

 $(\cos \theta_2, \sin \theta_2)$ has the correct angle, with the distance scaled to 1

$$d := \sqrt{\left(b-k\right)^2 + \left(a-h\right)^2}$$
 (the distance between (a,b) and (h,k))

the point would then be $(d\cos\theta_2, d\sin\theta_2)$

after shifting by (h, k) to counteract the shift from before it becomes $(h + d\cos\theta_2, k + d\sin\theta_2)$

$$= \left(h + \cos \left(\tan^{-1} (b - k, a - h) + \theta \right) \sqrt{(b - k)^2 + (a - h)^2}, \\ k + \sin \left(\tan^{-1} (b - k, a - h) + \theta \right) \sqrt{(b - k)^2 + (a - h)^2} \right)$$

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