PS 250 Final Exam Formula Sheet Plan

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NOTE: The LaTeX source has more context in comments for some formulas

1 Constants

$$\varepsilon_0 = 8.8541878188\text{E}{-12} \qquad \left[\frac{\text{F}}{\text{m}}\right] \tag{1}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\text{E}{+9} \qquad \left[\frac{\text{m}}{\text{F}}\right] \tag{2}$$

$$e = 1.602176634\text{E}{-19} \qquad [\text{C}] \tag{3}$$

$$m_e = 9.10938356\text{E}{-31} \qquad [\text{kg}] \tag{4}$$

$$m_p = 1.672621898\text{E}{-27} \qquad [\text{kg}] \tag{5}$$

$$m_n = 1.674927471\text{E}{-27} \qquad [\text{kg}] \tag{6}$$

$$\mu_0 = 4\pi \times 10^{-7} \qquad \left[\frac{\text{T m}}{\text{A}}\right] \tag{7}$$

$$c = 299\ 792\ 458 \qquad \left[\frac{\text{m}}{\text{s}}\right] \tag{8}$$

$$h = 2\pi\hbar = 6.62607015\text{E}{-34} \quad [\text{J s}] \tag{9}$$

$$\mu_B = 9.27401022241938\text{E}{-24} \qquad \left[\frac{\text{J}}{\text{T}}\right] \tag{10}$$

2 Units

$$\text{exponents (negative): } c, m, \mu, n, p = 2, 3, 6, 9, 12$$

$$\mathrm{C} = \mathrm{A\ s} \tag{1}$$

$$\mathrm{V} = \mathrm{A}\ \Omega = \frac{\mathrm{W}}{\mathrm{A}} = \frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{kg\ m^2}}{\mathrm{C\ s^2}} \tag{2}$$

$$\mathrm{N} = \frac{\mathrm{kg\ m}}{\mathrm{s^2}} \tag{3}$$

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}} \tag{4}$$

$$\mathrm{J} = \mathrm{N\ m} = \mathrm{V\ C} = \frac{\mathrm{kg\ m^2}}{\mathrm{s^2}} \tag{5}$$

$$\mathrm{eV} = |e|\ \mathrm{V} \tag{6}$$

$$\mathrm{F} = \frac{\mathrm{C}}{\mathrm{V}} = \frac{\mathrm{s^2\ C^2}}{\mathrm{kg\ m^2}} = \frac{\mathrm{C^2}}{\mathrm{J}} = \frac{\mathrm{s}}{\Omega} \tag{7}$$

$$\mathrm{T} = \frac{\mathrm{N}}{\mathrm{A\ m}} = \frac{\mathrm{kg}}{\mathrm{C\ s}} = 10^4\,\mathrm{G} \tag{8}$$

$$\mathrm{Wb} = \mathrm{V\ s} = \mathrm{T\ m^2} = \Omega\ \mathrm{C} = \frac{\mathrm{J}}{\mathrm{A}} \tag{9}$$

$$\mathrm{H} = \Omega\ \mathrm{s} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{s^2}}{\mathrm{F}} = \frac{\Omega\ \mathrm{s}}{\mathrm{rad}} \tag{10}$$

$$\frac{\mathrm{T\ m}}{\mathrm{A}} = \frac{\mathrm{H}}{\mathrm{m}} = \frac{\mathrm{N}}{\mathrm{A^2}} \tag{11}$$

$$\frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{N}}{\mathrm{C}} \tag{12}$$

3 Realistic Value Ranges

range	unit
$I \sim [10^{-4}, 10]$	A
$v_e \sim 10^6$	m/s
$v_d \sim 10^{-4}$	m/s
$n \sim 10^{28}$	m^{-3}
$C \sim [10^{-12}, 10^{-3}]$	F
$R \sim [1, 10^3]$	Ω
$V \sim [1, 200]$	V
$P \sim [10^{-3}, 10^4]$	W
$\rho \sim [10^{-9}, 10^{-6}]$	Ω m
$E_c \sim [10^3, 10^7]$	V/m
$E_w \sim [10^{-3}, 10]$	V/m
$R_w \sim [10^{-3}, 10^{-2}]$	Ω
$B \sim [10^{-6}, 10]$	T
$F_E \sim [10^{-12}, 10^2]$	N
$F_B \sim [10^{-12}, 10]$	N
$\Phi_E \sim [10^{-12}, 10]$	V m
$\Phi_B \sim [10^{-8}, 1]$	Wb
$\varepsilon/\varepsilon_0 \sim [1, 10]$	none
$\mu/\mu_0 \sim [1, 10^3]$	none
$N \sim [1, 10^3]$	none
$L \sim [10^{-9}, 10^2]$	H
$M \sim [10^{-9}, 10^{-3}]$	H

4 Resistivity Table

material	resistivity
silver	1.6E−8
copper	1.7E−8
gold	2.2E−8
aluminum	2.6E−8
iron	9.7E−8

5 Electric Field (E, F_E, Φ_E), Energy, Work, Voltage

5.1 General Formulas

$$F_E = k\frac{|q_1 q_2|}{r^2} \tag{1}$$

$$E = \frac{F}{q_t} = k\frac{|Q|}{r^2} = -\nabla V = -\frac{\mathrm{d}V}{\mathrm{d}x} \tag{2}$$

$$W = -\Delta U = q_t \Delta V = \int \mathbf{F} \cdot \mathrm{d}\mathbf{r} \tag{3}$$

$$W = q_t E d \quad \text{(uniform field. } d \text{ is the displacement)} \tag{4}$$

$$U = k\frac{q_1 q_2}{r} \tag{5}$$

$$U_{\text{sys}} = k\sum_{i<j} \frac{q_i q_j}{r_{i,j}} \tag{6}$$

$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \tag{7}$$

$$V = \sum k\frac{Q_i}{r_i} = \frac{U}{q_t} = \int \frac{\mathrm{d}q}{r} \tag{8}$$

$$E_{\text{tot}} = \sum_i E_i \tag{9}$$

5.2 Extended Objects

5.2.1 Voltage

$$\text{Conducting Cylinder radius } R\text{: } V = 2k\lambda \ln\frac{R}{d} \text{ if } d > R \text{ else } 0 \tag{1}$$

$$\text{Ring: } V = \frac{kQ}{\sqrt{d^2 + R^2}} \tag{2}$$

5.2.2 Electric Field

$$E = 0 \text{ inside a conducting region.}$$

$$\text{Line: } E = \frac{k\lambda}{r}\left[\frac{a}{\sqrt{a^2+r^2}} + \frac{b}{\sqrt{b^2+r^2}}\right] \approx \frac{2k\lambda}{r} \tag{1}$$

$$\text{Disk/Plate: } E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{d}{\sqrt{d^2+R^2}}\right] \approx \frac{\sigma}{2\varepsilon_0} = 2\pi k\sigma \tag{2}$$

$$\text{Ring: } E = \frac{kQd}{(d^2+R^2)^{3/2}}, Q = 2\pi R\lambda \tag{3}$$

$$\text{radius } R \text{ Sphere: } E = k\frac{Q}{r^2}, \text{ if } r \geq R \tag{4}$$

$$\text{inside radius } R \text{ insulating Sphere: } E = \frac{kQr}{R^3} \tag{5}$$

$$\text{radius } R \text{ cylinder: } E = \frac{2k\lambda}{r}, \text{ if } r \geq R \tag{6}$$

$$\text{inside radius } R \text{ insulating cylinder: } E = \frac{2k\lambda r}{R^2} \tag{7}$$

$$\text{equal, opposite plates: } E = \frac{\sigma}{\varepsilon_0} \text{ inside, else } 0 \tag{8}$$

6 Magnetism

6.1 General Formulas

$$\vec{F} = F_E + F_B = q(\vec{E} + \vec{v} \times \vec{B}) \implies |\vec{F}| = q(E + vB\sin\theta) \tag{1}$$

this next one assumes magnetic field is orthogonal to motion:

$$F_B = (\text{\# of charges})\left(\frac{\text{force}}{\text{charge}}\right) = (nA\ell)(qvB) = (nqv_dA)(\ell B) = I\ell B \tag{2}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} = I\vec{\ell} \times \vec{B} \tag{3}$$

$$\vec{\mu} = NI\vec{A} \;\; \text{(magnetic moment)} \tag{4}$$

$$\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B} \;\; \text{(torque)} \tag{5}$$

$$U = \int \tau \mathrm{d}\theta = -\vec{\mu} \cdot \vec{B} \;\; \text{(magnetic potential energy)} \tag{6}$$

$$\text{point charge: } B = 10^{-7}\frac{q\vec{v} \times \hat{r}}{r^2} \tag{7}$$

$$\text{point charge: } mv = |q|BR \tag{8}$$

$$\oint \vec{B} \cdot \mathrm{d}\ell = \mu_0 I_{\text{encl}} \tag{9}$$

$$\mathcal{E} = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -N\frac{\mathrm{d}(AB\cos\theta)}{\mathrm{d}t} = B\ell v\sin\theta = IR \tag{10}$$

$$\vec{B} = 10^{-7}\int \frac{I\mathrm{d}\ell \times \hat{r}}{r^2} \tag{11}$$

$$\Phi_B = \int B\cos\theta \mathrm{d}A = \int \vec{B} \cdot \mathrm{d}\vec{A} = AB\cos\theta \tag{12}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0 \tag{13}$$

$$\text{velocity selector: } F_E = F_B \implies v = \frac{E}{B} \tag{14}$$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \tag{15}$$

6.2 Ampere's Law Applications

magnetic field outside a solenoid is zero

$$\text{cylindrical solenoid: } B = \mu_0 n I = \mu_0 \frac{N}{\ell} I \tag{1}$$

$$\text{toroidal solenoid: } B = \frac{\mu_0}{2\pi} N \frac{I}{r}, r_{\text{min}} < r < r_{\text{max}} \tag{2}$$

$$\text{distance } r \text{ away from a radius-}R \text{ wire center: } B = \frac{\mu_0}{2\pi} \begin{cases} \frac{Ir}{R^2}, r < R \\ \frac{I}{r}, r \geq R \end{cases} \tag{3}$$

$$\text{torus of current at } a < r < b\text{: } B = \frac{\mu_0}{2\pi} \frac{I}{r} \begin{cases} 0, r < a \\ \frac{r^2 - a^2}{b^2 - a^2}, a < r < b \\ 1, b < r \end{cases} \tag{4}$$

7 General DC Circuit Formulas

$$V = \frac{Q}{C},\ I = C\frac{\mathrm{d}V}{\mathrm{d}t} \tag{1}$$

$$R = \frac{V}{I} = \frac{\rho\ell}{A} \tag{2}$$

$$L = \frac{N\Phi_B}{I} = \frac{ABN}{I} \tag{3}$$

$$M = \frac{N_2\Phi_{B2}}{I_1} = \frac{N_1\Phi_{B1}}{I_2} \tag{4}$$

$$\mathcal{E} = -L\frac{\mathrm{d}I}{\mathrm{d}t},\ \mathcal{E}_1 = -M\frac{\mathrm{d}I_2}{\mathrm{d}t},\ \mathcal{E}_2 = -M\frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{5}$$

equivalences	Series	Parallel
Capacitor	$[\sum 1/C_i]^{-1}$	$\sum C_i$
Resistor	$\sum R_i$	$[\sum 1/R_i]^{-1}$
Inductor	$\sum L_i$	$[\sum 1/L_i]^{-1}$

(6)

$$U_C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV,\ U_L = \frac{1}{2}LI^2 \tag{7}$$

$$u_C = \frac{\varepsilon_0}{2}\frac{V^2}{d^2} = \frac{\varepsilon_0}{2}E^2,\ u_L = \frac{B^2}{2\mu_0} \tag{8}$$

$$\text{dielectric: } C = KC_0,\ K = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0} \tag{9}$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = |q|nAv_d \tag{10}$$

$$J = \frac{I}{A} = \frac{E}{\rho}\ \ \text{(current density)} \tag{11}$$

$$P_R = VI = \frac{V^2}{R} = I^2R,\ P_L = IL\frac{\mathrm{d}I}{\mathrm{d}t} \tag{12}$$

$$I = L\frac{\mathrm{d}I}{\mathrm{d}t} \tag{13}$$

$$\text{parallel plate: } C = \frac{Q}{V} = \frac{\varepsilon A}{d}, E = \frac{V}{d} = \frac{Q}{Cd} = \frac{Q}{\varepsilon A}, d = \frac{\varepsilon A}{C} = \frac{\varepsilon AV}{Q} \tag{14}$$

$$\text{Junction/Current rule: } \sum I_{\text{in}} = \sum I_{\text{out}} \tag{15}$$

$$\text{Loop/Voltage rule: } \sum V = 0\ \ \text{(around a closed loop)} \tag{16}$$

charge is preserved in series, but split in sequence

$$\text{series: } Q_{\text{eq}} = C_{\text{eq}} V_{\text{eq}},\ V_i = \frac{C_{\text{eq}}}{C_i} V_{\text{eq}} \tag{17}$$

$$\text{parallel: } Q_i = C_i V_{\text{eq}} = \frac{C_i}{C_{\text{eq}}} Q_{\text{eq}},\ V_i = V_{\text{eq}} \tag{18}$$

$$\text{parallel, all equal: } Q_i = \frac{Q_{\text{eq}}}{n} \tag{19}$$

8 Inductors and Inductance Scenarios

8.1 Cylindrical Solenoid

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I \tag{1}$$

$$L = \frac{\mu_0 N^2 A}{\ell} \tag{2}$$

$$u_L = \frac{\mu_0 N^2 I^2}{2\ell^2} \tag{3}$$

8.2 Toroidal Solenoid

$$B = \frac{\mu_0}{2\pi} N \frac{I}{r}, r_{\text{min}} < r < r_{\text{max}} \tag{1}$$

$$L = \frac{\mu_0 N^2 A}{2\pi r_{\text{avg}}} \tag{2}$$

$$u_L = \frac{\mu_0 N^2 I^2}{8\pi^2 r^2} \tag{3}$$

8.3 Coaxial Cable

use Ampere's Law for B.

$$M = L = \frac{\mu_0}{2\pi} \ell \ln \frac{r_{\text{max}}}{r_{\text{min}}} \tag{1}$$

8.4 Adjacent Cylindrical Solenoids

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} \tag{1}$$

9 DC Transient Circuits

9.1 RC Circuit

$$\tau = RC \tag{1}$$

9.1.1 Charging RC Circuit

$$Q(t) = Q_f\left(1 - e^{-t/\tau}\right),\ Q_f = C\mathcal{E} \tag{1}$$

$$I(t) = I_0 e^{-t/\tau},\ I_0 = \frac{\mathcal{E}}{R} \tag{2}$$

$$V(t) = V_0\left(1 - e^{-t/\tau}\right),\ V_0 = \mathcal{E} \tag{3}$$

9.1.2 Discharging RC Circuit

$$Q(t) = Q_0 e^{-t/\tau} \tag{1}$$

$$I(t) = -\frac{Q_0}{\tau}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau} \tag{2}$$

$$V(t) = V_0 e^{-t/\tau} \tag{3}$$

9.2 LR Circuit

$$\tau = \frac{L}{R} \tag{1}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \qquad \text{(charging)} \tag{2}$$

$$I(t) = \frac{\mathcal{E}}{R}e^{-t/\tau} \qquad \text{(discharging)} \tag{3}$$

9.3 LC Circuit

$$\omega = \frac{1}{\sqrt{LC}} \tag{1}$$

$$Q(t) = Q_0 \cos \omega t \tag{2}$$

$$I(t) = -\omega Q_0 \sin \omega t \tag{3}$$

9.4 LRC Circuit

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{1}$$

For $R < 2\sqrt{\frac{L}{C}}$, $Q(t) = Q_0 \exp\left(-\frac{R}{2L}t\right)\cos\omega' t$. (under damped)
For $R = 2\sqrt{\frac{L}{C}}$, $Q(t) = Q_0 \exp\left(-\frac{R}{2L}t\right)$. (critically damped)
For $R > 2\sqrt{\frac{L}{C}}$, $Q(t) = Q_0 \exp\left(-\frac{R}{2L}t\right)\exp\left(-t\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)$. (over damped)

10 AC Circuits

$$X_L = \omega L \tag{2}$$

$$X_C = \frac{1}{\omega C} \tag{3}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{4}$$

$$X_L - X_C = R\tan\phi \tag{5}$$

$$V_{\text{RMS}} = \frac{V_{\max}}{\sqrt{2}},\ I_{\text{RMS}} = \frac{I_{\max}}{\sqrt{2}} \tag{6}$$

capacitor: I leads V, V lags I
resistor: in phase
inductor: I lags V, V leads I

$$P_{\text{avg}} = V_{\text{RMS}} I_{\text{RMS}} \cos\phi = \frac{1}{2} V_0 I_0 \cos\phi \tag{7}$$

$$V_L = I_0 X_L,\ V_R = I_0 R,\ V_C = I_0 X_C \tag{8}$$

$$V = IZ \tag{9}$$

$$\text{maximize power: } \omega = \frac{1}{\sqrt{LC}} \tag{10}$$

11 Hall Effect

idk man, just pray this is not on the exam.