MA 441 Formulas

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$$\langle A, B\rangle \equiv A \cdot B \tag{1}$$

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r} = \int_C M\mathrm{d}x + N\mathrm{d}y = \int_C \left(\vec{F} \cdot \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right) \mathrm{d}t \tag{2}$$

$$\int_{C_1+C_2} \vec{F} \cdot \mathrm{d}\vec{r} = \int_{C_1} \vec{F} \cdot \mathrm{d}\vec{r} + \int_{C_2} \vec{F} \cdot \mathrm{d}\vec{r} \tag{3}$$

$$\mathrm{d}\vec{r} = \langle \mathrm{d}x, \mathrm{d}y\rangle \quad \text{(in 2d)} \tag{4}$$

$$\mathrm{d}z = z_x \mathrm{d}x + z_y \mathrm{d}y \tag{5}$$

$$W = \int_C \nabla f \cdot \mathrm{d}\vec{r} = f(p_2) - f(p_1) \implies \oint_C \nabla f \cdot \mathrm{d}\vec{r} = 0 \tag{6}$$

$$N_x - M_y = 0 \iff \vec{F} = \nabla f \quad \text{(in 2d)} \tag{7}$$

$$\oint_C \vec{F} \cdot \mathrm{d}\vec{r} = \iint_D (N_x - M_y)\, \mathrm{d}A \tag{8}$$

$$\oint_{C_1} \nabla f \cdot \mathrm{d}\vec{r} = \oint_{C_2} \nabla f \cdot \mathrm{d}\vec{r} \tag{9}$$

$$f(x, y) = \int M\mathrm{d}x + \int N\mathrm{d}y \quad \text{(ignoring duplicate terms in the addition)} \tag{10}$$

$$\mathrm{curl}(\vec{F}) = \nabla \times \vec{F} = \vec{0} \iff \vec{F} = \nabla f \tag{11}$$

$$\text{Flux on } S = \iint \vec{F} \cdot \vec{n}\mathrm{d}s \tag{12}$$

$$\oiint_S \vec{F} \cdot \vec{n}\mathrm{d}s = \iiint_E \mathrm{div}(\vec{F})\mathrm{d}V = \iiint_E \nabla \cdot \vec{F}\mathrm{d}V \tag{13}$$

$$\vec{n}\mathrm{d}s = \pm \langle -f_x, -f_y, 1\rangle\, \mathrm{d}x\mathrm{d}y \ni z = f(x, y) \tag{14}$$

$$\vec{n}\mathrm{d}s = \frac{\vec{N}}{\vec{N} \cdot \hat{\imath}}\mathrm{d}y\mathrm{d}z = \frac{\vec{N}}{\vec{N} \cdot \hat{\jmath}}\mathrm{d}x\mathrm{d}z = \frac{\vec{N}}{\vec{N} \cdot \hat{k}}\mathrm{d}x\mathrm{d}y \tag{15}$$

$$\oint_C \vec{F}\cdot \mathrm{d}\vec{r} = \iint_S (\nabla \times \vec{F})\cdot \vec{n}\mathrm{d}s \tag{16}$$

$$u_t = k\left(u_{xx}+u_{yy}+u_{zz}\right) = -\mathrm{div}(\vec{F}) = \mathrm{div}(k\nabla u) \tag{17}$$

$$\int_C g(x,y,z)\mathrm{d}s = \int_C g(x,y,z)\frac{\mathrm{d}s}{\mathrm{d}t}\mathrm{d}t = \int_C g(x(t),y(t),z(t))\left|r'(t)\right|\mathrm{d}t \tag{18}$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = |r'(t)| = \sqrt{x'(t)^2+y'(t)^2+z'(t)^2} = \sqrt{r'(\theta)^2+r^2+z'(t)^2} \tag{19}$$

$$\iint_S g(x,y,z)\mathrm{d}s = \iint_D g(x,y,f(x,y))\sqrt{1+f_x^2+f_y^2}\,\mathrm{d}x\mathrm{d}y \tag{20}$$

$$\iint_S g(x,y,z)\mathrm{d}s = \iint_D g(x(u,v),y(u,v),z(u,v))\left|\vec{r}_u\times\vec{r}_v\right|\mathrm{d}u\mathrm{d}v \tag{21}$$

$$\iint_S g(x,y,z)\mathrm{d}s = \iint_D g(x,y,z)\frac{|\vec{N}|}{\vec{N}\cdot\hat{k}}\mathrm{d}x\mathrm{d}y \tag{22}$$

$$\text{vertices: } (x_0,0,0),(0,y_0,0),(0,0,z_0) \implies \vec{N} = \left\langle \frac{1}{x_0},\frac{1}{y_0},\frac{1}{z_0}\right\rangle \tag{23}$$

$$G(x,y,z)=0 \implies \vec{N} = \nabla G(x,y,z) \quad (G \text{ defines the surface}) \tag{24}$$

$$\hat{\imath}\times\hat{\jmath} = \hat{k} \qquad \hat{x}\times\hat{y} = \hat{z} \tag{25}$$

$$\hat{\jmath}\times\hat{k} = \hat{\imath} \qquad \hat{y}\times\hat{z} = \hat{x} \tag{26}$$

$$\hat{k}\times\hat{\imath} = \hat{\jmath} \qquad \hat{z}\times\hat{x} = \hat{y} \tag{27}$$

counterclockwise circle paramaterization: (cosine one) $\times$ (sine one) $=$ (missing one)
example: $\hat{x}\times\hat{y}=\hat{z} \implies x = r\cos\theta, y = r\sin\theta$
for clockwise, just switch them. (missing one) is the direction of the normal.

$$\int_C \mathrm{d}s = \text{length of curve } C \tag{28}$$

$$\iint_S \mathrm{d}s = \text{surface area of } S \tag{29}$$

$$\mathrm{d}s = r^2\sin\theta\mathrm{d}\theta\mathrm{d}\phi \quad \text{(on a sphere)} \tag{30}$$

$$\iint_S (x^n = y^n = z^n)\mathrm{d}s = \begin{cases} 0, n \text{ odd} \\ \dfrac{4\pi r^{n+2}}{n+1}, n \text{ even}\end{cases} \qquad S: x^2+y^2+z^2=r^2 \tag{31}$$

$$\iint_S z^n \mathrm{d}s = \frac{2\pi r^{n+2}}{n+1} \quad S : x^2 + y^2 + z^2 = r^2, z \geq 0 \tag{32}$$

$$\iint_S (x^n = y^n) \mathrm{d}s = \begin{cases} 0, n \text{ odd} \\ \dfrac{2\pi r^{n+2}}{n+1}, n \text{ even} \end{cases} \qquad S : x^2 + y^2 + z^2 = r^2, z \geq 0 \tag{33}$$

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n} \mathrm{d}s \text{ is surface independent}$$

$$\iint_S \vec{F} \cdot \vec{n} \mathrm{d}s \text{ is not surface independent}$$