Session 4

49-781 Data Analytics for Product Managers
Spring 2018

Section 1

Interactive Terms

Interactive Terms

So far, we have been using individual predictors as main terms.

It's possible that there is a joint effect of predictors, and that it's really the co-existence of two predictors that impacts the outcome.

An **interactive effect** between predictors is an additional change to the response that occurs at particular combination of predictors.

Interaction can be between categorical, numeric or a combination of variables. (There could also be more than two variables)

One Categorical, One Continuous

```
"","id","chol","stab.glu","hdl","ratio","glyhb","location","age","gender","height","weight","frame","bp.1s","bp.1d","bp
.2s","bp.2d","waist","hip","time.ppn"
"1",1000,203,82,56,3.5999999,4.30999994,"Buckingham",46,"female",62,121,"medium",118,59,NA,NA,29,38,720
"2",1001,165,97,24,6.9000001,4.44000006,"Buckingham",29,"female",64,218,"large",112,68,NA,NA,46,48,360
"3",1002,228,92,37,6.19999981,4.63999987,"Buckingham",58,"female",61,256,"large",190,92,185,92,49,57,180
"4",1003,78,93,12,6.5,4.63000011,"Buckingham",67,"male",67,119,"large",110,50,NA,NA,33,38,480
"5",1005,249,90,28,8.89999962,7.71999979,"Buckingham",64,"male",68,183,"medium",138,80,NA,NA,44,41,300
"6",1008,248,94,69,3.5999999,4.80999994,"Buckingham",34,"male",71,190,"large",132,86,NA,NA,36,42,195
"7",1011,195,92,41,4.80000019,4.84000015,"Buckingham",30,"male",69,191,"medium",161,112,161,112,46,49,720
"8",1015,227,75,44,5.19999981,3.94000006,"Buckingham",37,"male",59,170,"medium",NA,NA,NA,NA,34,39,1020
"9",1016,177,87,49,3.5999999,4.84000015,"Buckingham",45,"male",69,166,"large",160,80,128,86,34,40,300
"10",1022,263,89,40,6.5999999,5.78000021,"Buckingham",55,"female",63,202,"small",108,72,NA,NA,45,50,240
```

Let us fit a multiple linear regression model including a two way interaction between **age and frame**

Frame is a categorical variable with three levels (*small, medium and large*)

Age and Frame Only - No Interaction

```
==============================================================================
Dep. Variable:                   chol   R-squared:                       0.067
Model:                            OLS   Adj. R-squared:                  0.059
Method:                 Least Squares   F-statistic:                     9.175
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           7.07e-06
Time:                        07:22:30   Log-Likelihood:                -2013.8
No. Observations:                 390   AIC:                             4036.
Df Residuals:                     386   BIC:                             4051.
Df Model:                           3
Covariance Type:            nonrobust
===================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------
Intercept         176.7377      8.455     20.903      0.000     160.114     193.362
frame[T.medium]     9.7609      5.324      1.833      0.068      -0.708      20.229
frame[T.small]     -4.1812      6.107     -0.685      0.494     -16.188       7.826
age                 0.5916      0.139      4.256      0.000       0.318       0.865
==============================================================================
```

Age and Frame with Interaction

```
==============================================================================
Dep. Variable:                   chol   R-squared:                       0.079
Model:                            OLS   Adj. R-squared:                  0.067
Method:                 Least Squares   F-statistic:                     6.580
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           6.85e-06
Time:                        07:22:30   Log-Likelihood:                -2011.2
No. Observations:                 390   AIC:                             4034.
Df Residuals:                     384   BIC:                             4058.
Df Model:                           5
Covariance Type:            nonrobust
=======================================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------
Intercept             200.9110     14.653     13.711      0.000     172.100     229.722
frame[T.medium]       -16.3423     17.631     -0.927      0.355     -51.008      18.323
frame[T.small]        -44.9474     18.984     -2.368      0.018     -82.273      -7.621
age                     0.1341      0.266      0.505      0.614      -0.388       0.657
age:frame[T.medium]     0.4997      0.335      1.493      0.136      -0.158       1.158
age:frame[T.small]      0.8511      0.378      2.252      0.025       0.108       1.594
==============================================================================

Large frame: chol =200.9 + 0.1341 × age
Medium Frame: chol = 200.9 - 16.3423 + (0.1341 +0.4997) × age
Small Frame: chol = 200.9 - 44.9474 + (0.1341 +0.8511) × age
```

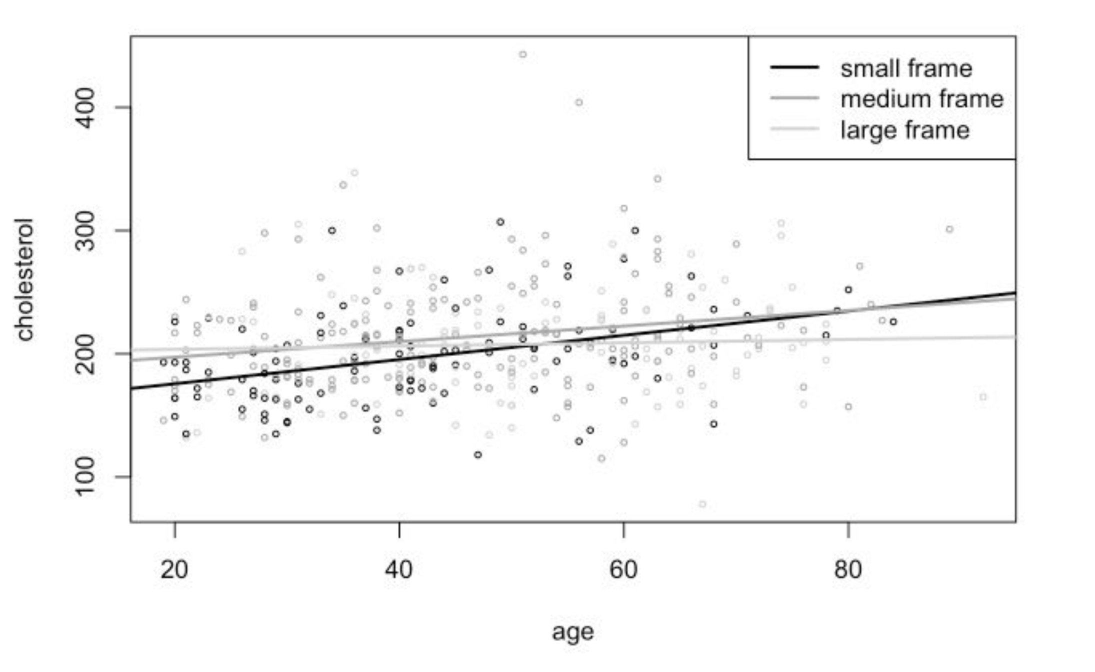
In addition to main coefficients for age and frame, we see additional terms for interaction of age with two non-reference levels of frame.

If at least one level is significant, we should treat the categorical variable as significant.

So there is statistically significant interaction between age and frame.

Plot them

```
Large frame: chol =200.9 + 0.1341 × age
Medium Frame: chol = 200.9 - 16.3423 + (0.1341 + 0.4997) × age
Small Frame: chol = 200.9 - 44.9474 + (0.1341 + 0.8511) × age
```



The non-parallel nature of the three lines reflects the effect of interaction.

Code (Lab later)

```
main = sm.ols(formula="chol ~ age+frame",data=d).fit()
print(main.summary())

inter = sm.ols(formula="chol ~ age*frame",data=d).fit()
print(inter.summary())
```

Two Categorical Variables

```
Gender and Frame: "chol ~ gender*frame"

==============================================================================
Dep. Variable:                   chol   R-squared:                       0.025
Model:                            OLS   Adj. R-squared:                  0.012
Method:                 Least Squares   F-statistic:                     1.947
Date:                Thu, 12 Apr 2018   Prob (F-statistic):             0.0858
Time:                        17:18:42   Log-Likelihood:                -2022.3
No. Observations:                 390   AIC:                             4057.
Df Residuals:                     384   BIC:                             4080.
Df Model:                           5
Covariance Type:            nonrobust
====================================================================================================
                                       coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------------
Intercept                          209.8810      6.722     31.222      0.000     196.664     223.098
gender[T.male]                      -3.1760      8.735     -0.364      0.716     -20.351      13.998
frame[T.medium]                      4.5759      7.845      0.583      0.560     -10.849      20.001
frame[T.small]                     -10.2433      8.526     -1.201      0.230     -27.007       6.520
gender[T.male]:frame[T.medium]       0.6897     10.981      0.063      0.950     -20.900      22.279
gender[T.male]:frame[T.small]       -3.3146     12.634     -0.262      0.793     -28.156      21.527
```

Two Continuous Variables

```
chol ~ height*weight

==============================================================================
Dep. Variable:                   chol   R-squared:                       0.010
Model:                            OLS   Adj. R-squared:                  0.003
Method:                 Least Squares   F-statistic:                     1.369
Date:                Thu, 12 Apr 2018   Prob (F-statistic):              0.252
Time:                        17:20:39   Log-Likelihood:                -2059.2
No. Observations:                 396   AIC:                             4126.
Df Residuals:                     392   BIC:                             4142.
Df Model:                           3
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept       360.5061    176.110      2.047      0.041      14.269     706.744
height           -2.5536      2.675     -0.955      0.340      -7.813       2.705
weight           -0.5511      0.987     -0.559      0.577      -2.491       1.389
height:weight     0.0097      0.015      0.651      0.516      -0.020       0.039
==============================================================================
```

Higher Order Interactions

```
chol ~ height*waist*hip

==================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept        2121.8158   1762.750      1.204      0.229   -1343.950    5587.581
height            -33.7402     27.064     -1.247      0.213     -86.951      19.470
waist             -43.6014     47.000     -0.928      0.354    -136.008      48.806
height:waist        0.7946      0.718      1.107      0.269      -0.617       2.206
hip               -46.1213     42.251     -1.092      0.276    -129.192      36.949
height:hip          0.7876      0.650      1.211      0.227      -0.491       2.066
waist:hip           1.0799      1.036      1.042      0.298      -0.958       3.118
height:waist:hip   -0.0189      0.016     -1.187      0.236      -0.050       0.012
============================================================================
```

Combine Main & Interactive Terms

```
chol ~ age+weight+age*weight

===============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------
Intercept    125.9460     28.568      4.409      0.000      69.782     182.110
age            1.4679      0.607      2.420      0.016       0.275       2.661
weight         0.2959      0.161      1.843      0.066      -0.020       0.612
age:weight    -0.0047      0.003     -1.369      0.172      -0.012       0.002
===============================================================================
```

Interactive Terms

Lab Review

Code for Slide Exercises

```
import numpy as np
import pandas as pd
import statsmodels.formula.api as sm

# # Multiple Logistic Regression
d=pd.read_csv("diabetes.csv")

main = sm.ols(formula="chol ~ age+frame",data=d).fit()
print(main.summary())

inter = sm.ols(formula="chol ~ age*frame",data=d).fit()
print(inter.summary())

inter = sm.ols(formula="chol ~ gender*frame",data=d).fit()
print(inter.summary())

inter = sm.ols(formula="chol ~ height*weight",data=d).fit()
print(inter.summary())

inter = sm.ols(formula="chol ~ age+weight+age*weight",data=d).fit()
print(inter.summary())
```

Section 2

Linear Model Selection

Linear Model Selection

Statisticians refer to the balancing act between **goodness-of-fit** and **complexity** as the principle of parsimony, where the goal of the associated model selection is to find a model that's as simple as possible (in other words, with relatively low complexity), without sacrificing too much goodness-of-fit.

General Guidelines

- For categorical variables, you cannot just remove some levels (You can remove the entire predictor)
- If an interaction is present in the fitted model, all lower-order interactions and main effects of the relevant predictors must remain in the model.
- In models where you've used a polynomial transformation of a certain explanatory variable, keep all lower-order polynomial terms in the model if the highest is deemed significant.

Celebrated statistician George Box (1919– 2013): "All models are wrong, but some are useful."

Linear Model Selection

Analysis of Variance

Nested Comparisons

In nested models, the smaller model is a reduced version of the bigger, more complex model.

Height~Wr.Hnd+Sex
Height~Wr.Hnd+Sex+Smoke

Test with Anova

Analysis of Variance is useful for comparing means from different models for statistical significance.

```
Height~Wr.Hnd+Sex
Height~Wr.Hnd+Sex+Smoke

   df_resid          ssr  df_diff    ss_diff         F    Pr(>F)
0     204.0  9959.181574      0.0        NaN       NaN       NaN
1     201.0  9914.305976      3.0  44.875598  0.303265  0.823015
```

The result of this particular test, is a high p-value of 0.823, suggesting no evidence against H_o.

This means that adding Smoke to the reduced model, which includes only the explanatory variables Wr.Hnd and Sex, offers no tangible improvement in fit when it comes to modeling student height.

Diabetes Data Set

cholesterol based on age and frame.

(Identify and delete any individuals with a missing value for age or for frame.)

- chol~1
- chol~age
- chol~age+frame
- chol~age*frame

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	389.0	747264.823077	0.0	NaN	NaN	NaN
1	388.0	712078.244044	1.0	35186.579033	19.630610	0.000012
2	386.0	697527.450662	2.0	14550.793383	4.058947	0.018010
3	384.0	688294.773581	2.0	9232.677081	2.575458	0.077433

Notes

(If you hadn't deleted the records containing missing values in those predictors, you would've received an error telling you that the data sets for the four models were not equal sizes.)

- including age provides a significant improvement to modeling chol;
- including a main effect for frame provides a further mild improvement;
- there's very weak evidence, if any, that including an interactive effect is beneficial to goodness-of-fit.

From this, you might prefer to use the main-effects-only model.

Quick Lab

```
d=pd.read_csv("diabetes.csv")
chol1 = sm.ols(formula="chol ~ 1",data=d).fit()
chol2 = sm.ols(formula="chol ~ age",data=d).fit()
chol3 = sm.ols(formula="chol ~ age+frame",data=d).fit()
chol4 = sm.ols(formula="chol ~ age*frame",data=d).fit()

print(sma.stats.anova_lm(chol1,chol2,chol3,chol4))
```

Linear Model Selection

Forward Selection

Model Selection: Forward Selection

Start with an intercept-only model

Perform a series of independent tests to determine which of your predictor variables significantly improves the goodness-of-fit.

Update your model object by adding that term

Execute the series of tests again for all remaining terms to determine which of those would further improve the fit.

Repeat the process until there aren’t any more terms that improve the fit in a statistically significant way.

Nuclear Dataset

cost
The capital cost of construction in millions of dollars adjusted to 1976 base.

date
The date on which the construction permit was issued. The data are measured in years since January 1 1990 to the nearest month.

t1
The time between application for and issue of the construction permit.

t2
The time between issue of operating license and construction permit.

cap
The net capacity of the power plant (MWe).

pr
A binary variable where 1 indicates the prior existence of a LWR plant at the same site.

ne
A binary variable where 1 indicates that the plant was constructed in the north-east region of the U.S.A.

ct
A binary variable where 1 indicates the use of a cooling tower in the plant.

bw
A binary variable where 1 indicates that the nuclear steam supply system was manufactured by Babcock-Wilcox.

cum.n
The cumulative number of power plants constructed by each architect-engineer.

pt
A binary variable where 1 indicates those plants with partial turnkey guarantees.

Base Model with no predictors

```
cost ~ 1
                            OLS Regression Results
==============================================================================
Dep. Variable:                   chol   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                       inf
Date:                Wed, 11 Apr 2018   Prob (F-statistic):                nan
Time:                        12:48:59   Log-Likelihood:                -2027.2
No. Observations:                 390   AIC:                             4056.
Df Residuals:                     389   BIC:                             4060.
Df Model:                           0
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    207.8385      2.219     93.647      0.000     203.475     212.202
==============================================================================
```

Compare against a model with each predictor

```
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  562837.107402      1.0  334335.201295  17.820531  0.000207   date
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  710188.747817      1.0   186983.56088   7.898614   0.00863   t1
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  897144.924936      1.0      27.383761   0.000916   0.97606   t2
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  697499.098161      1.0  199673.210536   8.588106  0.006414   cap
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  888135.636650      1.0    9036.672047   0.305246  0.584705   pr
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  768531.357071      1.0  128640.951626   5.021563  0.032589   ne
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  854130.070411      1.0   43042.238286   1.511792  0.228422   ct
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  880966.893535      1.0    6205.415162   0.551851   0.46334   bw
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  829234.216944      1.0   67938.091753   2.457861  0.127427   cumn
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      31.0  897172.308697      0.0            NaN        NaN       NaN
1      30.0  591838.528850      1.0  305333.779847  15.477217  0.000458   pt
```

New Base Model with date

cost ~ date

```
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.373
Model:                            OLS   Adj. R-squared:                  0.352
Method:                 Least Squares   F-statistic:                     17.82
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           0.000207
Time:                        15:03:36   Log-Likelihood:                -201.81
No. Observations:                  32   AIC:                             407.6
Df Residuals:                      30   BIC:                             410.5
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -6553.5666   1661.963     -3.943      0.000   -9947.748   -3159.386
date         102.2893     24.231      4.221      0.000      52.803     151.775
```

Compare with remaining predictors

```
   df_resid           ssr  df_diff       ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0           NaN       NaN       NaN
1      29.0  547515.249347     1.0  15321.858055  0.811546  0.375084     t1
   df_resid           ssr  df_diff       ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0           NaN       NaN       NaN
1      29.0  494675.912803     1.0  68161.194599  3.995898  0.055061     t2
   df_resid           ssr  df_diff        ss_diff          F    Pr(>F)
0      30.0  562837.107402     0.0            NaN        NaN       NaN
1      29.0  373105.360239     1.0  189731.747163  14.747096  0.000616     cap
   df_resid           ssr  df_diff      ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0          NaN       NaN       NaN
1      29.0  558809.947565     1.0  4027.159837  0.208993  0.650964     pr
   df_resid           ssr  df_diff       ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0           NaN       NaN       NaN
1      29.0  470580.704698     1.0  92256.402704  5.685392  0.023867     ne
   df_resid           ssr  df_diff       ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0           NaN       NaN       NaN     ct
1      29.0  508043.436220     1.0  54793.671182  3.127718  0.087491
   df_resid           ssr  df_diff      ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0          NaN       NaN       NaN     bw
1      29.0  561597.054694     1.0  1240.052708  0.064034  0.802015
   df_resid           ssr  df_diff      ss_diff         F    Pr(>F)
0      30.0  562837.107402     0.0          NaN       NaN       NaN     cumn
1      29.0  558178.895546     1.0  4658.211856  0.242016  0.626457
   df_resid           ssr  df_diff       ss_diff         F  Pr(>F)
0      30.0  562837.107402     0.0           NaN       NaN     NaN     pt
1      29.0  472250.424653     1.0  90586.682749  5.562756  0.0253
```

New Base Model with date & cap

```
cost ~ date+cap
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.584
Model:                            OLS   Adj. R-squared:                  0.555
Method:                 Least Squares   F-statistic:                     20.37
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           2.98e-06
Time:                        15:08:59   Log-Likelihood:                -195.23
No. Observations:                  32   AIC:                             396.5
Df Residuals:                      29   BIC:                             400.9
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -6790.8792   1377.668     -4.929      0.000   -9608.527   -3973.231
date         100.7764     20.070      5.021      0.000      59.729     141.823
cap            0.4132      0.108      3.840      0.001       0.193       0.633
==============================================================================
```

Compare with remaining predictors

```
   df_resid            ssr  df_diff      ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0          NaN       NaN       NaN
1      28.0  372148.158204      1.0   957.202034  0.072019  0.790387       t1
   df_resid            ssr  df_diff       ss_diff         F   Pr(>F)
0      29.0  373105.360239      0.0           NaN       NaN      NaN
1      28.0  359749.826705      1.0  13355.533533  1.039486  0.31667       t2
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0           NaN       NaN       NaN
1      28.0  354651.969151      1.0  18453.391088  1.456907  0.237521      pr
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0           NaN       NaN       NaN
1      28.0  278568.803905      1.0  94536.556334  9.502225  0.004575      ne
   df_resid            ssr  df_diff      ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0          NaN       NaN       NaN
1      28.0  324148.494509      1.0  48956.86573  4.228902  0.049169       ct
   df_resid            ssr  df_diff      ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0          NaN       NaN       NaN
1      28.0  365600.547942      1.0  7504.812297  0.574766  0.454705       bw
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0           NaN       NaN       NaN
1      28.0  345043.635875      1.0  28061.724364  2.277185  0.142493      cumn
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      29.0  373105.360239      0.0           NaN       NaN       NaN
1      28.0  277188.678320      1.0  95916.681918  9.688949  0.004243      pt
```

New Base Model with date, cap & pt

```
cost ~ date+cap+pt

==============================================================================
Dep. Variable:                   cost   R-squared:                       0.691
Model:                            OLS   Adj. R-squared:                  0.658
Method:                 Least Squares   F-statistic:                     20.88
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           2.64e-07
Time:                        16:14:29   Log-Likelihood:                -190.47
No. Observations:                  32   AIC:                             388.9
Df Residuals:                      28   BIC:                             394.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -4553.3078   1406.112     -3.238      0.003   -7433.598   -1673.018
date          68.5245     20.428      3.355      0.002      26.680     110.369
cap            0.4191      0.094      4.439      0.000       0.226       0.612
pt          -162.7636     52.290     -3.113      0.004    -269.875     -55.652
==============================================================================
```

Compare with remaining predictors

```
   df_resid           ssr  df_diff     ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0         NaN       NaN       NaN
1      27.0  277177.159655      1.0   11.518666  0.001122  0.973525       t1
   df_resid           ssr  df_diff        ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0            NaN       NaN       NaN
1      27.0  266759.995586      1.0   10428.682734  1.055535  0.313353    t2
   df_resid           ssr  df_diff       ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0           NaN       NaN       NaN
1      27.0  271171.408835      1.0   6017.269485  0.599128  0.445636     pr
   df_resid           ssr  df_diff        ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0            NaN       NaN       NaN
1      27.0  222616.935606      1.0   54571.742714  6.618711  0.015908    ne
   df_resid           ssr  df_diff        ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0            NaN       NaN       NaN
1      27.0  259069.239609      1.0   18119.438711  1.888394  0.180684    ct
   df_resid           ssr  df_diff     ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0         NaN       NaN       NaN
1      27.0  276534.594631      1.0   654.08369  0.063863  0.802406       bw
   df_resid           ssr  df_diff      ss_diff         F    Pr(>F)
0      28.0  277188.678320      0.0          NaN       NaN       NaN
1      27.0  276726.014154      1.0   462.664166  0.045142  0.833339      cumn
```

New Base Model with date, cap, pt & ne

```
cost ~ date+cap+pt+ne

                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.752
Model:                            OLS   Adj. R-squared:                  0.715
Method:                 Least Squares   F-statistic:                     20.45
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           7.51e-08
Time:                        16:16:38   Log-Likelihood:                -186.97
No. Observations:                  32   AIC:                             383.9
Df Residuals:                      27   BIC:                             391.3
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -4756.2145   1285.663     -3.699      0.001   -7394.177   -2118.252
date          71.0193     18.668      3.804      0.001      32.716     109.322
cap            0.4198      0.086      4.873      0.000       0.243       0.597
pt          -128.9438     49.498     -2.605      0.015    -230.506     -27.382
ne            99.3988     38.636      2.573      0.016      20.124     178.674
==============================================================================
```

Compare with remaining predictors

```
   df_resid            ssr  df_diff     ss_diff         F   Pr(>F)
0      27.0  222616.935606      0.0         NaN       NaN      NaN
1      26.0  222509.890380      1.0  107.045226  0.012508  0.91181        t1
   df_resid            ssr  df_diff       ss_diff         F  Pr(>F)
0      27.0  222616.935606      0.0           NaN       NaN     NaN
1      26.0  203387.016602      1.0  19229.919004  2.458259   0.129       t2
   df_resid            ssr  df_diff      ss_diff        F    Pr(>F)
0      27.0  222616.935606      0.0          NaN      NaN       NaN
1      26.0  217386.123321      1.0  5230.812285  0.62562  0.436123       pr
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      27.0  222616.935606      0.0           NaN       NaN       NaN
1      26.0  206852.279022      1.0  15764.656584  1.981516  0.171075     ct
   df_resid            ssr  df_diff     ss_diff         F    Pr(>F)
0      27.0  222616.935606      0.0         NaN       NaN       NaN
1      26.0  222168.964132      1.0  447.971474  0.052425  0.820687       bw
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      27.0  222616.935606      0.0           NaN       NaN       NaN
1      26.0  208796.998874      1.0  13819.936732  1.720898  0.201043     cumn
```

It appears that none of the remaining predictors would yield a statistically significant improvement in goodness-of-fit.

Final Model with date, cap, pt & ne

```
cost ~ date+cap+pt+ne

                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.752
Model:                            OLS   Adj. R-squared:                  0.715
Method:                 Least Squares   F-statistic:                     20.45
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           7.51e-08
Time:                        16:16:38   Log-Likelihood:                -186.97
No. Observations:                  32   AIC:                             383.9
Df Residuals:                      27   BIC:                             391.3
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -4756.2145   1285.663     -3.699      0.001   -7394.177   -2118.252
date          71.0193     18.668      3.804      0.001      32.716     109.322
cap            0.4198      0.086      4.873      0.000       0.243       0.597
pt          -128.9438     49.498     -2.605      0.015    -230.506     -27.382
ne            99.3988     38.636      2.573      0.016      20.124     178.674
==============================================================================
```

Summary

In forward selection, you start with an Intercept-only model and add each predictor by doing an Analysis of Variance on the model with and without the predictor.

You stop when there are no more significant predictors to add

Let’s Try a Lab

```
d=pd.read_csv("nuclear.csv")
d=d.rename(index=str,columns={"cum.n":"cumn"})
dia = []
dia.append(sm.ols("cost~date+cap+pt+ne", data=d).fit())
print(dia[0].summary())
dia.append(sm.ols("cost~date+cap+pt+ne+t1", data=d).fit())
dia.append(sm.ols("cost~date+cap+pt+ne+t2", data=d).fit())
dia.append(sm.ols("cost~date+cap+pt+ne+pr", data=d).fit())
dia.append(sm.ols("cost~date+cap+pt+ne+ct", data=d).fit())
dia.append(sm.ols("cost~date+cap+pt+ne+bw", data=d).fit())
dia.append(sm.ols("cost~date+cap+pt+ne+cumn", data=d).fit())
for i in np.arange(1,7):
    print(sma.stats.anova_lm(dia[0],dia[i]))
```

Linear Model Selection

Backward Selection

Backward Selection

backward selection starts with your fullest model and systematically drops terms.

Define the fullest model as that which predicts cost by main effects of all available covariates

Base Model with all predictors

```
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.839
Model:                            OLS   Adj. R-squared:                  0.763
Method:                 Least Squares   F-statistic:                     10.98
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           2.84e-06
Time:                        16:30:41   Log-Likelihood:                -180.00
No. Observations:                  32   AIC:                             382.0
Df Residuals:                      21   BIC:                             398.1
Df Model:                          10
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -8134.8823   2787.794     -2.918      0.008   -1.39e+04   -2337.347
date         115.4832     42.260      2.733      0.012      27.599     203.367
t1             5.9284     10.887      0.545      0.592     -16.712      28.569
t2             4.5709      2.243      2.038      0.054      -0.094       9.236
cap            0.4216      0.088      4.768      0.000       0.238       0.606
pr           -81.1211     40.769     -1.990      0.060    -165.905       3.662
ne           137.4502     38.690      3.553      0.002      56.989     217.911
ct            43.2733     34.307      1.261      0.221     -28.071     114.618
bw            -8.2384     51.884     -0.159      0.875    -116.136      99.659
cumn          -6.9886      3.822     -1.829      0.082     -14.936       0.959
pt           -19.2476     63.672     -0.302      0.765    -151.660     113.165
==============================================================================
```

Compare against a model without each predictor

```
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      22.0  195294.933636      0.0            NaN        NaN       NaN
1      21.0  144064.892149      1.0   51230.041487   7.467682   0.01247       date
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      22.0  146099.150012      0.0           NaN        NaN       NaN
1      21.0  144064.892149      1.0   2034.257864   0.296529  0.591803       t1
   df_resid            ssr  df_diff       ss_diff         F   Pr(>F)
0      22.0  172546.250258      0.0           NaN       NaN      NaN
1      21.0  144064.892149      1.0   28481.35811   4.15166  0.05439       t2
   df_resid            ssr  df_diff          ss_diff           F    Pr(>F)
0      22.0  300007.623455      0.0              NaN         NaN       NaN
1      21.0  144064.892149      1.0    155942.731307   22.731405  0.000104   cap
   df_resid            ssr  df_diff        ss_diff          F     Pr(>F)
0      22.0  171226.152965      0.0            NaN        NaN        NaN
1      21.0  144064.892149      1.0   27161.260816   3.959233   0.059794     pr
   df_resid            ssr  df_diff        ss_diff           F    Pr(>F)
0      22.0  230645.870888      0.0            NaN         NaN       NaN
1      21.0  144064.892149      1.0   86580.978739   12.620705  0.001883    ne
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      22.0  154979.922788      0.0           NaN        NaN       NaN
1      21.0  144064.892149      1.0   10915.03064   1.591058  0.221008       ct
   df_resid            ssr  df_diff      ss_diff          F    Pr(>F)
0      22.0  144237.861410      0.0          NaN        NaN       NaN
1      21.0  144064.892149      1.0   172.969261   0.025213  0.875354        bw
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      22.0  167004.306468      0.0           NaN        NaN       NaN
1      21.0  144064.892149      1.0   22939.41432   3.343824  0.081698       cumn
   df_resid            ssr  df_diff      ss_diff          F    Pr(>F)
0      22.0  144691.792614      0.0          NaN        NaN       NaN
1      21.0  144064.892149      1.0   626.900465   0.091382  0.765401        pt
```

it seems the predictor bw has the single least significant effect on reducing the goodness-of-fit,

New Base Model without bw

```
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.839
Model:                            OLS   Adj. R-squared:                  0.773
Method:                 Least Squares   F-statistic:                     12.76
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           7.66e-07
Time:                        16:34:46   Log-Likelihood:                -180.02
No. Observations:                  32   AIC:                             380.0
Df Residuals:                      22   BIC:                             394.7
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -7979.5131   2551.950     -3.127      0.005   -1.33e+04   -2687.093
date         113.1505     38.736      2.921      0.008      32.817     193.484
t1             6.6587      9.647      0.690      0.497     -13.347      26.665
t2             4.4461      2.054      2.165      0.042       0.186       8.706
cap            0.4235      0.086      4.940      0.000       0.246       0.601
pr           -80.1272     39.383     -2.035      0.054    -161.802       1.548
ne           137.1744     37.785      3.630      0.001      58.812     215.537
ct            44.2666     32.976      1.342      0.193     -24.120     112.654
cumn          -7.0778      3.696     -1.915      0.069     -14.742       0.586
pt           -22.3293     59.283     -0.377      0.710    -145.275     100.617
==============================================================================
```

Compare against a model without each predictor

```
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      23.0  200179.576049      0.0           NaN        NaN       NaN
1      22.0  144237.861410      1.0  55941.714639   8.532557  0.007913      date
   df_resid            ssr  df_diff      ss_diff          F    Pr(>F)
0      23.0  147361.703135      0.0          NaN        NaN       NaN
1      22.0  144237.861410      1.0  3123.841725  0.476467  0.497245      t1
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      23.0  174955.359915      0.0           NaN        NaN       NaN
1      22.0  144237.861410      1.0  30717.498505   4.685212  0.041546      t2
   df_resid           ssr  df_diff        ss_diff           F    Pr(>F)
0      23.0  304214.01552      0.0            NaN         NaN       NaN
1      22.0  144237.86141      1.0  159976.154111   24.400496  0.000061      cap
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      23.0  171377.366628      0.0           NaN        NaN       NaN
1      22.0  144237.861410      1.0  27139.505219   4.139476  0.054122      pr
   df_resid            ssr  df_diff       ss_diff           F    Pr(>F)
0      23.0  230645.874117      0.0           NaN         NaN       NaN
1      22.0  144237.861410      1.0  86408.012707   13.179454  0.001479      ne
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      23.0  156052.654523      0.0           NaN        NaN       NaN
1      22.0  144237.861410      1.0  11814.793114   1.802061  0.193153      ct
   df_resid           ssr  df_diff      ss_diff          F    Pr(>F)
0      23.0  168286.15768      0.0          NaN        NaN       NaN
1      22.0  144237.86141      1.0  24048.296271  3.667986  0.068557      cumn
   df_resid            ssr  df_diff     ss_diff          F    Pr(>F)
0      23.0  145167.987705      0.0         NaN        NaN       NaN
1      22.0  144237.861410      1.0  930.126296  0.141868  0.710039      pt
```

pt is the next most sensible main effect to drop.

Compare against a model without each predictor

```
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      24.0  238844.665951      0.0            NaN        NaN       NaN
1      23.0  145167.987705      1.0   93676.678245  14.841864  0.000811     date
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      24.0  148021.428849      0.0           NaN       NaN       NaN
1      23.0  145167.987705      1.0  2853.441144  0.452091  0.508043     t1
   df_resid            ssr  df_diff        ss_diff         F    Pr(>F)
0      24.0  176480.064481      0.0            NaN       NaN       NaN
1      23.0  145167.987705      1.0  31312.076776  4.960996  0.035996     t2
   df_resid            ssr  df_diff         ss_diff          F    Pr(>F)
0      24.0  310762.174057      0.0             NaN        NaN       NaN
1      23.0  145167.987705      1.0  165594.186351  26.236268  0.000034   cap
   df_resid            ssr  df_diff        ss_diff         F    Pr(>F)
0      24.0  175152.045770      0.0            NaN       NaN       NaN
1      23.0  145167.987705      1.0  29984.058065  4.750588  0.039782     pr
   df_resid            ssr  df_diff       ss_diff          F    Pr(>F)
0      24.0  258826.519005      0.0           NaN        NaN       NaN
1      23.0  145167.987705      1.0  113658.5313  18.007732  0.000307     ne
   df_resid            ssr  df_diff        ss_diff         F    Pr(>F)
0      24.0  160769.641361      0.0            NaN       NaN       NaN
1      23.0  145167.987705      1.0  15601.653656  2.471881  0.129556     ct
   df_resid            ssr  df_diff        ss_diff         F    Pr(>F)
0      24.0  193650.384331      0.0            NaN       NaN       NaN
1      23.0  145167.987705      1.0  48482.396626  7.681412  0.010856     cumn
```

t1 is the next most sensible main effect to drop.

Compare against a model without each predictor

```
   df_resid            ssr  df_diff        ss_diff          F        Pr(>F)
0      25.0  517039.978932      0.0            NaN        NaN           NaN
1      24.0  148021.428849      1.0  369018.550083  59.832183  5.697409e-08       date
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      25.0  176560.224962      0.0           NaN       NaN       NaN
1      24.0  148021.428849      1.0  28538.796113  4.627243  0.041748            t2
   df_resid            ssr  df_diff      ss_diff          F    Pr(>F)
0      25.0  310867.017049      0.0          NaN        NaN       NaN
1      24.0  148021.428849      1.0  162845.5882  26.40357  0.000029             cap
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      25.0  175230.053526      0.0           NaN       NaN       NaN
1      24.0  148021.428849      1.0  27208.624677  4.411571  0.046386            pr
   df_resid            ssr  df_diff        ss_diff          F    Pr(>F)
0      25.0  262135.460638      0.0            NaN        NaN       NaN
1      24.0  148021.428849      1.0  114114.031789  18.502299  0.000245          ne
   df_resid            ssr  df_diff      ss_diff         F    Pr(>F)
0      25.0  163221.598289      0.0          NaN       NaN       NaN
1      24.0  148021.428849      1.0  15200.16944  2.464536  0.129534             ct
   df_resid            ssr  df_diff       ss_diff         F    Pr(>F)
0      25.0  201309.521097      0.0           NaN       NaN       NaN
1      24.0  148021.428849      1.0  53288.092248  8.640061  0.007164            cumn
```

ct is the next most sensible main effect to drop.

Compare against a model without each predictor

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	537798.670515	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	374577.072226	57.372473	6.271558e-08	***date***

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	210296.772856	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	47075.174567	7.210316	0.012685	***t2***

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	320671.742805	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	157450.144516	24.116009	0.000047	***cap***

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	205491.553637	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	42269.955348	6.47432	0.017499	***pr***

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	290709.372718	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	127487.774429	19.526793	0.000168	***ne***

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)	
0	26.0	212897.740727	0.0	NaN	NaN	NaN	
1	25.0	163221.598289	1.0	49676.142438	7.608696	0.010703	***cumn***

All remaining predictors seem significant

Final Model

```
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.818
Model:                            OLS   Adj. R-squared:                  0.774
Method:                 Least Squares   F-statistic:                     18.74
Date:                Wed, 11 Apr 2018   Prob (F-statistic):           3.80e-08
Time:                        16:42:17   Log-Likelihood:                -182.00
No. Observations:                  32   AIC:                             378.0
Df Residuals:                      25   BIC:                             388.3
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -9701.5221   1294.355     -7.495      0.000   -1.24e+04   -7035.749
date         139.5909     18.429      7.574      0.000     101.635     177.546
t2             4.9051      1.827      2.685      0.013       1.143       8.667
cap            0.4137      0.084      4.911      0.000       0.240       0.587
pr           -88.5147     34.787     -2.544      0.017    -160.160     -16.869
ne           150.2262     33.996      4.419      0.000      80.210     220.243
cumn          -7.9194      2.871     -2.758      0.011     -13.832      -2.006
==============================================================================
```

Forward vs Backward Selection

Do they yield same models?

Forward: **cost~date+cap+pt+ne**

Backward: **cost~date+t2+cap+pr+ne+cumn**

What might cause these two to give different models?

Linear Model Selection

Lab Exercises

Use Diabetes Dataset

There are some missing values in diabetes that might interfere with model selection algorithms. Define a new version of the diabetes data frame that deletes all rows with a missing value in any of the following variables: chol, age, gender, height, weight, frame, waist, hip, location.

Use forward selection with a significance level of $\alpha = 0.05$ to choose a model, starting from intercept only model. Use the above variables.

Use backward selection with a conventional significance level of $\alpha = 0.05$ to choose a model, starting from the full model with above variables.

Section 3

Resampling Methods

Resampling Methods

Resampling involves repeatedly drawing samples from a training set and refitting a model of interest on each sample.

They are computationally expensive because they involve fitting the same method multiple times.

Training Error vs Test Error

Given a data set, the use of a particular statistical learning method is warranted if it results in a low test error. The **test error** can be easily calculated if a designated test set is available.

The **training error** can be easily calculated by applying the statistical learning method to the observations used in its training. The training error rate often is quite different from the test error rate.

When we do not have a designated test set that can be used to directly estimate the test error rate, some techniques can be used to estimate this quantity using the available training data.

Cross Validation

We instead consider a class of methods that estimate the test error rate by **holding out a subset of the training observations from the fitting process,** and then applying the statistical learning method to those held out observations.

Cross-validation can be used to estimate the test error associated with a given statistical learning method in order to evaluate its performance. The process of evaluating a model’s performance is known as **model assessment**, whereas the process of selecting the proper level of flexibility for a model is known as **model selection**.

Cross Validation

Validation Sets

Validation Set Approach

Validation set approach involves **randomly dividing** the available set of observations into a training set and a validation set.

We **fit the model on an a training set** and the fitted model is used to **predict the responses for observations in the validation set**.

The resulting **validation set error rate** is typically assessed using MSE and provides an estimate of the test error rate.

Validation Set Approach

Validation test error rate is highly variable, depending on which observations are included in the validation set.

Since a subset of observations are used to fit the model, and statistical methods tend perform worse when trained on fewer observations, the validation error rate may tend to overestimate the test error rate for the model fit on the entire data set.

Cross Validation

Leave-One-Out Cross-Validation (LOOCV)

Leave-One-Out Cross-Validation (LOOCV)

This approach involves splitting the set of observations into two parts. However, a single observation is used for validation, and the remaining observations are used for training.

We fit a model for n-1 observations, and prediction is made for the excluded observation.

The MSE on the test observation provides the estimate for test error.

We can repeat this procedure n times for n observations and compute n MSEs.

LOOCV estimate for the test MSE is the average of these n test errors.

LOOCV Logic

```
import statsmodels.sandbox.tools.cross_val as cross_val


d=pd.read_csv("auto.csv")
loo = cross_val.LeaveOneOut(len(d.index))
error_sum = 0
for train_index, test_index in loo:
        # print ("TRAIN:", train_index, "TEST:", test_index)
        a_train, a_test = cross_val.split(train_index,test_index,d)
        d_train = pd.DataFrame(a_train,columns=d.columns)
        d_test = pd.DataFrame(a_test,columns=d.columns)
        for x in d.columns:
            d_train[x] = d_train[x].astype(d[x].dtypes.name)
            d_test[x] = d_test[x].astype(d[x].dtypes.name)
        nuc = sm.ols("mpg~horsepower", data=d_train).fit()
        y = nuc.predict(d_test)
        error_sum+= (y[0] - d_test["mpg"][0])**2
print( "MSE= ", (error_sum/len(d.index)))
```

LOOCV

```
TRAIN:
[ True  True  True  True  True  True  True  True  True  True  True  True
  True  True  True  True  True  True  True  True  True  True  True  True
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  True  True  True  True  True  True  True  True  True  True  True  True
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```

Compare with Validation Set

In LOOCV, we repeatedly fit the method with n-1 data sets (or almost the same)

In contrast with Validation Set, which can yield different results owing to randomness in the test/training set selection, repeating LOOCV any number of times will yield same results.

Cross Validation

k-Fold Cross-Validation

k-Fold Cross-Validation

k-fold CV is an alternative to LOOCV. We randomly divide the set of observations into k groups or folds of approximately equal size

1. The first fold is treated as a validation set
2. The method is then fit on the remaining k-1 folds.
3. The MSE is computed on observations in the held-out fold
4. This process is repeated k times, each time picking a different group of observations as the validation set
5. This method gives k estimates of the test error, which are then averaged to calculate the k-fold CV estimate.

Note that LOOCV is a special case of k-fold CV when k equals n

K-fold Logic

```
d=pd.read_csv("auto.csv")
loo = cross_val.KFold(len(d.index),20)
error_sum = 0
for train_index, test_index in loo:
        # print ("TRAIN:", train_index, "TEST:", test_index)
        a_train, a_test = cross_val.split(train_index,test_index,d)
        d_train = pd.DataFrame(a_train,columns=d.columns)
        d_test = pd.DataFrame(a_test,columns=d.columns)
        for x in d.columns:
            d_train[x] = d_train[x].astype(d[x].dtypes.name)
            d_test[x] = d_test[x].astype(d[x].dtypes.name)
        nuc = sm.ols("mpg~horsepower", data=d_train).fit()
        y = nuc.predict(d_test)
        error_sum+= ((y - d_test["mpg"])**2).sum()/len(d_test.index)

print( "MSE= ", (error_sum/20))
```

LOOCV

```
TRAIN: [ True  True  True  True  True  True  True  True  True  True  True  True
  True  True  True  True  True  True  True  True  True  True  True  True
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```

Cross Validation

Cross Validation for Classification Problems

Cross Validation for Classification Problems

Cross-validation can also be a very useful approach in the classification setting when Y is qualitative.

Rather than using MSE to quantify test error, we instead use the number of misclassified observations.