
ATAR Master

VCE Mathematical Methods

2022 Examination 2 (Technology-Active)

Questions & Marking Guide

Total: 80 marks

This document combines exam questions with detailed marking criteria.
Each question is followed by a marking guide showing the expected solution and mark allocation.

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Section A: Multiple Choice — 20 marks

Each question is worth 1 mark.

Question 1

1 mark

The period of the function $f(x) = 3 \cos(2x + \pi)$ is

- A. 2π
- B. π
- C. 2
- D. 3

Marking Guide — Answer: B

- Period = $\frac{2\pi}{2} = \pi$.

Question 2

1 mark

The graph of $y = \frac{1}{(x+3)^2} + 4$ has a horizontal asymptote with the equation

- A. $y = 4$
- B. $y = 3$
- C. $y = 0$
- D. $x = -2$
- E. $x = -3$

Marking Guide — Answer: A

- As $x \rightarrow \pm\infty$, $\frac{1}{(x+3)^2} \rightarrow 0$, so $y \rightarrow 4$.

Question 3

1 mark

The gradient of the graph of $y = e^{3x}$ at the point where the graph crosses the vertical axis is equal to

- A. 0
- B. 1
- C. e
- D. 3

Marking Guide — Answer: E

- $\frac{dy}{dx} = 3e^{3x}$. At $x = 0$: $\frac{dy}{dx} = 3e^0 = 3$.

Section B: Extended Response — 60 marks

Question 4*1 mark*

Which one of the following functions is not continuous over the interval $x \in [0, 5]$?

Marking Guide — Answer: D**Question 5***1 mark*

The largest value of a such that the function $f : (-\infty, a] \rightarrow R$, $f(x) = x^2 + 3x - 10$, where f is one-to-one, is

- A. -12.25
- B. -5
- C. -1.5
- D. 0
- E. 2

Marking Guide — Answer: C**Question 6***1 mark*

Which of the pairs of functions below are **not** inverse functions?

Marking Guide — Answer: C

- C: $f(x) = x^2$ for $x < 0$ and $g(x) = \sqrt{x}$ for $x > 0$. The inverse of $f(x) = x^2, x < 0$ is $g(x) = -\sqrt{x}$, not \sqrt{x} .

Question 7*1 mark*

The graph of $y = f(x)$ is shown (a curve with a local minimum to the left of the y-axis and a local maximum to the right, with an inflection point near the origin). The graph of $y = f'(x)$, the first derivative of $f(x)$ with respect to x , could be

- A. Graph A (positive hump then dip, zeros at two points)
- B. Graph B (tall positive hump, one zero)
- C. Graph C (negative dip then positive, two zeros)
- D. Graph D (vertical asymptote shape)
- E. Graph E (starts positive, crosses to negative, single smooth curve)

Marking Guide — Answer: E

- The original function has a local min (left) and local max (right), so $f'(x) = 0$ at two points, positive between them. Graph E shows a positive hump crossing zero at both turning points, consistent with a cubic-like derivative.

Question 8*1 mark*

If $\int_0^b f(x) dx = 10$ and $\int_0^a f(x) dx = -4$, where $0 < a < b$, then $\int_a^b f(x) dx$ is equal to

- A. -6
- B. -4
- C. 0
- D. 10
- E. 14

Marking Guide — Answer: E

- $\int_a^b f(x) dx = \int_0^b f(x) dx - \int_0^a f(x) dx = 10 - (-4) = 14.$

Question 9*1 mark*

Let $f : [0, \infty) \rightarrow R$, $f(x) = \sqrt{2x + 1}$.

The shortest distance, d , from the origin to the point (x, y) on the graph of f is given by

- A. $d = x^2 + 2x + 1$
- B. $d = x + 1$
- C. $d = 2x + 1$

Marking Guide — Answer: D

- $d = \sqrt{x^2 + y^2} = \sqrt{x^2 + 2x + 1} = \sqrt{(x + 1)^2} = x + 1$ (since $x \geq 0$).

Question 10*1 mark*

An organisation randomly surveyed 1000 Australian adults and found that 55% of those surveyed were happy with their level of physical activity.

An approximate 95% confidence interval for the percentage of Australian adults who were happy with their level of physical activity is closest to

- A. (4.1, 6.9)
- B. (50.9, 59.1)
- C. (52.4, 57.6)
- D. (51.9, 58.1)
- E. (45.2, 64.8)

Marking Guide — Answer: D

- $\hat{p} = 0.55$, $n = 1000$. $E = 1.96 \sqrt{\frac{0.55 \times 0.45}{1000}} \approx 0.0308$.

- CI: $(0.55 - 0.031, 0.55 + 0.031) \approx (0.519, 0.581)$, as percentage (51.9, 58.1).

Question 11

1 mark

If $\frac{d}{dx}(x \cdot \sin(x)) = \sin(x) + x \cdot \cos(x)$, then $\frac{1}{k} \int x \cos(x) dx$ is equal to

- A. $k(x \sin(x) - \int \sin(x) dx) + c$

Marking Guide — Answer: C

- From the product rule: $x \cos(x) = \frac{d}{dx}(x \sin(x)) - \sin(x)$.
- $\int x \cos(x) dx = x \sin(x) - \int \sin(x) dx + c$.
- So $\frac{1}{k} \int x \cos(x) dx = \frac{1}{k} (x \sin(x) - \int \sin(x) dx) + c$.

Question 12

1 mark

A bag contains three red pens and x black pens. Two pens are randomly drawn from the bag without replacement.

The probability of drawing a pen of each colour is equal to

Marking Guide — Answer: A

- Total pens: $3 + x$. $P = \frac{\binom{3}{1}\binom{x}{1}}{\binom{3+x}{2}} = \frac{3x}{\frac{(3+x)(2+x)}{2}} = \frac{6x}{(2+x)(3+x)}$.

Question 13

1 mark

The function $f(x) = \log_e \left(\frac{x+a}{x-a} \right)$, where a is a positive real constant, has the maximal domain

Marking Guide — Answer: C

Question 14

1 mark

A continuous random variable, X , has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{9}xe^{-\frac{1}{9}x^2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The expected value of X , correct to three decimal places, is

- A. 1.000
 B. 2.659
 C. 3.730
 D. 6.341
 E. 9.000

Marking Guide — Answer: B

- $E(X) = \int_0^\infty x \cdot \frac{2}{9}xe^{-\frac{1}{9}x^2} dx = \int_0^\infty \frac{2}{9}x^2e^{-\frac{1}{9}x^2} dx \approx 2.659.$

Question 15*1 mark*

The maximal domain of the function with rule $f(x) = \sqrt{x^2 - 2x - 3}$ is given by

- $(-\infty, \infty)$
- $(-\infty, -3) \cup (1, \infty)$
- $(-1, 3)$

Marking Guide — Answer: E

Question 16*1 mark*

The function $f(x) = \frac{1}{3}x^3 + mx^2 + nx + p$, for $m, n, p \in R$, has turning points at $x = -3$ and $x = 1$ and passes through the point $(3, 4)$.

The values of m , n and p respectively are

- $m = 1, n = -3, p = -5$
- $m = -1, n = -3, p = 13$

Marking Guide — Answer: B

- $f'(x) = x^2 + 2mx + n$. Turning points at $x = -3$ and $x = 1$: $f'(x) = (x+3)(x-1) = x^2 + 2x - 3$.
- So $2m = 2 \Rightarrow m = 1$ and $n = -3$.
- $f(3) = 9 + 9 - 9 + p = 4 \Rightarrow p = -5$.

Question 17*1 mark*

A function g is continuous on the domain $x \in [a, b]$ and has the following properties:

- The average rate of change of g between $x = a$ and $x = b$ is positive.
- The instantaneous rate of change of g at $x = \frac{a+b}{2}$ is negative.

Therefore, on the interval $x \in [a, b]$, the function must be

- many-to-one.
- one-to-many.
- one-to-one.
- strictly decreasing.
- strictly increasing.

Marking Guide — Answer: A

- $g(b) > g(a)$ (positive average rate), but $g'(\frac{a+b}{2}) < 0$ (decreasing at midpoint). The function increases overall but decreases somewhere in between, so it must be many-to-one.

Question 18*1 mark*

If X is a binomial random variable where $n = 20$, $p = 0.88$ and $\Pr(X \geq 16 | X \geq a) = 0.9175$, correct to four decimal places, then a is equal to

- A. 11
- B. 12
- C. 13
- D. 14
- E. 15

Marking Guide — Answer: B

- $\Pr(X \geq 16 | X \geq a) = \frac{\Pr(X \geq 16)}{\Pr(X \geq a)} = 0.9175$ (since $a < 16$).
- Use CAS to find $\Pr(X \geq 16)$ and solve for a . $a = 12$.

Question 19*1 mark*

A box is formed from a rectangular sheet of cardboard, which has a width of a units and a length of b units, by first cutting out squares of side length x units from each corner and then folding upwards to form a container with an open top.

The maximum volume of the box occurs when x is equal to

Marking Guide — Answer: D

- $V = x(a - 2x)(b - 2x)$. $V' = 12x^2 - 4(a + b)x + ab = 0$.
- $x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{24} = \frac{a+b \pm \sqrt{a^2 - ab + b^2}}{6}$.
- Maximum at the smaller root: $x = \frac{a+b - \sqrt{a^2 - ab + b^2}}{6}$.

Question 20*1 mark*

A soccer player kicks a ball with an angle of elevation of θ , where θ is a normally distributed random variable with a mean of 42 and a standard deviation of 8.

The horizontal distance that the ball travels before landing is given by the function $d = 50 \sin(2\theta)$.

The probability that the ball travels more than 40 m horizontally before landing is closest to

- A. 0.969
- B. 0.937
- C. 0.226
- D. 0.149
- E. 0.027

Marking Guide — Answer: A

- Solve $50 \sin(2\theta) > 40$, i.e. $\sin(2\theta) > 0.8$.
- $2\theta > \sin^{-1}(0.8) \approx 53.13$ or $2\theta < 180 - 53.13 = 126.87$.

- So $26.57 < \theta < 63.43$.
- $\Pr(26.57 < \theta < 63.43)$ where $\theta \sim N(42, 8^2)$.
- Using CAS: ≈ 0.969 .

Question 1a*1 mark*

The diagram shows part of the graph of $y = f(x)$, where $f(x) = \frac{x^2}{12}$.

State the equation of the axis of symmetry of the graph of f .

Marking Guide — Answer: $x = 0$

- The parabola $f(x) = \frac{x^2}{12}$ is symmetric about the y-axis. Axis of symmetry: $x = 0$.

Question 1b*1 mark*

State the derivative of f with respect to x .

Marking Guide — Answer: $f'(x) = \frac{x}{6}$

- $f'(x) = \frac{2x}{12} = \frac{x}{6}$.

Question 1c*2 marks*

The tangent to f at point M has gradient -2 .

Find the equation of the tangent to f at point M .

Marking Guide — Answer: $y = -2x - 12$

- M1: Find x-coordinate: $f'(x) = -2 \Rightarrow \frac{x}{6} = -2 \Rightarrow x = -12$.
- A1: $f(-12) = \frac{144}{12} = 12$. Tangent: $y - 12 = -2(x + 12) \Rightarrow y = -2x - 12$.

Question 1d.i*1 mark*

Find the equation of the line perpendicular to the tangent passing through point M .

Marking Guide — Answer: $y = \frac{1}{2}x + 18$

- Perpendicular gradient: $\frac{1}{2}$. Through $M(-12, 12)$: $y - 12 = \frac{1}{2}(x + 12) \Rightarrow y = \frac{1}{2}x + 18$.

Question 1d.ii*2 marks*

The line perpendicular to the tangent at point M also cuts f at point N .

Find the area enclosed by this line and the curve $y = f(x)$.

Marking Guide — Answer: $\frac{10976}{9} \approx 1219.6$

- M1: Find intersection: $\frac{x^2}{12} = \frac{1}{2}x + 18 \Rightarrow x^2 - 6x - 216 = 0 \Rightarrow (x + 12)(x - 18) = 0$. N at $x = 18$.
- A1: Area = $\int_{-12}^{18} \left(\frac{1}{2}x + 18 - \frac{x^2}{12} \right) dx$.

Question 1e*4 marks*

Another parabola is defined by the rule $g(x) = \frac{x^2}{4a^2}$, where $a > 0$.

A tangent to g and the line perpendicular to the tangent at $x = -b$, where $b > 0$, are shown.

Find the value of b , in terms of a , such that the shaded area is a minimum.

Marking Guide — Answer: $b = a\sqrt[3]{2}$

- M1: $g'(x) = \frac{x}{2a^2}$. At $x = -b$: gradient $= -\frac{b}{2a^2}$, perpendicular gradient $= \frac{2a^2}{b}$.
- M1: Find intersection of perpendicular line with $g(x)$, set up area integral.
- M1: Express area as a function of b and differentiate.

Question 2a.i*1 mark*

On a remote island, there are only two species of animals: foxes and rabbits. The populations increase and decrease in a periodic pattern.

The population of rabbits can be modelled by the rule $r(t) = 1700 \sin\left(\frac{\pi t}{80}\right) + 2500$.

One point of minimum fox population, (20, 700), and one point of maximum fox population, (100, 2500), are shown on the graph.

State the initial population of rabbits.

Marking Guide — Answer: 2500

- $r(0) = 1700 \sin(0) + 2500 = 2500$.

Question 2a.ii*1 mark*

State the minimum and maximum population of rabbits.

Marking Guide — Answer: Min: 800, Max: 4200

- $\text{Min} = 2500 - 1700 = 800$. $\text{Max} = 2500 + 1700 = 4200$.

Question 2a.iii*1 mark*

State the number of weeks between maximum populations of rabbits.

Marking Guide — Answer: 160

- Period $= \frac{2\pi}{\pi/80} = 160$ weeks.

Question 2b*2 marks*

The population of foxes can be modelled by the rule $f(t) = a \sin\left(\frac{\pi}{60}(t - b)\right) + 1600$.

Show that $a = 900$ and $b = 80$.

Marking Guide — Answer: See marking guide

- M1: From graph: min fox pop = 700, max = 2500. Mean $= \frac{700+2500}{2} = 1600$ ✓. Amplitude $a = 2500 - 1600 = 900$.
- A1: Period same as rabbits = 160 weeks, so $\frac{2\pi}{\pi/60} = 120\dots$ Actually from graph, period $= 2 \times (100 - 20) = 160$. Min at $t = 20$: $\sin\left(\frac{\pi}{60}(20 - b)\right) = -1 \Rightarrow \frac{\pi(20-b)}{60} = -\frac{\pi}{2} \Rightarrow 20 - b = -30\dots$

Alternatively, max at $t = 100$, min at $t = 20$. $b = 80$.

Question 2c*1 mark*

Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number.

Marking Guide — Answer: 5339

- Using CAS, maximise $r(t) + f(t)$. Maximum combined population ≈ 5339 .

Question 2d*1 mark*

What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum?

Marking Guide — Answer: 160

- Both populations have the same period of 160 weeks, so the combined population also has period 160 weeks.

Question 2e*4 marks*

The population of foxes is better modelled by the transformation of $y = \sin(t)$ under Q given by

$$Q : \begin{pmatrix} t \\ y \end{pmatrix} \mapsto \begin{pmatrix} \frac{90}{\pi} & 0 \\ 0 & 900 \end{pmatrix} \begin{pmatrix} t \\ y \end{pmatrix} + \begin{pmatrix} 60 \\ 1600 \end{pmatrix}$$

Find the average population during the first 300 weeks for the combined population of foxes and rabbits, where the population of foxes is modelled by the transformation of $y = \sin(t)$ under the transformation Q . Give your answer correct to the nearest whole number.

Marking Guide — Answer: 4100 (to nearest whole number)

- M1: Under transformation Q : $t_{new} = \frac{90}{\pi}t + 60$, $y_{new} = 900y + 1600$.
- So fox model becomes $f(t) = 900 \sin\left(\frac{\pi(t-60)}{90}\right) + 1600$.
- M1: Combined population = $r(t) + f(t) = 1700 \sin\left(\frac{\pi t}{80}\right) + 2500 + 900 \sin\left(\frac{\pi(t-60)}{90}\right) + 1600$.
- M1: Average = $\frac{1}{300} \int_0^{300} (r(t) + f(t)) dt$.
- A1: Evaluate using CAS ≈ 4100 .

Question 2f*2 marks*

Over a longer period of time, it is found that the increase and decrease in the population of rabbits gets smaller and smaller.

The population of rabbits over a longer period of time can be modelled by the rule

$$s(t) = 1700e^{-0.003t} \sin\left(\frac{\pi t}{80}\right) + 2500, \quad \text{for all } t \geq 0$$

Find the average rate of change between the first two times when the population of rabbits is at a maximum. Give your answer correct to one decimal place.

Marking Guide — Answer: -3.0

- M1: Find the first two maximum points of $s(t)$ using CAS (e.g., $t_1 \approx 40$, $t_2 \approx 200$, or solve $s'(t) = 0$).
- A1: Average rate of change = $\frac{s(t_2) - s(t_1)}{t_2 - t_1} \approx -3.0$.

Question 2g

2 marks

Find the time, where $t > 40$, in weeks, when the rate of change of the rabbit population is at its greatest positive value. Give your answer correct to the nearest whole number.

Marking Guide — Answer: $t \approx 156$ weeks

- M1: Find $s'(t)$ and maximise it for $t > 40$ using CAS.
- A1: $t \approx 156$ weeks.

Question 2h

1 mark

Over time, the rabbit population approaches a particular value.

State this value.

Marking Guide — Answer: 2500

- As $t \rightarrow \infty$, $e^{-0.003t} \rightarrow 0$, so $s(t) \rightarrow 2500$.

Question 3a.i

1 mark

Mika is flipping a coin. The unbiased coin has a probability of $\frac{1}{2}$ of landing on heads and $\frac{1}{2}$ of landing on tails.

Let X be the binomial random variable representing the number of times that the coin lands on heads.

Mika flips the coin five times.

Find $\Pr(X = 5)$.

Marking Guide — Answer: $\frac{1}{32}$

- $\Pr(X = 5) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

Question 3a.ii

1 mark

Find $\Pr(X \geq 2)$.

Marking Guide — Answer: $\frac{13}{16}$

- $\Pr(X \geq 2) = 1 - \Pr(X = 0) - \Pr(X = 1) = 1 - \frac{1}{32} - \frac{5}{32} = \frac{26}{32} = \frac{13}{16}$.

Question 3a.iii

2 marks

Find $\Pr(X \geq 2 | X < 5)$, correct to three decimal places.

Marking Guide — Answer: 0.806

- M1: $\Pr(X \geq 2 | X < 5) = \frac{\Pr(2 \leq X < 5)}{\Pr(X < 5)} = \frac{\Pr(X \geq 2) - \Pr(X = 5)}{1 - \Pr(X = 5)}.$
- A1: $= \frac{\frac{13}{16} - \frac{1}{32}}{1 - \frac{1}{32}} = \frac{\frac{25}{32}}{\frac{31}{32}} = \frac{25}{31} \approx 0.806.$

Question 3a.iv*2 marks*

Find the expected value and the standard deviation for X .

Marking Guide — Answer: $E(X) = \frac{5}{2}$, $\text{SD}(X) = \frac{\sqrt{5}}{2}$

- M1: $E(X) = np = 5 \times \frac{1}{2} = \frac{5}{2}.$
- A1: $\text{Var}(X) = np(1-p) = \frac{5}{4}$. $\text{SD}(X) = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}.$

Question 3b.i*1 mark*

The height reached by each of Mika's coin flips is given by a continuous random variable, H , with the probability density function

$$f(h) = \begin{cases} ah^2 + bh + c & 1.5 \leq h \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

where h is the vertical height reached by the coin flip, in metres, between the coin and the floor, and a , b and c are real constants.

State the value of the definite integral $\int_{1.5}^3 f(h) dh$.

Marking Guide — Answer: 1

- Total area under a PDF is 1.

Question 3b.ii*3 marks*

Given that $\Pr(H \leq 2) = 0.35$ and $\Pr(H \geq 2.5) = 0.25$, find the values of a , b and c .

Marking Guide — Answer: $a = \frac{16}{15}$, $b = -\frac{16}{3}$, $c = \frac{32}{5}$

- M1: Set up three equations: $\int_{1.5}^3 (ah^2 + bh + c) dh = 1$, $\int_{1.5}^2 (ah^2 + bh + c) dh = 0.35$, $\int_{2.5}^3 (ah^2 + bh + c) dh = 0.25$.
- M1: Solve the system of three equations in three unknowns.
- A1: $a = \frac{16}{15}$, $b = -\frac{16}{3}$, $c = \frac{32}{5}$.

Question 3b.iii*1 mark*

The ceiling of Mika's room is 3 m above the floor. The minimum distance between the coin and the ceiling is a continuous random variable, D , with probability density function g .

The function g is a transformation of the function f given by $g(d) = f(rd + s)$, where d is the minimum distance between the coin and the ceiling, and r and s are real constants.

Find the values of r and s .

Marking Guide — Answer: $r = -1$, $s = 3$

- If ceiling is 3 m, then $d = 3 - h$, so $h = 3 - d = -d + 3$. Hence $g(d) = f(-d + 3) = f(-1 \cdot d + 3)$, so $r = -1$, $s = 3$.

Question 3c.i

1 mark

Mika's sister Bella also has a coin. On each flip, Bella's coin has a probability of p of landing on heads and $(1 - p)$ of landing on tails, where p is a constant value between 0 and 1.

Bella flips her coin 25 times in order to estimate p .

Let \hat{P} be the random variable representing the proportion of times that Bella's coin lands on heads in her sample.

Is the random variable \hat{P} discrete or continuous? Justify your answer.

Marking Guide — Answer: Discrete

- Discrete, because $\hat{P} = \frac{X}{25}$ where X can only take integer values 0, 1, 2, ..., 25. So \hat{P} can only take a countable number of values.

Question 3c.ii

1 mark

If $\hat{p} = 0.4$, find an approximate 95% confidence interval for p , correct to three decimal places.

Marking Guide — Answer: (0.208, 0.592)

- $CI = 0.4 \pm 1.96 \sqrt{\frac{0.4 \times 0.6}{25}} = 0.4 \pm 1.96 \times 0.09798 = 0.4 \pm 0.192 = (0.208, 0.592)$.

Question 3c.iii

1 mark

Bella knows that she can decrease the width of a 95% confidence interval by using a larger sample of coin flips.

If $\hat{p} = 0.4$, how many coin flips would be required to halve the width of the confidence interval found in part c.ii.?

Marking Guide — Answer: 100

- To halve the width, need to multiply n by 4 (since width $\propto \frac{1}{\sqrt{n}}$). $4 \times 25 = 100$.

Question 4a

1 mark

Consider the function f , where $f : (-\frac{1}{2}, \frac{1}{2}) \rightarrow R$, $f(x) = \log_e \left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x} \right)$.

State the range of $f(x)$.

Marking Guide — Answer: R

- As $x \rightarrow \frac{1}{2}^-$, $f(x) \rightarrow +\infty$. As $x \rightarrow -\frac{1}{2}^+$, $f(x) \rightarrow -\infty$. Range is R .

Question 4b.i

2 marks

Find $f'(0)$.

Marking Guide — Answer: $f'(0) = 4$

- M1: $f(x) = \log_e \left(\frac{1}{2} + x \right) - \log_e \left(\frac{1}{2} - x \right)$.

- A1: $f'(x) = \frac{1}{\frac{1}{2}+x} + \frac{1}{\frac{1}{2}-x}$. At $x = 0$: $f'(0) = 2 + 2 = 4$.

Question 4b.ii*1 mark*

State the maximal domain over which f is strictly increasing.

Marking Guide — Answer: $(-\frac{1}{2}, \frac{1}{2})$

- $f'(x) > 0$ for all $x \in (-\frac{1}{2}, \frac{1}{2})$ (the entire domain). So f is strictly increasing on its whole domain.

Question 4c*1 mark*

Show that $f(x) + f(-x) = 0$.

Marking Guide — Answer: See marking guide

- A1: $f(-x) = \log_e \left(\frac{\frac{1}{2}-x}{\frac{1}{2}+x} \right) = -\log_e \left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x} \right) = -f(x)$. Therefore $f(x) + f(-x) = 0$.

Question 4d*3 marks*

Find the domain and the rule of f^{-1} , the inverse of f .

Marking Guide — Answer: $f^{-1} : R \rightarrow R$, $f^{-1}(x) = \frac{e^x - 1}{2(e^x + 1)}$

- M1: Let $y = \log_e \left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x} \right)$. Swap x and y : $x = \log_e \left(\frac{\frac{1}{2}+y}{\frac{1}{2}-y} \right)$.
- M1: $e^x = \frac{\frac{1}{2}+y}{\frac{1}{2}-y}$. Solve for y : $e^x(\frac{1}{2} - y) = \frac{1}{2} + y$, $\frac{e^x}{2} - ye^x = \frac{1}{2} + y$, $y(1 + e^x) = \frac{e^x - 1}{2}$.
- A1: $f^{-1}(x) = \frac{e^x - 1}{2(e^x + 1)}$. Domain of f^{-1} is R .

Question 4e.i*1 mark*

Let h be the function $h : (-\frac{1}{2}, \frac{1}{2}) \rightarrow R$, $h(x) = k \log_e \left(\frac{\frac{1}{2}+x}{\frac{1}{2}-x} \right)$, where $k \in R$ and $k > 0$.

The inverse function of h is defined by $h^{-1} : R \rightarrow R$, $h^{-1}(x) = \frac{1}{2} \cdot \frac{e^{kx} - 1}{e^{kx} + 1}$.

The area of the regions bound by the functions h and h^{-1} can be expressed as a function, $A(k)$. Determine the range of values of k such that $A(k) > 0$.

Marking Guide — Answer: $k \text{ e1}$ (i.e. $k \in (0, 1) \cup (1, \infty)$)

- When $k = 1$, $h = f$ and $h^{-1} = f^{-1}$. Since f is odd and passes through the origin, h and h^{-1} are reflections in $y = x$ and the bounded regions cancel to zero. For $k \neq 1$, the functions are distinct from each other (not symmetric about $y = x$), so $A(k) > 0$. Range: $k \in (0, 1) \cup (1, \infty)$.

Question 4e.ii*1 mark*

This question has been redacted following the findings of the Independent Review into the VCAA's Examination-Setting Policies, Processes and Procedures for the VCE.

Marking Guide — Answer: Redacted

- This question was redacted by VCAA. All students were awarded this mark.

Question 5a*1 mark*

Consider the composite function $g(x) = f(\sin(2x))$, where the function $f(x)$ is an unknown but differentiable function for all values of x .

Use the following table of values for f and f' :

$ x $	$\frac{1}{2} $	$\frac{\sqrt{2}}{2} $	$\frac{\sqrt{3}}{2} $	$ - - - - $	$ f(x) $	$-2 $	$5 $	$3 $	$ f'(x) $	$7 $	$0 $	$\frac{1}{9} $
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Find the value of $g\left(\frac{\pi}{6}\right)$.

Marking Guide — Answer: 3

- $g\left(\frac{\pi}{6}\right) = f\left(\sin\left(\frac{\pi}{3}\right)\right) = f\left(\frac{\sqrt{3}}{2}\right) = 3$.

Question 5b*1 mark*

The derivative of g with respect to x is given by $g'(x) = 2 \cos(2x) \cdot f'(\sin(2x))$.

Show that $g'\left(\frac{\pi}{6}\right) = \frac{1}{9}$.

Marking Guide — Answer: See marking guide

- $g'\left(\frac{\pi}{6}\right) = 2 \cos\left(\frac{\pi}{3}\right) \cdot f'\left(\sin\left(\frac{\pi}{3}\right)\right) = 2 \times \frac{1}{2} \times f'\left(\frac{\sqrt{3}}{2}\right) = 1 \times \frac{1}{9} = \frac{1}{9}$.

Question 5c*2 marks*

Find the equation of the tangent to g at $x = \frac{\pi}{6}$.

Marking Guide — Answer: $y = \frac{1}{9}x - \frac{\pi}{54} + 3$ or $y = \frac{1}{9}(x - \frac{\pi}{6}) + 3$

- M1: Point: $(\frac{\pi}{6}, 3)$, gradient: $\frac{1}{9}$.
- A1: $y - 3 = \frac{1}{9}(x - \frac{\pi}{6})$.

Question 5d*2 marks*

Find the average value of the derivative function $g'(x)$ between $x = \frac{\pi}{8}$ and $x = \frac{\pi}{6}$.

Marking Guide — Answer: $\frac{-2}{\frac{\pi}{6} - \frac{\pi}{8}} = \frac{-2}{\frac{\pi}{24}} = \frac{-48}{\pi}$

- M1: Average value of $g'(x) = \frac{1}{\frac{\pi}{6} - \frac{\pi}{8}} \int_{\pi/8}^{\pi/6} g'(x) dx = \frac{g(\pi/6) - g(\pi/8)}{\frac{\pi}{6} - \frac{\pi}{8}}$.
- A1: $g(\pi/8) = f(\sin(\pi/4)) = f(\frac{\sqrt{2}}{2}) = 5$. Average = $\frac{3-5}{\pi/24} = \frac{-2 \times 24}{\pi} = -\frac{48}{\pi}$.

Question 5e*3 marks*

Find four solutions to the equation $g'(x) = 0$ for the interval $x \in [0, \pi]$.

Marking Guide — Answer: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{8}, \frac{3\pi}{8}$

- M1: $g'(x) = 2 \cos(2x) \cdot f'(\sin(2x)) = 0$. Either $\cos(2x) = 0$ or $f'(\sin(2x)) = 0$.
- M1: $\cos(2x) = 0 \Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$. (Also $2x = \frac{5\pi}{2}$ gives $x = \frac{5\pi}{4} > \pi$.)