$\boxed{\text{2014 AUMC Solutions}}$

1. Logarithms and exponents.

The solutions are $\boxed{x = 10^5 \text{ and } x = 10^{-5}}$. If we take base 10 logs on each side of the equation we get

$$2(\log_{10} x)(\log_{10} x) = 50,$$

so $(\log_{10} x)^2 = 25$ and $\log_{10} x = \pm 5$. Thus $x = 10^5$ or $x = 10^{-5}$.

2. Base b integers.

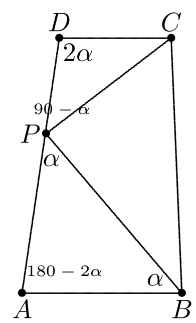
We show that $\boxed{b = 11 \text{ and } n = 1766}$ in base 10. We have $n = b^3 + 3b^2 + 6b + 6$, so $2n = 2b^3 + 6b^2 + 12b + 12$. Also, from the base b representation of $2n$ we have $2n = 2b^3 + 7b^2 + 2b + 1$. Equating the two expressions for $2n$ we get $b^2 - 10b - 11 = 0$; i.e., $(b - 11)(b + 1) = 0$. As b is positive, it follows that in base 10, $b = 11$, and $n = 1 \cdot 11^3 + 2 \cdot 11^2 + 6 \cdot 11 + 6 = 1766$

3. Lists of 2014 words.

It is $\boxed{1866}$. If the lists of A and B have no words in common, then the 114 words removed from A's list and the 34 removed from B's list constitute 148 distinct words, all of which had to be removed from C's list. In this case 1866 words remain on C's list, and it is clear that this is the smallest number which can remain there.

4. It's a right angle.

Let $\alpha = \angle APB$. Triangle PAB is isosceles with $AP = AB$, so $\angle ABP = \alpha$ also. Then $\angle BAP = 180° - 2\alpha$. Because $CD \parallel AB$, $\angle PDC = 2\alpha$. Triangle PDC is isosceles with $PD = DC$, so angles DPC and DCP are equal, with sum $180° - 2\alpha$, so each is $90° - \alpha$. Then $\angle DPC + \angle APB = (90° - \alpha) + \alpha = 90°$, so $\angle CPB = 90°$.



5. A quadratic with real roots.

Because r and s are solutions, $r^2 = ar - a$ and $s^2 = as - a$, so $r^2 + s^2 = a(r + s) - 2a$. But $r + s = a$ (the coefficient of x is the sum of the roots), so $r^2 + s^2 = a^2 - 2a$. Finally, because the roots are real, the discriminant is non-negative; i.e., $a^2 - 4a \geq 0$. Thus $a^2 \geq 4a$, and

$$r^2 + s^2 = a^2 - 2a \geq 4a - 2a = 2a = 2(r + s).$$

6. Floor function equation.

There are $\boxed{167}$ solutions. Because $\lfloor x \rfloor \leq x$ for all x, with equality if and only if x is an integer, the left member is smaller than the right unless all three of $n/2$, $n/3$ and $n/4$ are integers. This is the case if and only if n is a multiple of 12, and there are 167 multiples of 12 in the range from 1 to 2014.

7. Integral inequality.

For t in the interval $(5, 10)$, we have $t^2 - 24 > 0$, so

$$0 < \frac{t^3}{t^7 + t^2 - 24} < \frac{t^3}{t^7} = \frac{1}{t^4}.$$

Then

$$\begin{aligned} 0 &< \int_5^{10} \frac{t^3 dt}{t^7 + t^2 - 24} < \int_5^{10} t^{-4} dt \\ &= -\frac{1}{3}\left(\frac{1}{10^3} - \frac{1}{5^3}\right) = \frac{1}{3}\left(\frac{2^3}{10^3} - \frac{1}{10^3}\right) \\ &= \frac{7}{3 \cdot 10^3} < (2.5)10^{-3} = .0025. \end{aligned}$$

8. Coefficient of x^2, with constant term 2014.

The only value of k which satisfies these conditons is $\boxed{k = 23}$. Let the four roots be r, s, t, u, with $rs = -53$. Then $rstu = -2014$, so $tu = 38$. We also have

$$-1976 = rst + rsu + rtu + stu = rs(t + u) + tu(r + s) = -53(t + u) + 38(r + s),$$

and $39 = r + s + t + u$, so if $p = r + s$ and $q = t + u$ we have $-1976 = 38p - 53q$ and $p + q = 39$. These two linear equations yield $p = 1$ and $q = 38$. Finally, then,

$$k = rs + rt + ru + st + su + tu = -53 + (r + s)(t + u) + 38 = -15 + (1)(38) = 23.$$

9. The square of an integer.

The values are $\boxed{n = -3,\ -8,\ -11 \text{ and } -16}$. Here is one way to find them. The following equations are equivalent:

$$n^2 + 19n + 97 = m^2 = (n+k)^2;$$

$$\left(n + \frac{19}{2}\right)^2 + \frac{27}{4} = (n+k)^2;$$

$$\frac{27}{4} = (n+k)^2 - \left(n + \frac{19}{2}\right)^2 = \left(k - \frac{19}{2}\right)\left(2n + k + \frac{19}{2}\right);$$

$$(2k-19)(4n+2k+19) = 27.$$

Thus $2k - 19$ must be one of ± 1, ± 3, ± 9, ± 27.

If $2k - 19 = 1$ then $k = 10$, $4n + 2k + 19 = 27$, and $n = -3$, in which case $n^2 + 19n + 97 = 49 = 7^2$. Checking the other possibilities similarly leads to the following table of results:

$2k-19$	k	n	$n^2+19n+97$
1	10	-3	49
-1	9	-16	49
3	11	-8	9
-3	8	-11	9
9	14	-11	9
-9	5	-8	9
27	23	-16	49
-27	-4	-3	49

(1)

10. A global minimum.

The minimum value of $f(x)$ is $\boxed{-2}$. Substitute $\cos 2x = 2\cos^2 x - 1$ to obtain

$$f(x) = x^2 - 4x\cos x + 4\cos^2 x - 2 = (x - 2\cos x)^2 - 2.$$

From this it it clear that $f(x) \geq -2$ for all real x. We show that the value -2 occurs, and therefore it is the minimum value. If $g(x) = x - 2\cos x$, then $g(0) = -2$ and $g(\pi/2) = \pi/2$, so by the Intermediate Value Theorem for the continuous function g we conclude that $g(x) = 0$ for some x between 0 and $\pi/2$. At this x we have $f(x) = -2$, which is therefore the minimum value.