NAME:	CIRCLE:	Zweck 11:30am	Zweck 2:30pm

1 /10	2 /12	3 /8	4 /8	5 /12	
6 /10	7 /10	8 /10	9 /12	10 /8	T /100

MATH 2415 Final Exam, Fall 2015 (Zweck)

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. **Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts]
(a) Find the area of the parallelogram with vertices $(1, 1)$, $(3, 4)$, $(5, 6)$ and $(7, 9)$.

(b) Calculate the vector projection of $\mathbf{u} = (1, 2, -4)$ onto $\mathbf{v} = (3, -2, 1)$.

(2) [12 pts] Let C be the curve in $\mathbf{R}^2$ parametrized by $(x, y) = \mathbf{r}(t) = (3\cos t, 4\sin t)$ for $0 \leq t \leq \pi/2$.

(a) Sketch the curve C.

(b) Calculate $\int_C f\, ds$ where $f(x, y) = xy$.

(c) Let $\mathbf{F}(x, y) = y\mathbf{i} + x^2\mathbf{j}$. Find a function $g = g(t)$ and numbers a and b so that $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b g(t)\, dt$.

(3) [8 pts] Find the limit if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$

(4) [8 pts] Let $z = f(x, y) = 3x^2 + 4xy + 5y^2$.

(a) Calculate the equation of the tangent plane to the graph of f at $(x_0, y_0) = (2, -1)$.

(b) Suppose that an ant is walking on a hot plate in the xy-plane and that the function $z = f(x, y)$ given above is the temperature of the hot plate at the point (x, y). Suppose that at time $t = 0$ the position of the ant is $\mathbf{x} = (2, -1)$ and the velocity of the ant is $\mathbf{v} = (4, 3)$. What is the rate of change of the temperature of the ant's feet at time $t = 0$?

(5) [12 pts] Find the absolute maximum and absolute minimum of the function $f(x, y) = x + y - xy$ on the triangle in the xy-plane with vertices $(0, 0)$, $(4, 0)$, and $(0, 2)$.

(6) [10 pts] Use spherical coordinates to calculate the triple integral $\iiint_E z \, dV$, where E is the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.

(7) [10 pts]
Let E be the solid region in $\mathbf{R}^3$ bounded by the surfaces $z = 1 - y^2$, $y = x - 1$, $x = 0$, and $z = 0$.

(a) Sketch E. Is $\iiint_E y\, dV$ positive or negative? Why?

(b) Calculate $\iiint_E y\, dV$.

(8) [10 pts] Use the Change of Variables Theorem to evaluate the integral $\iint_R y\,dA$, where R is the quadrilateral region bounded by the lines $x + 2y = 2$, $x + 2y = 4$, $x = 0$, and $y = 0$. **Hint:** Let $u = x + 2y$ and $v = y$.

(9) [12 pts] Let $\mathbf{F}$ be the vector field in the plane given by $\mathbf{F}(x, y) = x^2y\mathbf{i} + (x^2 - y^2)\mathbf{j}$.
(a) Calculate the divergence of $\mathbf{F}$.

(b) Calculate the curl of $\mathbf{F}$.

(c) Is $\mathbf{F}$ conservative? Why?

(d) Suppose that the vector field $\mathbf{F}$ given above is the velocity vector field of a fluid flowing in the plane. On average is the fluid flowing in or out of a small disk centered at the point $(-1, 2)$? Why?

(10) [8 pts]
(a) Define what it means for a vector field to be conservative.

(b) Define what it means for the integral of a vector field to be independent of path.

(c) Prove that if $\mathbf{F}$ is a conservative vector field then $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________