

LAST NAME:	FIRST NAME:	CIRCLE:			
		Makhijani 8:30am	Makhijani 11:30am	Makhijani 2:30pm	Zweck 11:30am

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MATH 2415 [Spring 2019] Exam II, Apr 5th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [12 pts]

(a) Calculate the (level set) equation of the tangent plane to the graph of $z = f(x, y) = x^2 + 2y^2 + 3x + y$ at $(x, y) = (2, 1)$.

(b) Use your answer to (a) to estimate $f(2.1, 0.8)$.

(2) [12 pts] Let $f(x, y) = e^x - y^2$.

(a) What is the direction of steepest ascent at $(x, y) = (0, 1)$?

(b) Sketch the level curve $f(x, y) = 0$, together with the direction of steepest ascent of f at $(x, y) = (0, 1)$.

(c) In which directions is that rate of change of f equal to zero at $(x, y) = (0, 1)$?

(d) Let $(x, y) = \mathbf{r}(t)$ be a curve with $\mathbf{r}(2) = (0, 1)$ and $\mathbf{r}'(2) = (-2, 3)$. Let $z = f(\mathbf{r}(t))$. Find $\frac{dz}{dt}$ at $t = 2$.

(3) [12 pts]

(a) Show that

$$(x, y, z) = \mathbf{r}(u, v) = (v, 2 \cos u, 3 \sin u)$$

is a parametrization of an elliptical cylinder. **Hint:** Find an equation of the form $F(x, y, z) = 0$ for this surface by eliminating u and v from the equations above.

(b) Find a parametrization of the tangent plane to this surface at the point where $(u, v) = (\frac{\pi}{4}, 2)$.

(4) [12 pts] Let D be the triangular domain in the xy -plane with vertices $(0, 0)$, $(2, 2)$ and $(2, 4)$. Calculate $\iint_D (x^2 + y^2) dA$.

(5) [15 pts] Find and classify all critical points of $z = f(x, y) = y^3 - 6xy + 8x^3$.

(6) [12 pts] Use the Method of Lagrange Multipliers to find the absolute maximum and minimum of the function $z = x + 2y$ on the circle $(x - 1)^2 + (y - 3)^2 = 5$.