NAME:	CIRCLE: Turi	Zweck 10am	Zweck 4pm

1 /6	2 /6	3 /6	4 /12	5 /10	6 /10	
7 /10	8 /8	9 /6	10 /9	11 /7	12 /8	T /100

MATH 2415 Final Exam, Fall 2014

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. **Show all work and give complete explanations**. Don't spend too much time on any one problem. This 2 hours 45 mins exam is worth 100 points.

(1) [6 pts]
(a) Find the area of the triangle with vertices $(1, 2, 3)$, $(2, 0, 3)$, and $(0, 2, 0)$.

(b) Find a so that the line $\mathbf{r}(t) = (2 - t, 2 + at, 4 + 3t)$ is perpendicular to the plane $2x + 3y - 6z = 1$.

(2) [6 pts] Suppose $z = f(x, y)$ is a function such that $f(1, 2) = 3$, $\frac{\partial f}{\partial x}(1, 2) = 4$ and $\frac{\partial f}{\partial y}(1, 2) = -5$. Use a tangent-plane approximation to the graph of f to estimate the value of f at the point $(0.9, 2.2)$.

(3) [6 pts] Suppose that $(x, y, z) = \mathbf{r}(t)$ is a parametrized curve whose speed is constant. Show that the acceleration vector of the curve is always perpendicular to the velocity vector of the curve.

(4) [12 pts] Find the absolute maximum and absolute minimum of the function $f(x, y) = x + y - xy$ on the triangle in the xy-plane with vertices $(0, 0)$, $(4, 0)$, and $(0, 2)$.

(5) [12 pts] The intersection of the surfaces $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$ is a pair of curves.
(a) Find a parametrization for one of these curves.

(b) Show that the curve you found in (a) lies in a plane.

(c) Make a single sketch in three-dimensional space which shows the surface $y^2 + z^2 = 1$, the curve you found in (a), and the plane you found in (b).

(6) [10 pts] Let E be the solid region bounded by the surfaces $x = 0$, $y = 0$, $z = 0$, $y = 1 - x^2$, and $x + y + z = 5$. Find a function $g(x)$ and numbers a and b so that $\iiint_E x\,dV = \int_a^b g(x)\,dx$.

(7) [10 pts]
(a) Let C be the line segment from $(1,0,0)$ to $(4,1,2)$ and let $\mathbf{F}(x,y,z) = z^2\mathbf{i} + y^2\mathbf{k}$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(b) Let C be the circle of radius 3, center $(0,0)$, oriented clockwise and let $\mathbf{F}(x,y) = y^3\mathbf{i} - x^3\mathbf{j}$. Use Green's Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(8) [8 pts] Use the Change of Variables Theorem to find the area of the ellipse $(\frac{x}{a})^2 + \frac{y}{b})^2 = 1$. Hint: Let $(x, y) = (au, bv)$.

(9) [6 pts] Evaluate $\iiint_E z\, dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 \leq 4$, $z \geq 0$.

(10) [9 pts] Let S be the parametrized surface $x = \frac{1}{2}u\cos v$, $y = u\sin v$, $z = u^2$.
(a) Find a function f so that S is the graph of $z = f(x, y)$.

(b) Sketch the surface S together with the curve on S where $u = 1$ and the curve on S where $v = \pi/4$.

(c) Find a parametrization of the tangent plane to S at the point where $(u, v) = (1, \pi/4)$.

(11) [7 pts] Let S be the part of the plane $x + y + z = 4$ in the first octant (i.e., where $x > 0$, $y > 0$, and $z > 0$). Calculate $\iint z\,dS$.

(12) [8 pts] Let $\mathbf{r}$ be the position vector field, $\mathbf{r}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(a) Calculate $\nabla \cdot \mathbf{r}$

(b) Calculate $\nabla \times \mathbf{r}$

(c) Let $r = |\mathbf{r}|$. Calculate $\nabla \cdot (\nabla r^2)$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________