LAST NAME:	FIRST NAME:	CIRCLE:			
SOLUTIONS		Makhijani 8:30am	Makhijani 11:30am	Makhijani 2:30pm	Zweck 11:30am

1 /12	2 /12	3 /15	4 /12	5 /12	6 /12	T /75

MATH 2415 [Spring 2019] Exam I, Feb 22nd

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [12 pts] Let P, Q, and R be the points $P = (2, -1, 6)$, $Q = (3, 1, 6)$, and $R = (2, 5, 1)$.

(a) Calculate the scalar projection of the vector $\overrightarrow{PQ}$ onto the vector $\overrightarrow{PR}$

$$\vec{u} = \overrightarrow{PQ} = Q - P = (3,1,6) - (2,-1,6) = (1,2,0)$$

$$\vec{v} = \overrightarrow{PR} = R - P = (2,5,1) - (2,-1,6) = (0,6,-5)$$

$$\text{SPROJ}_{\vec{v}}\, \vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{(1,2,0)\cdot(0,6,-5)}{(0,6,-5)} = \frac{12}{\sqrt{36+25}}$$

(b) Calculate the area of the triangle with vertices P, Q, and R.

$$A = \frac{1}{2}|\vec{u} \times \vec{v}|$$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & 6 & -5 \end{vmatrix} = (-10, 5, 6)$$

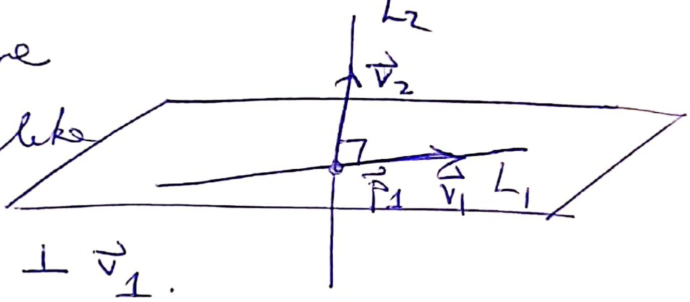
$$A = \frac{1}{2}\sqrt{100+25+36} = \frac{1}{2}\sqrt{161}$$

(2) [12 pts] Let L_1 and L_2 be the lines parametrized by

$$\mathbf{r}_1 = (-1 + 2t, 2 + 5t, 4 - 3t) \quad \text{and} \quad \mathbf{r}_2 = (-1 - 2t, 2 + t, 4 + 2t).$$
$$= \vec{p}_1 + t\vec{v}_1 \qquad\qquad = \vec{p}_2 + t\vec{v}_2$$

(a) Is there are plane that contains the line L_1 and is perpendicular to the line L_2? Justify your answer. If there is such a plane find an equation of the form $Ax + By + Cz = D$ for it.

If there was such a plane we would have a picture like this.



So we would have $\vec{v}_2 \perp \vec{v}_1$.

Now $\vec{v}_1 \cdot \vec{v}_2 = (2, 5, -3) \cdot (-2, 1, 2) = -5 \neq 0$

So $\vec{v}_1 \not\perp \vec{v}_2$.

So no such plane exists.

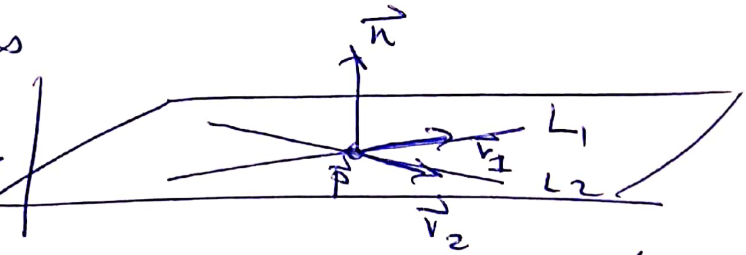
(b) Is there are plane that contains the lines L_1 and L_2? Justify your answer. If there is such a plane find an equation of the form $Ax + By + Cz = D$ for it.

The lines L_1 and L_2 both contain the point $\vec{p} = (-1, 2, 4)$ as $\vec{r}_1(0) = \vec{p} = \vec{r}_2(0)$.

Any time two non-parallel lines intersect w a point $\vec{p}$ the two lines lie in a common plane

The normal to this plane is

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & -3 \\ -2 & 1 & 2 \end{vmatrix}$$

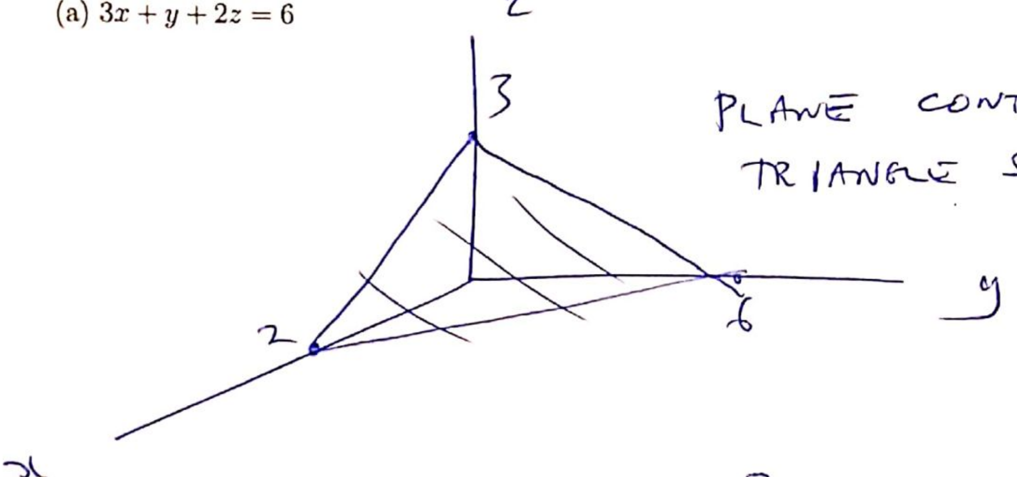


$\vec{n} = (13, 2, 12)$. Since $\vec{p}$ is a point in this plane the eqn is $0 = (\vec{r} - \vec{p}) \cdot \vec{n} = (x+1, y-2, z-4) \cdot (13, 2, 12)$

or $13x + 2y + 12z = 39$

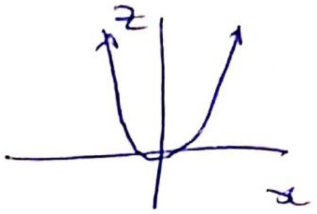
(3) [15 pts] Sketch the following surfaces. Make sure you label the axes and carefully show how you obtained your answers.

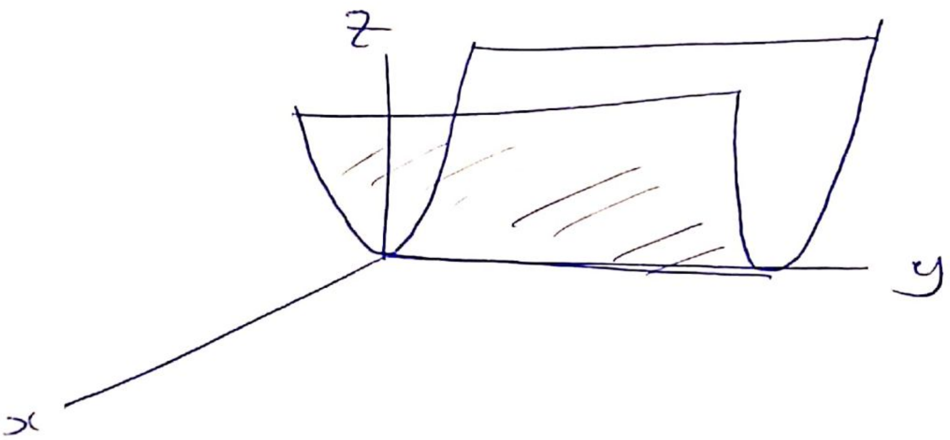
(a) $3x + y + 2z = 6$



PLANE CONTAINING TRIANGLE SHOWN.

(b) $z = x^2$



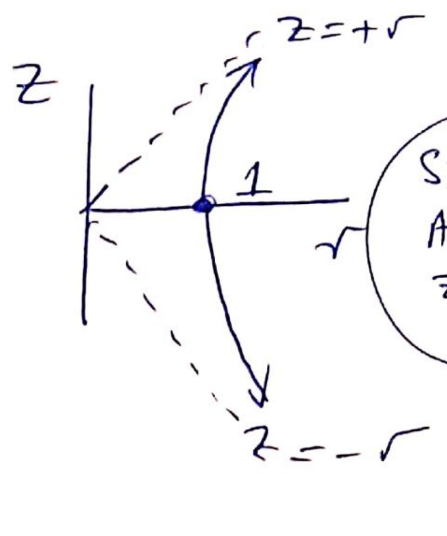


Translate along y-axis

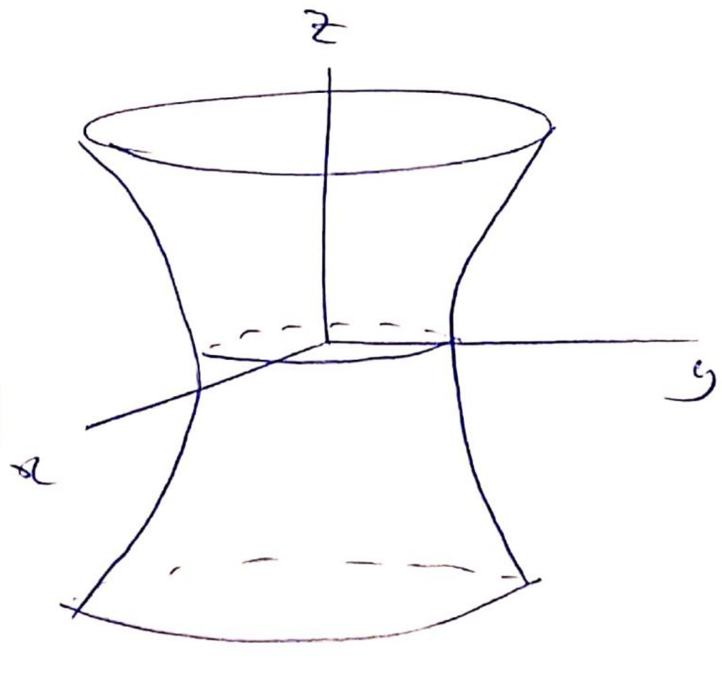
(c) $x^2 + y^2 - z^2 = 1$

CYL COORDS

$r^2 - z^2 = 1$



SPIN ABOUT Z-AXIS



(4) [12 pts] Let C be the curve parametrized by $\mathbf{r}(t) = (e^{-t}, \sqrt{2}t, e^{t})$.

(a) Find the length of the curve C from $(1, 0, 1)$ to $(e^{-1}, \sqrt{2}, e)$.

$$L = \int_0^1 |\vec{r}\,'(t)|\, dt$$

$$\vec{r}\,'(t) = (-e^{-t}, \sqrt{2}, e^{t})$$

$$|\vec{r}\,'(t)| = \sqrt{e^{-2t} + 2 + e^{2t}}$$

$$= \sqrt{(e^{-t} + e^{t})^2}$$

$$= e^{-t} + e^{t}$$

$$L = \int_0^1 e^{-t} + e^{t}\, dt$$

$$= \left[-e^{-t} + e^{t}\right]_0^1$$

$$= (-e^{-1} + e) - (-1 + 1) = e - \frac{1}{e}$$

(b) Find a parametrization of the tangent line to the curve C at $t = 0$.

$$\vec{r}\,'(0) = (-1, \sqrt{2}, 1)$$

$$\vec{r}(0) = (1, 0, 1)$$

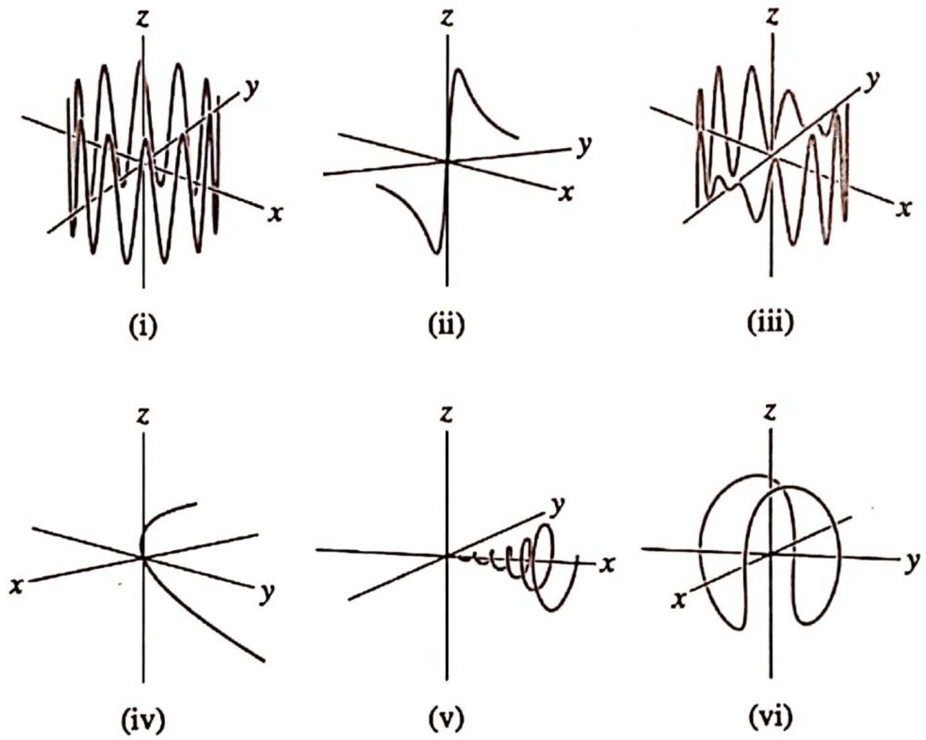
$$\vec{l}(s) = \vec{r}(0) + s\,\vec{r}\,'(0)$$

$$= (1, 0, 1) + s(-1, \sqrt{2}, 1)$$

$$= (1 - s, \sqrt{2}s, 1 + s)$$

(5) [12 pts]
Match the vector-valued functions with the space curves. Carefully justify your answers.

(A) $\mathbf{r}(t) = (t, t^2, 2t)$ (B) $\mathbf{r}(t) = (\cos t, \sin t, \sin 12t)$ (C) $\mathbf{r}(t) = (t, t\cos t, t\sin t)$



(A) = (iv) $\vec{r}(0) = (0,0,0)$

$y = t^2 > 0.$

$x = t,\ z = 2t$ here same sign

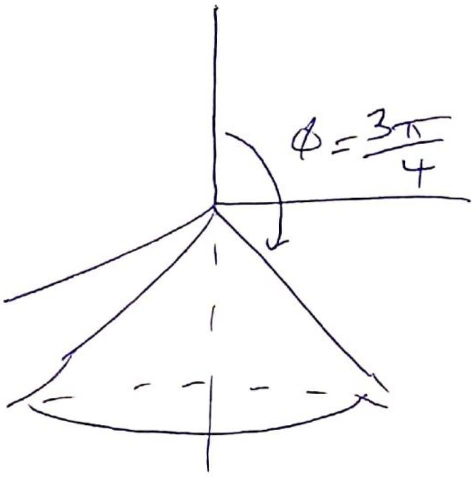
(B) = (i) Shadow on xy plane is circle $x^2+y^2=1$.

$z = \sin(12t)$ is periodic, period $\frac{2\pi}{12} = \frac{\pi}{6}$.

So 12 peaks as go around.

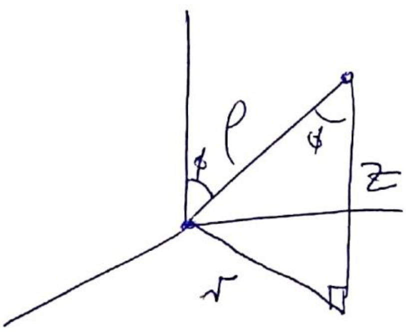
(C) = (v) $x^2 = y^2+z^2$. So curve lies on cone whose axis is x-axis

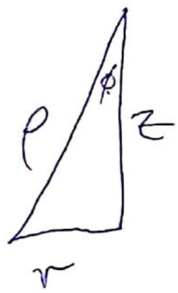
(6) [12 pts]
(a) Sketch the surface whose equation in the spherical coordinates is $\phi = 3\pi/4$.



CONE

(b) Let P be the point with spherical coordinates $(\rho, \theta, \phi) = (4, \pi/3, 3\pi/4)$. Find the cylindrical coordinates of P.





$$z = \rho \cos\phi$$

$$r = \rho \sin\phi$$

$$r = 4 \sin\frac{3\pi}{4} = \frac{4}{\sqrt{2}}$$

$$\theta = \pi/3$$

$$z = 4\cos\frac{3\pi}{4} = -\frac{4}{\sqrt{2}}$$