NAME:

1 /12	2 /10	3 /5	4 /10	5 /10	6 /8	
7 /10	8 /10	9 /5	10 /8	11 /8	12 /4	T /100

MATH 2415 (Spring 2014, **ZWECK**) Final Exam

No calculators, books or notes! Show all work and give **complete explanations**. This 2 hours 30 mins exam is worth 100 points.

(1) [12 pts] (a) Find the (level set) equation of the plane through the points $(2, 1, 1)$, $(1, 3, 0)$, and $(-2, 3, 1)$.

(b) Find a parametrization of the line segment from the point $(-2, 0, -1)$ to the point $(1, 2, 3)$.

(c) Make a single sketch that includes both the plane in (a) and the line segment in (b). Using your sketch, explain why the line segment intersects the plane in a point. Then calculate the coordinates of this point of intersection.

(2) [10 pts] Make labelled sketches of the traces of the surface

$$y^2 + \left(\frac{z}{2}\right)^2 - \left(\frac{x}{3}\right)^2 = 1.$$

in the planes $y = 0$, $z = 0$, and $x = k$ for a few appropriately chosen values of k. Then sketch the surface.

(3) [5 pts] Suppose that an ant is walking on a hot plate in the xy-plane and that the position of the ant at time t is $\mathbf{r}(t) = (\sin t, \cos 3t)$. Let $T = T(x, y)$ be the temperature at the point (x, y) on the hot plate. Suppose that $\frac{\partial T}{\partial x} = 1$ and $\frac{\partial T}{\partial y} = 3$ at the point $(x, y) = \mathbf{r}(\frac{\pi}{4})$. Is the temperature of the ant's feet increasing or decreasing at time $t = \frac{\pi}{4}$?

(4) [10 pts] Find the absolute maximum and absolute minimum of the function $z = f(x, y) = x^2 + y^2 - 2x$ on the closed triangular region with vertices $(2, 0)$, $(0, 2)$ and $(0, -2)$.

(5) [10 pts] Use a triple integral to calculate the volume of the solid region bounded by $z = y^2$, $x = 0$, and $z + x = 1$.

(6) [8 pts] Use spherical coordinates to calculate the triple integral $\iiint_E x^2 + y^2 + z^2 \, dV$, where E is the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = -\sqrt{x^2 + y^2}$.

(7) [10 pts] Use the change of variables $x = 2u + v$, $y = u + 2v$ to evaluate the integral $\iint_R (x - 3y)\, dA$, where R is the triangular region with vertices $(0, 0)$, $(2, 1)$, and $(1, 2)$.

(8) [10 pts]
Let S be the surface with parametrization

$$\mathbf{r}(u,v) = (u\cos v, 2u, u\sin v), \qquad \text{where } 0 < u < 2 \text{ and } 0 < v < \pi.$$

(a) Find parametrizations of the grid curve $u = 1$ and of the grid curve $v = \pi/3$.

(b) Show that the points on the surface S satisfy the equation $x^2 + z^2 = \frac{y^2}{4}$.

(c) Sketch the surface S together with the grid curves you found in (a).

(d) Find a parametrization of the tangent plane to S through the point where $u = 1$ and $v = \pi/3$.

(9) [5 pts] Let C be the oriented curve parametrized by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$, for $0 \le t \le \pi$ and let $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + xy\,\mathbf{k}$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(10) [8 pts]
(a) Give a careful statement of Green's Theorem.

(b) Use Green's Theorem to show that if $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ is a vector field on a region D in the plane then

$$\iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{k}\, dA \;=\; \int_{\partial D} \mathbf{F} \cdot d\mathbf{r}.$$

(11) [8 pts] Suppose that $\mathbf{F}$ is a vector field in the xy-plane such that $\nabla \times \mathbf{F} = (x^2 + y^2)\,\mathbf{k}$.

(a) Can you find a function $z = f(x, y)$ so that $\mathbf{F} = \nabla f$? Why?

(b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the circle $x^2 + y^2 = 4$.

(12) [4 pts] Suppose that $\mathbf{F} = \cos x\ \mathbf{i} + \sin y\ \mathbf{j}$ is the velocity vector field of a fluid flowing in the xy-plane. Let D be a small disc centered at the point $(1, 0)$. On average is fluid flowing into or out of D? Why?

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________