# Math 6301 Some Extra Problems on

## Abstract Measure Spaces, Fubini, $L^p$-spaces

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Note: For all Fubini problems you should carefully justify that Tonelli/Fubini actually applies or does not, as the case may be.

1. **J**.6D: 21

2. **J**.6E: 22

3. **J**.6E: 23

4. **J**.6E: 24

5. **J**.6E: 25

6. **J**.6F: 38

7. **J**.6F: 40

8. **J**.8: 1

9. **J**.8: 3

10. **J**.8: 7

11. **J**.8: 14

12. **J**.8: 15

13. **J**.8: 17

14. **A**.7A: 2

15. **A**.7A: 4

16. **A**.7A: 5

17. **A**.7A: 7

18. **A**.7A: 12

19. **A**.7A: 13

20. **A**.7A: 14

21. **A**.7A: 15

22. Suppose $p, q, r \in [1, \infty)$ and $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$. Prove that $\|fg\|_r \leq \|f\|_p \|f\|_q$.

23. Let $1 < p_0 < \infty$. Find an example of sequence $f_k$ so that $f_k \in L^p$ for all $1 \leq p < \infty$ and $f_k \to 0$ in $L^p$ for all $1 \leq p < p_0$, but $f_k$ does not converge in $L^{p_0}$.