NAME:		CIRCLE:	Zweck 11:30am	Zweck 2:30pm

1	/12	2	/15	3	/10	4	/12	5	/10	6	/4	7	/12	T	/75

MATH 2415 (Fall 2015) Exam II, Nov 6th

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. This is 75 minute exam.

(1) [12 pts]
(a) Calculate the directional derivative of the function $f(x, y) = x^2 y^3$ in the direction of the vector $\mathbf{v} = (3, 4)$ at the point $(x, y) = (1, 2)$.

(b) Let $f(x, y) = y - x^2$. Find the gradient of f at the point $(1, 3)$. Sketch the level curve of f through the point $(1, 3)$, together with the gradient at that point.

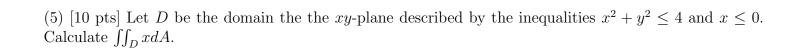(2) [15 pts] Suppose that $z = f(x, y)$ is a function such that

(a, b)	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	0	(0,0)	5	3	1
$(7, -2)$	0	(0,1)	5	3	1
$(3, 4)$	7	(0,0)	-5	-3	-2
$(5, -3)$	68	(0,0)	8	-4	2
$(2, 1)$	35	(0,0)	5	3	2

Identify any local maxima, minima, and saddle points of f. Explain the reasons for your answers.

(3) [10 pts]
(a) Find a parametrization of the line that contains the point $(1, 2, 3)$ and is perpendicular to the plane $6(x - 1) + 2(y + 3) + 4(z - 8) = 0$.

(b) Find an equation of the form $Ax + By + Cz = D$ for the plane that contains both the point $(2, 4, 6)$ and line with parametrization $\mathbf{r}(t) = (7 - 3t, 3 + 4t, 5 + 2t)$.

(4) [12 pts] Let D be the domain the the xy-plane that is bounded by the curves $y = x^2$ and $y = 2 - x$. Calculate $\iint_D x \, dA$.

(5) [10 pts] Let D be the domain the the xy-plane described by the inequalities $x^2 + y^2 \leq 4$ and $x \leq 0$. Calculate $\iint_D x \, dA$.

(6) [4 pts] Prove that the directional derivative of a function $z = f(x, y)$ at a point (x_0, y_0) is maximized when the derivative is taken in the direction of the gradient of f at (x_0, y_0).

(7) [12 pts] In this problem you will use the **method of Lagrange Multipliers two different ways** to solve the same problem. The problem is to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2$ on the ellipse $4(x - 1)^2 + y^2 = 16$. [*Hint: There are four critical points.*]

(a) First, solve the problem **graphically** by sketching the ellipse and some appropriately chosen level curves, $f(x, y) = k$. (This approach will enable you to find the approximate but not necessarily the exact locations and values of the maxima and minima.)

(b) Now, solve the problem exactly by setting up the appropriate **equations** and solving them algebraically.

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: _____