LAST NAME:	FIRST NAME:	CIRCLE:		
		Zweck 10:00am	Khafizov 11:30am	Khafizov 2:30pm

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Spring 2016

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Evaluate the double integral $\iint_D 2\frac{y}{x}\,dA$, where D is bounded by $y = x$ and $y = x^2$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(2) [10 pts] (a) Use the Chain Rule to find $\partial z/\partial t$ if $z = e^{xy}$, $x = \sin t$, $y = t^2$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(b) Suppose that $2x + 3y = 5$ is the tangent line to a curve $f(x, y) = 4$ at the point $(x_0, y_0) = (1, 1)$. Find a unit vector that is either in the direction or in the opposite direction of ∇f at the point (x_0, y_0). Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(3) [10 pts]
(a) Let $z = f(x, y)$ be a function of two variables. State the limit definition of the partial derivative of f with respect to y at a point (x_0, y_0).

(b) Let $z = f(x, y)$ be a function of two variables. Write a sentence and draw a picture that explains the geometrical meaning of the partial derivative of f with respect to y at a point (x_0, y_0).

(c) Now let $z = f(x, y) = e^{4y} \sin(x^2 + y^2)$. Calculate the partial derivative of f with respect to y at a point $(x_0, y_0) = (-4, 3)$.

(4) [10 pts] Find (a) the local maximum values, (b) the local minimum values and (c) saddle point(s) of $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$, if they exist. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(5) [10 pts] Use a triple integral to find the volume of the solid enclosed by surfaces $y = x^2$, $z = 0$, and $y + z = 1$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(6) [10 pts] Let D be the domain in the plane given by $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 \leq 1$. Use the Change of Variables Theorem to calculate $\iint_D 16x^2 + 9y^2\,dxdy$. Hint: Use the change of variables $u = \frac{x}{3}$, $v = \frac{y}{4}$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

(7) [10 pts] Let S be the surface parametrized by $(x, y, z) = \mathbf{r}(u, v) = (u \cos v, u \sin v, u)$. Find a parametrization for the tangent plane to S at the point where $(u, v) = (2, \frac{\pi}{3})$.

(8) [10 pts] Make sketches of parametrized curves below. Be sure to label the axes and indicate which sketch goes with which curve.

$$
\begin{aligned}
(x, y, z) &= \mathbf{r}_1(t) = (\cos t, \sin t, t) \qquad \text{for } 0 \le t \le 2\pi \\
(x, y, z) &= \mathbf{r}_2(t) = (\sin t, \cos t, t) \qquad \text{for } 0 \le t \le 2\pi
\end{aligned}
$$

(9) [10 pts]
(a) Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = z^2 y\,\mathbf{i} + (x^2 - z^2)\,\mathbf{j} + xz\,\mathbf{k}$. Calculate the curl of $\mathbf{F}$.

(b) Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = \dfrac{x}{(x^2+y^2+z^2)^{3/2}}\,\mathbf{i} + \dfrac{y}{(x^2+y^2+z^2)^{3/2}}\,\mathbf{j} + \dfrac{z}{(x^2+y^2+z^2)^{3/2}}\,\mathbf{k}$.
Show that $\operatorname{div}\mathbf{F} = 0$ everywhere it is defined.

(10) [10 pts] This problem is about the vector field $\mathbf{F}(x, y) = -y\,\mathbf{i} + x\,\mathbf{j}$.

(a) Let C be the unit circle oriented counter clockwise. Calculate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

(b) Use your answer to (a) to determine whether or not the vector field $\mathbf{F}$ is conservative.

(c) Use a different approach from (b) to determine whether or not $\mathbf{F}$ is conservative.

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________