NAME:

1	/10	2	/10	3	/13	4	/12	5	/15	6	/15	T	/75

MATH 2415 (Spring 2014) Exam II, April 3rd

Dr. Zweck's Class

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [10 pts] Let $\mathbf{r} : \mathbb{R} \to \mathbb{R}^3$ be the parametrized curve

$$\mathbf{r}(t) \;=\; (t^2, e^{3t}, \cos(4t))$$

and let $f : \mathbb{R}^3 \to \mathbb{R}$ be a function such that

$$f(0,1,1) = 5 \qquad f(0,3,0) = -2$$
$$\nabla f(0,1,1) = 2\mathbf{i} - 5\mathbf{j} + 7\mathbf{k} \qquad \nabla f(0,3,0) = -\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}.$$

Let $g(t) = f(\mathbf{r}(t))$. Find $g'(0)$.

(2) [10 pts] Let $z = f(x, y)$ be a function with table of values given by

		y		
		4	5	6
x	0	7	8	5
	1	6	9	12
	2	8	11	15

Estimate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(x, y) = (1, 5)$. Use your answer to estimate the directional derivative of f in the direction $\theta = \pi/3$ at the point $(1, 5)$.

(3) [13 pts]
(a) Calculate $\iint_D y^3\,dA$, where D is the triangle with vertices $(0,2)$, $(1,1)$, and $(3,2)$.

(b) Calculate $\iint_D \cos(x^2+y^2)\,dA$, where D is the region above the x-axis and within the circle $x^2+y^2=9$.

(4) [12 pts]
(b) Sketch the level curves of the function $z = f(x, y) = x - e^y$ at levels $k = -1$, $k = 0$, and $k = 1$. Also calculate the gradient of f at the origin, add it to your sketch, and explain how it is related to the level curve that passes through the origin.

(5) [15 pts] In this problem you will use the **method of Lagrange Multipliers two different ways** to solve the same problem. The problem is to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2$ on the ellipse $(x - 1)^2 + 4y^2 = 4$.

(a) First solve the problem **graphically** by sketching the ellipse and some appropriately chosen level curves, $f(x, y) = k$. [Hint: The ellipse is centered at $(1, 0)$.]

(b) Now solve the problem by setting up the appropriate **equations** and solving them algebraically.

(6) [15 pts] Suppose that $z = f(x, y)$ is a function with the following table of values.

(a, b)	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	0	(0,0)	5	3	1
$(7, -2)$	0	(0,1)	5	3	1
$(3, 4)$	7	(0,0)	-5	-3	-2
$(5, -3)$	68	(0,0)	8	-4	2
$(2, 1)$	35	(0,0)	5	3	2

Identify any local maxima, minima, and saddle points of f. Explain the reasons for your answers.

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: _____