LAST NAME:	FIRST NAME:	CIRCLE: Makhijani 8:30am / Makhijani 11:30am / Makhijani 2:30pm / Zweck 11:30am

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Spring 2019

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] (a) If $z = f(x, y)$, with $x = e^t$, $y = t^2 + 3t + 2$, $\nabla f = (2xy^2 - y, 2x^2y - x)$ find $z'(t)$ at $t = 0$.

(b) Parametrize the surface $(y - 2)^2 + (z - 3)^2 = 4$.

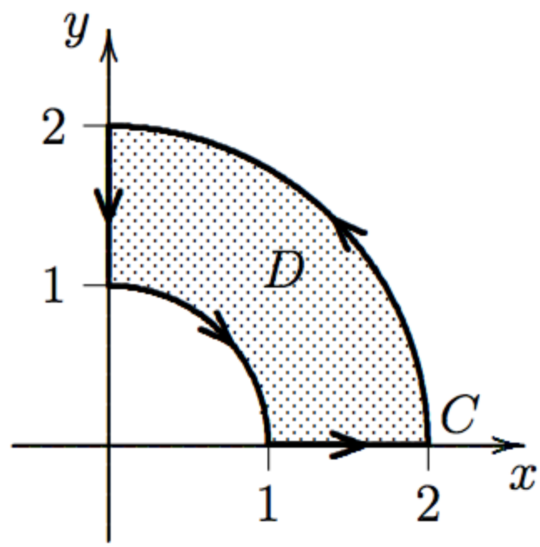
(2) [10 pts] Let $\mathbf{F}_1(x,y) = (2y - x^2e^{-y})\mathbf{i} + 2xe^{-y}\mathbf{j}$ and $\mathbf{F}_2(x,y) = 2xe^{-y}\mathbf{i} + (2y - x^2e^{-y})\mathbf{j}$

(a) One of these vector fields is conservative. Which one is it and why?

(b) Find a potential function for the conservative vector field.

(c) Evaluate $\int\limits_C \mathbf{G} \cdot d\mathbf{r}$ where C is the line segment from $(1,0)$ to $(2,1)$ and $\mathbf{G}$ denotes the conservative vector field you identified in (a).

(3) [10 pts] Use Green's theorem to evaluate $\int_C xy^2\,dx \; - \; x^2y\,dy$ where C is given in the figure.



(4) [10 pts] Evaluate $\int_{x=0}^{x=1} \int_{y=\sqrt{x}}^{y=1} \cos(y^3)\, dy dx$.

(5) [15 pts] Make a labelled sketch of the traces of the surface

$$y^2 - 4x^2 - z^2 = 1$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0$, ± 1, ± 2. Then sketch the surface.

(6) [10pts] Let $A = (1, 2, 5)$, $B = (2, 2, 7)$, and $C = (3, 5, 8)$ be three points in space.
(a) Let P_1 be the plane containing the points A, B, and C. Find an equation of the form $ax + by + cz = d$ for the plane P_1.

(b) Let P_2 be the plane containing the line segment AB and which is parallel to the normal vector to the plane P_1. Find a parametrization for the plane P_2.

(7) [10 pts] Let D be the closed triangular domain with verticies $(0,0)$, $(2,0)$, and $(0,4)$. Find the absolute maximum and minimum of the function $f(x,y) = xy - x - y$ on D.

(8) [10 pts] Evaluate $\iint_R (x+2y)(y-3x)\, dA$ where R is the parallelogram enclosed by the lines $x+2y=-4$, $x+2y=3$, $y-3x=-1$, $y-3x=5$.

(9) [10 pts] Let E be the solid region bounded by the surfaces $z = y$, $z = y^2$, $x = 0$, and $x + 2z = 2$. Sketch the region E and calculate $\iiint_E y\, dV$.

(10) [10 pts]
(a) Let $\mathbf{F}_1(x, y) = x^2\mathbf{i} + y\mathbf{j}$ be the velocity vector field of a fluid flowing in $\mathbb{R}^2$. On average, is the fluid flowing in or out of a small disc centered at the point $(-3, 1)$? Why?

(b) Let $\mathbf{F}_2(x, y) = y^2\mathbf{i} + x^2\mathbf{j}$ be the velocity vector field of a fluid flowing in $\mathbb{R}^2$. On average, is the fluid rotating clockwise or counter-clockwise around the point $(1, 2)$? Why?