## $\boxed{E}$ FOUR FUNDAMENTAL SUBSPACES OF $m \times n$ $A$ [M, 4.2]

$$F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$
$$\vec{x} \longmapsto A\vec{x}$$

$$G : \mathbb{R}^m \longrightarrow \mathbb{R}^n$$
$$\vec{y} \longmapsto A^T\vec{y}$$

$N(A)$, $R(A^T)$ are VSSs of $\mathbb{R}^n$
$R(A)$, $N(A^T)$ are VSSs of $\mathbb{R}^m$

$\boxed{R(A^T)}$

TRANSPOSE!

$$\underset{n \times 1}{R(A^T)} = \{ A^T\vec{y} \;/\; \vec{y} \in \mathbb{R}^m \}$$
$$= \{ \beta_1 (A^T)_{*1} + \dots \beta_m (A^T)_{*m} \;|\; \beta_j \in \mathbb{R} \}$$
$$= \left[ \{ \beta_1 A_{1*} + \dots + \beta_m A_{m*} \;|\; \beta_j \in \mathbb{R} \} \right]^T$$
$$= [\text{Span } \underset{1 \times n}{\text{Rows}} \text{ of } A]^T \leftarrow \text{MAKES } n \times 1 \checkmark$$

**PROP 24**

Let $E$ be a row echelon form of $A$
Then
$$R(A^T) = \text{Span of transposes of non-zero rows of } E.$$

EX (AS ABOVE)

$$R(A^T) = \text{Span}\{(1,3,5)^T, (0,1,1)^T\}$$