

①

16.1 VECTOR FIELDS

DEF A VECTOR FIELD,  $\vec{F}$ , on  $\mathbb{R}^n$  ASSIGNS A VECTOR,  $\vec{F}(\vec{x})$  to each point  $\vec{x} \in \mathbb{R}^n$ . So  $\vec{F}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

$\mathbb{R}^2$   $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$

$\mathbb{R}^3$   $\vec{F}(x,y,z) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$

$P, Q, R$  are functions from  $\mathbb{R}^n$  to  $\mathbb{R}$ .

EX  
① VELOCITY Vector Field of fluid/air

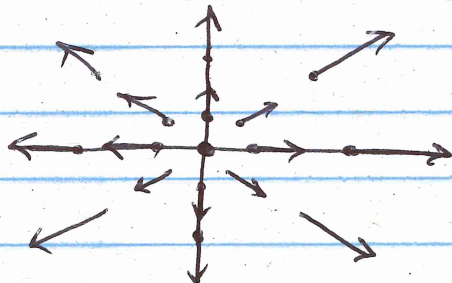
② FORCE vector field (gravity, electric, magnetic)

③ GRADIENT vector field

EX  $z = f(x,y) = x^2 + xy^3$   
 $\vec{F} = \nabla f = (2x + y^3)\vec{i} + 3xy^2\vec{j} = (2x + y^3, 3xy^2)$

VISUALIZING VECTOR FIELDS

EXS ①  $\vec{F}(x,y) = x\vec{i} + y\vec{j}$  POSITION V.F.



If translate  $\vec{F}(x,y)$  to start at  $\vec{0}$ , Then it ends at  $(x,y)$ .

(2)

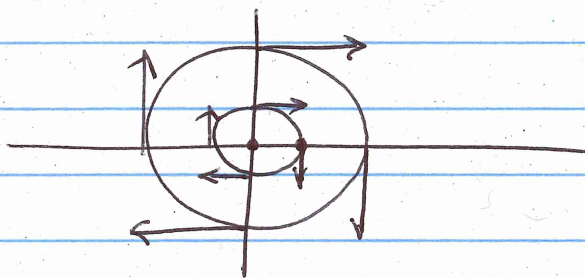
NB  $\vec{F} = \nabla f, \quad f = \frac{1}{2}(x^2 + y^2)$

(2)  $\vec{F}(x,y) = y\vec{i} - x\vec{j}$

•  $|\vec{F}(x,y)| = \sqrt{y^2 + x^2} = r$

•  $\vec{F} \perp \text{Path VF} \quad \text{as} \quad (y\vec{i} - x\vec{j}) \cdot (x\vec{i} + y\vec{j}) = yx - xy = 0.$

So

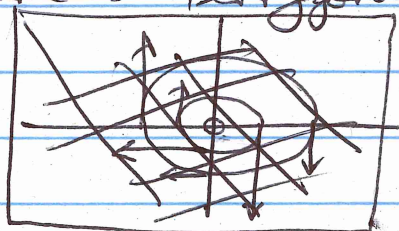


$\vec{F}(x,y) = \text{Velocity VF of Merrygo round.}$

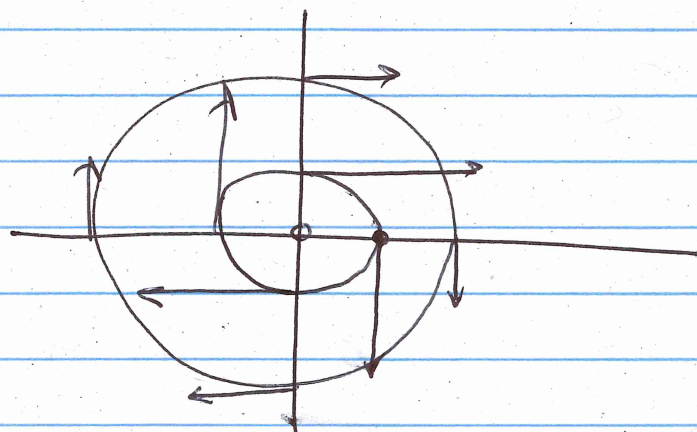
(3)  $\vec{F}(x,y) = \frac{y\vec{i} - x\vec{j}}{x^2 + y^2}$

•  $|\vec{F}(x,y)| = \frac{\sqrt{y^2 + x^2}}{(x^2 + y^2)^{3/2}} = \frac{1}{\sqrt{x^2 + y^2}} = \frac{1}{r}$

• DIRN Same as Merrygo round VF



"TORNADO VF"



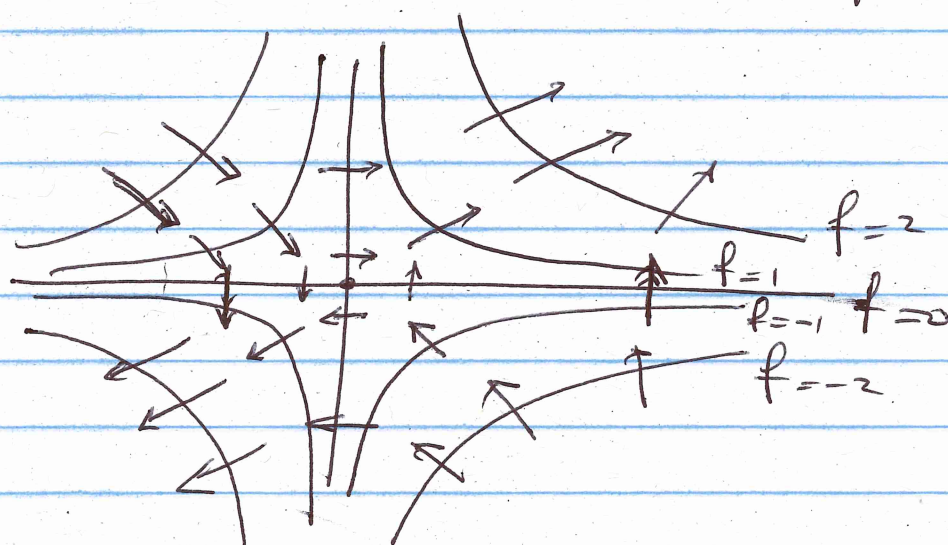


④  $f(x,y) = xy$

$\vec{F} = \nabla f = y\vec{i} + x\vec{j}$

$|\vec{F}| = \sqrt{x^2 + y^2} = r$

Dirn of  $\vec{F} \sim \perp$  Level Curves of  $f$



Can also plot points

EX  $\vec{F}(2,3) = (3,2)$

