LAST NAME:	FIRST NAME:	CIRCLE:		
SOLUTIONS		Martynova 8:30am	Martynova 1pm	Zweck

1 /10	2 /12	3 /12	4 /12	5 /12	6 /9	7 /8	T /75

MATH 2415 (Spring 2017) Exam I, Feb 17th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [10 pts] Let $P = (1, 1, 0)$ and $Q = (1, 2, 3)$ be two points and let $\mathbf{v} = -\mathbf{j} + 2\mathbf{k}$ be a vector.

(a) Find a parametrization for the line parallel to the vector $\mathbf{v}$ that passes through the point P.

$\vec{r}(t) = \vec{p} + t\vec{v}$ where $\vec{p} = \overrightarrow{OP} = (1, 1, 0)$

$\vec{v} = (0, -1, 2)$

So $\vec{r}(t) = (1, 1, 0) + t(0, -1, 2) = (1, 1-t, 2t)$

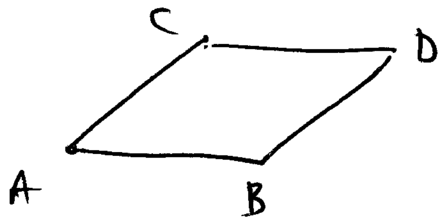
(b) Calculate the projection of the vector $\overrightarrow{PQ}$ onto the vector $\mathbf{v}$.

$\vec{u} = \overrightarrow{PQ} = Q - P = (1, 2, 3) - (1, 1, 0) = (0, 1, 3)$

$$\text{PROJ}_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \frac{\vec{v}}{|\vec{v}|} = \frac{(0, 1, 3) \cdot (0, -1, 2)}{\left(\sqrt{0^2 + 1^2 + 2^2}\right)^2} (0, -1, 2)$$

$$= \frac{5}{5}(0, -1, 2) = (0, -1, 2)$$

(2) [12 pts] Let $A = (1,3,0)$ and $B = (2,3,-4)$ and $C = (3,3,2)$ be three points.
(a) Find a point, D, so that A, B, C, and D are the vertices of a parallelogram.



$$D = C + \vec{AB}$$
$$= (3,3,2) + (1,0,-4)$$
$$= (4,3,-2)$$

(b) Find the area of the parallelogram in (a).

$$A = |\vec{AB} \times \vec{AC}| = \left\| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -4 \\ 2 & 0 & 2 \end{matrix} \right\|$$

$$= |0\vec{i} - (2+8)\vec{j} + (0)\vec{k}| = |-10\vec{j}| = 10$$

(c) Find a unit vector orthogonal to the plane containing the points A, B, and C.

$$\vec{n} = \pm \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \pm \frac{-10\vec{j}}{10} = \pm \vec{j}$$

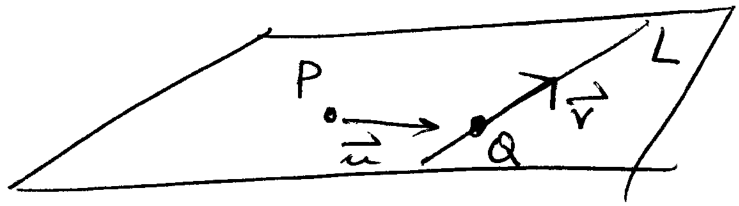
$$\boxed{\vec{n} = \pm \vec{j} \qquad \text{BOTH WORK}}$$

(3) [12 pts]

(a) Find an equation of the form $Ax+By+Cz=D$ for the plane that passes through the point $P=(1,3,4)$ and contains the line L parametrized by $x=3t$, $y=4t$, $z=2+2t$.

$$Q = \vec{r}(0) = (0,0,2)$$

$$\vec{u} = \overrightarrow{PQ} = (-1,-3,-2)$$



$$\vec{v} = \vec{r}\,'(t) = (3,4,2)$$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & -2 \\ 3 & 4 & 2 \end{vmatrix} = (2,-4,5)$$

Eqn of Plane is $(\vec{r}-\vec{p})\cdot\vec{n} = 0 \qquad \vec{r} = (x\,y\,z)$

$$(x-1,\ y-3,\ z-4)\cdot(2,-4,5) = 0$$

$$2(x-1) - 4(y-3) + 5(z-4) = 0 \quad \text{OR} \quad 2x-4y+5z = 10$$

(b) Find the point of intersection of the line $\mathbf{r}(t)=(t,t+1,t+2)$ and the plane $x+y+z=6$.

Put $x=t \quad y=t+1 \quad z=t+2$ into equation of plane:

$$6 = x+y+z = t + (t+1) + (t+2)$$

$$= 3t+3$$

$$\Rightarrow \boxed{t=1} \quad \text{So Point is}$$

$$P = \vec{r}(1) = (1,2,3)$$

(4) [12 pts] Let C be the curve with parametrization $x = t\sin 2t$, $y = t\cos 2t$, $z = t$ for $0 \le t \le 2\pi$.

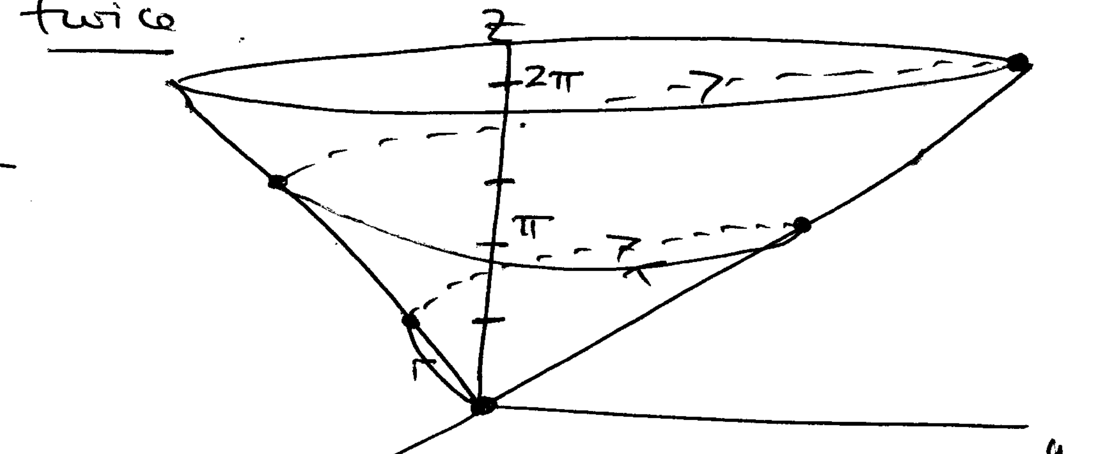
(a) Show that the curve, C, lies on the surface $z = \sqrt{x^2+y^2}$. Sketch the surface and the curve.

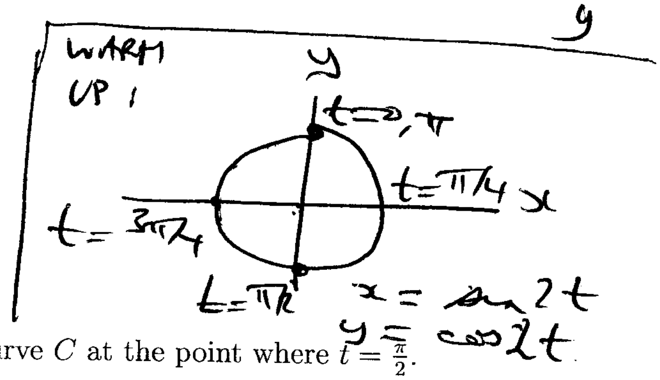
$$\sqrt{x^2+y^2} = \sqrt{t^2\sin^2 2t + t^2\cos^2 2t} = \sqrt{t^2} = t = z$$

Curve is a LEFT-HANDED SPIRAL HELIX ON CONE.

Goes around twice

t	x	y	z
0	0	0	0
$\pi/2$	0	$-\pi/2$	$\pi/2$
π	0	$-\pi$	π
$3\pi/2$	0	$-\frac{3\pi}{2}$	$3\pi/2$
2π	0	-2π	2π
$\pi/4$	$\pi/4$	0	$\pi/4$
$3\pi/4$	$-\frac{3\pi}{4}$	0	$\frac{3\pi}{4}$





(b) Calculate a parametrization of the tangent line to the curve C at the point where $t = \frac{\pi}{2}$.

$$\vec{r}(t) = (t\sin 2t,\ t\cos 2t,\ t)$$

$$\vec{r}(\pi/2) = (0,\ -\pi/2,\ \pi/2)$$

$$\vec{r}\,'(\pi/2) = (\sin 2t + 2t\cos 2t,\ \cos 2t - 2t\sin 2t,\ 1)$$

$$\vec{r}\,'(\pi/2) = (-\pi,\ -1,\ 1)$$

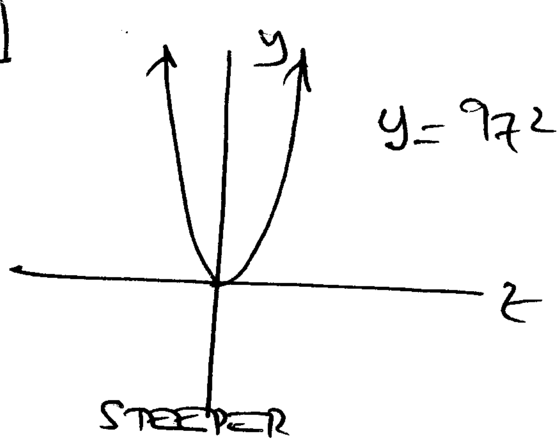
$$\vec{l}(s) = \vec{r}(\pi/2) + s\,\vec{r}\,'(\pi/2)$$
$$= (-s\pi,\ -\pi/2 - s,\ \pi/2 + s)$$

(5) [12 pts] Make labelled sketches of the traces (slices) of the surface

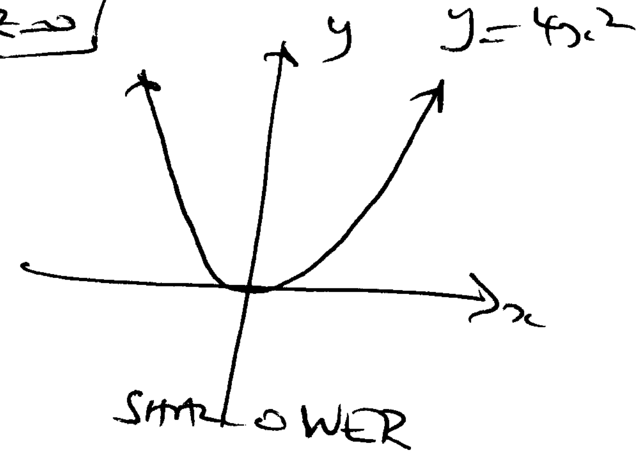
$$y = 4x^2 + 9z^2$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then make a labelled sketch of the surface.

$\boxed{x=0}$



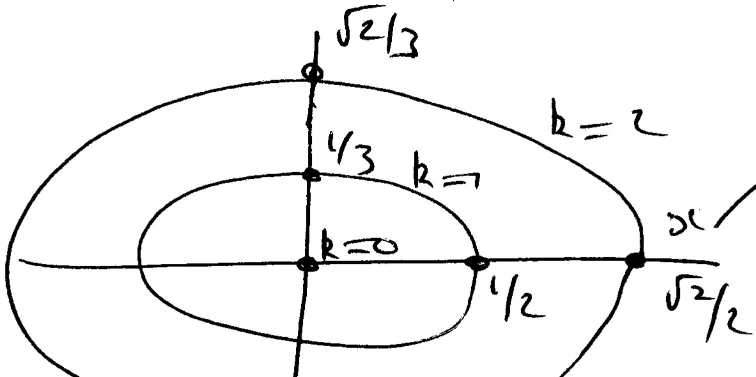
$\boxed{z=0}$

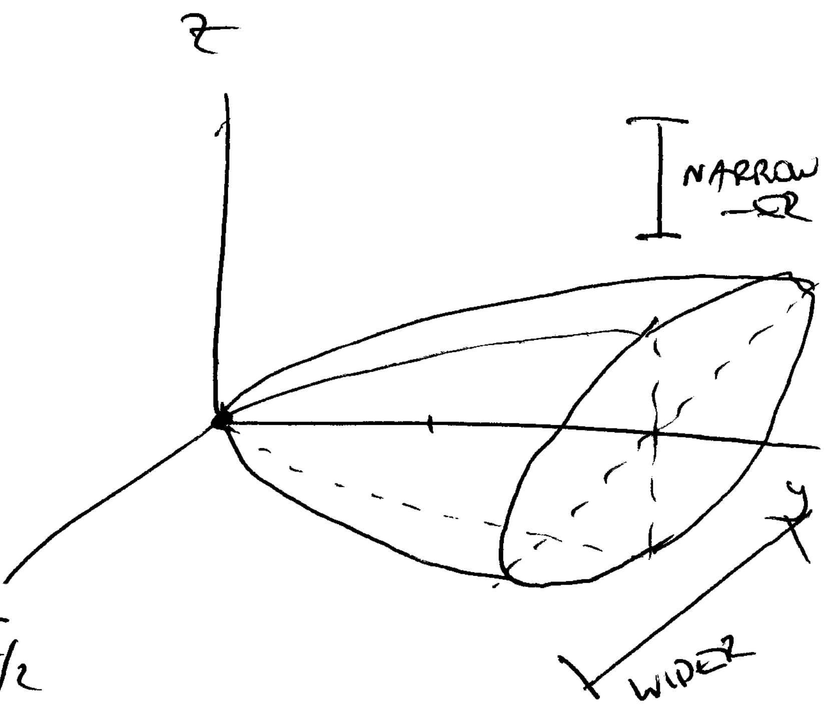


$\boxed{y=k}$

IF $k<0$: EMPTY SET

$k=0$ ORIGIN





(6) [9 pts] Sketch the level curves of the function $z = (y^2 - x)^3$ at levels $k = 0,\ k = \pm 8$.

$(y^2 - x)^3 = k$

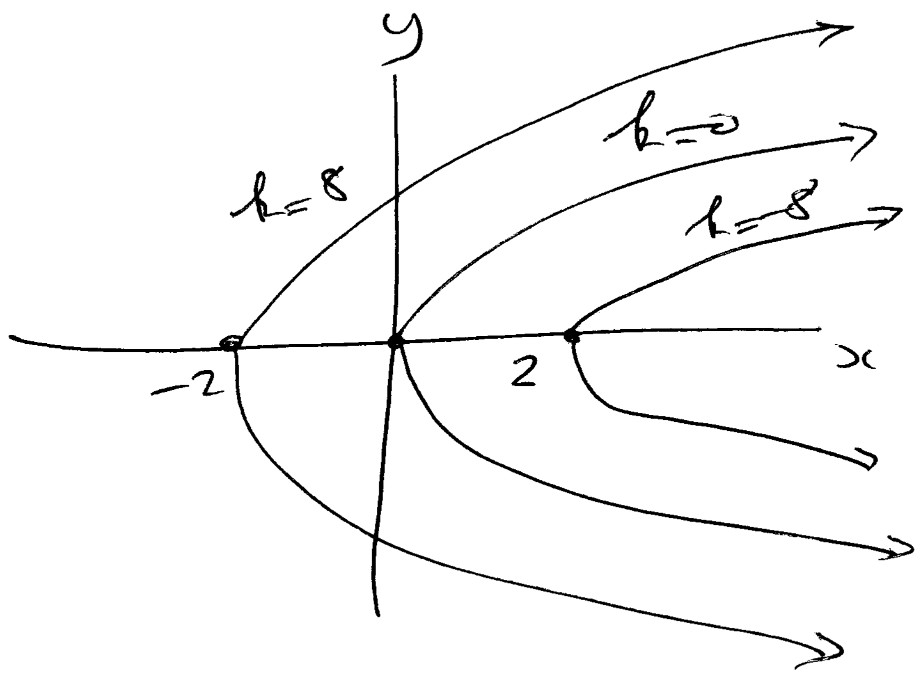
$y^2 - x = k^{1/3}$

$x = y^2 - k^{1/3}$

$\underline{k=0}$ $\quad x = y^2$

$\underline{k=8}$ $\quad x = y^2 - 2$

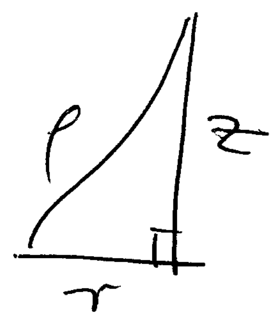
$\underline{k=-8}$ $\quad x = y^2 + 2$



(7) [8 pts] Convert the point with cylindrical coordinates $(r, \theta, z) = (3, \frac{\pi}{4}, 4)$ into spherical coordinates.

$\rho^2 = r^2 + z^2 = 3^2 + 4^2$

So $\rho = 5$.



$\theta = \pi/4$.

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\cos\phi = \dfrac{z}{\rho} = \dfrac{4}{5}$

$\phi = \arccos\left(\frac{4}{5}\right)$

$\underline{\text{OR}}$ $\quad \phi = \arctan\left(\frac{r}{z}\right) = \arctan\left(\frac{3}{4}\right)$