NAME:

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MATH 2415 (Fall 2012) Exam I, Oct 5

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 2 hour exam is worth 100 points.

(1) [16 pts]
(a) Is the line through the points $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the line through the points $(10, 18, 4)$ and $(5, 3, 14)$?

(b) Find parametric equations for the line through the point $(5, 1, 0)$ that is perpendicular to the plane $2x - y + z = 1$.

(2) [16 pts] (For Dr. Zweck's class: When we refer to the *equation* of a plane we mean a *level set equation.*)

(a) Find the equation of the plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, and $z = 7 + 4t$.

(b) Write down a general equation of a plane that involves the normal vector to the plane. Draw a picture that explains why this equation holds. Be sure to carefully label your picture.

(3) [16 pts]
(a) Find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ where $\mathbf{a} = (3, 6, -2)$ and $\mathbf{b} = (1, 2, 3)$.

(b) Use vector algebra to show that the vector $\mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}$ is orthogonal to $\mathbf{a}$. (Here $\text{proj}_{\mathbf{a}}\,\mathbf{b}$ is the projection of $\mathbf{b}$ onto $\mathbf{a}$.) [Note: you need to show this in general not for the specific vectors given in part (a).]

(4) [16 pts] Make a labelled sketch of the traces of the surface

$$y^2 - x^2 + 4z^2 = 1$$

in the planes $y = 0$, $z = 0$, and $x = k$ for $k = 0, \pm 1$. Then sketch the surface.

(5) [12 pts]
(a) Sketch the surface whose equation in cylindrical coordinates is $z = r - 2$.

(c) What are the rectangular coordinates of the point whose spherical coordinates are $(\rho, \theta, \phi) = (2, \pi, 3\pi/4)$?

(6) [12 pts] Match each of the functions below with both a contour plot and a graph. *To receive credit you must provide reasons for your answers.*

(a) $f(x, y) = e^{-y} \cos x$

(b) $f(x, y) = y^2 - y^4 - x^2$

(c) $f(x, y) = \frac{1}{4x^2+y^2}$

(7) [12 pts]
(a) Calculate a parametrization of the tangent line to the curve $\mathbf{r}(t) = (2\sin t, 3\cos t)$ at $t = \pi/3$

(b) Suppose that $\mathbf{r}$ is a parametrized curve in space with $\mathbf{r}'(0) = (1, 2, 3)$ and $\mathbf{r}''(0) = (4, 5, 6)$. Could the parametization $\mathbf{r}$ have constant speed? Explain!

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: _____