| LAST NAME: | FIRST NAME: | CIRCLE: Li Minkoff Zweck |
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# MATH 2415 Final Exam, Fall 2016

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Find a parametrization of the line which is given by the intersection of the planes $x+y+z=0$ and $x-2y+z=0$.

(2) [10 pts] (a) Sketch the curve with the parametrization $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$, for $0 \le t \le \pi$.

(b) Find the arc length of the curve $\mathbf{r}$ defined in part (a) from the point $(1,\, 0,\, 0)$ to the point $(-1,\, 0,\, \pi)$.

(3) [10 pts] Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$.

(4) [10 pts] Let $\mathbf{F}(x, y) = x^3y^4\mathbf{i} + x^4y^3\mathbf{j}$.

(a) Find a function $f$ so that $\mathbf{F} = \nabla f$.

(b) Use part (a) to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve

$$\mathbf{r}(t) = \sqrt{t}\ \mathbf{i} + (1 + t^3)\ \mathbf{j} \qquad \text{for } 0 \leq t \leq 1.$$

(5) [10 pts] Find the absolute maximum and minimum values of the function $f(x, y) = xy - x$ on the triangle with vertices $(-1, 0)$, $(-1, 3)$, and $(2, 0)$.

(6) [10 pts] Use Green's Theorem to evaluate the line integral

$$\int_C y^2 dx + xy dy,$$

where $C$ is the oriented curve consisting of the line segment from $(-2, 0)$ to $(2, 0)$ followed by the top half of the circle $x^2 + y^2 = 4$ traversed counter-clockwise.

(7) [10 pts]
(a) Let $C$ be the curve in the plane which is parametrized by $\mathbf{r}(t) = (3t, 4t+1)$, where $0 \leq t \leq 1$ and let $z = f(x, y) = xy$. Calculate $\int_C f\, ds$.

(b) Calculate the divergence of the vector field $\mathbf{F}(x, y, z) = e^x \sin y\, \mathbf{i} + e^x \cos y\, \mathbf{j} + z\, \mathbf{k}$.

(8) [10 pts] Let $D$ be the region in the first quadrant of the $xy$-plane bounded by the curves $y = \frac{x}{2}$, $y = x$, $xy = 4$ and $xy = 9$. Calculate $\iint_D x\, dxdy$. **Hint:** Use the change of variables $x = ve^u$, $y = ve^{-u}$.

(9) [10 pts] Let $E$ be the solid region in the first octant ($i.e.$, where $x \geq 0$, $y \geq 0$, $z \geq 0$) that is inside the cylinder $x^2 + y^2 = 1$ and below the plane $x + z = 1$. Sketch the solid $E$ and calculate $\iiint_E y\,dV$.

(10) [10 pts]
(a) Let $z = f(x, y)$ and $(x_0, y_0) = (1, 2)$. Let $C$ be the level curve of $f$ through the point $(x_0, y_0)$ and suppose that $\nabla f(x_0, y_0) = (3, 4)$. Find a parametrization of the tangent line to the curve $C$ at $(x_0, y_0)$.

(b) Prove that the directional derivative of a function $z = f(x, y)$ at a point $(x_0, y_0)$ is maximized when the derivative is taken in the direction of the gradient of $f$ at $(x_0, y_0)$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________