NAM	E:							CLA	ASS: 11	:30am	OR	4pm
1	/5	2	/10	3	/10	4	/10	5	/10	6	/10	
7	/10	8	/7	9	/10	10	/10	11	/8	Т	/100	

MATH 2415 (Fall 2012) Final Exam, Dec 14th

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 2 hour 30 minute exam is worth 100 points.

(1) [5 pts] Find an equation of the form ax + by + cz = d for the plane through the point (2,0,1) that is perpendicular to the line x = 3t, y = 2 - t, z = 3 + 4t.

(2) [10 pts] Make labelled sketches of the traces of the surface

$$x^{2} + \left(\frac{y}{2}\right)^{2} - \left(\frac{z}{3}\right)^{2} = -1.$$

in the planes x = 0, y = 0, and z = k for a few appropriately chosen values of k. Then sketch the surface.

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(a) Calculate a parametrization of the tangent line to the curve C at t = 1.

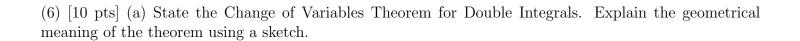
(b) Integrate the vector field $\mathbf{F} = y\mathbf{i} + x^2\mathbf{j}$ over the curve C. What is the physical meaning of this integral in the case that \mathbf{F} is a force field?

(4) [10 pts] Suppose that z = f(x, y) is a function with the following table of values.

(a,b)	f(a,b)	$\nabla f(a,b)$	$f_{xx}(a,b)$	$f_{xy}(a,b)$	$f_{yy}(a,b)$
(1,2)	0	(0,0)	5	3	1
(7, -2)	0	(0,1)	5	3	1
(3,4)	7	(0,0)	-5	-3	-2
(5, -3)	68	(0,0)	8	-4	2
(2,1)	35	(0,0)	5	3	2

Identify any local maxima, minima, and saddle points of f. Explain the reasons for your answers.

(5) [10 pts] Use a double integral to calculate the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the plane z = 4.



(b) Define what it means for vector field to be conservative.

(c) Let $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ be a vector field in the plane. Prove that:

If **F** is conservative then
$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$
.

(7) [10 pts] Use the change of variables $x=2u+3v,\,y=3u+5v$ to calculate the integral $\iint_R (7x+2y)\,dA$, where R is the parallelogram in the xy-plane with vertices $(0,0),\,(2,3),\,(3,5)$ and (5,8).

(8) [7 pts] Let \mathbf{F} be the vector field $\mathbf{F}(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$. Show that \mathbf{F} is conservative and calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is an arbitrary (unknown) curve from (0,0) to (1,1).

 (9) [10 pts] Let F be the vector field in the plane given by F(x, y) = x²yi + (x² - y²)j. (a) Calculate the divergence of F.
(b) Calculate the curl of F .
(c) Suppose that the vector field \mathbf{F} given above is the velocity vector field of a fluid flowing in the plane. On average is the fluid flowing in or out of a small disk centered at the point $(-1,2)$? Why?

(10) [10 pts] Let S be the surface that is parametrized by

$$\mathbf{r}(u,v) = (\cos v, u, \sin v)$$
 for $0 \le u \le 2$ and $0 \le v \le \pi$.

(a) Find an equation of the form F(x, y, z) = 0 for this surface.

(b) Sketch the graph of the surface. Also sketch the grid curves u=1 and $v=\frac{\pi}{4}$ on the surface together with the vector $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$ at the point on the surface where $(u,v)=(1,\pi/4)$.

(11) [8 pts] Calculate the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the surface parametrized by $\mathbf{r}(u, v) = v\mathbf{i} + uv\mathbf{j} + u\mathbf{k}$ for $0 \le u \le 3$ and $0 \le v \le 2$, and \mathbf{F} is the vector field $\mathbf{F} = x\mathbf{i} + xy^2\mathbf{j} - z\mathbf{k}$. The surface S is oriented so that the normal to the surface has a positive y component.
Please sign the following honor statement:
On my honor, I pledge that I have neither given nor received any aid on this exam.
Signature: