NAME:

1 /8	2 /12	3 /12	4 /12	5 /10	
6 /10	7 /12	8 /6	9 /10	10 /8	T /100

MATH 251 (Fall 2011) Final Exam, Dec 15th

No calculators, books or notes! Show all work and give **complete explanations**. This 120 min exam is worth 100 points.

(1) [8 pts] Let $z = f(x, y) = x^2y^3$.

(a) Calculate the directional derivative of f at the point $(-1, 2)$ in the direction of the vector $(3, 2)$.

(b) In which direction does f decrease the fastest at the point $(-1, 2)$?

(2) [12 pts]
(a) Let D be a region in the xy-plane. If D is the shape of a metal plate and $z = f(x, y)$ is mass per unit area of the plate at the point (x, y), what does $\iint_D f(x, y)\, dA$ represent, and why?

(b) Calculate $\iint_D y\, dA$, where D is the triangular region in the xy-plane with vertices $(0, 0)$, $(3, 0)$, and $(2, 1)$.

(c) Calculate $\iint_D x\, dA$, where D is the region in the xy-plane bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ with $x \geq 0$ and $y \leq 0$.

(3) [12 pts] One of the following two vector fields is conservative:

$$\begin{aligned} \mathbf{F}_1(x,y) &= (2y + 3x^2y^2 + 6x)\mathbf{i} \ + \ (2x^3y + 6y + 8x)\mathbf{j} \\ \mathbf{F}_2(x,y) &= (2x + 3x^2y^2 + 6y)\mathbf{i} \ + \ (2x^3y + 6x + 8y)\mathbf{j}. \end{aligned}$$

(a) Which vector field is conservative and which is not? Why?

(b) For the vector field that is conservative, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve parametrized by $\mathbf{r}(t) = (\cos(\frac{\pi t^2}{2}), t^3)$ for $0 \le t \le 1$.

(4) [12 pts] Let $\mathbf{F}(x, y) = (x^2 + 2y)\mathbf{i} + (y - 3x)\mathbf{j}$.

(a) Calculate the curl of $\mathbf{F}$.

(b) Calculate the divergence of $\mathbf{F}$.

(c) Suppose that $\mathbf{F}$ is the velocity vector field of a fluid flowing in the plane. Let C be a small circle centered at the origin.
(i) On average, is fluid flowing in or out of C? Why?

(ii) On average, is the fluid rotating counterclockwise or clockwise about C? Why?

(5) [10 pts] Let L be the line through the points $(1, 2, 4)$ and $(2, 1, 3)$, and let P be the plane parametrized by $\mathbf{r}(u, v) = (1 + 2u, 2 + v, 3 - u)$. Find the (x, y, z) coordinates of the point of intersection of the line L and the plane P.

(6) [10 pts] Let $z = f(x, y)$ be a function such that

(x, y)	$(-1, 2)$	$(1, -2)$	$(0, \sqrt{3})$	$(\sqrt{3}, 0)$
$\frac{\partial f}{\partial x}$	-4	1	6	0
$\frac{\partial f}{\partial y}$	10	4	8	0

Which of the (x, y) values in this table are candidates for the absolute maximum and absolute minimum of f subject to the constraint $4x^2 + 3xy + 2y^2 = 6$? *Carefully justify your answers!*

(7) [12 pts]
(a) Carefully state Stokes' Theorem. In particular, with the aid of a sketch, describe in words the relationship between the orientation on the surface and the direction to go around the boundary.

(b) Let C be the curve obtained by intersecting the surfaces $x^2 + z^2 = 1$ and $y = 2$. Go around C in the direction so that when you pass through the point $(1, 2, 0)$ you are heading in the positive z direction. Sketch the directed curve C. Also, identify and sketch an oriented surface, S, with $\partial S = C$.

(c) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the directed curve described in (b) and $\mathbf{F}(x, y, z) = x\mathbf{i} + y^2\mathbf{j} + z^3\mathbf{k}$.

(8) [6 pts] Let $z = f(x, y) = xy^2$, where $x = g(t) = t^2$ and $y = h(t)$. Suppose that $h(2) = -3$ and $h'(2) = 1$. Calculate $\frac{dz}{dt}$ at $t = 2$.

(9) [10 pts] Let E be the solid region that is above the triangle with vertices $(0,0,0)$, $(3,0,0)$ and $(0,2,0)$ and below the plane $x+y+z=6$.

(a) Sketch the solid E.

(b) Set up *but do not evaluate* an iterated triple integral for $\iiint\limits_E x^2\,dV$.

(10) [8 pts] Let S be the surface $y = x + 3z$ with $0 \le x \le 2$ and $0 \le z \le 1$. Calculate $\iint_S yz\,dS$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ____________________________________