NAME:

1 /10	2 /10	3 /10	4 /6	5 /8	6 /6	T /50

MATH 251 (Fall 2011) Exam II, Oct 27th

No calculators, books or notes! Show all work and give **complete explanations**. This 65 min exam is worth 50 points.

(1) [10 pts] Sketch the level curves (i.e. contours) of $z = f(x, y) = ye^{-x}$ at levels $k = -1$, 0, and 1.

(2) [10 pts] Consider the curve, C, in the plane parametrized by $(x, y) = \mathbf{r}(t) = (2\sin t, \cos t)$ for $0 \leq t \leq 2\pi$.

(a) Find $\mathbf{r}'(\pi/4)$.

(b) Find a parametrization for the tangent line to the curve, C, at $t = \pi/4$.

(c) Sketch the curve, C, and include in your sketch the vectors $\mathbf{r}(\pi/4)$ and $\mathbf{r}'(\pi/4)$.

(3) [10 pts]
(a) Let $z = f(x, y)$ be a function with table of values given by

		y		
		4	5	6
x	1	9	11	14
	2	4	7	9
	3	0	6	8

Estimate $\frac{\partial f}{\partial x}$ at the points $(x, y) = (2, 4)$ and $(2, 5)$. Uses these two estimates to estimate $\frac{\partial^2 f}{\partial y \partial x}$ at $(2, 4)$.

(b) Calculate the equation of the tangent plane to the graph of the function $f(x, y) = x^2 y^3$ at $(x, y) = (2, 1)$.

(4) [6 pts] Either calculate the following limit or prove that it does not exist: $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^3+5y^3}$.

(5) [8 pts] Parametrize that part of the surface $x^2 + y^2 + z^2 = 4$ that lies above the surface $z = x^2 + y^2$.

(6) [6 pts] If $\mathbf{r}(t) \neq \mathbf{0}$, show that

$$\frac{d}{dt}|\mathbf{r}(t)| \;=\; \frac{1}{|\mathbf{r}(t)|}\,\mathbf{r}(t)\cdot\mathbf{r}'(t).$$

Hint: $|\mathbf{r}(t)|^2 \;=\; \mathbf{r}(t)\cdot\mathbf{r}(t)$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____