

LAST NAME:	FIRST NAME:	CIRCLE:  Martynova   Martynova   Zweck 8:30am       1pm
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# MATH 2415 (Spring 2017) Exam I, Feb 17th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [10 pts] Let  $P = (1, 1, 0)$  and  $Q = (1, 2, 3)$  be two points and let  $\mathbf{v} = -\mathbf{j} + 2\mathbf{k}$  be a vector.

(a) Find a parametrization for the line parallel to the vector  $\mathbf{v}$  that passes through the point  $P$ .

(b) Calculate the projection of the vector  $\overrightarrow{PQ}$  onto the vector  $\mathbf{v}$ .

(2) [12 pts] Let  $A = (1, 3, 0)$  and  $B = (2, 3, -4)$  and  $C = (3, 3, 2)$  be three points.

(a) Find a point,  $D$ , so that  $A$ ,  $B$ ,  $C$ , and  $D$  are the vertices of a parallelogram.

(b) Find the area of the parallelogram in (a).

(c) Find a unit vector orthogonal to the plane containing the points  $A$ ,  $B$ , and  $C$ .

(3) [12 pts]

(a) Find an equation of the form  $Ax + By + Cz = D$  for the plane that passes through the point  $P = (1, 3, 4)$  and contains the line  $L$  parametrized by  $x = 3t$ ,  $y = 4t$ ,  $z = 2 + 2t$ .

(b) Find the point of intersection of the line  $\mathbf{r}(t) = (t, t + 1, t + 2)$  and the plane  $x + y + z = 6$ .

(4) [12 pts] Let  $C$  be the curve with parametrization  $x = t \sin 2t$ ,  $y = t \cos 2t$ ,  $z = t$  for  $0 \leq t \leq 2\pi$ .

(a) Show that the curve,  $C$ , lies on the surface  $z = \sqrt{x^2 + y^2}$ . Sketch the surface and the curve.

(b) Calculate a parametrization of the tangent line to the curve  $C$  at the point where  $t = \frac{\pi}{2}$ .

(5) [12 pts] Make labelled sketches of the traces (slices) of the surface

$$y = 4x^2 + 9z^2$$

in the planes  $x = 0$ ,  $z = 0$ , and  $y = k$  for  $k = 0, \pm 1, \pm 2$ . Then make a labelled sketch of the surface.

(6) [9 pts] Sketch the level curves of the function  $z = (y^2 - x)^3$  at levels  $k = 0$ ,  $k = \pm 8$ .

(7) [8 pts] Convert the point with cylindrical coordinates  $(r, \theta, z) = (3, \frac{\pi}{4}, 4)$  into spherical coordinates.