LAST NAME:	FIRST NAME:	CIRCLE: Martynova 8:30am / Martynova 1pm / Zweck

1 /7	2 /10	3 /12	4 /10	5 /12	6 /12	7 /12	T /75

MATH 2415 (Spring 2017) Exam II, Mar 31st

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [7 pts] Find the equation of the tangent plane to the surface $z = e^x \cos(xy)$ at $(x, y, z) = (1, \pi/2, 0)$.

(2) [10 pts] Let $f(x, y) = xy^2$.

(a) Find the direction in which f increases most rapidly at the point $(x, y) = (2, 3)$. What is the rate of change of f in this direction?

(b) In what directions is the rate of change of f equal to zero at the point $(2, 3)$?

(3) [12 pts] (a) Suppose that $z = f(x, y) = \sin(3x^2 + 4y^2)$ where $x = x(t)$ and $y = y(t)$. If $x(0) = 2$, $y(0) = 1$, $x'(0) = 3$, and $y'(0) = -4$, find $\frac{dz}{dt}$ at $t = 0$.

(b) Evaluate the iterated integral $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\, dx\, dy$ by reversing the order of integration.

(4) [10 pts] Find the volume of the solid under the surface $z = xy+1$ and above the region in the (x, y)-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 4$.

(5) [12 pts] Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = u \cos v\,\mathbf{i} + 2u \sin v\,\mathbf{j} + u\,\mathbf{k} \qquad \text{for } 0 \leq u \leq 2 \text{ and } 0 \leq v \leq \pi/2.$$

(a) Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the surface, S, together with the "grid" curves on S where (i) $u = 1$ and (ii) $v = \pi/4$.

(c) Calculate the tangent vector to the grid curve $u = 1$ at the point where $(u, v) = (1, \pi/4)$.

(6) [12 pts] Find and classify all critical points of $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

(7) [12 pts] Use the method of Lagrange multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2$ on the ellipse $x^2 + 2(y+1)^2 = 8$.