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MATH 2415 (Fall 2016) Exam II, Nov 4th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [9 pts] Find the equation of the tangent plane to the surface $z = x^2 + xy + 3y^2$ at the point (1, 1, 5).

$$\frac{\partial f}{\partial x} = 2x + y = 2 \times 1 + 1 = 3 \text{ @ } (1,1)$$

$$\frac{\partial f}{\partial y} = \cancel{2}x + 6y = 1 + 6 = 7 \text{ @ } (1,1)$$

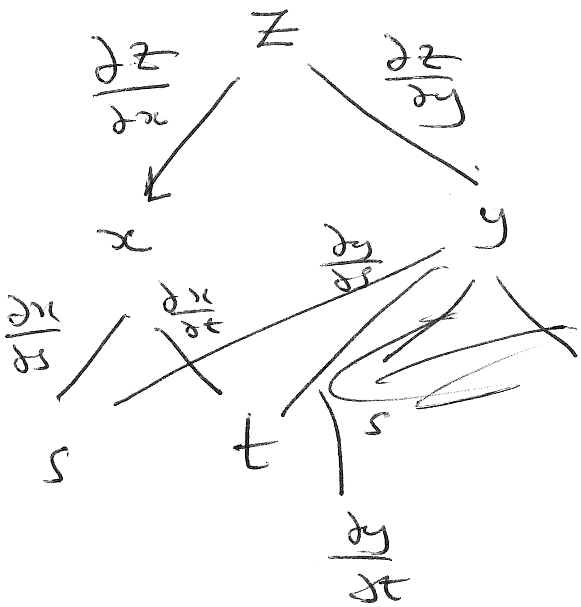
So equation of tangent plane ~~at~~ at $(x_0, y_0) = (1,1)$

$$z = f(x_0, y_0) + (x - x_0)\frac{\partial f}{\partial x}(x_0,y_0) + (y-y_0)\frac{\partial f}{\partial y}(x_0,y_0)$$

$$z = 5 + 3(x-1) + 7(y-1)$$

(2) [9 pts]

(a) Use a tree diagram to write out the Chain Rule for the composition $z = f(x, y)$, where $x = g(s, t)$ and $y = h(s, t)$.



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

(b) Use your answer to (a) to find $\frac{\partial z}{\partial t}$ at $(s,\, t) = (2,\, 0)$ where $z = e^{xy}$, $x = s + \cos t$ and $y = s - \sin t$.

$$\frac{\partial z}{\partial x} = y e^{xy} = (s \mp \sin t)\, e^{(s+\cos t)(s-\sin t)}$$

$$= 2 e^{(2+1)(2-0)} = 2e^{6} \quad @ \ (s,t) = (2,0)$$

$$\frac{\partial z}{\partial y} = x e^{xy} = (s+\cos t)\, e^{(s+\cos t)(s-\sin t)}$$

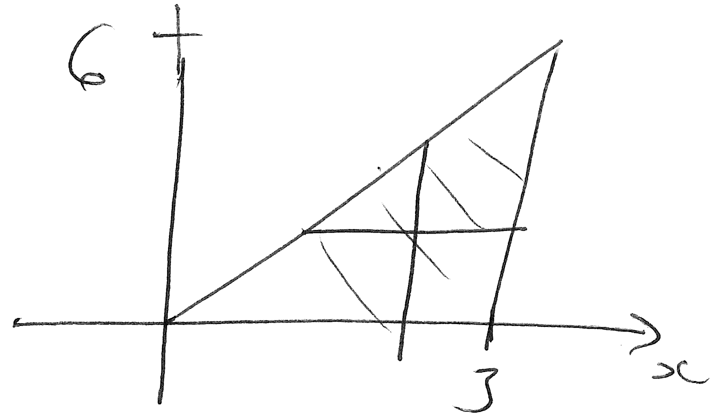
$$= 3e^{6} \quad @ \ (s,t) = (2,0)$$

$$\frac{\partial x}{\partial t} = -\sin t = 0 \quad @ \ t=0$$

$$\frac{\partial y}{\partial t} = -\cos t = -1 \quad @ \ t=0$$

$$\frac{\partial z}{\partial t} = 2e^{6}\cdot 0 + 3e^{6}(-1) = -3e^{6}$$

(3) [9 pts] Evaluate the double integral $\iint_D e^{-x^2}\,dA$, where D is the region in the xy-plane bounded by $y = 0$, $y = 2x$ and $x = 3$.



$0 \le y \le 6$

$\frac{y}{2} \le x \le 3.$

$$\iint_D e^{-x^2}\,dA = \int_{y=0}^{6}\int_{x=y/2}^{3} e^{-x^2}\,dx\,dy$$ IMPOSSIBLE

OTHER ORDER

$0 \le x \le 3$

$0 \le y \le 2x$

$$\iint_D e^{-x^2}\,dA = \int_{x=0}^{3}\int_{y=0}^{2x} e^{-x^2}\,dy\,dx$$

$$= \int_{x=0}^{3} e^{-x^2}\left[\int_{y=0}^{2x} 1\,dy\right]dx$$

$$= \int_{x=0}^{3} e^{-x^2}\cdot 2x\,dx = \int_{0}^{9} e^{-u}\,du$$

$\boxed{u = x^2}$

$$= \boxed{1 - e^{-9}}$$

(4) [12 pts] Let $f(x, y) = x^2 y$.

(a) Find the maximum rate of change of the function f at the point $(2, 1)$.

$$\nabla f = (2xy, x^2) = (4, 4) \quad @ (2,1)$$

$$\text{Max RofC} = |\nabla f(2,1)| = \sqrt{32}$$

(b) In which direction does this maximum rate of change occur?

$$\vec{u} = \frac{\nabla f(2,1)}{|\nabla f(2,1)|} = \frac{(4,4)}{\sqrt{32}}$$

(c) Find the directional derivative of f at the point $(2, 1)$ in the direction $\mathbf{i} + \mathbf{j}$.

$$(D_{\vec{u}} f)(\vec{x}) = \nabla f(\vec{x}) \cdot \vec{u}$$

$$= (4,4) \cdot \frac{(1,1)}{\sqrt{2}}$$

$$= \frac{8}{\sqrt{2}}$$

(5) [12 pts] Find the limit if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)(x^2+y^2)}{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} x^2-y^2 = 0$$

(b) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

Along $x = ky^2$

$$= \lim_{y\to 0} \frac{ky^2 \cdot y^2}{k^2y^4+y^4}$$

$$= \lim_{y\to 0} \frac{ky^4}{(1+k^2)y^4}$$

$$= \frac{k}{1+k^2}$$ depends on k

So LIMIT DNE.

(6) [12 pts] Identify the local maximum and minimum values and saddle points of the function

$$f(x,y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y.$$

$0 = \frac{\partial f}{\partial x} = 2x - 2y \implies y = x$ (1)

$0 = \frac{\partial f}{\partial y} = -2x + y^2 - 3$ (2)

Plug (1) into (2) to get

$0 = x^2 - 2x - 3 = (x-3)(x+1)$

So $x = 3$ or $x = -1$

$\boxed{x=-1}$ By (1) $\boxed{(x,y) = \cancel{(1,1)}}$ $\boxed{(-1,-1)}$

$\boxed{x=3}$ By (1) $\boxed{(x,y) = (3,3)}$

2nd Der Test

$D = \det \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{vmatrix} 2 & -2 \\ -2 & 2y \end{vmatrix} = 4y - 4 = 4(y-1)$

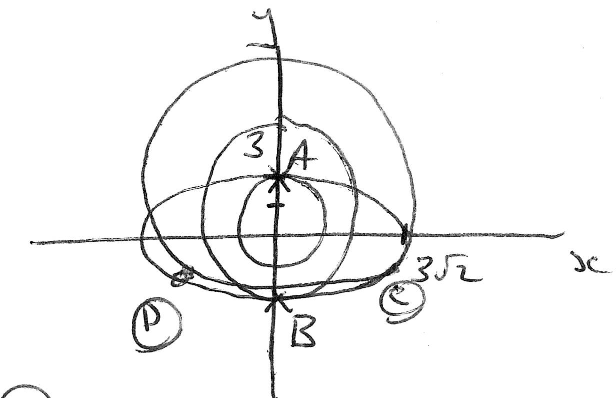
$f_{xx} = 2 > 0.$

$\boxed{(x,y) = (-1,-1)}$ $D = -8 < 0$ Saddle

$\boxed{(x,y) = (3,3)}$ $D = 4 \times 2 = 8 > 0$ $f_{xx} = 2 > 0$ Local Min

(7) [12 pts] Use the method of Lagrange Multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + (y-2)^2$ on the ellipse $x^2 + 2y^2 = 18$. [*Hint: There are 4 critical points.*]

$f = c$ IS CIRCLE CENTER $(0,2)$



$$\boxed{\begin{array}{l}\nabla f = \lambda \nabla g \\ g = k\end{array}}$$ becomes

$2x = \lambda 2x \implies x(\lambda - 1) = 0$ (1)

$2(y-2) = \lambda 4y \implies y - 2 = 2\lambda y$ (2)

$x^2 + 2y^2 = 18$ (3)

By (1) $x = 0$ OR $\lambda = 1$

$\boxed{x=0}$ By (3) $y = \pm 3$

By (2) $\pm 3 - 2 = 2\lambda(\pm 3) = \pm 6\lambda$

$\lambda = \frac{3-2}{6} = \frac{1}{6}$ OR $\lambda = \frac{-3-2}{-6} = \frac{5}{6}$

CPTS $(x, y, \lambda) = (0, 3, \frac{1}{6})$ (A) and $(0, -3, \frac{5}{6})$ (B)

$\boxed{\lambda = 1}$ By (2) $y - 2 = 2y \implies y = -2$

By (3) $x = \pm\sqrt{18 - 2\cdot(-2)^2} = \pm\sqrt{10}$

$(x, y, \lambda) = (\pm\sqrt{10}, -2, 1)$ (C), (D)

	x	y	λ	$f(x,y)$	
A	0	3	$\frac{1}{6}$	1	MIN
B	0	-3	$\frac{5}{6}$	25	
C	$\sqrt{10}$	-2	1	26	ABS MAX
D	$-\sqrt{10}$	-2	1	26	ABS MAX

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ______________________________