LAST NAME:	FIRST NAME:	CIRCLE:	Dahal 4pm	Li 1pm
		Li 5:30pm	Zweck 11:30am	Zweck 1pm

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Fall 2019

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Let D be the portion of the disc $x^2 + y^2 \leq 9$ that lies in the third quadrant of the xy-plane. Calcuate $\iint_D x\, dA$.

(2) [10 pts]
(a) Find the (level set) equation of the plane through the point $(1, 2, 3)$ that is perpendicular to the line with parametrization $\mathbf{r}(t) = (2 - 3t, 4 + 6t, -1 - t)$.

(b) Find parametrizations of **two** different lines that go through the point $(1, 0, 2)$ and are parallel to the plane $3x - 2y + 4z = 10$.

(3) [10 pts]
(a) Use vectors to find the area of the parallelogram with vertices $A = (1, 1)$, $B = (2, 6)$, $C = (3, 4)$ and $D = (4, 9)$.

(b) Make a sketch that shows how to project the vector $\mathbf{v} = \mathbf{i}$ onto the vector $\mathbf{w} = \mathbf{i} + \sqrt{3}\mathbf{j}$. Use your sketch to find the component of $\mathbf{v}$ in the direction $\mathbf{w}$.

(4) [10 pts] Let C be the half-circle given by $x^2 + y^2 = 4$ with $x \geq 0$, oriented counter-clockwise.

(a) Calcuate $\int_C x\, ds$.

(b) Without doing any calculation, find $\int_C y^3\, ds$. Explain your reasoning!

(c) Let $f(x, y) = xe^{x^2+y^2}$. Find $\int_C \nabla f \cdot d\mathbf{r}$.

(5) [10 pts]
(a) Sketch the surface $x^2 - y^2 + z^2 = 2$ for $0 \leq y \leq \sqrt{2}$. [Hint: Convert into an appropriate cylindrical coordinate system.]

(b) Show that the line through the point $(1, 0, 1)$ in the direction of the vector $(-1, \sqrt{2}, 1)$ lies on the surface in (a). Add this line to your sketch in (a).

(6) [10 pts] Find the absolute maximum and minimum of the function $z = f(x, y) = x^2 - 2x - 2y^2 + 8y$ on the triangular domain with vertices $(0, 0)$, $(4, 0)$, and $(0, 4)$.

(7) [10 pts] Sketch the solid, E, in the first octant that is bounded by the planes $x + z = 2$ and $2y + z = 2$. Calculate $\iiint_E z \, dV$.

(8) [10pts]

(a) Sketch the curve obtained by intersecting the surfaces whose equations in spherical coordinates are given by $\rho = 2$ and $\phi = \pi/3$. Write down a parameterization of the form $(x, y, z) = \mathbf{r}(t)$ for this curve.

(b) Let E be the solid region $x^2 + y^2 + z^2 \leq 16$. Calculate $\iiint_E z^4 \, dV$.

(9) [10 pts] Use an appropriate change of variables to evaluate $\iint_R x\,dA$, where R is the parallelogram bounded by the lines $x - 3y = 0$, $x - 3y = 2$, $2x - y = 1$, and $2x - y = 6$.

(10) [10 pts] Let $\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}$ be the velocity vector field of a fluid flowing in $\mathbb{R}^2$.

(a) Calculate $\nabla \cdot \mathbf{F}$.

(b) Calculate $\nabla \times \mathbf{F}$.

(c) On average, is the fluid rotating clockwise, counter-clockwise, or not rotating at all about the point $(1, 2)$? Why?

(d) On average, is the fluid flowing in, out, or neither in or out, of a small disc centered at $(1, 2)$? Why?