LAST NAME:	FIRST NAME:	CIRCLE: Li 2:30pm Li 5:30pm Zweck 10am Zweck 1pm

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Fall 2017

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Evaluate the integral $\iint_D x\,dA$ where D is that quarter of the annulus $1 \leq x^2 + y^2 \leq 16$ that is in the first quadrant.

(2) [10 pts] Let $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + xyz\mathbf{k}$. Calculate

(a) curl($\mathbf{F}$)

(b) div($\mathbf{F}$)

(3) [10 pts] Let $z = f(x, y) = x^2 + 4y^2$, and let C be the level curve $f(x, y) = 4$.
(a) Find a parametrization of the curve C.

(b) Use your answer to (a) to find a vector, $\mathbf{v}$, that is tangent to the curve C at the point $(x, y) = (1, \sqrt{3}/2)$.

(c) Find the directional derivative of f in the direction of the vector $\mathbf{v}$ in (b) at the point $(x, y) = (1, \sqrt{3}/2)$.

(4) [10 pts] Find an equation of the form $Ax+By+Cz = D$ for the plane that contains the line parametrized by $\mathbf{r}_1(t) = (1 + 2t, 3 + 4t, 5 - t)$ and that is parallel to the line parametrized by $\mathbf{r}_2(t) = (2 + t, 3, -1 + 4t)$.

(5) [10 pts]
(a) Let $u(x,t) = \sin(x + 2t)$. Show that u satisfies the wave equation $u_{tt} = 4u_{xx}$.

(b) Let $\mathbf{r}(t) = (t^2, t^3)$ and let $z = f(x, y)$ be a function so that $f(1, 1) = 6$, $\frac{\partial f}{\partial x}(1, 1) = 4$, and $\frac{\partial f}{\partial y}(1, 1) = 5$. Let $g(t) = f(\mathbf{r}(t))$. Calculate $g'(1)$.

(6) [10 pts] Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 2y$ on the upper half disk where $x^2 + y^2 \leq 4$ and $y \geq 0$.

(7) [10 pts] Let $\mathbf{F}(x, y) = y\mathbf{i} - x\mathbf{j}$ and let C be the unit circle oriented counter clockwise.

(a) Use a sketch to determine whether $\int_C \mathbf{F} \cdot d\mathbf{r}$ is positive, negative, or zero.

(b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using the definition of the line integral.

(c) Is $\mathbf{F}$ conservative? Justify your answer.

(8) [10 pts] Evaluate $\iint_R (x-y)^2 \mathrm{e}^{x+y}\, dxdy$ where R is the parallelogram bounded by $x+y=1$, $x+y=3$, $x-y=-2$ and $x-y=1$. **Hint:** Use the Change of Variables Theorem with $u=x+y$ and $v=x-y$.

(9) [10 pts] Let S be the surface parametrized by

$$(x, y, z) \;=\; \mathbf{r}(\theta, \phi) \;=\; (2\cos\theta\sin\phi, 3\sin\theta\sin\phi, 4\cos\phi).$$

Calculate a parametrization of the tangent plane to the surface S at the point where $(\theta, \phi) = (\pi/4, \pi/2)$.

(10) [10 pts] Let E be the solid region bounded by the planes $y = x$, $x = 1$, $y = 0$, $z = 1 + y$, and $z = 0$. Calculate $\iiint_E x\, dV$.

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________