NAME:	CLASS: 11:30am OR 4pm

1 /14	2 /12	3 /10	4 /12	
5 /15	6 /12	7 /15	8 /10	T /100

MATH 2415 (Fall 2012) Exam II, Nov 9th

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 2 hour exam is worth XX points.

(1) [14 pts] Let $z = f(x, y) = 1 + e^x + 3\sin y + x^2y^3$.

(a) Calculate the equation of the tangent plane to the graph of f at $(x, y) = (0, 0)$.

(c) What is the maximum rate of change of f at $(0, 0)$ and in which direction does it occur?

(2) [12 pts]
(a) Let $z = f(x, y)$ be a function so that

x	1	2	3	2	2
y	5	5	5	4	6
$f(x,y)$	3	4	6	2	7

Estimate $\frac{\partial f}{\partial y}$ at $(x, y) = (2, 5)$.

(b) Which of the following functions satisfies Laplace's equation $u_{xx} + u_{yy} = 0$?

(i) $u(x, y) = x^3 + 3xy$

(ii) $u(x, y) = e^{-y} \cos x$

(3) [10 pts]
(a) Suppose that $z = f(x, y)$ is a function and $(x, y) = \mathbf{r}(t)$ is a parametrized curve. State the version of the Chain Rule you would use to differentiate the composition $f \circ \mathbf{r}$.

(b) Let $z = f(x, y) = x^3y^2 + \ln(x^3)$ and suppose that $(x, y) = \mathbf{r}(t)$ is a parametrized curve so that

t	x	y	$\frac{dx}{dt}$	$\frac{dy}{dt}$
-1	0	0	3	-4
0	1	3	-2	5
1	3	2	5	4

Calculate $\frac{dz}{dt}(0)$.

(4) [12 pts] Consider the surface that is parametrized by

$$x = r\cos\theta,$$
$$y = r\sin\theta,$$
$$z = r,$$

for $1 \le r \le 3$ and $0 \le \theta \le 2\pi$.

(a) Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the graph of the surface. Also sketch the grid curves $\theta = \frac{\pi}{4}$ and $r = 2$ on the surface.

(5) [15 pts]

(a) Draw an example of a region that is Type II but not Type I.

(b) Calculate $\iint_D x^2y^3\,dA$, where D is the domain bounded by the curves $y = x$ and $x = y^4$.

(6) [12 pts] Find the local maxima, minima, and saddle points of the function $z = f(x, y) = y^3 - 12xy + 8x^3$.

(7) [15 pts] Use the method of Lagrange multipliers to find the absolute maximum and minimum of the function $z = f(x, y) = x^2y$ on the circle $x^2 + y^2 = 1$.

(8) [10 pts]
Let $z = f(x, y)$ be a function so that $\nabla f(0,0) = 3\mathbf{i}$.

(a) Let $\mathbf{u} = (\cos\theta, \sin\theta)$ be a unit length vector in direction given by an angle $\theta \in [0, 2\pi]$. For which values of θ is the directional derivative $D_{\mathbf{u}} f(0,0) < 0$?

(b) What is the equation of the tangent line to the level curve of f at the point $(0,0)$?

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ________________________________