NAME:	CIRCLE: Zweck 11:30am / Zweck 2:30pm

1 /9	2 /12	3 /10	4 /12	5 /12	6 /12	7 /8	T /75

MATH 2415 (Fall 2015) Exam I, Oct 9th

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [9 pts] Find a parametrization of the line which is given by the intersection of the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

(2) [12 pts] Let L_1 be the line with parametrization $\mathbf{r}_1(t) = (1 - 2t, 2 + 3t, 5 + 4t)$ and let L_2 be the line with parametrization $\mathbf{r}_2(t) = (3 + t, 4 + t, 2 - 2t)$.

(a) Find an equation of the form $Ax + By + Cz = D$ for the plane that contains the line L_1 and is parallel to the line L_2.

(b) How many planes contain the line L_1 and are perpendicular to the line L_2? Justify your answer.

(3) [10 pts] Let C be the curve with parametrization $\mathbf{r}(t) = (\cos t, \sin t, \frac{2\sqrt{3}}{\pi} t)$ and let S be the sphere of radius 2, centered at the origin.

(a) The curve, C, intersects the surface, S, in two points. Find the coordinates of these points.

(b) Find the arclength of that segment of the curve C that lies within the sphere S.

(4) [12 pts]
(a) Calculate the linearization of the function $z = f(x, y) = x^3y^2$ about the point $(x_0, y_0) = (2, -1)$.

(b) Let $z = f(x, y)$ and $g(u, v) = f(e^u + \sin v, e^u + \cos v)$. Using the table of values below, calculate the partial derivatives $g_u(0, 0)$ and $g_v(0, 0)$.

	f	g	f_x	f_y
$(0, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

(5) [12 pts] Make a labelled sketch of the traces of the surface

$$4x^2 - 2y^2 + z^2 = 1$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then sketch the surface.

(6) [12 pts] Find the limit if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{x^2+y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

(7) [8 pts]
(a) Let $\mathbf{v}$ and $\mathbf{w}$ be nonzero vectors in $\mathbf{R}^3$. Under what conditions is $|\mathbf{v} \times \mathbf{w}| = \mathbf{v} \cdot \mathbf{w}$?

(b) Suppose that $(x, y, z) = \mathbf{r}(t)$ is a parametrized curve whose speed is constant. Show that the acceleration vector of the curve is always perpendicular to the velocity vector of the curve, *i.e.*, that $\mathbf{r}'(t) \perp \mathbf{r}''(t)$.

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ________________________________