NAME:	CIRCLE: Turi	Zweck 10am	Zweck 4pm

1 /10	2 /12	3 /12	4 /15	5 /16	6 /10	T /75

MATH 2415 (Fall 2014) Exam II, Nov 7th

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [10 pts] Let $z = f(x, y) = 3x^2 + 2y^2 - xy$. Suppose that $(x, y) = \mathbf{r}(t)$ is a parametrized curve so that $\mathbf{r}(0) = (2, 3)$ and $\mathbf{r}'(0) = (-3, 4)$. Let $g(t) = f(\mathbf{r}(t))$. Find the slope of g at $t = 0$.

(2) [12 pts] Let $z = f(x, y) = 3x^2 + 2y^2 - xy$.
(a) Find the directional derivative of f in the direction of the vector $\mathbf{v} = (3, 4)$ at the point $(x, y) = (1, 2)$.

(b) Find the direction of steepest descent of f at the point $(x, y) = (1, 2)$. What is the rate of change of f in this direction?

(c) Find a parametrization for the tangent line to the level curve of f that passes through the point $(x, y) = (1, 2)$.

(3) [12 pts] Let $z = f(x, y) = x^3 + 3x^2 - y^2 + y$. Find all local maxima, minima, and saddle points of f.

(4) [15 pts] In this problem you will use the **method of Lagrange Multipliers two different ways** to solve the same problem. The problem is to find the absolute maximum and absolute minimum of the function $f(x, y) = x - y$ on the circle $x^2 + y^2 = 1$.

(a) First solve the problem **graphically** by sketching the circle and some appropriately chosen level curves, $f(x, y) = k$.

(b) Now solve the problem by setting up the appropriate **equations** and solving them algebraically.

(5) [16 pts]
(a) Let D be the region in the xy-plane that is bounded by the curves $y = x^3$ and $y = x^4$. Calculate $\iint_D y\, dA$.

(b) Evaluate the integral by reversing the order of integration:

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1}\, dydx.$$

(6) [10 pts] Find the volume of the solid bounded by the surfaces $z = 2x^2 + 2y^2$ and $z = 4 - x^2 - y^2$.

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: _____