NAM	IE:								CIRC	LE:	Turi		Zweck 10am	Zwe 4pm	
1	/10	2	/12	3	/10	4	/10	5	/12	6	/12	6	/9	Т	/75

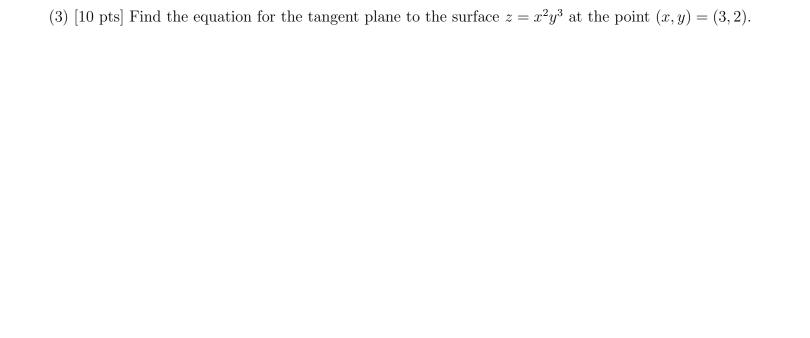
MATH 2415 (Fall 2014) Exam I, Oct 3rd

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

- (1) [10 pts]
- (a) Find a parametrization of the line, L, through the point (5,1,0) that is parallel to the line with parametrization $\mathbf{r}(t) = (3+4t, -2+7t, 1-6t)$.

(b) Find the point of intersection of the line L in (a) and the plane x+y+z=1.

(2) [12 pts] Find an equation of the form z = Ax + By + C for the plane that passes through the points (1,2,3) and (-2,3,6) and which is perpendicular to the plane 3x + 2y - z = 2.



(4) [10 pts] Let C be the curve that is parametrized by $(x,y,z)=\mathbf{r}(t)=(2t,t^2,\frac{1}{3}t^3)$. Find the arc length of C between the points P=(0,0,0) and $Q=(2,1,\frac{1}{3})$.

(5) [12 pts] Make a labelled sketch of the traces of the surface

$$4x^2 - y^2 + z^2 = -1$$

in the planes $x=0,\,z=0,$ and y=k for $k=0,\,\pm 1,\,\pm 2.$ Then sketch the surface.

- (6) [12 pts] Find the limit if it exists, or show that the limit does not exist. (a) $\lim_{(x,y)\to(0,0)} \frac{y^4-9x^2}{y^2-3x}$

(b) $\lim_{(x,y)\to(0,0)} \frac{y^4-9x^2}{y^2-3x^2}$

- (7) [9 pts] Let $\mathbf{r}(t) = \mathbf{p} + t\mathbf{v}$ be a parametrization of the line, L, through the point \mathbf{p} in the direction of the vector \mathbf{v} and let \mathbf{q} be a point that is not on the line L.
- (a) Show that the distance between the point \mathbf{q} and a point $\mathbf{r}(t)$ on the line is given by

$$D(t) = \sqrt{|\mathbf{p} - \mathbf{q}|^2 + 2t(\mathbf{p} - \mathbf{q}) \cdot \mathbf{v} + t^2 |\mathbf{v}|^2}.$$
 (1)

(b) Use Equation (1) above to show that the point on the line L that is closest to \mathbf{q} is given by

$$\mathbf{r}_* = \mathbf{p} + \operatorname{Proj}_{\mathbf{v}}(\mathbf{q} - \mathbf{p})$$

where $\text{Proj}_{\mathbf{b}}(\mathbf{a})$ is the vector projection of the vector \mathbf{a} onto the vector \mathbf{b} .