

LAST NAME:		FIRST NAME:		CIRCLE:  Li 2:30pm   Li 5:30pm   Zweek 10am   Zweek 1pm									
1	/12	2	/12	3	/15	4	/12	5	/12	6	/12	T	/75

MATH 2415 (Fall 2017) Exam II, Nov 3rd

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

- (1) [12 pts] Find the equation of the tangent plane to the surface  $z = x^2 + xy + 3y^2$  at the point  $(1, 1, 5)$ .

(2) [12 pts] Let  $f(x, y) = x^2y$ .

(a) Find the maximum rate of change of the function  $f$  at the point  $(2, 1)$ .

(b) In which direction does this maximum rate of change occur?

(c) Find the directional derivative of  $f$  at the point  $(2, 1)$  in the direction of the vector  $\mathbf{i} + \mathbf{j}$ .

(3) [15 pts] Suppose that  $z = f(x, y)$  is a function such that

$(a, b)$	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	0	$(0, 0)$	5	3	1
$(7, -2)$	0	$(0, 1)$	5	3	1
$(3, 4)$	7	$(0, 0)$	-5	-3	-2
$(5, -3)$	68	$(0, 0)$	8	-4	2
$(2, 1)$	35	$(0, 0)$	5	3	2

Identify any local maxima, minima, and saddle points of  $f$ . Explain the reasons for your answers.

(4) [12 pts] Let  $D$  be the domain in the  $xy$ -plane that is bounded by the curves  $y = x^2$  and  $y = 2 - x$ . Calculate  $\iint_D x \, dA$ .

(5) [12 pts] Use the method of Lagrange Multipliers to find the absolute maximum and absolute minimum of the function  $f(x, y) = x^2 + (y - 2)^2$  on the ellipse  $x^2 + 2y^2 = 18$ . [*Hint: There are 4 critical points.*]

(6) [12 pts] Let  $S$  be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = (u \cos v, u \sin v, u) \quad u \geq 0, \quad 0 \leq v \leq 2\pi.$$

(a) Find an equation of the form  $F(x, y, z) = 0$  for this surface by eliminating  $u$  and  $v$  from the equations for  $x$ ,  $y$ , and  $z$  above.

(b) Sketch the surface, together with the curve on the surface where  $u = 2$  and the curve where  $v = \pi/4$ .