

SOME ELEMENTARY FACTS

(A)

① Let $f : \mathbb{R} \rightarrow \mathbb{R}$

(a) f is EVEN if $f(-x) = f(x) \quad \forall x \in \mathbb{R}$

(b) f is ODD if $f(-x) = -f(x) \quad \forall x \in \mathbb{R}$

② If f is even, $\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx$
If f is odd, $\int_{-L}^L f(x) dx = 0$

③ $\int u^n \sin u du = -u^n \cos u + n \int u^{n-1} \cos u du$

$\int u^n \cos u du = u^n \sin u + n \int u^{n-1} \sin u du$

④ ~~§~~ $\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

So $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

~~⑤ $\sin(x+y) = \sin x \cos y + \cos x \sin y$~~

~~$\sin(x-y) = \sin x \cos y - \cos x \sin y$~~

So ~~$\sin x$~~


(B)

$$\textcircled{5} \quad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

Adding gives

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\textcircled{6} \quad \cos(k\pi) = (-1)^k$$


$$\sin\left(\frac{k\pi}{2}\right) = 0 \quad k \text{ even}$$

$$\sin\left(\frac{(2l+1)\pi}{2}\right) = (-1)^l$$