

LAST NAME:	FIRST NAME:	CIRCLE:
		<div>Zweck</div> <div>10:00am</div> <div>Khafizov</div> <div>11:30am</div> <div>Khafizov</div> <div>2:30pm</div>

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# MATH 2415 (Spring 2016) Exam II, Apr 1st

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [12 pts] Let  $z = f(x, y) = x^2 + 4xy - y^3 + x + y$ .

(a) Find the direction of steepest ascent and the maximum rate of change of  $f$  at  $(x, y) = (2, -1)$ .

Final Answer:

(b) Find the directional derivative of  $f$  in the direction of the vector  $\mathbf{v} = (-3, 4)$  at the point  $(x, y) = (2, -1)$ .

Final Answer:

(2) [12 pts] Find the limit if it exists, or show that the limit does not exist.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$

Final Answer:

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2y}{\sqrt{x^2 + y^2}}$

Final Answer:

(3) [12 pts] Find  $\iint_D xy^2 dA$  where  $D$  is the triangular domain with vertices  $(0,0)$ ,  $(2,0)$  and  $(2,4)$ .

Final Answer:

(4) [15 pts] Find the absolute maximum and absolute minimum of the following function

$$z = f(x, y) = 2xy - 2x^2 - 5y^2 + 4x + 4y - 4$$

on the triangular domain with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(0, 2)$ .

Final Answer:

Use this page if additional space is needed for the solution of Problem 4:

(5) [12 pts] Consider the surface that is parametrized by

$$(x, y, z) = \mathbf{r}(\theta, \phi) = (3 \sin \phi \cos \theta, 3 \sin \phi \sin \theta, 3 \cos \phi).$$

(a) Find an equation of the form  $F(x, y, z) = 0$  for this surface.

(b) Sketch the graph of that portion of the surface given above for which  $0 \leq \theta \leq \frac{\pi}{4}$  and  $0 \leq \phi \leq \frac{\pi}{2}$ . Also sketch the grid curves  $\theta = \frac{\pi}{8}$  and  $\phi = \frac{\pi}{4}$  on the surface.

(6) [12 pts] Use the method of Lagrange Multipliers to find the absolute maximum and absolute minimum of the function  $z = f(x, y) = x^2y^2$  subject to the constraint  $x^2 + 4y^2 = 1$ .

Please sign the following honor statement:

*On my honor, I pledge that I have neither given nor received any aid on this exam.*

Signature: \_\_\_\_\_