Math 2415
Paper Homework #1

Some of the paper homework problems in this course are "scaffolded". This means that we break a harder problem down into several manageable parts. When you are working on such problems it helps to work out how the answers to the earlier parts help you tackle the later parts. Schematic diagrams may help you solve each part and also connect different parts of the problem. A schematic diagram shows the relationships between the different nouns in the problem statement, but not in a completely realistic way.

1. (a) Describe and sketch the curve in $\mathbb{R}^2$ represented by $(x - 1)^2 + (y + 3)^2 = 4$.

   (b) Describe and sketch the surface in $\mathbb{R}^3$ represented by $(x - 1)^2 + (y + 3)^2 = 4$.

2. Suppose you have a parallelogram with vertices P, Q, R, and S so that the displacement vector from P to Q equals the displacement vector from S to R. If $P = (2, 0, 1)$, $Q = (3, 1, 0)$ and $R = (4, 3, 5)$ find the coordinates of the point, S. Draw a schematic diagram showing how you arrived at your answer.

3. In this problem you will find two unit vectors that each make an angle of $30°$ with the vector $\mathbf{v} = (3, 4)$.

   (a) Make a sketch that explains why there are precisely two such vectors $\mathbf{u}$.

   (b) Let $\mathbf{u} = (a, b)$ be one such vector. Write down an equation in terms of a and b that encodes the fact that $|\mathbf{u}| = 1$. Write down an equation in terms of a and b that encodes the fact that the angle between $\mathbf{u}$ and $\mathbf{v}$ is $30°$.

   (c) Now you have two simultaneous equations in two unknowns, a and b. Solve them by eliminating one of the variables to obtain a quadratic equation for the other.

   (d) Hence find the two unit vectors.

   (e) Check your answers are correct.

4. Suppose that AB is the diameter of a circle with center O and that C is a point on the circle, with C not equal to either A or B. (See figure below.) Use the following steps to prove that the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are orthogonal.

   (a) Why can you assume that the point O is the origin and that the line segment $\overline{AB}$ is on the x-axis?

   (b) Suppose that angle $\angle BOC = \theta$ and that the radius of the circle is R. Find a formula for the vector $\mathbf{c} = \overrightarrow{OC}$ in terms of R and θ.

   (c) Let $\mathbf{a} = \overrightarrow{AC}$ and $\mathbf{b} = \overrightarrow{BC}$. Find formulae for the vectors $\mathbf{a}$ and $\mathbf{b}$ in terms of R and θ.

   (d) Calculate the dot product of $\mathbf{a}$ and $\mathbf{b}$. What can you conclude?