LAST NAME:	FIRST NAME:	CIRCLE:	Dahal 4pm	Li 1pm	
		Li 5:30pm	Zweck 11:30am	Zweck 1pm	

## MATH 2415 [Fall 2019] Exam II, Nov 1st

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points. **Your points for each problem will be recorded on the top of the second page.** 

- (1) [12 pts]
- (a) Suppose that w = f(x, y, z), where x = x(t), y = y(t) and z = z(t). Use a tree diagram to write out a formula for  $\frac{dw}{dt}$ . Use this formula to find  $\frac{dw}{dt}$  when  $f(x, y, z) = \ln(x^2 + y^2 + z)$ ,  $x = t^3$ ,  $y = \sin t$  and z = 3t.

<sup>(</sup>b) Find the equation of the tangent plane to the graph of  $z = f(x, y) = y^2 e^x$  at (0, 1). Use this tangent plane to approximate f(0.2, 1.1).

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- (2) [12 pts] Let  $f(x,y) = ye^{2x}$ .
- (a) Find the gradient of f at the point (0,1).

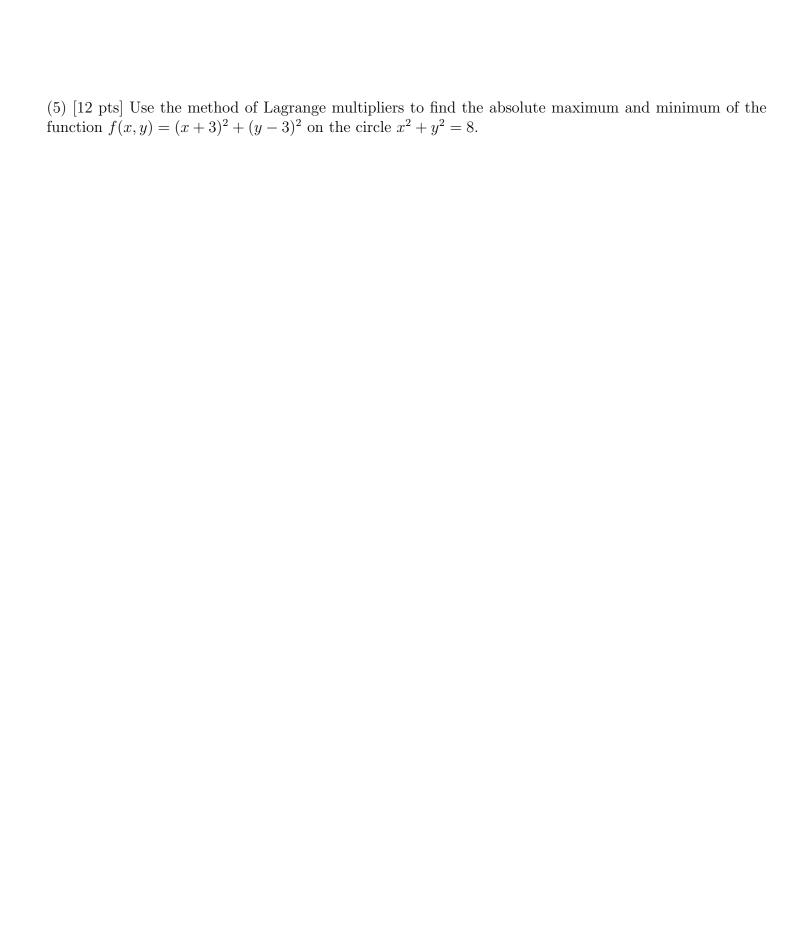
(b) Find the directional derivative of f at the point (0,1) in the direction of the vector  $\mathbf{v} = \mathbf{i} + \mathbf{j}$ .

(c) Find the direction of the minimum rate of change in f at (0,1). Also find the minimum rate of change.

(d) Sketch the level curve f(x,y) = 1. Add the vector  $\nabla f(0,1)$  to your sketch.

(3) [12 pts] Calculate  $\iint_D \cos(x^3 + 1) dA$  where D is the domain in the plane bounded by y = 0, x = 1, and  $y = x^2$ .

(4) [15pts] Find and classify all critical points of the function  $f(x,y)=2x^3-3x^2y+3y^2+12x^2$ .



(6) [12 pts] Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = (u \cos v, u \sin v, u^2),$$
 for  $0 \le u \le 3$  and  $0 \le v \le 2\pi$ .

(a) Show that S is part of a paraboloid. Hint: Find an equation of the form F(x, y, z) = 0 for this surface.

(b) Sketch the surface S, together with the grid curves where (i) u=2 and (ii)  $v=\frac{\pi}{4}$ . (Label these curves!)

(c) Calculate the tangent vector to the grid curve where  $v = \frac{\pi}{4}$  at the point  $\mathbf{r}(2, \frac{\pi}{4})$ .