NAME:

1	/12	2	/10	3	/5	4	/10	5	/10	6	/8		
7	/10	8	/10	9	/5	10	/8	11	/8	12	/4	Τ	/100

MATH 2415 (Spring 2014, **ZWECK**) Final Exam

No calculators, books or notes! Show all work and give **complete explanations**. This 2 hours 30 mins exam is worth 100 points.

(1) [12 pts] (a) Find the (level set) equation of the plane through the points (2, 1, 1), (1, 3, 0), and (-2, 3, 1).

(b) Find a parametrization of the line segment from the point $(-2, 0, -1)$ to the point $(1, 2, 3)$.
(c) Make a single sketch that includes both the plane in (a) and the line segment in (b). Using your sketch, explain why the line segment intersects the plane in a point. Then calculate the coordinates of this point of intersection.

(2) [10 pts] Make labelled sketches of the traces of the surface

$$y^2 + \left(\frac{z}{2}\right)^2 - \left(\frac{x}{3}\right)^2 = 1.$$

in the planes y = 0, z = 0, and x = k for a few appropriately chosen values of k. Then sketch the surface.

(3) [5 pts] Suppose that an ant is walking on a hot plate in the xy-plane and that the position of the ant at time t is $\mathbf{r}(t) = (\sin t, \cos 3t)$. Let T = T(x,y) be the temperature at the point (x,y) on the hot plate. Suppose that $\frac{\partial T}{\partial x} = 1$ and $\frac{\partial T}{\partial y} = 3$ at the point $(x,y) = \mathbf{r}(\frac{\pi}{4})$. Is the temperature of the ant's feet increasing or decreasing at time $t = \frac{\pi}{4}$?

(4) [10 pts] Find the absolute maximum and absolute minimum of the function $z = f(x, y) = x^2 + y^2 - 2x$ on the closed triangular region with vertices (2, 0), (0, 2) and (0, -2).

(5) [10 pts] Use a triple integral to calculate the volume of the solid region bounded by $z = y^2$, x = 0, and z + x = 1.

(6) [8 pts] Use spherical coordinates to calculate the triple integral $\iiint_E x^2 + y^2 + z^2 dV$, where E is the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = -\sqrt{x^2 + y^2}$.

(7) [10 pts] Use the change of variables x = 2u + v, y = u + 2v to evaluate the integral $\iint_R (x - 3y) dA$, where R is the triangular region with vertices (0,0), (2,1), and (1,2).

(8) [10 pts]

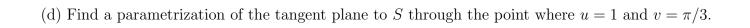
Let S be the surface with parametrization

$$\mathbf{r}(u, v) = (u \cos v, 2u, u \sin v),$$
 where $0 < u < 2$ and $0 < v < \pi$.

(a) Find parametrizations of the grid curve u=1 and of the grid curve $v=\pi/3$.

(b) Show that the points on the surface S satisfy the equation $x^2 + z^2 = \frac{y^2}{4}$.

(c) Sketch the surface S together with the grid curves you found in (a).



(9) [5 pts] Let C be the oriented curve parametrized by $\mathbf{r}(t) = \cos t \, \mathbf{i} + \sin t \, \mathbf{j} + t \, \mathbf{k}$, for $0 \le t \le \pi$ and let $\mathbf{F} = x \, \mathbf{i} + y \, \mathbf{j} + xy \, \mathbf{k}$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

- (10) [8 pts]
- (a) Give a careful statement of Green's Theorem.

(b) Use Green's Theorem to show that if $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a vector field on a region D in the plane then

 $\iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{k} \, dA \; = \; \int_{\partial D} \mathbf{F} \cdot d\mathbf{r}.$

(11) [8 pts] Suppose that ${f F}$ is a vector field in the xy -plane s	such that $\nabla \times \mathbf{F} = (x^2 + y^2) \mathbf{k}$.
--	---

(a) Can you find a function z = f(x, y) so that $\mathbf{F} = \nabla f$? Why?

(b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the circle $x^2 + y^2 = 4$.

(12) [4 pts] Suppose that $\mathbf{F} = \cos x \, \mathbf{i} + \sin y \, \mathbf{j}$ is the velocity vector field of a fluid flowing in the xy-plane. Let D be a small disc centered at the point (1,0). On average is fluid flowing into or out of D? Why?

Pledge: I have neither given nor received aid on this exam

Signature: