NAME:	CIRCLE: Turi	Zweck 10am	Zweck 4pm

1 /10	2 /12	3 /10	4 /10	5 /12	6 /12	6 /9	T /75

MATH 2415 (Fall 2014) Exam I, Oct 3rd

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [10 pts]
(a) Find a parametrization of the line, L, through the point $(5, 1, 0)$ that is parallel to the line with parametrization $\mathbf{r}(t) = (3 + 4t, -2 + 7t, 1 - 6t)$.

(b) Find the point of intersection of the line L in (a) and the plane $x + y + z = 1$.

(2) [12 pts] Find an equation of the form $z = Ax + By + C$ for the plane that passes through the points $(1, 2, 3)$ and $(-2, 3, 6)$ and which is perpendicular to the plane $3x + 2y - z = 2$.

(3) [10 pts] Find the equation for the tangent plane to the surface $z = x^2y^3$ at the point $(x, y) = (3, 2)$.

(4) [10 pts] Let C be the curve that is parametrized by $(x, y, z) = \mathbf{r}(t) = (2t, t^2, \frac{1}{3}t^3)$. Find the arc length of C between the points $P = (0, 0, 0)$ and $Q = (2, 1, \frac{1}{3})$.

(5) [12 pts] Make a labelled sketch of the traces of the surface

$$4x^2 - y^2 + z^2 = -1$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then sketch the surface.

(6) [12 pts] Find the limit if it exists, or show that the limit does not exist.

(a) $\lim\limits_{(x,y)\to(0,0)} \frac{y^4-9x^2}{y^2-3x}$

(b) $\lim\limits_{(x,y)\to(0,0)} \frac{y^4-9x^2}{y^2-3x^2}$

(7) [9 pts] Let $\mathbf{r}(t) = \mathbf{p} + t\mathbf{v}$ be a parametrization of the line, L, through the point $\mathbf{p}$ in the direction of the vector $\mathbf{v}$ and let $\mathbf{q}$ be a point that is not on the line L.

(a) Show that the distance between the point $\mathbf{q}$ and a point $\mathbf{r}(t)$ on the line is given by

$$D(t) \;=\; \sqrt{|\mathbf{p}-\mathbf{q}|^2 + 2t(\mathbf{p}-\mathbf{q})\cdot\mathbf{v} + t^2|\mathbf{v}|^2}. \tag{1}$$

(b) Use Equation (1) above to show that the point on the line L that is closest to $\mathbf{q}$ is given by

$$\mathbf{r}_* \;=\; \mathbf{p} \;+\; \mathrm{Proj}_{\mathbf{v}}(\mathbf{q}-\mathbf{p})$$

where $\mathrm{Proj}_{\mathbf{b}}(\mathbf{a})$ is the vector projection of the vector $\mathbf{a}$ onto the vector $\mathbf{b}$.