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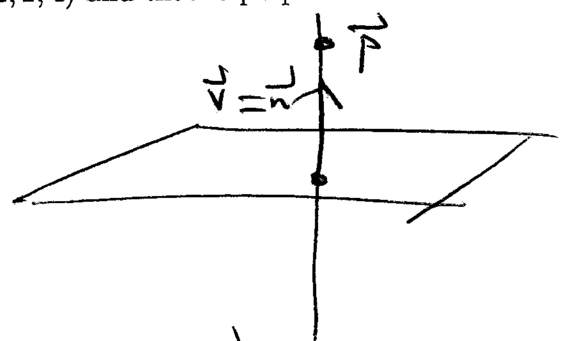
MATH 2415 (Fall 2016) Exam I, Sep 30th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [12 pts]
(a) Find a parametrization of the line that goes through the point $(-2, 2, 4)$ and that is perpendicular to the plane $2x - y + 5z = 12$.



$\vec{P} = (-2, 2, 4)$ is a point on line

$\vec{n} = (2, -1, 5)$ is normal to plane

$\vec{v} = \vec{n}$ is also a vector along line.

So $\vec{r}(t) = \vec{P} + t\vec{v} = (-2, 2, 4) + t(2, -1, 5)$

$= (-2 + 2t, 2 - t, 4 + 5t)$

(b) Calculate the vector projection of $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ onto $\mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

PROJECTION OF $\vec{a}$ ONTO $\vec{b}$

$$\text{PROJ}_{b}(\vec{a}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{(3, -2, 1) \cdot (1, 1, -2)}{1^2 + 1^2 + 2^2} (1, 1, -2)$$

$$= -\frac{1}{6} (1, 1, -2)$$

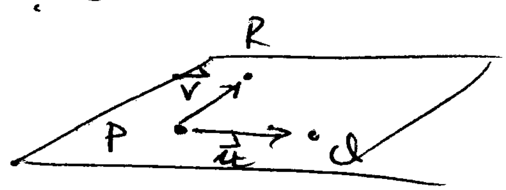
(2) [14 pts] Let $P = (3, -1, 1)$, $Q = (4, 0, 2)$, and $R = (6, 3, 1)$ be three points in space.

(a) Find a parametrization of the form $\mathbf{r}(s,t) = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$ for the plane containing P, Q, and R.

$$\vec{p} = P = (3, -1, 1)$$

$$\vec{u} = \overrightarrow{PQ} = (4, 0, 2) - (3, -1, 1) = (1, 1, 1)$$

$$\vec{v} = \overrightarrow{PR} = (6, 3, 1) - (3, -1, 1) = (3, 4, 0)$$



$$\vec{r}(s,t) = \vec{p} + s\vec{u} + t\vec{v}$$

$$= (3, -1, 1) + s(1, 1, 1) + t(3, 4, 0)$$

$$= (3 + s + 3t,\ -1 + s + 4t,\ 1 + s)$$

(b) Find an equation of the form $Ax + By + Cz + D = 0$ for the same plane as in (a).

Normal is $\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 3 & 4 & 0 \end{vmatrix}$

$$= (-4, 3, 1)$$

$\vec{p} = P = (3, -1, 1)$ is pt in plane

So equation is $(\vec{r} - \vec{p}) \cdot \vec{n} = 0$

$$(x-3, y+1, z-1) \cdot (-4, 3, 1) = 0$$

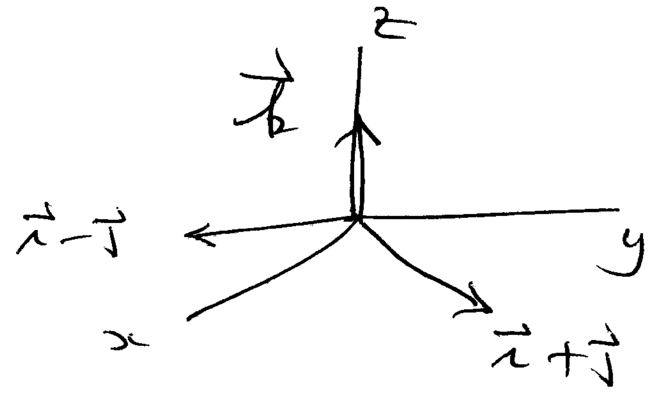
$$\boxed{-4x + 3y + z + 14 = 0}$$

(3) [7 pts] Find two unit vectors that are both perpendicular to $\mathbf{i}+\mathbf{j}$ and perpendicular to each other.

$\vec{i}+\vec{j} = (1,1,0)$

obviously $\boxed{\vec{u}_1 = \vec{k}}$

is $\perp$ to $\vec{i}+\vec{j}$.



Let $\vec{v} = (\vec{i}+\vec{j})\times\vec{k}$

$= \vec{i}\times\vec{k} + \vec{j}\times\vec{k}$

$= -\vec{j}+\vec{i}$

So set

$\vec{u}_2 = \frac{\vec{v}}{|\vec{v}|} = \boxed{\frac{\vec{i}-\vec{j}}{\sqrt{2}} = \vec{u}_2}$

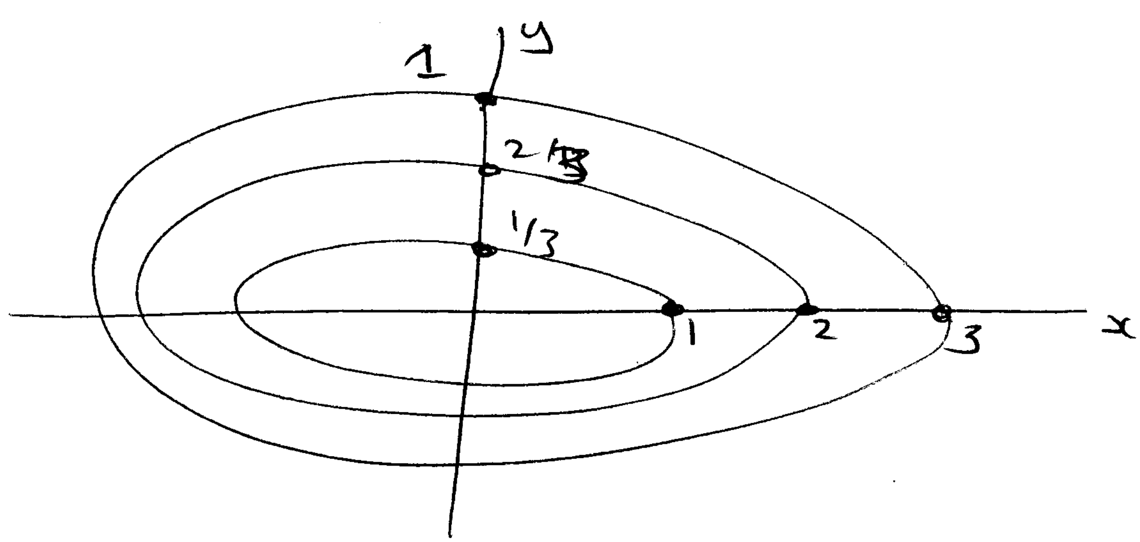
(4) [6 pts] Let $f(x, y) = \sqrt{x^2+9y^2}$. Sketch the level curves $f(x, y) = k$ for $k = 1, 2, 3$.

$\sqrt{x^2+9y^2} = k$

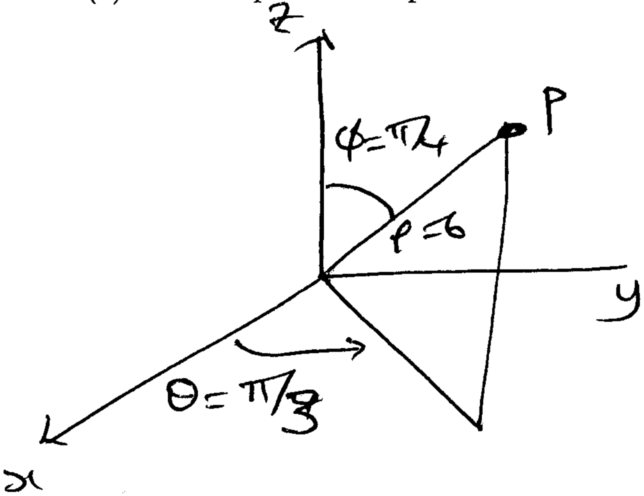
$x^2+9y^2 = k^2$ ELLIPSES

INTERCEPTS AT (1) $y=0$, $x = \pm k$

(2) $x=0$, $y = \pm k/3$



(5) [12 pts]
(a) Plot the point with spherical coordinates $(\rho, \theta, \phi) = (6, \frac{\pi}{3}, \frac{\pi}{4})$, and find its rectangular coordinates.



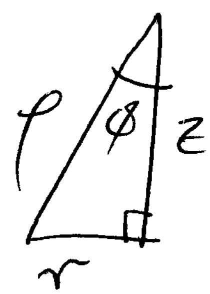
$$x = \rho \sin\phi \cos\theta$$
$$= 6 \sin \pi/4 \cos \pi/3$$
$$= 6 \frac{1}{\sqrt{2}} \frac{1}{2} = \frac{3}{\sqrt{2}}$$

$$y = \rho \sin\phi \sin\theta$$
$$= 6 \sin \pi/4 \sin \pi/3$$
$$= 6 \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} = 3\sqrt{\frac{3}{2}}$$

$$z = \rho \cos\phi = 6 \cos \pi/4 = \frac{6}{\sqrt{2}}$$

(b) The equation $z = r$ is given in cylindrical coordinates. Convert this equation to spherical coordinates.

$$z = r$$
$$\rho \cos\phi = \rho \sin\phi$$



$$\tan\phi = 1$$

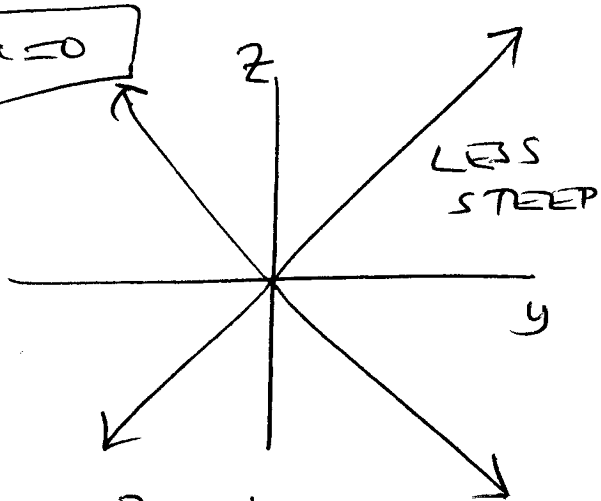
$$\boxed{\phi = \pi/4}$$

(6) [12 pts] Make a labelled sketch of the traces (slices) of the surface

$$4x^2 - y^2 + z^2 = 0$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then make a labelled sketch of the surface.

$\boxed{x=0}$



$z = \pm y$

as

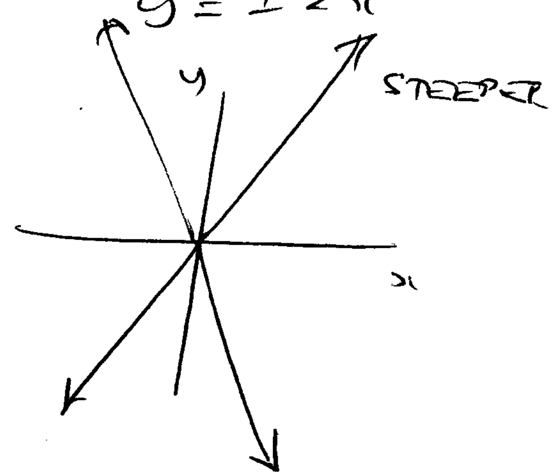
$-y^2 + z^2 = 0$

$\Rightarrow z^2 = y^2$

$\Rightarrow z = \pm y$

$\boxed{z=0}$ $4x^2 = y^2$

$y = \pm 2x$



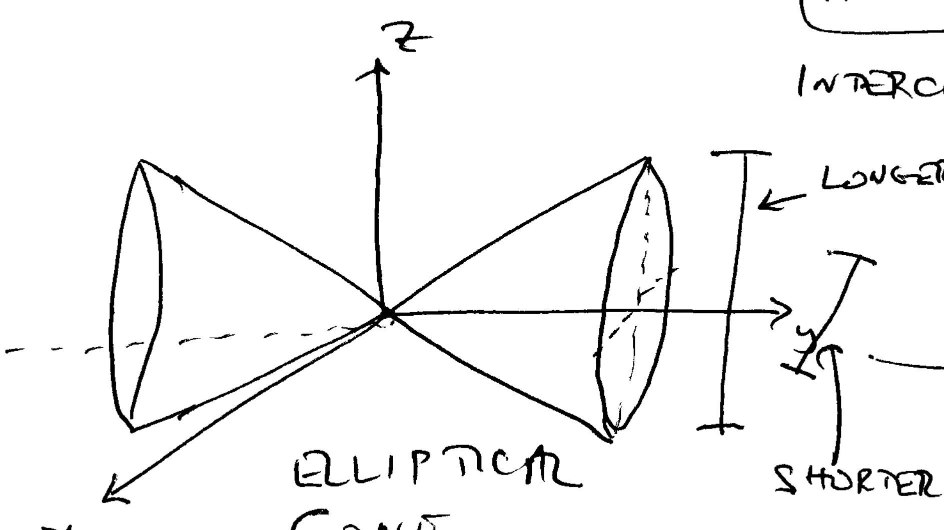
$\boxed{y=k}$ $4x^2 + z^2 = k^2$

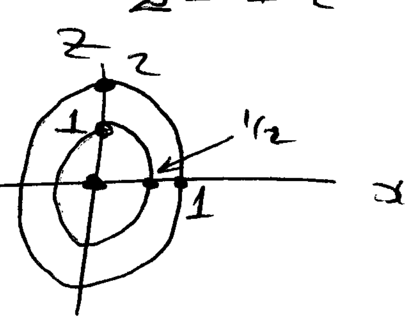
$\boxed{k=0}$ ORIGIN.

$\boxed{k = \pm 1, \pm 2}$ ELLIPSES

INTERCEPTS $x = \pm k/2$

$\cancel{z} = \pm k$





ELLIPTICAL CONE

ALIGNED WITH

y AXIS

(7) [12 pts] Let C be the curve parametrized by

$$x = t, \qquad y = \frac{t^2}{2}, \qquad z = \frac{t^3}{6}.$$

Find the length of C from the origin to the point $(6, 18, 36)$.

$$\vec{r}(t) = (t, t^2/2, t^3/6)$$

$$\vec{r}(0) = (0, 0, 0), \qquad \vec{r}(6) = (6, 18, 36)$$

$$\vec{r}\,'(t) = (1, t, t^2/2)$$

$$|\vec{r}\,'(t)| = \sqrt{1 + t^2 + \frac{t^4}{4}} = \frac{1}{2}\sqrt{4 + 4t^2 + t^4}$$

$$= \frac{1}{2}\sqrt{(2+t^2)^2} = \frac{1}{2}(2+t^2) = \frac{t^2}{2} + 1$$

So

$$L = \int_0^6 |\vec{r}\,'(t)|\, dt = \int_0^6 \left(\frac{t^2}{2} + 1\right) dt$$

$$= \left[t + \frac{t^3}{6}\right]_0^6$$

$$= \boxed{42}$$

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ______________________________