LAST NAME:	FIRST NAME:	CIRCLE: Martynova Zweck

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Spring 2017

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + 4\mathbf{j}$, and $\mathbf{w} = 2\mathbf{i} + a\mathbf{j} + 3\mathbf{k}$, for some scalar, a.

(a) Find a unit vector parallel to $\mathbf{u}$.

(b) Find a vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$

(c) Find the value of a that makes $\mathbf{w}$ perpendicular to $\mathbf{u}$.

(2) [10 pts] Find a parametrization of the line that contains the point $(1, 3, -2)$ and is parallel to both the plane $x + 2y + z = 4$ and the plane $2x - y + z = 1$.

(3) [10 pts] Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the vector field $\mathbf{F}(x, y) = y\,\mathbf{i} - x\,\mathbf{j}$ and where C is the circle $(x-2)^2 + (y+5)^2 = 9$.

(4) [10 pts] Determine whether or not the vector field $\mathbf{F}(x, y) = (y^2 - 2xy)\mathbf{i} + (2xy - x^2)\mathbf{j}$ is conservative. If it is conservative, find a potential function for $\mathbf{F}$.

(5) [10 pts] Find the absolute maximum and minimum values of the function $f(x, y) = xy - x$ on the triangle with vertices $(-1, 0)$, $(-1, 3)$, and $(2, 0)$.

(6) [10 pts] Use Green's Theorem to evaluate the line integral $\int_C \left(-\frac{1}{3}y^3 + x\right) dx + \left(\frac{1}{3}x^3 - y\right) dy$ where C is the boundary of the annulus, $4 \leq x^2 + y^2 \leq 9$. You should orient C so that the inner circle is traversed counterclockwise and the outer circle is traversed clockwise.

(7) [10 pts] Use cylindrical coordinates to find the volume of the solid that lies both within the cylinder $x^2 + y^2 = 3$ and the sphere $x^2 + y^2 + z^2 = 4$.

(8) [10 pts]
Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = \cos u \sin v\, \mathbf{i} + \sin u \sin v\, \mathbf{j} + 4 \cos v\, \mathbf{k} \qquad \text{for } 0 \leq u \leq 2\pi \text{ and } 0 \leq v \leq \frac{3\pi}{4}.$$

(a) Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the surface, S.

(c) Let $w = f(x, y, z) = xz$. Use the Chain Rule from Multivariable Calculus to calculate $\frac{\partial w}{\partial v}$.

(9) [10 pts] Use the change of variables $u = x + y$, $v = y - 2x$ to evaluate

$$\int_0^1 \int_0^{1-x} (y - 2x)^2 \sqrt{x + y}\, dy\, dx.$$

(10) [10 pts] Calculate the volume of the the solid region in the first octant that is bounded by the surfaces $x + y = 4$ and $x = 4 - z^2$.

Pledge: *I have neither given nor received aid on this exam*

Signature: _____