NAME:	CLASS: 11:30am OR 4pm

1 /5	2 /10	3 /10	4 /10	5 /10	6 /10
7 /10	8 /7	9 /10	10 /10	11 /8	T /100

MATH 2415 (Fall 2012) Final Exam, Dec 14th

No calculators, books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 2 hour 30 minute exam is worth 100 points.

(1) [5 pts] Find an equation of the form $ax + by + cz = d$ for the plane through the point $(2, 0, 1)$ that is perpendicular to the line $x = 3t$, $y = 2 - t$, $z = 3 + 4t$.

(2) [10 pts] Make labelled sketches of the traces of the surface

$$x^2 + \left(\frac{y}{2}\right)^2 - \left(\frac{z}{3}\right)^2 = -1.$$

in the planes $x = 0$, $y = 0$, and $z = k$ for a few appropriately chosen values of k. Then sketch the surface.

(3) [10 pts] Let C be the curve in the plane parametrized by $\mathbf{r}(t) = (t^2, t^3)$ for $0 \leq t \leq 2$.
(a) Calculate a parametrization of the tangent line to the curve C at $t = 1$.

(b) Integrate the vector field $\mathbf{F} = y\mathbf{i} + x^2\mathbf{j}$ over the curve C. What is the physical meaning of this integral in the case that $\mathbf{F}$ is a force field?

(4) [10 pts] Suppose that $z = f(x, y)$ is a function with the following table of values.

(a, b)	$f(a, b)$	$\nabla f(a, b)$	$f_{xx}(a, b)$	$f_{xy}(a, b)$	$f_{yy}(a, b)$
$(1, 2)$	0	(0,0)	5	3	1
$(7, -2)$	0	(0,1)	5	3	1
$(3, 4)$	7	(0,0)	-5	-3	-2
$(5, -3)$	68	(0,0)	8	-4	2
$(2, 1)$	35	(0,0)	5	3	2

Identify any local maxima, minima, and saddle points of f. Explain the reasons for your answers.

(5) [10 pts] Use a double integral to calculate the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the plane $z = 4$.

(6) [10 pts] (a) State the Change of Variables Theorem for Double Integrals. Explain the geometrical meaning of the theorem using a sketch.

(b) Define what it means for vector field to be conservative.

(c) Let $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ be a vector field in the plane. Prove that:

$$\text{If } \mathbf{F} \text{ is conservative then } \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}.$$

(7) [10 pts] Use the change of variables $x = 2u + 3v$, $y = 3u + 5v$ to calculate the integral $\iint_R (7x + 2y)\, dA$, where R is the parallelogram in the xy-plane with verticies $(0, 0)$, $(2, 3)$, $(3, 5)$ and $(5, 8)$.

(8) [7 pts] Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$. Show that $\mathbf{F}$ is conservative and calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is an arbitrary (unknown) curve from $(0, 0)$ to $(1, 1)$.

(9) [10 pts] Let $\mathbf{F}$ be the vector field in the plane given by $\mathbf{F}(x, y) = x^2y\mathbf{i} + (x^2 - y^2)\mathbf{j}$.
(a) Calculate the divergence of $\mathbf{F}$.

(b) Calculate the curl of $\mathbf{F}$.

(c) Suppose that the vector field $\mathbf{F}$ given above is the velocity vector field of a fluid flowing in the plane. On average is the fluid flowing in or out of a small disk centered at the point $(-1, 2)$? Why?

(10) [10 pts] Let S be the surface that is parametrized by

$$\mathbf{r}(u,v) = (\cos v, u, \sin v) \qquad \text{for} \quad 0 \le u \le 2 \quad \text{and} \quad 0 \le v \le \pi.$$

(a) Find an equation of the form $F(x,y,z) = 0$ for this surface.

(b) Sketch the graph of the surface. Also sketch the grid curves $u = 1$ and $v = \frac{\pi}{4}$ on the surface together with the vector $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$ at the point on the surface where $(u,v) = (1, \pi/4)$.

(11) [8 pts] Calculate the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the surface parametrized by $\mathbf{r}(u,v) = v\mathbf{i} + uv\mathbf{j} + u\mathbf{k}$ for $0 \leq u \leq 3$ and $0 \leq v \leq 2$, and $\mathbf{F}$ is the vector field $\mathbf{F} = x\mathbf{i} + xy^2\mathbf{j} - z\mathbf{k}$. The surface S is oriented so that the normal to the surface has a positive y component.

Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ________________________________