LAST NAME:	FIRST NAME:	CIRCLE:		
SOLUTIONS		Zweck 10:00am	Khafizov 11:30am	Khafizov 2:30pm

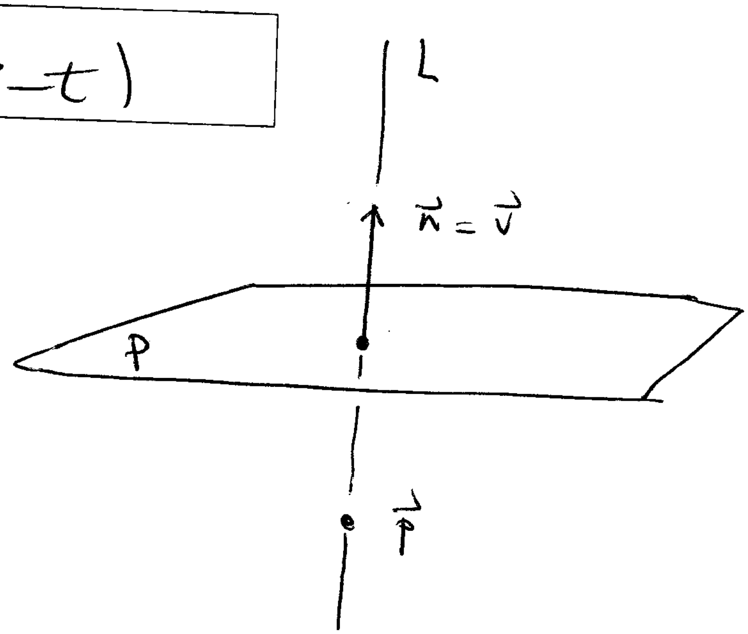
1 /9	2 /15	3 /15	4 /12	5 /12	6 /12	T /75

MATH 2415 (Spring 2016) Exam I, Feb 19th

No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 90 minute exam is worth 75 points.

(1) [9 pts] Find a parametrization of the line that goes through the point $(2, 1, 3)$ and that is perpendicular to the plane $z = 3x + 2y + 4$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:
$$\boxed{\vec{r}(t) = (2+3t,\ 1+2t,\ 3-t)}$$



L is the line through point $\vec{p} = (2, 1, 3)$ in direction of vector $\vec{v} = \vec{n}$ = NORMAL to plane P.

Since equation of plane is
$$3x + 2y - z = -4$$
we can read off from coefficients of x, y, z that $\vec{n} = (3, 2, -1)$

So parametrization of L is
$$\vec{r}(t) = \vec{p} + t\vec{v} = (2, 1, 3) + t(3, 2, -1)$$
$$= (2 + 3t,\ 1+2t,\ 3-t)$$

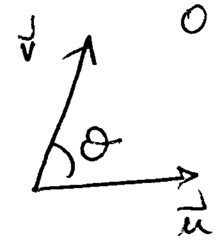
(2) [15 pts]

(a) Calculate the angle between the vectors $\mathbf{u} = (1, 0, 0)$ and $\mathbf{v} = (\sqrt{6}, 1, 1)$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

$$\boxed{\theta = \pi/6}$$

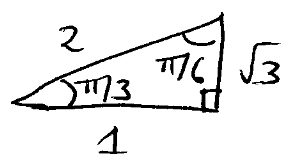
$0 \leq \theta \leq \pi$



$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta$$

So $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \dfrac{\sqrt{6}}{1 \cdot \sqrt{6+1+1}} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}$

So $\cos\theta = \dfrac{\sqrt{3}}{2}$



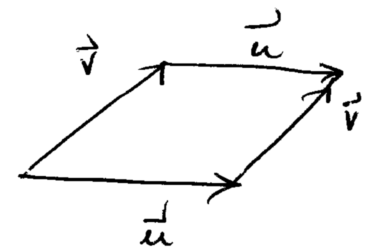
We know $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

So $\theta = \pi/6$

(b) Find the area of the parallelogram with vertices $\overset{P}{(2, 2, 3)}$, $\overset{Q}{(7, 3, 8)}$, $\overset{R}{(3, 4, 6)}$, and $\overset{S}{(6, 1, 5)}$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer:

$$\boxed{\text{AREA} = \sqrt{230}}$$

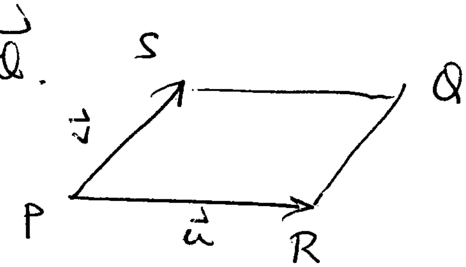


FIRST we have to two vectors $\vec{u}, \vec{v}$ that form the 2 sides of //gram as in the picture.

Notice: $\overrightarrow{PR} = R - P = (1, 2, 3)$ and $\overrightarrow{SQ} = Q - S = (1, 2, 3)$

while $\overrightarrow{PS} = S - P = (4, -1, 2) = \overrightarrow{RQ}$.

So //gram vertices can be labeled as



$$\begin{array}{r} 49 \\ 181 \\ \hline 230 \end{array}$$

which gives

$\vec{u} = \overrightarrow{PR} = (1, 2, 3)$

$\vec{v} = \overrightarrow{PS} = (4, -1, 2)$

Then Area $= |\vec{u} \times \vec{v}|$

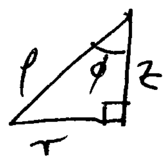
$$= \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & -1 & 2 \end{vmatrix} \right\| = |7\vec{i} + 10\vec{j} - 9\vec{k}| = \underline{\underline{\sqrt{230}}}$$

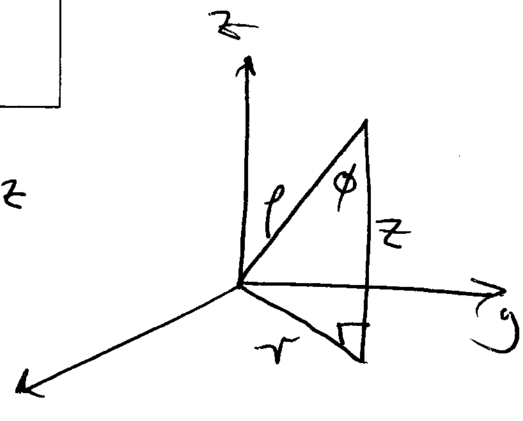
(3) [15 pts]

(a) Let P be the point with spherical coordinates $(\rho, \theta, \phi) = (2, \frac{\pi}{4}, \frac{\pi}{6})$. Find the cylindrical coordinates of P. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer: $(r, \theta, z) = (\sqrt{3}, \frac{\pi}{4}, 1)$

BY TRIANGLE ON RIGHT





$r = \rho \sin\phi = 2 \sin\frac{\pi}{6} = 1$

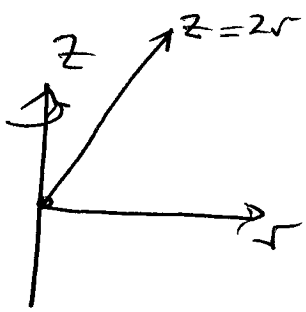
$z = \pm\sqrt{\rho^2 - r^2} = \pm\sqrt{2^2 - 1^2} = \pm\sqrt{3}$

$\theta = \frac{\pi}{4}$ is same as for spherical coordinates

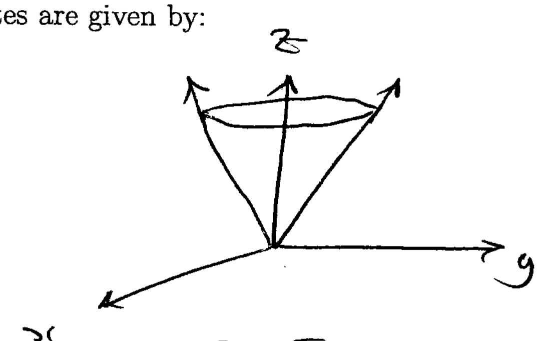
NOTE Since $\phi < \pi/2$, $z > 0$ must hold. So $z = \sqrt{3}$.

(b) Sketch the surfaces whose equations in cylindrical coordinates are given by:

(i) $z = 2r$

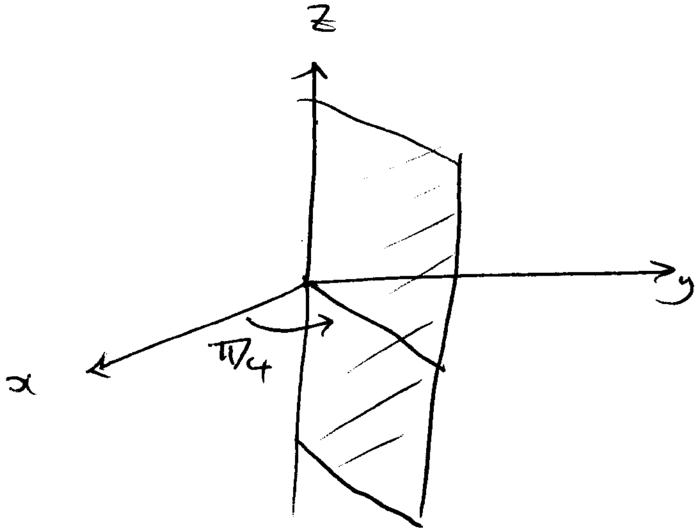


ROTATE $\longrightarrow$ AROUND z AXIS



CONE

(ii) $\theta = \frac{\pi}{4}$.



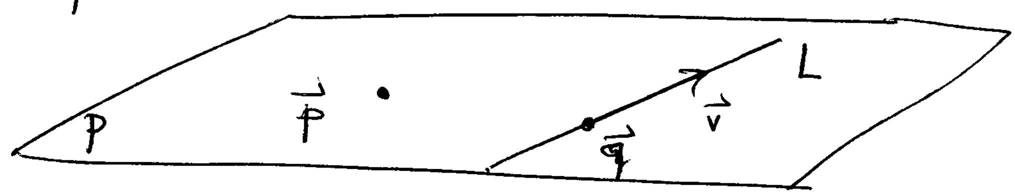
$\frac{1}{2}$ - PLANE

(4) [12 pts] Find the equation of the plane that goes through the point $(-1, 2, 5)$ that contains the line $x = 1 + 2t$, $y = -3 + 5t$, $z = 4t$. In the box, write your final answer in the form $Ax + By + Cz = D$. Explain the reasons for your answer in the space below.

Final Answer:

$$\boxed{5x - 18y + 20z = 59}$$

POINT $\vec{p} = (-1, 2, 5)$ is in plane P



LINE L lies in plane.

It has parametrization

$$\vec{r}(t) = \vec{q} + t\vec{v} = (1+2t, -3+5t, 4t)$$

$$= (1, -3, 0) + t(2, 5, 4)$$

So $\vec{q} = (1, -3, 0)$ is another point in plane P

and $\vec{v} = (2, 5, 4)$ is a vector lying in P

To get NORMAL vector $\vec{n}$ to P we need a 2nd vector lying in P. Since $\vec{p}, \vec{q}$ are points in P, we can choose this vector to be $\vec{w} = \vec{q} - \vec{p} = (2, -5, -5)$

Then

$\cancel{\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ 2 & -5 & -5 \end{vmatrix} = -5 \quad 18}$

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 4 \\ 2 & -5 & -5 \end{vmatrix} = (-5, 18, -20)$$

So equation of P is $\vec{n} \cdot (\vec{r} - \vec{p}) = 0$

$(-5, 18, -20) \cdot (x+1, y-2, z-5) = 0$

(5) [12 pts] Make a labelled sketch of the traces (slices) of the surface

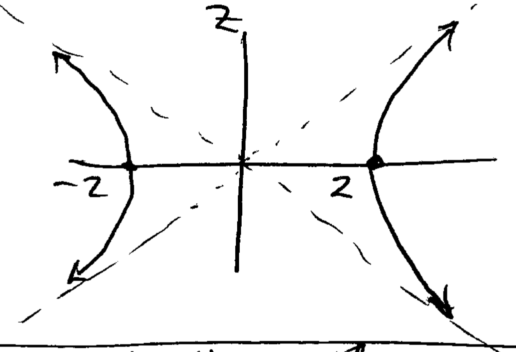
$$2x^2 - y^2 - 4z^2 = 8$$

in the planes $y = 0$, $z = 0$, and $x = k$ for $k = 0, \pm1, \pm2, \pm3$. Then sketch the surface.

$\boxed{y=0}$ $\quad x^2 - 2z^2 = 4$

ASYMPTOTES $\quad z = \pm \frac{1}{\sqrt{2}} x$

INTERCEPTS $\quad (x, z) = (\pm 2, 0)$

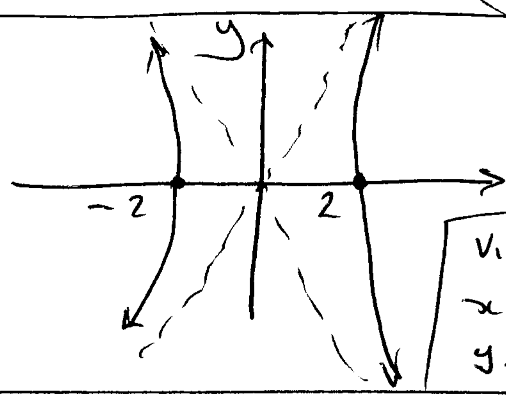


Hyperbola

$\boxed{z=0}$ $\quad 2x^2 - y^2 = 8$

ASYMPTOTES $\quad y = \pm\sqrt{2}\, x$

INTERCEPTS $\quad (x, y) = (\pm 2, 0)$



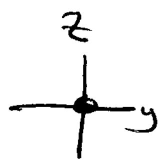
OPENS UP WIDER WHEN VIEWED FROM x AXIS THAN y=0 SLICE DOES

$\boxed{x=k}$ $\quad y^2 + 4z^2 = 2k^2 - 8 = -8, -6, 0, 10$ for $k = 0, \pm1, \pm2, \pm3$

SO $\boxed{x=0}$ NO SOLN

$\boxed{x=\pm 1}$ NO SOLN

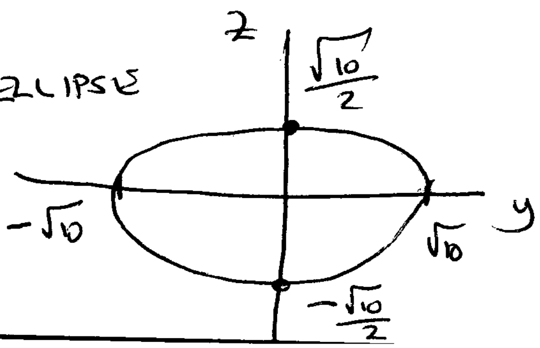
$\boxed{x=\pm 2}$ $\quad y^2 + 4z^2 = 0.$

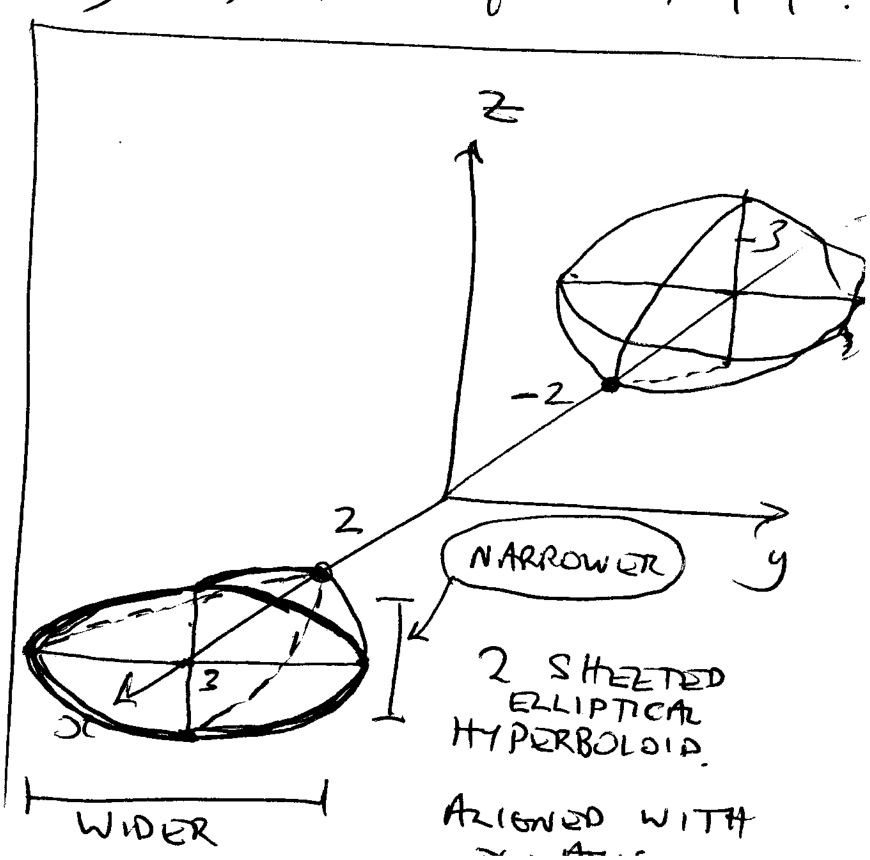


ORIGIN

$\boxed{x=\pm 3}$ $\quad y^2 + 4z^2 = 10$

ELLIPSE





2 SHEETED ELLIPTICAL HYPERBOLOID

ALIGNED WITH x AXIS

(6) [12 pts]
Let C be the curve with parametrization $(x, y, z) = \mathbf{r}(t) = (4t, 2\sin t, \cos t)$.

(a) Calculate the tangent vector to C when $t = \frac{\pi}{4}$. Write your final answer in the box, and explain the reasons for your answer in the space below.

Final Answer: $\vec{r}\,'(\pi/4) = \left(4, \sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

Tangent Vector to C at $t = \frac{\pi}{4}$ $= \vec{r}\,'\left(\frac{\pi}{4}\right)$

Now $\vec{r}\,'(t) = (4, 2\cos t, -\sin t)$

So $\vec{r}\,'\left(\frac{\pi}{4}\right) = \left(4, 2\cos\frac{\pi}{4}, -\sin\frac{\pi}{4}\right)$

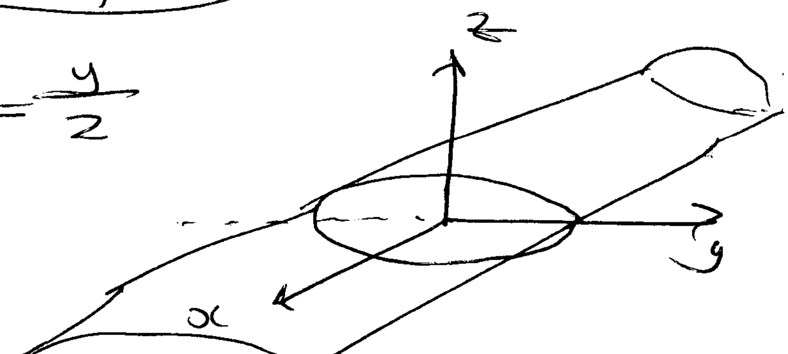
$= \left(4, \frac{2}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(b) Show that the curve C lies on a cylinder. an elliptical

SINCE

$y = 2\sin t \Rightarrow \sin t = \frac{y}{2}$

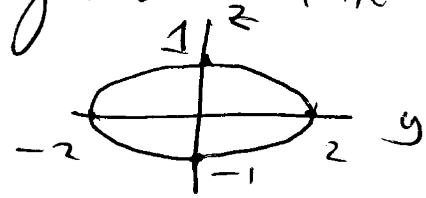
$z = \cos t$



we have

$1 = \cos^2 t + \sin^2 t = z^2 + \left(\frac{y}{2}\right)^2$

which is equation of elliptical cylinder in $\mathbb{R}^3$



Please sign the following honor statement:

On my honor, I pledge that I have neither given nor received any aid on this exam.

Signature: ____________________