Co-ordinate Geometry Exercises

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Semester 1

4.1 Given the following position vectors in $\mathbb{R}^3$

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}, \qquad \mathbf{d} = \begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix}.$$

find:

(a) the equation of the line that passes through $\mathbf{a}$ and $\mathbf{b}$;

(b) the equation of the line that passes through $\mathbf{c}$ and $\mathbf{d}$;

(c) the equation of the plane which passes through $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ lie;

(d) the equation of the plane upon which passes through $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$.

4.2 Find the equation of the line that passes through the point with position vector $(3, 2, 1)^\mathsf{T}$ which is parallel to $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

4.3 Find the equation of the plane that passes through the point with position vector $(3, 2, 5)^\mathsf{T}$ which has a normal vector $\mathbf{n} = (2, 1, 3)^\mathsf{T}$.

4.4 A plane has the equation $3x - 2y + z = 10$. Identify the normal to the plane and find the co-ordinates of 2 points on the plane having $z = 2$.

4.5 Two lines in $\mathbb{R}^3$ are defined by $\ell_1 : (1 + 2t, -t, 1 + 3t)^\mathsf{T}$ and $\ell_2 : (1 + 2t, 4, 7 - t)^\mathsf{T}$ respectively.

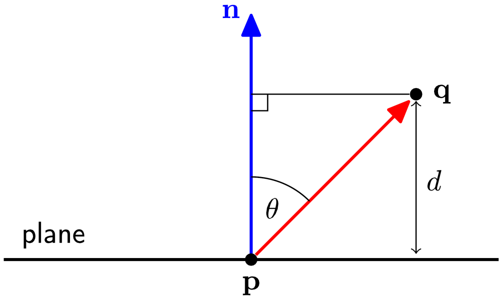
(a) find the intersection of the lines or show they are skew;

(b) find the distance between the point with position vector $\mathbf{p} = (0, -1, 3)^\mathsf{T}$ and ℓ_1;

(c) find the shortest distance between the lines.

4.6 Find the point where the line $\ell : (1 + 2t, 2 + t, -1 + 4t)^\mathsf{T}$ meets the plane $6x - y - 4z = 3$.

4.7 Consider the diagram below that shows a plane that passes through the point $\mathbf{p}$ and has normal vector $\mathbf{n}$ and the point with position $\mathbf{q}$ not on the plane.



Using the geometric definition of a dot product, derive an expression for calculating the shortest distance between a point and a plane. Use your expression to find the shortest distance from the point with position vector $(2, 4, -3)^\mathsf{T}$ to the plane $6x - y - 4z = 3$.