Systems of Linear Equations Exercises

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Semester 1

2.1 Solve the following linear system of equations using the inverse of the coefficient matrix.

(a)

$$-4x_1 + 2x_2 = -22,$$
$$3x_1 + 4x_2 = 11.$$

(b)

$$-4x_1 + 2x_2 = 6,$$
$$-x_1 - 3x_2 = -2.$$

(c)

$$-4x_1 - 4x_2 - 2x_3 = 16,$$
$$3x_1 + 4x_3 = -8,$$
$$x_1 = 0.$$

(d)

$$4x_1 - 4x_3 = 8,$$
$$4x_1 - x_2 + x_3 = -4,$$
$$3x_1 + x_2 + 2x_3 = -12.$$

2.2 Solve the following linear systems of equations using Cramer's rule.

(a)

$$x_1 + 4x_2 = -20,$$
$$-4x_1 + x_2 = -5.$$

(b)

$$x_1 + x_2 = 4,$$
$$4x_2 = 12.$$

(c)

$$3x_1 - 4x_2 - 4x_3 = 21,$$
$$-2x_1 - x_2 - x_3 = 8,$$
$$4x_1 - x_2 + 3x_3 = -14.$$

(d)

$$4x_1 + 4x_2 + x_3 = 5,$$
$$-2x_1 + x_2 + x_3 = -1,$$
$$-5x_1 - 4x_2 + 2x_3 = -14.$$

2.3 Solve the following linear systems of equations using Gaussian elimination.

(a)

$$-x_1 + 3x_2 = -2,$$
$$-2x_1 + x_2 = 1.$$

(b)

$$3x_1 + x_2 + 2x_3 = 11,$$
$$4x_1 - 4x_3 = -4,$$
$$4x_1 - 2x_2 + x_3 = 13.$$

(c)

$$-x_1 - 5x_2 - 2x_3 = -17,$$
$$2x_1 - 2x_2 - 3x_3 = -14,$$
$$3x_1 - x_2 + 4x_3 = -13.$$

(d)

$$-x_1 - 5x_2 - 2x_3 = -26,$$
$$2x_1 - 2x_2 - 3x_3 = -19,$$
$$3x_1 - x_2 - 4x_3 = -20.$$

2.4 Solve the linear system of equations from exercise 2.3 using Gaussian elimination with partial pivoting.

2.5 Solve the linear system of equations from exercise 2.3 using Gauss-Jordan elimination.

2.6 Use Gauss-Jordan elimination to calculate the inverse of the following matrices.

(a) $\begin{pmatrix} -4 & 2 \\ 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} -4 & 2 \\ -1 & -3 \end{pmatrix}$ (c) $\begin{pmatrix} -4 & -4 & -2 \\ 3 & 0 & 4 \\ 1 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 4 & 0 & -4 \\ 4 & -1 & 1 \\ 3 & 1 & 2 \end{pmatrix}$

2.7 For the following linear systems of equations, determine the rank of the coefficient matrix and the augmented matrix and classify the system is consistent, inconsistent or indeterminate and calculate the solution (if possible).

(a)   (b)   (c)

$$x_1 - x_2 + 2x_3 = 2,$$ $$x_1 - x_2 + 2x_3 = 3,$$ $$x_1 + x_2 - 2x_3 = 1,$$
$$2x_1 + x_2 + 4x_3 = 7,$$ $$2x_1 - 3x_2 + 7x_3 = 4,$$ $$2x_1 - x_2 + x_3 = 9,$$
$$4x_1 + x_2 + x_3 = 4.$$ $$-x_1 + 3x_2 - 8x_3 = 1.$$ $$x_1 + 4x_2 - 7x_3 = 2.$$