Vectors Exercises

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Semester 1

3.1 The points U, V and W have the following position vectors:

$$\mathbf{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}.$$

Find:

(a) $2\mathbf{u} + \mathbf{w}$

(b) $\mathbf{w} - \mathbf{u}$

(c) a unit vector pointing in the same direction of $\mathbf{u}$

(d) a unit vector pointing in the opposite direction of $\mathbf{v}$

(e) a vector pointing in the same direction as $\mathbf{v}$ but half its length

(f) the vector pointing from U to V

(g) the vector pointing from U to W

(h) $\mathbf{u} \cdot \mathbf{w}$

(i) the angle $\angle VUW$

(j) show that $\mathbf{u}$ is at right angles to $\mathbf{v}$

(k) $\mathbf{v} \times \mathbf{w}$

3.2 Write $\mathbf{u} = (2, 7, 1)^\mathsf{T}$ as:

(a) a linear combination of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$

(b) a linear combination of vectors $\mathbf{f}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \mathbf{f}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ and $\mathbf{f}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

3.3 Find k such that the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular:

(a) $\mathbf{u} = \begin{pmatrix} 1 \\ k \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix}$ in $\mathbb{R}^3$

(b) $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ k+2 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ k \\ -2 \\ 1 \\ 2 \end{pmatrix}$ in $\mathbb{R}^5$

3.4 Which pair of the following vectors is perpendicular? For the remaining pairs, what is the angle between them?

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$