# Vector Spaces Exercises

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5.1 Prove that the axioms of the vector space $\mathbb{R}^3$ hold.

5.2 Using the subspace condition, determine whether the following subsets of $\mathbb{R}^3$ are subspaces:

(a) $U = \{(x, y, 0)^\mathsf{T} : x, y \in \mathbb{R}\}$ (b) $V = \{(1, 2, 0)^\mathsf{T}\}$

(c) $W = \{(0, y, 0)^\mathsf{T} : y \in \mathbb{R}\}$ (d) $X = \{(x, y, z)^\mathsf{T} : y = |x|, x, y, z \in \mathbb{R}\}$

5.3 Which of the following sets are subspaces of $M_2(\mathbb{R})$?

(a) $A = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R}, a + b = 1, \right\}$

(b) $B = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R}, a = c = d \right\}$

(c) $C = \{A \in M_{2\times 2} : A^2 = A\}$

5.4 Prove that $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \right\}$ is a basis for $\mathbb{R}^3$ and represent the vectors $\begin{pmatrix} 0 \\ 13 \\ 17 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ with respect to this basis.

5.5 Extend $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -5 \\ 2 \end{pmatrix} \right\}$ to a basis of $\mathbb{R}^4$.

5.6 Suppose that $W$ is a subspace of $\mathbb{R}^n$ generated by the vectors

$$\left\{ \mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 2 \\ 1 \\ 5 \\ 3 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 4 \end{pmatrix} \right\}.$$

Find a basis for $W$ and determine $\dim(W)$.