Binary Logistic DrPH (Epidemiology)

Kamarul Imran M 1 February 2016

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Modeling Binomial Data

- 1. Describe data
- 2. Explore data Exploratory Data Analysis
- 3. Estimate parameters
- 4. Make Inference
- 5. Make Prediction
- 6. Interpretation

Locate files

- Browse your folders.
- · Look for the files.
- Check the path to the folder containing the files

Set the folder

Set our working directory. REMEMBER! your working directory (working folder) is different from my working directory

```
# this is my working directory. You have to specify yours
setwd("E:/Epi_Stat_Matters/LectureNotes2015/binary-logistic/binary-logistic-DrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/BinaryLogisticDrPH-2015/B
```

Read data

- Read our data in the working folder
- Then, save as a csv file in our working directory

```
mydata <- read.csv("http://www.ats.ucla.edu/stat/data/binary.csv")
write.csv2(mydata,'logistic.csv')</pre>
```

Describe data

```
#observe data
head(mydata,10)
```

Rank is taken as numerical variable which does not make sense. We need to convert it to a categorical (factor) variable

```
summary(mydata)
```

```
##
       admit
                          gre
                                          gpa
                                                         rank
  Min.
          :0.0000
                    Min.
                           :220.0
                                    Min.
                                          :2.260
                                                    Min.
                                                            :1.000
   1st Qu.:0.0000
                    1st Qu.:520.0
                                     1st Qu.:3.130
                                                    1st Qu.:2.000
## Median :0.0000
                    Median :580.0
                                    Median :3.395
                                                    Median :2.000
          :0.3175
                                          :3.390
## Mean
                    Mean
                           :587.7
                                    Mean
                                                    Mean
                                                           :2.485
## 3rd Qu.:1.0000
                    3rd Qu.:660.0
                                    3rd Qu.:3.670
                                                    3rd Qu.:3.000
## Max.
          :1.0000
                    Max.
                           :800.0
                                    Max.
                                            :4.000
                                                    Max.
                                                           :4.000
```

```
mydata$rank<-factor(mydata$rank)</pre>
summary(mydata$rank)
       2
##
     1
             3
  61 151 121 67
More fancy, we can use psych::describe function
library(psych)
describe (mydata)
##
                              sd median trimmed
         vars
                    mean
                                                   mad
                                                           min max range skew
               n
## admit
            1 400
                    0.32
                           0.47
                                    0.0
                                           0.27
                                                   0.00
                                                          0.00
                                                                     1.00
            2 400 587.70 115.52 580.0 589.06 118.61 220.00 800 580.00 -0.14
## gre
## gpa
            3 400
                    3.39
                           0.38
                                    3.4
                                           3.40
                                                  0.40
                                                          2.26
                                                                     1.74 - 0.21
                           0.94
                                           2.48
## rank*
            4 400
                    2.48
                                    2.0
                                                  1.48
                                                          1.00
                                                                 4
                                                                     3.00 0.10
##
         kurtosis
                    se
            -1.39 0.02
## admit
            -0.36 5.78
## gre
            -0.60 0.02
## gpa
## rank*
            -0.91 0.05
```

Explore data

Use plots like * Histogram for numerical variables * and barplot for categorical variables, at least.

Estimate parameters

- we estimate the logit or the log odds.
- We used summary to see the results stored as mylogit
- We used **coefficients** to examine the regression coefficients

```
mylogit <- glm(admit~gre+gpa+rank,family = 'binomial'(link = logit),data=mydata)
summary(mylogit)</pre>
```

```
##
## glm(formula = admit ~ gre + gpa + rank, family = binomial(link = logit),
##
       data = mydata)
##
## Deviance Residuals:
##
      Min
                 10
                      Median
                                   30
                                           Max
## -1.6268 -0.8662 -0.6388
                              1.1490
                                        2.0790
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) -3.989979 1.139951 -3.500 0.000465 ***
## gre
               0.002264
                           0.001094
                                    2.070 0.038465 *
## gpa
               0.804038
                           0.331819
                                     2.423 0.015388 *
## rank2
              -0.675443
                           0.316490 -2.134 0.032829 *
## rank3
              -1.340204
                           0.345306 -3.881 0.000104 ***
                           0.417832 -3.713 0.000205 ***
## rank4
              -1.551464
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
##
       Null deviance: 499.98 on 399 degrees of freedom
## Residual deviance: 458.52 on 394 degrees of freedom
## AIC: 470.52
##
## Number of Fisher Scoring iterations: 4
coefficients(mylogit)
    (Intercept)
                                                   rank2
                                                                rank3
                          gre
                                       gpa
## -3.989979073
                 0.002264426
                              0.804037549 -0.675442928 -1.340203916
##
          rank4
## -1.551463677
To obtain the odds ratios and their 95% CI, we need to exponentiate using exp the regression coefficients or
the betas
exp(coefficients(mylogit))
## (Intercept)
                                               rank2
                                                           rank3
                                                                        rank4
                        gre
                                    gpa
     0.0185001
                 1.0022670
                              2.2345448
                                          0.5089310
                                                       0.2617923
                                                                    0.2119375
##
exp(confint(mylogit))
## Waiting for profiling to be done...
##
                      2.5 %
                               97.5 %
## (Intercept) 0.001889165 0.1665354
## gre
               1.000137602 1.0044457
               1.173858216 4.3238349
## gpa
## rank2
               0.272289674 0.9448343
## rank3
               0.131641717 0.5115181
## rank4
               0.090715546 0.4706961
```

Make inference

Here, we examine the p-values (hypothesis testing) and the confidence intervals.

- First, using the method of maximum likelihood
- Next, using the SE method (function **confint.default**)

confint(mylogit)

```
## Waiting for profiling to be done...

## 2.5 % 97.5 %

## (Intercept) -6.2716202334 -1.792547080

## gre 0.0001375921 0.004435874

## gpa 0.1602959439 1.464142727

## rank2 -1.3008888002 -0.056745722

## rank3 -2.0276713127 -0.670372346

## rank4 -2.4000265384 -0.753542605
```

confint.default(mylogit)

```
##
                        2.5 %
                                    97.5 %
   (Intercept) -6.2242418514 -1.755716295
  gre
                0.0001202298
                              0.004408622
  gpa
                0.1536836760
                               1.454391423
               -1.2957512650 -0.055134591
## rank2
## rank3
               -2.0169920597 -0.663415773
## rank4
               -2.3703986294 -0.732528724
```

Calculate the fitted values

The fitted values are the expected values of the model. These expected values are the predicted probability for each observation (each patient) in the dataset

fitted(mylogit)

```
2
                                    3
##
  0.17262654 0.29217496
                          0.73840825
                                      0.17838461 0.11835391 0.36996994
                         8
                                    9
##
                                               10
                                                           11
   0.41924616 0.21700328 0.20073518 0.51786820 0.37431440 0.40020025
                        14
                                               16
                                                           17
                                                                       18
##
            13
                                   15
##
   0.72053858 0.35345462 0.69237989 0.18582508 0.33993917 0.07895335
            19
                       20
                                   21
                                               22
                                                           23
                                                                       24
   0.54022772
               0.57351182
                          0.16122101
                                      0.43727108
                                                  0.12837525
##
                                                              0.19204860
                                                           29
##
            25
                       26
                                   27
                                               28
                                                                       30
              0.68229503
   0.43759396
                          0.57848091
                                      0.20475422
                                                  0.42307349 0.45829857
##
##
            31
                       32
                                   33
                                               34
                                                           35
                                                                       36
   0.21765393
               0.28583616
                          0.22481919
                                      0.42494837 0.34296523 0.21293277
##
           37
                       38
                                   39
                                               40
                                                           41
                                                                       42
                          0.26569575 0.11942769
                                                  0.18975965 0.33567002
   0.48413281
               0.13931720
                                   45
##
           43
                       44
                                               46
                                                           47
                                                                       48
##
   0.31560404
               0.17702923
                           0.32817441
                                      0.18025548
                                                  0.36121718 0.11699101
##
           49
                       50
                                   51
                                               52
                                                           53
                                                                       54
                           0.31488795
                                      0.11624726
   0.07235381
              0.15047417
                                                  0.23936553 0.37838478
##
           55
                       56
                                               58
                                                           59
                                                                       60
                                   57
   0.24045684 0.39213236
                          0.18283980
                                      0.10853139 0.30472142 0.12837525
##
##
            61
                       62
                                   63
                                               64
                                                           65
                                                                       66
   0.33078459
              0.16742893
                          0.28289780
                                      0.33295972 0.30988311 0.39645173
           67
                       68
                                   69
                                               70
                                                           71
                                                                       72
##
##
   0.27784995
               0.51681586
                           0.57206626
                                      0.69436828
                                                  0.33966212 0.07486000
                       74
                                   75
##
           73
                                               76
                                                           77
                                                                       78
              0.46607599
                          0.24284830
                                      0.38139149
##
   0.15073716
                                                  0.20415281 0.42494837
##
           79
                       80
                                   81
                                               82
                                                           83
                                                                       84
   0.43570986 0.65251556 0.16456653 0.31150713 0.20517359 0.08776685
##
##
           85
                       86
                                   87
                                               88
                                                           89
                                                                       90
##
   0.21358749
              0.25126279
                          0.34584314 0.37549461
                                                  0.55783057 0.51131037
           91
                       92
                                               94
##
                                   93
                                                           95
                                                                       96
##
   0.49978497
              0.63809471
                          0.57000341 0.26968427
                                                  0.40010880 0.37907977
##
           97
                       98
                                   99
                                              100
                                                          101
                                                                      102
  0.22063013 0.33002244 0.31762762 0.14640896 0.11633954 0.24114689
##
##
                                  105
                                              106
                                                          107
                                                                      108
## 0.11883427 0.28100436 0.50126183 0.35394219 0.61241920 0.25695415
```

```
112
          109
                     110
                                111
                                                       113
## 0.11218813 0.30904921 0.17869743 0.13603549 0.10881750 0.48942091
          115
                     116
                                 117
                                            118
                                                       119
## 0.35153649 0.32780508 0.29004920 0.47768876 0.68922540 0.09863460
          121
                     122
                                 123
                                            124
                                                        125
## 0.38205848 0.19283124 0.13456621 0.14161529 0.35890251 0.16784107
          127
                     128
                                 129
                                            130
                                                        131
## 0.55353632 0.29761787 0.29364378 0.12270194 0.32900715 0.27429792
                     134
                                 135
                                            136
                                                        137
                                                                   138
          133
## 0.35016196 0.15167362 0.26397051 0.20956391 0.16855273 0.37076538
          139
                     140
                                 141
                                            142
                                                        143
                                                                   144
## 0.37104174 0.56147017 0.48592324 0.24487554 0.27496207 0.21702497
                     146
                                 147
                                            148
                                                       149
          145
                                                                   150
## 0.18326999 0.15292361 0.30053113 0.13202601 0.36278299 0.58590453
          151
                     152
                                 153
                                            154
                                                        155
                                                                   156
## 0.69607194 0.26076336 0.48793196 0.22533437 0.27701027 0.12691355
          157
                     158
                                 159
                                            160
##
                                                        161
                                                                   162
## 0.20243105 0.49385024 0.40979572 0.33767745 0.31214097 0.40081797
                                            166
                                                                   168
          163
                     164
                                 165
                                                        167
## 0.44572710 0.21536268 0.33209361 0.69237989 0.12564635 0.33881603
          169
                     170
                                 171
                                            172
                                                        173
                                                                   174
## 0.27253083 0.25713529 0.16766865 0.13610230 0.27045353 0.47601029
                                 177
                                            178
                                                        179
          175
                     176
                                                                   180
## 0.17207711 0.36543032 0.20079352 0.20929210 0.22290898 0.09702710
          181
                     182
                                 183
                                            184
                                                        185
## 0.29173405 0.21592659 0.53390445 0.41213948 0.10284874 0.51016205
          187
                     188
                                 189
                                            190
                                                        191
                                                                   192
## 0.23875288 0.26184001 0.28313813 0.30160149 0.29894660 0.33797096
          193
                     194
                                 195
                                            196
                                                        197
## 0.29780561 0.14252603 0.37361105 0.37499458 0.20306181 0.11520619
          199
                     200
                                 201
                                            202
                                                        203
                                                                   204
## 0.25867413 0.23203530 0.29790835 0.31450637 0.69237989 0.19176895
          205
                     206
                                 207
                                            208
                                                        209
                                                                   210
## 0.62160882 0.37552455 0.62994688 0.59336886 0.17269671 0.36867073
          211
                     212
                                 213
                                            214
                                                        215
                                                                   216
## 0.23500145 0.28417171 0.21145148 0.23806753 0.39069474 0.18303592
          217
                     218
                                 219
                                            220
                                                        221
## 0.29144726 0.49458858 0.36532833 0.37499458 0.18691983 0.35841190
                     224
                                 225
                                            226
          223
                                                        227
## 0.38346629 0.32549498 0.37234438 0.29200523 0.40539785 0.13119209
          229
                     230
                                 231
                                            232
                                                        233
## 0.30562595 0.42917277 0.17040039 0.20845157 0.25212831 0.09688336
          235
                     236
                                 237
                                            238
                                                        239
                                                                   240
## 0.65921863 0.30806878 0.40979572 0.41039144 0.10815929 0.27465027
          241
                     242
                                 243
                                            244
                                                        245
## 0.19001218 0.56239934 0.19616746 0.33794240 0.41996550 0.40736827
          247
                     248
                                 249
                                            250
                                                        251
                                                                   252
## 0.39171070 0.24596016 0.29657173 0.29278619 0.20011793 0.17414395
          253
                     254
                                 255
                                            256
                                                        257
                                                                   258
## 0.43247252 0.18780755 0.26200847 0.23371984 0.30267400 0.32075797
          259
                     260
                                 261
                                            262
                                                        263
                                                                   264
## 0.33944941 0.46187255 0.34863249 0.24298996 0.16969339 0.32075797
                                 267
                                            268
                                                                   270
##
          265
                     266
                                                       269
## 0.26562483 0.14378335 0.15865328 0.26021896 0.41492493 0.12579904
```

```
##
          271
                      272
                                  273
                                              274
                                                          275
                                                                      276
  0.48994106 0.19310678 0.45641226 0.54337733 0.27302605 0.28684953
          277
                      278
                                  279
                                              280
                                                          281
                                                                      282
                                                  0.49925174 0.13172559
   0.22143462
              0.55028996
                          0.16945136 0.34384116
##
##
           283
                      284
                                  285
                                              286
                                                          287
                                                                      288
                                                  0.60122274 0.25502619
##
  0.21874547 0.13337693
                          0.28021662 0.17925207
           289
                      290
                                  291
                                              292
                                                          293
  0.23197657 0.05878643 0.38047126 0.35008696
                                                  0.46240272 0.73372225
##
           295
                      296
                                  297
                                              298
                                                          299
                                                                      300
   0.29885443 0.17659931
                          0.45483793 0.23950580
                                                  0.34785059 0.27566478
           301
                      302
                                  303
                                              304
                                                          305
                                                                      306
   0.36288468
               0.28067279
                          0.22671860
                                      0.51860565
                                                  0.07198547
                                                              0.19060160
          307
                      308
                                  309
                                              310
                                                          311
                                                                      312
   0.44561844 0.37054412
                          0.28373804 0.12588934
                                                  0.30028221 0.44520022
##
           313
                      314
                                  315
                                              316
                                                          317
                                                                      318
   0.30907647 \ 0.19322270 \ 0.17701800 \ 0.15412239
                                                  0.18491373 0.29806393
                                                                      324
##
                      320
                                  321
                                              322
                                                          323
           319
   0.18670880
               0.46755914
                          0.14630641 0.32183935
                                                  0.12035456 0.17486941
                                                                      330
##
          325
                      326
                                  327
                                              328
                                                          329
   0.12112920
               0.66498227
                          0.38597852
                                      0.35450549
                                                  0.33926538 0.11370930
##
           331
                      332
                                  333
                                              334
                                                          335
                                                                      336
  0.39213236 0.27905234
                          0.34097123 0.21344965
                                                  0.20393972 0.59795326
##
           337
                      338
                                  339
                                              340
                                                          341
##
   0.16520993 0.16070084 0.45158492 0.26006097 0.14037382 0.12659514
##
          343
                      344
                                  345
                                              346
                                                          347
                                                                      348
   0.22560760
              0.29075910
                          0.18859648 0.14657301
                                                  0.35132030 0.42636137
           349
                      350
                                              352
                                                          353
                                                                      354
                                  351
                                      0.23714608
   0.25767548 0.27488628
                          0.57858815
                                                  0.18120291 0.43779599
                                                                      360
          355
                      356
                                  357
                                              358
                                                          359
   0.40050290 0.49758253 0.38909423 0.57487559
                                                  0.25063922 0.37007654
           361
                      362
                                  363
                                              364
                                                          365
                                                                      366
  0.59956970 0.50972425 0.35412991 0.29777892 0.49491656 0.11836196
           367
                      368
                                  369
                                              370
                                                          371
                                                                      372
                                                              0.31708679
   0.12645014
               0.26745319
                          0.63170496
                                      0.56803162
                                                  0.39857395
           373
                      374
                                  375
                                              376
                                                          377
                                                                      378
                          0.41142403 0.18735742
                                                  0.41512421 0.58958954
##
  0.37650752 0.53085361
           379
                      380
                                  381
                                              382
                                                          383
  0.20223990 0.21896113 0.46366743 0.34602886
                                                  0.34967678 0.67275941
           385
                      386
                                  387
                                              388
                                                          389
                                                                      390
  0.18665107\ 0.35189341\ 0.52842881\ 0.34287938\ 0.33908140\ 0.40275050
                                  393
                                              394
                                                                      396
          391
                      392
                                                          395
   0.40093595 0.48719398
                          0.22202911 0.43872524
                                                  0.25342327 0.48866999
           397
                      398
                                  399
## 0.16550430 0.18106222 0.46366743 0.30073055
```

Make prediction

Similarly, one of the important objectives in modelling is to perform prediction based on the model using new data.

We can perform these predictions:

1. Predict the log odds for having the outcome

2. Predict the conditional probability for having the outcome

```
Let us say we have these data
gre = 380 gpa = 3.61 rank = 3
First, we create a data frame
new_datal <- data.frame( gre = 380, gpa = 3.61, rank = '3')</pre>
Now, we can perform the prediction
pred.logit<-predict(mylogit,newdata = new_datal, type='link')</pre>
pred.logit
##
## -1.567126
We can confirm this by calculate this
-3.99+0.00226*380+0.8041*3.61-1.34
## [1] -1.568399
Notice, that similarity between predict(x, type='response' and fitted Remember, we can calculate the
conditional probability of having the outcome
pred.prob<-predict(mylogit, newdata = new_datal, type='response')</pre>
pred.prob
##
## 0.1726265
We can verify this by two ways
head(fitted(mylogit))
## 0.1726265 0.2921750 0.7384082 0.1783846 0.1183539 0.3699699
# calculate the logistic probability
\exp(-1.567)/(1+\exp(-1.567))
## [1] 0.1726445
```

Compare models

summary(mylogit)

We compare a model with **vs** and without **gre**. This is done using the deviance

```
##
## Call:
  glm(formula = admit ~ gre + gpa + rank, family = binomial(link = logit),
##
       data = mydata)
##
## Deviance Residuals:
       Min
                 1Q
                      Median
                                   3Q
                                           Max
## -1.6268 -0.8662 -0.6388
                             1.1490
                                        2.0790
##
## Coefficients:
```

```
Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.989979 1.139951 -3.500 0.000465 ***
                          0.001094
## gre
              0.002264
                                   2.070 0.038465 *
               0.804038
                         0.331819
                                    2.423 0.015388 *
## gpa
## rank2
              -0.675443
                         0.316490 -2.134 0.032829 *
              -1.340204
## rank3
                          0.345306 -3.881 0.000104 ***
## rank4
              -1.551464
                          0.417832 -3.713 0.000205 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 499.98 on 399 degrees of freedom
## Residual deviance: 458.52 on 394 degrees of freedom
## AIC: 470.52
##
## Number of Fisher Scoring iterations: 4
mylogit2 <- glm(admit~gpa+rank,family = 'binomial'(link = logit),data=mydata)</pre>
summary(mylogit2)
##
## Call:
## glm(formula = admit ~ gpa + rank, family = binomial(link = logit),
      data = mydata)
##
## Deviance Residuals:
      Min
                10
                    Median
                                  30
                                          Max
## -1.5055 -0.8663 -0.6590 1.1505
                                       2.0913
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.4636
                          1.1003 -3.148 0.001645 **
                1.0521
                           0.3102
                                   3.392 0.000694 ***
## gpa
                           0.3141 -2.168 0.030181 *
## rank2
               -0.6810
## rank3
               -1.3919
                           0.3419 -4.071 4.68e-05 ***
                           0.4152 -3.840 0.000123 ***
## rank4
               -1.5943
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
      Null deviance: 499.98 on 399 degrees of freedom
## Residual deviance: 462.88 on 395 degrees of freedom
## AIC: 472.88
## Number of Fisher Scoring iterations: 4
anova(mylogit,mylogit2,test = 'Chisq')
## Analysis of Deviance Table
##
## Model 1: admit ~ gre + gpa + rank
## Model 2: admit ~ gpa + rank
## Resid. Df Resid. Dev Df Deviance Pr(>Chi)
```

```
## 1
          394
                 458.52
## 2
          395
                 462.88 -1 -4.3578 0.03684 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Linearity in logits

```
gre is tested for linearity in logit. gre is linear but it is rescaled to produce less decimals
The linearity of logits is tested using library mfp
library(mfp)
## Loading required package: survival
mylogit3 <- mfp(admit~fp(gre)+gpa+rank,family = 'binomial'(link = logit),data=mydata,verbose=T)</pre>
##
##
   Variable
                 Deviance
                               Power(s)
## Cycle 1
##
   rank2
##
                 463.096
##
                 458.517
                               1
##
##
##
##
    rank3
##
                  474.043
##
                 458.517
                               1
##
##
##
##
    rank4
##
                 473.551
##
                 458.517
                               1
##
##
##
##
    gpa
                  464.532
##
##
                  458.517
##
##
##
##
    gre
                 462.875
##
##
                  458.517
##
                  458.415
                               0
##
                  458.366
                               -2 -2
##
##
## Tansformation
         shift scale
```

```
## rank2
## rank3
             0
## rank4
             0
             0
                   1
## gpa
## gre
             0 1000
##
## Fractional polynomials
         df.initial select alpha df.final power1 power2
## rank2
                  1
                         1 0.05
                                        1
                                               1
                         1 0.05
## rank3
                  1
                                        1
                                               1
## rank4
                  1
                         1 0.05
                                        1
                                               1
                         1 0.05
                                        1
## gpa
                  1
                                               1
##
                  4
                         1 0.05
                                        1
  gre
##
##
## Transformations of covariates:
##
                formula
## gre I((gre/1000)^1)
## gpa
                    gpa
## rank
                   rank
##
##
## Deviance table:
             Resid. Dev
## Null model
                 499.9765
## Linear model 458.5175
## Final model
                 458.5175
summary(mylogit3)
##
## Call:
## glm(formula = admit ~ rank + gpa + I((gre/1000)^1), family = binomial(link = logit),
##
      data = mydata)
##
## Deviance Residuals:
##
      Min
                      Median
                                   3Q
                 1Q
                                           Max
                                        2.0790
## -1.6268 -0.8662 -0.6388
                               1.1490
##
## Coefficients:
##
                   Estimate Std. Error z value Pr(>|z|)
                   -3.9900
                                1.1400 -3.500 0.000465 ***
## (Intercept)
## rank2
                    -0.6754
                                0.3165 -2.134 0.032829 *
## rank3
                    -1.3402
                                0.3453 -3.881 0.000104 ***
                                0.4178 -3.713 0.000205 ***
## rank4
                    -1.5515
                     0.8040
                                0.3318
                                        2.423 0.015388 *
## gpa
## I((gre/1000)^1)
                   2.2644
                                1.0940
                                        2.070 0.038465 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
       Null deviance: 499.98 on 399 degrees of freedom
## Residual deviance: 458.52 on 394 degrees of freedom
## AIC: 470.52
```

```
##
## Number of Fisher Scoring iterations: 4
mylogit3$fptable
##
         df.initial select alpha df.final power1 power2
## rank2
                   1
                           1
                              0.05
                                            1
## rank3
                   1
                           1
                              0.05
                                            1
                                                   1
## rank4
                   1
                              0.05
                                            1
                                                   1
                              0.05
## gpa
                   1
                           1
                                            1
                                                   1
## gre
                   4
                              0.05
                                                   1
```

Diagnostics for a model with a binomial response

To do these diagnostics, you need to load library('LogisticDx').

First, we produce the diagnostic measures for a binary regression model by covariate pattern

Next, we produce the Goodness-of-fit for binomial regression. Usually, the number of groups (quantiles) equal 10 to perform the Hosmer-Lemeshow test. At the same time, we plot the ROC curve

Similarly, we can check the auc value

```
library('LogisticDx')
dx(mylogit2,byCov=T)
```

```
##
                      gpa rank2 rank3 rank4 y
                                                        P n
        (Intercept)
                                                                 yhat
##
     1:
                   1 3.94
                              1
                                     0
                                           0 1 0.5001242 2 1.0002483
##
     2:
                   1 3.30
                              1
                                     0
                                           0 1 0.3378663 3 1.0135990
##
     3:
                   1 3.99
                              0
                                           0 1 0.3412259 3 1.0236777
                                     1
##
     4:
                   1 3.35
                              1
                                     0
                                           0 1 0.3497327 3 1.0491980
##
     5:
                   1 3.17
                              1
                                           0 1 0.3079792 3 0.9239375
##
## 256:
                   1 3.52
                              0
                                           1 2 0.2051122 3 0.6153366
                              0
                                           0 1 0.1154721 1 0.1154721
## 257:
                   1 2.68
                                     1
## 258:
                   1 3.00
                              0
                                     0
                                           1 1 0.1299149 1 0.1299149
## 259:
                   1 2.42
                              0
                                     0
                                           0 1 0.2854368 1 0.2854368
##
  260:
                   1 2.65
                              0
                                     1
                                           0 1 0.1122874 1 0.1122874
##
                                   dr
##
     1: -0.0003511985 -0.0003511985 0.014377039 -0.0003537507 -0.0003537506
     2: -0.0165997514 -0.0166181074 0.006902268 -0.0166573376 -0.0166757573
##
##
     3: -0.0288329487 -0.0288874508 0.015618469 -0.0290607843 -0.0291157171
##
     4: -0.0595625368 -0.0597856465 0.006844794 -0.0597674360 -0.0599913131
##
         0.0951239167 \quad 0.0944359041 \ 0.007474218 \quad 0.0954814103 \quad 0.0947908121
##
## 256:
         1.9798618855
                       1.7253451406 0.017098521
                                                   1.9970084425
                                                                  1.7402874601
         2.7676886108
                        2.0778480687 0.011720502
                                                   2.7840519420
                                                                  2.0901328741
         2.5879228606
                       2.0203343433 0.013448875
                                                   2.6055027084
                                                                  2.0340585431
   259:
         1.5822145401
                       1.5834991249 0.034641074
                                                   1.6103525417
                                                                  1.6116599714
## 260:
         2.8117113691
                       2.0912646219 0.011927131
                                                   2.8286306943 2.1038486967
##
                              dDev
              dChisq
                                           dBhat
     1: 1.251395e-07 1.251395e-07 1.825379e-09
##
     2: 2.774669e-04 2.780809e-04 1.928462e-06
##
##
     3: 8.445292e-04 8.477250e-04 1.339953e-05
     4: 3.572146e-03 3.598958e-03 2.461912e-05
```

```
## 5: 9.116700e-03 8.985298e-03 6.865333e-05

## ---

## 256: 3.988043e+00 3.028600e+00 6.937586e-02

## 257: 7.750945e+00 4.368655e+00 9.192235e-02

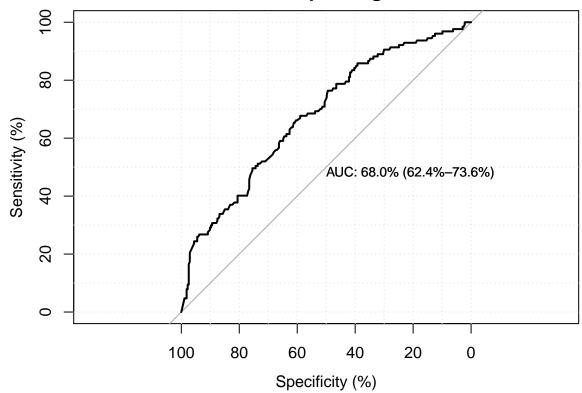
## 258: 6.788644e+00 4.137394e+00 9.254425e-02

## 259: 2.593235e+00 2.597448e+00 9.305602e-02

## 260: 8.001152e+00 4.426179e+00 9.658274e-02

fit.mylogit2<-gof(mylogit2,g=10,plotROC = T)
```

Receiver Operating Curve

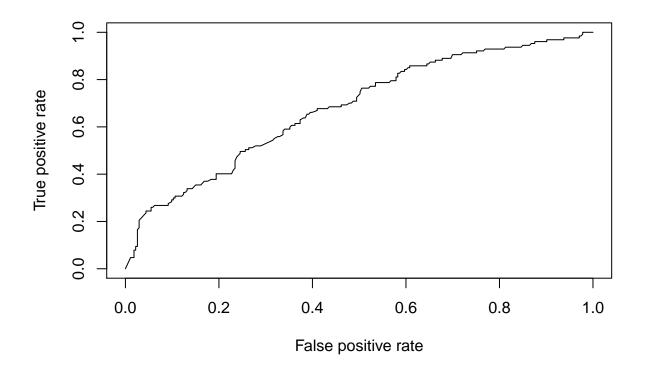


fit.mylogit2

```
##
         chiSq df
## PrI 400.78 395 0.409645
## drI
       462.88 395 0.010379 *
## PrG
       255.36 255 0.481870
## drG 294.60 255 0.044609
## PrCT 255.36 255 0.481870
## drCT 294.60 255 0.044609 *
##
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
                         val df
                                    pVal
## HL chiSq
                    8.190458
                             8 0.415090
## mHL F
                    1.484491
                             9 0.153918
## OsRo Z
                    0.027903 NA 0.977740
## SstPgeq0.5 Z
                    0.217564 NA 0.827769
## SstP10.5 Z
                    0.422859 NA 0.672398
```

```
## SstBoth chiSq
                   0.226144 2 0.893086
## SllPgeq0.5 chiSq 0.047739 1 0.827045
## SllPl0.5 chiSq 0.176745 1 0.674186
## SllBoth chiSq
                    0.177398 2 0.915121
#area under curve
fit.mylogit2$auc
##
            auc lower 95% CI upper 95% CI
##
       68.01073
                   62.39737
                             73.62409
## attr(,"interpret")
## [1] "auc = 0.5
                       --> useless"
                                       "0.7 < auc < 0.8 --> good"
## [3] "0.8 < auc < 0.9 --> excellent"
#chi square test for gof
fit.mylogit2$chiSq
             chiSq df
      test
                             pVal
## 1: PrI 400.7833 395 0.40964543
## 2: drI 462.8753 395 0.01037867
## 3: PrG 255.3601 255 0.48187033
## 4: drG 294.6050 255 0.04460896
## 5: PrCT 255.3601 255 0.48187033
## 6: drCT 294.6050 255 0.04460896
#contigency table for HL test
fit.mylogit2$ctHL
##
                    y1hat y0
          P y1
                               y0hat n
   1: 0.149 6 5.037833 34 34.96217 40 0.1259458
## 2: 0.192 5 6.824095 35 33.17591 40 0.1706024
## 3: 0.219 7 8.260683 33 31.73932 40 0.2065171
## 4: 0.258 11 9.327547 28 29.67245 39 0.2391679
## 5: 0.293 12 11.338710 29 29.66129 41 0.2765539
## 6: 0.34 15 12.556219 25 27.44378 40 0.3139055
## 7: 0.376 14 13.796927 25 25.20307 39 0.3537674
## 8: 0.437 14 16.561867 27 24.43813 41 0.4039480
## 9: 0.511 13 18.253216 25 19.74678 38 0.4803478
## 10: 0.678 30 25.042903 12 16.95710 42 0.5962596
#GOF test
fit.mylogit2$gof
##
            test stat
                              val df
                                          pVal
## 1:
             HL chiSq 8.19045835 8 0.4150903
## 2:
            \mathtt{mHL}
                  F 1.48449073 9 0.1539182
## 3:
                    Z 0.02790263 NA 0.9777398
           OsRo
## 4: SstPgeq0.5
                     Z 0.21756392 NA 0.8277689
       SstPl0.5
## 5:
                    Z 0.42285933 NA 0.6723979
        SstBoth chiSq 0.22614408 2 0.8930863
## 7: SllPgeq0.5 chiSq 0.04773921 1 0.8270450
## 8:
       SllPl0.5 chiSq 0.17674490 1 0.6741857
## 9:
        SllBoth chiSq 0.17739848 2 0.9151208
We can also perform model fitness by using ROCR package
library(ROCR)
```

```
## Loading required package: gplots
##
## Attaching package: 'gplots'
   The following object is masked from 'package:stats':
##
##
##
       lowess
pred.prob2<-predict(mylogit2, type='response')</pre>
head(pred.prob2)
                      2
                                 3
                                            4
                                                      5
##
           1
                                                                 6
## 0.2577653 0.2700256 0.6779993 0.1542276 0.1218149 0.2712217
pred.prob22<-prediction(pred.prob2, mydata$admit )</pre>
pred.prob22f<-performance(pred.prob22, measure='tpr', x.measure='fpr')</pre>
plot(pred.prob22f)
```



Using ROCR package, we can also calculate the AUC

```
auc2<-performance(pred.prob22, measure='auc')
auc2@y.values[[1]]</pre>
```

[1] 0.6801073

Another package is *MKmisc* package, to perform the Hosmer-Lemeshow test of GOF library (MKmisc)

##

```
## Attaching package: 'MKmisc'
## The following object is masked from 'package:psych':
##
##
       corPlot
HLgof.test(fit = fitted(mylogit2), obs = mydata$admit)
## $C
##
   Hosmer-Lemeshow C statistic
##
##
## data: fitted(mylogit2) and mydata$admit
## X-squared = 9.8564, df = 8, p-value = 0.2752
##
##
## $H
##
##
   Hosmer-Lemeshow H statistic
##
## data: fitted(mylogit2) and mydata$admit
## X-squared = 7.6802, df = 8, p-value = 0.4653
```

Diagnostic plot

Will return many diagnostic plot

```
plot(mylogit2)
```

Using K-fold validation. Will not discuss here.

Interaction

Let use see how we deal an interaction. First, read data from this text file.

Columns (variables) no 2, and from 5 to 10 need to be converted to categorical (factor) variables

```
data.l<-read.table("LOWBWT.txt",header=T)
data.l[,c(2,5:10)]<-lapply(data.l[,c(2,5)],factor)</pre>
```

To simulate a binary predictor variable, we now recode LWT to LWD (LWT<110 vs >=110)

```
## [1] 120 130 187 105 85 150 97 128 132 165
str(data.l$LWD)
```

```
## Factor w/ 2 levels "less 110",">=110": 2 2 2 1 1 2 1 2 2 2 ...
Model the relationship; outcome (LOW=0,1) with predictors of LWD and AGE interact with each other.
You may try with using #
mod.lwd.age<-glm(LOW~LWD*AGE,family = binomial(link =logit ),data=data.l)</pre>
summary(mod.lwd.age)
##
## glm(formula = LOW ~ LWD * AGE, family = binomial(link = logit),
##
       data = data.1)
##
## Deviance Residuals:
##
       Min
                 1Q
                      Median
                                    3Q
                                            Max
## -1.4257 -0.8554 -0.6960
                               1.1602
                                         2.0329
##
## Coefficients:
##
                Estimate Std. Error z value Pr(>|z|)
                            1.46515 -0.798
## (Intercept)
               -1.16959
                                               0.4247
## LWD>=110
                 1.94409
                            1.72481
                                       1.127
                                               0.2597
## AGE
                 0.05262
                            0.06449
                                       0.816
                                               0.4145
## LWD>=110:AGE -0.13220
                            0.07570 - 1.746
                                               0.0807 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
       Null deviance: 234.67 on 188 degrees of freedom
## Residual deviance: 221.14 on 185 degrees of freedom
## AIC: 229.14
##
## Number of Fisher Scoring iterations: 4
Predict our model using new data. Before doing so, we need to create a dataset containing new data
newdata.2<-data.frame(AGE=c(15,15,20,20),LWD=rep(c("less 110",">=110"),2))
Now let us predict the log odds
predict(mod.lwd.age,newdata=newdata.2)
## -0.3802252 -0.4190874 -0.1171023 -0.8169483
newdata.2
     AGE
              LWD
##
## 1 15 less 110
## 2 15
            >=110
## 3 20 less 110
## 4 20
            >=110
Can you prove these?
-1.1696+0+0.0526*15+0
```

[1] -0.3806

```
-1.1696+1.944*1+0.0526*15-0.1322*1*15

## [1] -0.4196
-1.1696+0+0.0526*20+0

## [1] -0.1176
-1.1696+1.944*1+0.0526*20-0.1322*1*20

## [1] -0.8176
```

Resources

- $1.\ http://www.ats.ucla.edu/stat/r/dae/logit.htm$
- 2. https://cran.r-project.org/web/packages/HSAUR/vignettes/Ch_logistic_regression_glm.pdf