ST2334 Midterm cheatsheet

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Chapter 1: Probability

More Event Operations

(a) $A \cap A' = \emptyset$ (b) $A \cap \emptyset = \emptyset$

(c) $A \cup A' = S$ (d) $(A')' = A$

(e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive law

(f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) $A \cup B = A \cup (B \cap A')$

(h) $A = (A \cap B) \cup (A \cap B')$

Draw venn diagram for proof

De Morgan's Law

For any n events $A_1, A_2, \ldots, A_n$,

(i) $(A_1 \cup A_2 \cup \ldots \cup A_n)' = A_1' \cap A_2' \cap \ldots \cap A_n'$.

A special case: $(A \cup B)' = A' \cap B'$.

(j) $(A_1 \cap A_2 \cap \ldots \cap A_n)' = A_1' \cup A_2' \cup \ldots \cup A_n'$.

A special case: $(A \cap B)' = A' \cup B'$.

Axioms of Probability

Probability, denoted by $P(\cdot)$, is a **function** on the collection of events of the sample space S, satisfying:

Axiom 1. For any event A,

$$0 \le P(A) \le 1.$$

Axiom 2. For the sample space,

$$P(S) = 1.$$

Axiom 3. For any two mutually exclusive events A and B, that is, $A \cap B = \emptyset$,

$$P(A \cup B) = P(A) + P(B).$$

Proposition 5

For any two events A and B,

$$P(A) = P(A \cap B) + P(A \cap B').$$

P(AUB) = P(A) + P(B) - P(A⋂B)

<u>robability:</u>

$$P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

Multiplication Rule

Starting from the definition of conditional probability, and rearranging the terms, we have

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A), \quad \text{if } P(A) \neq 0 \\ \text{or} \quad P(A \cap B) &= P(B)P(A|B), \quad \text{if } P(B) \neq 0. \end{aligned}$$

This is known as the **Multiplication Rule**.

$\rightarrow$ $P(A \cap B \cap C) = P(A)\,P(B|A)\,P(C|AB)$

then perform algebraic simplification

Inverse Probability Formula

The multiplication rule together with the definition of the conditional probability gives us:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}.$$

This is known as the **Inverse Probability Formula**.

<u>Independence of events:</u>

Events A and B are independent iff P(A⋂B) = P(A) * P(B)

<u>Law of total probability:</u>

Special case: Law of Total Probability

For any events A and B, we have

$$P(B) = P(A)P(B|A) + P(A')P(B|A').$$

$P(A \cap B) + P(A' \cap B)$

<u>Bayes Theorem:</u>

Used to flip a conditional probability from P(A|B) to P(B|A)

Special case: Bayes' Theorem

Let us consider a special case of Bayes' Theorem when $n = 2$.

$\{A, A'\}$ becomes a partition of S, and we have

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A')P(B|A')}.$$

$= P(B)$ law of total probability

<u>Extra Notes:</u>

Chapter 2: Random variables

Discrete random variable:

- Probability mass function

DEFINITION 3 (PROBABILITY MASS FUNCTION)
For a discrete random variable X, define

$$f(x) = \begin{cases} P(X = x), & \text{for } x \in R_X; \\ 0, & \text{for } x \notin R_X. \end{cases}$$

Then $f(x)$ is known as the ***probability function (pf)***, *or* ***probability mass function (pmf)*** *of X.*

The collection of pairs $(x_i, f(x_i)), i = 1, 2, 3, \dots$, is called the ***probability distribution*** *of X.*

PROPERTIES OF THE PROBABILITY MASS FUNCTION
The probability mass function $f(x)$ of a discrete random variable **must** satisfy:

(1) $f(x_i) \geq 0$ for all $x_i \in R_X$;

(2) $f(x) = 0$ for all $x \notin R_X$;

(3) $\sum_{i=1}^{\infty} f(x_i) = 1$, or $\sum_{x_i \in R_X} f(x_i) = 1$.

For any set $B \subset \mathbb{R}$, we have

$$P(X \in B) = \sum_{x_i \in B \cap R_X} f(x_i).$$

- Can also be shown in a table form:

The probability mass function of X can be summarized by

x	0	1	2
$f(x)$	1/2	1/6	1/3

Continuous random variable:

- Probability density function (pdf)

DEFINITION 4 (PROBABILITY DENSITY FUNCTION)
The ***probability density function*** *of a continuous random variable X, denoted by $f(x)$, is a function that satisfies:*

(1) $f(x) \geq 0$ for all $x \in R_X$; and $f(x) = 0$ for $x \notin R_X$;

(2) $\int_{R_X} f(x)\,dx = 1$;

(3) For any a and b such that $a \leq b$,

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx.$$

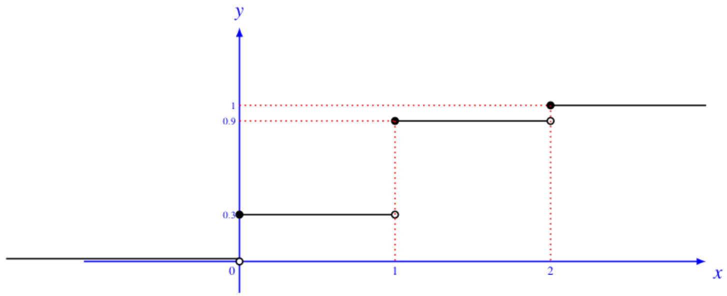
** Basically area under curve

Cumulative distribution function(cdf)

DEFINITION 5 (CUMULATIVE DISTRIBUTION FUNCTION)
For any random variable X, we define its ***cumulative distribution function (cdf)*** *by*

$$F(x) = P(X \leq x).$$

$$F(x) = \begin{cases} 0, & x < 0, \\ 0.3, & 0 \leq x < 1, \\ 0.9, & 1 \leq x < 2, \\ 1, & 2 \leq x. \end{cases}$$



For continuous random variable:

CDF: CONTINUOUS RANDOM VARIABLE
If X is a continuous random variable,

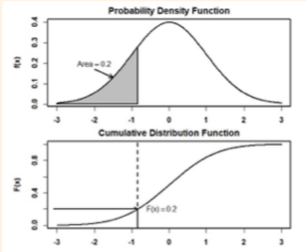
$$F(x) = \int_{-\infty}^{x} f(t)\,dt,$$

and

$$f(x) = \frac{dF(x)}{dx}.$$

Further

$$P(a \leq X \leq b) = P(a < X < b) = F(b) - F(a).$$



Expectation and Variance:

- **Discrete random variable**

$$E(X) = \sum_{x_i \in R_X} x_i f(x_i).$$

- **Continuous random variable**

$$\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{x \in R_X} x f(x)\,dx.$$

Properties of expectation:

$$E(aX + b) = aE(X) + b.$$

$$E(X + Y) = E(X) + E(Y).$$

Let $g(\cdot)$ be an arbitrary function.

- If X is a **discrete** random variable with probability mass function $f(x)$ and range R_X,
$$E[g(X)] = \sum_{x \in R_X} g(x) f(x).$$
- If X is a **continuous** random variable with probability density function $f(x)$ and range R_X,
$$E[g(X)] = \int_{R_X} g(x) f(x)\,dx.$$

Properties of variance:

Discrete random variable:

$$V(X) = \sum_{x \in R_X} (x - \mu_X)^2 f(x).$$

Continuous random variable:

$$V(X) = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x)\,dx.$$

Preferred method:

$$V(X) = E(X^2) - [E(X)]^2.$$

Standard deviation =sqrt(variance)