ABSTRACT

FIVE-DIMENSIONAL FLOW IN SOLID FUEL ROCKET ENGINES – FUN WITH LaTeX

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Northern Illinois University, 2010
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Solid fuel rocket engines are one of the most reliable and efficient propulsion systems used to lift payloads into orbit, in terms of $\lambda = (\Box + \Diamond)\psi$. Used throughout the astrodynamics community, the theory of the flow within the motor chamber is in fact a black art which defies all attempts at analysis.

The present work (no exception to the statement above) contains a theoretical and numerical approach to the flow of the gases within the motor chamber. The shape of the chamber and original fuel configuration, and the patterns of combustion and flow/expulsion of gases, are modelled by a system of thirty fourth-degree differential equations.

$$f_i^{(34)}(x, y, z, t) = \sum_{j=0}^{33} a_{ij} f_i^{(\mathrm{j})}(x, y, z, t)$$

Acceptable numerical solutions would require one thousand pentium processors working day and night for $10^{11.2}$ years.

NORTHERN ILLINOIS UNIVERSITY
DE KALB, ILLINOIS

MAY 2010

FIVE-DIMENSIONAL FLOW IN SOLID FUEL ROCKET ENGINES – FUN WITH LaTeX

BY

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A DISSERTATION SUBMITTED TO THE GRADUATE SCHOOL

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE

DOCTOR OF PHILOSOPHY

DEPARTMENT OF ROCKET SCIENCE

Dissertation Director:
Wernher von Braun

ACKNOWLEDGEMENTS

Here’s where you acknowledge folks who helped. Here’s where you acknowledge folks who helped. Here’s where you acknowledge folks who helped.

DEDICATION

To all of the fluffy kitties. To all of the fluffy kitties. To all of the fluffy kitties. To all of the fluffy kitties.

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CHAPTER 1

INTRODUCTION

This sample document illustrates how to use the `niuthesis` class. We always put a bit of text in between the heading commands to see how it goes. *We always put a bit of text in between the heading commands to see how it goes.*

1.1 This Is a Section (Level 2)

First we want to see how the sectioning commands work on different levels of sectioning. We always put a bit of text in between the heading commands to see how it goes. **We always put a bit of text in between the heading commands to see how it goes.**

1.1.1 So Here We Have a Subsection (Level 3)

We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes.

1.1.1.1 And Here a LaTeX Subsubsection (Level 4)

We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes.

And Here Yet Lower Sectioning, a Paragraph (Level 5). We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes. We always put a bit of text in between the heading commands to see how it goes.

1.2 This Is a Section (Level 2)

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1.3 This Is a Section (Level 2)

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1.4 This Is a Long Section Title That Needs to Be Broken Over Two Lines – or May Be Three? We Will See ...

Some requirements of the Graduate School are written into that file; page size, line spacing, appropriate placement of captions for tables and figures, etc. Other tasks of conforming to the requirements are left to other existing LaTeX packages.

1.4.1 Question: What Are the Issues in Studying This Subject?

A major goal in studying solid fuel rocket motors is to create a model of the dynamics of a motor chamber. This involves two major goals: the combustion zone and the acoustic zone.

The combustion zone consists of the thin layer above the solid fuel where the gasification of the fuel takes places. The zone is very reactive and highly turbulent. The acoustic-vortical zone is the volume of gas above the combustion zone. Within this zone, the gas is non-reactive and contains acoustic waves and vorticity. The work presented here[1] is an extension of Lao [6] and Lao et al. [7]. The driving frequency is on the order of the inverse of the axial acoustic time scale, $t'_A = L'/C'_0$, where L' is the length of the cylinder and C'_0 is the reference speed of sound.[2] Radial and azimuthal velocities are found to vanish exponentially fast in the downstream direction, as suggested by Table 1.1.

These results provide an analytical explanation of those found from computational analysis by Fabnis et al. [4]. The non-axisymmetric flow near the endwall contains cross-sectional velocity patterns that include flow across the cylinder axis. A viscous boundary layer ad-

[1] Footnotes are handled neatly by LaTeX.

[2] Remember the traditional method of calculating the distance of lightning? See the flash, count seconds until you hear the thunder, divide by five, that's the number of miles. That assumes $C_0 = \frac{1mi.}{5s}$.

Table 1.1: Here is an example of a table with its own footnotes. Don't use the `\footnote` macro if you don't want the footnotes at the bottom of the page. Also, note that in a thesis the caption goes *above* a table, unlike figures.

wave form	S (kVA)	P (kW)	Q^* (kVAr)	$D^\dagger$ (kVAd)
	25.87	25.83	1.3	≈ 0
	25.48	25.00	-2.82	4.03
	25.11	18.02	-9.75	14.52
Table 2	24.98	22.26	9.19	6.64
Fig. 2.3	23.48	15.00	6.59	16.82
Fig. 2.1	24.64	22.81	-0.44	9.3
Fig. 2.2	23.03	18.01	3.36	13.95

*kVAr means reactive power.
†kVAd means distortion power.

jacent to the sidewall and near the endwall is studied to find the transition between the transient core flow and the no-slip condition on the sidewall. It is found, as in Lao et al. [7], that the azimuthal component of the vorticity is proportional to the inverse of the Mach number. In addition, the axial component of the vorticity driven by the non-axisymmetric boundary condition at the endwall is also found to be proportional to the the inverse of the Mach number.

CHAPTER 2

MATHEMATICAL FORMULATION

The objective of this fake thesis document is to demonstrate some LaTeX features as well as features specific to the thesis class. We start by giving one short formula,

$$A = \pi r^2, \tag{2.1}$$

and one big hairy multi-line formula (one of the non-dimensional Navier-Stokes equations):

$$\begin{aligned}
\rho \left[\frac{DV_r}{Dt} - M\epsilon^2 \frac{V_\theta^2}{r} \right] \quad = \quad & -\frac{\delta^2}{\gamma\ M} \frac{\partial P}{\partial r} + \frac{M\ \delta^2}{Re} \left\{ 2 \frac{\partial}{\partial r} \left[\mu \left(\frac{\partial V_r}{\partial r} - \frac{1}{3} \nabla \cdot \overline{\mathbf{V}} \right) \right] \right. \\
& + \frac{1}{r} \frac{\partial}{\partial \theta} \left[\mu \left(\frac{1}{r} \frac{\partial V_r}{\partial \theta} + \epsilon \frac{\partial V_\theta}{\partial r} - \epsilon \frac{V_\theta}{r} \right) \right] \\
& + \frac{\partial}{\partial z} \left[\mu \left(\frac{1}{\delta^2} \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right) \right] \\
& \left. + 2\ \frac{\mu}{r} \left[\frac{\partial V_r}{\partial r} - \frac{\epsilon}{r} \frac{\partial V_\theta}{\partial \theta} - \frac{V_r}{r} \right] \right\},
\end{aligned} \tag{2.2}$$

The latter equation is non-dimensionalized using the following definitions:

$$r = \frac{r'}{R'}, \quad z = \frac{z'}{L'}, \quad t = \frac{t'}{t'_a}, \quad \kappa = \frac{\kappa'}{\kappa'_0}, \quad \mu = \frac{\mu'}{\mu'_0}, \quad C_V = \frac{C'_V}{C'_{V0}},$$

where P'_0 is the initial static pressure in the cylinder, and ρ'_0 and T'_0 are the density and temperature of the fluid being injected from the sidewall. The aspect ratio is given by $\delta = \frac{L'}{R'}$, where $\delta \gg 1$. The induced characteristic axial velocity and the characteristic endwall velocity disturbance V'_{z0} is defined with respect to the injection reference sidewall

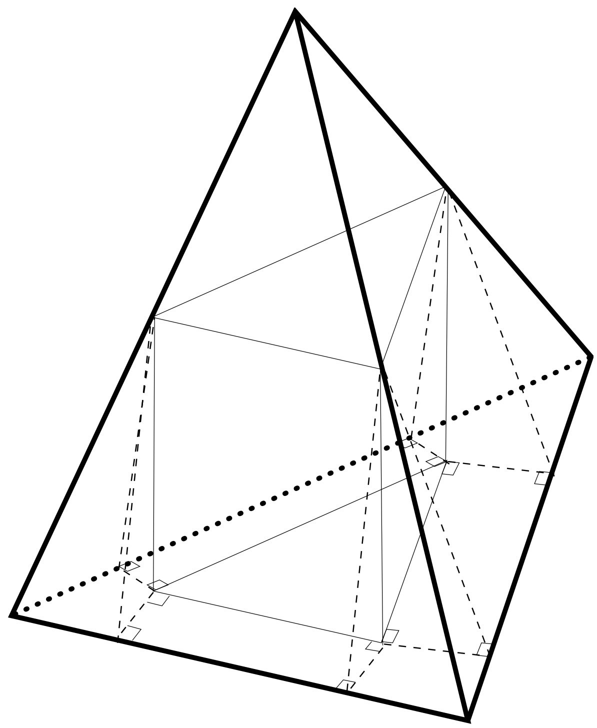

Figure 2.1: A triangular pyramid may be cut up as shown, to yield one top pyramid (with one-eighth the volume of the full pyramid), three bottom corner pyramids (which, when joined, are congruent to the top pyramid), three prisms along the bottom edges (the area of whose bottom faces total $B/2$) and the large central prism (volume = $(B/4)(h/2) = Bh/8$). The image, from PostScript file "pyr.eps", was read in using the `\includegraphics` command, from the `graphicx` package.

velocity, V'_{r0} by overall mass conservation, $\frac{V'_{z0}}{V'_{r0}} = \delta$. The size of the initially unknown reference azimuthal velocity $V'_{\theta 0}$ is related to V'_{r0} by $\frac{V'_{\theta 0}}{V'_{r0}} = \epsilon$. Later, it is shown that $\epsilon = 1$.

The time is non-dimensionalized using the axial acoustic time scale, $t'_a = \frac{L'}{C'_0}$, where $C'_0 = (\gamma \mathcal{R}' T'_0)^{\frac{1}{2}}$ is the speed of sound,[1] $\mathcal{R}'$ is the gas constant, and γ is the ratio of specific heats. Also the Reynolds number, Wrenchl number, and Mock number are defined as

$$Re = \frac{\rho' V'_{z_0} L'}{\mu'_0}, \quad Wr = \frac{\mu'_0 C'_{p_0}}{\kappa'_0}, \quad M = \begin{pmatrix} V'_{z_0} \\ C'_0 \end{pmatrix} \cdot \begin{pmatrix} 8a & z_0 - \rho \\ 2 & z_0 - \mu \end{pmatrix} \begin{pmatrix} Wr \\ p - 7 \end{pmatrix},$$

[1] In air at 1 atm., $\frac{1mi.}{5s}$.

Table 2.1: This is a table constructed with LaTeX commands in the `tabular` environment.

n	n^2	n^3	n^4	n^7	n^{13}
2	4	8	16	128	8192
3	9	27	81	2187	1594323
4	16	64	256	16384	67108864
5	25	125	625	78125	1220703125
6	36	216	1296	279936	13060694016
7	49	343	2401	823543	96889010407

Table 2.2: Table from a PostScript file. This table wasn't constructed with LaTeX commands, but resides in a PostScript file (`tableD.eps`) created by some other software.

n	$\mathbf{n^2}$	$\mathbf{n^3}$	$\mathbf{n^4}$	$\mathbf{n^7}$	$\mathbf{n^{13}}$
2	4	8	16	128	8192
3	9	27	81	2187	1594323
4	16	64	256	16384	67108864
5	25	125	625	78125	1220703125
6	36	216	1296	279936	13060694016
7	49	343	2401	823543	96889010407

where $Re \ll 1$, $M \gg 1$, and $Wr = O(1)$.

Here is an example of using the `singlespace` environment.

This paragraph was surrounded by the `singlespace` environment. The Mock number is chosen as a small parameter to model the small magnitude found in a typical rocket motor chamber, as opposed to the rocket nozzle where larger values are possible.[2] The aspect ratio, δ, is taken to be a large parameter, because many chambers have aspect ratios between 15 and 50.

The following table is created using the LaTeX `tabular` environment.

However, sometimes you want to use a table produced by some other software, such as Excel. If the table is saved to a PostScript file, then it can be displayed using the `\includegraphics` macro inside a `table` environment:

[2]Not just possible, desirable!

2.1 Conditions for Catastrophic Combustion

Initially, a steady flow is generated by the sidewall injection, $V_r = -V_{rws}(z)$. The subscript srw is used to mean that there is a **s**teady **r**adial **w**all velocity. The sign is negative due to the injection toward centerline. At $t = 0^+$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_z = \widetilde{F}_{rw}(r, \theta, t)$, for $\omega = O(1)$.

Some of the boundary conditions are:

$$z = 0; \qquad V_z = \begin{cases} 0, & t \leq 0 \\ \\ \widetilde{F}_{zw}(r, \theta, t), & t > 0 \end{cases} \tag{2.3}$$

$$z = 0; \qquad V_\theta = V_r = 0, \tag{2.4}$$

$$r = 0; \qquad P, \rho, T, V_r, V_\theta, V_z \text{ finite}, \tag{2.5}$$

$$r = 1; \qquad V_r = F_{rws}(z), \tag{2.6}$$

$$r = 1; \qquad V_z = V_\theta = 0, \tag{2.7}$$

and solutions must be periodic in θ.

2.2 More Boundary Conditions

Initially, a steady flow is generated by the sidewall injection, $V_r = -V_{rws}(z)$. The sign is negative due to the injection toward centerline.[3] At $t = 0^+$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_z = \widetilde{F}_{rw}(r, \theta, t)$, for $\omega = O(1)$. The

[3] This convention was suggested by Goddard and Smythe.

frequency condition chosen represents the first few axial acoustic modes observed in high aspect ratio chambers.[4]

The full boundary conditions include:

$$z = 0; \qquad V_\theta = V_r = 0, \tag{2.8}$$

$$r = 1; \qquad V_r = \begin{cases} F_{rws}(z), & t < 0, \\ F_{rws}(z) + \widetilde{F}_{rw}(z, \theta, t), & t \geq 0 \end{cases} \tag{2.9}$$

$$r = 1; \qquad V_z = V_\theta = 0, \tag{2.10}$$

and solutions must be periodic in θ.

If you don't believe this stuff, check out Mulick [9] and Baylor [1].

2.2.1 Just Meaningless Text to Test Lines per Page

Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page. Just meaningless text to test lines per page.

[4] Toy rockets, the kind you used to shoot off with your dad in the park, typically have only two significant modes.

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2.2.2 This Is a Subsection

This is a subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

2.2.3 This Is Another Subsection

This is another subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

This Is a Paragraph. It used a `\paragraph{}` header, which are always inlined (with extra space) and boldfaced.

Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

What Is It?. This is a paragraph. The heading of the paragraph is emphasized. This is a paragraph. The heading of the paragraph is emphasized.

2.3 Sideways Tables

In Table 2.3 on page 13 we show on a separate page how LaTeX can handle big tables that need to be put sideways. All we need to do is to put the table into a `sidewaystable` environment. The rest is handled by LaTeX. In particular, we can easily make sure that the table will take just the available space. That's neat, isn't it? Just make sure you load the `rotating` package that defines the `sidewaystable` environment.

2.4 Sideways Figures

The `rotating` package also defines a `sidewaysfigure` environment. This works well with big figures – convince yourself by looking at Fig. 2.2 on page 14!

2.5 The End

Finally, this is the end. The bibliography starts on the next page.

Table 2.3: A big table that needs to go sideways – easy with LaTeX.

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

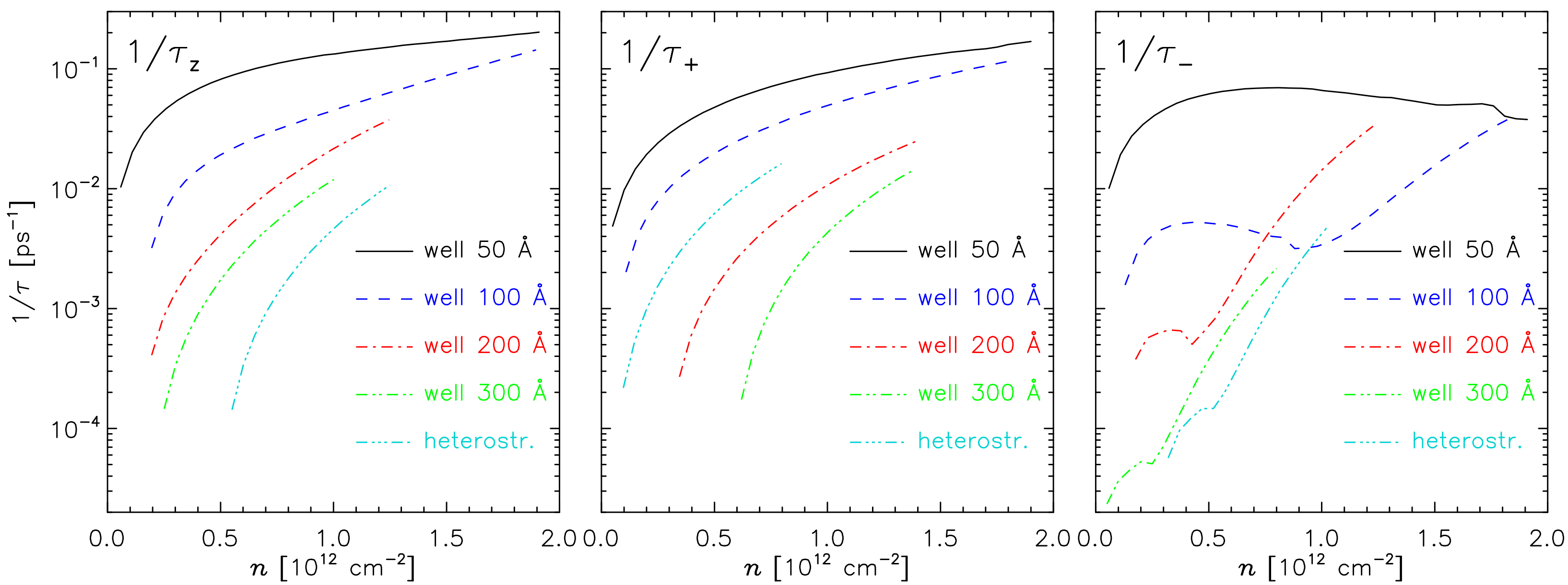

Figure 2.2: A big figure that needs to go sideways.

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APPENDIX A

OBJECTIVE SYMPTOMS

Appendices follow the same page-numbering rules as regular chapters. The first page of a multi-page appendix is not numbered. But the page of a single-page appendix *is* numbered.

Are they slow learners or is it a *REAL* problem? These are classic findings in the hopelessly computer challenged.

1. Can't copy from hard drive to disk.

2. Can't eject disks.

3. The word "disk" has thousands of meanings to them. None are correct.

4. Saving a document in any form is a concept totally unexplainable to them.

5. Desktop covered with Untitled Folders - look again, untitled folders are everywhere.

6. "Lost" documents found often in the Apple Menu.

7. Trash always full. Claim they don't know how to place things in trash.

8. Mysterious things happen to their documents or computer when they are not present. AKA "computer victims".

9. Highlighting = deleting. Dragging = Oblivion.

10. Selecting, double-clicking a problem? They will always say their mouse is broken.

11. Their double- click mechanics wants you to send them to a neurologist.

12. Computer always on due to fear of having to restart it.

13. Have never read their QuickMail - will say "I prefer a phone call".

14. Have magical beliefs about what computers do.

15. Describes some flaky way computers could REALLY help them, but is not yet available.

16. Constantly saying they need more “memory”.

17. Requests gizmos and gadgets, i.e., “mouse leash” or “disk cozy”.

18. Avoids eye contact when talking about computers.

APPENDIX B

ODE TO SPOT

(Data, Stardate 1403827). Throughout the ages, from Keats to Giorchamo, poets have composed "odes" to individuals who have had a profound effect upon their lives. In keeping with that tradition I have written my next poem ... in honor of my cat. I call it... Ode... to Spot. (Shot of Geordi and Worf in audience, looking mystified at each other.)

Felus cattus, is your taxonomic nomenclature
an endothermic quadruped, carnivorous by nature?
Your visual, olfactory, and auditory senses
contribute to your hunting skills, and natural defenses.
I find myself intrigued by your sub-vocal oscillations,
a singular development of cat communications
that obviates your basic hedonistic predilection
for a rhythmic stroking of your fur to demonstrate affection.
A tail is quite essential for your acrobatic talents;
you would not be so agile if you lacked its counterbalance.
And when not being utilized to aid in locomotion,
It often serves to illustrate the state of your emotion.

(Commander Riker begins to applaud, until a glance from Counselor Troi brings him to a halt.) Commander Riker, you have anticipated my denouement. However, the sentiment is appreciated. I will continue.

O Spot, the complex levels of behavior you display
connote a fairly well-developed cognitive array.
And though you are not sentient, Spot, and do not comprehend
I nonetheless consider you a true and valued friend.