A. Pillepich
Block Course Cosmology
Heidelberg University

Problem Sheet #4 (Hand out: Wed => hand in: Fri)

Solve the exercises below.

1. The abundance of dark-matter haloes from the Illustris and TNG simulations

1.1 How many haloes above a certain mass are there in the TNG100-3-Dark simulation at z=0? Plot the cumulative dark-matter halo mass functions

N(>MHalo) vs. MHalo and
n=N/V(>MHalo) vs. MHalo

for different choices of MHalo = M_fof, M200c, M200m, M500c
Hint: use log10 scales. Request: use Msun units, not 10^10Msun or 10^10Msun/h.
Write down the number of haloes that are more massive than 10^12 Msun for the three different definitions of halo masses. Comment on such number in relation to the volume encompassed by the simulation and the halo mass definitions.
The halo of the Milky Way is thought to have a mass of about 10^12 Msun: how more numerous are Milky-Way like haloes in comparison to haloes of 10^14 Msun?
How rare are 10^13 Msun haloes in comparison to 10^11 Msun haloes?
What is the minimum halo mass you can plot? Why?

1.2 Compare the cumulative and differential dark-matter halo mass functions at z=0 between TNG100-3-Dark and TNG300-3-Dark. For the latter (differential mass function), use the following operational definition:

f(M,z) = dN/dM/dV [1/Msun/Mpc^3] or dN/dlog10M/dV [1/dex/Mpc^3].

How many objects above 10^14 solar masses are there in TNG100-3-Dark? How many in TNG300-3-Dark? What is their typical number density today?

1.3 [Optional] Compare the dark-matter halo mass functions at z=0 between TNG100-3-Dark and Illustris-3-Dark. Are there any differences? Due to what?

1.4 Quantify the redshift evolution of the dark-matter halo mass function using TNG300-3-Dark and quantify how more rare 10^14 solar mass haloes are at z=2 in comparison to z=0.
In practice, plot dn/dM vs. M in log10 scales and n=N/V(>MHalo) vs. MHalo at different redshifts: one curve for each mass function definition per redshift.

Note: the differential mass function at the high mass end is very steep. To get accurate estimates of the mass function, you need to choose very narrow logarithmic bins in halo mass. In each bin, measure the number of haloes in the given volume and divide by the volume: this gives you the y value for every bin. What is the "x" value of the bin? A priori, you may be tempted to take the mid point of the mass bin, or the mean. The correct result is obtained by taking for the x value of each bin the median or mean value of the masses of those haloes that fall in that bin.

2. Build your own Python Cosmological Calculator (Part III)

Write a Python function that returns

- the (linear) Power Spectrum
- the mass variance of the linear density field

as a function of k (the wavenumber, e,g. in 1/Mpc) at any redshift and for any cosmological model with a cosmological constant Lambda, i.e. for any combinations of the cosmological parameters Omega_m, Omega_r, Omega_Lambda, h, …

Make the following plots of P(k) and comment on them:

1. P(k, z) vs. k in LCDM at three redshifts, z=0, 1, 4
2. P(k) vs. k at z=0 in an Einstein de Sitte Universe, in a Low-density Universe and in a LCDM Universe, as in Exercise 2 of Problem Sheet 2
3. P(k) vs. k for a LCDM with sigma8 = 0.7, 0.8 and 0.9

Make the following plots of sigma(M, z) and comment on them:

4. sigma(M, z) vs. M at z=0, 1, 4 in LCDM
5. sigma(M, z) vs. M at z=0 in a LCDM universe with varying Omega_m (and hence Omega_Lamda): e.g. 0.2, 0.3 and 0.4

Note: you need a Transfer function analytical formula or numerical prescription. If you could fix the cosmological model, you could use an online calculator like CAMB, online: https://lambda.gsfc.nasa.gov/toolbox/camb_online.html (if you do not know the meaning of the parameters, leave their default values).

However, we want the Transfer function to be allowed to vary based on the values of the cosmological parameters. We can use analytical approximations to the Transfer function for cold dark matter in mixture cosmological models:
e.g. Bardeen et al. 1986 aka BBKS T(k)
e.g. Eisenstein & Hu 1998: https://arxiv.org/pdf/astro-ph/9709112.pdf eqs. 26+28-31
The BBKS T(k) formula is found in the Lectures scripts.

Note: here the functions you are coding up could take as input: array of k, a choice for the smoothing scale (M or R), the redshift z, and the set of cosmological parameters

3. Analytical vs. Numerical formulas for the dark-matter halo mass functions.

3.1 Compare the dark-matter halo mass functions at z=0 by Press & Schechter (1974, analytically-derived), Sheth & Tormen (1999, analytically-derived), Tinker et al. 2008 (numerically-derived), and (optional) TNG300-3-Dark, in the 10^10-15 Msun mass range, for a choice of halo mass M200c.

Here let us express the mass function in terms of the mass variance of the liner density field \sigma(M,z):

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,z) = f(\sigma)\,\frac{\bar{\rho}_{\mathrm{m}}}{M}\,\frac{\mathrm{d}\ln[\sigma^{-1}(M,z)]}{dM}\,. \tag{2}$$

where $\bar{\rho}_{\mathrm{m}}$ is the mean background matter density today, and $\sigma^2(M, z)$ is the variance of the linear density field

$$\sigma^2(M,z) = \frac{1}{2\pi^2}\int_0^\infty k^2\,P_{\mathrm{lin}}(k,z)\,W^2(k,M)\,\mathrm{d}k, \tag{3}$$

Namely, plot dn/dM vs. M at z=0: one curve per formula, in log10 scale. The formulas are given in the lectures. How different are they? To quantify, make a ratio plot of each formulas above with respect to Tinker et al. What relative systematic uncertainties i.e. deviations do you notice?

3.2 Understand what it means that the mass function expressed in terms of the mass variance of the linear density field is universal, e.g. does not depend on redshift. Take any of the above fitting functions and plot f(sigma) vs. 1/sigma for a given range of halo masses at different redshifts. E.g. consider the 10^10-15 Msun range at z = 0,1, and 4 and plot the corresponding f(sigma) vs. 1/sigma function: one curve per redshift choice.
What do you notice?

Hint: you need to build the mapping between Halo mass and sigma(M,z) — see exercise 2. At different z, the same halo mass range corresponds to different ranges of values for sigma. The same halo mass at different redshifts corresponds to very different values of sigma and its differently rare.

3.3 [Optional] Show that the mass function expressed in terms of the mass variance of the linear density field is universal, e.g. does not depend on redshift. Namely, take TNG300-3-Dark, associate to any halo its smoothed variance of the linear density field: namely build the mapping between Halo mass and sigma(M,z) — see exercise 2; e.g. associate to any halo its sigma value by interpolating the sigma(M,z)-M relation. Measure f(sigma) at different redshifts. e.g. z= 0, 1 and 4 and plot the results: use different symbols for different redshifts.

Note: here you do the binning in sigma (logarithmic) and count the number of haloes per bin in sigma to obtain f(sigma), ….

4. [Optional] The spatial distribution of dark-matter haloes.
Estimate the two-point correlation function of haloes from the TNG300-3-Dark simulation at z=0, 1, 4 in four bins of dark-matter halo mass as a function of separation.
At fixed halo mass, what is the dependence of the clustering on separation?
Which haloes are more clustered? Compare with the findings of Exercise 1.1 in Problem Sheet #1.

Hint: use the estimator seen in the class: \xi ~ DD/DR -1 for different bins in 3D halo separation. Here, DD = number of pairs of haloes in the simulation at a given separation; DR = number of pairs at a given separation between a halo in the simulation and a halo in a random uniform realization of the same simulation.
To build the random/uniform distributions of haloes, take the halo catalog from the simulation, and build a second halo catalog with the same number of haloes as in the simulated one but where you assign to each halo a set of random values for the position x,y, z uniformly distributed between 0 and the size of the box. Make a map of such randomized simulated catalog to convince yourself about the procedure.