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Problem Sheet #2 (Hand out: Fri => hand in: Mon)

1. Build your own Python Cosmological Calculator (Part I)

1.1 Assume an Einstein-deSitter Universe (Omega_m = 1, Omega_Lambda = 0, i.e. flat with k=0), and write Python functions that return

- The Hubble Factor
- The Density parameters Omega_X = rho_X/rho_crit
- The proper and comoving distances
- The cosmic time

as a function of redshift.

Plot H(z), Omega_X(z), dCom(z), dPhys(z), t(z) vs. redshift z and a(t) vs. t in different panels in the case of the Einstein-deSitter Universe. Confirm the behaviors and findings discussed in the lectures.

1.2 What can you say about the evolution of a matter-dominated Universe for different values of Omega_m?
Namely, plot H(z), Omega_X(z), dCom(z), dPhys(z), t(z) for a handful of different cosmological scenario: e.g. Omega_m =1, Omega_m = 0.7, Omega_m = 0.3.

1.3 Generalize the above functions to any non-flat cosmological model with a cosmological constant Lambda, i.e. to any combinations of the cosmological parameters Omega_m, Omega_r, Omega_Lambda, h, …
Compare the functional forms of H(z), Omega_X(z), dCom(z), dPhys(z), t(z) vs. redshift z and a(t) vs. t for an Einstein-deSitter Universe (Omega_m = 1, Omega_Lambda = 0) and a concordance model LCDM (aka standard cosmological model LCDM).

Note: the currently accepted values for the cosmological parameters in our Universe are in the table: the current Universe is compatible with a flat LCDM model. Please use these values with you solve your functions numerically and want actual values The density parameter of the radiation can be taken to be:

$$\Omega_{\rm rad} = 2.47 \times 10^{-5} h^{-2} .$$

Table 8.2 Summary of the cosmological parameters as determined by the Planck 2013 data, in combination with low-ℓ polarization measurements from WMAP (WP)

Ω_Λ	0.685 ± 0.018
$\Omega_{\rm m}$	0.315 ± 0.018
$\Omega_{\rm b} h^2$	0.02205 ± 0.00028
$H_0 ({\rm km\,s^{-1}\,Mpc^{-1}})$	67.3 ± 1.2
$z_{\rm ion}$	11.1 ± 1.1
τ	0.089 ± 0.014
σ_8	0.829 ± 0.012
$Y_{\rm P}$	0.24770 ± 0.00012
t_0 (Gry)	13.817 ± 0.048
$z_{\rm rec}$	1090 ± 0.54
$z_{\rm eq}$	3391 ± 60

2. The evolution of the densities of the various components of the Universe.
2.1 How do the densities of matter, radiation, and dark energy (i.e. of a cosmological constant, for now) evolve with redshift in the following Cosmological scenarios?

- Einstein - DeSitter Universe (Omega_m = 1, Omega_Lambda = 0)
- Low-density Universe (Omega_m = 0.3, Omega_Lambda = 0)
- Standard Cosmological Universe (Omega_m = 0.3, Omega_Lambda = 0.7, h=0.7 or Table above)

Express everything in units of the critical density and make plots of the three density parameters as a function of redshift in the three aforementioned cosmological scenarios.

2.2 In the standard cosmological scenario, what is the redshift of the three equivalence epochs? (The equivalence between energy component i and j occurs when rho_i = rho_j, where i= matter, radiation, cosmological constant). Identify the radiation, matter and dark-energy dominated eras: what redshift ranges do they correspond to? At what redshift was the Sun born? At what redshift did the dinosaurs disappear?
Solve this by interpolating the required function as a function of time/redshift.

3. The age of the Universe and the standard cosmological scenario
3.1 Our universe today is flat, with measured values of the cosmological parameters today being consistent with :

$$\boxed{\Omega_{\mathrm{m}} \sim 0.3\,;\quad \Omega_{\Lambda} \sim 0.7\,;\quad h \sim 0.7}\,.$$

By using the values in the inset or those of the table, please answer the following questions.

Determine:

- The age of the Universe.
- The age at which the expansion of the Universe first started to accelerate.
- The age at which the energy density in matter and the cosmological constant were equal.

3.2 A cosmologically-ignorant civilization measures h=0.7 but refuses to countenance the possibility of a cosmological constant. How old would they believe their Universe to be if they assumed it was flat with Omega_m = 1?

4. Galaxies, redshift, and cosmic time
Galaxies are observed nowadays all the way up to z=13. How old was the Universe at the time from which we see such most distant galaxies? What fraction of the universe history does that correspond to? How far in comoving and physical distance are those galaxies from us?

NOTE: the tools developed to solve this Problem Sheet will be needed for Problem Sheets #3, #4, and #5.
NOTE: you may not use Astropy or similar cosmology-related existing Python libraries to solve these exercises.