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Problem Sheet #3 (Hand out: Fri => hand in: Tue)

1. Evolution of density contrast on different spatial scales

2.1 Consider the output of an N-body simulation (e.g. TNG100-3-Dark or TNG50-4-Dark) and visualize the evolution of the matter density in the simulated box as a function of time i.e. redshift (namely, do sample 3-5 snapshots from very high z to z=0; e.g. z>10, 5, 2, 0). What are the typical matter column densities probed by the simulation in Msun/Mpc^2? What are the ranges of 3D matter densities in Msun/Mpc^3? What is the 3D average matter density of the box, at different redshifts?

Hint: in practice make a 2D histogram of the particle distribution with grid cells ~a few hundred kpc or larger. The particle data loading and handling may be quite memory intensive. You may consider down sampling. Namely you can keep and work with a random subsample of the DM particles. Here, however, you need to "re-define" the mass of each particle to maintain the total matter content of the box. If e.g. Ndownsampling = 1/100 Nsimulation, m_DM_downsampling = 100 x m_DM_simulation.

2.2 Measure the density contrast field in the simulation at a given snapshot, say z=0:

$$\boxed{\delta(\boldsymbol{r},t) := \frac{\rho(\boldsymbol{r},t) - \bar{\rho}(t)}{\bar{\rho}(t)}}$$

This requires a ``smoothing'' procedure, i.e. measuring densities as here requires defining a spatial scale or an aperture within which these are taken: as a first approach, you can measure the density on 3D cubes of various sizes: e.g. 100 kpc, 1 Mpc, 10 Mpc? The density contrast follows from eq. above. You can also characterize the density contrast in 2D density projections.
What is the probability distribution of the density contrast? Plot it in log10 scale. How can it be described? Is it symmetric? What maximum values of the density contrast do you estimate? What changes for different smoothing spatial scales?

2.3 How does the probability distribution of the smoothed density contrast change with time, i.e. as the Universe ages? Namely, repeat the measurement above at very different redshifts: z>10, 10, 5, 2, 0 and plot the renormalized pdfs of the density contrast in the same plot. What happens to the high density contrast tails of the matter distribution as the Universe ages?

2.4 Now, compare the probability distribution of the density contrast smoothed on a fixed spatial scale between two N-body simulation boxes of that encompass different volumes: e.g. TNG100-3-Dark and TNG300-3-Dark. What do you notice? What are the maximum values of the density contrast probed by the two volumes?

2. Gravitational collapse and the formation of haloes

2.1 Clusters of galaxies today are observed to have masses as high as 10^15 solar masses in regions of about 2-3Mpc of size. Assuming clusters are spherical and have a radius of 2 Mpc, what is their typical over density today? How rare such density contrast values are? Compare to the pdfs of Exercise 1.

3. The distribution of material within dark-matter haloes

3.1 Choose a halo from the halo catalog. Measure and plot its 3D matter density profile. What shape does it assume?
Note: The halo catalog already tells you its mass and radius (e.g. M200c, R200c). You can use the extent of the halo to decide a binning in radius: r_min>0.03R200c, r_max = R200c, n_bins, logarithmic. Measure rho [Msun/kpc^3] vs. radius [kpc], namely rho =dM/dV of shells of Volume dV at distance r and plot it in log-log scales.

3.2 Repeat the profile measurement for many haloes in a few bins of halo mass (M200c).
Let us for example choose three regimes: haloes of 10^11, 10^12 and 10^13 and plot the profiles of haloes in mass bins of 0.2dex. Compare the typical halo profiles across the three studied mass bins (e.g. plot the "average" profiles in one only plot, or put three plots side by side keeping the axis limits the same). What are the typical densities at the centers of the haloes? How does the innermost density and the shape of the profile change with halo mass? Now, make the same plot by normalizing the radius by the viral radius R200c: what do you notice?

3.2 Fit the density profiles of simulated haloes with the Navarro-Frenk-White or Einasto formula and record for a large number of simulated haloes their scale radius. What relation exists between scale radius and halo mass?

Hint: periodic boundary conditions need to be taken into account when centering the dark matter particles of a halo in the system of coordinates defined by its center.

4. Build your own Python Cosmological Calculator (Part II)

Write a Python function that returns the Growth Factor D+ as a function of redshift for any cosmological model with a cosmological constant Lambda, i.e. for any combinations of the cosmological parameters Omega_m, Omega_r, Omega_Lambda, h, …
We discussed the growth factor at the blackboard.

Plot the evolution with redshift of D+ for three reference flat cosmological models: e.g. Einstein-DeSitter, Low-density Universe (Omega_m = 0.3, Omega_Lambda = 0), and a Standard Cosmological Model.

Note: feel free to use the Carroll+1992 fitting formula